Multiphysical tolerance analysis – Assessment technique of the impact of the model parameter imprecision

Dantan, Jean-Yves; Eifler, Tobias; Homri, Lazhar

Published in:
Procedia CIRP

Link to article, DOI:
10.1016/j.procir.2020.05.192

Publication date:
2020

Document Version
Publisher's PDF, also known as Version of record

Available online at www.sciencedirect.com

ScienceDirect

Procedia CIRP 92 (2020) 206–211

www.elsevier.com/locate/procedia

16th CIRP Conference on Computer Aided Tolerancing (CIRP CAT 2020)

Multiphysical tolerance analysis – Assessment technique of the impact of the model parameter imprecision

Jean-Yves DANTAN[a]*, Tobias EIFLER[b], Lazhar HOMRI[a]

[a] *LCFC, Arts et Métiers, Université de Lorraine, 4 rue A Fresnel, 57070 METZ, FRANCE*
[b] *Technical University of Denmark (DTU), Kgs. Lyngby, 2800, Denmark*

* Corresponding author. Tel.: +33 387 375 441. *E-mail address:* jean-yves.dantan@ensam.eu

Abstract

Tolerance analysis is a well-accepted key element in industry for ensuring product quality as well as for reducing manufacturing costs. At the same time, and particularly in light of the recent advances in simulation technology, tolerancing decisions are also becoming increasingly important during earlier stages of design. One critical task hereby is the simulation of the "real-world" behavior of the product with minimum uncertainty, i.e. the calculation how geometrical deviations impact the mechanical behavior and/or multiple simultaneous physical phenomena in a multiphysical system. Given the short iterations in design, this usually represents a compromise between two contradictory requirements: an acceptable computation time and the accuracy of the results. The presented paper addresses this challenge by presenting a framework to assess the impact of model parameter uncertainty of the multiphysical system behavior on the accuracy of the results. The framework integrates evidence and probability theories to propagate geometrical variability and model imprecision for tolerance analysis. The information regarding geometrical variability is modelled using probability distributions; and the information regarding the model imprecision is more faithfully modelled using families of probability distributions encoded by probability-boxes (upper & lower cumulative distribution functions). Monte Carlo simulation is used for probabilistic analysis while nonlinear optimization is used for interval analysis.

Keywords: Tolerancing, Tolerance analysis, Uncertainty propagation

1. Introduction

As performance requirements are continuously tightened, the cost and the required precision of assemblies increase as well. For industry, this implies a need for a coherent tolerance design approach as a key element for improving product quality and decreasing the manufacturing cost. However, particularly in the quick iteration cycles of early product design, it is still a significant challenge to simulate the "real-world" behavior of the product **with the minimum of uncertainty** [3] [22]

In light of the above, one of the main scientific challenges concerns the development of approaches for propagating the impacts of geometrical deviations on the mechanical behavior or/and multiple simultaneous physical phenomena, hereinafter referred to as multiphysics tolerance analysis. For this purpose, it is necessary to simulate the influences of component deviations on the geometrical behavior and subsequently on the multiphysical behavior of the overall product (multiphysics refers to simulations that involve multiple physical models or multiple simultaneous physical phenomena). These behavior models need to include geometrical deviations of each component (position, orientation, size and form deviations) and relative displacements between components according to the gap [9] Furthermore, these models depend on a set of parameters: parameters of the multiphysics behavior laws and external parameters defined by the external environment. As the latter is often not controllable by the designer, the concrete values of these parameters are usually subject to significant imprecisions or uncertainties with a high influence on the accuracy of results.

