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Time domain simulation of focused waves by High-Level Irrotational Green-Naghdi equations and Harmonic Polynomial Cell method

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Abstract

Wave focusing is one of the main mechanisms that can generate freak waves in deep water. The strongly nonlinear High-Level Irrotational Green-Naghdi (HLIGN) equations and the Harmonic Polynomial Cell (HPC) method are used here to simulate the focused waves. Numerical simulations are performed in intermediate water depth as well as in deep water. In intermediate water, the HLIGN free surface and wave kinematics are then compared with the well-accepted laboratory data and the results of the HPC method. The effect of water depth on focused waves is also carried out. It is demonstrated that both the HLIGN equations and the HPC method can accurately simulate the surface elevation and particle velocity of focused waves compared with the experiments. The velocity distribution is distinct for the focused waves with the same crest elevations but in different water depths. The HLIGN equations are considered to be an accurate and efficient wave model to investigate focused waves in the intermediate and deep water cases.

Keywords: High-Level Irrotational Green-Naghdi equations, Harmonic Polynomial Cell method, Focused Wave Groups, Finite Difference Method

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1. Introduction

The mechanisms and the great damage of the extreme ocean waves have been comprehensively investigated and summarized by many researchers (see e.g., Pelinovsky et al., 2000; Kharif and Pelinovsky, 2003; Dysthe et al., 2008; Onorato et al., 2013). The wave focusing is always regarded as a significant mechanism for the formation of "short-lived" extreme waves, and this attracts many experimental and numerical investigations.

Laboratory studies were carefully performed to provide a benchmark to investigate the focused waves (Baldock et al., 1996; Baldock and Swan, 1996; Johannessen and Swan, 2001; Grue et al., 2003; Grue and Jensen, 2006; Touboul et al., 2006; Ma et al., 2010). Baldock et al. (1996) investigated the surface profile of focused wave and velocity field beneath the focused crest in an exhaustive experiment. The focused waves are generated from the linear superposition of the initial wave phase. It is widely used as the benchmark of the two dimensional focused wave in deep water (Bai and Taylor, 2007; Zhao et al., 2009; Ai et al., 2014). Further investigation on the focused wave in shallow waters was presented by Baldock and Swan (1996). Johannessen and Swan (2001) performed 3-D experiments on the directional focused wave. The stronger directional spreading makes the wave energy divergent, leading to a lower focused crest in comparison with the unidirectional focused waves.

Some efforts are also made in developing an accurate numerical model to describe the mechanism of focused waves. The widely used method known as the Boundary Element Method (BEM) can well model the focused waves (Fochesato et al., 2007; Ning et al., 2008, 2009). The wave profiles and velocity distributions are investigated with a range of steepness in Ning et al. (2009). However, the expensive CPU time cost is one of the shortages for the BEM because the full matrix has to be solved in the BEM. The Finite Element method (FEM) is also used to establish a numerical wave tank (Yan and Ma, 2009, 2010). Yan and Ma (2009) showed their FEM was much faster than the BEM used by Fochesato et al. (2007). The High Order Spectral (HOS) method is an efficient method to simulate the focused waves (Ducrozet, Bonnefoy, Touzé and Ferrant, 2012) despite the fact that the results are strongly dependent on the choice of the order of truncated nonlinearity for the HOS model (Onorato et al., 2007; Ducrozet, Bingham, Engsig-Karup, Bonnefoy and Ferrant, 2012). For the non-hydrostatic model developed by Ai et al. (2014), the efficiency remains to be improved despite the impressive agreements with some laboratory data because the 3-D Poisson equation has to be

solved in this model which is derived from the incompressible Navier-Stokes equations. The Nonlinear Schrödinger (NLS) equations give the approximate analytical results on the focused waves (Adcock and Taylor, 2009; Adcock et al., 2012; Adcock and Taylor, 2016). However, Adcock and Taylor (2016) showed that the modified NLS equation failed to accurately reproduce the maximum wave amplitude of the unidirectional focused wave groups. Some analytical investigations on the wavefronts can be used to generate larger focused waves (Chen and Li, 2019).

To investigate the mechanism of focused waves, it is important to simulate them accurately and efficiently. The Harmonic Polynomial Cell (HPC) method is already developed in marine hydrodynamics. In the HPC system, the velocity potential approximated by series of harmonic polynomials satisfies the Laplace equation automatically. And the flow field is discretized by a number of quadrilateral cells. The HPC method is originally derived by Shao and Faltinsen (2012) in two dimensions. The comparison with other potential solvers shows that the HPC method has good accuracy and efficiency. Then, Shao and Faltinsen (2014a) applied this method to investigate some simple marine hydrodynamic problems in two dimensions. Shao and Faltinsen (2014b) extended the HPC method to three dimensions and verified it with some relevant studies. Further developments on the HPC method such as the implementation of the Immersed Boundary Method (Hanssen et al., 2015), the coupling strategy with a local potential solver (Liang et al., 2015) and the improvement in mesh algorithm (Zhu et al., 2017) have been carried out in recent years.

