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ABSTRACT
Transport in the edge and scrape-off layer mediated by turbulent fluctuations is often studied using drift fluid models. In this work, we expand previous work on a two-fluid single ion species drift model to a multi-ion-species model that incorporates collisional interactions between the individual species while conserving energy. The model is simplified into a set of equations that are computationally realizable. This is used to study the dependency of seeded blob propagation on different mixes of deuterium and tritium isotopes in the background and blob, respectively. We find that the background mix is initially the dominant driver that determines propagation, but the blob mix becomes the dominating factor for continued evolution. It is found that the maximum velocity of the blob scales stronger with the initial blob mix than the background mix.

I. INTRODUCTION
A fusion plasma is inherently composed of multiple ion species due to the fusion of two particles into a third of a different type. In future fusion reactor plasmas, the reaction will be based on deuterium and tritium that fuse into helium. How species of all kinds interact and how these interactions carry over to transport coefficients, particularly in regard to impurity transport, has been studied theoretically. The importance of multi-component plasmas has become evident from experimental results that show how transport, and consequently confinement, is changed when the ion mix changes. Numerically, this has been studied using gyro-kinetic codes by comparing simulations with different single species main ions and in some cases also with heavy impurities. Drift wave turbulence modeling has also been employed, but again, this was only with a single main ion species. True multi-component mixtures have been numerically studied with a focus on impurities and their influence on turbulent behavior using a drift fluid approach but only with lower-order drifts in the direction perpendicular to the magnetic field. Other approaches have used single ion species models to simulate mixes by using an effective mass and charge. This limits the studies to plasmas with a spatially uniform ion species models to simulate mixes by using an effective mass and charge. This limits the studies to plasmas with a spatially uniform.
Sec. III. The model is tested by simulating a range of DT mixes in seeded blobs in Sec. IV. Finally, in Sec. V, we summarize and discuss our findings.

II. MULTISPECIES DRIFT FLUID MODEL WITH COLLISIONAL EFFECTS

The starting point for the derivation of the multispecies drift fluid model is the general momentum equation for an arbitrary plasma species, \( i \), denoted by the subscript \( s \),

\[
m_i \frac{d}{dt} \mathbf{u}_s = n_i q_i (E + \mathbf{u}_s \times \mathbf{B}) + \mathbf{R}_s - \nabla p_s - \nabla \cdot \mathbf{\pi}_s, \tag{1}
\]

where \( m_i, n_i, q_i \), and \( \mathbf{u}_s \) are the mass, charge, density, and fluid velocity of species \( s \), \( \frac{d}{dt} = \partial_t + \mathbf{u}_s \cdot \nabla \) is the total derivative, \( E \) and \( \mathbf{B} \) are the electric and magnetic fields, respectively, \( p_s \) is the scalar pressure, and \( \mathbf{\pi}_s \) is the viscous stress tensor. The quantity \( \mathbf{R}_s \) is the total resistive force acting on the species due to collisions with all other species. In general, we can split this resistive force into a frictional part \( \mathbf{R}_{s,f} \) and a thermal part \( \mathbf{R}_{s,t} \). The perpendicular frictional part for a species \( s \) acting on species \( s' \) is given by

\[
\mathbf{R}_{s,f_{s,s'}} = -n_s m_s \nu_{s,s'} (\mathbf{u}_{s} - \mathbf{u}_{s'}) \tag{2},
\]

while the thermal force is given by

\[
\mathbf{R}_{s,t_{s,s'}} = \frac{3}{2} m_s n_s \nu_{s,s'} \mu_{s,s'} \left( \frac{\mathbf{b} \times \nabla T_s}{m_s q_i B} - \frac{\mathbf{b} \times \nabla T_{s'}}{m_s q_i B} \right), \tag{3}
\]

where \( T_s \) is the species temperature, \( \nu_{s,s'} \) is the collision frequency between two species defined below, and \( \mu_{s,s'} = m_s m_{s'}/(m_s + m_{s'}) \). Furthermore, we have introduced the magnetic unit vector \( \mathbf{b} = \mathbf{B}/B \), where the magnetic field in a Cartesian coordinate system is assumed to be of the form

\[
\mathbf{B}(x) = \frac{B_0 R}{R + r + \bar{z}}, \tag{4}
\]

where \( B_0 \) is the magnetic field at the major radius \( R \), \( r \) is the minor radius, and \( \bar{z} \) is the unit vector pointing in the \( z \)-direction parallel to the magnetic field. We assume the magnetic field to be straight. Assuming a tokamak with a large aspect ratio and a small \( q \)-factor, this can be viewed as an approximation of the magnetic field at the outboard midplane. Furthermore, due to the large aspect ratio, this does not take into account the magnetic field shear. Summing over all species, \( s' \), gives the total resistive force acting on species \( s \). In the case of electron–ion interactions, we will consider the large mass ratio and make appropriate approximations. For the resistive force, this means neglecting the ion temperature gradient term in Eq. (3) when dealing with ion–electron resistive forces. In the derivation of the total resistive force, it is assumed that \( T_e - T_i \ll T_s \) for ion–ion interactions. Throughout this paper, we use the species indices \( i \) and \( j \) for ions and electrons alike, while \( x \) and \( \beta \) will be used for ions only and likewise \( e \) for electrons. This is to distinguish terms where there are different approximations for ions and electrons due to, e.g., the mass ratio.

The viscous stress tensor \( \mathbf{\pi} \) is comprised of a parallel component, a perpendicular collisional component, and a gyro viscous component. Since we only consider a 2D slab geometry, we leave out the parallel component of the viscous stress tensor. With \( \mathbf{b} \) along the \( z \)-axis in an \((x, y, z)\) coordinate system, the perpendicular collisional part of the ion viscous stress tensor is given by

\[
\mathbf{\pi}_{\perp,s} = -\eta_{\perp,s} \left[ \begin{array}{ccc} \partial_x u_{s,x} - \partial_y u_{s,y} & \partial_x u_{s,y} + \partial_y u_{s,x} \\ \partial_x u_{s,x} + \partial_y u_{s,y} & -\partial_x u_{s,y} + \partial_y u_{s,x} \end{array} \right], \tag{5}
\]

where the coefficient is given by summing over all ion species

\[
\eta_{\perp,s} = \sum_{\beta} \frac{n_s m_s \nu_{s,s}}{\Omega_{s,s}^2} \left( \frac{6 m_{\beta} Z_{\beta}^2}{5 m_s} + 2 - \frac{4 m_{\beta} Z_{\beta}^2}{5 m_s Z_{\beta}} \right), \tag{6}
\]

with the ion gyro-frequency

\[
\Omega_{s,s} = \frac{Z_s e B}{m_s}, \tag{7}
\]

where \( Z_s \) is the charge number. We have here only considered the ion viscous stress tensor due to the mass dependency. The electron viscous stress tensor is therefore neglected.