10.1016/j.procir.2020.05.192

A significant amount of research has been devoted to the definition and classification of the uncertainty, often separated into two types: aleatory and epistemic [1] [10] [11] [22] [25] [26] Epistemic uncertainty is due to the lack of knowledge or the incompleteness. Some studies on Probability Risk Assessment propose to split the epistemic uncertainty further into the following three categories: parameter, model, and completeness uncertainty [2]

Parameter uncertainty *describes limited knowledge about parameter values due to expert judgments or small samples of recorded data.*

Model uncertainty *describes the inaccuracy of the used model, including (1) Indefiniteness in its comprehensiveness, (2) Indefiniteness in its characterization*

Completeness uncertainties *describes whether all significant phenomena and relationships have been considered in the PRA (Probabilistic Risk Analysis).*

Despite numerous classifications available, a systematic approach to consider epistemic uncertainty in a multiphysics tolerance analysis is still lacking. This contribution addresses this challenge by proposing a corresponding assessment framework. Using the example of parameter uncertainty from the classification above, the aim is twofold: (1) to illustrate the extent to which the accuracy of a tolerance analysis depends on the given epistemic uncertainty; and (2) to suggest a potential calculation procedure for an efficient analysis.

Section 2 deals with the taxonomy of tolerance analysis issues, their mathematical formulations and the specific issue of a multiphysics tolerance analysis. Section 3 presents the proposed framework to assess the impact of the parameter imprecisions based on the probability-boxes (upper & lower cumulative distribution functions). In section 4, an application of the framework is demonstrated through an industrial case study, followed by a conclusion in section 5.

2. Tolerance Analysis vs Multiphysics Tolerance Analysis

In 2012, we provided an overview of tolerance analysis formulations and a classification of corresponding issues based on the type of the underlying behavior model [9] The behavior model is understood as the assembly response function, representing the deviation accumulation based on a parameterization of deviations from a theoretically ideal geometry of each individual component. The deviations and gaps are described by:

- $X=\{x_1, x_2, ..., x_n\}$ are the parameters which represent each deviation of the components making up the mechanism;
- $G=\{g_1, g_2, ..., g_m\}$ are the parameters which represent each gap between components.

The analytic formulation of tolerance analysis takes into account the influence of geometrical deviations on the geometrical behavior of the mechanism and on the geometrical product requirements; all geometrical phenomena are modeled by constraints [10] **Erreur ! Source du renvoi introuvable.**[18] :

- $C_c(X,G)= 0$ Composition relations of displacements in the various topological loops that express the geometrical behavior of the mechanism;
- $C_i(X,G) \leqslant 0$ and $C_{i*}(X,G) = 0$ Interface constraints that limit the geometrical behavior and characterize non-interference between substitute surfaces, which are nominally in contact;
- $C_f(Y,X,G) \leqslant 0$ Functional requirement that limits the geometrical deviations between surfaces, which are in functional relation.

In the case of analytic formulation, many works have been devoted to tolerance analysis techniques: worst-case techniques and probabilistic techniques **Erreur ! Source du renvoi introuvable.**[22]

In the case of a multiphysics system, the geometrical deviations influence some non-geometrical functional requirements. To propagate the geometrical deviations on these requirements, an analytic formulation cannot possibly be used. Therefore, the estimation of the functional characteristics Y is computed by the numerical simulation:

$$Y=f_{numerical\ simulation}(X) \text{ or } Y=f_{numerical\ simulation}(X,G) \quad (1)$$

In this case, there exists a strong need for statistical tolerance analysis to estimate two probabilities with high-precision [10]

- P_A: the probability of the assemblability. The assemblability condition describes the essential condition for the existence of gaps that ensure the assembly of the components in the presence of part deviations:

$$P_A = \text{Proba}(C_i(f_{ns}(X,G)) \leqslant 0) \quad (2)$$

- P_{FR}: the probability of fulfilling the functional requirements. The functional condition describes that the functional requirements must be verified for all gap configurations:

$$P_{FR} = \text{Proba}(C_f(f_{ns}(X,G)) \leqslant 0, \forall G \in \{G \in \boldsymbol{R}^m : \textit{behavior conditions are respected}\}) \quad (3)$$

Currently, many studies deal with tolerance analysis based on numerical simulation:

In 1997, Ceglarek et al [5] proposed a mechanical FEM modeling of compliant parts to predict assembly variation, using the influence coefficients method to evaluate the parts' and whole assembly's sensitivity matrix. Then, they proposed a methodology aimed to the prediction of dimensional variation considering complex hierarchical assembly trees. The results highlights that the propagation of variation during assembly is driven by the stiffness of the parts and sub-assemblies being assembled.

