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Relationship between inclination angle and friction factor of chevron-type plate heat exchangers

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Abstract

The inclination angle is a key design parameter of chevron-type plate heat exchangers. An increase of the inclination angle can possibly raise the flow friction by two orders of magnitude. According to our review, previous works concerning experimental measurements and correlations of the friction factor in plate heat exchangers are highly inconsistent. In order to establish a benchmark, we use the Large Eddy Simulation to model the fully-developed flow in cross-corrugated channels of the plate heat exchanger, and then calculate the friction factor for various conditions with the inclination angle ranging from 18° to 72°, and the Reynolds number varying from 10 to 6000. Based on the numerical data, well-defined equations are derived to correlate the friction factor with the inclination angle in a general way for both laminar and turbulent flow regimes. Moreover, we derive novel correlations to predict the critical point of laminar-to-turbulent transition in the plate heat exchanger. Then we use these established correlations to draw a friction factor diagram for plate heat exchangers. The diagram maps the relationship between the friction factor and the inclination angle for a wide Re range, which resembles the Moody diagram. Furthermore, the vortex structure and the mean flow properties for different inclination angles are analysed. The results suggest that a larger inclination angle leads to more intensified vortexes and span-wise secondary flows in the channel, resulting in a larger friction factor.

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1. Introduction

The chevron-type plate heat exchanger (PHE) is a prevalent heat transfer equipment widely used in food and chemical industries, and within the energy sector. The core of the PHE comprises numbers of cross-corrugated channels. The channel is constructed by two corrugated plates attached in criss-cross fashion, as illustrated in Fig. 1. The attachment between the opposite corrugations creates multiple contact corners, rendering the heat exchanger its high compressive strength. As described in Fig. 1, the geometry of the cross-corrugated channel is governed by three parameters, namely, the corrugation pitch (A), the corrugation height (b), and the inclination angle (β). Moreover, a dimensionless parameter, usually termed as the surface enlargement factor φ, is commonly used to characterize the steepness of the corrugation. The φ can be calculated by [1]

\[
\phi = \frac{1}{6} \left( 1 + \sqrt{1 + \left( \frac{b\pi}{A} \right)^2} + 4 \sqrt{1 + \frac{1}{2} \left( \frac{b\pi}{A} \right)^2} \right)
\] (1)

Among the three geometric parameters, the inclination angle (β), defined as half of the angle between the opposite corrugation profiles, has been proved to be the most influential parameter to the thermal-hydraulic performance of a PHE [1,2]. The degree of complexity of the flow path increases with the β; hence, the flow friction and the heat transfer strongly rely on the β. According to Martin’s analysis [1], at Re=2000, the friction factor and heat transfer coefficient of the PHE increase about 100 times and 3 times, respectively, when the β is increased from 15° to 75°. Therefore, it is of crucial importance to grasp the relationship between the thermal-hydraulic performance and the inclination angle.

Numerous experimental studies have been reported which measured the performance of industrial series PHEs with various β, usually ranging 30° ≤ β ≤ 65°. Fig. 2 shows a comparison of the friction factor data reported in different papers [3–14]. The friction factor data cited in this paper were all converted into the Darcy’s form, defined as:

\[
f = \frac{2\rho D_h \Delta P}{G^2}
\] (2)

The purpose of the comparison in Fig. 2 is to assess the consistency of these published data. In order to make a fair comparison, the experimental datasets of the PHEs with approximately equiv-
alent characteristic geometries ($\phi = 1.17 \pm 0.08$, $\beta = 30^\circ$ and $60^\circ$) were selected. It can be observed that the datasets are highly inconsistent, showing deviations exceeding one order of magnitude. The inconsistency of the experimental data may be attributed to the following reasons: (i) The flow maldistribution effect in commercial PHEs can lead to a certain degree of increase in the flow friction [15], which was commonly overlooked in experimental studies; and (ii) most of the experimental studies did not consider the pressure drop at PHE's inlet and outlet distribution areas, while they usually account for a considerable share of the total pressure drop [16]. The measurement uncertainty may be an additional source of error in experimental studies.