The non-collisional gyro-viscous part of the tensor is given by

\[
\mathbf{\pi}_{\parallel,s} = -\eta_{\parallel,s} \left[ \begin{array}{ccc} \partial_x u_{s,x} + \partial_y u_{s,y} & \partial_x u_{s,y} - \partial_y u_{s,x} \\ \partial_x u_{s,x} - \partial_y u_{s,y} & \partial_x u_{s,y} + \partial_y u_{s,x} \end{array} \right], \tag{8}
\]

with the coefficient given by

\[
\eta_{\parallel,s} = \frac{n_s \nu_{i,s}}{2 \Omega_{i,s}^2}. \tag{9}
\]

Again, the electron viscous stress tensor is small compared to the ion viscous stress tensor due to the small mass ratio and so it is neglected.

For the collisional parts of the resistive force and the viscous stress tensor, the collision frequency is defined as

\[
\nu_{c_{s,s'}} = \frac{2}{\pi} \frac{n_e Z_i^2 Z_e^2 e^4}{m_i m_e} \ln \Lambda_{i,e} \left( 1 + \frac{m_e}{m_i} \right) \frac{T_i^{3/2}}{m_i} \frac{T_e^{3/2}}{m_e}, \tag{10}
\]

The Coulomb logarithm is defined such that

\[
\Lambda_{i,e} = \frac{12 n_0 \mu_{i,e}}{|Z_i Z_e e^2|} \gamma_{i,e} \gamma_{e,i} \lambda_d, \tag{11}
\]

with \( \gamma_{i,e} = \gamma_i / (\gamma_i + \gamma_e) \), where \( \gamma_i = m_i / T_i \). Finally, the Debye length is given as

\[
\lambda_d = \left( \sum_i n_i Z_i^2 e^2 / \epsilon_0 T_i \right)^{1/2}. \tag{12}
\]

Numerically solving the momentum equation requires resolving all time and lengths scales including the electron gyro-motion scales. However, turbulent transport occurs on time and length scales much larger than these scales. Hence, we use a reduced fluid model that resolves the proper scales and captures the physics of interest while being numerically accessible. With all elements defined, we follow Ref. 14 and proceed to the drift ordering. Here, we employ the usual
assumptions of the characteristic timescales being longer than the reference ion (such as deuterium) gyro period and perpendicular lengths scales being longer than the ion gyro radius,
\[
\frac{t_0}{\Omega_{\text{Ref}}} \ll 1 \quad \text{and} \quad \frac{\rho_b^2}{L_{\perp}^2} \ll 1,
\]
where
\[
\rho_b = \sqrt{\frac{T_a}{m_a\Omega_{\perp}^2}}.
\]
The lowest order drifts following this are the electric and diamagnetic drifts, which are given by
\[
\begin{align*}
\mathbf{u}_{e,0} &= \mathbf{u}_{p,e} + \mathbf{u}_{D,e} + \mathbf{u}_{R,e}, \\
\mathbf{u}_{i,0} &= \mathbf{u}_{p,i} + \mathbf{u}_{D,i} + \mathbf{u}_{R,i}.
\end{align*}
\]
The plasma at the outboard midplane is weakly collisional, meaning the resistive force can be assumed to be the next order (denoted order 1). Following Ref. 16, the same holds for the viscous stress tensor. Consequently, for the next order drift, we find the polarization, resistive, and viscous drifts
\[
\begin{align*}
\mathbf{u}_{e,1} &= \mathbf{u}_{p,e} + \mathbf{u}_{D,e} + \mathbf{u}_{R,e}, \\
\mathbf{u}_{i,1} &= \mathbf{u}_{p,i} + \mathbf{u}_{D,i} + \mathbf{u}_{R,i}.
\end{align*}
\]
The resistive and viscous drifts are expressed in terms of the velocity \(\mathbf{u}_e\) but the respective drifts are first order, meaning we use only zeroth order drifts \(\mathbf{u}_{i,0}\) when evaluating them.

We now turn our attention to the equations for density, vorticity, and pressure. Inserting the expression for the fluid velocities into the continuity equation yields the perpendicular density equation for ions,
\[
\frac{\partial}{\partial t} n_x + \nabla \cdot \left( n_x (\mathbf{u}_e + \mathbf{u}_{D,e} + \mathbf{u}_{R,e} + \mathbf{u}_{e,s}) \right) = 0.
\]
We impose quasi-neutrality \(\sum_i Z_i n_x = n_x\) meaning there is no need to solve an electron density equation explicitly. As a result, multiplying the density equation for all species by their charge state \(Z_i\) (with \(Z_e = -1\)) and summation under the quasi neutrality condition yields the vorticity equation
\[
\sum_i Z_i \left( \frac{\partial}{\partial t} n_x + \nabla \cdot (n_x \mathbf{u}_e) \right) = \sum_i Z_i \nabla \cdot (n_x \mathbf{u}_{D,e} + \mathbf{u}_{R,e} + \mathbf{u}_{e,s}) \quad - \nabla \cdot (n_x \mathbf{u}_{D,e}) = 0.
\]
Here, we have used that the viscosity and polarization terms are mass dependent and so are negligible for electrons. Also, we employed that the resistive force conserves momentum, \(\mathbf{R}_{e-\text{res}} = -\mathbf{R}_{e-\text{visc}}\) and consequently that all \(\nabla \cdot Z_i n_x \mathbf{u}_{e,s}\) terms cancel.

Finally, the pressure equations are found from the multispecies pressure moment equation. For electrons, this gives
\[
\begin{align*}
\frac{3}{2} \frac{\partial}{\partial t} p_e + \frac{3}{2} \nabla \cdot \left( p_e (\mathbf{u}_e + \mathbf{u}_{D,e} + \mathbf{u}_{R,e}) \right) + p_e \nabla \cdot (\mathbf{u}_e + \mathbf{u}_{D,e} + \mathbf{u}_{R,e}) + \nabla \cdot \mathbf{q}_e &= \bar{Q}_e,
\end{align*}
\]
and for each ion species \(z\),
\[
\begin{align*}
\frac{3}{2} \frac{\partial}{\partial t} p_z + \frac{3}{2} \nabla \cdot \left( p_z (\mathbf{u}_e + \mathbf{u}_{D,e} + \mathbf{u}_{R,e}) \right) + p_z \nabla \cdot (\mathbf{u}_e + \mathbf{u}_{D,e} + \mathbf{u}_{R,e} + \mathbf{u}_{z,s}) + \nabla \cdot \mathbf{q}_z &= \bar{Q}_z.
\end{align*}
\]
Here, the total collisional energy exchange is \(\bar{Q}_z = \sum_{x \neq z} \bar{Q}_{x-\text{visc}}\). The interspecies collisional energy exchange is given as \(\bar{Q}_{x-\text{visc}} = Q_{x-\text{visc}} - u_x \cdot \mathbf{R}_{x-\text{visc}}\) with \(Q_{x-\text{visc}} = -Q_{x-\text{visc}}\), which is required for energy conservation. As the resistive force is of order 1 due to low collisionality, it is evaluated using the lowest order drifts, which gives \(-u_x \cdot \mathbf{R}_{x-\text{visc}} \approx -u_{x,b} \cdot \mathbf{R}_{x-\text{visc}} = -u_{x,b} \cdot (q_x n_x \nabla \phi + \nabla p_x)\). Finally, the thermal energy exchange is given by
\[
\bar{Q}_{x-\text{visc}} = \frac{3n_i m_i \bar{Q}_{x-\text{visc}}(T_x - T_i)}{m_e + m_i}.
\]

