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# A review of integrated approaches for optimizing electric vehicle and crew schedules

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## Abstract

In the public bus transport industry, the vehicle and crew scheduling problems are the primary drivers of cost. Given a set of timetabled trips, the vehicle scheduling problem (VSP) assigns buses to the timetabled trips such that every trip is covered by a bus and the objective is to minimize the operational cost based on bus usage. A duty is defined as the work of a bus driver for a day and the crew scheduling problem (CSP) is concerned with determining sets of duties to cover all scheduled bus trips. The objective of the CSP is to minimize total wages paid to the drivers while satisfying numerous labor union rules. The multiple depot VSP and CSP are  $\mathcal{NP}$ -hard problems and have been extensively studied in the Operations Research (OR) literature. However, practitioners often apply a sequential approach that finds the vehicle schedule first before finding the crew schedule. Simultaneously handling the vehicle and crew scheduling aspects could lead to further reduction of operational cost. This paper discusses the models and solution methods reported in the OR literature for integrating the VSP and CSP. Furthermore, particular attention is given to electric buses. The use of electric bus technologies is expected to increase in the coming years due to its significant environmental benefits. The aim of this paper is to provide researchers and practitioners an overview of the existing research on integrating the two scheduling problems and the electric bus technologies that could offer valuable insights and perspectives for future research.

*Keywords:* Transportation, Vehicle Scheduling, Crew Scheduling, Electric buses, Literature Review

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## 1. Introduction

In 2018, the United Nations (UN) reported that 55% of the world population reside in urban areas, which is estimated to be 4.2 billion people (United Nations (2018)). By 2050, 68% of the world population is projected to be urban. The growth of the world's population and urbanization requires building sustainable cities that provide opportunities for social and economic development while reducing adverse impacts on the environment. Public transportation is recognized as a crucial backbone for sustainable urban development since it enhances mobility by providing infrastructure and services for the safe and efficient movement of people. A sustainable transport system prevents severe traffic congestion, road accidents, air and noise pollution. However, planning, operating and controlling a city's public transport system is known to be challenging due to the system's sheer size and complexity. Several stakeholders namely public authorities, public transport companies and users or passengers, with different goals are involved in the transport planning process. The passengers usually have varying socio-economic characteristics and expect a high level of service; i.e. the transport system should be safe, accessible, comfortable, affordable and provide the possibility of reaching destinations quickly. The objective of transport companies is to provide high quality service to the passengers while minimizing the overall operational cost (Desaulniers and Hickman

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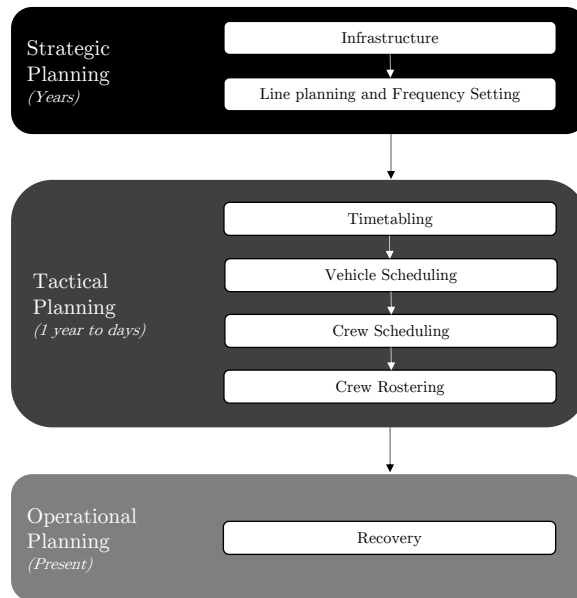
(2007) and Ibarra-Rojas et al. (2015)). A public transport system is typically designed with multiple modes of transport such as tram, metro, train and bus. The aim of the system is to seamlessly integrate the different services for a better passenger experience.

This paper focuses on the Operations Research (OR) literature that is related to improving the efficiency of bus services from the companies' perspective. For a bus company, maintenance and fuel consumption of buses and the wages paid to bus drivers are the main factors that contribute to the operational cost. The crew cost contributes to approximately 60% of the total operational cost for bus transport systems in Northern Europe (Perumal et al. (2019)). Reducing the operational cost directly influences the ridership cost and increases service attractiveness for the passengers. However, bus companies are challenged to create cost-effective vehicle and driver schedules for cities with large-scale transport systems. Several practical conditions such as the labor regulations that govern the working conditions of drivers and infrastructure properties have to be considered during the operational planning process. The bus companies and the industry in general are also affected by the climate agenda initiated by government and intergovernmental organizations. In accordance with the UN Paris Agreement (United Nations Climate Change (2015)), the European Union (EU) aims to create a climate-neutral economy by 2050 (European Union (2018)). Therefore, the EU has initiated strategies to reduce greenhouse gas emission, which also includes the modernization of the transport infrastructure. Most major cities in Europe have pledged to procure only zero-emission buses from 2025 as part of the C40 Fossil Fuel Free Street Declaration (C40 Cities (2017)). The use of electric bus technologies is expected to rise due to its significant environment benefits. For example, Paris and Copenhagen aim to electrify all their city buses already by 2025 (Transport and Environment (2018) and Copenhagen Capacity (2019)).

