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Flexible Control of Wastewater Aeration for Cost-Efficient, Sustainable Treatment

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Abstract: With increasing fluctuations in electricity production due to prioritization of renewable energy sources, new applications that can adjust quickly to changes in demand/supply will be needed. Wastewater treatment use a significant amount of electricity to reduce nutrients in wastewater before discharge. The treatment process demands electricity in some selected periods which can be controlled, and hence the time of consumption is changeable. Here we suggest a novel predictive control strategy which enhances the flexibility in electricity demand by accounting for electricity price and probability of up or down regulation. The strategy is demonstrated in simulation experiments, where the concept is illustrated and the potential savings are estimated. Furthermore flexibility is investigated as a function of regulating prices, and it is shown that when difference between electricity price and regulating price increases, so does flexibility of the system.

Keywords: Wastewater Treatment, Stochastic Model Predictive Control, Smart Energy Application, Balancing Market, Day-ahead market, Optimization, Price-based control

INTRODUCTION

Advancements in wind turbines and solar panels coupled with societal requests, declines in cost and favourable regulation have all contributed to a rapid increase in renewables in electricity grids (Ueckerdt et al., 2015). While this is an obligatory development to reach the desired fossil-free energy system (Jacobson and Delucchi, 2011), fluctuating renewable energy sources impose a challenge in securing sufficient supply to cover demand at all times. According to the EU, adaptation to the increasing amount of renewables requires massive development in smart energy systems. Hence, building heat, vehicle charging and industrial cooling (e.g. (Zemtsov et al., 2018)) have received attention in their ability to smartly prioritize electricity consumption in selected periods without significant loss of utility for the users. However, as more renewables penetrate the electricity grids, more smart applications will be needed to maintain a stable system (Morales et al., 2014).

Wastewater treatment plants (WWTP) use approximately 1% of a country's total electricity consumption (Shi, 2011) meaning that e.g. Germany and USA spend 4.4 and 30.2 TWh/yr on wastewater treatment respectively (Haberkern et al., 2008; Pabi et al., 2013). From this follows (i) that electricity is a major economic issue for plant operation corresponding to 25-50% of operational costs (e.g. (Huang et al., 2013)) and (ii) that the greenhouse gas emissions (GHG) related to electricity consumption of wastewater treatment are noteworthy (Mizuta and Shimada, 2010).

The most electricity consuming process on a WWTP is aeration which accounts for 40-75% of total electricity demand of a WWTP (Rosso et al., 2008). Aeration is typically carried out in large, engineered tanks where specialized bacteria need aerobic conditions (oxygen present) to convert ammonium from e.g. urine to nitrate. Then other bacteria convert nitrate to nitrogen gas under anoxic conditions (oxygen not present) and hence nitrogen is removed from the water. This implies, that the ideal process requires both aerobic and anoxic periods or areas in the tank. Hence advanced control of wastewater treatment aims at turning aeration on and off in feedback loops to secure good treatment (Zhao et al., 2004).

In this paper we suggest a novel optimization strategy which controls aeration with respect to both the nonlinear biochemical processes and the electricity market. We use Nonlinear Model Predictive Control (NMPC) methods to solve the control problem. In other words we satisfy the wastewater treatment requirements regarding treatment and equipment constraints while we control electricity demand in a flexible way that allows for trading electricity. Last, we show through an example study that the control leaves satisfactory effluent concentrations of the investigated nutrients, and that flexibility in power usage can be envoked. Finally we show how different regulating prices influence the flexibility and costs.

THEORY AND METHODS

We briefly describe how wastewater treatment can be modelled using Stochastic Differential Equations (SDE).

Furthermore we resume the Nordpool electricity market as an example case. Then, with respect to the market design, the optimal control problem is defined, and finally the numerical implementation is briefly mentioned.