Other technical papers addressing the problem of defining a methodology for tolerance stack-up analysis of compliant assembly were developed: [4] [7] [19] [20] [23]

The first step of current approaches is to define the compliance and sensitivity matrix of each part by means of influence coefficients. The sensitivity matrix is computed once for all by FEM, with the parts located on a set of isostatic locators and then applying a unit displacement to the overconstrained joint at each part. The forces and deformations are stored. The compliance matrix of the whole assembly is

computed similarly applying a unit force on each joint. The displacement resulting on the control points can be defined as a linear combination of the effects computed on parts and assembly [7]

While the approaches are well suited for Monte Carlo simulations, i.e. to calculate the probability function of variations and their contributors, this is usually a very time consuming step. Furthermore, the existing approaches essentially focus on the behavior model of flexible parts with applications in aerospace, automotive and appliance industries, without considering model imprecision or model uncertainty (the model of the physical behavior is supposed perfect). The following section therefore focuses on the impact of this imprecision and its assessment technique.

3. Assessment technique of the impact of the model parameter imprecision

The propagation of parameter uncertainty is an essential part of robust design approaches [6] including a clear distinction between internal, controllable parameters and external, non-controllable parameters. At the same time, this focus has been seldomly considered in tolerancing though. Adopting the Robust Design viewpoint, this contribution therefore considers both, parameter uncertainties and Model uncertainty, as a form of epistemic uncertainty [25] [26] defines the accuracy of a mathematical model to describe an actual physical system of interest. All models are unavoidably simplifications of the reality that leads to a disturbing conclusion: every model is definitely false. In this paper, we consider only the Parameter uncertainties.

Parameter uncertainty comes from the model parameters that are inputs to a multiphysics behavior model but whose exact values are unknown and cannot be controlled in reality. Differentiating uncertain model parameters from all geometrical deviations, the mathematical formulation of the response function is:

$$Y=f_{numerical\ simulation}(X,G,MP) \quad (4)$$

With:

X: all geometrical deviations of each component

G: all gaps (geometrical displacement between surfaces that are nominally in contact)

MP: all model parameters (parameters of the multiphysics behavior laws and external parameters (external parameters are defined by external environment in which the designer does not have control over))

Existing work on robust design predominately use classical probabilistic models and Monte Carlo simulations to assess the impact of parameter uncertainty [6] [8] also in form of usual sensitivity measures [17] These approaches require knowledge of the probabilistic distribution of each input variable, in order to specify the dependency structures between all input variables and to perform numerical analyses for evaluating the output uncertainty. That means that corresponding approaches on the one hand are based on a great amount of information and lengthy computations. On the other hand, they often require additional assumptions as a similar information level of all input factors is needed [16]

An alternative approach is to use interval analysis to calculate the propagation of uncertainty. Instead of a full probabilistic distribution, it is assumed that only the intervals in which each parameter may vary are known, resulting in a best and worst case calculation. Compared to probabilistic analysis, interval analysis can therefore be seen as a conservative analysis.

To cover both aspects in the analysis, the here suggested approach combines the probabilistic analysis and interval analysis. Based on the classification above, i.e. all geometrical parameters (X and G) as well as all further model parameters (MP), it can be stated that the mathematical formulation includes aleatory uncertainty, which is X, as well as parameter imprecision, which affects MP. An algorithm is proposed based on statistical sampling power of Monte Carlo simulation and on optimization to find the worst gap configuration in case of uncertaint MPs. A general flow chart describing the module for uncertainties propagation is shown in Figure 1.

