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Robust Distributed Coordination of Parallel Restored Subsystems in Wind Power Penetrated Transmission System

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Abstract—Several parallel restored subsystems exist in the last stage of the build-up restoration process of bulk power systems. The whole system restoration is completed when these subsystems are reconnected and their internal loads are recovered. This paper proposes a robust distributed coordination scheme to achieve reconnection and load recovery of parallel restored subsystems in wind power penetrated transmission systems. First, robust distributed restoration models are constructed considering individual subsystems under uncertain conditions. Then, an inner-outer nested upper-lower (IOUL) iterative algorithm is developed to solve the constructed models. The robust distributed coordination can be realized according to convergence. The proposed algorithm solves the complex non-convex model with uncertain variables by iteratively solving the small-scale mixed-integer linear programming (MILP) problem. The robust distributed restoration scheme allows independent decision-making within subsystems and ensures the feasibility of the distributed restoration strategy under uncertain conditions. The effectiveness of the proposed method is validated using three interconnected 6-bus systems and the IEEE 118-bus system, showing improved computational efficiency and restoration acceleration.

Index Terms—Bulk power system, distributed optimization, power system restoration, sectionalized restoration, wind power penetration.

I. INTRODUCTION

POWER system restoration is the recovery process after a complete or partial collapse of a power system [1]. Restoring power services quickly after a blackout is crucial for power system resilience [2]. In parallel restoration strategies, the large-scale transmission system restoration process needs to coordinate a number of subsystems during the build-up process. This is essential, since in the early stages of restoration, the blackout area is divided into several sectionalized subsystems to achieve their parallel power system restoration (PPSR)

[3]–[5]. Thus, at the end of the PPSR stage, several parallel restored subsystems exist. With high availability of distributed (RESs) resources with black-start capability, such as wind turbines [6] and micro-grids [7], the number of potential individual subsystems also increases. These separate energized subsystems need to be reconnected in the last restoration stage to rebuild the bulk system [1], [3]. Restoration conditions within the various subsystems differ according to their individual generator ramping capability, RES types and penetration and load recovery conditions. Effective coordination of parallel restored subsystems is of vital importance to achieve the bulk system reconnection and accelerate the whole restoration process.

The role of tie-lines is critical for the coordination of the energized subsystems [8], [9]. Ref. [8] identified these roles and proposes a tie-line utilization strategy for different restoration stages. In [9], a centralized method for tie-line utilization is employed based on a central coordination entity with explicit models of subsystems. However, the PPSR process involves several isolated individual grids [1], [3], [10] with restoration plans realized by different independent system operators (ISOs) [11]. Considering the incomplete data exchange among the different ISOs and the data processing burden of a central control center [12], [13], a distributed method is more suitable to coordinate the parallel restored subsystems. Using a distributed scheme, subsystems can make independent restoration decisions, while coordination based on boundary information on tie-lines is possible.

An important factor that should be considered in the restoration of power systems with high penetration of wind power is uncertainty [14]–[16]. Using stochastic programming [14], robust optimization [15] and risk constrained methods [16], uncertainty problem in the transmission restoration process is generally taken into account in a centralized way. To achieve a distributed scheme, the analytical target cascading (ATC) algorithm [17], [18], without requiring convexity of the model [19], is suitable for restoration models with integer variables. In the ATC based method, consistency of shared boundary variables between different subsystems is necessary for maintaining the feasibility of solutions [17], [18]. However, the boundary values obtained by the ATC method [12], [13], [19] may differ due to uncertainties in load and wind power output. Moreover, the step-by-step restoration process aggravates this mismatch [20] leading to not feasible results and security problems. Thus, for the wind power penetrated restoration, the feasibility of the distributed coordination scheme under uncer-

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tainties must be ensured requiring a robust distributed scheme able to guarantee the feasibility of the distributed coordination result under uncertainties.

In this paper, a robust distributed approach is proposed to coordinate parallel restored subsystems for wind power penetrated transmission system restoration. First, a coordinator-subsystem decision-making structure is built with robust distributed optimization models. Then, the ATC based algorithm is enhanced with an inner-outer nested upper-lower (IOUL) iterative loop to solve the proposed robust distributed models. Finally, the robust distributed restoration strategy is obtained by iteratively solving small-scale mixed-integer linear programming (MILP) models.

The contributions of this paper can be summarized as follows: 1) A robust distributed coordination scheme is proposed to coordinate parallel restored subsystems for the last stage of the build-up restoration process under uncertainties in wind power penetrated power systems. 2) The ATC based distributed method is improved to ensure the feasibility of the result, even in the worst-case uncertainties. 3) An IOUL algorithm is developed to iteratively solve the robust distributed restoration model. Accordingly, the large scale non-convex restoration model with uncertainty variables is transformed into small-scale parallel MILP models.

The rest of the paper is organized as follows: Section II provides compact forms of robust distributed restoration models with the coordinator-subsystem structure. Detailed models are built in Section III. Section IV presents the IOUL algorithm and information interaction process. Section V provides case study results and discussions, followed by the conclusions.

II. FRAMEWORK OF PARALLEL RESTORED SUBSYSTEM COORDINATION

This section introduces the framework of the parallel restored subsystem coordination. The structure of the parallel restored transmission system is described and the shared variables of the tie-lines are defined. The coordinator-subsystem structure is designed followed by robust distributed restoration models in a compact form.

A. Coordination of parallel restored subsystems

According to the parallel sectionalization approach, several individual subsystems exist in the PPSR process. At the last stage of PPSR process, these parallel subsystems have restarted their generators and energized their transmission lines [4], [5], [10]. However, their loads are not completely restored and the subsystems are isolated. As shown in Fig. 1, it is assumed that each subsystem contains traditional generators, wind farms, loads and a meshed network with different generator ramping up and load recovery conditions. The coordination of parallel restored subsystems aims to balance power supply and demand under different regional restoration conditions and rebuild the interconnected transmission system. Since information of each subsystem cannot be completely shared due to commercial or security consideration, a distributed coordination scheme is preferred.

Fig. 1. Boundary shared variables between the tie-line and subsystem.

