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Dahl, Nicolai Jerram; Iversen, Niels Elkjær; Lazarevic, Vlada; Andersen, Michael A. E.; Vasic, Miroslav

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Tracking Multi-Level Power Supplies for Class-D Audio Amplifiers

Nicolai J. Dahl\textsuperscript{1} Student Member, IEEE, Niels E. Iversen\textsuperscript{2} Member, IEEE, Vladan Lazarevic\textsuperscript{3} Student Member, IEEE, Michael A. E. Andersen\textsuperscript{1} Member, IEEE and Miroslav Vasic\textsuperscript{3} Senior Member, IEEE

Abstract—Switch-mode technology is the common choice for high efficiency audio power amplifiers. The dynamic nature of real audio reduces efficiency as less continuous output power can be achieved. Based on methods used for RF amplifiers, this paper proposes a Tracking Multi-Level Power Supply (TMLPS) in order to improve system efficiency. A simple prototype system is designed and tested up to 200W. Measured results show that employing a TMLPS can improve efficiency with a factor of 6 when playing music. In addition, the temperature rise of the amplifier is vastly reduced, especially for the switching power MOSFETs where it is halved from 100°C to 50°C. To further improve efficiency, an optimization method is proposed. The optimization considers the amplitude distribution of music signals and consumer behaviors. Simulations show that efficiency can be further improved by a factor of 1.23 with an optimized TMLPS compared to a simple uniform TMLPS solution.

Index Terms—Class-D, Tracking power supply, Optimization, Audio systems, Multilevel systems

I. INTRODUCTION

Switch-mode power audio amplifiers, also known as class-D amplifiers, have become the conventional choice for audio amplification as they have excellent audio performance with very low distortion [1–3], and superior efficiency compared to linear amplifiers such as class-A and class-AB [4]. However, the measurement technique for measuring efficiency usually utilizes sine waves. Sine waves are fundamentally different from dynamic music signals, and therefore, do not represent real audio signals very well [5, 6]. Real audio signals are much more dynamic and have a high peak-to-rms ratio, also known as the Crest Factor (CF). The highly dynamic nature of music signals causes a degradation of efficiency as less continuous output power can be achieved. The dominant losses in the amplifier at low output power are switching losses which directly relate to the supply voltage level. Low efficiency is a challenge as it indicates an excessive loss within the amplifier that can cause thermal stress on power stage components, increase the size of required heat sinks, and act as a limiting factor for playback time in battery-driven systems.

To overcome this challenge, one can use different types of dynamic voltage supplies for the amplifier. Previous studies [7–10] present multi level amplifier solutions where the dynamic voltage supply is a integrated part of the amplifier power stage improving low power efficiency. These solutions were implemented for low- and mid-power Integrated Circuits (IC) delivering up to 70W. Another study [11] proposes a different approach where dedicated power supply constantly adjust the supply voltage of a conventional Buck type Class-D amplifier also beneficial for the low power efficiency. This paper will use a Tracking Multi-Level Power Supply (TMLPS) similar to that but with discrete voltage levels as seen in [12–16] Radio Frequency (RF) amplifiers. A TMLPS is capable of toggling between multiple fixed voltage levels as depicted in Fig. 1 which shows the behaviour of the switching when used with audio. This work aims to apply TMLPS techniques for class-D audio amplifier applications to optimize efficiency and operating temperature when playing dynamic music signals. This technique is not to be confused with conventionally class-H and class-G topologies as those are characterized by having a linear power amplifier stage, as opposed to the switch-mode of class-D [17].

This work extends the previous published work from [18] and is split into three main sections, section II, III and IV. Section II and III enhance the original work performed in [18] by 1) performing an analysis of supply dependent loss mechanisms present in the class-D power stage and 2) propose an optimization method to determine the placements of a predetermined number of voltage levels of the TMLPS where both the distribution of music and consumer listening behavior

Fig. 1: Simple TMLPS supply voltage, $V_S$, and amplifier output in case dynamic music signal.
is taken into account. Finally section IV present the original implementation and measurements from [18] of a 50V class-D amplifier being supplied by a uniform TMLPS with equally spaced fixed voltage levels. Results are compared to the theoretical expectations and discussed.

II. SUPPLY VOLTAGE DEPENDANT LOSSES

Fig. 2 shows a full-bridge synchronous rectifier power stage of the Buck topology. This is a typical topology used for class-D amplifiers.

The power losses related to this topology can be modeled as the sum of multiple loss mechanism inherently present in the amplifier, i.e. conduction losses, reverse conduction losses, and switching losses [19]. To estimate the impact of applying a TMLPS to the class-D amplifier, it is relevant to find how these losses relate to the supply voltage. We will consider a Self-Oscillating (SO) class-D amplifier where the idle switching frequency is independent of the supply voltage. This can be achieved using modulator designs like the Astable Integrating Modulator (AIM) where the modulator can be self-contained [20]. The independence of the idle switching frequency is essential when used with a tracking power supply as it avoids the switching frequency dropping down to unacceptable levels. A property for many self-oscillating modulators is their ability to generate a constant ripple current for a constant supply voltage by changing the switching frequency with the audio signal. Because the ripple current is constant, its size can easily be determined by considering the ripple when the amplifier is in idle. Equation (1) shows the peak-to-peak ripple current, $I_D$. The 0.5 represent the duty cycle, indicating an idle state.

