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Published in:
Journal of Sound and Vibration

Link to article, DOI:
[10.1016/j.jsv.2020.115268](https://doi.org/10.1016/j.jsv.2020.115268)

Publication date:
2020

Document Version
Peer reviewed version

[Link back to DTU Orbit](#)

Citation (APA):
Berardengo, M., Høgsberg, J., Manzoni, S., Vanali, M., Brandt, A., & Godi, T. (2020). LRLC-shunted piezoelectric vibration absorber. *Journal of Sound and Vibration*, 474, [115268].
<https://doi.org/10.1016/j.jsv.2020.115268>

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LRLC-shunted piezoelectric vibration absorber

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Abstract

This paper addresses a new approach for mono-modal vibration reduction by means of a piezoelectric shunt. It is based on an innovative shunt impedance which allows to improve the attenuation performance and the robustness to mistuning compared to the use of the classical resonant shunt. This result is achieved by building a network, composed of two inductances, one capacitance and one resistance, which generates two resonances, instead of the single resonance imposed by the classical resonant shunt. All the theoretical results discussed in the paper are validated by an experimental campaign on a tailored set-up. These tests show a good agreement between theoretical and experimental results and thereby validate the benefits of the new approach.

Keywords: piezoelectric shunt, LRLC shunt, resonant shunt, vibration, damping

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1. Introduction

The use of piezoelectric actuators shunted to electric impedances to attenuate vibrations is a widely studied topic and has recently been thoroughly investigated (e.g. [1, 2, 3, 4]). In this control approach, the piezoelectric transducer acts at the same time as a sensor and as an actuator and the layout of the electric impedance depends on the required type of attenuation. As an example, an impedance composed of either the series or parallel connection of an inductance and a resistance (named resonant shunt or LR shunt) is effective for single-mode control [5, 6, 7, 8, 9, 10]. Conversely, in case of multi-mode control, the use of a resistance coupled to one or two negative capacitances offers good performance [11, 12, 13, 14]. Furthermore, other approaches, based on different and more complex networks, are also possible, as shown in [15, 16].

Among all the possible approaches, the use of an LR shunt (even with coupling to negative capacitances [17, 18]) has been found to be the most effective approach for controlling a single mode. Different methods have been proposed in the literature to set the values of the inductance L and the resistance R [19]. They can rely on the shape of the frequency response function (FRF) of the controlled system [6, 8] or exploit the pole placement theory, requiring defined conditions on the damping associated with the poles of the system [20, 21, 22]. Among the approaches of the latter category, the so-called balanced calibration stands out for desirable features such as its high attenuation levels and good robustness to possible mistuning thanks to the way of setting the value of the resistance R [21, 22]. The robustness of the control is very important in LR shunts because this type of control suffers from significant performance loss because of mistuning [19, 23, 24]. Indeed, the effect of the LR shunt can be seen as equivalent to that of tuned vibration absorbers (TVAs), with all the related advantages and drawbacks, such as sensitivity to possible mistuning and parameter uncertainties.

To overcome this limitation, this paper presents a new shunt impedance which uses similar principles as those adopted in [21, 22] for the balanced cali-

bration, while improving the attenuation performance both in tuning and with respect to mistuning. This is achieved by adding a capacitance and an inductance to the existing LR shunt (LRLC shunt, see Sections 3.1 and 3.2) to create an additional resonance for the whole electro-mechanical system, compared to the LR shunt (i.e. the LRLC shunt introduces two resonances in the system, while the LR shunt just one). This LRLC shunt is found to improve both the robustness and performance of the control action.

This approach already showed to be effective in the field of mechanical TVAs, where multiple TVAs are used to add more than one eigenfrequency to the whole system [25, 26] and are tuned for attenuating the vibrations of a single mode of the primary system. The technique, applied in different fields, such as for example civil engineering [27, 28, 29, 30] and acoustic control [31], has proven to be able to improve the attenuation provided by a single TVA. TVAs can be used with different layouts, such as series (e.g. [32, 33, 34]), parallel (e.g. [33, 34, 35, 36, 37, 38]) or other configurations (e.g. [39, 40]). Moreover, it is possible to base this technique on smart materials or electro-magnetic interactions (e.g. [41, 42, 43]), which allow to develop new configurations for devices able to add more than one eigenfrequency to the whole system. Unfortunately, most of the time, when smart materials are employed, the obtained equivalent mechanical schemes are different from those typical of multiple TVAs and thus, in these cases, specific optimisation procedures are needed to set the values of all the elements in the control system. Oftentimes, due to the complexity of the problem with additional resonances, these techniques rely on either numerical minimisation of target functions, without analytical formulas, or the numerical solution of a system of polynomial equations and the consequent numerical analysis of the obtained solutions.

The idea of the present work is to exploit the special features of piezoelectric materials for developing an LRLC shunt impedance that adds two eigenfrequencies to the whole system and aims at improving the attenuation performance and robustness of the resonant shunt. To do this, a specific procedure based on a mixed analytical-numerical approach is proposed for the tuning of the elec-

62 tric parameters, providing closed analytical formulations for the basic circuit
 63 components.

64 The structure of the paper is as follows: Section 2 presents the model used for
 65 describing the electro-mechanical structure and recalls the balanced calibration
 66 approach for setting the values of L and R presented in [21, 22] for the tradi-
 67 tional LR shunt. Section 3 presents the new shunt impedance together with its
 68 theoretical discussion. Section 4 describes a numerical case aimed at showing
 69 the advantages provided by the proposed approach and Section 5 explains how
 70 to predict the attenuation that it provides. Finally, Section 6 addresses the
 71 experiments carried out to validate the theoretical outcome.

72 2. The system model and the balanced calibration

Figure 1: Vibration attenuation by means of a piezoelectric shunt.

73 A generic structure excited by an external force f_e is considered. A piezo-
 74 electric actuator is bonded on it, and it is shunted with an electric impedance
 75 Z (see Fig. 1), composed of a resistance R and an inductance L which can be
 76 connected either in parallel or series (see Figs. 2a and b). The displacement U
 77 of any given degree-of-freedom x of the structure at time t can be represented
 78 in modal coordinates:

Figure 2: Traditional resonant shunt impedance in parallel (a) and series (b), and the new LRLC shunt impedance in parallel (c) and series (d).

$$U(x, t) = \sum_{s=1}^N \phi_s(x) u_s(t) \quad (1)$$

where N is the number of modes of the system, ϕ_s is the s -th eigenvector (scaled to the unit modal mass and with the piezoelectric actuator short-circuited) and u_s is the s -th modal coordinate.