On the other hand, the strongly nonlinear High-Level Irrotational Green-Naghdi (HLIGN) equations derived by Kim et al. (2001) are developed from the original Green-Naghdi (GN) fluid theory (Green et al., 1974; Green and Naghdi, 1976; Ertekin et al., 1986). For the irrotational flow, the converged Irrotational Green-Naghdi results are identical to the converged results obtained by the GN equations, but the HLIGN equations are more efficient compared with the original GN equations at the same level (Zhao, Ertekin and Duan, 2015). We should mention that Serre (1953) developed another branch of this fluid sheet theory named the Serre-Green-Naghdi (S-GN) equations which has equal form with Green et al. (1974). However, the S-GN equations assumed that the velocity has the depth averaged values (Viotti et al., 2014), which makes it become a strongly nonlinear but weakly dispersive wave model. The depth-dependent kinematics assumption contributes to the HLIGN equations to well simulate the strongly dispersive and broad-

band wave groups. As the development of HLIGN equations, several numerical simulations such as the solitary wave, the monochromatic wave and the irregular waves were performed (Kim and Ertekin, 2000; Kim et al., 2003; Zhao, Ertekin, Duan and Kim, 2015; Zhao, Ertekin and Duan, 2015). As the name indicates, the HLIGN equations are categorized into different levels for the representation of the horizontal and vertical variations in the wave kinematics. The higher level IGN equations are more accurate to describe the strongly nonlinear and strongly dispersive periodic waves without too much loss of efficiency (Zhao et al., 2016; Duan et al., 2017, 2018).

The first motivation of this paper is to investigate the wave kinematics for the focused waves with the same crest elevations but in different water depths. This investigation provides a new perspective on the velocity distributions in different water depths. Considering that the focused waves have not been investigated by the HLIGN equations or the HPC method before, the second motivation is to demonstrate that the HLIGN equations can accurately provide the time domain simulations of the unidirectional focused waves in the intermediate and deep water compared with the HPC method, which provides further verification for the usage of the HLIGN equations in the future.

Section 2 briefly describes the basic theory, the algorithm and the boundary conditions of the HLIGN equations in finite water depth. The brief introduction of the HPC method is followed in Section 3. The numerical cases including the focused wave in intermediate water and deep water depths as well the effect of water depth are presented in Section 4. Summaries and conclusions of this study are provided in Section 5.

2. The HLIGN Equations

The fluid is assumed to be incompressible and inviscid and the flow is irrotational. The HLIGN equations are briefly introduced in this section.

2.1. Basis of the HLIGN Equations

In two dimensions, the Hamilton's principle is utilized to derive the HLIGN equations (Kim et al., 2001, 2003, 2010). The velocity field (u, w) can be expressed by the stream function $\Psi(x, z; t)$.

$$u(x, z; t) = \frac{\partial \Psi(x, z; t)}{\partial z}, \quad w(x, z; t) = -\frac{\partial \Psi(x, z; t)}{\partial x}. \quad (1)$$

108 The stream function $\Psi(x, z; t)$ is expressed by a series of polynomials with
 109 stream function coefficients ψ_n ,

$$\psi(x, z; t) = \sum_{n=1}^K \psi_n(x; t) \cdot f_n(\gamma); \quad K = 1, 2, 3, \dots, \quad (2)$$

110 where $f_n(\gamma) = \gamma^{2n-1}$, $\gamma = [z + d(x)]/[\eta(x; t) + d(x)]$ and $d(x)$ represents the
 111 water depth. The free surface elevation is specified by $z = \eta(x; t)$. This
 112 approximation on wave kinematics could be raised to a higher level by in-
 113 creasing the order of polynomials K . The corresponding HLIGN equations
 114 are written as IGN- K , such as IGN-5. The higher level IGN equations are the
 115 strongly nonlinear and strongly dispersive wave model (Duan et al., 2017).

116 Based on the kinematic and dynamic free-surface conditions, the HLIGN
 117 equations (Ertekin et al., 2014; Zhao, Ertekin and Duan, 2015) are given by
 118 the two equations below.

$$\eta_{,t} + \hat{\Psi}_{,x} = 0, \quad (z = \eta) \quad (3a)$$

$$\hat{\Phi}_{,t} + \frac{\partial}{\partial x} \frac{\partial E_k}{\partial \eta_{,x}} - \frac{\partial E_k}{\partial \eta} + g\eta = 0, \quad (z = \eta), \quad (3b)$$

119 where $\hat{\Phi}(x, \eta; t)$ and $\hat{\Psi}(x, \eta; t)$ represent the velocity potential and the stream
 120 function on the surface, respectively. $\eta_{,t}$ and $\eta_{,x}$ are the time and the spatial
 121 derivatives of the free surface.

