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Thermo-optic instabilities in asymmetric dual-core amplifiers

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Dynamical thermo-optic instabilities in asymmetric dual-core fiber amplifiers are investigated by numerical simulations. A coupled-mode model capable of describing the asymmetric dual-core structure is derived, and structures with various levels of refractive-index asymmetry from $10^{-6}$ to $10^{-4}$ are studied. The most viable route to avoid thermally induced power signal field fluctuations is found to be core decoupling via large separation and strong refractive-index asymmetry. A design criterion for this strategy to be effective is suggested based on the numerical results. © 2020 Optical Society of America

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1. INTRODUCTION

Transverse mode instabilities (TMIs) arising from thermo-optic nonlinear effects are a major limitation to average-power scaling of large-core high-power fiber amplifiers [1,2]. Results from intense experimental studies since the first published report in 2011 [1] have been successfully modeled by both beam propagation and coupled-mode approaches [2–6]. Theory and experiments in single-core single-pass amplifiers have found TMI to be an inherently dynamical phenomenon. One way to picture it is as a Raman-like power transfer from the fundamental mode (FM) to the higher-order modes (HOMs) in the fiber, due to the delayed nature of the thermo-optic nonlinear response [6]. A different, though equivalent, picture due to Eidam et al. [1] and Smith and Smith [2] is that intermodal power transfer results from a self-written long-period grating, formed by an initial (small) population of the HOMs, which leads to a beating pattern in the transverse temperature profile and therefore the refractive index. The grating has by nature the correct periodicity to transfer power between the FM and HOMs, but will have the right phase to mediate power transfer into the HOMs only if the frequency of the HOM light is slightly downshifted, typically by a few kHz [2].

Recent theoretical developments within the framework of established coupled-mode theories for TMI have predicted that static or quasi-static mode deformations could arise in double-pass [7] or dual-core [8,9] amplifiers. In the double-pass case, where the predictions have seen some experimental confirmation [10], this is most easily understood in the grating picture, because light scattering on the grating set up by the counter-propagating signal can be effective even in the absence of a frequency shift. In the dual-core case, on the other hand, the results are most easily rationalized by an analogy with the combined action of Raman and four-wave mixing (FWM) processes in “ordinary” nonlinear fiber optics [9].

While the dual-core theory results have yet to be tested experimentally, they are of some practical relevance because power distribution over multiple amplifying cores is being considered as a strategy for scaling total amplifier output power beyond the single-core TMI limits [11–13]. While it is easier to maintain single-mode guidance in the small cores than in a single large core, if the cores are close enough to be strongly coupled, one has to consider thermo-optic couplings between the supermodes of the whole core array. Experiments with an array of 18 closely spaced cores indicated a low instability threshold [12], whereas arrays of well-separated cores have shown performance resembling a set of independent amplifiers [11]. The question then is how far the cores need to be separated to achieve decoupling, and what behavior one can expect if the cores are not decoupled. In [11], the problem of thermally decoupling multimoded cores was studied numerically in the limit where the cores were optically decoupled. In our own previous numerical work, we studied thermo-optic instabilities between two single-moded cores with a separation small enough to allow for optical coupling over the amplifier length. It was found that the stability properties depended strongly on the choice of supermode for amplification. The even supermode was found to suffer from...
very low static and dynamic instability thresholds for intermediate core separations, whereas when amplifying in the odd supermode, the cores decoupled much more rapidly.

The purpose of the present work is to extend the investigations of [9] to the case of an asymmetric dual-core structure, where the refractive indices of the two cores are not exactly identical. This is relevant because on one hand, there is a limit to the accuracy with which the refractive index can be controlled in the fabrication process, and for intermediate or large core separations, this may significantly affect the supermodes due to the weak intercore couplings. On the other hand, there may also be some room for making intentional core index differences if this could lead to improved stability properties.