$$I_D = \frac{V_s^2 - V_{spk}^2}{4LV_{swIdle}V_s} \bigg|_{V_{spk}=0} = \frac{0.5V_s}{2LV_{swIdle}}$$

(1)

Since the idle switching frequency, $f_{swIdle}$, is constant for all supply voltages, the ripple current will change with the supply voltage to uphold (1). Thus the modulation will follow the supply voltage such that the switching frequency will reach zero whenever $V_s = V_{spk}$. Equation (2) shows the described behavior of the switching frequency. The equation is a rewriting of (1) where the function for the ripple current evaluated at idle is inserted to obtain a simpler expression.

$$f_{sw} = \frac{V_s^2 - V_{spk}^2}{4LV_sI_D} = f_{swIdle} \frac{V_s^2 - V_{spk}^2}{V_s^2}$$

(2)

Looking at (2) it is seen that the switching frequency is dependent on the supply voltage when $V_{spk}$ has a non-zero voltage. However, when $V_{spk}$ is zero, the supply voltage cancels out, and the switching frequency becomes the idle frequency, $f_{swIdle}$.

$$f_{swIdle} = \frac{0.5V_s}{2LI_s}$$

(3)

With the ripple current and switching frequency determined, the losses of the class-D amplifier can be modeled. The switching losses occur whenever the MOSFETs are turned on or off, and can be divided into two separate losses. The first type of loss are switching losses due to the instantaneous presence of voltage and current in the MOSFET channel that happen during its rise and fall time where the drain current and drain-source voltage are ramping up and down. The second loss is due to the sudden charge/discharge of the parasitic output capacitance through the MOSFETs when turning on, and is the dominating loss at low power. The combined switching losses for a half-bridge are shown in (4).

$$P_{sw} = f_{sw} \left( \frac{1}{2} V_{ds} I_d (t_r + t_f) + C_{oss,q} V_{ds}^2 \right)$$

(4)

Where $C_{oss,q}$ and $V_{ds}$ are the charge equivalent parasitic capacitance and voltage between the drain and source of the MOSFET device, $I_d$ is the drain current at the switching time, and $t_r$ and $t_f$ are the rise- and fall time of the MOSFET respectively. $C_{oss,q}$ and the rise- and fall time are all non-linear and related to the value of $V_{ds}$ and temperature, making it difficult to model exactly. $V_{ds}$ is heavily dependent on the ratio between the ripple current and continuous current, $I_{out}$, and the dead-time. When properly designed, the ripple current can charge/discharge $C_{oss,q}$ during the dead-time period resulting in reduced voltage switching or zero voltage switching. This has been covered in depth in [21, 22]. Thus $P_{sw}$ is dependent on the ripple current, the switching frequency, and the supply voltage. If the dead-time period is too long reverse conduction losses in the MOSFET body diode will occur. Equation (5) describes the diode conduction losses. This loss only happens whenever the ripple current is at its peaks, and thus has a slightly different value for the high and low side MOSFET. The conduction time, $t_{cond}$ is the time remaining of the dead-time period after $C_{oss,q}$ has been fully charged/discharged. This remaining part of the dead-time is usually very short. Hence the diode losses will be small as well. In addition to the reverse conduction losses, the body diode generates a reverse recovery loss. The size of reverse recovery loss depends on the reverse recovery charge, reverse recovery time, the magnitude, and $dV/dt$ of the current flowing in the body diode, the dead time, and gate bias. Thus the loss is difficult to model precisely. However, this loss can be significant at high output powers [23, 24].

![Fig. 2: Full bridge buck topology. $V_s$ is supply rail voltage.](image)
\[ P_{\text{diode}} = V_d t_{\text{cond}} \left( I_{\text{out}} \pm \frac{I_\Delta}{2} \right) f_{sw} \]

The MOSFETs when on, and filter inductors both have a small resistance which leads to conduction losses. The inductor further has AC losses due to the skin effect and core losses due to eddy currents in the core material. Equation (6) shows the conduction loss for the MOSFET, and (7) for the inductor.

\[ P_{\text{con}} = I_{\text{rms}}^2 R_{\text{on}} \quad (6) \]

\[ P_{\text{wire}} = I_{\text{rms}}^2 R_{\text{dc}} + I_{\text{ac},\text{rms}}^2 R_{\text{ac}} \quad (7) \]

Both conduction losses use the RMS current, hence having a dependency on the ripple current. The core loss can be approximated by (8) and (9) under the assumption that the maximum current of the amplifier produces an magnetic field that is within the range where it remains approximately linear with the flux density. The losses in the core increases with frequency. Thus the audio signal, due to its low frequency, is disregarded. \( B_{\text{max}} \) is the flux density when the maximum ripple current \( I_{\Delta,\text{max}}/2 \) is passing through the inductor, and \( B_{\Delta} \) is the resulting peak-to-peak flux density for \( I_{\Delta} \). Finally, the core loss, \( P_{\text{core}} \) is determined as the relative core loss, \( P_{v} \) for the found \( B_{\Delta} \) multiplied by the volume of the core, \( v_{\text{core}} \). The relative core loss is usually provided by the datasheet for the selected core material.

\[ B_{\Delta} = B_{\text{max}} \frac{2I_{\Delta}}{I_{\Delta,\text{max}}} \quad (8) \]

\[ P_{\text{core}} = v_{\text{core}} P_v (B_{\Delta}) \quad (9) \]

With this, all the losses which are dependent on the supply voltage have been determined and the total power loss, \( P_{\text{loss}} \) can be found as the sum of all the individual losses.