122 Specifically, E_k is the kinetic energy given by

$$\begin{aligned} E_k = & \frac{1}{2} \sum_{m=1}^K \sum_{n=1}^K [(\eta + d)A_{mn}\psi_{m,x}\psi_{n,x} - 2\{(\eta + d)_{,x}B_{mn}^1 - d_{,x}B_{mn}\}\psi_{m,x}\psi_n + \\ & \frac{1}{\theta}\{(1 + d_{,x}^2)C_{mn} - 2(\eta + d)_{,x}d_{,x}C_{mn}^1 + (\eta_{,x} + d_{,x})^2C_{mn}^2\}\psi_m\psi_n], \end{aligned} \quad (4)$$

where $\theta = \eta + d$ and

$$A_{mn} = \int_0^1 f_m(\gamma) f_n(\gamma) d\gamma, \quad (5a)$$

$$B_{mn} = \int_0^1 f_m(\gamma) f'_n(\gamma) d\gamma, \quad B_{mn}^1 = \int_0^1 \gamma f_m(\gamma) f'_n(\gamma) d\gamma, \quad (5b)$$

$$C_{mn} = \int_0^1 f'_m(\gamma) f'_n(\gamma) d\gamma, \quad C_{mn}^1 = \int_0^1 \gamma f'_m(\gamma) f'_n(\gamma) d\gamma, \quad (5c)$$

$$C_{mn}^2 = \int_0^1 \gamma^2 f'_m(\gamma) f'_n(\gamma) d\gamma. \quad (5d)$$

123 The water depth is assumed constant and finite in this work, i.e., $d(x) = d$.
 124 Then, the bottom condition can be expressed as $\psi(x, -d; t) = 0$.

125 To complete the HLIGN equations, an additional equation is implemented
 126 (Kim et al., 2003):

$$f_n(1) \cdot \hat{\Phi}_{,x} = -\frac{\partial}{\partial x} \frac{\partial E_k}{\partial \psi_{n,x}} + \frac{\partial E_k}{\partial \psi_n} \quad (n = 1, 2, \dots, K). \quad (6)$$

127 Therefore, the relation between the surface potential and the stream function
 128 is finally obtained. The spatial derivative of Eq. (3b) gives one expression
 129 of $\hat{\Phi}_{,t;x}$. On the other hand, the temporal derivative of Eq. (6) leads to the
 130 other expression of $\hat{\Phi}_{,x;t}$. By matching these two equations, each levels of
 131 the stream function coefficients can be solved. We can then obtain the free
 132 surface and velocity field.

133 2.2. Solution Algorithm

134 As introduced in Zhao, Ertekin and Duan (2015), the Eqs. (3) and (6)
 135 of the HLIGN equations can be discretized by the spatial finite-difference
 136 method. The seven point central-difference method is utilized here to ob-
 137 tain sixth-order accuracy $O(\Delta x^6)$ for the first and second derivatives and
 138 fourth-order accuracy $O(\Delta x^4)$ for third derivatives. The fourth-order Adams
 139 predictor-corrector scheme is used to march in time. The HLIGN equations
 140 can, as a result, be expressed by

$$[A]\dot{\xi}_{,xx} + [B]\dot{\xi}_{,x} + [C]\dot{\xi} = f, \quad (7)$$

141 where $\xi = [\psi_1, \psi_2, \dots, \psi_K]^T$. The dot over ξ indicates the time derivative,
 142 and the subscript comma indicates the differentiation with respect to the

143 indicated variable. $[\mathbf{A}]$, $[\mathbf{B}]$, $[\mathbf{C}]$ are the matrices with dimensions $K \times K$,
 144 and $\dot{\boldsymbol{\xi}}$, $\mathbf{f}$ are the K -dimensional vectors. The time derivatives of the stream
 145 function coefficients represented by $\dot{\boldsymbol{\xi}}$ in Eq. (7) are finally solved (Zhao
 146 et al., 2014). Then, the time derivative of the free surface is obtained by Eq.
 147 (3a).

