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Hierarchical Optimal Control for Synthetic Inertial Response of Wind Farm Based on Alternating Direction Method of Multipliers

Sheng Huang, Qiuwei Wu*, Weiyou Bao, Nikos Hatziaargyriou, Lei Ding, and Fei Rong

Abstract—An ADMM-based hierarchical optimal active power control (HOAPC) scheme is proposed for synthetic inertial response of large-scale wind farms. The proposed scheme coordinates the active power outputs of wind turbines (WTs) inside the wind farm during the synthetic inertial response process, aiming to minimize the differences in the rotor speed of the WTs and the wind energy loss caused by the decrease in the rotor speed of the WTs. The optimal control is formulated based on the model predictive control (MPC) according to the local WT wind conditions. A hierarchical solution method based on the alternating direction method of multipliers (ADMM) is developed to solve the MPC-based optimization problem in a fast way without the loss of optimality. With the HOAPC, the wind farm controller guarantees the rotor speed stability of WTs inside the wind farm and reduces the wind energy losses during the synthetic inertial response process. Moreover, the optimization problem is decomposed to several optimization subproblems solved in parallel. As such, the computation burden of the wind farm central controller is significantly reduced, and the protection of the information privacy of the wind farm is improved. A wind farm with 100 WTs was used to validate the proposed HOAPC scheme.

Index Terms—alternating direction method of multiplier (ADMM), hierarchical control, model predictive control (MPC), synthetic inertial response, wind farm.

I. INTRODUCTION

With the increasing penetration of wind power in power systems, conventional power plants are increasingly replaced by wind farms. It is a big challenge to maintain frequency security with the decreasing inertia of the grid [1], [2]. Frequency is an indicator of the system balance between generation and load in a power system. The decreasing system inertia results in greater frequency deviation during load-generation mismatches. As the system inertia decreases, frequency security is becoming a concern of system operators [3]. Moreover, with the capacity of wind farms increasing, wind farms with power electronics interface are required to provide similar services as conventional power plants [4].

Many wind farms are equipped with a large number of variable-speed wind turbines (WTs) to achieve accurate tracking for optimal wind power extraction [5]. The large-scale replacement of synchronous generators with variable-speed WTs will decouple the WT dynamics from the grid frequency. Moreover, compared with conventional synchronous generators, the synthetic inertial response of wind farms is dependent on the stochastic and intermittent nature of wind energy [6], which creates challenges in estimating the necessary system reserve that must be maintained to ensure instantaneous supply-demand balance.

The synthetic inertial response of wind farms has motivated a great number of studies. The control schemes for the synthetic inertial response can be divided into three levels: the WT level, wind farm level, and system level [7]. The WT-level control strategy developed in recent years releases the energy stored in the rotating mass to regulate the generator power output. In [8], [9], inertia response, rotor speed control and pitch angle control of a WT were coordinated to achieve a synthetic inertial response. In [10], a frequency response strategy was proposed for DFIG-based WTs. The aims are to improve the secure operation of the WT frequency responses by modifying the classical droop and inertia control loops, controlling tip-speed ratio, and de-loading percentage of the WTs. Ref. [11] proposed a step over-production-based modified inertia-emulation scheme, which aims to optimize energy transfer and handle the variable stored kinetic energy. The optimal parameters of the scheme are generated by using the particle swarm optimization (PSO) algorithm.

In [12], [13], the synthetic inertial response was provided at the wind farm supervisory control level. In [14], a model of wind plant frequency response was developed, which takes into account variation of the wind speed to evaluate the potential for frequency support. Ref. [15] proposed a scheme to control and estimate the amount of stored kinetic energy in the WTs during power system frequency drops. In [16], the impact of the wake effect on the inertial response of the wind farm was modeled and quantified. In [17], an optimal coordinated operation strategy of wind farms was proposed for frequency response, which considers the wake effect. In [18], an active power control framework was proposed for doubly fed induction generator (DFIG) WTs to provide inertial response and primary frequency control, which estimates the maximum extraction energy of WTs and guarantees rotor speed security. Ref. [19] addressed the system level response by modifying the average power system frequency model to

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include the participation of wind farms in frequency control. Ref. [20] studied the dynamic frequency characteristics of the power system when the synthetic inertial response of DFIGs is used. In [21], the droop control strategy as an alternative to the synthetic inertia strategy was developed, thereby providing an inertial response for power systems of different configurations. The frequency nadir and rate of change of the frequency (ROCOF) are compared to evaluate the optimal parameter tuning of the droop control.

According to the above studies, the synthetic inertial response control scheme can be classified into the decentralized control scheme and centralized optimal control scheme. The PD-based control scheme is implemented in individual local controllers, which cannot obtain an optimal solution for the synthetic inertial response control of wind farms. For a large-scale wind farm, the WTs may operate in different wind conditions. The active power references can be optimized and redistributed among WTs according to local wind conditions to improve the synthetic inertial response performance. For the centralized optimal control scheme, the computation and communication burdens significantly grow with the size increase of the wind farm. Since the synthetic inertial response is a fast process, the centralized optimal control scheme may not be suitable for the large-scale wind farms. Currently, the distributed/hierarchical control for large-scale wind farm, which has advantages of scalability, flexibility, robustness, and information privacy, has motivated a great number of studies [22]- [24]. However, the distributed/hierarchical control has not been studied for the synthetic inertial response of large-scale wind farms.