The practical design of the PHE always demands a predictive $f$-correlation for quantifying the pressure drop. Many correlations have been proposed for the PHE, which were reviewed by Ayub [17] and Huang [10]. Most of those correlations were derived in the form of $f = c_0 \text{Re}^n + c_1$. The model constants $c_0$, $n$, and $c_1$, were obtained by fitting the experimental dataset into the equation. Such correlations, however, have very limited predictive capability, since the model constants are not able to be well-correlated with the PHE geometry and the flow regime. Martin [1] has derived a more mechanistic $f$ correlation for PHEs. Based on the flow visualization in cross-corrugated channels [18], Martin [1] decomposed the total flow friction into a stream-wise component and a furrow-wise component, and then provided approximated formulations to both. Martin’s model has been extensively used since published. Doivić et al. [19] derived an $f$ correlation for PHEs, which shares the common spirit of Martin’s correlation. Arsenyeva et al. [20] developed a new form of $f$ - correlation for PHEs. It was obtained by fitting a limited number of experimental data points to Churchill’s equation [21]. The correlation shows good agreement with the relevant experimental data.

The existing correlations from heavily cited literatures [1,7,8,20,22,23] are compared in Fig. 3. Provided a PHE with a fixed geometry, these correlations fail to yield a consistent prediction of the $f$. The maximum deviation among them is more than five-fold. This issue was also noted by Solotych et al. [24], who failed to match the existing correlations with their experimental results. In general, the correlations are derived by data-fitting of available experimental data. However, as pointed out previously, the existing experimental database of the $f$ is highly inconsistent, resulting in the inconsistency of the correlations.

Computational fluid dynamics (CFD) modelling is nowadays a powerful tool to assist the design of various kinds of heat exchangers. The thermal and flow fields in the heat exchanger can be resolved in the CFD simulation, so that the thermal-hydraulic performances can be obtained with lower costs (in terms of the financial and time investments) than the experimental approach. Moreover, the flow and heat transfer details in the heat exchanger could be better understood by analysing the CFD result. The CFD modelling of PHEs is especially meaningful because the numerical model can fully exclude the disturbing factors, like the flow maldistribution and measurement errors associated with the experiments, and hence, purely compute the performance of the cross-corrugated core. The flow turbulence in the PHE is non-trivial, and so must be treated carefully in order to get realistic solutions. Most of the available works modelled the turbulent flow in PHEs by solving Reynolds-averaged Navier–Stokes (RANS) equations (eg. [25–28]). The RANS-based simulation is economic, while it is not able to resolve well the detailed flow structures. Blomerius et al. [29] modelled the cross-corrugated channel flow by direct numerical simulations (DNS) for $Re \leq 2000$. In their simulations, the $\beta$ was fixed to $45^\circ$, while the $\phi$ was varied. The results suggest that the $\phi$ has slight influence on the $f$. It is known that DNS requires a very fine grid to resolve all scale structures, making it extremely costly especially for modelling high-$Re$ flows in complex wall-bounded flows.

An alternative way to model the turbulent channel flow is the large eddy simulation (LES), which directly resolves large-scale structures in the channel, while sub-grid, small-scale structures are modelled. In a few research papers [30–32], the LES was adopted to model the cross-corrugated channel flow in PHEs. Cifalo et al. [30] showed that the $f$ and Nusselt number resulting from LES are more precise than the RANS-based models. Lee et al. [31,32] performed LES for different cross-corrugated channels, and used the simulation output to optimize the PHE performance.

The inconsistency among experimental data and correlations concerning the $f$ versus the Reynolds number ($Re$) in PHEs suggests that there is the need for a comprehensive analysis on the topic. In this paper, the $f$ of the PHE was computed by meticulous CFD simulations. The LES technique was used to model the flow in cross-

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**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$b$</td>
<td>Corrugation height, m</td>
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<tr>
<td>$c_p$</td>
<td>Specific heat, J/kg K</td>
</tr>
<tr>
<td>$D_h$</td>
<td>Hydraulic diameter, $D_h = 2b/\phi$, m</td>
</tr>
<tr>
<td>$f$</td>
<td>Darcy friction factor</td>
</tr>
<tr>
<td>$G$</td>
<td>Flow mass flux, kg/m$^2$s</td>
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<tr>
<td>$P$</td>
<td>Pressure, Pa</td>
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<tr>
<td>$\Delta P_l$</td>
<td>Pressure drop per unit length, Pa/m</td>
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<tr>
<td>$Pr$</td>
<td>Prandtl Number</td>
</tr>
<tr>
<td>$Q$</td>
<td>Q-criteria, $1/s^2$</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds number, $Re = GD_h/\mu$</td>
</tr>
<tr>
<td>$S$</td>
<td>Strain rate, $1/s$</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature, $K$</td>
</tr>
<tr>
<td>$t$</td>
<td>Time, $s$</td>
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<tr>
<td>$u$</td>
<td>Velocity, $m/s$</td>
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**Greek symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>$\beta$</td>
<td>Corrugation angle</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Viscosity, Pa·s</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Fluid density, kg/m$^3$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Surface enlargement factor</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>Corrugation pitch, m</td>
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<tr>
<td>$\Omega$</td>
<td>Vorticity tensor, $1/s$</td>
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**Subscript**