By using the expression for the collision frequency in Eq. (10), it is easily verified that this satisfies the required energy conservation condition \(Q_{x-\text{visc}} = -Q_{x-\text{visc}}\). For the ion pressure equation (20), the collisional energy exchange and the resistive drifts can be combined to read
\[
\begin{align*}
\frac{3}{2} \nabla \cdot (p_x \mathbf{u}_{R,x}) + p_x \nabla \cdot \mathbf{u}_{R,x} - \bar{Q}_x &= \frac{5}{2} \nabla \cdot (p_x \mathbf{u}_{R,x}) - \sum_{x \neq z} Q_{x-\text{visc}} + u_x \cdot (q_x n_x \nabla \phi).
\end{align*}
\]

We now turn our attention to the heat flux \(\mathbf{q}_x\). In general, it can be split into a parallel \(q_{x,\parallel}\), perpendicular collisional \(q_{x,\perp}\), and diamagnetic heat flux \(q_{x,\perp}\). For ions, omitting the parallel component, these are given by
\[
\begin{align*}
q_{x,\perp} &= \frac{5 p_e}{2 q_e} \mathbf{b} \times \nabla T_x, \\
q_{x,\perp} &= \frac{p_x}{m_x} \sum_{\beta} \frac{\mu_{x,\beta} \beta z_x Z_x n_x}{n_x} \left( \frac{13 m_{\beta}}{4 m_x} + 4 + \frac{15 m_x}{2 m_{\beta}} \right) \nabla T_x - \frac{27 Z_x}{4 Z_{\beta}} \nabla T_{\beta}.
\end{align*}
\]

It should be noted that in the case of a single ion species, it reduces to the expression found in Braginskii. For electrons, similar expressions are given in Ref. 2. When using the mass disparity between ions and electrons, this can be written in a compact form,
\[
q_{x,\perp} = -\frac{5 p_e}{2 e B} \mathbf{b} \times \nabla T_e.
\]

From the resistive force, Eqs. (2) and (3), in the large mass ratio limit and using the lowest order drifts, the electron resistive drift becomes
\[ u_{R,T} = -\sum \frac{\nu_{3\rightarrow e}}{m_e \Omega_e} \frac{\nabla p_e}{n_e Z_e n_a} \] and \[ u_{R,\perp} = \frac{3}{2} \sum \frac{\nu_{3\rightarrow e}}{m_e \Omega_e} \nabla T_e. \] (27)

With this, it is instructive to write the perpendicular collisional heat flux as
\[ q_{\perp,e} = q_{\perp,ue} + q_{\perp,T} = -\frac{3}{2} p_e u_{R,\perp} - \sum p_e \left( \frac{13}{4} + \frac{\sqrt{2}}{Z_e} \right) \frac{2}{3} u_{R,T}. \] (28)

Combining Eq. (28) with the collisional energy exchange and the resistive drift terms in the electron pressure equation, we obtain
\[
\frac{3}{2} \nabla \cdot (p_e u_{R,e}) + p_e \nabla \cdot u_{R,e} + \nabla \cdot q_{\perp,e} - \dot{Q}_e = \\
\nabla \cdot (p_e u_{R,e}) - \nabla \cdot \sum p_e \left[ \frac{1 + \frac{\sqrt{2}}{Z_e}}{3} \right] u_{R,T} \\
- \dot{Q}_e + u_{R,e} \cdot \left( q_{\perp,e} \nabla \phi \right). 
\] (29)

Finally, we simplify the polarization term by employing the gyro-viscous cancelation by which part of the gyro viscous stress tensor component cancels out the diamagnetic contribution to the advection velocity in the polarization term giving a simpler expression. Since the gyro-viscous part of the viscous stress tensor is non-collisional, it is independent of other species and as such the cancelation holds regardless of the number of ion species. A commonly used form of this cancelation is expressed as
\[ \nabla \cdot n_s u_{D,s} = \nabla \cdot \left( n_s u_{D,s} \right) = 0. \] (30)

The term on the right-hand side represents the remainder of the gyro-viscous cancelation. As this is commonly neglected, which we will also do, we will not consider it in detail. The polarization term then reads
\[ \nabla \cdot \left( n_s u_{D,s} + u_{G,s} \right) = -\nabla \cdot n_s \frac{d}{dt} \left( \frac{\nabla \cdot \phi}{B} + \frac{\nabla \cdot p_s}{q_s n_s B} \right) + \nabla \cdot n_s u_{G,s}, \] (31)

where the term \( u_{G,s} \) is the drift associated with the remainder of the gyro-viscous cancelation stated in Eq. (30) and where the total time derivative is given by
\[ \frac{d}{dt} = \frac{\partial}{\partial t} + \left( u_{e} + u_{p,a} + u_{R,a} + u_{\perp,a} \right) \cdot \nabla. \] (32)

We note that due to the appearance of the polarization drift in the advective part of the total derivative, the drift is recursively defined by itself, making it impractical for numerical implementations. As such, it is commonly dropped. We do the same when setting up the numerically implementable model.