In case of low modal coupling, the motion of the system for $\omega \simeq \omega_s$ (where ω is the angular frequency and ω_s is the s -th eigenfrequency of the system with the piezoelectric actuator short-circuited) can be approximated as:

$$U(x, t) \simeq \phi_s(x) u_s(t) \quad (2)$$

and the equation of motion of the structure becomes:

$$(-\omega^2 m_s + i\omega c_s + k_s) u_s + f = f_{e,s} \quad (3)$$

where m_s , c_s and k_s are the modal mass, damping and stiffness ($\omega_s = \sqrt{k_s/m_s}$), respectively, and $f_{e,s}$ is the modal forcing. Furthermore, i is the imaginary unit. From here on, the modal mass will be set equal to 1 ($m_s=1$) and thus the eigenvector components are scaled to unit modal mass, as mentioned previously. Furthermore, f is the force exerted by the piezoelectric transducer on the structure and it is expressed as:

$$f = \theta_s V \quad (4)$$

where θ_s is the coupling coefficient and V is the voltage across the electrodes of the piezoelectric actuator (see Fig. 1).

The sensor equation couples the electric and mechanical behaviours:

$$Q = -\theta_s u_s + C_s V \quad (5)$$

where the charge Q on the transducer surfaces depends on two contributions: the mechanical deformation ($-\theta_s u_s$) and the capacitive effect ($C_s V$). For vibration damping of a flexible structure with multiple modes, the modal capacitance $C_s = C_0 + C'_s$ is composed of two contributions: the capacitance associated with constrained transducer boundaries C_0 , and a static correction term C'_s accounting for the contribution from higher modes [44, 45].

If the shunt impedance Z is considered as in Fig. 1, the equation linking the charge on the surfaces of the piezoelectric transducer and the voltage across its terminals can be written as:

$$V = -Z(\omega)I = -i\omega Z(\omega)Q \quad (6)$$

where I is the current flowing in the circuit (see Fig. 1). According to the type of shunt impedance used, the expression of Z changes. If the classical LR shunt is considered, Z either represents the series or parallel connection of a resistance R and an inductance L . These specific cases are shown in Figs. 2a and b and treated in the next subsection.

2.1. The classical resonant shunt and its balanced calibration

Relying on the electrical analogy of mechanical systems and knowing that an electrical series connection corresponds to a parallel mechanical connection and vice versa, it is possible to translate the electrical model of Fig. 3a (L and R connected in parallel) into an equivalent mechanical model, as shown in Fig. 3b (m and c in series). In this equivalent representation, the mechanical parameters

Figure 3: Circuit diagram of the parallel LR shunt (a) and its mechanical equivalent (b).

are obtained by multiplying the electrical parameters by θ_s^2 . A similar approach can be adopted for the series connection between L and R (see Figs. 4a and b).

Using Eqs. (4), (5) and (6) in Eq. (3), the FRF of the electro-mechanical system is obtained. According to the type of connection between inductance L and resistance R (i.e. parallel or series in Figs. 2a and b, respectively), the mathematical description of Z changes and two different FRFs are achieved. In the case of a parallel link, the FRF is:

$$\frac{u_s k_s}{f_{e,s}} = \frac{k_s(-\omega^2 + 2i\zeta_e \omega_e \omega + \omega_e^2)}{[-\omega^2 + 2i\zeta_s \omega_s \omega + (1 + \kappa_0^2)\omega_s^2](-\omega^2 + 2i\zeta_e \omega_e \omega + \omega_e^2) - \kappa_0 \omega_s^2(\omega_e^2 + 2i\zeta_e \omega_e \omega)} \quad (7)$$

while in case of a series link, the FRF is instead given as:

$$\frac{u_s k_s}{f_{e,s}} = \frac{k_s(-\omega^2 + 2i\zeta_e \omega_e \omega + \omega_e^2)}{[-\omega^2 + 2i\zeta_s \omega_s \omega + (1 + \kappa_0^2)\omega_s^2](-\omega^2 + 2i\zeta_e \omega_e \omega + \omega_e^2) - \kappa_0 \omega_s^2 \omega^2} \quad (8)$$

The symbol ζ_s indicates the non-dimensional damping ratio associated with the

Figure 4: Circuit diagram of the series LR shunt (a) and its mechanical equivalent (b).

124 structural eigenfrequency ω_s , while ω_e and ζ_e are the electric eigenfrequency
 125 and non-dimensional damping ratio, respectively. The electric eigenfrequency
 126 ω_e is related to L by the following relation:

$$\omega_e = \frac{1}{\sqrt{LC_s}} \quad (9)$$

127 The expression of ζ_e depends on the connection of L and R . For the parallel
 128 connection, its expression is:

$$\zeta_e = \frac{1}{2\omega_e RC_s} \quad (10)$$

129 whereas for the series connection, it is given as:

$$\zeta_e = \frac{R}{2\omega_e L} \quad (11)$$

Moreover, a normalised coupling coefficient κ_0 is expressed as (see Figs. 3b and 4b):

$$\kappa_0 = \frac{k_0}{k_s} = \frac{\theta_s^2}{C_s k_s} \quad (12)$$

It is noted that $\sqrt{\kappa_0}$ is the modal electro-mechanical coupling coefficient [21], which can be estimated as [44, 46, 47]:

$$\kappa_0 = \frac{\theta_s^2}{C_s \omega_s^2} \simeq \frac{\hat{\omega}_s^2 - \omega_s^2}{\omega_s^2} \quad (13)$$

where $\hat{\omega}_s$ is the system eigenfrequency for the piezoelectric actuator with open-circuited electrodes.

The FRFs in Eqs. (7) and (8) are characterised by four poles. More precisely, for low to moderate damping values, they appear as two pairs of complex conjugate eigenvalues. The balanced calibration, considered here as the starting point of the proposed method, is based on the requirement of equal modal damping of the eigenvalues. This tuning approach for the shunt impedance has already demonstrated to provide simultaneously high attenuation values (close to those provided by minimisation criteria on the FRF amplitude) and a high robustness to possible mistuning due to an increased value of R (see e.g. [21]).

As demonstrated in [21, 48, 49], plotting the absolute value of the real and imaginary parts of the eigenvalues (normalised by a real-valued reference frequency ω_0) is a good way to investigate whether the condition of equal modal damping is fulfilled. Indeed, the condition of equal modal damping is achieved when the normalised eigenvalues lie on the same line containing the origin of the complex plane (plotting them in terms of absolute value of the real and imaginary parts) and thereby appear as inverse points with respect to a unit circle. The value of ω_0 is automatically obtained when the equal modal damping condition is imposed (see [21]) and represents the anti-resonance frequency in the FRF when $\zeta_e=0$. For the parallel connection of L and R , the reference frequency is:

$$\omega_0 = \omega_s \quad (14)$$

155 while, for the series connection, it is:

$$\omega_0 = \omega_s \sqrt{1 + \kappa_0} \quad (15)$$

156 Following this tuning procedure, it is thus possible to derive the expressions
 157 of ω_e and ζ_e which secure equal modal damping, denoted as ω_e^{opt} and ζ_e^{opt} ,
 158 respectively. For the parallel connection of L and R , they are:

$$\omega_e^{\text{opt}} = \omega_s \quad (16)$$

$$\zeta_e^{\text{opt}} = \sqrt{\frac{\kappa_0}{2}} \quad (17)$$

159 while, for the series connection, their expressions are:

$$\omega_e^{\text{opt}} = \omega_s(1 + \kappa_0) \quad (18)$$

$$\zeta_e^{\text{opt}} = \sqrt{\frac{\kappa_0}{2(1 + \kappa_0)}} \quad (19)$$

160 From this tuning procedure, obtaining a robust LR shunt, the authors pro-
 161 pose a new layout for the shunt impedance Z , composed of two inductances (L
 162 and L_0), a capacitance C and a resistance R (see Figs. 2c and d). It is conceived
 163 to further improve the performance and the robustness of the LR shunt by in-
 164 troducing an additional resonance in the system. As mentioned, it is referred
 165 to as an LRLC shunt and introduced in the next section.