The model predictive control (MPC) is a widely used optimal control method, which is suitable for solving the multi-input and multi-output wind farm optimization problem. Moreover, the dynamic response of the WTs can be considered to the optimization problem to improve the control performance. The control input is obtained by solving a discrete-time optimal control problem over a given horizon [25]. In [26], an MPC-based control scheme was used to minimize fatigue loads of WTs. Ref. [27] proposed an MPC-based voltage control scheme to minimize the voltage fluctuations of buses inside the WF while considering the economic operation.

Therefore, an ADMM-based hierarchical optimal active power control (HOAPC) scheme is proposed for the synthetic inertial response of large-scale wind farms. In the synthetic inertial response process, the synthetic inertial controller generates an incremental active power reference for a large-scale wind farms. The MPC is used to minimize the differences of the rotor speeds of the WTs to guarantee their stability and minimize the wind energy loss caused by the decrease in rotor speeds while tracking the incremental active power reference from the synthetic inertial controller. A hierarchical solution method based on the alternating direction method of multipliers (ADMM) is developed to solve the optimization problem in a fast way. Each WT controller only communicates with the wind farm controller and exchanges a small amount of information. The main contributions of this paper can be summarized as follows:

An HOAPC scheme is designed for the synthetic inertial

response of wind farms. The MPC-based optimization problem is formulated based on local wind fluctuations to improve the secure operation of the wind farm and minimize the wind energy loss within the wind farm, as well as guaranteeing the performance of the synthetic inertial response. A hierarchical solution method based on the ADMM is proposed to solve the optimization problem in a fast way, which guarantees the optimality of the control performance. With the HOAPC, the optimization problem is decomposed into several optimization subproblems and solved in parallel. The computation burden of the wind farm central controller is significantly reduced. Moreover, the central controller and local WT controllers exchange the updated global, local, and dual variables, instead of explicit measurements and set-points. The protection of the information privacy of the wind farm is improved.

The rest of the paper is organized as follows. Section II provides an overview of the proposed control scheme. The synthetic inertial response model for a wind farm and the MPC-based objective function are presented in Section III. The ADMM-based hierarchical solution method for the synthetic inertial response is described in Section IV. Finally, simulation results are presented and discussed in Section V, followed by conclusions.

II. CONTROL ARCHITECTURE

A. Configuration of a Wind Farm

Fig. 1 shows the typical configuration of a wind farm, which is connected to the external AC grid through 380 kV transmission line. The nominal capacity of the wind farm is 500 MW. The wind farm consists of ten collector substations, each with 10×5 MW WTs.

Fig. 1. Configuration of a wind farm.

B. Control Concept

In this paper, an HOAPC for the synthetic inertial response of wind farm is designed as illustrated in Fig. 2. The wind farm is equipped with a wind farm controller and each WT is equipped with a WT controller. The proposed scheme coordinates all WTs within the wind farm and aims to minimize the energy losses of the WTs and the deviations of the rotor speeds from the average WT rotor speed while tracking the active power reference of the wind farm synthetic inertial

Fig. 2. Control structure.

response. The synthetic inertial controller is used to generate an incremental active power reference for the wind farm, which is described in section III. A. The incremental active power reference depends on the deviation of the external AC grid frequency from the nominal frequency. To efficiently solve the optimal control problem and improve the protection of the wind farm information privacy, a hierarchical control scheme based on the ADMM is used. The information is exchanged between the WT controllers and the wind farm controller, which includes the global, local, dual variables and rotor speeds. The computation task is decomposed to the individual WT controllers and wind farm controller. A simple augmented Lagrangian optimization problem with an equality constraint is solved in the wind farm controller. The optimization subproblem related to the WT model with local small-scale constraints is solved in the local WT controllers. Besides, the communication topology of the HOAPC scheme is the same as the conventional centralized control scheme, which makes it easy to adapt the conventional centralized control to hierarchical control structure.

III. SYNTHETIC INERTIAL RESPONSE MODEL FOR WIND FARM

A. Synthetic Inertial Response Controller

The synthetic inertial response controller in Fig. 2 is used to generate an active power reference to provide synthetic inertial support for the power system. The WTs inside the wind farm release kinetic energy stored in the rotating masses to increase the frequency nadir. The control diagram of the synthetic inertial response controller is shown in Fig. 3.

Fig. 3. Control diagram of the synthetic inertial response controller.

The synthetic inertial response controller consists of two control loops, the ROCOF loop and the droop loop, which compensate for each other's drawbacks [28]. K_d and K_p are the gains of the ROCOF and droop loops, respectively, T_L and T_H are the time constants of the low-pass filter and high-pass filter, f_{meas} is the measured power system frequency, and f_{nom} is the nominal power system frequency. ΔP_d is generated by the ROCOF loop, which can be expressed as,

$$\Delta P_d = -K_d f_{meas} \frac{df_{meas}}{dt}. \quad (1)$$

ΔP_p is generated by the droop loop, which can be expressed as,

$$\Delta P_p = -K_p (f_{meas} - f_{nom}). \quad (2)$$

The active power reference for the wind farm P_{WF}^{ref} can be expressed as,

$$P_{WF}^{ref} = P_{WF}^0 + \Delta P_d + \Delta P_p. \quad (3)$$

where P_{WF}^0 is the initial active power output of the wind farm.