It should be noted that due to the fluid drift expansion, and subsequent truncation, the ion density, ion pressure, and vorticity equations contain an asymmetry when adding the ion densities together. This means that multiple identical ion species with an equal temperature, but with different densities, do not add exactly to give the same as if they were represented by one combined density distribution unless \( n_s / n_e = \text{const.} \) everywhere. Correct summing of the ion species is a fundamental requirement for multispecies modeling, and so should always be considered carefully. Taking, e.g., the ion density equation, this would imply that
\[
\sum_s \left( \frac{\partial}{\partial t} n_s + \nabla \cdot n_s u_s \right) = \frac{\partial}{\partial t} n_i + \nabla \cdot n_i u_i, \] (33)

where \( n_i = \sum_s n_s \). For our drift fluid version of the continuity equation, this equality does not hold. The source of deviation occurs in the vorticity and ion pressure equations and is related to terms associated with polarization and viscosity drifts. This is because the terms are non-linear in \( n_s \) as, e.g.,
\[
\sum_s \nabla \cdot n_s u_{p,s} = \sum_s \nabla \cdot n_s \Omega_z b \times \frac{d}{dt} \left( \frac{b \times \nabla \phi}{B} + \frac{b \times \nabla p_s}{q_s n_s B} \right) \\
\neq \nabla \cdot \left( \sum_s n_s \Omega_z b \times \frac{d}{dt} \left( \frac{b \times \nabla \phi}{B} + \frac{b \times \nabla \sum_s p_s}{q_s \sum_s n_s B} \right) \right), \] (34)

which in turn is due to the inclusion of the diamagnetic drift as the lowest order drift as seen in Eq. (15). The validity of the occurrence of the diamagnetic drift at this order is discussed in Ref. 18. As the polarization and viscosity drift are of higher order, the effect is expected to be small and therefore negligible, which has been confirmed by numerical experiments (see the last paragraph of Sec. IV).

A. Energy conservation

In this section, we derive the energy theorem for the collisional multispecies drift fluid model expressed in Eqs. (17)–(20). Similar to Refs. 14 and 19, we start by multiplying the vorticity equation (18) by \( \psi \) followed by integrating over all space while neglecting the surface terms. Combining all this together yields the global fluid kinetic energy,
\[
\int dV \psi \sum_s \nabla \cdot Z_s n_s u_{p,s} + \phi \sum_s \nabla \cdot Z_s n_s u_{D,s} \\
- \psi \nabla \cdot n_s u_{D,s} + \phi \sum_s \nabla \cdot Z_s n_s u_{E,s} \\
= \int dV u_e \cdot \left( \sum_s n_s \frac{d}{dt} u_e + u_{D,s} \right) \\
+ u_e \cdot \sum_s \nabla \cdot \left( q_s n_s \nabla \phi \right) + Q_e = 0. \] (35)

The next part of the energy theorem is the thermal energy given by the pressures. Integrating the electron pressure equation (19) over space and again neglecting surface terms, gives
\[
\int dV \frac{3}{2} \frac{\partial}{\partial t} p_e - u_e \cdot \nabla p_e + u_{R,e} \cdot \left( q_e n_e \nabla \phi \right) - Q_e = 0. \] (36)

Finally, we perform the same procedure for the ion pressure equations, Eq. (20),
\[
\int dV \frac{3}{2} \frac{\partial}{\partial t} p_s + n_s m_s u_{D,s} \frac{d}{dt} u_e + u_{D,s} \\
- u_e \cdot \nabla p_s + \left( \nabla \cdot n_s \right) u_{D,s} + u_{R,s} \cdot \left( q_s n_s \nabla \phi \right) + n_s \cdot \nabla u_{s,0} - Q_s = 0. \] (37)
With the above equations at hand, we proceed to sum everything together. To this end, we first look at the interaction terms between ion species, i.e., the resistive and heat exchange terms. Starting with the resistive fluid drift for ions, we have
\[ u_{R,\alpha} = u_{R,\beta,\gamma} + \sum_{\beta} u_{R,\beta,\gamma} \]  
(38)
Evaluating the frictional force given in Eq. (2) with the lowest order drifts, the resistive drift is linear in \( n_i \) and that \( n_i u_{R,\gamma/\gamma} \) is antisymmetric in \( s \) and \( s' \). Consequently, all terms related to \( u_{R,\gamma} \) cancel when summing all pressure equations. As for the heat exchange term [Eq. (21)], since \( m_i n_i u_{\epsilon,\gamma/\gamma} = m_i n_i u_{s,\gamma/\gamma} \), it becomes evident that \( Q_{\epsilon/\gamma} = -Q_{s/\gamma} \), and consequently, all ion–ion heat exchange terms cancel when summed similar to ion–electron heat exchange. Integrating parts of the \( \pi_\alpha : \nabla u_{R,\gamma} \) term allows for the diamagnetic part to cancel the fourth term in Eq. (37). The total ion thermal energy contribution to the energy theorem then becomes
\[ \int \frac{dV}{dt} \frac{3}{2} p_i + \sum_s \left( \frac{3}{2} p_s + \frac{1}{2} m_s n_s u_{R,0}^2 \right) = 0. \]  
(39)
The final energy theorem is now obtained by adding the contributions from the vorticity (35), electron (36), and ion pressures (39), and invoking the density equation to put the density (17) under the time derivative. In conclusion, the energy theorem reads
\[ \frac{\partial}{\partial t} \int dV \left( \frac{3}{2} p_i + \sum_s \left( \frac{3}{2} p_s + \frac{1}{2} m_s n_s u_{s,0}^2 \right) \right) = 0. \]  
(40)
This is the general energy theorem and so the total energy is given as the sum of the thermal and fluid kinetic energy as we expect.

III. COMPUTATIONALLY IMPLEMENTABLE MODEL: MIHESEL-MODEL

The system of equations presented in Sec. II is numerically cumbersome to implement and so we seek to simplify the equations to get a numerically workable model. In this section, we also discuss the influence of the approximations on the energy theorem and the summability of the equations in regard to the asymmetry in the summation discussed earlier. As mentioned, we consider a 2D slab geometry with \( x \) and \( y \) as the radial and poloidal directions, respectively, with the magnetic field given by Eq. (4). The process of simplifying the model starts with the terms involving the polarization and gyro viscosity drifts. As mentioned in Eq. (30), the sum of these terms results in the gyro viscous cancelation. The remaining term \( u_{R,\gamma} \) from the cancelation is dropped in (32), and for computational convenience, we employ the thin layer approximation.14 The terms involving the polarization drift, which only concern ions, are thus reduced to
\[ \nabla \cdot \left( n_s \left( u_{p,s} + u_{s,g,y} \right) \right) \simeq - \frac{n_{s,0}}{\Omega_{s,0}^2} \nabla \cdot \left( d^0 \nabla \phi_s^0 \right), \]  
(41)
\[ \nabla \cdot \left( p_s \left( u_{p,s} + u_{s,g,y} \right) \right) \simeq - \frac{\rho_s}{\Omega_{s,0}^2} \nabla \cdot \left( d^0 \nabla \phi_s^0 \right), \]  
(42)
with
\[ \phi_s^0 = \frac{\phi}{B_0} + \frac{p_s}{q_i n_s B_0} \quad \text{and} \quad d^0 = \partial_t + u_{R,0} \cdot \nabla. \]  
(44)
Here, \( n_{i,0} \) is a characteristic reference value for the density of species \( s \).
The expression in Eq. (42) is formally small14 and is consequently left out of the ion pressure equation. The exclusion of this term does not violate the energy theorem. The similarity of the approximations in Eqs. (41) and (43) is required for energy conservation.