166 3. The LRLC shunt

167 The LRLC shunt proposed in this paper is calibrated based on the require-
 168 ment of equal modal damping, as introduced for the classical resonant shunt in
 169 Section 2.1. The shunt circuit, its coupling with the electro-mechanical struc-
 170 ture and its tuning procedure are presented in this section. Particularly, two

different electrical circuits will be considered. The parallel LRLC and the series LRLC, discussed in Sections 3.1 and 3.2, respectively. The absorber system is similar to a mechanical vibration absorber, suspended by either a shunted piezoelectric [50] or electromagnetic [43] transducer. However, in the present case, the absorber is realized entirely by a shunt and thus without a physical vibratory mass.

3.1. The parallel LRLC

Figure 5: Circuit diagram of the parallel LRLC shunt (a) and its mechanical equivalent (b).

The shunt impedance discussed in this section has the layout shown in Fig. 2c. When it is connected to the piezoelectric actuator, the electrical model of Fig. 5a is obtained. Figure 5a shows that a null value of R short-circuits its branch. Therefore, no current flows through L and C (i.e. the current flows through the short-circuited branch) and, consequently, only a single resonance is created by the transducer capacitance C_s and the leading shunt inductance L_0 .

Conversely, when R is not null, current flows through the L and C branches and, thus, an additional resonance is introduced. In comparison with the classical LR shunt, the proposed layout adds a supplementary resonance that will improve performance and robustness, when calibrated properly.

3.1.1. FRF

As already mentioned for the LR shunt, using the impedance analogy between electrical and mechanical systems, the electrical model of Fig. 5a may be translated into the equivalent mechanical model shown in Fig. 5b. The applied electro-mechanical equivalence is similar to that employed in the previously referenced works [21, 22] related to the balanced calibration of the LR shunt, in order to obtain a parallelism between the analytical treatments. Thus, the mechanical parameters are again simply obtained by multiplying the electrical parameters by θ_s^2 .

Using the approach employed in Section 2.1, and taking into account the parallel impedance layout in Eq. (6), the FRF of the parallel LRLC system can be derived as:

$$\frac{u_s k_s}{f_{e,s}} = \frac{(-r^2 \mu_0 + \kappa_0)G - r^2 \mu \kappa}{(-r^2 + 2ir\zeta_s + 1)\{(-r^2 \mu_0 + \kappa_0)G - r^2 \mu \kappa\} - r^2 \kappa_0(\mu_0 G + \kappa \mu)} \quad (20)$$

where the frequency function G for the parallel circuit in Fig. 5a is:

$$G = -r^2 \mu + ir \frac{\mu \kappa}{\beta} + \kappa \quad (21)$$

while r is the normalised frequency:

$$r = \frac{\omega}{\omega_s} \quad (22)$$

The remaining system ratios in Eq. (20) are:

$$\kappa_0 = \frac{k_0}{k_s}, \mu_0 = \frac{m_0}{m_s}, \kappa = \frac{k}{k_s}, \mu = \frac{m}{m_s}, \beta = \frac{c}{\sqrt{m_s k_s}} \quad (23)$$

where m_0 , k_0 , m , c and k are the parameters of the equivalent mechanical representation of the LRLC circuit, as defined in Fig. 5b.

205 The denominator of the FRF in Eq. (20) is of sixth order and, for low
 206 to moderate values of damping, three complex conjugate pairs of eigenvalues
 207 thereby exist. For the balanced calibration of the circuit, it is possible to follow
 208 the same procedure used for the LR shunt calibration, briefly summarised in
 209 Section 2.1.

210 3.1.2. Characteristic equation

211 This tuning technique is based on the pole placement principle and the
 212 optimal parameters for the electrical circuit are derived by imposing some con-
 213 straints on the system poles when neglecting structural damping ($\zeta_s=0$). More
 214 specifically, the first requirement is to have two pairs of eigenvalues with equal
 215 modal damping. As mentioned, according to [22], this implies that two eigenval-
 216 ues are inverse points with respect to a circle of radius ω_0 in the complex plane.
 217 This condition is secured by the following characteristic polynomial equation
 218 [49]:

$$\omega^4 - 2(1 + 2\chi^2)\omega_0^2\omega^2 + \omega_0^4 - 4i\chi\tau\omega_0\omega(\omega^2 - \omega_0^2) = 0 \quad (24)$$

219 where χ and τ are parameters depending on the frequency and the damping
 220 associated with the eigenvalues.

221 The second requirement on the pole positions is to have the last pair of poles
 222 at a frequency value equal to ω_0 , thus leading to the following polynomial form:

$$-\omega^2 + 2i\zeta_3\omega_0\omega + \omega_0^2 = 0 \quad (25)$$

223 where ζ_3 is the non-dimensional damping ratio associated with the considered
 224 eigenvalues. Using the conditions of Eqs. (24) and (25), the following resulting
 225 equation can be constructed as:

$$[\omega^4 - 2(1 + 2\chi^2)\omega_0^2\omega^2 + \omega_0^4 - 4i\chi\tau\omega_0\omega(\omega^2 - \omega_0^2)](-\omega^2 + 2i\zeta_3\omega_0\omega + \omega_0^2) = 0 \quad (26)$$

226 Equation (26) can also be expressed as a function of the dimensionless root
 227 $\xi = \omega/\omega_0 = r/\Omega_0$ with $\Omega_0 = \omega_0/\omega_s$, leading to:

Table 1: Conditions on the coefficients of Eq. (27)

quantity	value
ratio between the coefficients of the 6-th and 0-th order terms of Eq. (27)	-1
ratio between the coefficients of the 5-th and 1-st order terms of Eq. (27)	1
ratio between the coefficients of the 4-th and 2-nd order terms of Eq. (27)	-1

$$\begin{aligned}
& -\xi^6 + \xi^5(2i\zeta_3 + 4i\chi\tau) + \xi^4[1 + 8\zeta_3\chi\tau + 2(1 + 2\chi^2)] + \\
& \xi^3[-8i\chi\tau - 2i\zeta_3(2 + 4\chi^2)] + \xi^2[-1 - 8\zeta_3\chi\tau - 2(1 + 2\chi^2)] + \\
& \xi(2i\zeta_3 + 4i\chi\tau) + 1 = 0
\end{aligned} \tag{27}$$

3.1.3. Equal damping calibration

From Eq. (27), it is evident that the condition of equal modal damping for four of the poles and the condition related to the value of the eigenfrequency on the other two translate into the three conditions gathered in Tab. 1.