The rest of this paper is structured as follows: in Section 2, we develop our coupled-mode formalism, emphasizing mainly the modifications relative to [9] necessary to describe the asymmetric coupler case. In Section 3, we present results for nine different dual-core structures. Section 4 provides a short unifying discussion of these results, and Section 5 summarizes our conclusions.

2. FORMAL THEORY

A. Coupled-Mode Formalism

The formal theory of this work is overall similar to the one developed in Refs. [8,9]; however, a few improvements have been necessary to properly describe the case of asymmetric dual-core amplifiers. As before, our overarching objective is to obtain predictions for the TMI instability thresholds. In our previous work, a simple amplifier model based on rate equations was developed in Refs. [8,9]; however, a few improvements have been indicated that the mode profiles in the structure considered can be written as [14]

\[ \delta \mathbf{P} = \delta \mathbf{P}_g + \delta \mathbf{P}_h, \]  

where \( \delta \mathbf{P}_g \) expresses amplifier gain, while \( \delta \mathbf{P}_h \) describes the thermo-optic nonlinearity. We neglect thermo-optic modifications to the transverse mode profiles, which may have some impact on the quantitative results, leading to an overestimate of the intercore coupling. However, a recent finite-element study indicated that the mode profiles in the structure considered were not strongly modified at thermal loads of a few tens of W/m [15], which is the regime we are considering in most of the simulations.

Since we will consider fibers with weak index contrasts, the full-vectorial modes are approximated by real linearly polarized scalar modes:

\[ \mathbf{c}_m(r_\perp) = \frac{1}{\sqrt{2\pi\varepsilon_0}} \Psi_m(r_\perp) \hat{\mathbf{e}}, \]  

\[ \int dr_\perp \Psi_m(r_\perp) \Psi_n(r_\perp) = \delta_{mn}. \]  

In writing Eq. (6), the effective index of the mode in question has been approximated by the cladding index \( n_0 \). Due to the very small index variations in the considered fiber structures, such an approximation is permissible except when the difference between modal effective indices is an important parameter. The signal intensity can now be written as

\[ I(r, t) = \sum_{mn} \frac{1}{(2\pi)^2} \int d\Omega_1 \int d\Omega_2 A^*_m(z, \Omega_1) A^*_n(z, \Omega_2) \times \Psi_m(r_\perp) \Psi_n(r_\perp) e^{i(\beta_m - \beta_n)z} e^{i(\Omega_2 - \Omega_1)t}. \]  

In this equation, integration has been restricted to positive optical frequencies and centered on the signal frequency. Due to the slow response of the thermo-optic nonlinearity, this frequency integration can be restricted to the few-kHz range (\pm 10 kHz in the simulations), and these “slow” frequency variables are designated by capital omegas, \( \Omega_1, \Omega_2, \) etc.

The power gain coefficient at a particular point in the amplifier can be expressed as

\[ g(r, t) \approx \frac{g_0(z, t)}{1 + \frac{g_0(z, t)}{I_{sat}(z, t)}}, \]  

where \( g_0(z, t), I_{sat}(z, t) \) can be calculated from the Yb dopant concentration and the pump intensity, which is assumed to be always uniform across the pump cladding. To simplify the numerics, the intensity in each core is approximated constant over the core cross section, \( I_c \approx \frac{P_c}{A_c} \), where \( A_c \) is the area of one core, and \( P_c \) is the total power in this core. Contrary to Refs. [8,9], however, the formalism developed in this paper allows for different power levels in the two cores. The local gain becomes

\[ g(r, t) \approx \frac{g_0(z, t)}{1 + \frac{P_c(z, t)}{P_{sat}(z, t)}}. \]
for \( r \) in core \( k \), and zero elsewhere. The functions \( g_0 \) and \( P_{\text{sat}} \) are calculated from the steady-state solutions to the Yb rate equations:

\[
g_0(z, t) = N_{\text{Yb}} \frac{P_p(z, t)}{A_{\text{cl}}} (\sigma_{\text{en}}\sigma_{\text{en}} - \sigma_{\text{em}}\sigma_{\text{em}}) - P_t\sigma_{\text{em}},
\]

\[
P_{\text{sat}}(z, t) = \frac{A_{\lambda_p} P_p(z, t) (\sigma_{\text{en}} + \sigma_{\text{em}}) + P_t}{\lambda_p (\sigma_{\text{en}} + \sigma_{\text{em}})}.
\]