### III. Optimization

This section presents an optimization method for selecting the optimal values for a fixed number of voltage levels for a TLMPs such that losses in the audio amplifier are minimized. Table VI shows the specification for the class-D audio amplifier used in both the optimization and simulation.

#### A. Data representation

The optimization method considers only the distribution of the voltage values in the music signals. Hence all information about frequency content and time dependent properties, like transients and hold time, are lost. Despite this, the distribution will be used due to the great reduction in the problem size which it provides, making the problem fast to solve since multiple music tracks can be comprised in a single distribution analog to what have been made in [6, 17].

Fig. 3 shows the estimated Probability Density Function (PDF) of the voltage values for 256 music tracks of various genres. The PDF is used to ensure that the distribution does not scale with the resolution of the distribution. Each music track is normalized, and the first and last seconds are removed to make sure the tailing zeros does not bias the distribution. Looking at the distribution, it is seen that the audio stays at small values most of the time. However, large peaks in the limits of the distribution also exist, indicating that some of the music tracks are mixed so loud that clipping occurs regularly. The normalization results in the expectation that the audio is always played at the same sound level which is rarely the case. According to [4] and [25], the average consumer tends to listen to audio at certain power levels which can be categorized by four distinct listening levels. These power levels will naturally vary depending on the sensitivity of the loudspeaker, room size etc, but are in the general case true. The amount of time the average consumer will listen to the subjective listening levels follows a distribution that should be taken into account when optimizing the efficiency of the audio system. Table I shows the typical power levels for the subjective listening levels with the corresponding utilization of the 50V supply voltage in percent.

#### TABLE I: Typical audio listening levels with the time distribution for each power level and the corresponding utilization of the 50V supply voltage in percent.

<table>
<thead>
<tr>
<th>Subjective Level</th>
<th>Listening Distribution</th>
<th>Peak Power</th>
<th>Voltage [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clipping</td>
<td>0.001%</td>
<td>312.5W</td>
<td>100%</td>
</tr>
<tr>
<td>Party</td>
<td>0.999%</td>
<td>45W</td>
<td>37.9%</td>
</tr>
<tr>
<td>Listening</td>
<td>10%</td>
<td>8W</td>
<td>16%</td>
</tr>
<tr>
<td>Background</td>
<td>89%</td>
<td>1.3W</td>
<td>6.4%</td>
</tr>
</tbody>
</table>
With this in mind, the distribution from Fig. 3 is weighted by the listening distribution in Table I to match the suggested average use case. Fig. 4 shows the weighted PDF. It is noticed that the large peaks at the limits in Fig. 3 are mapped to the peak voltage of the different consumer listening levels.

Due to symmetry and since only positive voltage rails will be considered, the negative side is flipped over to obtain the one-sided distribution of the audio distribution weighted for the average use case. Fig. 5 shows the final one-sided distribution that will be used for the optimization.

From the distribution in Fig. 5, it is clear to see that the audio content mainly is experienced at low amplitudes and rarely appear above the 37.9% mark i.e. above the “Party” listening level. This observation is especially of interest for the influence of the average consumer’s listening behavior.

**B. Cost Function**

With the data representation in place, the optimization procedure can be designed. This mainly consists of determining the cost function that is to be minimized, and the selection of an efficient optimization routine based on the structure of the problem.

1) **Direct Optimization:** The simplest and most direct cost function would be to minimize the area between the tracking voltage, \( \alpha \) and the voltage out of the amplifier, \( v \) (10).

\[
\min_{\alpha} \phi(\alpha) = \int \alpha(t) - v(t) dt \quad \text{(10)}
\]

\[\text{s.t. } v(t) \leq \alpha(t) \quad \forall t\]

This can be rewritten to use a distribution by considering the sum of the costs (11) for operating between each pair of discrete supply voltages (12).

\[
\min_{\alpha} \phi(\alpha) = \sum_{\alpha} \phi_{\alpha_i} \quad \text{(11)}
\]

\[
\phi_{\alpha_i} = \frac{1}{N} \sum_{n=1}^{N} (\alpha_{i+1} - v_n) f_p(v_n) \quad \text{(12)}
\]

\[\text{s.t. } \alpha_i \leq v_n < \alpha_{i+1}\]

\( f_p \) is the density of the distribution at the specified voltage. \( \alpha \) is the voltage levels provided by the power supply containing \( m \) different levels such that:

\[
\alpha = [\alpha_1 \alpha_2 \ldots \alpha_m] \quad \text{(13)}
\]

\[\text{s.t. } \alpha_1 < \alpha_2 < \ldots < \alpha_m\]

Where the \( m \)’th level will be the maximum level of the supply voltage. In (12) it is noticed that the size of the summation for each \( \phi_{\alpha_i} \) varies with the placement of the voltages rails, \( \alpha \), making the optimization difficult to solve efficiently. The cost functions in (11), (12) will minimize the losses introduced by the voltage overhead, which in the case of class-G amplifiers would be the desired minimization. By using (11) with the distribution in Fig. 3 solutions for the voltage levels equivalent to the ones found in [17] is obtained. These solutions are general and are, therefore, independent of the amplifier design e.g. maximum supply voltage and nominal load.