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<th>Symbol</th>
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<tr>
<td>$c$</td>
<td>Critical</td>
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<tr>
<td>$t$</td>
<td>Turbulent</td>
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corrugated channels over a wide $Re$ range ($10<Re<6000$), covering laminar and turbulent regimes. The $\beta$ was varied from $18^\circ$ to $72^\circ$. This range is wider than the design scope of most commercial PHEs, for which usually $30^\circ \leq \beta \leq 65^\circ$. The aspect ratio, i.e. the ratio of the corrugation pitch to the height was fixed to 1 in this study, corresponding to a $\phi = 1.46$ according to Eq. (1). Based on numerous simulations, a refined database of $f$ for cross-corrugated channels was produced, benchmarking the $f$ of fully developed flow in cross-corrugated channels. The critical point for laminar-to-turbulent transition was identified and then correlated with $\beta$. In addition, generalized correlations between the $f$ and the $\beta$, for both laminar and turbulent flow regimes, were derived based on the numerical data. Furthermore, the vortex structure and the flow statistics were analysed in order to unveil fundamentally the effect of different inclination angles on the fluid hydraulics inside the PHE.

The major contribution of this paper is that the generalized relationship between the inclination angle (as the key design parameter of PHEs) and the friction factor is established. We extend the use of Colebrook equation to PHEs, by making the hypothesis that the inclination angle can be regarded as a kind of roughness of the channel. The developed friction factor correlations can be practically used for the PHE design and optimization. Meanwhile, we found the dependence of the flow transition on the $\beta$. Novel correlations were formulated, which can be used to discriminate whether the flow is laminar or turbulent in a PHE. Overall, our results will support both industry and academia in designing more cost-efficient and compact PHEs.

The paper is organized as follows. In Section 2, the numerical models are presented. In Section 3.1, the $f$ data resulted from the simulation is presented and validated; then generalized correlations between the $f$ and the $\beta$ are derived; this is then followed by a study of the flow transition criteria inside the PHE in Section 3.2; by warping up all the established correlations, a friction factor diagram for the PHE is created in Section 3.3; finally in Section 3.4, the effect of the inclination angle on the friction factor is analysed.
and interpreted from the fluid dynamic perspective. In Section 4, the conclusions are summarized.

2. Numerical models

2.1. Governing equations

In this study, the incompressible flow of Newtonian fluid (water) in cross-corrugated channels was modelled based on the LES technique. The filtered governing equations of mass, momentum and energy are written in the following form:

\[
\frac{\partial \rho}{\partial x_i} = 0
\]

\[
\rho \frac{\partial u_i}{\partial t} + \rho \frac{\partial u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{\partial \tau_{ij}}{\partial x_j} \right]
\]

(4)

\[
\rho \frac{\partial T}{\partial t} + \rho \frac{\partial u_i T}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ \mu T \frac{\partial T}{\partial x_j} + \frac{\partial \tau_{ij}}{\partial x_j} \right] - q_i + \frac{\partial}{\partial x_j} (\kappa \frac{\partial T}{\partial x_j})
\]

(5)

The sub-grid turbulent stress in Eq. (4) was approximated according to the Boussinesq hypothesis in the form of \( \tau_{ij} = -2 \mu_t \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} \). The sub-grid turbulent heat flux \( q_i \) is expressed as \( q_i = -\frac{\partial \bar{T}}{\partial x_i} \), with turbulent Prandtl number \( Pr_t = 0.85 \). The eddy viscosity was modelled by using an Algebraic Wall-Modelled LES (WMLES) Formulation [33]

\[
\mu_t = \rho \cdot \min \left[ (\kappa d_w)^2, (C_{Smag} \Delta)^2 \right] \cdot S \cdot \left[ 1 - \exp \left( -y^+ / 25 \right)^3 \right]
\]

(6)

where \( d_w \) is the wall distance, \( S \) the strain rate, \( \kappa = 0.41 \) and \( C_{Smag} = 0.2 \). \( y^+ \) is the normal to the wall inner scaling, and \( \Delta \) is the local grid scale. The laminar flow in PHEs is mainly in the low-Reynolds number range. The corresponding strain rate \( S \) of the laminar shear is small, so is the turbulent viscosity \( \mu_t \) predicted by Eq. (6). Therefore, it is appropriate to use the LES model to predict the laminar flow hydraulics in PHEs.

