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Mode-resolved Directional Enhancement of Spontaneous Emission Inside/Outside Finite Multilayer Hyperbolic Metamaterials

Maryam Mahmoodi^a, Seyed Hassan Tavassoli^{a,*}, Andrei V. Lavrinenko^b

^aLaser and Plasma Research Institute, Shahid Beheshti University, Velenjak, Tehran, Iran

^bTechnical University of Denmark, Dept. of Photonics Engineering, Ørstedss plads, bldg. 345V, 2800 Kgs. Lyngby, Denmark

Abstract

The concept of large photonic density of states associated with high-k modes of Hyperbolic Metamaterials (HMMs) plays a significant role in the interests behind the HMM notion. Despite the theoretical predictions, it is proved that photonic Local Density of States (LDOS) is not infinite in practical multilayer HMMs. In the present study, a developed theory based on the effective medium approximation is used to probe the finite number of high-k modes in finite multilayer HMMs. The High-k modes are classified in two categories as short-range and long-range propagating modes, strongly dependent on the number of periods. Introducing a mode-resolved transition rate enhancement factor, emission coupling of a quantum emitter, placed inside and in near vicinity of HMM slabs, to the high-k modes is calculated. The results show that the short-range and long-range modes have the commanding influence on transition rate enhancement outside and inside the slabs, respectively. The results point out that HMM slabs having lower number of periods provide higher total LDOS giving rise to larger transition rate enhancement. The results prove the capability of engineering photonic density of states in finite HMMs and pave the way for application-orientated HMMs design.

Keywords: Local Photonic Density of States, Spontaneous Emission Transition Rate, Hyperbolic Metamaterials, High-k Modes

1. Introduction

Hyperbolic Metamaterials (HMMs) hold promise to embody the boundless possibilities in light-matter interaction on nanoscale[1]. Artificial materials having hyperbolic dispersion have been designed for various applications such as nano focusing [2], sensing [3, 4], wave front manipulation [5], sub diffraction imaging [6, 7], emission enhancement of quantum emitters [8, 9] and many more. HMMs have attracted a great attention for spontaneous emission enhancement due to the large photonic Local Density of States (LDOS) associated with their unique modes having large wave vector, so called high-k modes[10, 11]. The significance of LDOS in quantum photonics is evidenced in the Fermi's golden rule expressing that manipulation of photonic local density of states can change the spontaneous emission lifetime (transition rate) of a Quantum Emitter (QE) [12]. Directional propagation of the high-k modes is proved as another unique feature of the hyperbolic metamaterials[13] promising for improving emission collection of the QEs coupled to the HMM modes[14]. Artificial materials having nanometer-size building blocks like multilayer stacks [15, 16, 17] and nanowires [18, 19] in hyperbolic regime, have shown capabilities in engineering directional enhancement of quantum emitters spontaneous emission[20].

Based on the conventional Effective Medium Approximation (EMA)[21], the open form iso-frequency surface (ideal hyperboloid) of the HMMs have no upper limit for the wave vector of propagating waves. It leads to the conclusion that HMMs could

provide infinite density of states. Yet, in practice, the constraints imposed by thickness of the building blocks and nonlocal effects proved that the LDOS of HMMs must be finite [22, 23]. Affiliation of LDOS to the high-k modes and the relevance of such modes to the number of periods in multilayer structures[24], creates an urge to study the individual high-k modes in finite thickness structures. Since the number of periods in HMM structures is a very practical question, an analysis of contribution of individual modes of finite thickness HMMs in total LDOS is needed to investigate the necessary number of periods required for providing a large LDOS in application-orientated multilayer HMMs. While the Green's function calculations[25, 26] and reflection coefficients of multilayer structures[27, 28] as largely performed methods in literature do not provide an insight into the individual high-k modes LDOS, here we report the first demonstration of a mode-resolved LDOS and propagation direction calculation in multilayer hyperbolic metamaterials.