2) **Power Optimization:** While losses for linear amplifiers are linear dependent with the voltage overhead, this is not the case for class-D amplifiers. As was seen in section II, both the ripple current and the switching frequency vary with the supply voltage, thus creating polynomial dependencies for the losses. This results in the cost function in (12) becoming a poor choice for finding the optimal voltage levels. Fortunately, the structure of the cost function can be reused with the power losses from (4) to (7) to create the cost function in (14).

This cost function directly minimizes the power losses of the amplifier instead of the voltage overhead like the one in (12).

\[
\phi_{\alpha_i} = \frac{1}{N} \sum_{n=1}^{N} P_{\text{loss}}(v_n) f_p(v_n) \quad \text{(14)}
\]

\[\text{s.t. } \alpha_i \leq v_n < \alpha_{i+1}\]

\( P_{\text{loss}}(v_n) \) is the sum of all the power losses which are dependent on the supply voltage. A swift look at all the equations in section II clearly shows that this cost function does not have a single set of solutions for all amplifiers like
we saw for the previous cost function. Thus the optimal placement of the voltage levels for the multilevel power supply is heavily dependent on the design choices made for the class-D amplifier, making the minimizer to the cost function unique to each amplifier design. The solutions obtained from minimizing (14) are not useful for practical implementations as the function assumes that a switch between two voltage levels is instantaneous and has no power consumption. Thus the effects of transition time and hold time related to the switch is ignored. This leads to the found voltage levels being close to zero as most of the audio reside here. However, these voltages will rarely be reached due to the before mentioned effects, thereby rendering the found voltage levels useless. A way to introduce the time dependent effects into the cost function is to include a switch cost for going from one voltage level to another. The switch cost will use the density at each voltage level \(\alpha_i\) as a measure for the amount of switching needed at the selected levels. Equation (15) shows the cost function with the switch cost, \(\xi\) introduced.

\[
\phi_{\alpha_i} = \frac{1}{N} \sum_{n=1}^{N} P_{\text{loss}}(v_n) f_p(v_n) + f_p(\alpha_i)\xi \\
\text{s.t. } \alpha_i \leq v_n < \alpha_i + 1 \\
0 \leq \xi
\]  

The switch cost, \(\xi\), represent the dissipated energy due to a switch between two voltage levels. The considered energy both comes from energy dissipated in component due to the switching, but also the increased energy usage due to the non-instantaneous transitions and the minimum time a voltage level needs to be kept to avoid clipping. Since \(\xi\) is the sum of multiple different energy loss mechanisms it has lost its physical meaning. Therefore, \(\xi\) should be considered as a tuning parameter for the optimization to steer the optimization towards solutions with the desired number of voltage levels.

C. Method

To minimize the derived cost functions, an optimization routine is needed. As the cost function is dynamic in size depending on the distance between \(\alpha_i\) and \(\alpha_{i+1}\), it is not possible to state the optimization problem on a standard form for gradient based solvers. This combined with the fact that the problem is non-convex suggest that a heuristic method is needed to find the global minimum. One of the most common heuristics is the Particle Swarm (PSO). The PSO is based on the principles of swarm intelligence [26]. A population of particles is generated and the entire population will converge toward the particle with the lowest function value. This results in a high population density around the current minimum, which increases the likelihood of discovering a close by global minimum. Thus, the PSO is very efficient in cases where it is expected that the minima will be close together.

In our case, it can be expected that the minima are close to each other as small deviations in the voltage levels can result in one of the terms reaching its minimum, which is likely to be a local minimum. Hence the PSO will be used for finding the minimum of the derived cost functions.

D. Performance

To investigate the performance of the derived cost function in (15), a simulation of the efficiency is made. The efficiency of a class-D amplifier is simulated for three different configurations of the TMLPS: the optimal selection of voltages, equally spaced voltages, and without tracking. The optimal and equally spaced solution will have four different voltage levels. Four voltage levels is selected based on previous findings which show that a significant efficiency increment is obtained going from 3 to 4 levels while going above 4 only provide diminishing returns [14, 27]. The reduced benefits from using more supply levels are among others caused by the increased size and complexity of power supplies and control circuits resulting in higher power draw, hence lower efficiency. With the number of voltage levels determined, \(\xi\) is determined by running the optimization with different values of \(\xi\) until a range of \(\xi\), where the optimization results in the correct number of supply levels, are found. Thus, to determine a proper selection of the switching penalty, \(\xi\) is swept from 0 to 0.5, and the optimization routine is performed to obtain an overview of all the possible configurations of the power supply voltage levels.