It should be noted that although this paper does not discuss the heat transfer aspect, the energy equation was sloved as well. Results about the heat transfer will be presented in a separate paper.

2.2. Channel geometry and computational domain

Focke et al. [34] measured the \( f \) in cross-corrugated channels for a wide range of \( \beta \) and \( Re \). A test section was customized to allow for uniform flow to be developed in the cross-corrugated geometry. Therefore, the measurements by Focke et al. [34] should represent well the hydraulics of fully developed flows in the cross-corrugated channel, which were hence chosen as references to validate our numerical results. The corrugation pitch and height for the present numerical study are identical to the experiment of Focke et al. [34], as listed in Table 1. In order to understand better the effect of \( \beta \) on the flow hydraulics, seven cases with different \( \beta \) (18°, 30°, 38°, 45°, 52°, 60° and 72°) are investigated.

The cross-corrugated channel of the PHE consists of repetitive geometric units; therefore, it is possible to reduce the computational cost by only considering a few respective units of the channel. Before doing so, the sensitivity of the simulation result on the domain size should be examined at the first place. In Fig. 4, two different sized domains are foregrounded, which are both repetitive elements extracted from a cross-corrugated channel. The larger one (labelled “L”) is four times the size of the smaller one (labelled “S”). Fig. 4 shows the comparison of the time history of the stream-wise velocity resulting from two different sized domains. They are close to each other in terms of their instantaneous fluctuation and the mean value, which suggests that the simulation results are not strongly influenced by the domain size, and therefore, the smaller domain is chosen for the simulation in the present study.

The hexahedral grid was designed for the domain as illustrated in Fig. 5. The near wall grids were carefully refined to ensure \( y^+ < 1 \) for all simulations. Four grid sizes, labelled respectively as G1 (0.2 million elements), G2 (0.5 million elements), G3 (1.24 million elements) and G4 (2.08 million elements), were tested to ensure the grid independency. Fig. 5(a) shows the friction factor from different grid sets for \( \beta = 60^\circ \) case. Apparently, the G2-grid with 0.5 million elements is ample for the case of \( Re = 1263 \), while the G3-grid with 1.24 million elements satisfies the case of \( Re = 3597 \). Since we performed in total 58 case studies, it was not feasible to do a respective grid test for each case. It was assumed that the grid test for \( \beta = 60^\circ \) applies for the other \( \beta \) as well. In this respect, the G2-grid was applied for the cases if \( Re < 1263 \); otherwise, the G3-grid was used.

2.3. Boundary conditions and numerical methods

The periodic boundary condition was applied to both stream-wise and span-wise directions. A pressure gradient was imposed

![Fig. 4.](image-url) (a) The extraction of computational domains from a cross-corrugated channel. The “S” and “L” represent a small and a large domain, respectively. (b) The time history of the stream-wise velocity at \( Re = 1000 \) in the “S” and “L” domains are compared. Note that the velocity is monitored in the middle cross-section of the domains.
on the stream-wise direction to drive the fluid flow, while zero pressure gradient was applied to the span-wise direction. The non-slip boundary condition was assigned to the bounded walls. For the thermal boundary condition, a constant and uniform heat flux was applied to the corrugated walls.

The governing equations incorporated with assigned boundary conditions were solved by a finite-volume based solver, ANSYS Fluent 18.2. The velocity-pressure coupling was achieved by using the PISO algorithm. A second-order accurate central differencing scheme was used for spatial discretization. The temporal discretization was based on a second-order backward differencing scheme.

3. Results and discussion

3.1. Friction factor: numerical data and generalized correlations

The friction factor $f$ was calculated based on the CFD results, with the pressure gradient $\Delta P_L$ a time-averaged value. 

Fig. 5. (a) The dependence of the simulation results on the number of total grid elements at two different Reynolds numbers for the $\beta = 60^\circ$ case. (b) The details of the G3-grid ($\beta = 45^\circ$); the zoomed images show the grid refinement around the contact corner and in the wall boundary layer.

Fig. 6. Validation of the numerical results against the experimental results measured by Focke et al. [34]. Solid marks represent experimental results, while empty marks represent our CFD results.

Fig. 7. The symbols represent the $f$ calculated from the CFD simulations. The pink lines and blue lines plot the solutions of Eq. (8) and Eq. (11), respectively. The red dots with the bar indicate the $Re$ range wherein the critical transition point is located. The red dashed line corresponds to Eq. (13).