To account for the practical finite number of periods, a theory based on the optical waveguide rules is developed for the finite thickness HMM slabs homogenized using a nonlocal EMA. We managed to find the dispersion relations of the individual high-k modes in any practical multilayer structures. Two classes of high-k modes in HMMs are distinguished and named as short-range (SR) and long-range (LR) propagating modes. A quantum emitter is considered inside and nearby the HMM slabs to investigate contribution of the LR and SR high-k modes to transition rate enhancement and directionality of the spontaneous emission using a pragmatic theoretical approach. Our findings demonstrate that the lower the number of period is, the larger the LDOS of HMM slabs will be. Meanwhile, the SR

*h-tavassoli@sbu.ac.ir

modes provide larger enhancement factors for QEs placed outside the slabs and the LR modes take the role for QEs located inside the HMM slabs.

2. Theory

2.1. High-k modes in finite HMMs

Multilayer hyperbolic metamaterials are depicted in sub-wavelength periodic meta-dielectric layers with complex permittivity ε_m , ε_d and thickness t_m , t_d ($\ll \lambda$), respectively. The conventional effective medium approximation is widely used to predict the HMMs optical behavior by a uniaxial permittivity tensor[29] with ordinary and extraordinary components as

$$\varepsilon = \begin{bmatrix} \varepsilon_o & 0 & 0 \\ 0 & \varepsilon_o & 0 \\ 0 & 0 & \varepsilon_e \end{bmatrix} \quad (1)$$

It is proved that nonlocal effects in nanometer-size structures is considerable and drastically affects the precision of theoretical studies[22, 30, 31]. Thus, in utilizing effective medium approximation, a nonlocal homogenization should be taken into consideration. The operator effective medium theory[32] considers a power series dependency of the primitivity components on $k_0 t$ to account for the nonlocality, where k_0 is the vacuum wavenumber and p is the metal fill fraction in the unit cell of thickness $t = t_m + t_d$. For a bilayer unit cell of metal-dielectric, the operator approach EMT is used in the present paper with the following nonlocal permittivity components taken up to $(k_0 t)^2$.

$$\varepsilon_o^{nl} = \varepsilon_o + \frac{(k_0 t)^2}{6} \sigma_{\parallel} f(\beta) \quad (2a)$$

$$\varepsilon_e^{nl} = \left\{ \frac{1}{\varepsilon_e} + \frac{(k_0 t)^2}{6} \sigma \left(\frac{2p-1}{\varepsilon_r} - \frac{f(\beta)}{\varepsilon_{\perp}} \right) \right\}^{-1} \quad (2b)$$

The familiar extraordinary ε_e and ordinary ε_o local permittivity components as well as the other parameters are defined below, where contributing the metal's complex permittivity generates complex values.

$$\begin{aligned} \varepsilon_o &= p \varepsilon_m + (1-p) \varepsilon_d & \varepsilon_e &= \left(\frac{p}{\varepsilon_m} + \frac{1-p}{\varepsilon_d} \right)^{-1} \\ \varepsilon_{\parallel} &= p \varepsilon_m - (1-p) \varepsilon_d & \varepsilon_{\perp} &= \left(\frac{p}{\varepsilon_m} - \frac{1-p}{\varepsilon_d} \right)^{-1} \\ \varepsilon_r &= \left(\frac{1}{\varepsilon_m} + \frac{1}{\varepsilon_d} \right)^{-1} & \sigma &= p(1-p)(\varepsilon_d - \varepsilon_m) \\ f(\beta) &= \frac{\beta^2}{\varepsilon_r} - 1 \end{aligned}$$

In a cartesian system that z-axis coincides with the optical axis of multilayer HMMs, the well-known dispersion equation of HMMs[21] considering the nonlocal parameters is written as

$$\frac{k_x^2 + k_y^2}{\varepsilon_{e^{nl}}} + \frac{k_z^2}{\varepsilon_{o^{nl}}} = k_0^2 \quad (3)$$

Figure 1: Finite thickness multilayer HMMs considered as waveguide slabs confining the propagating high-k modes.