Fig. 6 shows the number of supply voltages and their levels for different \(\xi\) except for the highest supply voltage, which is the fixed rail voltage 50V. The obtained solutions can be divided into three sections: \(\xi\) less than 0.18, \(\xi\) greater than 0.28, and everything in between. For \(\xi < 0.18\) four levels a visible in the figure. This shows that a solution with 5 supplies is feasible, indicating the switch penalty is selected too small. For \(\xi > 0.28\), the third voltage level becomes within 1V of the upper supply voltage of 50V, indicating that the two voltage levels...
levels can be merged to a single level for a better solution with 3 voltage levels. This indicates that the switch penalty from this point is selected too high. Lastly, there is the region in-between. In this region all solutions matches a configuration with exactly 4 supply voltage levels. The solutions close to the lower bound have lower voltage levels and can be considered as more aggressive solutions that benefits from faster tracking while the solutions close to the upper bound is more conservative and better suited for slower tracking. The fluctuations seen on especially the highest voltage levels are the product of the low probability that audio will be at these voltage levels. Hence the exact placement of the voltage level has little effect when evaluating the cost function. Selecting the proper value of $\xi$ within the acceptable range is a matter of trial and error to see if the tracking speed is sufficiently fast for a low value of $\xi$. In practice this is achieved by performing a simulation of the efficiency for each value of $\xi$ where the desired hold time is included in the simulation. The final value of $\xi$ is then the value that provides the overall best efficiency. A switching penalty of $\xi = 0.25$ is found to be a reasonable choice. Table II shows all the voltage levels of the TMLPS for each configuration.

**TABLE II: The voltage levels of the TMLPS for the different configurations.**

<table>
<thead>
<tr>
<th>Method</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal</td>
<td>3.5V</td>
<td>14.7</td>
<td>46.8V</td>
<td>50V</td>
</tr>
<tr>
<td>Equal Spaced</td>
<td>12.5V</td>
<td>25V</td>
<td>37.5V</td>
<td>50V</td>
</tr>
<tr>
<td>No Tracking</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>50V</td>
</tr>
</tbody>
</table>

The efficiency simulations uses the component values listed in Table VI and are performed for the three music tracks presented in Table III. The tracks are selected outside of the pool of tracks used for the optimization, and is selected to represent the different dynamical characteristics that come with different music genres. The differences in the dynamics between the three songs are seen on the corresponding crest factors listed in Table IV.

**TABLE III: The three audio tracks used for simulation and measurement.**

<table>
<thead>
<tr>
<th>Name</th>
<th>Artist</th>
<th>Genre</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can’t We Be Friends</td>
<td>Ella &amp; Louis</td>
<td>Jazz</td>
</tr>
<tr>
<td>Redneck</td>
<td>Lamb of God</td>
<td>Metal</td>
</tr>
<tr>
<td>Get Lucky</td>
<td>Daft Punk</td>
<td>Pop</td>
</tr>
</tbody>
</table>

**TABLE IV: Crest factor for the three audio tracks and for the first 20 seconds.**

<table>
<thead>
<tr>
<th>Name</th>
<th>Entire track</th>
<th>First 20 sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can’t We Be Friends</td>
<td>20.4 dB</td>
<td>22.3 dB</td>
</tr>
<tr>
<td>Redneck</td>
<td>8.9 dB</td>
<td>11.2 dB</td>
</tr>
<tr>
<td>Get Lucky</td>
<td>13.0 dB</td>
<td>13.6 dB</td>
</tr>
</tbody>
</table>

To limit the simulation time, only the first 20 seconds of each number is considered. The simulation works by first subdividing the audio track sample-by-sample into the sections where the difference supply voltage levels are active. A hold time of 50ms on the supply voltages are assumed to account for short transition times when toggling between two voltage levels. Next, the losses are calculated based on the equations in section II for each supply voltage level and sample before taking the average of the losses. Hence the average loss for the selected average output power is determined. The simulation differs from the optimization by considering a single audio track through time, thereby preserving the time dimension in the data which are used to calculate the efficiency reduction due to the hold time of the supply voltages. Fig. 7 (a) to (c) shows the simulated efficiencies for the three music pieces with the three tracking methods.

Looking at Fig. 7 (a) to (c) it is observed that the equally spaced voltages and the single voltage method both are very consistent in their performance across the three music pieces. Also, both of the tracking methods perform better than using a single supply, with the optimal performing better than the equally spaced up to about 0.6W in all cases. The large fluctuations in the efficiency at the higher power levels (>0.6W) for the optimal selection of voltages is due to the hold time. Since the optimization routine does not take the time dependencies into account, solutions with large changes in the voltage levels can be found. These solutions are the optimal solutions when only the amplitude distribution is considered, however, once the time dependencies are introduced the large changes in the supply voltage levels becomes expensive as the increased losses, due to the high voltage, persist for a longer time because of the hold time. This results in a sharp reduction in the efficiency at the higher power levels as seen in Fig. 7. Thus the optimal solution for the simplified problem using the audio distribution is not a truly optimal solution to the actual problem but still likely to perform well. The optimization routine may be improved by adding an additional penalty term, penalizing the distance between the voltage levels in the solution. This way the hold time will have less of an impact on the performance. However, this has not been further investigated.

To assess whether it is the optimal or the equally spaced solution that provides the most overall improvement in the efficiency, the efficiency is evaluated for the power levels listed in Table I and weighted according to the time distribution of the subjective listening levels. This provides an efficiency measure that is true to the typical use of a consumer amplifier. Table V shows the average efficiency for each of the music pieces with the total average for each method.