B. WT Model

The dynamics of the rotor speed of a WT are described by the two-mass model,

$$\begin{aligned} 2H_t \frac{d\omega_t}{dt} &= \frac{P_m}{\omega_t} - K_s \theta_s - D_s (\omega_t - \omega_r) - D_t \omega_t \\ 2H_g \frac{d\omega_r}{dt} &= K_s \theta_s + D_s (\omega_t - \omega_r) - D_g \omega_r - \frac{P_e}{\omega_r} \\ \frac{d\theta_s}{dt} &= \omega (\omega_t - \omega_r) \end{aligned} \quad (4)$$

where P_m is the mechanical power, P_e is the electromagnetic power, H_t and H_g are the inertial constants of the WT and generator mass, respectively, ω_t and ω_r are the angular speed of the WT and generator rotor, respectively, D_t and D_g are the damping constants of WT and generator rotor, respectively, K_s is the shaft stiffness, D_s is the damping constant, θ_s is the torsional twist, and ω is the base angular speed. ω_r^0 and ω_t^0 are the initial angular speed of WT and generator rotor at the control period, and Δ operator denotes the change in a variable between control periods. Based on this, the following can be obtained,

$$\begin{aligned} \omega_t &= \omega_t^0 + \Delta \omega_t, \\ \omega_r &= \omega_r^0 + \Delta \omega_r, \\ \theta_s &= \theta_s^0 + \Delta \theta_s. \end{aligned} \quad (5)$$

Note that (4) is nonlinear, and it can be approximated near the operating point from its Taylor series expansion. Thus, (4)

can be rewritten as,

$$\frac{d\omega_t}{dt} = \frac{d\Delta\omega_t}{dt} = \left(\frac{P_m}{2H_t(\omega_t^0)^2} - \frac{D_s + D_t}{2H_t} \right) \Delta\omega_t + \frac{D_s}{2H_t} \Delta\omega_r - \frac{K_s}{2H_t} \Delta\theta_s + \xi_t \quad (6)$$

$$\frac{d\omega_r}{dt} = \frac{d\Delta\omega_r}{dt} = \left(-\frac{P_e}{2H_g(\omega_r^0)^2} - \frac{D_s + D_g}{2H_g} \right) \Delta\omega_r + \frac{D_s}{2H_g} \Delta\omega_t + \frac{K_s}{2H_g} \Delta\theta_s - \frac{1}{2H_g\omega_r^0} \Delta P_e + \xi_g \quad (7)$$

$$\frac{d\theta_s}{dt} = \frac{d\Delta\theta_s}{dt} = \omega(\Delta\omega_t - \Delta\omega_r) + \xi_\theta \quad (8) \quad \text{with}$$

with

$$\begin{aligned} \xi_t &= \frac{P_m}{2H_t\omega_t^0} - \frac{K_s\theta_s^0}{2H_t} - \frac{D_s(\omega_t^0 - \omega_r^0)}{2H_t} - \frac{D_t\omega_t^0}{2H_t} \\ \xi_g &= -\frac{P_e^0}{2H_g\omega_r^0} + \frac{K_s\theta_s^0}{2H_g} + \frac{D_s(\omega_t^0 - \omega_r^0)}{2H_g} - \frac{D_g\omega_r^0}{2H_g} \\ \xi_\theta &= \omega(\omega_t^0 - \omega_r^0) \end{aligned}$$

where P_e^0 is the initial mechanical electromagnetic power at the operating point. The input mechanical power delivered to the WT is a function of the wind speed, tip speed ratio and blade pitch angle, which can be expressed by,

$$P_m = \frac{1}{2} \rho \pi R^2 C_P(\lambda, \beta) v_w^3 \quad (9)$$

where ρ is the air density, R is the radius of the blade, v_w is the wind speed, C_P is the aero-dynamic power efficiency, β is the blade pitch angle, and λ is the tip speed ratio. C_P is a function of the tip speed ratio which contains the blade pitch angle as a parameter. For simplicity, C_P can be fitted by a third-order polynomial function as,

$$C_P = m_1 \lambda^3 + m_2 \lambda^2 + m_3 \lambda + m_4 \quad (10)$$

where m_1, m_2, m_3 , and m_4 are the polynomial coefficients. Then, P_m can be rewritten as,

$$P_m = n_1 \omega_r^3 + n_2 \omega_r^2 v_w + n_3 \omega_r v_w^2 + n_4 v_w^3 \quad (11)$$

where $n_1 = \frac{1}{2} \pi R^5 m_1$, $n_2 = \frac{1}{2} \pi R^4 m_2$, $n_3 = \frac{1}{2} \pi R^3 m_3$, and $n_4 = \frac{1}{2} \pi R^2 m_4$.