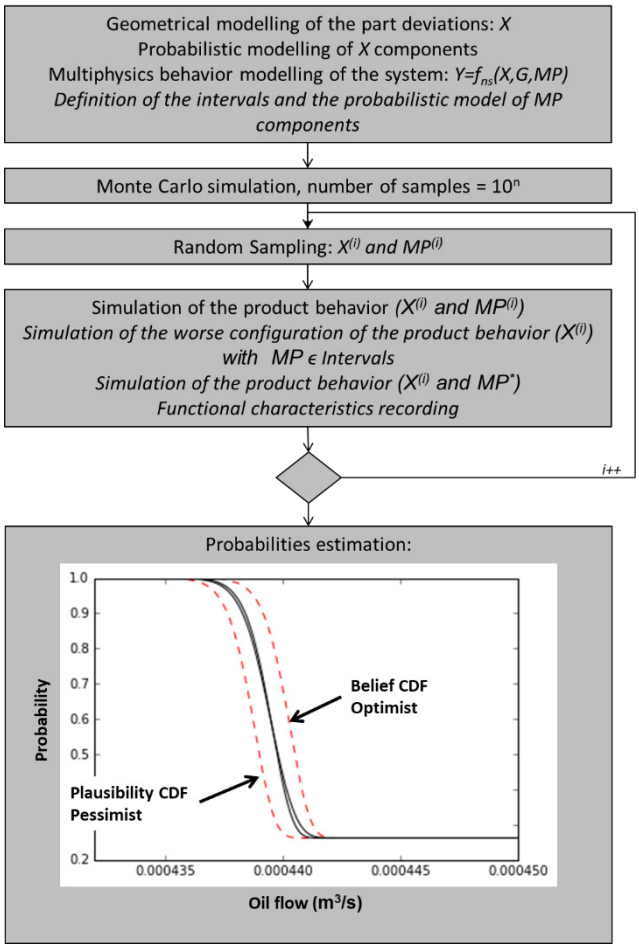

Fig. 1. General scheme of Tolerance analysis and Parameter uncertainty assessment by Monte Carlo simulation.

The suggested approach adopts the idea of belief and plausibility measures from evidence theory [12] [14] [24]. While geometrical variability is modelled using probability distributions resulting in a cumulative distribution function (CDF) for the investigated oil flow, the model imprecision is

modelled more faithfully using families of probability distributions encoded by probability-boxes. These results are represented as cumulative belief and plausibility functions, shown as dotted lines in Fig. 1, and can be interpreted as the lower and upper bounds of a probability measure.

In summary, the proposed framework includes:

- Statistical tolerance analysis without taking into account the parameter imprecision, (CDF without consideration of epistemic uncertainty)
- Statistical tolerance analysis and probabilistic propagation of the MP imprecision (CDF with probabilistic consideration of uncertainty),
- Statistical tolerance analysis coupled with optimization to perform the evaluation of the cumulative belief and plausibility functions (Plausibility CDF & Belief CDF to represent the pessimistic and the optimistic case).

The outcome is the non-conformance rate (in ppm) for the assemblability as well as functional requirement. The framework is based on one Monte Carlo simulation; so that all rates are estimated with the same sample. Therefore, the differences between all results only depend on the impact of the chosen techniques to propagate the model parameter imprecision.

The following section illustrates these approaches on an industrial use case, an external gear pump.

4. Application: Tolerance analysis of a gear pump

An important research trend is to significantly reduce power consumption of machines, by reducing idle power, increasing efficiency, and optimizing duty cycles. In the case of an external oil gear pump, the importance of tolerances allocation on the final performance of the product can be illustrated. Reduction of losses can be achieved through better choices of component quality. Tolerances are acting in two different ways: guidance precision and flow leakage control, both based on the resulting distances between gears, shaft, as well as pump housing. The bearing assembly for example prevents the shaft to move against the casing, a motion that would cause the rotor to contact static parts, hence increasing friction and wear, modifying air gap and generating more vibrations. In common mechanical systems, the gap could be increased to avoid this problem, but for pumps, reducing the space between moving parts is the only way to ensure a limited flow leakage, hence to allow for better performance.

A compromise has to be found to ensure the best global efficiency: limited friction but efficient fluid dynamics (which tends to increase during use as pressure may contribute to accidental contact between housing and gears, usually observed near the input port).

A multiphysics model of the pump can allow the designer to determine pressure distribution in the whole circumference of rotor, deformable behavior of elements, and influence of part tolerances allocation on these functional specifications.