To derive the tuning conditions it is sufficient to apply the abovementioned requirements about the pole locations to the actual poles of the characteristic polynomial, which can be obtained from the denominator of Eq. (20). Indeed, the denominator of Eq. (20) can be expressed in terms of ξ and posed equal to zero in order to derive the eigenvalues in terms of the lumped physical parameters:

$$\begin{aligned}
& -\xi^6 + \xi^4 \frac{1}{\Omega_0^2} \left(1 + \kappa_0 + \frac{\kappa + \kappa_0}{\mu_0} + \frac{\kappa}{\mu}\right) - \xi^2 \frac{1}{\Omega_0^4} \left[\frac{\kappa}{\mu}(1 + \kappa_0) + \frac{\kappa_0}{\mu_0} \left(1 + \frac{\kappa}{\kappa_0} + \frac{\kappa}{\mu} + \kappa\right)\right] + \\
& \frac{\kappa\kappa_0}{\Omega_0^6\mu\mu_0} + i\xi \frac{\kappa}{\Omega_0\beta} \left[\xi^4 - \xi^2 \frac{1}{\Omega_0^2} \left(1 + \frac{\kappa_0}{\mu_0} + \kappa_0\right) + \frac{\kappa_0}{\Omega_0^4\mu_0}\right] = 0
\end{aligned} \tag{28}$$

Requiring the equality between the coefficients of Eqs. (28) and (27) (thus obtaining six equations: from the 5-th order to the 0-th order) allows to find the

values of the physical electrical parameters, which thereby satisfy the condition of equal modal damping. Three of these equations can be replaced by imposing the three conditions stated in Tab. 1, simplifying the solution process. Applying these three conditions to the coefficients of Eq. (28) (terms of order 0, 1, 2, 4, 5 and 6), the following three equations are obtained:

$$\frac{\kappa\kappa_0}{\Omega_0^6\mu\mu_0} = 1 \quad (29)$$

$$\frac{\kappa_0}{\Omega_0^4\mu_0} = 1 \quad (30)$$

$$\frac{1}{\Omega_0^2}\left(1 + \kappa_0 + \frac{\kappa + \kappa_0}{\mu_0} + \frac{\kappa}{\mu}\right) = \frac{1}{\Omega_0^4}\left[\frac{\kappa}{\mu}(1 + \kappa_0) + \frac{\kappa_0}{\mu_0}\left(1 + \frac{\kappa}{\kappa_0} + \frac{\kappa}{\mu} + \kappa\right)\right] \quad (31)$$

whose solution leads to the following parameter relations:

$$\mu_0 = \frac{\kappa_0}{(1 + \kappa_0)^2}, \mu = \frac{\kappa}{1 + \kappa_0}, \Omega_0^2 = 1 + \kappa_0 \quad (32)$$

The value of κ_0 depends on the physical properties of the electro-mechanical system and is therefore considered known, while the value of μ_0 (thus L_0) can be readily found from Eq. (32). When deriving the value of μ (thus L) as a function of κ (thus C) using Eq. (32), five parameters are still unknown: κ , β , ζ_3 , χ and τ . Three equations out of the six original equations of the problem have been already used to derive Eq. (32) (see Tab. 1). Hence, the problem is overdetermined and two parameters must be chosen by the user or derived by adding additional constraints to the problem. In this paper, it is chosen to formulate an additional control target to derive the values of κ and β . Their tuning procedure is described in the next subsection.

3.1.4. Amplitude minimisation

The additional requirement in this case is the minimisation of the H_∞ norm of the FRF. Indeed, the value of β that guarantees, for a given value of κ , the minimisation of the peak of the dynamic amplification (i.e. H_∞ control) has

Figure 6: Trends of the absolute values of the real and imaginary parts of the squared roots (LRLC in parallel layout): $\kappa = 0.10\kappa_0$ (a), $\kappa = 0.15\kappa_0$ (b) and $\kappa = 0.20\kappa_0$ (c). The value of κ_0 is 0.02.

260 been chosen in this paper. To find this optimal value, a numerical minimisation
 261 of the maximum of the FRF amplitude of the electro-mechanical system must be
 262 carried out. Indeed, an analytical solution is not straightforward to be found. It
 263 is also noticed that the user can choose the β value according to another control
 264 target (e.g. H_2 control) or to the desired level of vibration mitigation. Once the
 265 conditions for equal modal damping are set, providing closed-form analytical
 266 formulas for the values of μ_0 and μ , the subsequent numerical minimisation
 267 finds the optimal κ and β values according to the desired performance target
 268 (H_∞ optimisation in this case).

269 The choice of the optimal κ value arises from considerations on the shape of
 270 the root locus of the controlled system as a function of this parameter. Figure 6
 271 shows the complex root trajectories in the ξ^2 -plane for three different values of
 272 κ/κ_0 , chosen as an example. In the plots of Fig. 6, the red crosses indicate the
 273 roots for $\beta \rightarrow \infty$, the circles are for $\beta = 0$, while the asterisks represent the roots
 274 obtained with the optimal β value from an H_∞ -norm optimisation. As expected
 275 from the requirements imposed on the pole positions in Eqs. (27) and (28), one
 276 of the squared poles lies on a circle with unit radius, while the other two are

277 inverse points with respect to this circle. It is possible to demonstrate that,
 278 if the normalised eigenvalues lie on the same line containing the origin of the
 279 complex plane, and thus represent inverse points with respect to a circle of unit
 280 radius, the same holds for the squared eigenvalues. Looking at the three loci of
 281 Fig. 6 (that are all characterised by the equal modal damping condition), it can
 282 be noticed that, by decreasing the value of κ/κ_0 , the trajectories described by
 283 two of the squared poles (those not on the circle with unitary radius) separate
 284 into two different side lobes (see Fig. 6a) passing through a bifurcation condition
 285 (see Fig. 6b), where all three squared poles coincide for a certain value of β .
 286 Although this bifurcation point looks as the best choice from the modal damping
 287 point of view, it actually leads to a high modal coupling and thereby to a non-
 288 optimal solution in terms of dynamic amplification, as explained in [21, 48].
 289 Furthermore, a shunt without roots near a bifurcation point in the complex
 290 plane is expected to be robust with respect to calibration because the roots are
 291 well separated. Therefore, the optimal solution in terms of H_∞ optimisation is
 292 achieved with a κ value such that the complex roots are sufficiently separated,
 293 which is furthermore expected to provide good robustness.

294 3.1.5. Bifurcation point

295 The condition mentioned above (to guarantee the desired amount of modal
 296 damping, while keeping the complex roots sufficiently separated) occurs for a κ
 297 value that is lower than for the bifurcation point, as indicated by the asterisks
 298 in Fig. 6 (optimal by the H_∞ norm of Eq. (20)) and their respective FRF
 299 amplitudes shown in Fig. 7. The dynamic amplification for the optimal β value
 300 and $\kappa = 0.1\kappa_0$ (see Fig. 7a) is indeed lower than for the other two cases in Figs.
 301 7b and c. From this analysis, it can be concluded that the optimal value of κ
 302 must be searched numerically among all the values lower than that leading to
 303 the bifurcation point of the three squared roots. This threshold $\kappa_{\text{thr,p}}$ can be
 304 derived analytically by noticing that all the three squared poles have the same
 305 frequency and damping values in the bifurcation point and thus are all placed
 306 on the circle with unitary radius in the complex ξ^2 -plane. Therefore, they must

Figure 7: Amplitude of the FRF (LRLC in parallel layout, see Eq. (20)): $\kappa = 0.10\kappa_0$ (a), $\kappa = 0.15\kappa_0$ (b) and $\kappa = 0.20\kappa_0$ (c). The value of κ_0 is 0.02 and the β value is set according to the H_∞ -norm optimisation (see the asterisks in Fig. 6).