With power electronic converters, the WTs can regulate active power and reactive power independently as well as provide active and reactive power support for the power system. The dynamic response of the WT converter is considered in this paper. The dynamic behaviour and communication delay of the active power control loops of WTs can be described by a first-order lag function [29], [30],

$$\Delta \dot{P}_e = \frac{1}{1 + sT_e} \Delta P_e^{\text{ref}} \quad (12)$$

where T_e is the time constant, and P_e^{ref} is the active power reference for the WT converter. Accordingly, the continuous state space model of a WT can be formulated in a matrix form,

$$\Delta \dot{x}_t = \mathbf{A}_t \Delta x_t + \mathbf{B}_t \Delta u_t + \mathbf{E}_t \quad (13)$$

where

$$\Delta x_t = [\Delta\omega_t, \Delta\omega_r, \Delta\theta_s, \Delta P_e]^T, \Delta u_t = \Delta P_e^{\text{ref}}$$

$$\mathbf{A}_t = \begin{bmatrix} \vartheta_t & \frac{D_s}{2H_t} & -\frac{K_s}{2H_t} & 0 \\ \frac{D_s}{2H_g} & \vartheta_r & \frac{K_s}{2H_g} & -\frac{1}{2H_t\omega_t^0} \\ \omega & -\omega & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{T_e} \end{bmatrix}$$

$$\mathbf{B}_t = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{T_e} \end{bmatrix} \quad \mathbf{E}_t = \begin{bmatrix} \xi_t \\ \xi_r \\ \xi_\theta \\ 0 \end{bmatrix}$$

$$\vartheta_t = \frac{P_m}{2H_t(\omega_t^0)^2} - \frac{D_s + D_t}{2H_t}$$

$$\vartheta_r = -\frac{P_e}{2H_g(\omega_r^0)^2} - \frac{D_s + D_g}{2H_g}$$

C. Wind Energy Loss

In the synthetic inertial response process of the wind farm, a decrease in the rotor speed of a WT will result in a decrease in the kinetic energy stored in the rotating mass. The kinetic energy loss in a control period can be expressed as,

$$E_{\text{loss}} = \int_t^{t+\Delta t} \Delta P_m dt = \frac{1}{2} \rho \pi R^2 v_w^3 \int_t^{t+\Delta t} \Delta C_P dt \quad (14)$$

where Δt is the time of the control period. According to [31], ΔC_P can be approximated near the operating point according to its Taylor series expansion,

$$\Delta C_P \approx \left. \frac{\partial C_P}{\partial t} \right|_{t=0} t + \left. \frac{\partial^2 C_P}{\partial t^2} \right|_{t=0} \frac{t^2}{2} \quad (15)$$

Then, the relationship between the wind energy loss E_{loss} , the rotor speed deviation $\Delta\omega_r$ and the time of the control period Δt can be obtained,

$$\begin{aligned} E_{\text{loss}} &= \frac{1}{4} \rho \pi R^3 v_w^2 \left. \frac{\partial C_P}{\partial \lambda} \right|_{t=0} \Delta t \Delta\omega_r \\ &+ \frac{1}{12} \rho \pi R^4 v_w \left. \frac{\partial^2 C_P}{\partial \lambda^2} \right|_{t=0} \Delta t \Delta\omega_r^2 \end{aligned} \quad (16)$$

where

$$\begin{aligned} \left. \frac{\partial C_P}{\partial \lambda} \right|_{t=0} &= 3m_1 \lambda_0^2 + 2m_2 \lambda_0 + m_3 \\ \left. \frac{\partial^2 C_P}{\partial \lambda^2} \right|_{t=0} &= 6m_1 \lambda_0 + 2m_2 \end{aligned}$$

with

$$\lambda_0 = \frac{\omega_r^0 R}{v_w}$$

D. Wind Farm Model

Denote $\mathcal{N}_w$ as the WT set in a wind farm, the continuous state space model of a wind farm can be formulated as,

$$\begin{aligned} \Delta \dot{x} &= \mathbf{A}x + \mathbf{B}\Delta u + \mathbf{E} \\ y &= \mathbf{C}\Delta x \end{aligned} \quad (17)$$

where

$$\Delta x = [\Delta x_{t,1}, \Delta x_{t,2}, \Delta x_{t,3}, \dots, \Delta x_{t,|\mathcal{N}_w|}]^T,$$

$$\begin{aligned}
\Delta u &= [\Delta u_{t,1}, \Delta u_{t,2}, \Delta u_{t,3}, \dots, \Delta u_{t,|\mathcal{N}_w|}]^T, \\
y &= [\Delta \omega_{r,1}, \Delta \omega_{r,2}, \Delta \omega_{r,3}, \dots, \Delta \omega_{r,|\mathcal{N}_w|}]^T, \\
\mathbf{A} &= \text{diag} [\mathbf{A}_{t,1}, \mathbf{A}_{t,2}, \mathbf{A}_{t,3}, \dots, \mathbf{A}_{t,|\mathcal{N}_w|}], \\
\mathbf{B} &= \text{diag} [\mathbf{B}_{t,1}, \mathbf{B}_{t,2}, \mathbf{B}_{t,3}, \dots, \mathbf{B}_{t,|\mathcal{N}_w|}], \\
\mathbf{E} &= \text{diag} [\mathbf{E}_{t,1}, \mathbf{E}_{t,2}, \mathbf{E}_{t,3}, \dots, \mathbf{E}_{t,|\mathcal{N}_w|}], \\
\mathbf{C} &= \text{diag} [\mathbf{C}_{t,1}, \mathbf{C}_{t,2}, \mathbf{C}_{t,3}, \dots, \mathbf{C}_{t,|\mathcal{N}_w|}],
\end{aligned}$$

with

$$\mathbf{C}_{t,i} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \forall i \in \mathcal{N}_w$$