Due to symmetry in x and y axes, without loss of generality, the wave vector $\mathbf{k}$ is taken as $\mathbf{k} = \beta k_0 \hat{\mathbf{x}} + k_z \hat{\mathbf{z}}$, where $\beta = \frac{k_x}{k_0}$ is the normalized transverse wavevector. Thus, the vertical component of the wavevector, k_z , can be considered as

$$k_z = k_0 \sqrt{-\varepsilon_o^{nl} \left(\frac{\beta^2}{\varepsilon_e^{nl}} - 1 \right)} \quad (4)$$

Generally speaking, k_z is a complex parameter. As the nonlocal permittivities are dependent on β , the k_z of high-k waves in the HMMs will be a nonlinear function of β . Based on the relation between ε_e^{nl} and β , for HMMs type II with the condition of $\varepsilon_o^{nl} < 0$ and $\varepsilon_e^{nl} > 0$, two classes of propagating waves can be conceived as

- Long-Range (LR) high-k waves
The k_z of these waves will be largely a real value under the condition of $\varepsilon_e^{nl} < \beta^2$. Therefore, the high-k waves will be propagating along the z axis with a minor damping caused by the imaginary part.
- Short-Range (SR) high-k waves
Under the condition of $\varepsilon_e^{nl} > \beta^2$ large imaginary part of k_z results in a high damping for the bulk waves and leads to the kind of waves having short propagation length in the HMM.

To take into account the limited number of periods or equivalently limited thickness of the effective medium, we need to consider the boundary effects. Thus, the effective medium calculated out of the multilayer structure is deliberated as a finite thickness single layer sandwiched between two semi-infinite media as substrate and superstrate shown in Fig. 1. Here, the middle homogenous and uniaxial layer is considered as a waveguide for the SR and LR propagating high-k waves, being confined within the slab and attenuating outside in the superstrate and substrate. Defining β as the effective index of the propagating waves inside the HMM slabs, the constrain of $\sqrt{\varepsilon_1} \leq \sqrt{\varepsilon_2} < \beta$ is satisfied for the asymmetric optical slab waveguide; $\varepsilon_1 \leq \varepsilon_2$ is taken for simplicity. Solving Maxwell equations for the three homogenous layers, dispersion equations of high-k modes in finite thickness HMMs is calculated

Figure 2: Bulk high-k modes dispersion in HMM slabs consisting of a) 3, b) 5, c) 8 periods of 15-25 nm thick Au – SiO₂ layers. TMM calculations in the background, theoretical SR high-k modes in green, even and odd LR modes in blue and red dashed lines, respectively.

as;

$$\tanh(\mathbb{K}_{\text{SR}} k_0 h) = \frac{\mathbb{K}_{\text{SR}}(\varepsilon_1 \mathbb{K}_2 + \varepsilon_2 \mathbb{K}_1)}{\frac{\varepsilon_1 \varepsilon_2}{-\varepsilon_0^{\text{nl}}} \mathbb{K}_{\text{SR}}^2 - \varepsilon_0^{\text{nl}} \mathbb{K}_1 \mathbb{K}_2} \quad (5a)$$

$$\tan(\mathbb{K}_{\text{LR}} k_0 h) = \frac{\mathbb{K}_{\text{LR}}(\varepsilon_1 \mathbb{K}_2 + \varepsilon_2 \mathbb{K}_1)}{\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_0^{\text{nl}}} \mathbb{K}_{\text{LR}}^2 - \varepsilon_0^{\text{nl}} \mathbb{K}_1 \mathbb{K}_2} \quad (5b)$$

where h stands for the thickness of slab, $h = n \times t$, here n is the number of period. $\mathbb{K}_1 = \sqrt{\beta^2 - \varepsilon_1}$ and $\mathbb{K}_2 = \sqrt{\beta^2 - \varepsilon_2}$ are the normal components (in the z direction) of the normalized wavevectors of evanescent waves in the substrate and superstrate, $\mathbb{K}_{\text{SR}} = \sqrt{-\varepsilon_0^{\text{nl}}(1 - \frac{\beta^2}{\varepsilon_{\text{enl}}})}$ and $\mathbb{K}_{\text{LR}} = \sqrt{-\varepsilon_0^{\text{nl}}(\frac{\beta^2}{\varepsilon_{\text{enl}}} - 1)}$ are those of SR and LR high-k modes in the HMM slab respectively[24].