Here \( P_p \) is the pump power, \( A_{\text{cl}} \) the area of the pump cladding, \( N_{\text{Yb}} \) the concentration of Yb ions, \( \sigma_{\text{en}}, \sigma_{\text{em}}, \sigma_{\text{en}} \) absorption cross sections at pump and signal wavelengths, \( \lambda_p, \lambda_s \), respectively, \( \sigma_{\text{en}}, \sigma_{\text{em}} \), emission cross sections at pump and signal wavelengths, and \( P_t = \frac{hc}{\lambda_p} \), where \( \tau \) is the Yb upper-state lifetime, and \( h \) is Planck’s constant.

The evolution equation for the time-dependent pump power is

\[
\partial_z P_p(z, t) = -\frac{A_{\lambda_p}}{A_{\text{cl}}} N_{\text{Yb}} \sum_{k=1}^2 \frac{(\sigma_{\text{en}}\sigma_{\text{en}} - \sigma_{\text{em}}\sigma_{\text{em}}) P_p(z, t)}{(\sigma_{\text{en}} + \sigma_{\text{em}}) P_p(z, t) + P_t} + \sigma_{\text{en}} P_t \lambda_p + (\sigma_{\text{en}} + \sigma_{\text{em}}) P_p(z, t) \frac{\lambda_p}{A_{\lambda_p}} + \frac{P_t}{A_{\lambda_p}}.
\]

The use of steady-state rate equations is of course not fully correct when allowing for a time-varying pump, but it has been found numerically that this quasi-static approach works well for the slow time variations considered here. In addition, the actual pump power variations with time are found to be quite small, on the order of a few percent when the signal field distribution is strongly fluctuating, and negligible otherwise.

The gain term in the propagation equations can then be written as

\[
-i \omega e^{i\beta_m z} \int d\tau e^{-i\omega\tau} g_0(z, t) \cdot \delta P_{\text{th}}(r, t)
\]

\[
\approx \frac{1}{2} \int d\tau e^{-i\omega\tau} \sum_n A_n(z, t) e^{i(\beta_m - \beta_n)z} \sum_{k=1}^2 \frac{g_0(z, t)}{1 + \frac{P_p(z, t)}{P_{\text{sat}}(z, t)}} O_{mn}^{(k)}
\]

\[
A_n(z, t) = \frac{1}{2\pi} \int d\Omega A_n(z, \Omega) e^{i\Omega z},
\]

\[
O_{mn}^{(k)} = \int d\Omega \Psi_m(r_\perp) \Psi_n(r_\perp),
\]

where \( \int_k \) signifies integration over core \( k \) only. For the gain term, it is unproblematic to include time variations in this way. However, to get a manageable expression for the thermo-optic nonlinearity, it is necessary to perform a first-order expansion of the gain saturation term [16]. The thermo-optic contribution to \( \delta P \) can be written as

\[
\delta P_{\text{th}}(r, t) = \varepsilon_0 E(r, t) \Delta \varepsilon(r, t),
\]

\[
\Delta \varepsilon(r, t) = \frac{1}{2\pi} \int d\Omega e^{i\Omega^2} \Delta \varepsilon(r, \Omega),
\]

\[
\Delta \varepsilon(r, \Omega) = 2n_0 \frac{\eta}{\kappa} \int d\Omega' G(r_\perp, r_\perp', \Omega) q(r', \Omega),
\]

\[
q(r, \Omega) = q_d \int d\tau e^{-i\Omega t} I(r, t) g(r, t),
\]

\[
g(r, \Omega) = \frac{g_0(z)}{1 + \frac{P_p(z)}{P_{\text{sat}}(z)}} q_d \left( \frac{\lambda_s}{\lambda_p} - 1 \right).
\]