Then, the discrete state-space model with the sampling time ΔT_p can be derived from the continuous model, which is given by,

$$\begin{aligned}
\Delta x(k+1) &= \mathbf{A}_d \Delta x(k) + \mathbf{B}_d \Delta u(k) + \mathbf{E}_d, \\
y(k+1) &= \mathbf{C} \Delta x(k+1)
\end{aligned} \tag{18}$$

where

$$\mathbf{A}_d = e^{A\Delta T_p}, \mathbf{B}_d = \int_0^{\Delta T_p} e^{At} B dt, \mathbf{E}_d = \int_0^{\Delta T_p} e^{At} E dt$$

E. Objective Function

Before the synthetic inertial response process, each WT in a wind farm contains a different level of releasable kinetic energy because of the intermittent nature of wind energy and the wake effect. The WTs inside the wind farm may be operated at different rotor speeds due to local wind fluctuations. When the synthetic inertial response process starts, if the wind farm controller dispatches the same active reference or proportional distribution-based active reference to the WTs, the rotor speeds of WTs at low wind speed conditions may hit the minimum rotor speed first. WTs terminate the synthetic inertia control and have a risk of being tripped as wind speed decreases. Therefore, the first objective is to minimize the differences in the rotor speeds of the WTs to guarantee the rotor speed stability of each WT inside the wind farm. The second objective is to minimize the wind energy loss caused by the decrease in the rotor speeds of the WTs. The third objective function is to reduce the variations of the rotor speed, which can damp the oscillations of the rotor speed caused by the first and second objective. Then, the objective function can be written as,

$$\min \sum_{i=1}^{|\mathcal{N}_T|} \sum_{k=1}^{N_p} \|\Delta \omega_{r,i}^*(k)\|_{Q_w}^2 + \|E_{\text{loss},i}(k)\|_{Q_l} + \|\Delta \omega_{r,i}(k)\|_{Q_d}^2 \tag{19}$$

where N_p is the prediction steps, Q_w , Q_l , and Q_d are the weighting factors for minimizing the differences in the rotor speed of the WTs and wind energy loss, and damping the oscillations of the rotor speed, respectively, and $\Delta \omega_{r,i}^*$ can be expressed as,

$$\Delta \omega_{r,i}^* = \omega_{r,i}^0 + \Delta \omega_{r,i} - \omega_r^{\text{avr}} \tag{20}$$

where $\omega_r^{\text{avr}} = \frac{1}{|\mathcal{N}_T|} \sum_{i=1}^{|\mathcal{N}_T|} \omega_{r,i}$ is the average rotor speed of the WTs, and $\omega_{r,i}^0$ is the i th WT initial wind speed at the operating point, which are calculated in the rotor speed deviation calculation block in Fig. 2. For a WT, the active power output in the synthetic inertial response process cannot exceed the rated capacity of the WT,

$$0 \leq P_{e,i}^0 + \Delta P_{e,i}^{\text{ref}} \leq P_{\text{wt},i}^{\text{rated}}, \quad \forall i \in \mathcal{N}_w. \tag{21}$$

where $P_{\text{wt},i}^{\text{rated}}$ is the rated capacity of the i th WT converter. For the wind farm, the total incremental power of the wind farm should track the reference which is calculated by synthetic inertia controller,

$$\sum_{i=1}^{|\mathcal{N}_w|} \Delta P_{e,i}^{\text{ref}} = \Delta P_{\text{in}} + \Delta P_d \tag{22}$$

IV. HIERARCHICAL SOLUTION METHOD FOR THE SYNTHETIC INERTIAL RESPONSE

A. ADMM-based Hierarchical Optimization Problem Formulation

$\|\Delta \omega_{r,i}^*(k)\|_{Q_w}^2$ is a quadratic programming (QP) problem with Δu_t , and $E_{\text{loss},i}$ is a QP problem with incremental rotor speed $\Delta \omega_r$. Thus, the MPC-based optimization problem (17) can be rearranged into a compact QP form as follows,

$$\begin{aligned}
\min_x \Phi(x) &= \frac{1}{2} x^T \mathbf{H} x + \mathbf{g}^T x \\
s.t. \quad &\underline{x} \leq x \leq \bar{x} \\
&\mathbf{A} x = \mathbf{b}
\end{aligned} \tag{23}$$

where $x = [\Delta u(1), \Delta u(2), \dots, \Delta u(N_p)]^T$, $x \in \mathbb{R}^{n \times 1}$, is the decision variable vector, $\mathbf{H} \succeq 0$, $\mathbf{H} \in \mathbb{R}^{n \times n}$ is a matrix and $\mathbf{g} \in \mathbb{R}^{n \times 1}$ is a vector. $\underline{x} \in \mathbb{R}^{n \times 1}$, and $\bar{x} \in \mathbb{R}^{n \times 1}$ are the lower and upper limits for the decision variable vector, which are related to constraints (19), $n = N_p \times |\mathcal{N}_w|$, and $\mathbf{A}$ and $\mathbf{b}$ are matrixes for equality constraints, which can be expressed as,

$$\mathbf{A} = \text{diag}[\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3, \dots, \mathbf{A}_{N_p}],$$

$$\mathbf{b} = [\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, \dots, \mathbf{b}_{N_p}]^T.$$

with

$$\mathbf{A}_i = \mathbf{1}^{1 \times |\mathcal{N}_w|}, \quad \forall i = 1, 2, \dots, N_p,$$

$$\mathbf{b}_i = \Delta P_{\text{in}} + \Delta P_d, \quad \forall i = 1, 2, \dots, N_p,$$