**TABLE V: Weighted efficiency based on the subjective listening levels for the three music pieces and the average across the three tracks.**

<table>
<thead>
<tr>
<th>Method</th>
<th>Ella</th>
<th>Lamb</th>
<th>Daft</th>
<th>Avg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal</td>
<td>3.98%</td>
<td>23.60%</td>
<td>18.18%</td>
<td>15.25%</td>
</tr>
<tr>
<td>Equal Spaced</td>
<td>2.57%</td>
<td>20.56%</td>
<td>13.79%</td>
<td>12.31%</td>
</tr>
<tr>
<td>No Tracking</td>
<td>0.50%</td>
<td>5.22%</td>
<td>3.22%</td>
<td>2.98%</td>
</tr>
</tbody>
</table>
From Table V it is clear to see that the optimal selection of voltages has better overall efficiency than the competing solution. The efficiency is improved by more than 23% compared to the equal spaced voltages and more than 5.1 times compared to the single voltage supply. By looking at the simulation results, it can be concluded that the improved efficiency, compared to the equally spaced voltages, mostly comes from the fact that the listening behaviors of consumers are considered, and not that the audio is more tightly tracked. This is visible in the way the efficiency drops below the efficiency for the equally spaced once the toggling between the 14.7V and 46.8V rail starts occurring due to the hold time. The decreased efficiency at high power suggests that the optimization traded the efficiency at power levels above normal listening levels for an increased efficiency at more likely listening levels. If shorter hold times can be achieved the efficiency improvements from the audio tracking will have a larger impact, increasing the efficiency at the higher power levels. However, the trade-off will remain, just less prominent.

IV. SIMPLE TMLPS FOR CLASS-D

This section serves to show that efficiency gains can be achieved in practice through the use of TMLPS. To illustrate the possible improvements, a simple TMLPS solution for class-D audio amplifiers with equally spaced voltage levels is implemented. Both concept and experimental results are presented.

A. Concept

The simple TMLPS utilizes an analog multiplexer which selects the amplifier supply voltage, $V_S$, from four evenly spaced predefined voltage levels. Fig. 8 shows how a conventional Flyback converter can be used for this purpose. Having the secondary windings stacked and with same number of turns enables fairly easy transformer design. The only drawback is slightly lower fill factor of the winding window due to more outlets which will lower converter efficiency slightly, mainly at higher output powers. Changing the turns ratio between secondary windings allows for fine tuning of voltages but increase the complexity of the transformer design. This selection is based on an analysis of the amplitude of the music input signal, which ensures that the amplified music signal, generated by the amplifier, is never clipped to the supply voltage. Fig. 9 (a) shows a simplified schematic of the analog multiplexer. The control signals, $S_1$ – $S_3$, is generated from an analog High Precision Peak Detection (HPPD) circuit, shown in Fig. 9 (b), which tracks the rectified audio input signal. The output of the HPPD, $V_{in,pk}$, is fed to three comparators with hysteresis and individual thresholds, $v_{th1}$–$3$. The control signals, $S_1$–$S_3$, are enabled when the audio input triggers these

![Fig. 7: Simulated efficiency for the three songs using the two tracking methods and no tracking. The grey lines indicates the power levels of the subjective listening levels from Table I.](image-url)

![Fig. 8: Flyback converter with stacked secondary windings enabling multiple output voltages.](image-url)
thresholds. The speed of the tracking is set by the time constant formed by $C_1$ and $R_2$. As was seen in section III, the tracking speed does not need to be very fast to produce efficiency improvements, but faster tracking will reduce the needed hold time thereby reducing the dips in the efficiency observed in Fig. 7. However, excessive switching in the multiplexer should be avoided as this will result in increased losses.

### B. Experimental results

A prototype system is designed consisting of the simple TMLPS and a 50V class-D amplifier. The class-D amplifier is designed conventionally in accordance with [28] and will be tested up to 200W peak power. The amplifier regulated, i.e. a closed loop configuration, making the gain of the amplifier independent of supply voltage level. In addition to that it is implemented as a full bridge amplifier similar to Fig. 2 switching at 500kHz. A full bridge configuration is preferred for this application since issues with supply rail pumping is avoided [29]. Table VI shows the bill of materials.

**TABLE VI: Specifications for the audio amplifier used in the optimization and simulation.**

<table>
<thead>
<tr>
<th>Type</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutoff freq. ($f_c$)</td>
<td>40</td>
<td>kHz</td>
</tr>
<tr>
<td>Filter ind. ($L_1$)</td>
<td>22</td>
<td>uH</td>
</tr>
<tr>
<td>Core type</td>
<td>RM8-N87</td>
<td></td>
</tr>
<tr>
<td>Filter cap ($C_1$)</td>
<td>350</td>
<td>nF</td>
</tr>
<tr>
<td>Idle switch freq. ($f_{sw,idle}$)</td>
<td>500</td>
<td>kHz</td>
</tr>
<tr>
<td>Max. supply voltage ($V_s$)</td>
<td>50</td>
<td>V</td>
</tr>
<tr>
<td>Load</td>
<td>8</td>
<td>Ohm</td>
</tr>
<tr>
<td>MOSFET</td>
<td>Infineon - BSZ150N10LS3</td>
<td></td>
</tr>
<tr>
<td>Dead time ($t_{dt}$)</td>
<td>10</td>
<td>ns</td>
</tr>
</tbody>
</table>

The amplifier performs equivalently with the amplifier model used in section III. Four fixed voltage levels are provided to the multiplexer circuit evenly spaced from 0 to 50V, i.e. 12.5V, 25V, 37.5V, and 50V. For measurements external laboratory power supplies was used to provide the voltage levels to the analog multiplexer. The input signal range of the amplifier is ±1V. The thresholds, $v_{th,1-3}$ of the comparator circuit, are set to 0.2V, 0.4V, and 0.6V respectively, meaning that the multiplexer provides 50V when the input signal exceeds ±0.6V. These thresholds are selected to ensure some headroom to the supply voltage levels, thus avoiding significant distortion on the amplified audio signal. Furthermore, the time constant of the RC-network in the HPPD is selected such that the maximum resulting hold time becomes 160ms. For implementation this yields $C_1 = 12uF$ and $R_2 = 1M\Omega$. Fig. 10 shows the implanted TMLPS connected to an amplifier.