Following the derivations of [9], we separate the intensity into its time average and a small time-fluctuating term, to arrive at

\[
q(r, \Omega) \approx q_d \int d\tau e^{-i\Omega t} I(r, t) \frac{g_0(z)}{1 + \frac{P_p(z)}{P_{\text{sat}}(z)}}
\]

for \( r \) in core \( k \), and zero elsewhere. In this equation, \( P_k(z) \) is the time-averaged power in core \( k \), the saturated gain term has been expanded to leading order in the dynamic intensity, and a zero-frequency term has been neglected in accordance with [9]. The constants \( \kappa, \eta \) are the silica heat conductivity and thermo-optic coefficient, respectively, so that \( n = n_0 + \eta \Delta T \), where \( \Delta T \) is the difference in some reference temperature, where \( n = n_0 \).

The time-dependent functions \( g_0(z), P_{\text{sat}}(z) \) are obtained from the expressions (11), (12), replacing the time-dependent pump power with its average value. This formalism is valid for weak temporal power fluctuations, i.e., in the region of transition from stable behavior to TMI, but not necessarily deeply into the unstable region.

Inserting Eq. (22) into Eqs. (17)–(19), we obtain

\[
-i \omega e^{i\beta_m z} \int d\tau e^{-i\omega\tau} \int d\Omega \Psi_m(r_\perp) \Psi_n(r_\perp) \Delta \varepsilon(r, \Omega)
\]

\[
= \frac{\eta}{\kappa} \sum_{\beta m q} \frac{1}{(2\pi)^3} \int d\Omega_1 d\Omega_2 A_m(z, \Omega_1) A^*_n(z, \Omega_2)
\]

\[
\times A_q(z, \Omega - \Omega_1 + \Omega_2) e^{i\theta_{mnpq} z}
\]

\[
\times \sum_{k=1}^2 \frac{g_0(z)}{1 + \frac{P_p(z)}{P_{\text{sat}}(z)}} G_{mnpq}^{(k)}(z, \Omega - \Omega_1),
\]

\[
\Gamma = \frac{2\eta \Delta T}{\kappa \lambda_s} \theta_{mnpq} = \beta_m - \beta_n + \beta_p - \beta_q,
\]

\[
G_{mnpq}^{(k)}(z, \Omega) = \int d\Omega \Psi_m(r_\perp) \Psi_n(r_\perp)
\]

\[
\times \int d\Omega' G(r_\perp, r_\perp', \Omega) \Psi_p(r_\perp') \Psi_q(r_\perp').
\]

Due to the asymmetry of the dual-core structure, none of the nine inequivalent \( G_{mnpq}^{(k)}(z, \Omega) \) functions can be eliminated by symmetry considerations. They are all included in the simulations, regardless of their phase-mismatching terms.
The scalar mode fields $\Psi_m$ have been estimated as the dominant polarization components of full-vectorial solutions obtained with a commercial full-vector modal solver based on the finite element method [15,17,18], with subsequent renormalization according to Eq. (7). The $G^{(0)}_{\text{npq}}$ double integrals are evaluated directly on a rectangular integration grid, using the Greens function expansions given in [3]. The $z$-stepping is done by a fourth-order Runge–Kutta method with adaptive step size [19]. A frequency grid spanning $20\, \text{kHz}$ with a resolution of $1\, \text{Hz}$ is used (real frequency, not angular frequency), and this was found to yield good convergence of the results.