The ADMM is a computational framework for solving the optimization problem and is suitable for solving the convex optimization problem in parallel. By the decomposition and coordination process, the ADMM decomposes the large global problem into several small local subproblems and obtains the solution of the large global problem by coordinating the solutions of the sub-problems. Since the WT synthetic inertial response is carried out independently of the individual WTs, the optimization problem (21) can be decomposed into $|\mathcal{N}_w| + 1$ subproblems. The equality constraint is solved in the wind farm central unit by using the KKT condition. The individual WT optimization problems and their corresponding small-scale inequality constraints are solved in parallel in the

individual WT controllers. Thus, the optimization problem in (21) can be rewritten in an ADMM form as,

$$\begin{aligned} \min_x & \Phi(x) + I_{\mathcal{X}}(x) \\ \text{s.t.} & \quad x - z = 0 \\ & \quad \underline{x} \leq x \leq \bar{x} \\ & \quad \mathbf{A}z = \mathbf{b} \end{aligned} \quad (24)$$

where x is the local variables, z is the global variables, and $I_{\mathcal{X}}(\cdot)$ is the indicator function of the inequality constraints, such that $I_{\mathcal{X}} = 0$ for $\underline{x} \leq x \leq \bar{x}$, else $I_{\mathcal{X}} = \infty$. The augmented Lagrangian can be expressed as,

$$\mathcal{L}^{\text{ADMM}} = \Phi(x) + I_{\mathcal{X}}(x) + y^T(x - z) + \frac{\rho}{2} \|x - z\|_2^2 \quad (25)$$

where y is the dual variables vector, and $\rho > 0$ is the augmented Lagrangian parameter.

B. Hierarchical Solution method Based on ADMM

The ADMM algorithm is an iterative algorithm. Each controller updates one type of variables while holding the other variables constant. With the ADMM, controllers obtain the optimal solution for the synthetic inertial response of the wind farm by solving the optimization problem in a hierarchical manner. The ADMM-based synthetic inertial response is solved by the following steps:

1) Initialize: Assign 0 to all local, global, and dual variables on the first iteration.

$$x^{[1]} = 0, \quad z^{[1]} = 0, \quad y^{[1]} = 0 \quad (26)$$

2) Update z : Since the optimization problem (22) is separable, and the inequality constraints are all local constraints, the optimization subproblems with inequality constraints can be distributed to the individual WT controllers. However, the equality constraint $\mathbf{A}z = \mathbf{b}$ cannot be directly decomposed. Thus, the task of the wind farm central unit in Fig. 2 is to guarantee wind farm active power tracking from the synthetic inertial controller. The central unit minimizes the augmented Lagrangian with global variables z as,

$$\begin{aligned} z^{[r+1]} = \arg \min_z & y^{T[r]}(x^{[r]} - z^{[r+1]}) \\ & + \frac{\rho^{[r]}}{2} \|x^{[r]} - z^{[r+1]}\|_2^2 \\ \text{s.t.} & \quad \mathbf{A}z = \mathbf{b} \end{aligned} \quad (27)$$

3) Update x : After updating z , the central unit sends z_i to the corresponding WT controller. Each WT controller in Fig. 2 solves the optimization subproblem with its local constraints in parallel. For the i th WT controller, the augmented Lagrangian can be decomposed into,

$$\begin{aligned} \mathcal{L}_i^{\text{ADMM}} = & \Phi_i(x_i) + I_{\mathcal{X}_i}(x_i) + y_i^{T[r]}(x_i - z_i^{[r+1]}) \\ & + \frac{\rho^{[r]}}{2} \|x_i - z_i^{[r+1]}\|_2^2 \end{aligned} \quad (28)$$

Define $\mathbf{I}$ as the identity matrix. $\mathbf{H} + \rho\mathbf{I} \succ 0$, which is invertible. The local variable x_i can be updated in each WT

local controller by minimizing (26). $x_i^{[r+1]}$ can be obtained by,

$$x_i^{[r+1]} = \min\{\max\{\mathbf{H}_i + \rho^{[r]}\mathbf{I}^{-1}(\mathbf{g}_i + y_i^{[r]} - \rho^{[r]}z^{[r+1]}), \underline{x}_i\}, \bar{x}_i\}, \quad (29)$$

4) Update y : Once each WT controller obtains the $x_i^{[r+1]}$ and $z_i^{[r+1]}$, the dual variable $y_i^{[r+1]}$ can be updated in the local WT controller by,

$$y_i^{[r+1]} = y_i^{[r+1]} + \rho(x_i^{[r+1]} - z_i^{[r+1]}) \quad (30)$$

5) The synthetic inertial response of the wind farm is a rapid process, implying that the control periods of the wind farm controllers should be set as short as possible to achieve the best performance. To limit the online computation time and guarantee the accuracy of the calculation, a set of stopping criteria must be defined. Define the primal residuals h and dual residuals j as,

$$\begin{aligned} h^{[r]} &= x^{[r]} - z^{[r]} \\ j^{[r]} &= \rho^{[r]}(z^{[r]} - z^{[r-1]}) \end{aligned} \quad (31)$$

The iteration will be stopped when $h^{[r]}$, $j^{[r]}$ satisfy,

$$\begin{aligned} \|h^{[r]}\|_2^2 &\leq \epsilon^{\text{pri}} \\ \|j^{[r]}\|_2^2 &\leq \epsilon^{\text{dual}} \end{aligned} \quad (32)$$

where $\epsilon^{\text{pri}} > 0$, and $\epsilon^{\text{dual}} > 0$ are constants for the feasibility tolerances of the primal and dual feasibility conditions, respectively. A fixed number of iterations should also be set for the controllers. If the iteration number reaches the maximum, even if the primal residuals and dual residuals do not satisfy (30), the iteration will be stopped. This step is carried out in the central unit.