As shown in Fig.1, the key to realize distributed coordination is the decoupling of shared boundary variables between the tie-line and subsystem. The shared variables to avoid the excessive standing phase angle (SPA) problem [21], are the phase angle θ at the interconnecting bus and the active power p of the tie-lines. Besides the phase angle, the frequency and voltage differences should also be within acceptable limits. The frequency difference requirement can be satisfied by using the isochronous machine in each subsystem, while local voltage regulation can be used to meet the voltage difference requirement since voltage constraints are included in the optimization model. In Fig. 1, p_c , p_j and p_k are the active power from the tie-line, subsystem j and subsystem k , respectively, and θ_j and $\theta_{c,j}$ represent phase angles at the interconnecting bus of subsystem j and the tie-line connecting subsystem j . ‘SUB j ’ stands for subsystem j . For effective parallel subsystem restoration, all the loads should be restored, as soon as possible, to complete the restoration process, i.e. the load recovery at each step of the restoration process should be maximized. [3]. Model (1) represents the load restoration coordination model for the whole system consisting of several parallel subsystems.

$$\begin{aligned}
 & \text{Max} \sum_{j \in N_{\text{Sub}}} \sum_{i \in N_{L,j}} (c_{L,j,i} E_{L,j,i} x_{L,j,i}) \\
 & \text{s.t.} \quad \mathbf{g}_j(\mathbf{x}_{L,j}, \mathbf{p}_{G,j}, \mathbf{p}_{L,j}, \mathbf{p}_{RE,j}, \mathbf{S}_{B,j}) \leq 0 \\
 & \quad \mathbf{h}_j(\mathbf{x}_{L,j}, \mathbf{p}_{G,j}, \mathbf{p}_{L,j}, \mathbf{p}_{RE,j}, \mathbf{S}_{B,j}) = 0 \\
 & \quad \mathbf{g}_{C_i}(\mathbf{S}_{C_i,j}, \mathbf{S}_{C_i,k}) \leq 0 \\
 & \quad \mathbf{h}_{C_i}(\mathbf{S}_{C_i,j}, \mathbf{S}_{C_i,k}) = 0 \\
 & \quad \boldsymbol{\sigma}_j = \mathbf{S}_{C_i,j} - \mathbf{S}_{B,j} = 0 \\
 & \quad \boldsymbol{\sigma}_k = \mathbf{S}_{C_i,k} - \mathbf{S}_{B,k} = 0 \\
 & \quad \boldsymbol{\sigma}_{C_i} = [\boldsymbol{\sigma}_j, \boldsymbol{\sigma}_k] \\
 & \quad \forall j, k \in N_{\text{Sub}}, j \neq k, C_i \in N_{\text{tie}}
 \end{aligned} \tag{1}$$

where $x_{L,j,i}$, $E_{L,j,i}$ and $c_{L,j,i}$ are the binary variable of load pickup decision, expected load amount and the corresponding weighting coefficients of load i at subsystem j , $\mathbf{x}_{L,j}$, $\mathbf{p}_{G,j}$, $\mathbf{p}_{L,j}$ and $\mathbf{p}_{RE,j}$ are sets of load pickup decision variables, active power outputs of generators, uncertain load amounts and uncertain active outputs of wind farms. $\mathbf{S}_{B,j}$ is the set of shared variables θ and p for subsystem j , $\mathbf{S}_{C_i,j}$ and $\mathbf{S}_{C_i,k}$ contain shared variables of the tie-line C_i connecting subsystems j and k , respectively, $\boldsymbol{\sigma}_j$ and $\boldsymbol{\sigma}_k$ are sets of shared variable mismatches at one side of the tie-line connecting subsystems j and at tie-line C_i . $\mathbf{g}_j()$ and $\mathbf{h}_j()$ represent the inequality and equality constraints of generator outputs, AC power flow, boundary variables, voltage and frequency of subsystem j , and $\mathbf{g}_{C_i}()$ and $\mathbf{h}_{C_i}()$ are boundary variable limits and variable consistency con-

straints of tie-line C_i . N_{Sub} , $N_{L,j}$ and N_{tie} are sets of subsystems, loads in subsystem j and tie-lines, respectively. Note that the load amount variables in $\mathbf{p}_{L,j}$ and wind farm output variables in $\mathbf{p}_{\text{RE},j}$ are all uncertain variables integrated in constraints. $E_{L,j,i}$ with a fixed value is just used in the objective function.

B. Robust distributed coordination structure

Based on Model (1), the optimization model can be divided according to parallel subsystems if tie-line constraints are separated. Using the Lagrangian decomposition method [17], the constraint set $\sigma=0$ is transformed into $\Phi(\sigma)$ and is further added to the objective function. The augmented Lagrangian method with a quadratic penalty term [18] is used to form $\Phi(\sigma)$ where $\mathbf{v}_j$ and $\mathbf{w}_j$ are sets of penalty multipliers for boundary variables decoupling of subsystem j and are updated during the iterative process. The symbol $\circ$ represents the Hadamard product which is an entry-wise multiplication of two vectors.

$$\Phi(\sigma) = \mathbf{v}\sigma^T + \|\mathbf{w} \circ \sigma^T\|_2^2 = \sum_{j \in N_{\text{Sub}}} \left(\mathbf{v}_j \sigma_j + \|\mathbf{w}_j \circ \sigma_j\|_2^2 \right) \quad (2)$$

With uncertain variables $\mathbf{p}_{L,j}$ and $\mathbf{p}_{\text{RE},j}$, Model (1) is transformed into robust distributed models (3)-(4). Model (3) is the tie-line optimization model which can be regarded as the coordinator, and Model (4) is the optimization model of parallel subsystems. Model (4) is a typical two-stage robust optimization model. The first stage includes the first *min* term of the objective in (4) and load pickup variable related constraints, while the second stage contains the *max-min* term in the objective and all the rest constraints. The subsystem model (4) maximizes regional load recovery at the first stage and minimizes the mismatch of boundary shared variables in the worst-case under which uncertain wind power output and load amount deteriorate the boundary consistency at the second stage. With a boundary consistency threshold, the feasibility of the distribution method under uncertain conditions can be guaranteed.