Wasterwater Treatment: Applied SDE Modelling

SDEs are used in a wide range of applications. A general form of an SDE is

$$x(t) = x_0 + \int_0^t f(x(\tau), u(\tau))\,\mathrm{d}\tau + \int_0^t g(x(\tau), u(\tau))\,\mathrm{d}\omega(\tau),$$

or in short

$$\mathrm{d}x(t) = f(x(t), u(t))\,\mathrm{d}t + g(x(t), u(t))\,\mathrm{d}\omega(t), \tag{1}$$

where $x\colon \mathbb{R}\to\mathbb{R}^{n_x}$ denotes the states, x_0 is the initial distribution of the states, $u\colon \mathbb{R}\to\mathbb{R}^{n_u}$ is the input variables and $\omega\colon \mathbb{R}\to\mathbb{R}^{n_\omega}$ denotes a standard (possibly multivariate) Brownian motion. Brownian motion is defined by its independent increments which satisfy that for each $s, t \in \mathbb{R}$, $\omega(t) - \omega(s)$ is normally distributed with zero mean and covariance $I(t-s)$. $f : \mathbb{R}^{n_x}\times\mathbb{R}^{n_u}\to\mathbb{R}^{n_x}$ is often referred to as the drift function while $g : \mathbb{R}^{n_x}\times\mathbb{R}^{n_u}\to\mathbb{R}^{n_x}\times\mathbb{R}^{n_\omega}$ is called the diffusion function.

SDEs provide a powerful stochastic continuous-time modelling framework which can be used for both parameter and state estimation (Kristensen et al., 2004). This framework has been applied to wastewater treatment in Halvgaard et al. (2017); Stentoft et al. (2018) where the activated sludge model in Henze et al. (2000) is reduced to a lower-order SDE model. Here, we use the model

$$\begin{aligned}
\mathrm{d}x_1(t) &= a_1(a_2 - x_1(t))\,\mathrm{d}t \\
&\quad - u(t)a_3\frac{x_1(t)}{a_4 + x_1(t)}\,\mathrm{d}t + \sigma_1\,\mathrm{d}\omega_1(t), \\
\mathrm{d}x_2(t) &= a_1(a_5 - x_1(t))\,\mathrm{d}t + a_3u(t)\frac{x_1(t)}{a_4 + x_1(t)}\,\mathrm{d}t \\
&\quad - (1 - u(t))a_6\frac{x_2(t)}{a_7 + x_2(t)}\,\mathrm{d}t + \sigma_2\,\mathrm{d}\omega_2(t),
\end{aligned} \tag{2}$$

Table 1. Example parameters of (2).

Shorthand	Description	Value
$x_1(0)$	Initial ammoinum conc.	1.12
$x_2(0)$	Initial nitrate conc.	0.87
a_1	Incoming wastewater rate	0.00067
a_2	Mean incoming ammonium	36.9
a_3	Nitrification rate	0.073
a_4	Monod kinetic constant	0.1
a_5	Mean incoming nitrate	2.00
a_6	Denitrification rate	0.300
a_7	Monod Kinetic constant	7.84
σ_1	Ammonium noise parameter	0.0085
σ_2	Nitrate noise parameter	0.026

where the parameters are estimated from online measurements from a WWTP using the estimation method given in Kristensen et al. (2004) and applied to wastewater treatment in Stentoft et al. (2018). The parameter estimates and a short description are presented in Table 1.

The input function, u, in (2) is a binary-valued function, which is 1 if the aeration system is activated and 0 otherwise. This means that the system may be characterized as a switched dynamical system where the optimal control is related to control of switching times between the two systems (Egerstedt et al., 2003).

The initial values, $x(0) = (x_1(0), x_2(0))^\top$, will in an online implementation be estimated when new measurements become available. However, in the simulations carried out in the later case-study, these values will constitute the initial values in the open-loop control problem.

The Nordic Electricity Market

In Northern Europe, electricity is traded in a common market called Nord Pool which consists of 15 interconnected price areas. The market which trades with the largest volume is called the day-ahead market. Here, electricity is purchased and sold for the upcoming day. When the day-ahead market closes, the intra-day market opens. In this market, electricity can be traded until 45 minutes prior to the operating hour.