Fig. 11 shows the efficiency of the amplifier with and without the tracking power supply. The efficiency is measured conventionally by using a 1kHz sine wave. The input and output power is then measured using digital multimeters, Agilent 34401A. The measured efficiency is compared to the loss model established in section II and good correlation is noticed. As expected, the efficiency is greatly improved in the low power region due to the TMLPS. At 1W the efficiency is improved from 30% to 65%. For high output levels, the measured efficiency of the system with the TMLPS has 3% to 5% lower efficiency than the model. This mismatch between measurement and model is due to some non-linearities in...
the losses not included in the model. These include the on resistance $R_{ds}$ and the body diode of the MOSFETs, both of which depend on the temperature and drain current and increases with temperature and current thereby increasing the losses at high power levels. Working with high crest factor signals these losses becomes difficult to model precisely and therefore they were not included in the model.

In order to evaluate the efficiency when playing audio, the three tracks used for the simulation are used. To ensure that the tracks are perceived equally loud, they have been loudness normalized in accordance with the EBU-R128 recommendation [30, 31]. For measurement reasons, only the first twenty seconds of each track has been considered. Five loudness levels are used going from low volume up to clipping. The corresponding average output power can be found in Table VII. A DPO3014 oscilloscope from Tektronix equipped with high precision current- and voltage probes is used to capture the highly dynamic input and output voltages/currents. With a record length of 10 million points, this oscilloscope ensures a sufficient sampling frequency when considering a 20 second audio signal.

Table VII: Output power for different loudness levels.

<table>
<thead>
<tr>
<th>Track</th>
<th>Loudness level</th>
<th>First 20 seconds</th>
<th>Full track</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Ella &amp; Louis</td>
<td>0.06 W</td>
<td>0.20 W</td>
<td>0.56 W</td>
</tr>
<tr>
<td>Lamb of God</td>
<td>0.08 W</td>
<td>0.25 W</td>
<td>0.71 W</td>
</tr>
<tr>
<td>Daft Punk</td>
<td>0.14 W</td>
<td>0.47 W</td>
<td>1.34 W</td>
</tr>
</tbody>
</table>

Fig. 12 shows the measured supply voltage as well as amplifier output playing ”Daft Punk - Get Lucky” at loudness level 5. Although the tracking speed is slow, it is clear that the proposed method ensures that the supply voltage is vastly lowered for the majority of the audio signal. This will inevitably lower the losses in the power stage. Fig. 13 shows the measured and simulated efficiency of each track for the different loudness levels. There is a good correlation between the measured results and simulation. It is observed that for all tracks and loudness levels, the efficiency of the designed system is vastly improved when the TMLPS is enabled. At loudness 1 the average efficiency across the three tracks goes from 3.3% without tracking to 15.7% with tracking, and at loudness 5 the average efficiency goes from 39.7% to 65.7% with tracking. The largest improvement in efficiency is seen at loudness level 3 where the average efficiency goes from 24.3% without tracking to 57.3% efficiency with tracking. In terms of power losses, a reduction from 2.5W to only 0.5W is observed. The track, ”Get Lucky” marked by ”∗” in the figure, experiences the greatest efficiency improvement across all loudness levels.

The operating temperatures were measured using a thermal camera for a prototype with a maximum hold time of 850ms. Fig. 14 shows the measured operating temperature of the audio amplifier with and without the tracking power supply for the “Daft Punk - Get Lucky” track at loudness level 5, and Table VIII provides a summary of all the measurements. It is observed that in general, the operating temperature is greatly reduced. Reductions are especially seen at the four
switching MOSFETs in the amplifier power stage where the temperatures are halved from approximately 100°C to approximately 50°C across all tracks. In the same manner, the inductor temperature is reduced from approximately 50°C to approximately 35°C. The reduction in MOSFET operating temperatures are obtained because the switching losses are reduced significantly. The reduction of the temperature in the filter inductor is related to a smaller ripple current when the supply voltage is low, thus causing less AC winding- and core losses.

Finally, we performed a crude listening test to evaluate the audio quality when the tracking power supply is enabled. From the listening sessions, it is clear that the level-shifting, introduced by the TMLPS, generates undesired audible clicks, and therefore degrades the audio listening experience. These clicks originates from the fast transition in the supply voltage when jumping from one voltage level to another. They can be reduced by having a higher power supply rejection in the amplifier or a lower slew rate in the analog multiplexer. The latter could be achieved with a filter similar to the works in [32–34].