Figure 2 shows the use case: External gear pump. The manufacture of the current oil pump expects an oil flow of 4.35×10^{-4} m3/s. The designers know that the efficiency and oil flow of the pump is related to different backlashes. These backlashes are between the gears and the casing as well as between the gears and shafts. Too small backlashes will result in friction and too much of them will result in internal flow loss and therefore performance reduction. Achieving precise backlashes is the result of manufacturing precision to obtain tight tolerances.

Table 1 summarizes several considered geometrical characteristics, their nominal values, their tolerances and their standard deviations. The used geometrical model of this external gear pump was detailed in **Erreur ! Source du renvoi introuvable.**The estimation of the leakage rate is based on a surrogate model which depends on the clearance between gears, pump housing and the shaft.

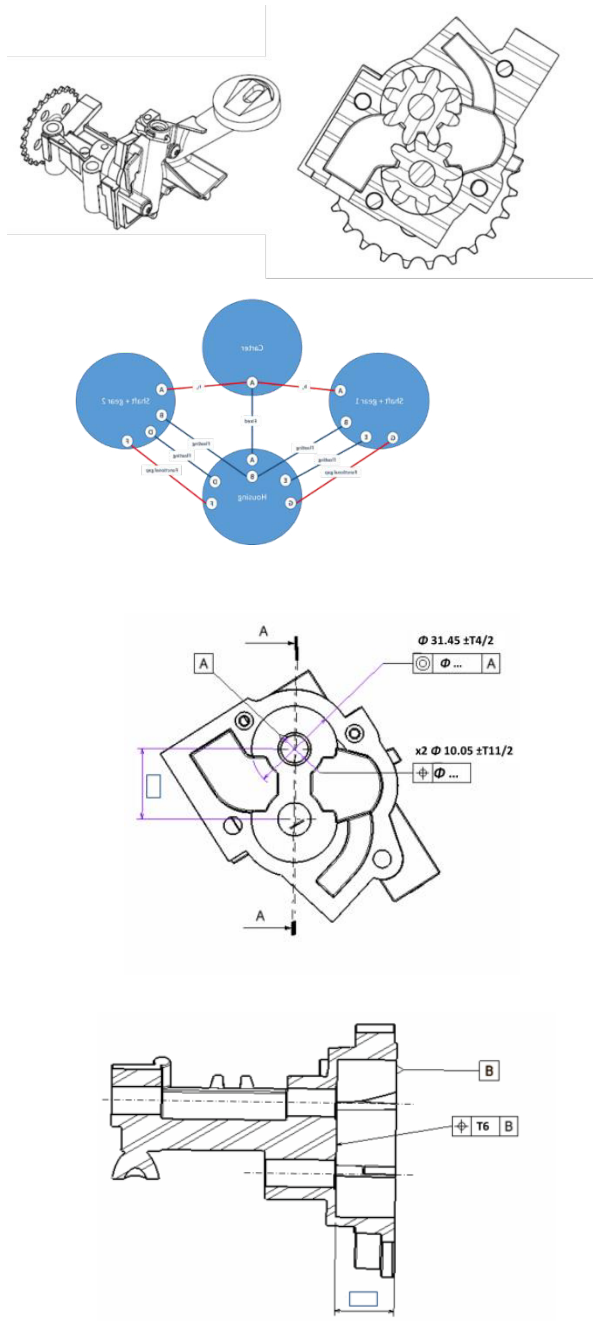

Fig. 2. Use case: Renault external gear pump.

Random Sampling of the geometrical deviations: $\dot{X}$

$$\begin{aligned} &\text{Max } G \\ &\text{s.t. } Cc(\dot{X},G) = 0 \\ &\quad Ci(\dot{X},G) \leqslant 0 \\ &\quad Ci^*(\dot{X},G) = 0 \end{aligned} \tag{5}$$

With:

$Cc(\dot{X},G)$ represents the displacements accumulation

$Ci(\dot{X},G)$ and $Ci^*(\dot{X},G)$ represents the contact constraints

$$\text{Oil Flow} = f_{surrogate\ model}(G,a,b,c,d) \tag{6}$$

With:

G: all gaps (geometrical displacement between surfaces that are nominally in contact)

a, b, c, d: model parameters (table 2)

Table 1. Geometrical characteristics of the use case.