Moving on to the collisional terms and starting with the frictional resistive component, we evaluate them using the lowest order drifts \( u_{R,\gamma} \) and \( u_{R,0} \). In this case, the electric drifts cancel, leaving only the diamagnetic contribution. The total resistive drift is then approximated as
\[ u_{R,\gamma} = - \frac{\nu_{\gamma/\gamma}}{m_s \Omega_{s,0}^2} \left( \nabla p_i - q_i \nabla \phi \right) = \frac{3}{2} \sum_s m_s n_s u_{R,0}^2 \left( \nu_{s,s/\gamma} + \nu_{s,0/\gamma} \right) = u_{R,\gamma,0}. \]  
(45)
Similar to Ref. 14, we see that the thermal gradient terms partially cancel, and so we neglect them giving the approximation
\[ u_{R,\gamma} \simeq - \frac{\nu_{\gamma/\gamma}}{m_s \Omega_{s,0}^2} \left( T_s \nabla n_s - q_s \nabla \phi \right), \]  
(46)
\[ \simeq - \frac{D_{s,s/\gamma}}{n_s n_s} \left( T_s \nabla n_s - q_s \nabla \phi \right) = u_{R,\gamma,0}. \]  
(47)
where we define the diffusion coefficient as
\[ D_{s,s/\gamma} = \nu_{s,s/\gamma}. \]  
(48)
For the density equation and pressure equation, this results in
\[ \nabla \cdot n_i u_{R,\gamma} \simeq \nabla \cdot n_i u_{R,\gamma,0} \]  
\[ = - \sum_s \frac{D_{s,s/\gamma}}{n_s n_s} \left( T_s \nabla n_s - q_s \nabla \phi \right), \]  
(49)

\[ u_{R,\gamma,0} \cdot \left( q_i n_i \nabla \phi \right) \simeq u_{R,\gamma,0} \cdot \left( q_i n_i \nabla \phi \right), \]  
(50)

\[ = - \sum_s \frac{D_{s,s/\gamma}}{n_s n_s} \left( T_s n_s \nabla n_s - q_s \nabla \phi \right) \cdot \left( q_i n_i \nabla \phi \right). \]  
(51)
These approximations do not change the energy conservation as the two first terms appear as surface terms in the integral and as such are integrated out. Equation (51) is symmetric in indices and all terms cancel when summing over all species. For the electron pressure equation, the resistive contributions from Eq. (29) are approximated as

\[ \mathbf{v} \cdot (p_e \mathbf{u}_{e,r}) - \mathbf{v} \cdot \sum_x p_x \left( 1 + \frac{\sqrt{2} \gamma}{Z_a} \right) \frac{2}{3} \mathbf{u}_{e,T,r} \]

\[ \simeq \mathbf{v} \cdot (p_e \mathbf{u}_{e,c0}) - \sum_x D_{e \rightarrow 0} \left( 1 + \frac{\sqrt{2} \gamma}{Z_a} \right) \mathbf{v} \cdot (n_x \nabla T_e). \]  

(52)

The ion heat conduction is approximated as

\[ \mathbf{v} \cdot \left[ \sum_x D_{i \rightarrow 0} \left( 1 + \frac{\sqrt{2} \gamma}{Z_b} \right) \mathbf{v} \cdot (n_x \nabla T_i) \right] \]

\[ \simeq \mathbf{v} \cdot \sum_x D_{i \rightarrow 0} \left( 1 + \frac{\sqrt{2} \gamma}{Z_b} \right) \mathbf{v} \cdot (n_x \nabla T_i). \]  

(53)

Finally, this does not alter the energy theorem.

Finally, we consider terms related to the ion viscous stress tensor. When calculating the viscous stress tensor, we evaluate the velocities using the lowest order drifts and use the same approximation used for the polarization drift. Furthermore, the viscosity coefficients \( \eta_{i,x} \) are computed using reference values for the pressure and magnetic field. The tensor finally reads

\[ \pi_{1,x} \simeq -\eta_{1,x} \left[ -2 \partial_{\alpha \gamma} \phi^*_{\alpha} \left( \partial_{\beta \alpha} - \partial_{\beta \gamma} \right) \phi^*_{\gamma} + 2 \partial_{\alpha \beta} \phi^*_{\alpha} \right]. \]  

(54)

In the approximations of terms including the viscous drift, we evaluate \( \eta_{1,x} \) and \( n_x \) in the denominator at reference values as done in, e.g., Eqs. (56) and (57). For the dyadic product of the viscous stress tensor and the zeroth order drifts entering the ion pressure equation, the perpendicular collisional part is then approximated using reference values for pressure, collision frequency, and magnetic field in the viscosity coefficient. Likewise, the lowest order drift is approximated as \( u_{i,x,0} = b \times \nabla \phi^*_{\alpha} \). Finally, we obtain

\[ \pi_{1,x} : \nabla u_{i,x,0} \simeq -\sum_b m_b n_b D_{\beta \rightarrow 0} \frac{\partial_{\alpha \beta}}{m_b + m_b} \]

\[ \left[ \left( \partial_{\alpha \gamma} \phi^*_{\alpha} - \partial_{\beta \gamma} \phi^*_{\beta} \right)^2 + 4 \left( \partial_{\beta \gamma} \phi^*_{\beta} \right)^2 \right], \]  

\[ \pi_{1,x} : \nabla u_{i,x,0} = 0, \]  

(56)

where the viscous stress tensor diffusion coefficient is defined as

\[ D_{\beta \rightarrow 0} = \frac{m_b n_b}{m_b + m_b} \left( \frac{3 m_b}{10 m_b} + \frac{1}{2} \frac{m_b}{m_b Z_a} \right). \]  

(57)

The remaining terms involving the viscous stress tensor are approximated as

\[ \nabla \cdot n_x \mathbf{u}_{e \rightarrow \alpha} \simeq \sum_{\beta} n_{\beta,0} D_{e \beta \rightarrow 0} \frac{2}{3} \nabla^2 \phi^*_{\alpha}, \]  

(58)

\[ \nabla \cdot p_x \mathbf{u}_{e \rightarrow \alpha} \simeq \sum_{\beta} p_{\beta,0} D_{e \beta \rightarrow 0} \frac{2}{3} \nabla^2 \phi^*_{\alpha}, \]  

(59)

\[ p_x \nabla \cdot \mathbf{u}_{e \rightarrow \alpha} \simeq p_x \sum_{\beta} D_{e \beta \rightarrow 0} \frac{2}{3} \nabla^2 \phi^*_{\alpha}. \]  

(60)

The term given in Eq. (59) is left out as \( \nabla \cdot (p_x \mathbf{u}_{e \rightarrow \alpha}) / \nabla \cdot \mathbf{q}_e \sim \rho_e^2 / L_e^2 \ll 1 \). The two other terms in Eqs. (58) and (60) are approximated, so that together with Eq. (56), energy is conserved.