One of the primary challenges when operating transmission systems is to guarantee grid stability. The Nordic Transmission System Operators (TSOs) have many methods for dealing with this challenge - one of them being a common balancing market (EnergiNet, 2016). In the balancing market, market participants have the option to make a bid that defines how much a participant is willing to change their production- or consumption schedule in a given operating hour. The balancing market also closes 45 minutes prior to the operating hour. Hence, when approaching the operating hour, the TSO has the possibility to activate balancing bids ahead of time and avoid possible imbalances. Three scenarios can take place in the balancing market:

- (↑) If the imbalance is negative, there is a deficit of electricity in the price area and hence an increase in the production or a decrease in the consumption is needed. This is called up regulation.
- (↓) If the imbalance is positive, there is a surplus of electricity in the price and hence a decrease in the production or an in the consumption is needed. This is called down regulation.
- (−) If the imbalance is too small or the duration is too short, the imbalance is not offered in the balancing market.

In a situation with up regulation, electricity is sold while in the situation with down regulation, electricity is purchased. The structure of the balancing market requires that the up regulation price is greater than the day-ahead price while the down regulation price is lower than the day-ahead price. Let $p(t)$ denote the day-ahead price, $p^{(\uparrow)}(t)$ the up regulation price and $p^{(\downarrow)}(t)$ the down regulation price. We then have that

$$p^{(\downarrow)}(t) \le p(t) \le p^{(\uparrow)}(t). \tag{3}$$

In Fig. 1 a Nord Pool price example of these prices is shown. In the price area DK1, a large share of the total production comes from wind turbines - this is a source of energy which is very difficult to predict and hence one of the primary reasons to imbalances - in Parbo (2014) it is suggested that approximately 65% of the total imbalances are due to forecast errors of the wind power production.

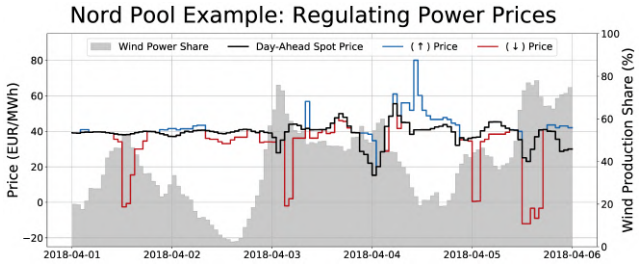

Fig. 1. Nord Pool price example for DK1 for the first five days of April 2018.

The Optimal Control Problem

In this section we define an optimization problem, which can compute the optimal input signal, u. We will assume that u can be parameterized by

$$\tau = \{(\tau_{\texttt{off},k}, \tau_{\texttt{on},k})\}_{k=0}^{N}, \tag{4}$$

such that

$$u(t;\tau) = \begin{cases} 1, \ t \in [\tau_{\texttt{on},k-1}, \tau_{\texttt{off},k}[, & k = 1, ..., N \\ 0, \ t \in [\tau_{\texttt{off},k}, \tau_{\texttt{on},k+1}[, & k = 0, ..., N-1 \end{cases}, \tag{5}$$

where we impose the cycle structure that we start with the aeration being deactivated (i.e. that $\tau_{\texttt{off},0} = 0$). The cycle parameters are ordered such that

$$\tau_{\texttt{on},k} \leq \tau_{\texttt{off},k}, \quad k = 0, ..., N, \tag{6a}$$
$$\tau_{\texttt{on},k+1} \geq \tau_{\texttt{off},k}, \quad k = 0, ..., N-1. \tag{6b}$$

The main objective of a WWTP is to remove nutrients from incoming wastewater before discharging it back to environment at a sustainable level at minimum operational cost. A sustainable level is arguably depending on the vulnerability of the environment where the wastewater is discharged, and hence it is typical for a WWTP to implement hard constraints on ammonium- and total-N concentrations (here x_1 and $x_1 + x_2$) in the effluent. In this study hard constraints on concentrations are not imposed, but rather, we define a cost on the discharge of total-nitrogen to the environment which means that large concentrations are preferably avoided. This is similar to the current Danish legislation where discharge of total-N is taxed by 30 DKK/kg-N. In addition to the cost related to total-N, the cost of electricity related to the aeration process is considered in the minimization of total cost where we assume that the WWTP has the possibility to bid (and be activated) in the balancing market.