V. CONCLUSION

This paper has investigated the effect of using Tracking Multi-Level Power Supplies (TMLPS) for class-D audio amplifiers. A simple TMLPS was proposed and implemented on a prototype 50V class-D amplifier where testing up to 200W peak power was conducted. The simple TMLPS toggles between equally spaced voltage levels. A power supply capable of delivering multiple voltage output may be configured as a Flyback converter with stacked secondary windings. Having same number of turns of secondary windings enables simple transformer design and equally spaced voltages. It is future work to investigate more complex transformer design for fine tuning of voltage levels. Measured results show that systems using the tracking power supply achieve significantly higher efficiencies compared to using a single supply voltage. In addition to this, it is observed that the amplifier operating temperature is strongly reduced, especially for the switching power MOSFETs where it was halved from 100°C to 50°C.

To further improve the efficiency, an optimization method for finding the voltage levels for TMLPS was proposed. The method considers the voltage value distribution of the audio signal. This distribution was made combining the distribution of music with information about listening behaviors for consumers. Based on a derived cost function, optimal voltage values for a fixed amount of levels for a TMLPS were found. Simulated results showed more than 23% improvements in efficiency when the listening behavior was considered making the optimal solution superior compared to using the simple TMLPS with equally spaced voltage levels. It was found that it is relevant to consider the listening behavior and not only the audio distribution when designing a TMLPS for consumer audio systems.

TABLE VIII: Summary of temperature measurements at loudness level 5.

<table>
<thead>
<tr>
<th>Input</th>
<th>Mean temperatures (°C)</th>
<th>w/o tracking</th>
<th>w. tracking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Idle</td>
<td>94.5°C</td>
<td>46.9°C</td>
<td>37.2°C 29.1°C</td>
</tr>
<tr>
<td>Ella &amp; Louis</td>
<td>97.5°C</td>
<td>51.6°C</td>
<td>46.2°C 33.3°C</td>
</tr>
<tr>
<td>Lamb of God</td>
<td>98.3°C</td>
<td>52.8°C</td>
<td>55.0°C 35.9°C</td>
</tr>
<tr>
<td>Daft Punk</td>
<td>98.2°C</td>
<td>52.5°C</td>
<td>53.8°C 35.7°C</td>
</tr>
</tbody>
</table>


Nicolai J. Dahl (S’19) is a Ph.D. student at the electronics group at the Technical University of Denmark, Kgs. Lyngby, Denmark. He received both his B.Sc. degree and M.Sc. degree from the Technical University of Denmark in June 2016 and August 2018 respectively. His research interest include control theory, system modeling, signal processing, and optimization, which he has applied in the fields of switch-mode power amplifiers, resonant converters, and his current research topic, time-based control.

Niels E. Iversen (M’17) is an industrial post doc at the Danish amplifier company ICEpower A/S collaborating with the Technical University of Denmark (DTU). He received his B.Sc. degree in electrical engineering from DTU in 2012 and completed his M.Sc. education in December 2014. In May 2018, he was awarded the Ph.D. degree for his research on efficient audio systems. During his studies, he has submitted several papers for international conventions, conferences and the Journals all concerning electrical aspects of audio such as amplifier design, measurement techniques and transducer modeling. His research as post doc focuses on high power amplifiers and efficiency optimization of electric acoustic transducers.

Vladan Ž. Lazarević (S’17) was born in in Bijeljina, Bosnia and Herzegovina, in 1992. He received the B.Sc. degree in electrical engineering from the University of Belgrade, Belgrade, Serbia, in 2015 and the M.Sc. degree in industrial electronics from the Universidad Politécnica de Madrid (UPM), Madrid, Spain, in 2016. Since 2015, he has been with the Center of Industrial Electronics (CEI), UPM, where he is currently working toward his Ph.D dissertation. His current research interests include design, optimization and digital control of fast, highly efficient and compact power amplifiers for industrial applications, based on GaN wide bandgap semiconductor technology, and design of SiC power modules for medium voltage applications.

Michael A. E. Andersen (M’88) received the M.Sc.E.E. and Ph.D. degrees in power electronics from the Technical University of Denmark, Kongens Lyngby, Denmark, in 1987 and 1990, respectively. He is currently a Professor of power electronics at the Technical University of Denmark, where since 2009, he has been the Deputy Head of the Department of Electrical Engineering. He is the author or coauthor of more than 300 publications. His research interests include switch-mode power supplies, piezoelectric transformers, power factor correction, and switch-mode audio power amplifiers.

Miroslav Vasić (M’10-SM’19) was born in Serbia, in 1981. He received the B.E. degree from the University of Belgrade, Belgrade, Serbia, in 2005 and the M.S. and Ph.D. degrees from the Universidad Politécnica de Madrid (UPM), Madrid, Spain, in 2007 and 2010, respectively. He advised two Ph.D. dissertations. He is currently an Assistant Professor with UPM. He has authored over 50 papers in IEEE journals and conferences. He holds two patents. His current research interests include dc–dc converters, power converters for RF applications, and optimization of converter topologies. Dr. Vasić received the SEMIKRON Innovation Award for the team work on “RF Power Amplifier with Increased Efficiency and Bandwidth,” in 2012, a medal from the Spanish Royal Academy of Engineering for his research trajectory as a Young Researcher, in 2015, and the Best Young Researcher Award from the Universidad Politécnica de Madrid in 2016. He has been cooperating with the IEEE and other professional associations as a Reviewer and a Session Chair.