Having done all the approximations above, we employ neoclassical corrections to the equations by multiplying the diffusion coefficients by a correction factor \( \lambda \)

\[ D_{e \rightarrow 0} \rightarrow \frac{1}{1 + \frac{R}{R_0}} D_{e \rightarrow 0}, \]  

(61)

where \( R \) is the major radius, \( r \) is the minor radius, and \( q_0 \) is the safety factor. In principle, the neoclassical correction should only be applied to the averaged fields on closed field lines as it assumes longer timescales than what is found in intermittent turbulent transport. However, to simplify the implementation of open and closed field lines, the correction is kept for the whole domain.

For numerical convenience, we perform gyro-Bohm normalization giving the dynamic variables,

\[ \frac{T_i}{T_{c,0}} \rightarrow \frac{T_{i,x}}{T_{c,0}} \xrightarrow{\rho_D} \frac{T_{c,0}}{x}, \quad \Omega_{e,i,D}, \rightarrow \frac{t}{\tau}, \quad \frac{e\phi}{T_{c,0}} \rightarrow \psi, \quad \frac{n_x}{n_{i,0}} \rightarrow \frac{n_x}{n_{i,0}}. \]  

(62)

and the static variables,

\[ \frac{m_i}{m_D} \rightarrow \frac{m_i}{m_D}, \quad \frac{n_{i,0}}{n_{i,0}} \rightarrow \frac{n_{i,0}}{n_{i,0}}, \quad \frac{T_{c,0}}{T_{c,0}} \rightarrow \frac{T_{c,0}}{T_{c,0}}, \quad \frac{\partial_{\alpha \beta}}{m_b + m_b} \]  

(63)

Here, \( \Omega_{e,i,D} = \frac{eB_0}{m_D}, \rho_D = \sqrt{T_{c,0} / m_D \Omega_E^2}, \) with the subscript \( D \) indicating deuterium. In general, it can be any normalization, but for this work, we will use deuterium as the normalizing species. We can now write down the final MIHESEL-model equations

\[ \frac{d}{dt} n_s + n_s \mathbf{v} \cdot \mathbf{E} \left( \frac{1}{Z_s} \right) \mathbf{E} \left( \frac{p_s}{Z_s} \right) - a_s \frac{\mu_s}{Z_s} \nabla \cdot \left( \frac{d}{dt} \nabla \phi^*_{\alpha} \right) = \Lambda_{n_s}, \]  

(64)

\[ \sum_x a_{i,s} \frac{\mu_s}{Z_s} \nabla \cdot \left( \frac{d}{dt} \nabla \phi^*_{\alpha} \right) - \mathbf{E} \cdot \left( \sum_x p_x + p_c \right) = \Lambda_{a_s}. \]  

(65)

\[ \frac{3}{2} \frac{d}{dt} p_b + 5 \frac{p_b}{Z_a} \mathbf{E} \cdot \mathbf{E} \left( \frac{5}{2} \frac{1}{Z_a} \right) \]  

\[ - p_b \frac{\mu_b}{Z_a} \nabla \cdot \left( \frac{d}{dt} \nabla \phi^*_{\alpha} \right) = \Lambda_{p_b}, \]  

(66)

\[ \frac{3}{2} \frac{d}{dt} p_c + 5 \frac{p_c}{Z_a} \mathbf{E} \cdot \mathbf{E} \left( \frac{5}{2} \frac{1}{Z_a} \right) = \Lambda_{p_c}. \]  

(67)

Here, \( \phi^+_s = \phi + p_s / Z_s a_s \) is the generalized potential. The right-hand side contains all terms related to collisions and viscosity and are given by
where
\[ f = \frac{a}{\Omega_x, \beta} \frac{\nabla^2 \phi}{\phi^*} \]
\[ A_\alpha = \sum_s \sum_s Z_s a \frac{D_s \beta = 0}{a_s L_s} \left( T_s \nabla n_s \cdot \nabla n_s + n_s \nabla T_s \cdot \nabla n_s + T_s n_s \nabla^2 n_s - \frac{Z_s}{Z} \left[ T_s \nabla n_s \cdot \nabla n_s + n_s \nabla T_s \cdot \nabla n_s + T_s n_s \nabla^2 n_s \right] \right) - \sum_s \sum_s a \frac{D_s \beta = 0}{a_s L_s} \frac{\Omega_x, \beta}{\phi^*} \frac{\nabla^2 \phi}{\phi^*}. \]
(68)
\[ A_w = \sum_s \sum_s Z_s a \frac{D_s \beta = 0}{a_s L_s} \frac{\nabla^2 \phi}{\phi^*}, \]
(69)
\[ A_p = \frac{5}{2} \sum_s \sum_s Z_s a \frac{D_s \beta = 0}{a_s L_s} \left( T_s \nabla n_s \cdot \nabla n_s + n_s \nabla T_s \cdot \nabla n_s + T_s n_s \nabla^2 n_s - \frac{Z_s}{Z} \left[ T_s \nabla n_s \cdot \nabla n_s + n_s \nabla T_s \cdot \nabla n_s + T_s n_s \nabla^2 n_s \right] \right) - \sum_s \sum_s a \frac{D_s \beta = 0}{a_s L_s} \frac{\Omega_x, \beta}{\phi^*} \frac{\nabla^2 \phi}{\phi^*}, \]
(70)
\[ \frac{d}{dt} \left( \frac{3}{2} \frac{P_s}{\rho_s} + \frac{3}{2} \frac{P_e}{\rho_e} + \frac{3}{2} \frac{P_{s,e}}{\rho_{s,e}} + \frac{3}{2} \frac{T_s}{\rho_s} + \frac{3}{2} \frac{T_e}{\rho_e} + \frac{3}{2} \frac{T_{s,e}}{\rho_{s,e}} \right) = 0. \]
(74)

The advective derivatives are defined as
\[ \frac{d}{dt} = \frac{\partial}{\partial t} + \frac{1}{B} \{ \phi \} \cdot \mathbf{v} \text{ and } \frac{d^0}{dt} = \frac{\partial}{\partial t} + \{ \phi \} \cdot \mathbf{v}. \]
(72)
where \( \{ f, g \} = \partial f / \partial g - \partial g / \partial f \) is the Poisson bracket. The curvature operator is given as
\[ \mathcal{Q}(f) = \nabla \cdot \left( \frac{\mathbf{b} \times \nabla f}{B} \right) = - \frac{P_d}{B} \partial f. \]
(73)