$$\begin{aligned} \text{Coordinator: Min } & \sum_{\forall C_i \in N_{\text{tie}}} \Phi_C(\sigma_{C_i}) \\ \text{s.t. } & \mathbf{g}_{C_i}(\mathbf{S}_{C_i,j}, \mathbf{S}_{C_i,k}) \leq 0 \\ & \mathbf{h}_{C_i}(\mathbf{S}_{C_i,j}, \mathbf{S}_{C_i,k}) = 0 \\ & j, k \in N_{\text{Sub}}, j \neq k, \forall C_i \in N_{\text{tie}} \\ \text{Subsystem: Min-} & \sum_{\mathbf{x}_{L,j}} \sum_{i \in N_{L,j}} (\mathbf{c}_{L,j,i} E_{L,j,i} \mathbf{x}_{L,j,i}) + \text{Max}_{\mathbf{p}_{L,j}, \mathbf{p}_{\text{RE},j}} \text{Min}_{\mathbf{S}_{B,j}, \mathbf{p}_{\alpha,j}} \Phi_j(\sigma_j) \\ \text{s.t. } & \mathbf{g}_j(\mathbf{x}_{L,j}, \mathbf{p}_{G,j}, \mathbf{p}_{L,j}, \mathbf{p}_{\text{RE},j}, \mathbf{S}_{B,j}) \leq 0 \\ & \mathbf{h}_j(\mathbf{x}_{L,j}, \mathbf{p}_{G,j}, \mathbf{p}_{L,j}, \mathbf{p}_{\text{RE},j}, \mathbf{S}_{B,j}) = 0 \\ & j \in N_{\text{Sub}} \end{aligned} \quad (3) \quad (4)$$

Fig. 2. The coordinator-subsystem structure

The coordination of parallel subsystems relies on an iterative information interaction process. As shown in Fig. 2, a coordinator-subsystem framework is designed based on the robust distributed models. In this framework, parallel subsys-

tems make independent restoration decisions according to Model (4), and submit boundary results to the coordinator. With the boundary information of subsystems, the coordinator performs Model (3) and provides coordinated boundary results to subsystems. The information interaction process continues until the coordinator and subsystems reach boundary consensus. The practical coordinator can be the upper-level dispatch center for different regional ISOs in the bulk system with knowledge of tie-lines.

III. MODELING OF ROBUST DISTRIBUTED COORDINATION

Considering uncertainties, the overall problem is formulated as distributed robust optimization models in this Section. First, the detailed restoration models of the coordinator and the subsystems are built. Then, the coordinator-subsystem models are transformed into MILP problems.

A. Robust distributed coordinator-subsystem models

First, the model of the coordinator is formulated.

1) *Objective*: the objective of the upper-level coordinator is to coordinate all the subsystems by pursuing the consistency of boundary variable values between tie-lines and parallel subsystems. In (5), $p_{C_i,j}$ and $\theta_{C_i,j}$ are the boundary active power and phase angle of tie-line C_i between tie-line C_i and subsystem j , $p_{C_i,k}$ and $\theta_{C_i,k}$ are the ones of tie-line C_i between tie-line C_i and subsystem k , p_{B,j,C_i}^* , p_{B,k,C_i}^* , θ_{B,j,C_i}^* and θ_{B,k,C_i}^* are boundary active power and phase angles provided by subsystem j and k .

$$\text{Min } \sum_{C_i \in N_{\text{tie}}} \left[\begin{aligned} & \mathbf{v}_j (p_{C_i,j} - p_{B,j,C_i}^*) + \|\mathbf{w}_j \circ (p_{C_i,j} - p_{B,j,C_i}^*)\|_2^2 \\ & + \mathbf{v}_j (\theta_{C_i,j} - \theta_{B,j,C_i}^*) + \|\mathbf{w}_j \circ (\theta_{C_i,j} - \theta_{B,j,C_i}^*)\|_2^2 \\ & + \mathbf{v}_k (p_{C_i,k} - p_{B,k,C_i}^*) + \|\mathbf{w}_k \circ (p_{C_i,k} - p_{B,k,C_i}^*)\|_2^2 \\ & + \mathbf{v}_k (\theta_{C_i,k} - \theta_{B,k,C_i}^*) + \|\mathbf{w}_k \circ (\theta_{C_i,k} - \theta_{B,k,C_i}^*)\|_2^2 \end{aligned} \right] \quad (5)$$

2) *Constraints of tie-lines*. Constraints (6)-(7) comprise active power limits of tie-lines. A maximum phase angle difference is required in (8) to avoid excessive SPA problems between neighboring subsystems.

$$p_{C_i,j} = -p_{C_i,k} \quad (\forall C_i \in N_{\text{tie}}) \quad (6)$$

$$|p_{C_i,j}| \leq p_{C_i,\text{max}} \quad (\forall C_i \in N_{\text{tie}}) \quad (7)$$

$$|\theta_{C_i,j} - \theta_{C_i,k}| \leq \theta_{jk,\text{max}} \quad \forall C_i \in N_{\text{tie}} \quad (8)$$

Without loss of generality, the load restoration oriented robust distribution model of subsystem j is constructed as follows.

1) *Objective*: The objective of the subsystem is to restore its own load considering coordination from the coordinator. The first term of (9) is the weighted load restoration amount and the latter term represents coordination with tie-lines. The subsystem model is a two-stage robust optimization with a polyhedral uncertainty set Φ (10) for uncertain wind farm output and load amount [22]. In (10), $p_{\text{low},i}$ and $p_{\text{up},i}$ are lower and upper active powers of the uncertain variable related to node i .

$$\begin{aligned} & \text{Min} - \sum_{x_{L,j,i}} (c_{L,j,i} E_{L,j,i} x_{L,j,i}) \\ & + \text{Max}_{(P_{L,j}, P_{W,j}) \in \Phi} \text{Min}_{C_i \in \text{Sub}_j} \sum_{C_i \in \text{Sub}_j} \left(v_j (p_{C_i,j}^* - p_{B,j,C_i}) + v_j (\theta_{C_i,j}^* - \theta_{B,j,C_i}) \right. \\ & \left. + \left\| w_j \circ (p_{C_i,j}^* - p_{B,j,C_i}) \right\|_2^2 + \left\| w_j \circ (\theta_{C_i,j}^* - \theta_{B,j,C_i}) \right\|_2^2 \right) \end{aligned} \quad (9)$$

$$\Phi = \{ (P_{L,j}, P_{W,j}) : p_{low,i} \leq p_i \leq p_{up,i}, i \in N_{L,j} \& N_{W,j} \} \quad (10)$$

2) *Power balance constraints*: The power supply contains the output of traditional generators and wind farms. In the load restoration stage, the conventional generators ramp up with ramp rate r_i between upper and lower limits in (11) [23]. Constraint (12) ensures spinning reserve which survives the largest loss of energy contingency. The limit of the restoration step time is shown in (13). Equalities (14) and (15) determine the active and reactive power of each bus in subsystem j , respectively. Constraints (16)-(22) are the linearized AC power flow which performs polyhedral relaxation of cosine function and linearization by Taylor series expansion, as detailed in [24].