To contain all of this into a univariate function, we propose a scenario-based structure, consisting of:

- an up regulation scenario,
- a down regulation scenario,
- a neutral scenario, where no regulation is activated,

where each of the scenarios contain a set of cycle parameters: $\tau^{(\uparrow)}$, $\tau^{(\downarrow)}$ and τ, respictively. In Fig. 2 we have made a schematic overview of these cycle parameters.

Invariant of which scenario is active, the WWTP has to trade electricity such that a nominal operation strategy can be deployed. We define this nominal strategy as the decision relating to the neutral scenario. Hence, the electricity cost for each of the scenarios may be defined as

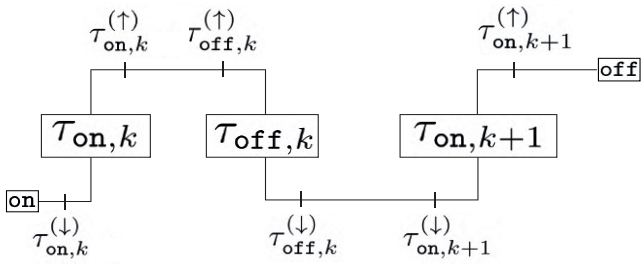

Fig. 2. Schematic overview of the cycle parameters, τ, $\tau^{(\uparrow)}$ and $\tau^{(\downarrow)}$.

$$c_p^{(0)}(\tau) = \sum_{k=1}^{N} \int_{\tau_{\texttt{on},k-1}}^{\tau_{\texttt{off},k}} p(t)\,\mathrm{d}t, \tag{7a}$$

$$c_p^{(\uparrow)}\left(\tau, \tau^{(\uparrow)}\right) = \sum_{k=0}^{N} \left[\int_{\tau_{\texttt{on},k}}^{\tau^{(\uparrow)}_{\texttt{on},k}} p^{(\uparrow)}(t)\,\mathrm{d}t + \int_{\tau^{(\uparrow)}_{\texttt{off},k}}^{\tau_{\texttt{off},k}} p^{(\uparrow)}(t)\,\mathrm{d}t \right], \tag{7b}$$

$$c_p^{(\downarrow)}\left(\tau, \tau^{(\downarrow)}\right) = \sum_{k=0}^{N} \left[\int_{\tau^{(\downarrow)}_{\texttt{on},k}}^{\tau_{\texttt{on},k}} p^{(\downarrow)}(t)\,\mathrm{d}t + \int_{\tau_{\texttt{off},k}}^{\tau^{(\downarrow)}_{\texttt{off},k}} p^{(\downarrow)}(t)\,\mathrm{d}t \right], \tag{7c}$$

where $c_p^{(0)}(\tau)$ denotes the electricity cost for the nominal strategy, $c_p^{(\uparrow)}\left(\tau, \tau^{(\uparrow)}\right)$ denotes the electricity cost in the up regulation scenario and $c_p^{(\downarrow)}\left(\tau, \tau^{(\downarrow)}\right)$ denotes the electricity cost in the down regulation scenario. Hence, the expected cost, c_p, when participating in the balancing market is

$$\begin{aligned} c_p\left(\tau, \tau^{(\uparrow)}, \tau^{(\downarrow)}\right) = c_p^{(0)}(\tau) - \theta^{(\uparrow)} c_p^{(\uparrow)}\left(\tau, \tau^{(\uparrow)}\right) \\ + \theta^{(\downarrow)} c_p^{(\downarrow)}\left(\tau, \tau^{(\downarrow)}\right), \end{aligned} \tag{8}$$

where $\theta^{(\uparrow)}$ and $\theta^{(\downarrow)}$ denote the probability of the up- and down regulation scenarios, respectively. Note that we have, with probability one, to pay the cost $c_p^{(0)}(\tau)$ for the nominal strategy, since this defines the consumption schedule from which we can participate in the balancing market. Similarly, we define the taxation cost (cost of effluent load) for the scenarios as