3. NUMERICAL RESULTS

In the numerical calculations, we consider a set of nine dual-core amplifier structures, with three different core separations and three values for the refractive-index difference $\Delta n$ between the two cores. The core separation is quantified by the center-to-center distance $d$ between the cores. The effective-index differences between the two supermodes for the different fiber parameters as well as for $\Delta n = 0$ are listed in Table 1. Both cores have a core radius $a = 10\, \mu m$, and the core with the largest refractive index has a numerical aperture of 0.038, in accordance with [8,9]. This implies that the $V$ parameter of the high-index core is 2.3, and so both cores are individually single-moded. In high-power operation second-order modes might be confined by thermal lensing, but this effect is not considered in the present work, which focuses on coupling between the FMs in the two cores. Other amplifier parameters are identical to those used in [9] and are reported in Table 2.

Before going into the nonlinear simulations, it is of interest to discuss the purely linear propagation properties of the asymmetric dual-core structures. In a symmetric dual-core structure, the even and odd supermodes will both have an equal amount of power in the two cores, but with asymmetric cores, the supermodes will also become asymmetric. In the numerical simulations, we determine input supermode coefficients so that the total power is equally distributed over the two cores, with a phase difference of either zero or $\pi$, which in the limit of $\Delta n \rightarrow 0$ is equivalent to seeding the even or odd supermode, respectively. We will refer to these incoupling conditions as even or odd seed fields. Due to the asymmetry, power will fluctuate between the cores during propagation. In Fig. 1, this fluctuation is illustrated for two of the dual-core structures investigated. The case of $d = 22\, \mu m, \Delta n = 10^{-6}$ is close to a symmetric structure, because $\Delta n$ is much lower than the $n_{\text{eff}}$ difference between even and odd supermodes in the perfectly symmetric structure (see Table 1). In this case, only a small fluctuation is seen. On the other hand, there are strong fluctuations for $d = 30\, \mu m, \Delta n = 10^{-5}$, where $\Delta n$ is comparable to $\Delta n_{\text{eff}}$ in the symmetric structure.

To further quantify these fluctuations, we calculate the root-mean-square (RMS) fluctuation between the two cores as a function of $z$. Since the initial modal amplitudes, $A_0, A_1$, can be chosen real when the relative field phase on input is zero or $\pi$, we can write the power in core $k$ as

$$P_k(z) = A_k^2 O_{0k}^{(d)} + A_k^2 O_{11}^{(d)} + 2 A_0 A_1 O_{12}^{(d)} \cos \beta z,$$  \hspace{1cm} (26)

and we define the relative RMS fluctuation in core 1 as

$$\text{RMS}_x = \sqrt{\frac{1}{L} \int_0^L \text{d}z \left( \frac{P_1^2(z) - \bar{P}_1^2}{\bar{P}_1} \right)},$$  \hspace{1cm} (27)

$$\bar{P}_1 = \frac{1}{L} \int_0^L \text{d}z P_1(z).$$  \hspace{1cm} (28)

In Fig. 2, we plot RMS$_x$ versus the ratio between $\Delta n$ and $\Delta n_{\text{eff}}^{(d)}$, the effective-index difference between the even and odd supermodes for a given value of $d$ and $\Delta n = 0$. It can be seen...
that the results fall on a curve peaked around $\frac{\Delta n}{\Delta n_{\text{eff}}} = 1$. For $\Delta n \ll \Delta n_{\text{eff}}^{(0)}$, the power is essentially coupled into one almost symmetric/antisymmetric supermode. In the opposite limit of $\Delta n \gg \Delta n_{\text{eff}}^{(0)}$, power is almost equally distributed between two supermodes, which are both highly localized in one core, and minimal power exchange occurs. Thus, it is for $\Delta n \sim \Delta n_{\text{eff}}^{(0)}$ that we see strong fluctuations between the cores.

The power fluctuations between cores imply a partial localization of signal power that increases intensity and thereby Kerr and Raman nonlinear effects. Thus, these fluctuations are undesired, even if the output field is temporally stable. As we shall see, thermo-optic nonlinear effects significantly modify the power distribution between the cores, but not necessarily in a desirable manner.