Finally, we revisit the energy conservation. We have already discussed how none of the approximations lead to a violation of the energy conservation, and redoing the calculation of the energy theorem with these approximations yields

\section{IV. Simulations of Seeded Blob Dynamics}

As a first test and demonstration of the model, we solve the above equations with seeded blobs as the initial condition. This means we introduce a Gaussian perturbation in the density,
\[ n_s(x, y, z, t = 0) = n_{s,bg} + n_{s,b} \exp \left( -\frac{(x - x_c)^2}{2\sigma_{x,s}^2} - \frac{(y - y_c)^2}{2\sigma_{y,s}^2} \right). \]
(75)

Here, \( n_{s,bg} \) is the background density and \( n_{s,b} \) is the blob amplitude. For the width of the blob, we use \( \sigma_{x,s} = \sigma_{y,s} = 10 \rho_s \) for all species. For reference values, we have \( n_{s,0} = 1.5 \times 10^{19} \text{m}^{-3} \), \( T_{s,0} = T_{e,0} = 20 \text{eV} \), and \( B_0 = 2T \). In the poloidal \( (y) \) direction, we employ periodic boundaries while we use Dirichlet and Neumann boundaries for the inner and outer radial \( (x) \) boundaries, respectively. The equations are numerically solved using the FELTOR discontinuous Galerkin library. Here, we have \( N_a = 2N_y = 192 \) grid cells and use three polynomial coefficients for a third order method. The box size is \( L_x = 2L_y = 200 \rho_D \). Time integration is performed using a third order implicit backward differentiation formula (BDF) method with a time step of \( dt = 0.5 \). The resulting non-linear system of equations is solved.

\begin{table}[h]
\centering
\caption{Table of setups for mixes in the background and perturbation with 49 unique initial conditions in total.}
\begin{tabular}{cccccccccccc}
\hline
D & T & D & T & D & T & D & T & D & T & D & T \\
\hline
Background mix & 12.5 & 87.5 & 25 & 75 & 37.5 & 62.5 & 50 & 50 & 62.5 & 37.5 & 75 & 25 & 87.5 & 12.5 \\
Perturbation mix & 12.5 & 87.5 & 25 & 75 & 37.5 & 62.5 & 50 & 50 & 62.5 & 37.5 & 75 & 25 & 87.5 & 12.5 \\
\hline
\end{tabular}
\end{table}

reduce to a single ion species term each. In the latter case, one has \( x = \beta \), meaning that, e.g., \( \mu_x = \mu_{x,\beta} \).
using Anderson acceleration,\(^{21}\) where for each Anderson iteration, we solve the system of equation for the partial derivative. We thus need to perform an inner inversion of the potential and ion pressure equations since these are intricately coupled in the time derivative. To do so, we cast them in the form

\[
\begin{pmatrix}
-\sum_{a} a \mu_{a} \nabla^{2} - \frac{\mu_{1}}{Z_{1}^{2}} \nabla^{2} - \frac{\mu_{2}}{Z_{2}^{2}} \nabla^{2} \\
- \frac{\mu_{1}}{Z_{1}^{2}} \nabla^{2} + \frac{3}{2} \frac{1}{p_{1}} - \frac{\mu_{1}}{Z_{1}^{2} a_{1}} \nabla^{2} - \frac{3}{2} \frac{1}{p_{2}} - \frac{\mu_{2}}{Z_{2}^{2} a_{2}} \nabla^{2} \\
\vdots
\end{pmatrix}
\begin{pmatrix}
\partial \xi \\
\partial p_{1} \\
\partial p_{2} \\
\vdots
\end{pmatrix}
= 
\begin{pmatrix}
- (\Lambda_{\phi} + \phi (\sum_{a} a \mu_{a} \nabla \cdot \{\phi, \nabla \phi_{a}^{*}\}) - \sum_{a} a \mu_{a} \nabla \cdot \{\phi, \nabla \phi_{a}^{*}\}) \\
\left(\Lambda_{p_{1}} - \frac{5}{2} \frac{1}{p_{1}} \phi (\phi) - \frac{5}{2} \frac{1}{Z_{1}} \phi \left(\frac{p_{1}^{2}}{n_{1}}\right) + \frac{1}{p_{1}} \frac{\mu_{1}}{Z_{1}} \nabla \cdot \{\phi, \nabla \phi_{1}^{*}\}\right) / p_{1} \\
\left(\Lambda_{p_{2}} - \frac{5}{2} \frac{1}{p_{2}} \phi (\phi) - \frac{5}{2} \frac{1}{Z_{2}} \phi \left(\frac{p_{2}^{2}}{n_{2}}\right) + \frac{1}{p_{2}} \frac{\mu_{2}}{Z_{2}} \nabla \cdot \{\phi, \nabla \phi_{2}^{*}\}\right) / p_{2} \\
\vdots
\end{pmatrix}
\]

(76)

Because of the complex form of the matrix, we invert this system with a matrix-free approach. We consider a two ion-species plasma of deuterium and tritium, which is split into different configurations for the background and perturbation mix. The collection of configurations is all possible combinations of “background mix” and “perturbation mix” given in Table I. The charge weighted sum of ion densities is such that in normalized units, \(n_{\text{tot}} = \sum_{a} Z_{a} n_{a} = 1\) and \(n_{\text{tot}, b} = \sum_{a} Z_{a} n_{a,b} = 1\) for all runs.

An example of the evolution of the electron density profile is shown in Fig. 1 for the case of 50%–50% DT in both the background and perturbation. The mass normalized to the case of 50%–50% DT in both the background and blob. The center of mass is defined as

\[X_{c} \equiv \frac{1}{M} \int \frac{x}{X} \sum_{a} m_{a}(n_{a} - n_{a,b})dV, \]  

(77)

where \(M\) is the mass of the blob given as

\[M = \int \sum_{a} m_{a}(n_{a} - n_{a,b})dV. \]  

(78)

The electron contribution to the blob mass is neglected since it is much smaller. From the left figure, we observe that the blobs overall follow the same trend with the light mixes moving faster. This picture eventually becomes less clear as light blobs in heavy backgrounds appear to have moved further at the end of the simulation. From the right figure, we see that the initial development is grouped within the seven distinct background mixes (see Table I) but quickly separates according to the perturbation mix. At around \(\Omega_{\text{CD}} \sim 1800\), this picture is switched as we observe a grouping corresponding to the perturbation mix after which they will start to separate again. Overall, it is evident that initial mixes with more deuterium (green) in general

**FIG. 1.** Electron density for the case of 50% D, 50% T in both the background and perturbation taken at different snapshots with the center of mass superimposed as magenta crosses.
move faster radially than with tritium (purple), while the dependency on the perturbation mix increases over time.