$$c_t^{(0)}(\tau) = \int_{\tau_{\texttt{off},1}}^{\tau_{\texttt{on},N}} \left(x_1^{(0)}(t) + x_2^{(0)}(t) \right) \mathrm{d}t \tag{9a}$$

$$c_t^{(\uparrow)}\left(\tau, \tau^{(\uparrow)}\right) = \int_{\tau^{(\uparrow)}_{\texttt{off},1}}^{\tau^{(\uparrow)}_{\texttt{on},N}} \left(x_1^{(\uparrow)}(t) + x_2^{(\uparrow)}(t) \right) \mathrm{d}t \tag{9b}$$

$$c_t^{(\downarrow)}\left(\tau, \tau^{(\downarrow)}\right) = \int_{\tau^{(\downarrow)}_{\texttt{off},1}}^{\tau^{(\downarrow)}_{\texttt{on},N}} \left(x_1^{(\downarrow)}(t) + x_2^{(\downarrow)}(t) \right) \mathrm{d}t, \tag{9c}$$

where $c_t^{(0)}(\tau)$ denotes the taxation cost for the nominal strategy, $c_p^{(\uparrow)}\left(\tau, \tau^{(\uparrow)}\right)$ denotes the taxation cost in the up regulation scenario and $c_p^{(\downarrow)}\left(\tau, \tau^{(\downarrow)}\right)$ denotes the taxation cost in the down regulation scenario. Note that this formulation of the taxation amount assumes that the flow of the incoming/outgoing water is constant over time. The expected taxation cost is then given by

$$\begin{aligned} c_t\left(\tau, \tau^{(\uparrow)}, \tau^{(\downarrow)}\right) = \left(1 - \theta^{(\uparrow)} - \theta^{(\downarrow)}\right) c_t^{(0)}(\tau) \\ + \theta^{(\uparrow)} c_t^{(\uparrow)}\left(\tau, \tau^{(\uparrow)}\right) \\ + \theta^{(\downarrow)} c_t^{(\downarrow)}\left(\tau, \tau^{(\downarrow)}\right). \end{aligned} \tag{10}$$

Note that the taxation cost of the nominal strategy is multiplied by $1 - \theta^{(\uparrow)} - \theta^{(\downarrow)}$, since we might be activated for

up or down regulation. Thus, the total cost of operating the WWTP can be modelled as

$$c\left(\tau,\tau^{(\uparrow)},\tau^{(\downarrow)}\right)=c_t\left(\tau,\tau^{(\uparrow)},\tau^{(\downarrow)}\right)+c_p\left(\tau,\tau^{(\uparrow)},\tau^{(\downarrow)}\right). \tag{11}$$

However, from an optimization perspective, c, is not a suitable choice for objective function; c will prefer cycle parameters which are as small as possible to minimize the optimization horizon. To eliminate this preference, we consider the time-averaged analogues of (7a)-(7c) and (9a)-(9c). We define these time-averaged versions according to

$$\hat{c}_p^{(0)}(\tau)=\frac{c_t^{(0)}(\tau)}{\tau_{\mathtt{on},N}}, \tag{12a}$$

$$\hat{c}_p^{(\uparrow)}\left(\tau,\tau^{(\uparrow)}\right)=\frac{c_p^{(\uparrow)}\left(\tau,\tau^{(\uparrow)}\right)}{\tau_{\mathtt{on},N}^{(\uparrow)}}, \tag{12b}$$

$$\hat{c}_p^{(\downarrow)}\left(\tau,\tau^{(\downarrow)}\right)=\frac{c_p^{(\downarrow)}\left(\tau,\tau^{(\downarrow)}\right)}{\tau_{\mathtt{on},N}^{(\downarrow)}}, \tag{12c}$$

and

$$\hat{c}_t^{(0)}(\tau)=\frac{c_t^{(0)}(\tau)}{\tau_{\mathtt{on},N}}, \tag{13a}$$

$$\hat{c}_t^{(\uparrow)}\left(\tau,\tau^{(\uparrow)}\right)=\frac{c_t^{(\uparrow)}\left(\tau,\tau^{(\uparrow)}\right)}{\tau_{\mathtt{on},N}^{(\uparrow)}}, \tag{13b}$$

$$\hat{c}_t^{(\downarrow)}\left(\tau,\tau^{(\downarrow)}\right)=\frac{c_t^{(\downarrow)}\left(\tau,\tau^{(\downarrow)}\right)}{\tau_{\mathtt{on},N}^{(\downarrow)}}. \tag{13c}$$