Geometrical characteristics	Nominal value	Tolerance	Standard deviation
Gear			
Head length of the teeth	1,8 mm	0,4	0,05
Primitive length of the teeth	4,9 mm	0,4	0,05
Base length of the teeth	9,8 mm	0,4	0,05
Gear thichness	21,4 mm	0,08	0,01
Gear diameter	56,5 mm	0,08	0,01
Root diameter	19 mm	0,08	0,01
Tooth depth	6 mm	0,4	0,05
Pump housing			
Diameter of the housing pocket for the gear	56,8 mm	0,08	0,01
Depth of the housing pocket for the gear (Localization)	21,5 mm	0,08	0,01
Bearing diameter	10,03 mm	0,015	0,002
Localization of the bearings		0,03	0,004
Shaft			
Shaft diameter	10 mm	0,015	0,002
Shaft length (localization)	22 mm	0,08	0,01
Coaxiality		0,03	0,004

Table 2. Model parameters of the use case.

Model Parameter	Nominal value	Intervals
a	2.75	[2.72, 2.78]
b	235	[234.6, 235.4]
c	500000	[497000,503000]
d	19.8e-3	[19.3e-3, 20.3e-3]

The proposed approach was implemented in Python. The Monte Carlo simulation was performed with the OPENTURNS library (*Python library including algorithms dedicated to the treatment of uncertainties*). The chosen algorithm for the optimization (worst gap configuration identification) is *Truncated Newton* algorithm. Several algorithms, e.g. *Sequential Least Squares Programming*, are tested and provide similar results. The sample size of the Monte Carlo simulation is 10^6. The confidence interval of these probability estimations by Monte Carlo Simulation is ±8 ppm. All geometrical deviations are considered as Gaussian random variables, and all model parameter imprecisions as Uniform random variables or as intervals. The Figure 3 shows the result of the tolerance analysis and the uncertainty propagation: the cumulative belief and plausibility functions (Belief CDF and Plausibility CDF), and the cumulative distribution functions (continuous curves – CDF of classical tolerance analysis and CDF of tolerance analysis with the probabilistic propagation of the model parameter imprecision). The differences between the Belief CDF and Plausibility CDF represent the impact of the model parameter imprecision on the probability of oil flow requirement. The small differences between the two continuous curves represent the impact of the random propagation of the model parameter imprecision.

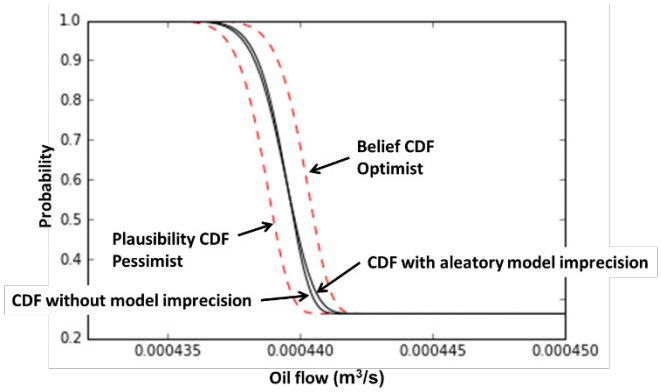

Fig. 3. Cumulative belief and plausibility functions (Belief CDF and Plausibility CDF – CDF: cumulative distribution function).).

While Fig. 3 clearly illustrates the importance of considering the uncertainty of MPs, the corresponding difference around the minimal acceptable oil flow level (4.35×10^{-4} m3/s) appears to be negligible. For this reason, the visualization is detailed by an overview of probabilities of conformance and all non-conformance rates in Table 3. Calculated with epistemic uncertainty, the optimistic result of the analysis is 6 ppm, and the pessimistic result is 410 ppm. Disregarding the uncertainty of MPs, the classical result is 68 ppm, while the result with probabilistic propagation of the model parameter imprecision is 112 ppm. In the automotive industry, the target of the statistical tolerance analysis is around 20 ppm. As only, the optimistic result verifies this expected rate, the analysis shows clearly that a careful consideration of behavior models and model uncertainty should be a decisive step of every analysis. If the designer only considers the other three results, decision-making would most likely be biased towards a reduction of tolerances for each component instead.