2.2. Mode-resolved Photonic Local Density of States

Many studies in literature are reported on multilayer HMMs application in spontaneous emission enhancement of quantum emitters placed inside [14] and outside [16, 33] the structures. Fabricating emitters in precise locations from practical point of view is a challenging procedure that make the need for a comprehensive study on the location of emitters for better performance. Besides that, out coupling of emission coupled to the HMM modes requires some extraction methods such as grating [34] and nanopatterning of the HMMs [9]. In all cases identifying the wave vector of the mode which is coupled to emitter is necessary for designing the extraction method. Here, realizing the number and the nature of high-k modes in finite thickness HMMs, we aim to compute a mode-resolved local density of states using an analytical formula to calculate the LDOS portion of each HMMs waveguide mode; contributing in the transition rate of an emitter placed nearby and inside the HMM slabs. Generally speaking, radiation occurs from an atom whose nucleus is localized in space. Typically, the wavelength of light emitted by an atom is several orders of magnitude greater than the spatial extend of the electron orbits about the nucleus. Therefore, the emission occurring can be thought of redistribution of charges (dipole formation) in the emitter during the

transition. In the spontaneous emission process, the electric dipole moment plays the strongest and main contributing part. In this regard, an electric dipole is chosen as an emitter for the further theoretical investigations. From the Fermi's golden rule developed for a slab waveguide[35], the transition rate of an emitter is given by

$$\gamma_{wg}(\omega) = 2\pi |g(r_{ed}, \omega)|^2 D_{2D}(\omega) \quad (6)$$

Here $D_{2D}(\omega)$ is defined as the local density of states for each mode of the slab in 2D space and $|g(r_{ed}, \omega)|^2$ is the coupling strength between the mode and the dipole at position r_{ed} , in a medium of refractive index $\sqrt{\varepsilon_{ed}}$ [36],

$$|g(r_{ed}, \omega)|^2 = \frac{\omega |\mu_0|^2}{2 \hbar \varepsilon_{ed} \varepsilon_0 V_{eff}} \quad (7a)$$

$$D_{2D} = \frac{l^2 \omega}{2\pi V_g(\omega) V_p(\omega)}, \quad (7b)$$

where V_{ph} and V_{gr} stand for the phase and group velocity of high-k waveguide modes and are calculated based on $V_{ph} = \frac{c}{\beta}$ and $V_{gr} = V_{ph} - \lambda \frac{\partial V_{ph}}{\partial \lambda}$. Here, β is obtained from solving the HMM's dispersion relations derived in Eq. (5). $V_{eff} = l^2 L_{eff}$ is the effective mode volume assigned to each high-k mode where l is an arbitrary quantization length in the $x - y$ plane and L_{eff} is the effective mode length in the z direction defined as

$$L_{eff} = \frac{1}{2} \frac{\int_{-\infty}^{\infty} [\epsilon(z) |E(z, \omega)|^2] dz}{\varepsilon_{ed} |E(z_{ed}, \omega)|^2} \quad (8)$$

To find the transition rate Enhancement Factor, EF , the transition rate of a QE coupled to each HMM mode, $\gamma_{wg}(\omega)$, needs to be compared to that of the isotropic media, $\gamma_{iso}(\omega) = \frac{\sqrt{\varepsilon_{ed}} \omega^3 |\mu_0|^2}{3 \hbar \pi \varepsilon_0 c^3}$. In case the QE is located outside the slab, the enhancement factor do not necessarily indicate that QE emission into HMM modes prevail the modes in the other half space. Therefore, another coefficient as coupling efficiency or Coupling Factor, CF needs to be defined as the followings.

$$EF = \frac{\gamma_{wg}(\omega)}{\gamma_{iso}(\omega)} \quad (9a)$$

Figure 3: Decay rate enhancement factor of electric dipole emission in vicinity of HMM slabs having 3, 5 and 8 periods, coupled to the a) LR and b) SR high-k modes. Slabs with 3, 5 and 8 periods have 2 (green lines, square), 3 (red lines, circle) and 5 (blue lines, triangle-asterisk) LR modes respectively, and 1 SR (diamonds) mode. c) Coupling factor of the dipole emission into the individual SR and LR modes.