Thus, we can define the optimal control problem as

$$\min_{\tau,\tau^{(\uparrow)},\tau^{(\downarrow)}}\hat{c}\left(\tau,\tau^{(\uparrow)},\tau^{(\downarrow)}\right), \tag{14a}$$

subject to

$$\left(\tau,\tau^{(\uparrow)},\tau^{(\downarrow)}\right)\in\mathcal{T}, \tag{14b}$$

$$\dot{x}^{(0)}(t)=f\left(x^{(0)}(t),u\left(t;\tau\right)\right),\qquad t\in[0,\tau_{\mathtt{on},N}], \tag{14c}$$

$$\dot{x}^{(\uparrow)}(t)=f\left(x^{(0)}(t),u\left(t;\tau^{(\uparrow)}\right)\right),\quad t\in\left[0,\tau_{\mathtt{on},N}^{(\uparrow)}\right], \tag{14d}$$

$$\dot{x}^{(\downarrow)}(t)=f\left(x^{(0)}(t),u\left(t;\tau^{(\downarrow)}\right)\right),\quad t\in\left[0,\tau_{\mathtt{on},N}^{(\downarrow)}\right], \tag{14e}$$

where $\mathcal{T}$ defines the set of permissible cycle structures, which might be constraints such as minimum- and maximum cycle length and minimum- and maximum levels of flexibility bid to the balancing market via $\tau^{(\uparrow)}$ and $\tau^{(\downarrow)}$. f is the system model which is given from (2). In an actual application of the framework presented above we would need to apply the principle of Nonlinear Model Predictive Control as described in Brok et al. (2018) to obtain a closed-loop control system, where we would solve an optimization problem of the form (14a)-(14e) each time we receive new measurements from the WWTP.

Numerical Implementation

The optimization problem (14a)-(14e) has been solved in `julia` using `ipopt` (Bezanson et al., 2017; Wächter and Biegler, 2006). `ipopt` is a gradient-based optimization method - hence, the derivatives of the optimization problem (14a)-(14e) have to be provided. The dynamical systems (14c)-(14e) are switched dynamical systems which implies that the gradient of the objective function can be computed using the adjoint equations of the optimal control problem (14a)-(14e) (Egerstedt et al., 2003). The numerical method we have implemented is in the literature often referred to as a single shooting method (Bock and Plitt, 1984).

EXAMPLES AND DISCUSSION

The control strategy is tested with respect to the model presented in (2) and parameters in Table 1. We assume that $\mathcal{T}$ in (14b) is defined as the set containing the cycle parameters satisfying the constraints defined in Table 2. Note that these constraints are also implemented across scenarios. We also assume that the following data is given:

(1) the day-ahead electricity price, $p(t)$,
(2) up regulation price and probability, $p^{(\uparrow)}$ and $\theta^{(\uparrow)}$,
(3) down regulation price and probability, $p^{(\downarrow)}$ and $\theta^{(\downarrow)}$.

Table 2. Cycle parameter constraints.

Constraint Description	Value
Minimum aeration length	6.0
Minimum no-aeration length	20.0
Maximum aeration length	60.0
Maximum no-aeration length	120.0

We will in the following simulation study assume that p is a constant function of time with value 10 price units. Hence we optimize costs with known prices and probabilities of up- or down regulation. We also assume that it is only the first cycle shift from `on` to `off` and the second cycle shift from `off` to `on` that can be bid to the balancing market. This means that $\mathcal{T}$ also imposes the constraints $\tau_{\mathtt{on},k}=\tau_{\mathtt{on},k}^{(\uparrow)}=\tau_{\mathtt{on},k}^{(\downarrow)}$ for $k\in\{0,1,3,...,N\}$ and $\tau_{\mathtt{off},k}=\tau_{\mathtt{off},k}^{(\uparrow)}=\tau_{\mathtt{off},k}^{(\downarrow)}$ for $k\in\{0,2,3,...,N\}$.