148 2.3. Focusing Wave-maker Boundary Conditions

149 The inlet boundary conditions are utilized to generate focused waves. The
 150 surface elevation η_i and the stream function Ψ_i of each wave components are
 151 obtained by the Airy wave theory expressed by Eq. (8).

$$\eta_i = a_i \cos[\omega_i(t - t_f) - k_i(x - x_f)], \quad (8a)$$

$$\Psi_i = \frac{ga_i \sinh k_i(z + d)}{\omega_i \cosh k_i d} \cos[\omega_i(t - t_f) - k_i(x - x_f)]; \quad (8b)$$

$$i = 1, 2, \dots, N_w.$$

152 N_w is the number of wave components, a_i , k_i and ω_i are the wave amplitude,
 153 wave number and wave frequency of each component, respectively. And the
 154 acceleration of gravity are represented by g . The initial phase is adjusted
 155 by the theoretical focal time and focal position, t_f and x_f . Consequently,
 156 the surface elevation $\eta(x; t)$, the stream function $\Psi(x, z; t)$ and their time
 157 derivatives are determined through the linear superposition. The stream
 158 function coefficients $\psi_n(x; t)$ are then obtained by the least square matching
 159 method.

160 2.4. Wave-absorber Boundary Conditions

161 We refer the reader to Zhao et al. (2014) to obtain an exhaustive ex-
 162 pression of wave-absorber boundary condition. This condition applies to the
 163 region near the outlet boundary to prevent significant reflections back into
 164 the numerical wave tank. As shown in Eq. (9), the surface elevation and
 165 the stream function coefficients are refreshed by multiplying them with a
 166 damping function varying from 1 to 0.

$$\eta' = \eta \cdot c_{damp}, \quad (9a)$$

$$\psi'_n = \psi_n \cdot c_{damp} \quad (n = 1, 2, \dots, K). \quad (9b)$$

167 The damping function c_{damp} is written as

$$c_{damp} = 126(x')^5 - 420(x')^6 + 540(x')^7 - 315(x')^8 + 70(x')^9$$

$$x' = \frac{L - x}{L_2}, (0 \leq x' \leq 1); \quad (10)$$

168 where L_2 and L represent the length of wave-absorbing region and the length
 169 of wave flume, respectively.

170 3. The HPC method

171 3.1. Basis of the HPC method

172 The two-dimensional flow field is assumed to be incompressible, inviscid
 173 and irrotational. If the flow field is discretized by quadrilateral cells, the
 174 local Cartesian coordinates could be established at the central 9th node with
 175 8 boundary nodes surrounding it, see Fig. 1.

Figure 1: The local index for a cell in HPC method

176 Then the velocity potential at any location within the cell can be approxi-
 177 mated by superposition of 8 harmonic polynomials as follows:

$$\Phi(\tilde{x}, \tilde{z}) = \sum_{j=1}^8 b_j f_j(\tilde{x}, \tilde{z}), \quad (11)$$

178 where $\tilde{x}$ and $\tilde{z}$ are the local coordinates relative to the stencil centre. The b_j is
 179 the unknown coefficients and the eight harmonic polynomials $f_j(\tilde{x}, \tilde{z})$ are de-
 180 fined as $f_1 = 1$, $f_2 = \tilde{x}$, $f_3 = \tilde{z}$, $f_4 = \tilde{x}^2 - \tilde{z}^2$, $f_5 = \tilde{x}\tilde{z}$, $f_6 = \tilde{x}^3 - 3\tilde{x}\tilde{z}^2$, $f_7 =$
 181 $3\tilde{x}^2\tilde{z} - \tilde{z}^3$, $f_8 = \tilde{x}^4 - 6\tilde{x}^2\tilde{z}^2 + \tilde{z}^5$ (Shao and Faltinsen, 2014b). The har-
 182 monic polynomials automatically satisfy the governing equation, the Laplace
 183 equation, everywhere. The higher-order harmonic polynomials contribute
 184 to reducing the free surface wave dispersion errors in time domain simu-
 185 lations (Shao and Faltinsen, 2012). The b_j can be determined through a
 186 boundary-value problem. Then the velocity potential is expressed by the
 187 eight boundary velocity potentials $\Phi_i(\tilde{x}, \tilde{z})$, ($i = 1, 2, \dots, 8$):

$$\Phi(\tilde{x}, \tilde{z}) = \sum_{i=1}^8 \left[\sum_{j=1}^8 c_{j,i} f_j(\tilde{x}, \tilde{z}) \right] \Phi_i, \quad (12)$$

188 where the $c_{i,j}$ is the elements of the inverse of the harmonic polynomials
 189 matrix. The f_j ($j = 2, 3, \dots, 8$) equals to zero considering the coordinates
 190 of the central node is (0,0). Therefore, the velocity potential on the 9th
 191 central node is specified as the linear combination of the surrounding velocity
 192 potentials:

$$\Phi_9 = \sum_{i=1}^8 c_{1,i} \Phi_i(\tilde{x}, \tilde{z}). \quad (13)$$

193 In each cell, this expression is enforced on each central node which will then
 194 be used as the boundary node of a neighboring cell.