5) Update ρ : If the iteration continues, the central unit updates the penalty parameter to reduce the iteration number, as well as to make the convergence less dependent on the initial penalty parameter. This step is conducted in the penalty calculation block in Fig. 2. The varying penalty parameter can be expressed as,

$$\rho^{[r]} = \begin{cases} \vartheta \rho^{[r]} & \text{if } \|h^{[r]}\|_2^2 \leq \kappa \|j^{[r]}\|_2^2 \\ \rho^{[r]}/\varpi & \text{if } \|j^{[r]}\|_2^2 \leq \kappa \|h^{[r]}\|_2^2 \\ \rho^{[r]} & \text{otherwise;} \end{cases} \quad (33)$$

where $\vartheta > 1$, $\varpi > 1$, and $\kappa > 1$ are coefficients.

V. CASE STUDY

A. Test System

In this section, the IEEE 39 bus system is used for the case study. The configuration of this system is shown in Fig. 7. The electrical system is modelled in Digsilent/PowerFactory and the ADMM-based HOAPC scheme is carried out in MATLAB. In the test system, a wind farm with 100×5 MW WTs, is connected to Bus23 to verify the proposed control scheme. The configuration of the wind farm is illustrated in Fig. 1. A period of 0.5 s was chosen to test the synthetic

inertial response process of the wind farm. The control period T_c and the prediction horizon T_p are set as 0.5 s and 5 s, respectively. At 20 s, a frequency disturbance occurred at Bus23. Before the frequency disturbance, the wind farm was operated in maximum power point tracking (MPPT) mode and each of the 10 WT's shared the same wind condition. To examine the performance of the proposed control method, the simulation results are compared with those based on the centralized optimal active power control scheme (COAPC) and those based on a conventional proportional differential (PD) control scheme. In the conventional PD-based control scheme, each synthetic inertial controller measures the WT terminal bus frequency and generates an incremental active power reference by the PD controller. The parameters of the WT are listed in Table I.

Fig. 4. IEEE 39 bus system.

TABLE I
WT PARAMETERS

Symbol	Description	value
S	Rated Apparent Power (kVA)	5556
P_m^{rated}	Rated Mechanical Power(kW)	4869.553
ω^{rated}	Nominal Speed(rpm)	1485.153
H_t	Inertial constant of WT ($kg.m^2$)	2230
H_g	Inertial constant of generator($kg.m^2$)	170
D_t	damping constant of WT	1.5
D_g	damping constant of generator	0.1
	Stator Resistance (p.u.)	0.01
	Stator Reactance (p.u.)	0.1
	Magnetizing Reactance (p.u.)	3.5
	Rotor Resistance (p.u.)	0.056
	Rotor Reactance (p.u.)	0.056

B. Control Performance

The total simulation time is set as 40 s. Fig. 5 shows the wind speed condition. Each ten WT's share the same wind speed condition. The wind speed fluctuates from 8.5 m/s to 11.5 m/s. The frequency performance is shown in Fig. 6. Before the frequency disturbance, the grid frequency is maintained at 60Hz. At 20s, the synthetic inertial response process of the wind farm starts with a decrease in frequency. With the PD-based control scheme, the incremental active

power references for each WT's are continuously generated. The optimal incremental active power references with the HOAPC scheme and the COAPC scheme are updated every 0.5 s. The frequency nadir with the PD-based scheme is slightly higher than that with the HOAPC scheme and the COAPC scheme by approximately 0.01 Hz. The frequency performance with the HOAPC scheme is the same as with the COAPC scheme. The three control schemes are compared with the ones without synthetic inertial response of the WT's. The frequency nadir with the three control schemes is much higher than that without synthetic inertial response.

Fig. 5. Wind speed variation at each WT group.

Fig. 6. Frequency performance.

Fig. 7 shows the incremental active power output of the wind farm. Before the synthetic initial response process, the incremental active power with the three control scheme is zero. After 20 s, the wind farm incremental active power output with the HOAPC scheme is similar to that with the COAPC scheme. The grid frequency with the HOAPC and COAPC scheme is slightly lower than with the PD-based scheme. The synthetic inertial controller with the HOAPC and the COAPC generate a higher incremental active power reference than that with the PD-based control scheme.