The friction factor in the channel flow is presented in Fig. 7. The symbols represent the $f$ obtained from the CFD simulations. They are compared with the experimental results measured by Focke et al. [34], which show a nice agreement except for cases with smaller $\beta$. In order to confirm further the validity of the numerical model, the fully developed laminar flow in a double-sine channel is simulated. The double-sine channel is an limiting case of the cross-corrugated channel ($\beta = 0^\circ$). The numerical results suggest an inverse proportional relationship between $f$ and $Re$, which is $f \propto Re = 62.8$. This agrees with the analytical solution of the friction factor in the double-sine channel ($f \propto Re = 62.3$), derived by Ding and Manglik [35].

In Fig. 7, all the calculated $f$ data for the cross-corrugated channels are presented. The relationship between $Re$ and $\beta$ is now comprehensively revealed. The data trend shown in Fig. 7 resembles the Moody diagram [36], a well-known graph that relates the $f$ of the pipe flow with the $Re$ and the surface roughness. Given the resemblance between Fig. 7 and the Moody diagram, we hypothesized that the mathematical formula underlying the Moody diagram could be shared by the cross-corrugated channel flow. Based
on this idea, we derived the correlations among \( f, \) \( Re \) and \( \beta \), as presented below.

At low \( Re \), Fig. 7 displays a zone wherein \( \log f \) is almost linearly correlated with \( \log Re \); that is

\[
f = cRe^n \tag{7}
\]

For each case located in the linear zone, the simulation was able to reach steady state, i.e. the resolved flow field has become independent of the time. Therefore, the flow remains laminar and steady in this linear zone. Eq. (7) is valid for fully developed laminar pipe flow, too, for which \( f = 64Re^{-1} \). This also recalls the \( f - Re \) correlation for the double-sine channel (\( \beta = 0 \)) shown in Fig. 6, which is \( f = 62.8Re^{-1} \). This verifies the fact that in Eq. (7), \( n = -1 \) always holds true for a straight channel, whereas the constant \( c \) depends on the shape of the channel cross-section. However, this is not the case for the cross-corrugated channel with \( \beta \neq 0 \^\circ \). The exponent \( n \) and the model coefficient \( c \) are both \( \beta \) dependent when \( \beta \neq 0 \^\circ \). This is because the cross-corrugated channels can produce a complex laminar flow pattern due to the flow separation and recirculation [37]. By fitting the \( f \) data in the linear zone into Eq. (7), the \( c \) and the \( n \) for each \( \beta \) were obtained, which are displayed in Fig. 8. It is interesting to see that \( c \) and \( n \) can be nicely expressed as a function of \( \sin \beta \). Therefore, a generalized Eq. (8) is obtained for predicting the \( f \) for steady laminar flow in cross-corrugated channels. The predictions of Eq. (8) are plotted in Fig. 7 (pink lines) for validation, which nicely match the CFD results.

\[
f = e^{(1.13(\sin \beta)^3 + 4.13)}Re^{0.43(\sin \beta)^3} - 0.92 \tag{8}
\]

When the Reynolds number is beyond a threshold value, the \( f \) abruptly deviates from the linear slope, which indicates the onset of flow unsteadiness. Zhu et al. [37] suggests that the flow transition from steady to unsteady is caused by the instability associated with the curved, free shear layers established over the lip of the corrugation crests. When the Kelvin–Helmholtz instability becomes critical, the free shear layers are destabilized, corresponding to the onset of flow unsteadiness. When the flow becomes unsteady, the flow enters the transition zone; it is no longer able to correlate \( f \) with \( \beta \) using a simple formula like Eq. (7). It is known that the plots in the Moody diagram for turbulent flows are calculated from the Colebrook equation [38], written as

\[
\frac{1}{\sqrt{f}} = -2.0 \log_{10} \left( \frac{f}{Re} + 2.51 \sqrt{\frac{Re}{f}} \right) \tag{9}
\]

where \( \epsilon \) is the surface roughness, and \( D \) is the inner diameter of the pipe. Based on the resemblance between the Moody diagram and Fig. 7, we extended the use of the Colebrook equation to PHEs, by making the hypothesis that the inclination angle can be regarded as a kind of roughness of the channel. The postulated formula of \( f \) for the cross-corrugated channel is

\[
\frac{1}{\sqrt{f}} = -2.0 \log_{10} \left( a_1 + \frac{a_2}{Re \sqrt{f}} \right) \tag{10}
\]

where \( a_1 \) and \( a_2 \) are model constants, both should depend on the \( \beta \). By fitting the \( f \) data on left side of the red dashed line in the Fig. 7 into Eq. (10), \( a_1 \) and \( a_2 \) were obtained for each \( \beta \) case; then, they were correlated with the \( \sin \beta \) and \( \cos \beta \). The final expression of \( f \) for the cross-corrugated channel is given as

\[
\frac{1}{\sqrt{f}} = -2.0 \log_{10} \left( 1.48 \sin \beta^{0.85} \cos \beta^{0.45} + \frac{60 \sin 2\beta^2 \cos \beta^5 + 16}{Re \sqrt{f}} \right) \tag{11}
\]