195 3.2. Algorithm and Boundary conditions

196 As mentioned above, the computational domain is discretized by quadri-
 197 lateral cells. The spatial derivative of the free surface is calculated by cubic-
 198 order polynomial fitting technique. The fourth-order Runge-Kutta scheme is
 199 utilized to perform the time-domain analysis.

200 At the boundary nodes, the Dirichlet boundaries are known since the ve-
 201 locity potential is specified. The Neumann boundary conditions are obtained
 202 by differentiating Eq. (12) in the normal direction:

$$\frac{\partial \Phi}{\partial n}(\tilde{x}, \tilde{z}) = \sum_{i=1}^8 \left[\sum_{j=1}^8 c_{j,i} \nabla f_j(\tilde{x}, \tilde{z}) \cdot \mathbf{n} \right] \Phi_i, \quad (14)$$

203 where $\mathbf{n}$ is the normal vector on the boundary nodes. Applying the continuity
 204 constraints given in Eq. (13) and the boundary conditions, a global matrix

linear algebraic equation with a sparse matrix can be obtained. The linear algebraic equation is then solved by the Generalized Minimal Residual solver.

The Mixed Eulerian-Lagrangian method is adopted to make the free-surface elevation satisfy the fully nonlinear free-surface condition:

$$\frac{\delta\eta}{\delta t} = \frac{\partial\Phi}{\partial z} - \frac{\partial\Phi}{\partial x} \frac{\partial\eta}{\partial x}, \quad (z = \eta) \quad (15a)$$

$$\frac{\delta\Phi}{\delta t} = -g\eta - \frac{1}{2}\nabla\Phi \cdot \nabla\Phi + \frac{\delta\eta}{\delta t} \frac{\partial\Phi}{\partial z}, \quad (z = \eta), \quad (15b)$$

where $\frac{\delta}{\delta t}$ is the differentiation in time for variables (wave elevation and velocity potential) on instantaneous free surface with a fixed horizontal position. The impenetrability condition is adopted on the sea bottom. Similar to the wave-maker implemented in the HLIGN equations, the superposed velocity-inflow boundary is utilized to generate focused waves in the HPC method. A numerical damping zone is also applied as a numerical beach. We refer the reader to Shao and Faltinsen (2012, 2014a,b) to obtain more details of the HPC method.

4. Numerical Tests

Baldock et al. (1996) performed an experimental study on the unidirectional focused wave in intermediate water depth of $d = 0.7$ m. We reproduced these focused waves by using the HLIGN equations and the HPC method. The focused waves are also simulated when $d = 1.4$ m and 2.1 m, to investigate the influence of water depth. All of these simulations are performed in two dimensions.

4.1. Comparison in intermediate water depth $d = 0.7$ m; (Baldock et al., 1996)

Baldock et al. (1996) conducted an experimental study of focused waves in a wave flume. Their wave flume is 20 m long, 0.3 m wide and 0.7 m deep. A bottom-hinged paddle is utilized to generate waves in this physical flume. Their laboratory data are widely used to verify the ability of numerical models (Bai and Taylor, 2007; Zhao et al., 2009; Ai et al., 2014).

In the numerical wave flume, the length and depth are the same as in their tank. The third dimension in width is not necessary because the numerical study is only performed in two dimensions. Despite four period ranges are considered, only two series of laboratory data are presented by Baldock et al.

(1996) corresponding to the broader band ("Case B") and the narrower band ("Case D"). In our numerical study, these two typical period ranges are selected. The characteristics of focused wave groups are shown in Table 1. T_i is the wave period of each wave component. Both of the wave groups are generated from 29 individual wave components ($N_w = 29$), each component has the same amplitude. The input amplitude $A = 29a_i$ is set to 22 mm, 38 mm and 55 mm. The water depth times the wave number, kd , indicates the dispersivity of each wave component that depends upon the water depth.

Table 1: Input characteristics

Case name	period range (s)	kd
Case B	$0.6 \leq T_i \leq 1.4$	$1.57 \leq kd \leq 7.83$
Case D	$0.8 \leq T_i \leq 1.2$	$2.03 \leq kd \leq 4.40$

In the numerical simulations, the theoretical focal position and focal time are specified as $x_f = 8.0$ m and $t_f = 20$ s, respectively. The convergence study, including the grid size, the time step and the levels, is performed in each simulation. The spatial and temporal resolutions ($\Delta x, \Delta t$) are set as $\Delta x / \lambda_{min} = 0.03$, $(\Delta t \cdot c_{max}) / \Delta x = 0.25$, where λ_{min} and c_{max} are the wavelength of the shortest component and the phase speed of the longest component in each wave group, respectively. These grid size and time step are converged enough to be utilized in both the HLIGN and the HPC simulations. Self-convergency study should also be performed in each HLIGN simulation (Zhao, Ertekin and Duan, 2015; Duan et al., 2017). In the HLIGN simulations, the IGN-5 equations are regarded as the converged one.