$$\max \{ p_{G,i,ini} + r_i T_m, p_{G,i,min} \} \leq p_{G,i} \leq \min \{ p_{G,i,ini} + r_i T_m, p_{G,i,max} \} \quad (11)$$

$$p_{G,i} \leq \sum_{i' \in N_{G,j}, i' \neq i} (r_{i'} T_{max} + p_{G,i'}) + \sum_{i \in N_{W,j}} p_{W,i} - \sum_{C_i \in \text{Sub}_j} p_{B,j,C_i} - \sum_{i \in N_{L,j}} x_{L,i} p_{L,i} \quad (12)$$

$$T_{min} \leq T_m \leq T_{max} \quad (13)$$

$$p_n = -x_{L,i} p_{L,i} (i \in N_{L,j}) - p_{B,j,C_i} (C_i \in \text{Sub}_j) + p_{W,i} (i \in N_{W,j}) + p_{G,i} (i \in N_{G,j}) + 0 \quad (14)$$

$$q_n = -x_{L,i} q_{L,i} (i \in N_{L,j}) + q_{G,i} (i \in N_{G,j}) + q_{B,i} (i \in N_{B,j}) + 0 \quad (15)$$

$$p_n = \sum_{\langle n,m \rangle \in L_j} P_{nm} \quad \forall n \in N_{Sub_j} \quad (16)$$

$$q_n = \sum_{\langle n,m \rangle \in L_j} Q_{nm} \quad \forall n \in N_{Sub_j} \quad (17)$$

$$P_{nm} = g_{nm} - g_{nm} \cos^* \theta_{nm} - b_{nm} (\theta_n - \theta_m) \quad (18)$$

$$Q_{nm} = -b_{nm} - g_{nm} (\theta_n - \theta_m) + b_{nm} \cos^* \theta_{nm} - b_{nm} (\delta_n - \delta_m) \quad (19)$$

$$\cos^* \theta_{nm} \geq \cos \theta_{\Delta}^0 \quad (20)$$

$$\cos^* \theta_{nm} \leq -\sin(v\delta - \theta_{\Delta}^0)(\theta_n - \theta_m - v\delta + \theta_{\Delta}^0) + \cos(v\delta - \theta_{\Delta}^0) \quad (21)$$

$$v = 1, 2, \dots, 2h$$

$$V_n = 1 + \delta_n \quad (\text{Volts p.u.}) \quad (22)$$

Variables $p_{G,j}$, $p_{G,j,ini}$, $p_{G,j,min}$ and $p_{G,j,max}$ are the active power output, initial active output in the current step, minimum and maximum output in the whole restoration process of generator at node i , respectively, $x_{L,i}$ is the binary variable of the load pickup decision, $p_{L,i}$ and $p_{W,i}$ are uncertain load amount and wind farm output at node i , p_{B,j,C_i} is the boundary active power between subsystem j and tie-line C_i , T is the load pickup time, $q_{L,i}$, $q_{G,i}$ and $q_{B,i}$ are reactive power of the load, generator and boundary node i , respectively, p_n and q_n are the active and reactive power injection into bus n , P_{nm} and Q_{nm} are the active and reactive power flow on line nm , g_{nm} and b_{nm} are the conductance, susceptance and phase angle difference of line nm , $\cos^* \theta_{nm}$ is the approximate variable of $\cos \theta_{nm}$, and V_n and δ_n are voltage magnitude and voltage magnitude deviation variable of bus n . $N_{G,j}$, $N_{L,j}$, $N_{W,j}$, $N_{B,j}$, N_{Sub_j} and L_j contain generator nodes, load nodes, wind farm nodes, boundary nodes, all the nodes and all the lines in subsystem j , respectively. In power balance constraints, define phase angle $\theta_n = 0$ when the

corresponding bus is the slack bus. Note that there is just one slack bus for all the subsystems.

3) *Operation constraints*: The linearized branch flow limits (23)-(26), and voltage and frequency constraints (27) and (28). Constraint (28) is a linear frequency limit considering the total capacity of restored generators s and their frequency response constant ε [15]. Inequalities (12) and (28) are the reserve constraints in the restoration process considering the $N-1$ security criterion of generators. Note that the original quadratic constraint of branch flow $-S_{nm,max}^2 \leq P_{nm}^2 + Q_{nm}^2 \leq S_{nm,max}^2$ is linearized by (23)-(26).

$$-S_{nm,max} \leq P_{nm} \leq S_{nm,max} \quad (23)$$

$$-S_{nm,max} \leq Q_{nm} \leq S_{nm,max} \quad (24)$$

$$-\sqrt{2}S_{nm,max} \leq P_{nm} + Q_{nm} \leq \sqrt{2}S_{nm,max} \quad (25)$$

$$-\sqrt{2}S_{nm,max} \leq P_{nm} - Q_{nm} \leq \sqrt{2}S_{nm,max} \quad (26)$$

$$V_{n,min} < V_n < V_{n,max} \quad \forall n \in N_{Sub_j} \quad (27)$$

$$\Delta f = \left| \sum_{i \in N_{L,j}} x_{L,i} p_{L,i} - p_L^0 - \sum_{C_i \in \text{Sub}_j} p_{B,j,C_i} \right| \quad (28)$$

$$+ p_B^0 - \sum_{i \in N_{W,j}} p_{W,i} + p_W^0 \bigg| \bigg/ \sum_{i' \in N_{G,j}, i' \neq i} \frac{S_{i'}}{\varepsilon_{i'}} \leq \max \Delta f$$

B. Linearization of robust distributed models

The distributed subsystem models are coordinated by the coordinator using Model (29) which is corresponding to (5)-(8).

$$\begin{aligned} & \text{Min} \sum_{C_i \in N_{tie}} \left[(S_{C_i} - S_{B,C_i}^*) w_{C_i} (S_{C_i} - S_{B,C_i}^*)^T + v_{C_i} (S_{C_i} - S_{B,C_i}^*) \right] \\ & \text{s.t. } S_{C_i} = [S_{C_i,j}, S_{C_i,k}] = [p_{C_i,j}, \theta_{C_i,j}, p_{C_i,k}, \theta_{C_i,k}] \end{aligned} \quad (29)$$

$$(6)-(8)$$

Model (29) has linearized constraints and a quadratic objective function. As shown in Fig. 3, by introducing binary variables u , the objective function can be transformed into piecewise linearization functions (30) with constraints (31) [25]. N_{C_i} is the set of the boundary node of tie-line C_i . Finally, the coordinator model becomes the MILP form in (32) where $U_{C_i,i}$, $S_{C_i,i}$, $O_{C_i,i}$ and $O_{C_i,i+1}$ are the set of $u_{C_i,i,o}$, $s_{C_i,i,o}$, $O_{C_i,i,o}$ and $O_{C_i,i,o+1} u_{C_i,i,o}$, respectively.