Eq. (11) is an implicit formula, which requires an iterative procedure to solve it. The solutions are presented in Fig. 7 for validation. The comparison with the CFD results verifies that Eq. (11) can well approximate the \( f \) for the PHE with different \( \beta \). Although the equation is not posed for the laminar regime, it behaves surprisingly well in the laminar regime as well, as evidenced in Fig. 7, which implies that Eq. (11) can be practically applied to all flow regimes.

As the \( Re \) increases, the second term in the right-hand side parentheses of Eq. (11) becomes insignificant, which means the \( f \) becomes less dependent on the \( Re \). Fig. 9 plots the asymptotic convergence of \( f \) to the \( f(Re \to \infty) \) as \( Re \) is increased. Some experimental results from different literatures are also displayed in the figure as a proof of our numerical results. It can be seen that the trend
of $f$ versus $\beta$ predicted by Eq. (11) for $Re = 2000$ agrees excellently with the experimental outcomes. The plot indicates that the $f$ is more influenced by the $\beta$ at high $Re$ flow. For instance, at $Re=100$ (laminar flow), $f_{\beta=72}/f_{\beta=18} \approx 11$, while this ratio can increase up to 208 for $Re \to \infty$. The $f$ is independent on the $Re$ for the extreme condition with $Re = \infty$, then it is purely defined by $\beta$

$$f(Re \to \infty) = \left[-2.0 \log_{10}(1.48 \sin \beta^{4.85} \cos \beta^{0.45} \right]^{-2}$$  \hspace{1cm} (12)

From Fig. 9, it is seen that the larger the $\beta$ is, the faster the $f$ converges to the $f(Re \to \infty)$.

### 3.2. Criteria for the flow transition

The inclination angle $\beta$ has a profound impact on the laminar-to-turbulent transition in PHEs. Focke and Knibbe [34] observed that the flow in the cross-corrugated channel with $\beta = 80^\circ$ was already turbulent at $Re_h = 109$. Blomerius et al.[41] suggested that the transition in a $\beta = 45^\circ$ channel taking place at $Re_h = 245$. From the experimental data published by Thonon et al. [42], it can be estimated that the critical $Re$ for a PHE with $\beta = 30^\circ$ is around 500. Apart from those results, the relationship between the flow transition and the $\beta$ of PHEs has not been investigated in the past. As a consequence, there is no well-established rule or equation that can be used to discriminate whether the flow is laminar or turbulent inside a PHE. In this section, the critical pressure gradient and the critical Reynolds number for the onset of flow transition are unveiled by analysing the CFD results, and a set of transition criteria are established for the PHE, which can be used to determine the flow characteristics in a PHE with any $\beta$ value.

By extensively screening the flow state at different $Re$, the interval of $Re$ in which the flow transition occurred were narrowed down. As a result, the possible location of the critical $Re$ for the onset of flow turbulence were identified for each $\beta$ case, as marked by the red dots and bars in Fig. 7. It can be observed that all these transition points approximately align on a line that can be expressed as

$$f_c = e^{0.97}Re_c^{-1.75}$$  \hspace{1cm} (13)

The equation approximately defines the boundary between the laminar zone and the transition zone. It can be slightly adjusted to

$$f_c = e^{11.4}Re^{-2}$$  \hspace{1cm} (14)

Since we want to cancel the $Re$ term by substituting the above equation into Eq. (2), the following correlation is then obtained

$$\langle \Delta P_r \rangle_c = \frac{e^{7.5}P_c}{2D_h\rho}$$  \hspace{1cm} (15)

The $\langle \Delta P_r \rangle_c$ is the critical pressure gradient of the PHE. When the imposed pressure gradient along the channel stream-wise direction is larger than the $\langle \Delta P_r \rangle_c$, the flow transition is most likely to occur. Eq. (15) suggests that $\langle \Delta P_r \rangle_c$ is governed by the channel hydraulic diameter $D_h$ and the fluid properties. This implies that the flow transition in PHEs is primarily determined by the imposed pressure gradient, while the role of the $\beta$ is the trivial to the flow transition from this perspective. Eq. (15) is particularly useful in engineering practice and in the experimental study, since the flow regime (laminar or turbulent) in the PHE can be easily predicted based purely on the pressure difference measured between the inlet and outlet of the PHE.