Figure 4: Decay rate enhancement factor of a dipole placed inside the HMM slabs, coupled to individual LR and SR high-k modes. Slabs with 3, 5 and 8 periods have 1 SR and 2, 3 and 5 LR modes respectively.

$$CF = \frac{\gamma_{wg}(\omega)}{\gamma_{total}(\omega)} = \frac{\gamma_{wg}(\omega)}{\gamma_{iso}(\omega) + \gamma_{wg}(\omega)} \quad (9b)$$

The modal LDOS is necessary to be investigated for the two classes of high-k modes in any arbitrary multilayer HMMs. But, from practical point of view, the transition rate enhancement factor is defined by contribution of all available modes in the HMMs. Therefore, we introduce the total enhancement factor, *TEF*, as

$$TEF = \frac{\sum_{LR}[EF_{jn} \times CF_{jn}] + \sum_{SR}[EF_n \times CF_n]}{\sum_{LR}[CF_{jn}] + \sum_{SR}[CF_n]} \quad (10)$$

summing over enhancement factor of each mode weighted by its coupling factor, where EF_{jn} and CF_{jn} are enhancement and coupling factors of j th LR mode of the HMM slab having n period. Correspondingly, EF_n and CF_n are enhancement and coupling factors of SR modes of the corresponding slab having n period.

2.3. Mode-resolved Directional Emission

According to the hyperboloid isofrequency surface of HMMs type II, Poynting vector S lies within a region ($\theta_c \leq \theta < 90$) so-called the resonance cone [37]. The ratio between the power flow in the transverse and vertical directions determine the

cone's half angle. Based on the nonlocal homogenization and waveguide definition for the HMM slabs, we define the mode-resolved half-angle of resonance cone given by

$$\tan(\theta_c) = \frac{S_x}{S_z} = \frac{Re(\frac{\beta}{\epsilon_{nl}})}{Re(\frac{\mathbb{K}_{SR/LR}}{\epsilon_{nl}})} \quad (11)$$

One can infer from Eq. (11) that spontaneous emission from a dipole coupled to each high-k mode should have different resonance cone because of its dependency on the effective mode index β .

3. Results and Discussion

To examine the mode-resolved directional enhancement theory, a quantum emitter is chosen as a perpendicularly oriented electric dipole placed inside and in near vicinity of a finite HMM. A configuration comprising a 15-25 nm thick Au and SiO₂ layers respectively, are chosen as unit cell of the HMM. Structures consisting of a substrate (silica glass) – multilayer stacks – and a superstrate (air) are considered as the HMM slabs. In our recent study, it is proved that HMMs having 3 periods and onward, show less than 5% mean-square error between modes dispersion of the multilayer structures and their corresponding effective medium response [38]. Therefore, slabs consisting of basic-3, intermediate-5 and higher-8 number of periods are chosen for the following characterizations. According to the EMA, the corresponding effective medium of such multilayer stacks operate at wavelengths above $\lambda = 530$ nm in the hyperbolic regime type II [39] that is called as transition wavelength to the hyperbolic regime.

High-k modes dispersion calculations shows 1 SR and 2, 3 and 5 LR high-k modes in the HMMs having 3, 5 and 8 periods, respectively as shown in Fig. 2(a-c). The green dashed lines denote the SR mode, blue and red dashed lines represent the odd and even LR modes [24]. LR propagating modes possess high effective indices, on the other side, the SR mode has the lowest effective index fading as a wideband mode for thicker HMMs. Mode calculations of the multilayer HMMs are also carried out by the Transfer Matrix Method (TMM) and shown in the background of Fig. 2, for comparison. In TMM approach, dips in

Figure 5: Electric field energy density of the first LR (1st row) and the SR (2nd row) modes in HMM slabs having 3 (a, d), 5 (b, e) and 8 (c, f) periods and in 300 nm distance from the slab into substrate (left side) and superstrate (right side), calculated over all bandwidth. The dotted lines highlight electric field energy density at wavelength 630 nm and the solid line highlight that of 1400 nm.

the reflection spectrum correspond to the modes of the multi-layer structures. Figure 2 depicts a perfect matching between the theoretical and numerical methods. By adding more periods to the HMM slabs number of LR modes gets increased and their effective index shifts to the lower values. Nonlocal effects and boundary conditions induce an upper cut-off for the wave vector and limits the number of possible LR modes [24]. As number of high-k modes in an HMM slab is finite, LDOS of the HMMs would be finite as well.