### A. $d = 22 \ \mu m$

In the case of $\Delta n = 0$, the fiber with $d = 22 \ \mu m$ displayed a behavior similar to what is found in ordinary single-core amplifiers, namely, intermodal coupling associated with a downshift in frequency, i.e., the Raman regime [8,9]. This is due to the relatively large $\Delta \beta$ values, which suppress the FWM effect arising from the single phase-mismatched term present in the coupled-mode equations for the symmetric dual-core amplifier. Since the asymmetry in the core index serves only to increase $\Delta \beta$, one may expect this conclusion to hold also in the asymmetric case. However, for strongly asymmetric cores, the input signal will be in a superposition of the two eigenmodes, and the analogy to the perfectly symmetric case is then not straightforward.

To quantify the onset of dynamic fluctuations, we calculate the RMS fluctuation of the temporal output power in each core over the full time window of a simulation (1 s). We shall denote this quantity $\text{RMS}_p$. Fig. 3 shows the results as a function of output signal power for the three $\Delta n$ levels considered here. A $\text{RMS}_p$ value of 1% indicated by the horizontal line is taken as a criterion for temporal instability. It can be seen that the amplifiers with $\Delta n = 10^{-6}$ and $10^{-5}$ have similar behavior, reaching the 1% threshold at $\sim 370 \ W$ and $435 \ W$ for even and odd inputs, respectively. The curves display the exponential growth of fluctuations with signal power that is characteristic of the Raman regime of TMI. To put the chosen instability criterion into perspective, Fig. 4 shows the temporal power fluctuation between the cores in a 20 ms time frame for the case of $\Delta n = 10^{-6}$, even seed distribution, and output power of $377 \ W$, i.e., at the instability threshold.

For $\Delta n = 10^{-4}$ on the other hand, an increase in threshold to $\sim 415 \ W$ (even) and $\sim 465 \ W$ (odd) is seen, showing that a decoupling of the cores begins to set in. At the same time, the behavior of the curve for the even seed field deviates markedly from a purely exponential growth, showing that more complex nonlinear coupling processes are now occurring. Indeed, this value of $\Delta n$ slightly exceeds $\Delta n_{\text{eff}}^{(0)}$, which means that a linear combination of eigenmodes is excited on input, and significant power transfer between the cores occurs during propagation. So the price for the slightly enhanced dynamical TMI threshold is a strong increase in static core-to-core fluctuations during propagation. Fig. 5 quantifies the time-averaged power in each of the active cores for $\Delta n = 10^{-6}$ and $\Delta n = 10^{-4}$. While the fiber with $\Delta n = 10^{-6}$ behaves almost like a perfect symmetric fiber,
there is strong fluctuation between the cores for $\Delta n = 10^{-4}$. It can even be noted in this case that the power localizes in core 1 during the first half of the propagation, indicating that nonlinear effects significantly affect the power distribution. In summary, the numerical results show that the $d = 22 \mu m$ design is resilient to index variations on the order of $10^{-5}$ but is negatively affected by a variation of $10^{-4}$, even if the threshold for temporal instability is slightly increased by the asymmetry.

B. $d = 30 \mu m$

For $d = 30 \mu m$, the perfectly symmetric amplifier was found to have a strongly reduced threshold for dynamical fluctuations due to the onset of FWM effects when seeding the even supermode. In addition, the amplifier was found to be unstable towards static spatial fluctuations, meaning that a slight imbalance in the seed level between the two cores leads to a strong imbalance in the average output power. Note that this effect differs from purely dynamic instabilities where the time-averaged power output of each core is the same, although the instantaneous power distribution fluctuates. On the other hand, when seeding the odd supermode, the dynamical threshold was high, and the output spatial profile was stable towards static deformations.