307 satisfy the following condition:

$$\begin{aligned}
 &(-\xi^2 + 2i\xi\zeta + 1)^3 = 0 \Rightarrow -\xi^6 + 6\zeta i\xi^5 \\
 &+ (3 + 12\zeta^2)\xi^4 - (12\zeta + 8\zeta^3)i\xi^3 - (3 + 12\zeta^2)\xi^2 + 6\zeta i\xi + 1 = 0
 \end{aligned} \tag{33}$$

308 If the first, second and third order terms of Eq. (33) are equated to those of
 309 Eq. (28), using the expressions in Eq. (32), the value of $\kappa_{\text{thr,p}}$ is found as:

$$\kappa_{\text{thr,p}} = \frac{8\kappa_0^2}{1 + \kappa_0} \tag{34}$$

310 The optimal κ value is then in the following chosen less than $\kappa_{\text{thr,p}}$ to avoid
 311 the bifurcation point (see Section 3.3).

312 3.2. The series LRLC

313 The shunt discussed in this section has the layout shown in Fig. 2d and
 314 the approach used to derive the FRF of the controlled system is the same as
 315 described for the parallel shunt in Section 3.1. As in the case of the parallel

Figure 8: Circuit diagram of the series LRLC shunt (a) and its mechanical equivalent (b).

316 LRLC, the electrical model of Fig. 8a can be converted into the equivalent
 317 mechanical model shown in Fig. 8b.

318 Figure 8a shows that, for $R \rightarrow \infty$, a single resonance is created by the
 319 transducer capacitance C_s and the leading shunt inductance L_0 , while, for finite
 320 values of R (i.e. decreasing R from ∞ to zero), an additional resonance is
 321 introduced by the shunt components C and L .

3.2.1. FRF

323 Using the same approach as employed in Section 3.1, the FRF of the system
 324 can be derived:

$$\frac{u_s k_s}{f_{e,s}} = \frac{(-r^2 \mu_0 + \kappa_0)E - r^2 \mu_0 \kappa_0}{(-r^2 + 2ir\zeta_s + 1)\{(-r^2 \mu_0 + \kappa_0)E - r^2 \mu_0 \kappa_0\} - r^2 \kappa_0 \mu_0 E} \quad (35)$$

where the frequency function E is defined as:

$$E = -r^2 \mu + ir\beta + \kappa \quad (36)$$

The parameters κ_0 , μ_0 , κ , μ and β are defined as in Eq. (23). Furthermore, by looking at the functions E and G (see Eq. (21)), it is possible to notice that the damping term is defined differently in the series and parallel layouts.

As already mentioned, the optimal shunt parameters are derived from conditions on the system poles. The system eigenvalues can be found by expressing the denominator of the FRF in Eq. (35) as a function of the dimensionless frequency ξ (neglecting the mechanical damping) and letting it equal zero:

$$-\xi^6 + \xi^4 \frac{1}{\Omega_0^2} \left(1 + \kappa_0 \left(1 + \frac{1}{\mu_0} + \frac{1}{\mu}\right) + \frac{\kappa}{\mu}\right) - \xi^2 \frac{1}{\Omega_0^4} \left[\frac{\kappa_0 \kappa}{\mu_0 \mu} + \kappa_0 \left(\frac{1}{\mu_0} + \frac{1 + \kappa}{\mu}\right) + \frac{\kappa}{\mu}\right] + \frac{\kappa \kappa_0}{\Omega_0^6 \mu \mu_0} + i\xi \frac{\beta}{\Omega_0 \mu} \left[\xi^4 - \xi^2 \frac{1}{\Omega_0^2} \left(1 + \kappa_0 + \frac{\kappa_0}{\mu_0}\right) + \frac{\kappa_0}{\Omega_0^4 \mu_0}\right] = 0 \quad (37)$$

3.2.2. Equal damping calibration

The three conditions of Tab. 1 can then be applied to the coefficients (of the terms of order 0, 1, 2, 4, 5 and 6) of Eq. (37) to require equal modal damping, leading to the following three equations:

$$\frac{\kappa \kappa_0}{\Omega_0^6 \mu \mu_0} = 1 \quad (38)$$

$$\frac{\kappa_0}{\Omega_0^4 \mu_0} = 1 \quad (39)$$

$$\frac{1}{\Omega_0^2} \left(1 + \kappa_0 \left(1 + \frac{1}{\mu_0} + \frac{1}{\mu}\right) + \frac{\kappa}{\mu}\right) = \frac{1}{\Omega_0^4} \left[\frac{\kappa_0 \kappa}{\mu_0 \mu} + \kappa_0 \left(\frac{1}{\mu_0} + \frac{1 + \kappa}{\mu}\right) + \frac{\kappa}{\mu}\right] \quad (40)$$

The solution to these equations gives the following three parameter relations:

$$\mu_0 = \kappa_0, \mu = \kappa, \Omega_0^2 = 1 \quad (41)$$

As in the case of the parallel LRLC shunt, the value of μ_0 (and thus of L_0) can be set according to Eq. (41), while the other parameters must be derived solving the overdetermined system of three equations and five unknowns. Therefore, also in this case, the values of β and κ need to be set by a numerical minimisation.

3.2.3. Amplitude minimisation

Figure 9: Trends of the absolute values of the real and imaginary parts of the squared roots (LRLC in series layout): $\kappa = 5\kappa_0$ (a), $\kappa = 10\kappa_0$ (b) and $\kappa = 20\kappa_0$ (c). The value of κ_0 is 0.02.

The value of β is obtained from the same criterion as before, looking for the minimisation of the peak of the dynamic amplification for a given value of κ relative to κ_0 . To set the κ value, it is again possible to look at the system root locus. Figure 9 shows, for the series LRLC, the trajectories of the squared roots in the complex plane for three different values of κ/κ_0 with respect to varying values of β . The crosses represent the roots when $\beta \rightarrow \infty$, while the circles are the roots when β is null. Moreover, the asterisks are related to the optimal β value. Also in this case, a condition where the complex poles follow separated trajectories is achievable by increasing the κ value above a threshold

Figure 10: Amplitude of the FRF (LRLC in series layout, see Eq. (35)): $\kappa = 5\kappa_0$ (a), $\kappa = 10\kappa_0$ (b) and $\kappa = 20\kappa_0$ (c). The value of κ_0 is 0.02 and the β value is set according to the H_∞ -norm optimisation (see the asterisks in Fig. 9).

value represented by the bifurcation of the roots (see Fig. 9b). By looking at the optimal positions of the roots in Fig. 9 (i.e. the asterisks) and their corresponding FRF amplitudes in Fig. 10, it is found that the situation characterised by two separated side lobes provides the highest attenuation level and the desired separation of the roots relative to the bifurcation point. Therefore, the ratio $\kappa/\kappa_0=20$ secures the desired damping and response mitigation, as shown in Figs. 9c and 10c.