QE placed outside HMMs. For a dipole placed outside the HMMs, 10 nm above in the superstrate, the mode-resolved transition rate enhancement factor is calculated and shown in Fig. 3(a). The coupled emission into the individual high-k modes indicates that LR modes of the 3-period slab provide the highest enhancement amongst all three configurations. The EF of the 5-period slab is also greater than that of the 8-period slab. Fig. 3(b) illustrates that SR modes enhancement factor of the 8-period slab is the strongest amongst the 3 and 5-period HMMs in the total bandwidth. Coupling factor of each LR and SR modes of the 3, 5 and 8-period HMM slabs are shown in fig. 3(c). It is clear that the SR modes have the strongest coupling factor around 50%. While wideband LR modes have strong coupling near the transition wavelength to the hyperbolic regime, epsilon-near-zero (ENZ) zone; they do not preserve the same coupling factor in the wideband and CF get decreased in tails of the modes.

QE placed inside HMMs. The enhancement factor calculations for a QE placed inside the HMMs are depicted in Fig. 4. It indicates that LR modes generally provide much higher enhancement factors than SR modes of the HMM slabs. On the contrary to the previous case, EF increases far from transition wavelength, in tail of the wideband long-range modes. But in the

same way, the 3-period HMM slab provides the strongest EF amongst all three configurations. To elaborate on the mode-resolved enhancement factors, following discussions should be taken into considerations.

3.1. Modal Energy Density Distribution.

It is clear from Eq. (7a) and Eq. (8) that the transition rate of a dipole emission into the HMM slab modes is directly proportional to electric field energy density in the location of the dipole $|E_{ed}(z, \omega)|^2$ and inversely proportional to the volume energy density in the slab and surroundings, $|E(z, \omega)|^2$. For slabs having 3, 5 and 8 periods the electric field intensity in the HMM slabs and surroundings are calculated for the first LR and the SR modes and depicted in fig. 5(a-c), and 5(d-f), respectively. It can be deduced from Fig. 5(a-c) that field intensity of the LR propagating modes is intense and localized inside the slabs. Outside fields intensity, in the substrate and superstrate, is much weaker and clearly decaying. On the contrary to the LR modes, the SR modes have stronger field intensity in the surroundings than in the slabs, inferred from fig. 5(d-f). The maximum field intensity outside in the superstrate is provided by the 8-period slab. The asymmetric surrounding of the slab is distinguished in the field intensities, emphasizing the role of substrate and superstrate materials in modifying field distribution and consequently the LDOS. As it is expected for the first LR modes, there is one antinode (maximum intensity) in the middle of slabs. It should be mentioned that for the second, third and higher order LR modes, respectively, there are two, three and higher number of peaks in the field intensity distribution in the slabs as is described in our previous study [24]. Therefore, the position of emitter inside the slabs is chosen based on the location of maximum electric field energy density (correspondingly the maximum local density of states) of each mode for calculating the modal enhancement factor. The plots in Fig. 5 are

Figure 6: Group velocity of LR and SR modes in HMM slabs consisting of 15-25 nm thick Au – SiO₂ layers as well as group velocity of the S-SPP mode in 15 nm thick Au layer. Slabs with 3, 5 and 8 periods have 1 SR and 2, 3 and 5 LR modes, respectively.

calculated over the whole bandwidth of the modes. To take a look at a specific frequency LDOS, two chosen wavelengths are highlighted. The field intensity corresponding to the LDOS at $\lambda = 630$ nm is emphasized with the dotted lines in Fig. 5 and that of $\lambda = 1400$ nm is highlighted using the solid lines. It is clear that LDOS of the LR modes at the tails ($\lambda = 1400$ nm) is too strong in the HMM slabs and almost too weak in the surroundings. While at $\lambda = 630$ nm (near the transition) LDOS of the LR modes is strong in the surrounding media. The same holds true for SR modes at $\lambda = 630$ nm. Our findings illustrate the localization of energy by LR high-k modes inside the HMMs, in accordance to the bulk Bloch waves or correspondingly named Bulk Plasmon Polariton (BPP) modes prediction in HMMs (the LR correspondence)[40]. Furthermore, the results indicate that the SR modes are responsible for localization of energy in nearfield distance outside the HMM slabs, where both classes of modes, in terms of number and effective mode index, are related to the number of periods.