The $RMS_P$ values versus output signal power are shown in Fig. 6. As for $d = 22 \mu m$, there is little difference in the threshold between $\Delta n = 10^{-4}$ and $\Delta n = 10^{-5}$. The other hand, a strong difference in threshold between even and odd seed fields is seen, in accordance with what was found for the symmetric amplifier. However, for $\Delta n = 10^{-4}$, a strong increase in threshold is seen, especially for the even seed distribution, and the difference between even and odd seeding is very much reduced—in fact, even seeding seems to be slightly preferable. For this case, there is no penalty in terms of longitudinal fluctuations. Fig. 7 shows the average power distribution between the cores versus propagation distance at power levels close to threshold for $\Delta n = 10^{-5}$ and $10^{-4}$. The absolute magnitude of the fluctuations is larger for $\Delta n = 10^{-4}$, but in relative terms, they are lower for this case since the signal output power is $\sim 3$ times larger than for $\Delta n = 10^{-5}$. For $\Delta n = 10^{-6}$ (not shown), the fluctuations between the cores were found to be quite small.

C. $d = 40 \mu m$

For the case of $d = 40 \mu m$, the results for a perfectly symmetrical dual-core amplifier were found to be similar to those of the $d = 30 \mu m$ case, with an even more pronounced difference between even and odd supermodes, and a large instability threshold for the latter. With this core separation, the effective-index difference between even and odd supermodes is as low as $2.4 \cdot 10^{-6}$, which means that even $\Delta n = 10^{-6}$ leads to significant asymmetry in the eigenmodes, and thus a sizeable power fluctuation between the cores during propagation. On the other hand, for $\Delta n = 10^{-4}$, each eigenmode is localized largely in one core, and fluctuations are minimal.

Fig. 8 depicts $RMS_P$ as a function of signal power for $d = 40 \mu m$. In this case, there are large differences between the various $\Delta n$ values, as well as incoupling conditions. For $\Delta n = 10^{-6}$, there is great disparity between even and odd incoupling conditions. For even incoupling, a threshold power
as low as 155 W is observed, whereas for odd incoupling, the threshold is at 655 W. This difference is in line with the findings for a perfectly symmetric dual-core amplifier seeded in either the even or odd supermode [8,9]. For \( \Delta n = 10^{-6} \), the threshold for odd seeding is similar to that for \( \Delta n = 10^{-6} \), albeit with a somewhat more complicated power dependence of RMSP. However, the even-seed threshold power is now increased to 485 W, so the difference between even and odd seeding is strongly diminished. Finally, for \( \Delta n = 10^{-4} \), power outputs up to 1360 W were obtained without significant fluctuations for both even and odd seed fields, indicating complete decoupling of the cores. This is reflected in the power distribution along the fiber length, which in Fig. 9 is shown for the three different \( \Delta n \) values with a pump power of 1000 W and odd seed fields, which is close to threshold for the two lowest \( \Delta n \) values. In the case of \( \Delta n = 10^{-6} \), the power is initially evenly distributed between the cores, but about halfway through the amplifier, strong localization sets in, and on output, the power is essentially localized in the low-index core. Obviously, such a localization is highly detrimental to the nonlinear performance of the amplifier. For \( \Delta n = 10^{-5} \), an opposite trend is seen, where the light initially localizes in the low-index core but returns to a more even distribution in the last half of the fiber, albeit with some fluctuations. For \( \Delta n = 10^{-4} \), however, an almost even power distribution is seen, with only small fluctuations between the cores.

It should be mentioned that for \( \Delta n = 10^{-4} \), the output power of 1360 W was reached at a pump power of 2400 W, which implies that the selected amplifier is too short, or too weakly doped, to obtain good efficiency at these power levels. So a longer or more strongly doped amplifier would need to be considered in order to investigate whether an instability threshold exists for this case. Since a short length and limited doping level are desired for suppression of nonlinear effects and photodarkening, respectively, we have not done further investigations along this line. We should also point out that the neglect of thermo-optic modifications to the mode profiles may not be fully justified at these power levels; however, including such effects would make the modes contract, thus making the decoupling of the cores even more efficient.