3.2.4. Bifurcation point

As in the case of the parallel LRLC, it is possible to find the threshold value of $\kappa = \kappa_{\text{thr},s}$, above which the separation of the side lobes in Fig. 9c is guaranteed. This threshold value is the κ value for which it is possible to have the bifurcation point, where all the roots exhibit the same frequency and damping and lie on the circumference with a unitary radius in the complex ξ^2 -plane. This condition is achieved if the poles of the system, expressed by Eq. (37), satisfy the requirement of Eq. (33). Therefore, by equating the first, second and third order terms of these two equations and then applying the equal

369 modal damping conditions in Eq. (41), the value of $\kappa_{\text{thr},s}$ becomes:

$$\kappa_{\text{thr},s} = \frac{1}{8} \quad (42)$$

370 It should be noticed that in this case the value of κ must be increased
 371 above this value for the two desired individual loci to appear (as in Fig. 9c).
 372 Conversely, in the parallel case, a decrease of the κ value leads to the desired
 373 separation of the loci. According to Eqs. (32) and (41), this implies that the
 374 optimal values of the inductance L will be larger in the series than in the parallel
 375 LRLC shunt.

376 3.3. The optimal value of κ

377 Sections 3.1 and 3.2 showed that the optimal value of κ must be sought in a
 378 range of values either below or above a certain bifurcation threshold, according
 379 to the circuit layout considered (parallel or series). The optimal κ value will be in
 380 the considered range that guarantees the maximum attenuation level when the
 381 corresponding optimal β value minimises the H_∞ -norm of the FRF amplitude
 382 in Eqs. (20) or (35). For the case of the parallel LRLC, the value of κ must be
 383 smaller than $\kappa_{\text{thr},p}$, and thus obtained between zero and $\kappa_{\text{thr},p}$. The problem
 384 is more complicated in the case of the series LRLC. Indeed, from the analysis
 385 performed in Section 3.2, it has emerged that the value of κ must be larger
 386 than $\kappa_{\text{thr},s}$ but no information is available regarding a possible upper bound.
 387 However, it is possible to find the optimal value of κ searching in a range such
 388 that the value of $1/\kappa$ is between zero and $1/\kappa_{\text{thr},s}$, which is numerically more
 389 convenient.

390 Section 4 will show a numerical simulation where the classical LR shunt
 391 with balanced calibration and the new LRLC shunt will be compared in terms
 392 of attenuation performance and robustness with respect to potential mistuning.

393 4. The performances of the LRLC shunt

394 This section presents a numerical analysis in which the LRLC shunt is com-
 395 pared to the classical balanced calibration of the LR shunt in terms of vibration

Figure 11: FRF amplitude. $\kappa = 40\kappa_0$ for the LRLC shunt. Series configuration for both the LRLC and LR shunts.

Table 2: Values of the system parameters

$\omega_s/(2\pi)$ [Hz]	$\hat{\omega}_s/(2\pi)$ [Hz]	ζ_s	$\sqrt{\kappa_0}$	C_s [nF]	$ \theta_s $ [$\text{kg}^{-(1/2)}\text{NV}^{-1}$]
100.00	100.50	$3.0 \cdot 10^{-3}$	0.1	40.0	0.0126

attenuation. This comparison has been carried out for both perfect tuning and mistuning. Indeed, the situation of mistuning is likely to be faced in real applications due to changes of the parameters of either the shunt impedance or the primary system to be damped by, for example, thermal shifts. The performance analysis in mistuned conditions also allows to validate the overall robustness of the proposed LRLC shunt.

The analysis is presented here considering a specific system chosen as an example, whose characteristics are gathered in Tab. 2. Nevertheless, the outcomes of the analysis can be generalised to any mechanical system equipped with a piezoelectric actuator; just a single example is shown here for the sake of conciseness. Figure 11 shows the amplitude of the FRF in perfect tuning for the LRLC shunt (series layout) with $\kappa = 40\kappa_0$. This value is very close, but not exactly equal, to the optimal value of κ . However, this difference is negligible in terms of attenuation performance, as the attenuations achieved by $\kappa = 35\kappa_0$

Figure 12: The value of $A_{dB,num}$ as a function of α . The values used for κ in the LRLC shunt are: $35\kappa_0$, $40\kappa_0$ and $45\kappa_0$. Series configuration for both the LRLC and LR shunts.

and $\kappa = 45\kappa_0$ are marginally worse than that for $\kappa = 40\kappa_0$. Therefore, a more detailed search for the actual optimum of κ is practically useless. This point will be addressed again at the end of this section.

Looking at the curve related to the LRLC shunt in Fig. 11, it can be noticed that the presence of an additional resonance (compared to the classical LR shunt) characterises the proposed circuit and provides an improvement in terms of attenuation performance over the traditional LR shunt. Figure 11 shows the FRF amplitudes for both the LRLC and the LR shunt with balanced calibration and in series layout, clearly indicating the improved performance in terms of H_∞ control of the LRLC shunt, with an increase in attenuation of approximately 2.3 dB for this specific case.

Considering the robustness analysis with respect to possible mistuning of the shunt, a parameter α has been used and defined as:

$$\alpha = \frac{\omega_{cal}}{\omega_s} \quad (43)$$

where ω_{cal} is the frequency to which the shunt impedance has been tuned. The parameter α thus represents the amount of mistuning experienced by the system.

Figure 12 shows the attenuation value as a function of α , where the attenuation is expressed as:

$$A_{\text{dB,num}} = 20 \log_{10} \frac{H_{\text{sc}}}{H_{\text{shunt,num}}} \quad (44)$$

where H_{sc} is the maximum of the amplitude of the system FRF with the piezoelectric actuator short-circuited, while $H_{\text{shunt,num}}$ is the maximum of the amplitude of the system FRF with the considered shunt circuit in the simulated mistuned condition.

Figure 12 shows that the LRLC shunt improves the attenuation compared to the LR shunt, even in presence of mistuning for reasonable values of α . The higher robustness of the LRLC shunt is demonstrated by the trend of the curves for α values between 0.99 and 1. Indeed, in this range, the LRLC shunt curves show an almost flat plateau, while the classical LR shunt exhibits steep curves with a very local optimum. Therefore, the LRLC shunt appears more robust than the LR shunt, thanks to the additional resonance introduced by the new circuit, which allows to obtain a wider and flatter shape of the FRF of the controlled system. It is also noticed that a range of α of approximately 1%, which corresponds to the plateau of the LRLC curves, is close to the natural uncertainty that can be encountered in real applications and to the possible bias effects due to, for example, environmental changes. Furthermore, the width of the flat plateau increases significantly when the value of κ_0 increases, as evidenced with the system used in the experiments of Section 6. Therefore, when the characteristics of the piezoelectric actuator are optimised for controlling a given mode, which is a reasonable assumption in practical applications, the coupling factor is large [46] and the LRLC shunt becomes consistently more robust than the corresponding LR shunt.