Nominal Control Strategy

In this, first, simulation, we assume that $\theta^{(\uparrow)}=\theta^{(\downarrow)}=0$. The resulting simulation is shown in Fig. 3. We observe that when the aeration is `on`, the ammonium concentration, x_1, decreases while the nitrate concentration, x_2, increases and vice versa when the aeration is `off`. From 3 we also observe that the accumulated cost is increasing steadily over time with approximately 70% of the total cost associated with the price of electricity. Finally, we note that after the first cycle, the aeration cycles are repeated with almost the same period. This is expected since the price of electricity and the flow-rate are constant.

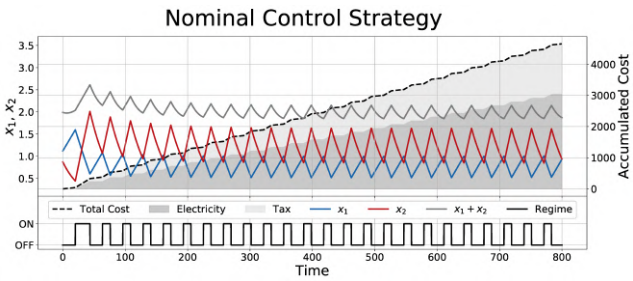

Fig. 3. Baseline simulation showing the optimal control given constant electricity price and no regulation. Note that regime refers to the aeration state (on/off).

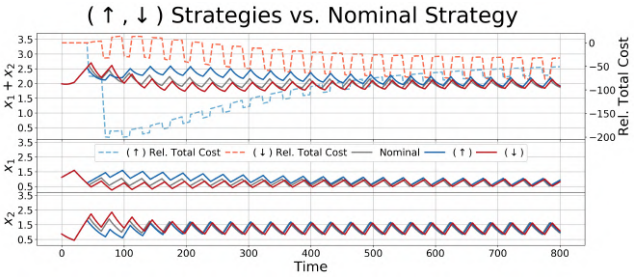

Fig. 4. Scenarios where regulation is expected and activated vs the nominal strategy as shown before.

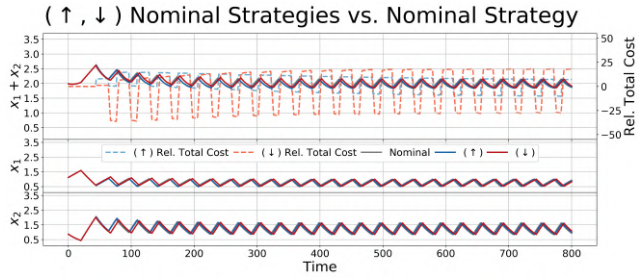

Fig. 5. Scenarios where regulation is expected but not activated vs the nominal strategy as shown before.

Up and Down Regulation Scenarios

In the following simulations, we have two different scenarios:

(↑) A scenario where the probability of being activated for up regulation is $\theta^{(\uparrow)} = 1/10$ at an up regulation price of $p^{(\uparrow)}(t) = 20$.

(↓) A scenario where the probability of being activated for down regulation is $\theta^{(\downarrow)} = 1/10$ at an down regulation price of $p^{(\downarrow)}(t) = 0$.

Note that for the (↓)-scenario, there is a small probability that the WWTP can consume electricity for free.

From Fig. 4 we see that for both scenarios it is a long term beneficial strategy to participate in the balancing market. We observe that the total cost, relative to the nominal strategy shown in Fig. 3, is ≈ 50 price units less than the nominal cost for the nominal strategy without probability of activation. From Fig. 4 we can also note that after the initial activation, the strategies re-prioritize reducing the total concentration of the nutrients and that the concentrations converge towards the result of the nominal strategy. In Fig. 5 we see that the nominal strategies with balancing market participation have a total cost which is very close to the cost of the nominal strategy with no participation. This is a desired property, since we don't want to impose a risk of huge total cost of the nominal strategies which primarily will define the state of operation.