Fig. 3. Diagram of the piecewise linearization

$$\sum_{C_i \in N_{tie}} \sum_{i \in N_{C_i}} \sum_{o=1}^{D-1} w_{C_i,i} (s_{C_i,i,o} k_{C_i,i,o} + m_{C_i,i,o} u_{C_i,i,o}) + \sum_{C_i \in N_{tie}} v_{C_i} S_{C_i} \quad (30)$$

$$\begin{aligned} & \text{s.t. } \sum_{o=1}^{D-1} u_{C_i,i,o} = 1 \\ & \sum_{o=1}^{D-1} s_{C_i,i,o} = s_{C_i,i} \\ & O_{C_i,i,o} u_{C_i,i,o} \leq s_{C_i,i,o} \leq O_{C_i,i,o+1} u_{C_i,i,o} \end{aligned} \quad (31)$$

The coordinator model

$$\begin{aligned} \text{Min}_{S_{C_i}} & \left[\sum_{C_i \in N_{tie}} w_{C_i} (S_{C_i} K_{C_i}^T + M_{C_i} U_{C_i}^T) + \sum_{C_i \in N_{tie}} v_{C_i} S_{C_i}^T \right] \quad (32) \\ \text{s.t.} & \begin{cases} S_{C_i} = [\text{sum}(S'_{C_{i,1}}), \dots, \text{sum}(S'_{C_{i,i}}), \dots] \\ \text{sum}(U_{C_{i,i}}) = 1 \\ \text{sum}(S'_{C_{i,i}}) = S_{C_{i,i}} \\ O_{C_{i,i}} U_{C_{i,i}}^T \leq S_{C_{i,i}}^T \leq O'_{C_{i,i}} U_{C_{i,i}}^T \\ (6) - (11) \quad (\forall i \in N_{C_i}, C_i \in N_{tie}) \end{cases} \end{aligned}$$

The subsystem model is a two-stage robust optimization in (33). The first stage addresses load pickup decisions and the second stage handles recourse decisions to reveal the uncertainty [26]. With the Min-Max-Min form of the objective function, the optimal solution maximizes load pickup amount and minimizes the boundary mismatch in the worst-case. $S_{C_j}^*$ is the set of $p_{C_j}^*$ and $\theta_{C_j}^*$.

$$\begin{aligned} \text{Min}_{X_{L,j}} & -X_{L,j} E_{L,j}^T + \text{Max}_{P_{L,j}, Y_j, P_{W,j}} \text{Min}_{S_{C_j}^*, S_{B,j}} (S_{C_j}^* - S_{B,j}) w_j (S_{C_j}^* - S_{B,j})^T \\ & + v_j (S_{C_j}^* - S_{B,j}) \\ \text{s.t.} & \Delta = \{X_{L,j}\}, \quad (33) \\ \Psi = & \left\{ \begin{aligned} & (S_{B,j}, Y_j) : (11) - (28) \\ & Y_j = (p_{G,j}, p_n, q_n, P_{nm}, Q_{nm}, \delta_n, \theta_n, \text{app}(\cos \theta_{nm})) \\ & i \in N_{G,j}, n \in N_{Sub_j}, C_i \in Sub_j \end{aligned} \right\} \\ \Phi = & \{(P_{L,j}, P_{W,j}) : p_{low,i} \leq p_i \leq p_{up,i}, i \in N_{L,j} \& N_{W,j}\} \end{aligned}$$

Because all the constraints in (33) are linear, the inner-level Max-Min problem is converted into a compact form with linearized equality and inequality (34).

$$\begin{aligned} \text{Max}_{(P_{L,j}, P_{W,j}) \in \Phi} & \text{Min}_{S_{B,j}, Y_j} (S_{C_j}^* - S_{B,j}) w_j (S_{C_j}^* - S_{B,j})^T + v_j (S_{C_j}^* - S_{B,j}) \\ \text{s.t.} & AS_{B,j} + BY_j + DX_{L,j}^* P_{L,j} + EP_{W,j} + F = 0 \\ & GS_{B,j} + HY_j + I \leq 0 \end{aligned} \quad (34)$$

When load pickup decision variables in $X_{L,j}$ are determined in the first stage, the Min problem in (34) is a convex optimization with a quadratic objective function and linear constraints. By applying the KKT condition, the solution of the Min problem can be obtained [27]. Then, Model (34) is further transformed into Model (35).

$$\begin{aligned} \text{Max}_{(P_{L,j}, P_{W,j}) \in \Phi} & (S_{B,j} - S_{C,j}^*) w_j (S_{B,j} - S_{C,j}^*)^T \\ & + v_j (S_{B,j} - S_{C,j}^*) \\ \text{s.t.} & \begin{cases} 2QS_{B,j}^T + C + \lambda^T A + \mu^T G = 0 \\ \lambda^T B + \mu^T H = 0 \\ AS_{B,j} + BY_j + DX_{L,j}^* P_{L,j} + EP_{W,j} + F = 0 \\ GS_{B,j} + HY_j + I \leq 0 \\ \mu^T (GS_{B,j} + HY_j + I) = 0 \\ \mu^T \geq 0 \\ 0 \leq \mu \leq MZ \\ M(Z - I) \leq GS_{B,j} + HY_j + I \leq 0 \end{cases} \quad (35) \end{aligned}$$