The time history of the free-surface elevation at a probed point of the focal position is displayed here. Both the HLIGN and the HPC numerical surface elevations are presented in Fig. 2 and compared with the experimental data obtained from Baldock et al. (1996). It should be noted that the coordinate is established at the focal time for the numerical results as well as the laboratory data. The 2nd-order solutions are insufficient to describe the strongly nonlinear wave-wave interactions (Baldock et al., 1996), which can easily be captured by the HLIGN equations and the HPC method. Good agreement is obtained between these two methods for different bandwidth and various input amplitudes. It is shown that the HLIGN equations are as accurate as the fully nonlinear HPC method, considering the HPC is a highly accurate field solver (Shao and Faltinsen, 2012). In addition, good

264 agreement is also observed between the numerical results and the labora-
265 tory data. This agreement could be quite obvious if the input amplitude
266 equals to 22 mm and 38 mm. However, the HLIGN and the HPC results
267 are slightly over predicted in comparison with the laboratory data for $A =$
268 55 mm. This deviation is possibly coming from different wave generation
269 mechanisms between the numerical inflow boundary and the displacement
270 of the wave-maker. In addition, the wall dissipation and the errors due to
271 measurement in the physical experiments may also result in the discrepancy
272 between the numerical results and the laboratory data.

273 For the wave profile, the surface elevations are symmetric according to the
274 origin of coordinates which represents the focal time. The profiles of broad-
275 band wave groups have flatter wave trough and broader distances between
276 the focused crest and the adjacent crests in comparison with the narrow-band
277 wave groups. On the other hand, the narrow-band wave groups have higher
278 crest elevation for either the focused crest or the adjacent crests.

(a) Case B, $A = 22$ mm

(b) Case B, $A = 38$ mm

(c) Case B, $A = 55$ mm

(d) Case D, $A = 22$ mm

(e) Case D, $A = 38$ mm

(f) Case D, $A = 55$ mm

Figure 2: Time history at the focal position

279 The relationship between the input amplitude and the maximum crest
 280 elevation is presented in Fig. 3. The input signal from 5 mm to 60 mm

281 are considered in the HLIGN simulations. The maximum crest elevation of
282 HLIGN agrees with the HPC result within the same input amplitude. And
283 both of the numerical results agree well with the laboratory data. To be
284 consistent with Baldock et al. (1996), the linear solutions shown here are
285 the summation of the amplitudes of all wave components. The maximum
286 crest elevations, no matter obtained from the numerical results or from the
287 laboratory data, actually deviate from the linear solutions with the increasing
288 of the input amplitudes. Moreover, the deviation is more obvious for the
289 narrow-band focused waves (Case D) than the broadband wave groups (Case
290 B).

(a) Case B

(b) Case D

Figure 3: Relationship between the input amplitude and the maximum crest elevation

291 The relationship between the input amplitudes and the exact focal posi-
 292 tion, measured from (x_f, t_f) , is presented in Fig. 4. The exact positions of the

293 focal point obtained from the HLIGN and the HPC simulations are shown in
294 Fig. 4(a) and good agreements are achieved. The numerical results can well
295 predict the downstream shift of the exact focal location which has also been
296 observed by Baldock et al. (1996). Figure. 4(b) displayed the relationship
297 between the input amplitudes and the exact focal time. It is believed that the
298 scatter appeared in experimental data of focal time results from the internal
299 clock of the PC used for control of the wave paddle (Baldock et al., 1996).
300 Figure. 4 shows that the downstream shift of the exact focal position and
301 focal time are quickly increasing with larger input amplitude and decreasing
302 with broader bandwidth. Yet again, the close agreement of the results
303 of the HLIGN and HPC models with the laboratory measurements reveals
304 their applicability in simulating accurately the generation and propagation
305 of nonlinear wave groups.