Table 3. Non-conformance rates of expected value of the oil flow (4.35×10-4 m3/s)

	Probability of conformance	Non conformance rate in ppm
$Proba_{optimist}$(Q>0,000435 & assembly)	0,999656	344
$Proba_{pessimist}$(Q>0,000435 & assembly)	0,999252	748
$Proba_{without}$(Q>0,000435 & assembly)	0,999594	406
$Proba_{with}$(Q>0,000435 & assembly)	0,99955	450
Proba (assembly)	0,999662	338
$Proba_{optimist}$(Q>0,000435 | assembly)	0,999994	**6**
$Proba_{pessimist}$(Q>0,000435 | assembly)	0,99959	**410**
$Proba_{without}$(Q>0,000435 | assembly)	0,999932	**68**
$Proba_{with}$(Q>0,000435 | assembly)	0,999888	**112**

In this case the computing time of a classical tolerance analysis is 22min (Processor: Intel Core i5 2.5GHz, RAM: 16Go) and the computing time of the four coupled tolerance analysis is 59min. The model parameter imprecision propagation has a significant impact on the computing time.

This approach provides all information to the designer. The designer can modify the tolerance allocation or can reduce the model parameter imprecision by new data collection.

5. Conclusion

As mentioned in the CIRP keynote paper [22] *tolerance analysis and variation simulation involves a large number of uncertainties that can be categorized according to 1) Tolerance models for representing the geometrical deviations on individual parts, 2) System behavior models for representing how variation propagates in a product or an assembly and 3) Tolerance and variation analysis techniques. It is essential to be aware of both modelling assumptions and simplifications, as well as model resolution.* This paper focus on the type 2 and propose a framework to assess the impact of the model parameter uncertainty of the multiphysics system behavior on the accuracy of the tolerance analysis results.

The proposed framework merges Tolerance Analysis techniques and Quantification of Margins and Uncertainty (QMU) techniques to manage heterogeneous uncertainties. In fact, tolerance analysis focuses on the manufacturing imperfections (aleatory uncertainty) propagation and QMU focuses on the model error (epistemic uncertainty) propagation.

This framework provides all information about the impact of the model parameter imprecision on the accuracy of the tolerance analysis. The results obtained for the industrial use case (external gear pump) illustrate that the impact of the model parameter imprecision on the estimated non-conformance rates is not negligible.

Acknowledgements

The authors would like to acknowledge the French higher education ministry and the Arts et Métiers foundation for their financial support under the sabbatical year of Prof. Jean-Yves DANTAN at DTU.

References

[1] Antonsson, E., Otto, K., 1995, Imprecision in engineering design. ASME Journal of Mechanical Design, 117:25-32.

[2] Bedford, T., Cooke, R., 2001, Different types of uncertainty. Probabilistic Risk Analysis: Foundations, Methods, Cambridge University Press, Cambridge, United Kingdom, 17-38.

[3] Boorla, S.M., Bjarklev, K., Eifler, T., Howard, T.J., McMahon, C.A., 2019, Industry 4.0 - A challenge for variation simulation tools for mechanical assemblies, Advances in Computational Design, 4 (1), pp. 43-52.

[4] Camelio, J., S. J. Hu, D. Ceglarek, 2003, Modeling variation propagation of multi-station assembly systems with compliant parts, Journal of Mechanical Design, 125/4:673-681.

[5] Ceglarek, D., J. Shi, 1997, Tolerance analysis for sheet metal assembly using a beam-based model, Proceedings Concurrent Product Design, Environmentally Conscious Manufacturing.

[6] Choi, H.-J., 2005, A robust design method for model, propagated Uncertainty, PhD diss., Georgia Institute of Technology.

[7] Dahlström, S., L. Lindkvist, 2004, Variation simulation of Sheet Metal Assemblies Using the Method of Influence Coefficients with Contact Modeling. ASME 2004 International Mechanical Engineering Congress, Exposition, Anaheim, California.