The subsystem model

$$\begin{aligned} \text{Min}_{X_{L,j}} & -X_{L,j} E_{L,j}^T + \text{Max}_{S_{B,j}, Y_j, P_{L,j}, P_{W,j}} [w_j (S_{B,j} K_{B,j}^T + M_{B,j} U_{B,j}^T) + v_j S_{B,j}^T] \\ \text{s.t.} & \Delta = \{X_{L,j} : (11)\} \\ \Psi' = & \left\{ \begin{aligned} & S_{B,j} = [\text{sum}(S'_{B_{j,1}}), \dots, \text{sum}(S'_{B_{j,i}}), \dots] \quad (\forall i \in Sub_j) \\ & \text{sum}(U_{B_{j,i}}) = 1 \quad (\forall i \in Sub_j) \\ & \text{sum}(S'_{B_{j,i}}) = S_{B_{j,i}} \\ & O_{B_{j,i}} U_{B_{j,i}}^T \leq S_{B_{j,i}}^T \leq O'_{B_{j,i}} U_{B_{j,i}}^T \quad (\forall i \in Sub_j) \\ & 2QS_{B,j}^T + C + \lambda^T A + \mu^T G = 0 \\ & \lambda^T B + \mu^T H = 0 \\ & AS_{B,j} + BY_j + DX_{L,j}^* P_{L,j} + EP_{W,j} + F = 0 \\ & 0 \leq \mu \leq MZ \\ & M(Z - I) \leq GS_{B,j} + HY_j + I \leq 0 \\ & (P_{L,j}, P_{W,j}) \in \Phi \end{aligned} \right\} \quad (37) \end{aligned}$$

The last three constraints in (35) are the complementary slackness condition which can be further linearized. Introducing binary variables in Z and the constant M with a sufficient large value linearizes the complementarity slackness condition as constraints (36). However, it adds binary variables and makes (35) become an MIQP model. The quadratic term in the objective function is transformed into (37) with the same piecewise linearization method of the coordinator model (32). Accordingly, the inner-level Max-Min optimization becomes an MILP problem (37).

By now, the robust distributed restoration models for the coordinator (32) and subsystems (37) are all converted into MILP form and are solved by an iterative parallel calculation process shown in the next section.

IV. SOLUTION METHOD OF ROBUST DISTRIBUTED MODELS

This section develops an IOUL algorithm to solve the robust distributed models in Section III. The inner-out (IO) iteration achieves independent restoration decision-making of regional grids by solving the subsystem problems in parallel. The global coordination is achieved by the upper-lower (UL) iteration between the coordinator and subsystem models. Through tie-line information exchange, the coordinated parallel restoration strategy is obtained according to convergence.

A. Solving coordinator-subsystem model by IOUL algorithm

With all the models in MILP forms, the solving process using the IOUL algorithm is shown as follows. Note that the lower level loop Steps 1 - 4 are for each subsystem and are performed in parallel.

Step 0: Set $L_{upp} = 1$, upper-lower convergence thresholds $\varepsilon_2 \geq 0$ and $\varepsilon_3 \geq 0$. Choose the initial values of shared variables in $S_{C_j}^{L_{upp}}$ for each subsystem, and define the initial values of the penalty multipliers in $v^{L_{upp}}$ and $w^{L_{upp}}$.

Step 1: Set $LB = -\infty$, $UB = +\infty$, the inner-outer tolerance $\varepsilon_1 \geq 0$, $L_{low} = 1$ and arbitrarily select a feasible $X_{L,j}^{L_{low}} \in \Delta$, $X_{L,j}^* = X_{L,j}^{L_{low}}$.

Step 2: Solve the inner-level problem (38) of the subsystem model (37) and obtain the worst-case wind output $P_{W,j}^*$ and load

amount $P_{L,j}^*$. Update $UB = \min \{UB, obj_{in} + X_{L,j}^{Low}\}$ where obj_{in} is the objective function value of (38) in the L_{low} -th iteration.

$$\begin{aligned}
 & \text{Max}_{S_{B,j}, Y_j, P_{L,j}, P_{W,j}} \left[w_j (S_{B,j} K_{B,j}^T + M_{B,j} U_{B,j}^T) + v_j S_{B,j}^T \right] \\
 \text{s.t. } & S_{B,j} = [\text{sum}(S'_{B,j,1}), \dots, \text{sum}(S'_{B,j,i}), \dots] \quad (\forall i \in \text{Sub}_j) \\
 & \text{sum}(U_{B,j}) = 1 \quad (\forall i \in \text{Sub}_j) \\
 & \text{sum}(S'_{B,j,i}) = S_{B,j,i} \\
 & O_{B,j,i} U_{B,j,i}^T \leq S_{B,j,i}^T \leq O'_{B,j,i} U_{B,j,i}^T \quad (\forall i \in \text{Sub}_j) \\
 & 2QS_{B,j}^T + C + \lambda^T A + \mu^T G = 0 \\
 & \lambda^T B + \mu^T H = 0 \\
 & AS_{B,j} + BY_j + DX_{L,j}^* P_{L,j} + EP_{W,j} + F = 0 \\
 & 0 \leq \mu \leq MZ \\
 & M(Z - I) \leq GS_{B,j} + HY_j + I \leq 0 \\
 & (P_{L,j}, P_{W,j}) \in \Phi
 \end{aligned} \tag{38}$$

Step 3: Solve the outer-level model (39) and obtain the optimal result $X_{L,j}^l$ ($l \in \{1, \dots, L_{low}\}$), let $X_{L,j}^{Low} = X_{L,j}^*$. Update $LB = \max \{LB, obj_{out}\}$ where obj_{out} is the objective function value of (39) in the L_{low} th iteration.

$$\begin{aligned}
 & \text{Min}_{\theta_j, X_{L,j}, S_{B,j}^l, Y_j^l} -X_{L,j} E^T + \theta_j \\
 \text{s.t. } & X_{L,j} \in \Delta \\
 & \theta_j \geq w_j (S_{B,j} K_{B,j}^T + M_{B,j} U_{B,j}^T) + v_j S_{B,j}^T \\
 & S_{B,j}^l = [\text{sum}(S'^l_{B,j,1}), \dots, \text{sum}(S'^l_{B,j,i}), \dots] \quad (\forall i \in \text{Sub}_j) \\
 & \text{sum}(U_{B,j}^l) = 1 \quad (\forall i \in \text{Sub}_j) \\
 & O_{B,j,i} U_{B,j,i}^T \leq S_{B,j,i}^{lT} \leq O'_{B,j,i} U_{B,j,i}^T \quad (\forall i \in \text{Sub}_j) \\
 & AS_{B,j}^l + BY_j^l + DX_{L,j}^* P_{L,j}^* + EP_{W,j}^* + F = 0 \\
 & GS_{B,j}^l + HY_j^l + I \leq 0 \\
 & \forall l \in \{1, 2, \dots, L_{low}\}
 \end{aligned} \tag{39}$$

Step 4: If $UB - LB \leq \varepsilon_1$, obtain the result $S_{B,j}^{Low}$ and go to *Step 5*. Otherwise, $L_{low} = L_{low} + 1$ and go to *Step 2*.