Furthermore, it is noted that, if it is desired to lift the LRLC curves of Fig. 12 far from $\alpha=1$, it is sufficient to slightly decrease the value of κ (using the corresponding optimal value of β), accepting a slight (and often negligible) decrease in attenuation for $\alpha=1$. In addition, this increases the robustness of

453 the shunt due to a corresponding reduced slope of the curves.

454 Figure 12 also shows that for a value of $\kappa = 35\kappa_0$ (thus, lower than the value
455 of $\kappa = 40\kappa_0$ used in Fig. 11) and for a value of $\kappa = 45\kappa_0$ (thus a higher value
456 than in Fig. 11), the attenuation performance for $\alpha = 1$ (i.e. perfect tuning) is
457 sufficiently close to that achieved with $\kappa = 40\kappa_0$, which is therefore considered
458 optimal, as mentioned at the beginning of this section.

459 Outcomes similar to those presented so far for the series layouts are also
460 found in case of a comparison between the classical parallel LR shunt and the
461 new LRLC shunt in its parallel layout.

462 The tuning methods for both the LR and LRLC shunts are developed under
463 the hypothesis of low modal coupling, as mentioned in Section 2. This means
464 that the contribution of the out-of-band modes to the mechanical behaviour of
465 the electro-mechanical system dynamics is neglected. Instead, the contribution
466 of the out-of-band modes is taken into account in the electrical behaviour of
467 the electro-mechanical system by the term with C_s in the model of Section 2.
468 However, it is important to underline that in case the contribution of the out-of-
469 band modes to the mechanical part of the electro-mechanical system dynamics
470 is not negligible, because they are close in frequency to the target mode, the
471 effects on the attenuation provided by the two different shunts is expected to
472 be similar. Therefore, this additional effect will not affect the results of the
473 comparison between the two shunt impedances and the outcome of the analysis.
474 Moreover, looking at Fig. 11, it is evident that the FRFs related to the two
475 different shunts differ in a frequency range of about ± 10 to 20% of ω_s (see also
476 Section 6). Therefore, the out-of-band modes can change the results of the
477 comparison between the LR and LRLC shunts only in case they are very close
478 to the targeted mode, whereby the low modal coupling hypothesis (which is
479 the foundation of the proposed method) is not applicable anymore. However,
480 since the robustness of the LRLC shunt is higher than that of the LR shunt,
481 the LRLC shunt is expected to still provide a higher attenuation level compared
482 to the LR shunt, even in case of high influence from the out-of-band modes.
483 Finally, it is worth evidencing that even in the case the hypothesis of low modal

coupling is not satisfied, the result of the proposed LRLC shunt optimisation procedure (as well as in the case of an LR shunt) can be used as the starting point for a minimisation aimed at tuning the LRLC impedance using a multi-degree-of-freedom model like that described in [15].

5. Attenuation by the LRLC shunt

Figure 13: The value of $A_{dB,num}$ as a function of $\sqrt{\kappa_0}$ for different values of ζ_s (10^{-5} , 10^{-4} , 10^{-3} , 10^{-2} and $5 \cdot 10^{-2}$) (a) and the corresponding attenuation improvement provided by the LRLC shunt (b). Series configuration for both the LRLC and LR shunts.

The results shown in Fig. 11, obtained for a system chosen as an example, can be generalised noticing that the FRFs associated to the classical LR shunt (in normalised form using r in place of ω in Eq. (8)) and the new LRLC shunt (see Eq. (35)) are dependent on only two parameters of the electro-mechanical system: κ_0 and ζ_s . Therefore, it is possible to numerically compute the attenuation provided by the two different impedance layouts as a function

Table 3: Values of the experimental parameters

$\omega_s/(2\pi)$ [Hz]	$\hat{\omega}_s/(2\pi)$ [Hz]	ζ_s	$\sqrt{\kappa_0}$	C_s [nF]	$ \theta_s $ [$\text{kg}^{-(1/2)}\text{NV}^{-1}$]
34.29	35.44	$4.5 \cdot 10^{-3}$	0.2602	39.92	0.0112

of κ_0 and ζ_s . For κ_0 , the authors chose to consider values from 0.01^2 (very low value) to 0.31^2 (close to the largest value encountered in practice [46]). For ζ_s , the authors have used five different values (from very low to very high for typical mechanical systems).

Figure 13a shows the resulting attenuation values $A_{\text{dB,num}}$ (in tuning) for the two different shunts. Usually, the LRLC shunt allows to have attenuation improvements between 2 and 2.5 dB (see Fig. 13b), except for low values of κ_0 coupled to high values of ζ_s where the improvement decreases. However, this improvement is still significant since also the overall attenuation decreases in this case. Therefore, Fig. 13 can be used as an abacus for predicting the improvement provided by the LRLC shunt compared to the classical LR shunt with equal modal damping calibration.

The improvement provided by the LRLC shunt is most of the time larger than 2 dB, and thus, in percentage, the LRLC shunt allows to further decrease the peak of the FRF amplitude of more than 25% (and many times of more than 30%). This result is in accordance with the results usually obtained from multiple-TVA configurations when compared to situations where a single TVA is used (e.g. [40, 43]). The result is remarkable, especially in light of the fact that the improvement is obtained by a completely passive approach. **However, it is important to remember that, in case operational amplifiers (OP-AMP) are needed to build the inductances of the shunt circuit because of their high values (see Section 6), the approach can be considered as passive from a dynamical point of view, although not strictly passive with respect to power consumption (i.e. OP-AMPs need a power supply).**

Figure 14: The experimental set-up.

Figure 15: The electrical layout used for the LRLC shunt.

519 6. Experiments

520 This section presents the experimental tests carried out with the aim of
 521 validating the theoretical outcomes shown previously. The set-up used was
 522 made from a stainless steel cantilever beam with a length equal to 180 mm, a
 523 width of 30.5 mm and a thickness of 1.1 mm. Two piezoelectric patches (length
 524 70 mm, width 30.0 mm and thickness 0.55 mm, material PIC 151) were bonded
 525 at the clamped end of the beam (one on each side of the beam) and electrically
 526 connected in series. The system was forced by means of a contactless magnetic

Figure 16: Experimental and numerical FRF amplitudes (in terms of displacement over force) in short-circuit and in tuned condition.

527 actuator [51] and the corresponding vibration response was collected by using
528 a laser velocimeter. This set-up, shown in Fig. 14, is the same already used in
529 [18], from which interested readers can find more details.