[8] Daneshkhah, AR., 2004, Uncertainty in probabilistic risk assessment. A review. Working Paper, University of Sheffield, UK.

[9] Dantan, J.Y., Gayton, N., Dumas, N., Etienne, A., Qureshi, A.J., 2012, Mathematical issues in Mechanical Tolerance Analysis, AIP PRIMECA 2012, France.

[10] Dantan JY, Gayton N, Qureshi AJ, et al., 2012. Tolerance analysis approach based on the classification of uncertainty (aleatory/ epistemic). In: Proceedings of the 12th CIRP conference on computer aided tolerancing, Huddersfield, UK.

[11] Dantan, J.-Y., Qureshi, A.J., Antoine, J.-F., Eisenbart, B., Blessing, L.T.M., 2013, Management of product characteristics uncertainty based on formal logic, characteristics properties model, Annals of CIRP, 62/1/147–150.

[12] Dempster, A.P. 1967. "Upper and Lower Probabilities Induced by a Multivalued Mapping." The Annals of Mathematical Statistics 38 (2): 325–39.

[13] Der Kiureghian, A., Ditlevsen, O., 2009, Aleatory or epistemic? Does it matter?, Struct. Saf., 31/ 2:105–112.

[14] Du X., 2006, Uncertainty Analysis with Probability and Evidence Theories" Proceedings of ASME 2006 International Design Technical Conferences & Computers and Information in Engineering Conference, Sep 10-13, Philadelphia, Pennsylvania.

[15] Dumas, A., Dantan, J.-Y., Gayton, N., 2015, Impact of a behavior model linearization strategy on the tolerance analysis of over-constrained mechanisms, CAD Computer Aided Design, 62, pp. 152-163.

[16] Eifler, T., 2014, Modellgestützte Methodik zur systematischen Analyse von Unsicherheit im Lebenslauf technischer Systems, PhD Thesis, Technische Universität Darmstadt.

[17] Göhler, S.M., Eifler, T., Howard, T.J., 2016, Robustness Metrics: Consolidating the Multiple Approaches to Quantify Robustness, J. Mech. Des, 138(11)

[18] Goka, E., Beaurepaire, P., Homri, L., Dantan, J.-Y., 2019,Probabilistic-based approach using Kernel Density Estimation for gap modeling in a statistical tolerance analysis, Mechanism and Machine Theory, 139, pp. 294-309.

[19] Jareteg, C., K. Wärmefjord, R. Söderberg, L. Lindkvist, J. S. Carlson, C. Cromvik, F. Edelvik, 2014, Variation simulation for composite parts , assemblies including variation in fiber orientation , thickness, CIRP CATS 2014.

[20] Lindau, B., K. Wärmefjord, L. Lindkvist, R. Söderberg, 2014, Method for Handling Model Growth in Nonrigid Variation Simulation of Sheet Metal Assemblies, Journal of Computing and Information Science in Engineering, 14/3:031004.

[21] Malmiry, R.B., Pailhès, J., Qureshi, A.J., Antoine, J.F., Dantan, J.Y., 2016, Management of product design complexity due to epistemic uncertainty via energy flow modelling based on CPM. Annals of CIRP, 65/1:169-172.

[22] Morse, E., Dantan, J.-Y., Anwer, N., Söderberg, R., Moroni, G., Qureshi, A., Jiang, X., Mathieu, L., 2018, Tolerancing: Managing uncertainty from conceptual design to final product, CIRP Annals, 67 (2), pp. 695-717.

[23] Schleich, B., Wartzack S., 2014, A discrete geometry approach for tolerance analysis of mechanism, Mech. Mach. Theory 77, 148–163.

[24] Shafer G., 1976, A Mathematical Theory of Evidence, Princeton, NJ.

[25] Thunnissen, D.P., 2005. Propagating and mitigating uncertainty in the design of complex multidisciplinary systems, California Institute of Technology.

[26] Walter, M., Storch, M., Wartzack, S., 2014, On uncertainties in simulations in engineering design: A statistical tolerance analysis application, Simul. Trans. Soc. Model. Simul. Int., 90/ 5:547–559.