Figs 8-10 show the rotor speeds corresponding to the three control schemes. Before the frequency disturbance, the rotor speeds of all three control schemes are the same and all WT's are operated in the MPPT mode. After 20s, because kinetic energy stored in the rotors is released equally to support the grid frequency, the rotor speeds with the PD-based control scheme decrease with a similar ramp. Since the COAPC and HOAPC schemes generate optimal active power references for the WT's to guarantee the rotor speeds stability, WT's that store more kinetic energy release more energy. The rotor speeds with the COAPC and the HOAPC schemes decreased with

Fig. 7. The incremental active power output of the wind farm.

different ramps. The speeds of WTs at the high wind speed condition decreased faster than those at the low wind speed condition. The rotor speed performance with the HOAPC scheme is similar as the one of the COAPC scheme. In order to better demonstrate the benefits of the HOAPC scheme, the minimum rotor speeds of WT1-WT100 with three control schemes are shown in Fig. 11. The minimum rotor speeds corresponding to the COAPC scheme and HOAPC scheme are similar and much higher than those of the PD-based control scheme. At the end of the synthetic inertial response, the minimum rotor speed with the PD-based method decreases to 0.85 p.u., and the minimal rotor speed with the COAPC scheme and HOAPC scheme is approximately 0.94 p.u.. In Fig. 12, the COAPC and HOAPC schemes can achieve more secure performance than the PD-based control scheme with regard to the synthetic inertial response of the wind farm.

Fig. 8. Rotor speeds with the PD-based control scheme

Fig. 9. Rotor speeds with the COAPC scheme.

Fig. 10. Rotor speeds with the HOAPC scheme.

Fig. 11. The minimum rotor speeds of WT1-WT100 with three control methods

Fig. 12. Average wind energy loss of each WT group.

Fig. 13. Convergence performance.

Fig. 12 shows the average wind energy loss of each WT group during the synthetic inertial response process of the wind farm. Since the controller updates every 0.5 s, there are 20 control periods in the synthetic inertial response process. The wind energy loss with the COAPC and HOAPC schemes is lower than that with the PD-based control scheme during most of the control periods. The wind energy loss with the COAPC scheme is similar as that of the HOAPC scheme.

Fig. 13 shows the convergence performance of the HOAPC scheme. The local variables $x_1 - x_{10}$ and the global variables $z_1 - z_{10}$ are selected as representative variables to illustrate the performance of the convergence. The local variables and their respective global variables converge at the same value after approximately 30 iterations, which can be seen in Fig. 13. The comparison of the calculation time with the COAPC and HOAPC at one control period are shown in Table III. The total calculation time with the HOAPC is reduced by 18.37% compared to those with the COAPC.

TABLE II
AVERAGE WIND ENERGY LOSS OF EACH WT GROUP (kWh)

Control periods	PD	COAPC	HOAPC
1-10	0.5586	0.2701	0.2709
11-20	0.7541	0.5803	0.5801
21-30	0.5788	0.4167	0.4157
31-40	0.5374	0.3964	0.3971

TABLE III
THE COMPARISON OF CALCULATION TIME

	COAPC	HOAPC
Central controller	0.2683 s	0.0032 s
WT controller	0 s	0.0041 s
Iteration number	1	30
Total computation time	0.2683 s	0.2190 s
Reduced percentage	0	18.37%

The control performances are also tested under the constant wind speed near to the rated wind speed. The wind speed of each WT group is shown in Table IV.

TABLE IV
WIND SPEED (M/S)

	Wind speed	Wind speed
1-10 WTs	9.000	51-60 WTs 10.000
11-20 WTs	9.201	61-70 WTs 10.200
21-30 WTs	9.398	71-80 WTs 10.402
31-40 WTs	9.597	81-90 WTs 10.602
41-50 WTs	9.798	91-100 WTs 10.799

The performances of the rotor speed and energy losses are similar as the ones under the varied wind speed. The rotor speed performances with the HOAPC and COAPC schemes are better than those with the PD-based controller. The rotor speed performances with the HOAPC is similar as those with the COAPC schemes. Compared with the PD-based control scheme, the energy losses can be reduced efficiently by using the HOAPC and COAPC schemes. The HOAPC is suitable for the constant wind speed condition.

Fig. 14. Rotor speeds with the PD-based control scheme under constant wind speeds.

Fig. 15. Rotor speeds with the COAPC scheme under constant wind speeds.

Fig. 16. Rotor speeds with the HOAPC scheme under constant wind speeds.

Fig. 17. Wind energy loss under constant wind speeds.

The case study results show that the COAPC scheme and HOAPC scheme can efficiently coordinate the active power outputs of the WTs within the wind farm during the synthetic inertial response process by minimizing the differences in the rotor speeds of the WTs, as well as the energy losses. Compared with the PD-based control scheme, although the COAPC and HOAPC schemes are complicated, the operation performance of the wind farm is improved efficiently. The

frequency support capability of the COAPC and HOAPC schemes are similar to that of the PD-based control methods. Moreover, the ADMM-based hierarchical solution method provides better scalability and can maintain the information privacy of the wind farm.

VI. CONCLUSION

In this paper, a hierarchical optimal control scheme based on the ADMM is proposed for the synthetic inertial response of large-scale wind farms. An MPC-based optimization problem is formulated to minimize differences in the rotor speeds of the WTs to guarantee the rotor speed stability as well as to minimize the wind energy loss caused by the decrease in rotor speeds of the WTs. A hierarchical solution method based on the ADMM is developed to solve the optimization problem in a fast way, which reduces the computation burden of the large-scale wind farm central controller and protects the information privacy of the wind farm. As verified by the case studies, the control scheme can efficiently reduce the differences in the rotor speeds of the WTs and reduce the energy loss caused by the decreases in the rotor speeds. The hierarchical method achieves the same performance as the centralized control method. Due to its advantages with regard to scalability, flexibility and information privacy, the hierarchical optimal control scheme is preferred for applications related to the synthetic inertial response of large-scale wind farms.

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