In Fig. 7, the critical Reynolds number $Re_c$ of the flow transition can be readily located by calculating the intersection point between the laminar-to-turbulent boundary line (red dashed line) and the $f$ curves (blue or pink lines). Accordingly, the $Re_c$ can be derived by substituting Eq. (13) into Eq. (11), or by substituting a modified transitional boundary line, $f_c = e^{0.75}Re^{-1.75}$, into

### 3.3. Friction factor diagram for plate heat exchangers

According to the Moody diagram, the entire $f$-$Re$ diagram for circular pipes can be divided into three regions based on the flow characteristics, which are termed as: laminar flow, transition zone and complete turbulence, respectively. By wrapping up Eqs. (8), (11), (13) and (18), a $f$-$Re$ diagram for the fully developed cross-corrugated channel flow is created; see Fig. 11. This diagram follows the same principles as those of the Moody diagram for the pipe. In Fig. 11, the friction factor is mapped for the PHE with different $\beta$ for $10 \leq Re \leq 10^5$, and is divided into three regions. The laminar flow and transition zone are divided by the Eq. (13), while the boundary line between the transition zone and the complete turbulence can be defined as follows:

$$f_{c2} = 0.98 f(Re \to \infty)$$  \hspace{1cm} (18)

This equation states that the flow is assumed to be complete turbulence if the $f$ is 2 percent less than the $f(Re \to \infty)$. Hence, the $f$ curve in the complete turbulence zone is almost horizontal, implying that the viscous effect on the flow friction is trivial in this zone.

The similarity and the difference between Fig. 11 and the Moody diagram are worthwhile to be elaborated here. It can be recognized in Fig. 11 that the $\beta$ plays a similar role as what the surface roughness ($\varepsilon$) in Eq. (9)) does in the Moody diagram. Then virtually, the inclination angle $\beta$ can be analogized as a type of “roughness element” of the cross-corrugated channel. From this
Fig. 11. The friction factor diagram for cross-corrugated channels of the PHE. The pink and blue lines are resulted from Eq. (8) and Eq. (11), respectively. The dashed lines are boundaries between the neighbouring zones, which are calculated from Eq. (13) and Eq. (18), respectively.

Fig. 12. The vortex structure in cross-corrugated channels described by the iso-contour of the Q-criteria. (a) $\beta = 60^\circ$, $Re = 1784$; (b) $\beta = 45^\circ$, $Re = 1934$; (c) $\beta = 30^\circ$, $Re = 3601$. Three values of $Q$ are depicted in the figure using different colors: $Q = 1.6 \times 10^3$ $s^{-2}$ is green, $Q = 8 \times 10^3$ $s^{-2}$ is white, and $Q = 4 \times 10^5$ $s^{-2}$ is pink (with 50% transparency).

perspective, a larger $\beta$ corresponds to a rougher channel geometry, and hence, a larger flow friction factor. Practical proofs of this analogy will be presented in Section 3.4.

The $f$ strongly depends on the $\beta$ in the laminar zone, whereas the $f$ is independent from the surface roughness for pipe flows. This is because the $\beta$ can affect the laminar flow pattern in the cross-corrugated channel by causing the flow separation and recirculation, while the surface roughness cannot make any change to the laminar pipe flow. The transition zone in Fig. 11 denotes that the flow friction comes from both the turbulent stress and the viscous stress. When the flow enters complete turbulence zone, the flow friction is dominated by the turbulent stress, so that the $f$ is almost independent from the $Re$, and can be directly calculated from Eq. (12). In a PHE’s cross-corrugated channel, the strong turbulence can be promoted by the channel geometry even at moderate $Re$. The friction resulting from the turbulent structure is the main source of the friction in the turbulent flow regime. Although the friction due to the viscous stress in a PHE is probably still there at a moderate $Re$, the friction resulting from the strong turbulence is much larger than the viscous friction, making the latter insignificant. This explains why the $f$ in a PHE with large inclination angle is almost independent of the $Re$, even when the $Re$ is only a few thousand.