Taking the time derivative of the radial center of mass position yields the radial center of mass velocity. This is related to the flux given by

$$\Gamma(t) = \sum m_i n_i u_i dV = MV_C = M \frac{d}{dt} X_C. \quad (79)$$

Consequently, it is interesting to look at the (maximum) velocity as it indicates the (maximum) flux. The radial velocity is shown in Fig. 3. Here, it is observed that mixes with more deuterium, especially in the blob, have higher initial radial velocity and reach their maximum earlier than for tritium dominated simulations. However, over time, this picture will reverse with tritium dominated blobs having higher velocity in the end. Taking a closer look at the maximum velocity in Fig. 3(right), it becomes clear that the moment at which the maximum value is obtained is highly dependent on the background density illustrated by the seven groupings. Comparing each group, we see that overall the maximum velocity decreases as the amount of tritium in the background increases. Within each group, it is clear that the amount of tritium in the perturbation significantly affects the maximum velocity.

To quantitatively estimate the influence of the mixture, we perform a simple scaling analysis. From previous scaling studies of the maximum blob velocity, we know that it scales as $v_{\text{max}} \propto m^{0.3}$ (see, e.g., Ref. 13),

$\text{FIG. 2.}$ Left: radial center of mass of electron density. Right: radial center of mass normalized to 50% D, 50% T.

$\text{FIG. 3.}$ Left: radial center of mass velocity. Right: maximum radial velocity against the time of occurrence.
thus we expect something similar in our case. However, since the mix in the background can differ from the blob mix, we use a composite scaling of the form $v_{\text{max}} \propto m_{\text{eff}} b + m_{\text{eff}} g$ where we have introduced an ion effective mass given by

$$m_{\text{eff}} = \frac{\sum_{a} n_{a} m_{a}}{\sum_{a} n_{a}}.$$  

(80)

With this, the dependency of the effective mass of the initial blob and background on the maximum radial velocities is found to scale as

$$V_{\text{max}}(m_{\text{eff}} b, m_{\text{eff}} g) = 0.055(\pm 0.001) \left( m_{\text{eff}} b^{-0.34(\pm 0.01)} + m_{\text{eff}} g^{-0.25(\pm 0.01)} \right).$$  

(81)

From this velocity scaling, we observe that not only the maximum radial velocity scales the strongest with the perturbation mix, but also the background mix plays a significant role. It should be noted that in the case of a uniform initial ratio, the background and the perturbation effective masses are the same. With this, we get a scaling of $V_{\text{max}}(m_{\text{eff}}) \propto m_{\text{eff}}^{-0.5}$. This differs slightly from the expected $m_{\text{eff}}^{-0.3}$ scaling; however, the magnitude of the blob put us in an energy-limited regime, as discussed in Ref. 28, giving a slightly higher mass dependency. Naturally, this scaling only works with the initial conditions presented earlier. The dependency of the effective mass in the blob is expected to also scale with, e.g., the magnitude of the blob.

Finally, we consider how the mix in the blob evolves. Within the model equations, we have included interactions between all species through collisions, which are contained within the resistive drift terms as seen in Eq. (43). Here, it is observed that the resistive force will result in an equilibration of the mix such that $n_{a} \rightarrow c_{a} n_{b}$, where $c_{a}$ is some constant. We examine this by looking at the ratio of tritium in the blob over time normalized to the background ratio with the result being shown in Fig. 4. In this figure, there is a clear tendency for all mixes to converge to the background mix as a consequence of the resistive force. As a result, it is to be expected that in a plasma where this is a dominating force, the plasma will tend toward a homogeneously mixed plasma that can be described by an effective mass approach. Up to now, we have only considered mass propagation. We now discuss the differences in the density distribution for the 50%D 50%T case. In Fig. 5, we show the difference between the blob and without the terms involving ion–ion resistivity at different points in time. We notice that, regardless of the resistivity, the difference between the two species is small compared to the initial blob perturbation meaning there is little separation between the two species. Comparing the two cases with and without the resistive drift (top and bottom), it is evident that the resistive force has a smoothing effect on the species as the differences in the resistive scheme are a factor of five smaller than the non-resistive counterpart. This supports our earlier observation in Fig. 4 where we observed a tendency to equilibrate toward a global ratio. It should be noted that the evolution of the electron density distribution is the same for both cases as it equals the sum of ion densities and ion-ionic resistive drifts cancel each other.

Earlier in Sec. II, we discussed the issue with asymmetry in $n_{a}$ when adding densities [see Eq. (38)] that could cause different outcomes when splitting a single species into two. We have therefore performed simulations similar to the above, but with a mix of pure deuterium in order to gauge the error introduced by this non-physical artifact. From these simulations, the error amounts to small deviations in the position of the blob over time. The errors are of a magnitude of below $10^{-4}$ when normalized to a single species simulation. In regard to the DT runs, this can be considered negligible and cannot be attributed to the mass effects we observe. Furthermore, runs with the same mix ratio in the blob and background show errors up to $10^{-7}$. As the resistive drift drives the ratio of the blob and background mix toward equality, the error introduced by the asymmetry in summation will diminish over time in general and so the error level is considered acceptable.

V. DISCUSSION

Based on the Zhdanov 21 moment closure, we have derived a drift ordered fluid model that incorporates multiple ion species and describes the evolution of density, vorticity, and pressure. The model includes consistent ion–ion and electron–ion collisional interactions while maintaining energy conservation. Based on the drift fluid equations for density, vorticity, and pressure, a computationally workable model was derived and studied through seeded blobs with different mixtures of deuterium and tritium in both the background and blob. We discussed how the mix in the background and blob affects the propagation of the blob individually and found that, initially, the deuterium dominated mixes developed the fastest with the background mix playing a leading role. However, as time progressed, the mixture in the blob became dominant. It was found that both parameters are important for the scaling of the maximum velocity but with the blob mix playing a larger role. Furthermore, we discussed how the resistive forces between the mixes tend to equilibrate so that the mixing ratio is the same everywhere in the plasma. As time progresses, the plasma is expected to converge to a system that can be modeled with an effective mass. The asymmetry in summation discussed in Sec. II was examined and found to be inconsequential.
The mass dependency on the blob propagation suggests that radial transport will decrease as the isotope mix increases the overall mass. For future studies, we plan to investigate the isotope dependency on cross field particle and energy transport in the edge in a fully developed turbulent plasma. In these studies, we will furthermore account for the parametrization of parallel losses as in Ref. 15.

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