3.2. Modal Group Velocity.

Another important parameter affecting the transition rate enhancement is the group velocity of high-k modes. Equation (7) illustrates that the lower the group velocity of the mode is,

Figure 7: Coupling angle of the SR and first LR high-k modes in HMM slabs consisting of 3, 5 and 8 periods of Au – SiO₂ layers working as a HMM type II.

the bigger the LDOS associated with it would be. Since V_{gr} can be thought as the velocity at which energy is conveyed, the modes having slower group velocities provide stronger light-matter interaction and consequently larger LDOS. Group velocities of SR and LR modes of HMM slabs having 3, 5 and 8 periods, depicted in Fig. 6, indicate that LR modes possess high group velocities near the transition wavelength, decreasing on the tails of modes. But the SR mode has a lower group velocity near the transition point increasing at longer wavelengths. Therefore, near the transition wavelength, SR modes will dominate in providing the large LDOS, but far from the ENZ regime, the broadband LR high-k modes will represent the larger LDOS. In the considered HMM slabs, the mode number 2 of the 3-period HMM that exhibits the lowest group velocity has the maximum enhancement factor amongst the others, shown in Fig. 3(a).

Hence, in order to engineer LDOS in HMM slabs the number of periods in the HMMs, working frequencies as well as the emitter location will be of essential matters. For wavelengths near the transition, the SR modes play the key role in transition rate enhancement of QEs placed outside the HMM slabs. Inversely, for QEs located inside the HMM slabs and far from the transition zone, LR high-k modes need to be engineered for manipulating the transition rate enhancement.

3.3. Modal Emission Direction

A half-angle of preferential emission direction of a dipole into the SR and first LR modes of the HMM slabs with 3, 5 and 8 periods is shown in Fig. 7. For HMMs type II, the bigger θ_c results in energy flow closer to the surface of slabs as it is obvious in the iso-frequency surface depicted in the inset of Fig. 7. Energy coupling to the SR and LR high-k modes of the HMM slabs occurs in a small and almost the same emission cones at wavelengths near the transition zone. Moving farther from the ENZ, emission coupled to different modes starts to diverge. In longer wavelengths, energy mostly flows near the surface of the slab because of the high coupling angles for both the LR and SR modes, yet the LR modes show higher coupled angles in comparison to the SR mode. A small emission cone at wavelengths near the transition zone, for both SR and LR modes indicate the best working frequencies. In near-fields applications, emission at angles close to the vertical direction will be desirable. Using objectives, the more the vertical emission is, the lower the objective aperture can be used. For bigger half angles emission direction falls out of cones aperture and reduces the photon collection. Therefore, working at shorter wavelengths (near the transition point) will be more desirable. It is worth mentioning that emission outcoupling to far-field needs some approaches to provide k-matching conditions[41]. Modes having lower effective indices will make the outcoupling more facile. Accordingly, the SR modes having lower effective index, have advantages over LR high-k modes close to the transition point and even far from it.

3.4. Thin vs. thick HMMs.

Investigation on individual high-k modes of the HMM slabs indicates that the modes related to thinner slabs provide larger

Figure 8: a) Theoretical and b) simulation calculation of total decay rate enhancement factor of electric dipole in vicinity of HMM slabs having 3, 5 and 8 periods; summing over contribution of the SR and all LR high-k modes.

Figure 9: a) Short range surface plasmon polariton (S-SPP) dispersion of a single Au layer of 15 nm thick Au sandwiched between SiO_2 substrate and air superstrate. b) Transition rate enhancement and coupling factor for an electric dipole placed 10 nm above the metal layer in superstrate.