Flexibility Diagram

To illustrate how flexibility can be invoked in the control, the effect of different up- and down regulation prices is investigated. In Fig. 6 we have shown a flexibility diagram. This diagram shows how much flexibility the method builds into the system for different up- and down regulation prices and how these additional strategies affect the nominal cost, expected cost and regulating cost. The total flexibility is defined as the amount the system is willing to change the nominal schedule given the up or down regulation price. Hence, it defines the available flexibility in the system. The flexibility diagram is generated based on:

(↑) In the up regulation region, the probability of being activated for up regulation is $\theta^{(\uparrow)} = 1/10$.

(↓) In the down regulation region, the probability of being activated for down regulation is $\theta^{(\downarrow)} = 1/10$.

We observe in fig. 6 that larger savings are expected as the regulation prices becomes more favorable (relative to the fixed day-ahead price) and that the nominal cost is flat with increased sensitivity towards down regulation.

The flexibility diagram depends on the parameters used in the prediction model in (2) and the probabilities and prices used. Thus in an online application, parameters are expected to be frequently updated and hence the flexibility diagram will also change. Also, the framework presented in this paper assumes that the scenario probabilities are invariant over time, this is however by no means a necessary assumption. Hence, we imagine that these could be generated from forecasts of the balancing market (even though this is a very difficult market to predict). For closed-loop strategies, where we rely on feedback to get efficient operation, we imagine that flexibility diagrams might be key tool in aggregation of multiple WWTPs where the total consumption and flexibility is traded in the day-ahead, intra-day and balancing markets. In applications of the optimization method, we would also need to consider how to realistically mimic the market structure of the Nordic electricity markets.

Lastly, we remark that the presented optimization method only considers biological nitrogen removal. An extension to include biological phosphorous removal would be useful for many WWTPs. Furthermore, legislation in many countries requires that ammonium and total-nitrogen are kept below a certain limit (i.e. hard constraints on x_1 and $x_1 + x_2$). For improved applicability this should also be included in future implementations.

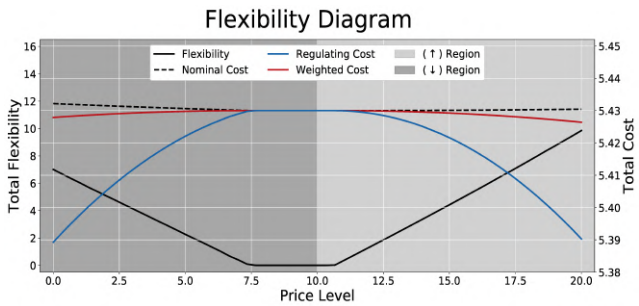

Fig. 6. Flexibility diagram showing the effect of changing the up- and down regulation price. Price level 10.0 corresponds to no regulation (or regulation price equal zero).

CONCLUSION

In this paper we presented a novel stochastic model predictive control strategy of wastewater aeration for cost-efficient and sustainable treatment. The optimization model trade-off taxation cost of nutrients discharged into the environment with the cost of operating the WWTP.

The output from the optimization model is a nominal strategy and strategies for up- and down regulation, respectively. We show via a simulation study, that by allowing the optimization model to utilize the balancing market, the operational costs can be decreased compared to only considering a nominal strategy with no participation in the balancing market. The simulation study also shows that after the activation period, the regulation-based strategies converges (over time) to the nominal strategy. We furthermore suggest a flexibility diagram which shows how different up and down regulation price levels influence the operational costs and flexibility. Furthermore it is suggested that this can be a tool for aggregation of multiple WWTPs.

Finally we consider this paper as a step towards integrating wastewater treatment in the balancing market(s). Ultimately, our approach can help wastewater treatment operation adjust to rapid changes in electricity supply/prices and thereby make them more resilient to increasing amounts of renewables in the transmission grid.

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