Step 5: Collect $S_{B,j}^{Low}$ from the lower level $S_B^{L_{low}} = [\dots, S_{B,j}^{Low}, \dots]$. Solve the coordinator model (32) with $S_B^* = S_B^{L_{low}}$. Obtain $S_C^{L_{low}+1}$.

Step 6: Check convergence by the necessary consistency equations (40) and sufficient stopping criteria (41). If they are not satisfied and the iteration limit (42) is not reached, proceed to *Step 7*. Otherwise, stop and obtain the final result.

$$|S_{B,j}^{L_{low}} - S_{C,j}^{L_{low}}| \leq \varepsilon_2 \quad \forall j \in N_{\text{Sub}} \tag{40}$$

$$|(f_j^{L_{low}} - f_j^{L_{low}-1})/f_j^{L_{low}}| \leq \varepsilon_3 \quad \forall j \in N_{\text{Sub}} \tag{41}$$

$$L_{low} \geq L_{\max} \tag{42}$$

Step 7: Update penalty parameters $v_j^{L_{low}}$ and $w_j^{L_{low}}$ according to (43) and (44). Set $L_{low} = L_{low} + 1$ and go to *Step 1*.

$$v_j^{L_{low}+1} = v_j^{L_{low}} + 2(w_j^{L_{low}}) Q(w_j^{L_{low}})^T (S_{B,j}^{L_{low}} - S_{C,j}^{L_{low}}) \quad \forall j \in N_{\text{Sub}} \tag{43}$$

$$w_j^{L_{low}+1} = \beta w_j^{L_{low}} \quad (\beta \geq 1) \quad \forall j \in N_{\text{Sub}} \tag{44}$$

The algorithm starts with an inner-outer iterative calculation (*Step 1-Step 4*) according to the column-and-constraint generation (C&CG) method [28] which is proven to converge for the MILP problem [22], [26]. The whole IO loop belongs to the

lower level of an upper-lower iteration which is based on the augmented Lagrangian ATC method [18] with good convergence characteristics, even for non-convex models [13], [19]. The detailed convergence characteristic of the ATC method is provided in [29]. Therefore, the IOUL algorithm generally converges for the robust distributed restoration model.

B. Coordination process in iteration of IOUL algorithm

Fig. 4 shows the structure of the whole IOUL loop and the boundary information exchange process. The IO loop (*Step 1-Step 4*) solves the subsystem model (37) in parallel. The inner optimization (*Step 2*) returns the worst-case conditions ($P_{W,j}^*$, $P_{L,j}^*$) to the outer optimization (*Step 3*), while the outer optimization determines the load pickup variables X_L^* for the inner optimization. The whole IO loop belongs to the lower level of the UL loop. Results in $S_{B,j}^*$ from the IO loop are delivered to the upper level optimization (*Step 5*) to solve the coordinator model (32). The final coordinated parallel restoration strategy is obtained when the IOUL algorithm converges.

Fig. 4. Structure of the IOUL loop

The robust distributed coordination is achieved with the IOUL algorithm. The parallel restored subsystems maximize regional load pickup amount as well as minimize the worst-case of boundary variable mismatch. With the tie-line information, the coordinator coordinates these independent restoration strategies of subsystems to pursue a global restoration optimization. In this scheme, each subsystem makes restoration decisions independently and is coordinated by the boundary variable information exchange. The independent subsystem restoration strategy is the result of the IO loop convergence and the coordination is achieved when the UL loop converges. Even in the worst-case, the boundary mismatch of the robust distributed restoration strategy can be maintained within ε_2 . That means the feasibility of the corresponding distributed coordination scheme can always be guaranteed in the uncertain condition.

V. CASE STUDY

A three-area 18-bus system [13] and a sectionalized IEEE118-bus system [30] are used to demonstrate the effectiveness of the proposed method. The convergence thresholds

of the IOUL algorithm are 0.5%. The initial values of boundary variables are set as 0, and the initial penalty values in $\mathbf{v}$ and $\mathbf{w}$ are set as 0 and 1, respectively. β is set as 1.8. 10% and 15% deviations from the expected values are used to determine the bound of polyhedral uncertainty sets of load and wind power output, respectively. All case studies were conducted using GUROBI V7.5.1 on a computer with Intel(R) Core(TM) i7-8550U CPU and 16 GB RAM.

A. Performance of IOUL algorithm in 18-bus system

Fig. 5. Three-area 18-bus system

Fig. 5 shows the three-area 18-bus system and Fig. 6 presents the iteration process of obtaining the robust distributed coordination strategy. There are 6 UL loops. The IO loops are 26 for SUB1, 6 for SUB2 and 10 for SUB3, respectively. As shown in Fig. 6, boundary variables of SUB1 $p_{B,1,C1}$ and $p_{B,1,C2}$ are used in the IO loop and the result of the IO loop is used as the initial values for the UL loop. The Tie-line variables $p_{C1,2}$ in Tie-line1 and $p_{C2,3}$ in Tie-line2 are used in the UL loop. In the UL loop, the IO loop is implemented for each subsystem. The coordination among subsystems is realized by the UL loop. Finally, the largest difference of the power exchange between SUB1 and tie-line 1 is below 0.5%, and so does the one between SUB1 and tie-line 2. Besides, the phase angle difference (Theta) is below 0.34 rad to avoid the SPA problem.

Fig. 6. IOUL iteration process of the distributed robust scheme

To demonstrate the advantages of the proposed method, the results are compared in Table I with the results of standard ATC [19] and centralized robust method [16]. In Table I, Objective (MW) is the expected value of load recovery amount, while Objective (p.u.) is the value of objective function with the piecewise linear approximation of quadratic terms. The power exchange between SUB1 and SUB2 is 7.43 MW, and the one between SUB1 and SUB3 is 17.57 MW. It shows that using the proposed robust ATC method, the boundary mismatch is below the threshold 0.5%. Under uncertain conditions, the result obtained with the standard ATC show an unacceptable 27.8% mismatch in the worst-case. Although more loads are restored, the standard ATC based coordination scheme is infeasible, because the unacceptable boundary power mismatch easily leads to security violation. The proposed method obtains the same objective value with the centralized robust optimization method [16]. This means that the proposed method can achieve global optimal coordination, although

subsystems make independent restoration decisions. Overall, the proposed method maintains the feasibility of the distributed scheme under uncertain conditions and achieves coordination of the whole system.