LDOS. For HMM slabs with 3, 5 and 8 periods the total enhancement factor for a dipole in the vicinity of HMM slabs, based on the eq. 10, is also calculated and shown in Figure 8(a). It proves that multilayer stacks with a lower number of periods provide the higher total enhancement factor. Larger number of modes in a thick slab (having multiple nodes and antinodes in the slab) facilitates their competition, which leads to the certain degradation of local density of states. To validate the theoretical calculations, we conducted a 3D FDTD simulation using commercially available Lumerical software. Simulation results shown in figure 8(b) affirm our theoretical derivations with the same conclusion. It is worth mentioning that the TEF calculations of a dipole outside the slabs indicate the maximum enhancement appears near the transition wavelength, in total

accordance with the above mode-resolved enhancement factor calculations as well as the experimental results reported in literature [29, 42, 43].

Investigation on LDOS of HMM slabs indicating the strongest transition rate enhancement is attached to the structures with smaller number of periods, raises a question about preferences of using a single metal layer (SML) instead of multilayer stacks. To provide a comparison, we considered a single Au layer with 15 nm thickness on a glass substrate, placed in free space (air as superstrate), the same as HMM slabs configurations. Looking for modes of the metal layer using TMM and theoretical calculation, we realize only one mode having high wave vector, shown in Fig. 9(a), familiar as a short range surface plasmon polariton (S-SPP). The enhancement factor as well as coupling factor of a dipole placed 10 nm above the layer, are calculated and shown in figure 9(b). In comparison to enhancement factor of SR mode in the 3-period HMM shown in figure 3(b), the single metal layer mode shows higher enhancement factor. Although coupling factor of the S-SPP at shorter wavelengths is comparable to that of the SR mode, it decreases by increasing the wavelength and seems weaker compared to the high-k SR mode. Group velocity of the S-SPP mode in SML is shown in Fig. 6, to be easily comparable to that of the high-k modes in the HMM slabs. It shows a lower group velocity at shorter wavelengths increasing fast at the longer wavelengths (decreasing LDOS), where the enhancement factor of the mode drops quickly.

It is clear that the high-k SR mode possesses lower effective index than the SML mode. As dispersion of the SR-SPPs in a single metal layer has lower effective index in comparison to a pure surface plasmon polariton (a bulk metal), adding more layers, the coupled SPPs shifts more to lower effective indices. Since the in/out coupling of energy to a structure's mode in addition to LDOS engineering is important and very promising for improvement of numerous near-field technologies, HMM structures hold advantages over SML in this regard. Besides that, the transition wavelength to the hyperbolic regime with the maximum enhancement factor for HMMs, can be tuned easily by engineering the unit cell; while such tunability for SMLs is not available[44]. It also should be noted that the max-

imum LDOS not necessarily gives rise to the maximum radiative emission enhancement for QEs. In transition rate enhancement, the portion of radiative and nonradiated LDOS needs to be considered separately, based on the application. For the SR-SPP mode existing at the metal-dielectric interfaces, there is stronger damping and correspondingly nonradiative transitions for a QE placed close to the metal rather than HMMs that behaves like dielectrics in two direction[21].

4. conclusion

In this study, a mode-resolved photonic local density of states and energy flow directionality of finite thickness HMM slabs are investigated. The results demonstrate that the LDOS enhancement elucidated through a quantum emitter coupled to the high-k modes of HMM slabs, is indeed totally dependent on the number of periods in HMMs as well as position of the emitter. The introduced SR and LR high-k modes play the most important role in LDOS engineering inside and in the near vicinity of the HMM slabs, respectively. In both positions, HMM slabs having smaller number of periods show stronger transition rate enhancement near the transition wavelength to the hyperbolic regime. Furthermore, SR high-k modes show lower coupling angles for directional emission of the coupled quantum emitters. In general, a low-period multilayer metamaterial seems promising for the hyperbolic LDOS engineering and optimal directional emission of quantum emitters. The results confirm that applicability of hyperbolic metamaterials in spontaneous emission enhancement is dependent on fully understanding of high-k modes of the HMMs to properly engineer application-oriented HMM slabs.

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