530 The first mode of the structure was considered for the tests because its
531 amplitude was higher than that of the other modes in the low frequency range.
532 Its eigenfrequencies with the patches in short- (ω_s) and open-circuit ($\hat{\omega}_s$), as well
533 as the mechanical non-dimensional damping ratio ζ_s , were identified by means of

Table 4: Values of the electrical parameters of the shunt impedances

type of shunt	L [H]	R [k Ω]	C [nF]	L_0 [H]
LR shunt (balanced calibration)	472.9	38.89	–	–
LRLC	3100.7	367.0	6.95	539.7

Figure 17: Experimental and numerical attenuation as a function of α .

an experimental modal analysis (see Tab. 3). The value of θ_s was estimated by means of Eq. (13), where knowledge of the C_s value is required. This value was estimated by measuring the capacitance of the piezoelectric patch as a function of ω with an LCR meter (see [18] for more details). All the estimated system parameters are gathered in Tab. 3.

The classical LR shunt with balanced calibration (see Section 2.1) and the new LRLC shunt (see Section 3) were compared in series layout. All the inductances were built using OP-AMPs with the Antoniou's circuit [6, 52] because of the high inductance values required. The whole electrical layout employed for the LRLC impedance is shown in Fig. 15. It is noticed that the layout of L_0 is different from that of L . Indeed, in the circuit of L_0 , there is the variable resistance $P_{2,0}$, which is used to compensate for the parasitic resistances usually present when employing OP-AMPs to build inductances [6]. The presence of this potentiometer for L_0 is important because L_0 is the only element on its branch (see Fig. 2d) and thus the parasitic resistance must be minimised. The value of $P_{2,0}$ was set in order to have a parasitic resistance slightly positive and not exactly null, as this would have increased the risk of instabilities from e.g.

551 a thermal shift imposing a negative parasitic resistance. The value of the par-
 552 asitic resistance was estimated close to 500Ω . Numerical simulations showed
 553 that such a resistance would not cause significant changes in the attenuation
 554 performance compared to the ideal case without parasitic resistance. The vari-
 555 able resistance $P_{2,0}$ was not necessary for L because this inductance is in series
 556 with the resistance R and thus it was easy to compensate the presence of the
 557 parasitic resistance by changing the value of R accordingly. All the OP-AMPs
 558 (OPA 445 type) were supplied with a constant voltage of ± 30 V.

559 Figure 16 shows the numerical and experimental FRF amplitudes (in terms
 560 of displacement over force) in the tuned condition for the classical LR shunt with
 561 balanced calibration and the new LRLC impedance. The match between exper-
 562 imental and numerical data is good. Furthermore, the LRLC shunt achieves,
 563 as expected, a higher attenuation performance over the classical LR shunt (ex-
 564 perimental improvement of approximately 2.1 dB). The values of the various
 565 electrical parameters used for the shunts are provided in Tab. 4. It is noticed
 566 that the value of the capacitance C for the LRLC shunt is not optimal (opti-
 567 mal value is 7.36 nF and used value is 6.95 nF). The use of this non-optimal
 568 value was due to the available capacitors in the laboratory. However, this dif-
 569 ference does not cause any significant change in terms of vibration attenuation
 570 (as evidenced by numerical simulations) and thus it was considered acceptable.

571 In order to validate the results related to the robustness of the LRLC shunt,
 572 also tests in mistuned conditions were performed, as shown in Fig. 17. This
 573 figure presents the value of the attenuation $A_{\text{dB,num}}$ as a function of the mistun-
 574 ing index α (see Eq. (43)). It is worth highlighting that there is a difference in
 575 the way of causing mistuning in the experiments with respect to what has been
 576 described in Section 4, where the mistuning was obtained by simulating a shift
 577 of the actual eigenfrequency value of the system (i.e. simulating a plausible real
 578 situation). Conversely, the mistuning is here obtained by changing the frequency
 579 value to which the shunt is optimised, while the actual eigenfrequency of the
 580 structure does not change. However, the meaning of the coefficient α does not
 581 differ and it is still an index of mistuning. As can be noticed by looking at the

experimental points in Fig. 17, also in this case there is a good match between numerical and experimental results. Moreover, the improved performance of the new LRLC shunt is experimentally demonstrated, as the LRLC shunt provides higher attenuation values compared to the classical LR shunt for all the tested values of α . The higher robustness of the LRLC shunt is also demonstrated by the trend of the theoretical curves in Fig. 17 for α values between 0.95 and 1, within which the LRLC shunt curve has an almost flat plateau, while the classical LR shunt exhibits steep curves with a very local optimum.

7. Conclusion

The paper has presented a new type of impedance to be employed when mono-modal vibration control is carried out with a piezoelectric shunt. The new impedance can have two different layouts. However, in both cases the driving idea is that the shunt impedance must be such that it generates two different resonances. To this purpose, it is composed of two inductances, one capacitance and one resistance, comprising the resulting LRLC shunt.

Guidelines are provided for setting all the electrical parameters of the shunt impedance. The new shunt network is found to be reliable and to provide better attenuation performance than the classical LR shunt based on balanced calibration. The benefits of the newly proposed impedance are evident in both tuned and mistuned situations.

The theoretical outcomes have been validated by means of a test set-up in which the inductances used for the shunt have been built by OP-AMPs. Good agreement between theory and experiments has been obtained, validating the proposed shunt concept. A future study should address the power consumption by the augmented LRLC shunt when using OP-AMPs to build inductances.

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774 8. List of table captions

775 Table 1: Conditions on the coefficients of Eq. (27).

776 Table 2: Values of the system parameters.

777 Table 3: Values of the experimental parameters.

778 Table 4: Values of the electrical parameters of the shunt impedances.

779 9. List of figure captions

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781 Figure 2: Traditional resonant shunt impedance in parallel (a) and series
 782 (b), and the new LRLC shunt impedance in parallel (c) and series (d).

783 Figure 3: Circuit diagram of the parallel LR shunt (a) and its mechanical
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785 Figure 4: Circuit diagram of the series LR shunt (a) and its mechanical
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787 Figure 5: Circuit diagram of the parallel LRLC shunt (a) and its mechanical
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789 Figure 6: Trends of the absolute values of the real and imaginary parts of
 790 the squared roots (LRLC in parallel layout): $\kappa = 0.10\kappa_0$ (a), $\kappa = 0.15\kappa_0$ (b)
 791 and $\kappa = 0.20\kappa_0$ (c). The value of κ_0 is 0.02.

792 Figure 7: Amplitude of the FRF (LRLC in parallel layout, see Eq. (20)):
 793 $\kappa = 0.10\kappa_0$ (a), $\kappa = 0.15\kappa_0$ (b) and $\kappa = 0.20\kappa_0$ (c). The value of κ_0 is 0.02 and
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796 Figure 8: Circuit diagram of the series LRLC shunt (a) and its mechanical
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798 Figure 9: Trends of the absolute values of the real and imaginary parts of the
799 squared roots (LRLC in series layout): $\kappa = 5\kappa_0$ (a), $\kappa = 10\kappa_0$ (b) and $\kappa = 20\kappa_0$
800 (c). The value of κ_0 is 0.02.

801 Figure 10: Amplitude of the FRF (LRLC in series layout, see Eq. (35)):
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803 β value is set according to the H_∞ -norm optimisation (see the asterisks in Fig.
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807 Figure 12: The value of $A_{\text{dB,num}}$ as a function of α . The values used for κ
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817 Figure 17: Experimental and numerical attenuation as a function of α .