4. DISCUSSION

While the behavior of the asymmetric dual-core amplifiers is in some cases complex, the results obtained allow us to tentatively draw some simple and intuitively reasonable conclusions. These may guide the design of dual- and multi-core amplifiers without resorting to complicated numerical modeling of the kind presented here, which would likely be prohibitive for amplifiers with many cores.

First, our results for \( \Delta n = 10^{-6} \) demonstrate that when \( \Delta n < \Delta n_{\text{eff}}^{(0)} \), the behavior in terms of dynamical threshold is similar to what was found for the perfectly symmetric case [8,9]. For \( d = 22 \mu \text{m} \), the thresholds for even and odd input fields are not so different, whereas for the larger \( d \) values, seeding with an odd input field is clearly advantageous. When \( \Delta n \) exceeds \( \Delta n_{\text{eff}}^{(0)} \), the dynamical thresholds go up, and the even and odd thresholds approach each other for the two larger core separations, whereas for \( d = 22 \mu \text{m} \), the difference between even and odd seed fields persists. However, in this case, we reach only a moderate value of \( \sim 1.6 \) for \( \frac{\Delta n}{\Delta n_{\text{eff}}^{(0)}} \). Even for \( \frac{\Delta n}{\Delta n_{\text{eff}}^{(0)}} \sim 7 \) (\( d = 30 \mu \text{m}, \Delta n = 10^{-4} \)), dynamical thresholds around 700 W were still found. Only in the case of \( \Delta n_{\text{eff}}^{(0)} \sim 40 \) (\( d = 40 \mu \text{m}, \Delta n = 10^{-4} \)) was complete decoupling observed, with no significant dynamical effects occurring within the pump power range that could reasonably be applied to this amplifier. Thus, it appears that full dynamical stability requires at least \( \frac{\Delta n}{\Delta n_{\text{eff}}^{(0)}} > 10 \).

Similar conclusions are reached when considering the distribution of power between the cores during propagation. The impacts of thermo-optic nonlinearities vary, but they tend to promote localization in one core over the whole or a part of the propagation length. As is the case without nonlinearities and gain, a nearly equal power distribution between the cores is found for either \( \frac{\Delta n}{\Delta n_{\text{eff}}^{(0)}} < 0.1 \) or \( \frac{\Delta n}{\Delta n_{\text{eff}}^{(0)}} > 10 \).

As was shown earlier [9], the thresholds for strongly coupled cores in the symmetric limit are lower than corresponding thresholds in a few-moded single-core fiber with the same core area and NA. So the most attractive limit for power scaling appears to be that of well-separated cores with \( \frac{\Delta n}{\Delta n_{\text{eff}}^{(0)}} > 10 \),
which yields both dynamical stability and an even power distribution between the cores. Although we have not tested it numerically, we expect this criterion to be more robust than a criterion in terms of core separations, because $\Delta n_{\text{eff}}(0)$ depends quite strongly on the $V$ parameter of an individual core.

5. CONCLUSION

We have numerically investigated dynamical mode couplings from thermo-optic nonlinear effects in asymmetric dual-core high-power fiber amplifiers. A coupled-mode theory suitable for this problem has been derived, and dual-core fibers with various core separations and refractive-index asymmetries between $10^{-6}$ and $10^{-4}$ have been investigated. We find that the refractive-index difference between the cores must exceed the effective-index difference between supermodes in the perfectly symmetric structure by at least an order of magnitude in order for decoupling of the cores to take place. Intermediate cases where these two quantities are of a comparable magnitude suffer from significant power fluctuations between the cores, and thus increased effects of Kerr, Raman, and/or Brillouin nonlinearities.

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REFERENCES