3.4. Influence of inclination angle on the flow dynamics

In this section, the vortex structure and the mean flow properties in cross-corrugated channels are examined, aiming to explain why the $\beta$ influences the flow friction in such a profound way. The vortexes are identified by the iso-contour of the Q-criterion, which is defined as

$$Q = \frac{1}{2} \left( \| \Omega^2 - S^2 \| \right)$$  \hspace{1cm} (19)

where $\Omega$ and $S$ are the vorticity tensor and the rate-of-strain tensor, respectively. A positive value of $Q$ identifies vortexes as the
regions where the vorticity magnitude is greater than the magnitude of the rate-of-strain. Snapshots of vortices for three different $\beta$ are displayed in Fig. 12. The vortex strength of $\beta = 60^\circ$ case is remarkably stronger than that of the $\beta = 45^\circ$ case at an equivalent level of $Re$. Although the $Re$ for the $\beta = 30^\circ$ case almost doubles the $Re$ for the $\beta = 60^\circ$ case, the vortex strength is not as intense as the latter. Therefore, it can be concluded that the increase of $\beta$ significantly intensifies the vortex production in PHEs, and hence increases the flow friction.

The mean flow velocity profile in the middle of the computational domain is examined. For clarity, Fig. 13(a) specifies the location at which the flow statistics was performed. Fig. 13 (b-d) shows the laminar velocity profiles for three $\beta$ values at $Re = 29$. The velocity profile is semi-parabolic for the $\beta = 30^\circ$. The velocity profile is deformed with the increase of $\beta$; and the velocity gradient near the wall is apparently increased, which probably results in a higher viscous shear stress and larger friction factor for the larger $\beta$ case. Fig. 13 (e-g) shows the mean velocity profiles of turbulent flows for the three $\beta$ values at $Re = 2000$. The profiles are no longer semi-parabolic. It can be seen that the span-wise components $u_x$ and $u_z$ increase remarkably with the $\beta$. For the $\beta = 60^\circ$, the span-wise component $u_x$ is even larger than the stream-wise component $u_z$, indicating that span-wise momentum is significant in this case. This suggests that a larger $\beta$ can lead to stronger crosswise flows in the cross-corrugated channel. It should be noted that the flow velocity profile varies significantly from one location to another in the cross-corrugated channel. Therefore, Fig. 13 is deemed to be a local description of the mean flow characteristics in such kind of channels. It quantitatively reflects the fact that the $\beta$ can alter the strength of span-wise secondary flows in the cross-corrugated channel.

4. Conclusions

This paper presented a comprehensive numerical study of the flow friction factor $f$ in cross-corrugated channels. The study addressed the question of how and why the inclination angle $\beta$ influences the $f$ of plate heat exchangers. The major findings/outcomes of the paper are the following:

1. The simulations yielded a high-fidelity database of the $f$ for PHEs with enlargement factor $\phi = 1.46$, $18^\circ \leq \beta \leq 72^\circ$, and $10 \leq Re \leq 6000$. The results serve as a benchmark for the $f$ of fully developed flows in cross-corrugated channels.

2. Based on the CFD results, correlations are developed to describe the relationship between the $f$ and the $\beta$ for both laminar and turbulent flow regimes. The results for the laminar zone indicate that the logarithmic $f$ is linearly related with logarithmic $Re$. Based on this observation, the laminar model is developed. The correlation for turbulent flow is established based on the form of Colebrook equation. The correlation is able to predict well the $f$ of a PHE with any $\beta$ value, and can be extendedly used for laminar flow regime as well.

3. The critical pressure gradients and critical Reynolds numbers for laminar-to-turbulent transition in cross-corrugated channels are identified based on the simulations. Novel correlations are established for estimating the critical pressure gradients and the critical Reynolds number. These correlations can help to discriminate whether the flow is laminar or turbulent in a PHE.

4. A friction factor diagram is developed for the cross-corrugated channel flows, which maps the relationship between $f$ and $\beta$ over a wide $Re$ range. The diagram resembles the well-known Moody diagram, which is divided into laminar, transition and complete turbulence zones. The $\beta$ can be analogized as a type of “roughness element” of the channel. Based on this analogy, it can be interpreted that a larger $\beta$ corresponds to a “rougher” channel, and therefore a higher friction factor. The diagram can be straightforwardly used to look up $f$ of a PHE.

5. The flow field analysis indicates that a larger $\beta$ leads to strengthened vortices and span-wise secondary flows, therefore causing a higher flow friction.
Declaration of Competing Interest

The authors declare that there is no conflict of interest.

CRedIT authorship contribution statement

Xiaowei Zhu: Conceptualization, Methodology, Investigation, Data curation, Formal analysis, Writing - original draft. Fredrik Haglind: Conceptualization, Methodology, Supervision, Writing - review & editing.

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