(a) Position of the focal point

(b) Time of focusing (at focal position)

Figure 4: Relationship among the input amplitude, the focal position and the focal time

306 The wave kinematics information is crucial to calculate the wave loads.
 307 Baldock et al. (1996) also provided the measurement of the particle velocity of

308 the focused wave groups in the physical tank. Therefore, we can compare our
 309 numerical predictions on wave kinematics with their data. As we mentioned
 310 previously, the numerical results appear to over-predict the maximum crest
 311 elevation because of the difference in wave generation methods between the
 312 experimental and numerical investigations. However, the wave kinematics are
 313 very sensitive to the surface elevation. To compare the numerical horizontal
 314 velocity with the measurement of Baldock et al. (1996), we make the HLIGN
 315 and HPC focused crests (η_{c1}) match with the experimental crest elevations
 316 (η_{c2}) by optimizing the input amplitude (shown in Table A-1). The vertical
 317 distribution of horizontal velocity beneath the focused crest is shown in Fig.
 318 5. It is demonstrated that the agreement of velocity profiles between the
 319 numerical results and the experimental data is reasonable, considering there
 320 is only slightly over-prediction close to the free surface. And the numerical
 321 velocities of the HLIGN equations and the HPC method shown in Fig. 5
 322 agree quite well.

(a) Case B, $\eta_{c2} = 22$ mm

(b) Case B, $\eta_{c2} = 42$ mm

(c) Case B, $\eta_{c2} = 65$ mm

(d) Case D, $\eta_{c2} = 22$ mm

(e) Case D, $\eta_{c2} = 47$ mm

(f) Case D, $\eta_{c2} = 72$ mm

Figure 5: Horizontal velocity beneath the focused crest

323 As shown in Table 1, the longest wave component is in intermediate water
 324 depth. This is why the horizontal velocity beneath the focused crest has not

325 decreased to zero on the bottom in Figs. 5. Further investigation of the
 326 focused wave in deep water is presented in the next section.

327 4.2. Comparison in different water depths, $d = 0.35, 0.7, 1.4$ and 2.1 m

328 The dispersion relationship of each wave component depends upon the
 329 water depth. It is well known that a wave component with $kd \geq 3.14$ can
 330 be regarded as a deep water wave (water depth is larger than half of the
 331 wavelength). The range of kd calculated for different water depths, $d = 0.35,$
 332 $0.7, 1.4$ and 2.1 m, is shown in Table. 2. As shown in Table. 2, the majority
 333 components of focused waves are propagated in deep water for $d = 2.1$ m.

Table 2: The range of kd

Case name	period range (s)	d (m)	kd
Case B_0	$0.6 \leq T_i \leq 1.4$	0.35	$0.96 \leq kd \leq 3.92$
Case B_1		0.7	$1.57 \leq kd \leq 7.83$
Case B_2		1.4	$2.89 \leq kd \leq 15.65$
Case B_3		2.1	$4.31 \leq kd \leq 23.48$
Case D_0	$0.8 \leq T_i \leq 1.2$	0.35	$1.18 \leq kd \leq 2.25$
Case D_1		0.7	$2.03 \leq kd \leq 4.40$
Case D_2		1.4	$3.92 \leq kd \leq 8.80$
Case D_3		2.1	$5.87 \leq kd \leq 13.21$

334 As mentioned in Section 2.1, the level of the HLIGN equations play a
 335 important role on the nonlinearity and dispersivity of this wave model. The
 336 level of the HLIGN equations has to be raised because the lower order poly-
 337 nomials is insufficient to represent the vertical distribution of velocity field
 338 due to the increase in the water depth. The convergency study, especially
 339 the convergent level of the HLIGN equations, is carefully tested in each sim-
 340 ulation. It is shown that the IGN-9 equations are well enough to simulate
 341 both the surface elevation and the wave kinematics of the deep water focused
 342 waves. So the IGN-9 equations are used in all the simulations of deep water
 343 focused waves. All the simulations in the previous section demonstrate that
 344 the HLIGN calculations are identical to the surface elevations and velocity
 345 distributions obtained by the HPC method. As a result, only the HLIGN
 346 equations are utilized here.

347 The simulations are performed to investigate the effect of water depth on
 348 focused wave. As we know, the wave profile, phase speed and wave kinematics

are deeply affected by the water depth even if the same input conditions are applied. To focus our attention on the effect of water depth, the numerical focused crest elevation η_{c1} is precisely specified as 22 mm, 55 mm by carefully optimizing the input amplitudes (shown in Table A-2). The effects of water depth on the surface elevation are not obvious. As an example, the time history at the focal position for is shown in Figs. 6 and 7. Despite we specified the η_{c1} , the profile of the focused wave is not constrained. It is shown that the focused wave profile are the same for different water depth although the nonlinearity is strong. Only a small difference is observed on the adjacent peak and trough among different d in Fig. 7(b).