TABLE I
COMPUTATION EFFICIENCY OF DIFFERENT METHOD

	Objective (p.u.)	Load recovery (MW)	Power exchange (MW)	Boundary mismatch
IOUL	-2.41	250	Tie-line1: 7.43 Tie-line2: 17.57	0.3%
ATC [19]	-2.94	300	Tie-line1: 66.5 Tie-line2: 77	27.8%
Centralized [16]	-2.50	250	Tie-line1: 35 Tie-line2: 70	/

B. Restoration performance in IEEE 118-bus system

Fig. 7. IEEE-118 bus system with five subsystems

According to [30], the IEEE-118 bus system is divided into five subsystems which can balance power supply and demand by themselves. The computation efficiency of the IOUL method is compared with the centralized robust optimization in [19] which solves the whole system restoration model with the C&CG method. The restoration performance of the proposed coordination scheme is compared with the restoration of five parallel restored subsystems in [30].

TABLE II
COMPARISONS OF DIFFERENT SCHEMES

	IOUL based Coordinated scheme	Separated scheme [30]	Centralized scheme [16]
Calculation time	203.87s	17.24s	1057.96s
Load recovery	3187MW	3030MW	3195MW
Optimization gap	0.25%	5.44%	/
Restoration time	30.44min	39.07min	/

As shown in Table II, compared with the centralized robust optimization method, the proposed method reduces the calculation time from 1057.96s to 203.87s with an acceptable optimization gap. This is because the proposed method performs parallel optimization; thus, the recorded time of the lower iteration is the most time consuming subsystem IO loop calculation. The IO loop is based on the C&CG algorithm. Since the distributed coordination scheme reduces the calculation size of the subsystem robust restoration optimization, the additional variables and constraints from the C&CG algorithm are largely reduced, which further reduces the calculation time. Compared with the separated scheme, the coordinated parallel restoration scheme can restore all the loads in 30.44 min with 5

steps, while the separated one picks up 96.25% loads in 39.07 min with 7 steps.

Fig. 8. Multi-step restoration conditions in the subsystem separated scheme (a) and coordinated scheme (b).

With the load pickup amount ‘SUB_L’, generator ‘Gen’ and wind power output ‘RES’, Fig. 8 demonstrates the 5-step co-ordinated parallel restoration process and the 7-step separated restoration process. When the separated scheme chooses to restore the loads in their own area as ‘SUB_SL’ in Step1 in Fig. 8 (a), the coordination scheme makes SUB3 support its neighboring grids first. As shown in Fig. 8 (b), although no load restored in SUB3, the generators in SUB3 still ramp up to support SUB2, SUB4 and SUB 5.

Fig. 9. Power exchange between subsystems in the coordinated scheme

Coordination of the whole system is achieved by the optimized power exchange among subsystems. Fig. 9 presents the optimized power exchange of four tie-lines in five restoration steps. Fig. 10 shows the voltage and phase angle at the boundary buses of subsystems. Besides, the boundary voltage and phase angle differences between neighboring subsystems are presented with ‘distance’ in (a) and (b), respectively. All the phase angle differences are below the required 0.34 rad (19.48 degree) to avoid the SPA problem. The boundary voltage values are obtained when reactive power of corresponding buses is regarded as zeros. The local voltage regulation can be performed to adjust boundary voltages if the voltage difference requirement is not satisfied.

Fig. 10. Boundary voltage (a) and boundary phase angle (b) conditions of neighboring subsystems in the multi-step restoration process

Results in Fig. 9 correspond to the converged results in Fig. 10. Taking SUB1 and tie-line between SUB1 and SUB2 for example, Fig. 11 demonstrates the IOUL iteration number of each step. As can be seen, the whole process has 110 IO loops and 55 UL loops. The UL loop continues as long as the dis-

tance of the inner and outer objective values of subsystem ‘UB-LB’ is below the threshold ε_1 , and the step-by-step restoration strategies are obtained when the boundary variables $p_{B,1,C1}$ in subsystem1, $p_{B,2,C1}$ in subsystem2 and p_{C1} in the tie-line become consistent.

Fig. 11. Boundary variable iterations between SUB1 and SUB2

Note that the separated scheme cannot pick up the load block at Bus 18 in SUB1 using the robust optimization. This means that SUB1 has the risk of security violation to restore all the loads alone under the uncertain condition. In the coordinated scheme, the tie-line flow ‘PF-2’ in Step 5 in Fig. 9 is negative, which means SUB2 supplies power to SUB1, so that all the loads in SUB1 can be restored. Overall, the proposed scheme not only allows independent restoration of subsystems but also accelerates the restoration process and assists subsystems to handle the worst-condition under the uncertainty.

VI. DISCUSSION

This paper uses a steady-state model to optimize load pickup and subsystem reconnection decision-making for the end of the PPSR stage. The frequency and voltage stability of subsystems can be controlled by the frequency response of generators and voltage regulation within individual subsystems considering boundary power exchanges. A linearized frequency constraint is incorporated into the subsystem model to ensure the frequency stability of the obtained result. For the voltage stability, the method in [31] can be applied to provide transient voltage constraints for each subsystem. Besides, extra simulations considering N-1 stability analysis can be performed before the implementation of the optimization result to maintain the stability in individual subsystems and during their coordinated restoration. More exact dynamic constraints for the frequency and voltage stabilities can be integrated into the model, and the proposed robust distributed method is feasible as long as these constraints are convex.

VII. CONCLUSION

This paper presents a new coordination scheme for parallel restored subsystems in the build-up restoration for wind power penetrated transmission systems. The robust distributed optimization models are constructed with the coordinator-subsystem framework. By applying the developed IOUL algorithm, a robust distributed coordination strategy is obtained by iteratively solving small-scale MILP models. The proposed scheme maintains the information privacy of subsystems by independent decision-making, coordinates the whole system restoration by boundary information interaction and provides robustness to handle the uncertain condition. The case study results show the computation efficiency benefits and good restoration performance of the proposed method. The robust distributed method ensures boundary consistency, even in the

worst-case, and largely reduces the calculation burden of the robust restoration optimization without loss of optimality. The coordination of subsystems accelerates the restoration process and ensures complete regional grid restoration without security violation under uncertain conditions.

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