(a) $\eta_{c1} = 22$ mm

(b) $\eta_{c1} = 55$ mm

Figure 6: Time history at the focal position, Case B with different water depth (B_0, B_1, B_2, B_3)

(a) $\eta_{c1} = 22$ mm

(b) $\eta_{c1} = 55$ mm

Figure 7: Time history at the focal position, Case D with different water depth (D_0 , D_1 , D_2 , D_3)

360 and 9. In order to show the difference more clearly, the detailed view of the
 361 velocity closed to the focused crest is displayed as a sub-figure. It is shown
 362 that the water depth affects the wave kinematics significantly, even through
 363 the focused crest elevation η_{c1} are specified for different water depths. It is
 364 quite clear that the horizontal velocity does not decrease to zero for either
 365 broad-band or narrow-band wave groups if d equals to 0.35 m and 0.7 m. The
 366 horizontal velocity on the bottom abruptly stops at certain values because
 367 the particle motion near the bottom can not be neglected for the long wave
 368 component with a wavelength larger than $2d$. For both $d = 1.4$ m and 2.1 m,
 369 the horizontal particle velocity quickly decrease almost exponentially when it
 370 approaches the sea bottom. These focused waves for $d = 1.4$ m and 2.1 m can
 371 be regarded as in deep water. This coincides with the previous analysis of the
 372 data in Table 2. The reduction of the water depth increases the horizontal
 373 velocity at the focused crest, seen in Figs. 8 and 9. The velocity for $d = 0.35$
 374 m is significantly larger than that for the other water depth.

(a) $\eta_{c1} = 22$ mm

(b) $\eta_{c1} = 55$ mm

Figure 8: Horizontal velocity beneath the focused crest, Case B with different water depth (B_0, B_1, B_2, B_3)

(a) $\eta_{c1} = 22$ mm

(b) $\eta_{c1} = 55$ mm

Figure 9: Horizontal velocity beneath the focused crest, Case D with different water depth (D_0 , D_1 , D_2 , D_3)

375 5. Summary and Conclusions

376 In this paper, the High-Level Irrotational Green-Naghdi equations and
377 the Harmonic Polynomial Cell method are used to simulate the intermediate
378 and deep water focused wave.

379 Some effects on the focused wave are analyzed by specifying the same
380 crest elevations in four different water depth, see Figs. 6-9. It provides a
381 new perspective on the investigation of the focused wave kinematics. For
382 the focused waves with the same crest elevations in different water depths,
383 the velocity distribution is significantly distinct but the wave profiles are the
384 same. The reduction of the water depth increases the horizontal velocity at
385 the focused crest.

386 The HLIGN results are verified by the experimental data (Baldock et al.,
387 1996) and by the fully nonlinear HPC method, see Figs. 2-5. Considering
388 the HPC is a highly accurate field solver (Shao and Faltinsen, 2012), the
389 good agreements show that the HLIGN equations are as accurate as the
390 fully nonlinear HPC method. Both of these models can simulate the cases
391 in a couple of minutes. We also point out that the HPC method is not
392 only designed for solving the nonlinear wave propagation problems, but also
393 applied to other marine hydrodynamics problems such as the sloshing and
394 diffraction problems and other potential flow characteristics at sharp corners.
395 Wave breaking is not considered in this work, but could be studied in the
396 future by following Baquet et al. (2017).

397 The experimental data of Baldock et al. (1996) is widely used as the
398 benchmark of unidirectional focused wave in deep water (Baldock and Swan,
399 1996; Grue et al., 2003; Bai and Taylor, 2007; Zhao et al., 2009; Ai et al., 2014;
400 Sriram et al., 2015). However, the water depth in Baldock et al. (1996) is not
401 deep enough since the horizontal velocity on the bottom has not decreased
402 to zero. We provide the wave profile and the horizontal velocity distributions
403 in deep water in this work, see Figs. 6 to 9.

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567 Appendix A

Table A-1: Input amplitudes for the simulations in Fig. 5. ($d = 0.7$ m)

Case name	A (mm)	η_{c1} (mm)	η_{c2} (mm)
	Input amplitudes	Numerical focused crest	Experimental focused crest
Case B	21	22	22
	36	42	42
	52	65	65
Case D	21	22	22
	37	47	47
	52	72	72

Table A-2: Input amplitudes for the simulations in Figs. 6, 7, 8 and 9.

Case name	d (m)	A (mm)	η_{c1} (mm)
	Water depth	Input amplitudes	Numerical focused crest
Case B_0 -22	0.35	21	22
Case B_1 -22	0.7	21	22
Case B_2 -22	1.4	21	22
Case B_3 -22	2.1	21	22
Case B_0 -55	0.35	47	55
Case B_1 -55	0.7	46	55
Case B_2 -55	1.4	46	55
Case B_3 -55	2.1	46	55
Case D_0 -22	0.35	21	22
Case D_1 -22	0.7	21	22
Case D_2 -22	1.4	21	22
Case D_3 -22	2.1	21	22
Case D_0 -55	0.35	46	55
Case D_1 -55	0.7	44	55
Case D_2 -55	1.4	43	55
Case D_3 -55	2.1	42	55