### 1.1. Bus Transportation Planning Process

Providing a bus service involves several stages of planning. They range from making long-term decisions such as investment in infrastructure to short-term decisions on how to execute day-to-day operations. The entire planning process of bus transportation is computationally intractable and cannot be solved in one integrated step. Hence, it is divided into several problems which are solved in a sequential manner as shown in Figure 1. The different planning problems are discussed in Desaulniers and Hickman (2007), Schöbel (2012) and Ibarra-Rojas et al. (2015). The planning process is also similar to other transport industries such as railways (see e.g. Lusby et al. (2018)). The *infrastructure* is represented as a bus transportation network that describes the streets and bus stops of a city. In a tram or a railway system, the network represents the track system. A *line* is defined as a path or a route in the city along which a bus service is offered and the *frequency* of a line refers to how often the service is offered along the line within a given time period (e.g. one hour). The *line planning* and *frequency setting problem* determine the lines and their frequencies based on forecast passenger demand. The frequency setting problem also takes the demand patterns during different periods (morning, afternoon, evening) of operation into account. *Timetabling* is the process of defining arrival and departure times at all bus stops in the city network in order to meet the given frequency and level of service of each line. The emphasis is on passenger service and the objective, most commonly, is to minimize travel or transfer times for passengers. A timetable corresponds to a set of *trips* with arrival and departure bus stops and times. The *vehicle scheduling problem* (VSP) assigns buses to the timetabled trips such that every trip is covered by a bus. The objective is to minimize the operational cost based on bus usage. In a bus transportation setting, only one type of crew, i.e. the bus drivers, is required to perform the services, whereas drivers, conductors and catering staff are required in a train or airline setting. A *duty* is defined as the work of a bus driver for a day and the *crew scheduling problem* (CSP) is concerned with determining sets of duties to cover all scheduled vehicle trips. The objective of the CSP is to minimize total wages paid to the drivers and the duties are subject to a wide range of labor union rules and regulations. The *crew rostering problem* consists of constructing and assigning weekly or monthly work schedules (called rosters) from the anonymous daily duties to the available drivers. The validity of the rosters is also restricted by labor union rules and regulations. During operation of transport systems, uncertain elements such as vehicle breakdowns or extreme weather conditions

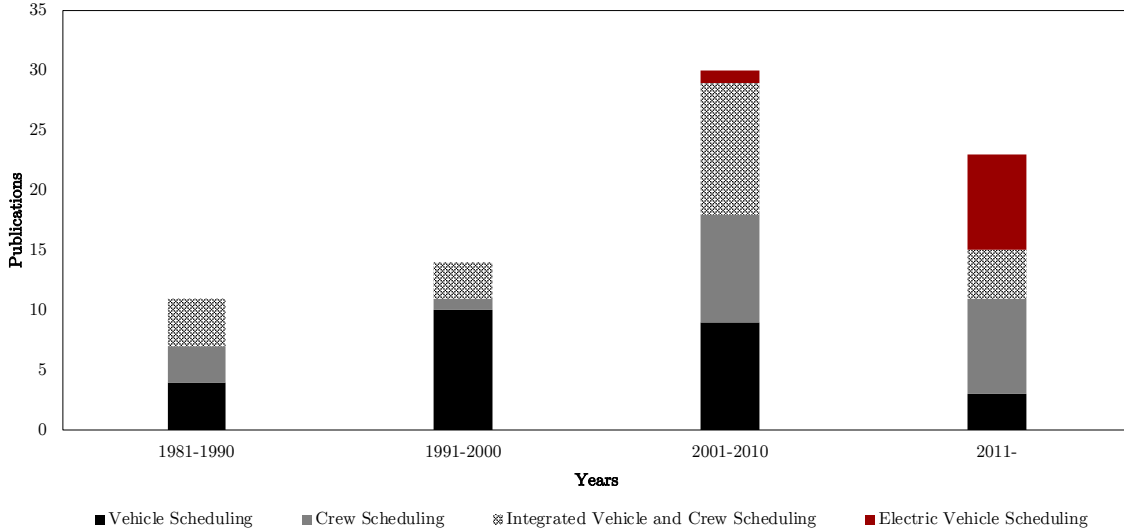
can severely disrupt the planned activities of vehicles and crew. *Recovery plans* and real-time control strategies are often implemented to reduce the impact of disruptions.



**Figure 1:** Bus Transportation Planning Process.

Figure 1 gives an overview of the different problems in bus transportation at the *strategic*, *tactical* and *operational* stages. The figure is similar to the figure presented in Lusby et al. (2018) for railway systems. The figure also indicates an estimate of the time each planning stage is considered before the day-of-operation. The infrastructure is rarely changed and potentially remains the same for many years. The timetabling process is typically carried out a year in advance and the timetables are known to be different on weekdays, weekends and public holidays. Public authorities are often responsible for the timetabling process. The bus companies construct vehicle and crew schedules for the different timetables. Desrochers and Soumis (1989) state that a bus company in Montreal, Canada usually used a crew schedule for about half a year. At Nederlandse Spoorwegen, the largest passenger railway operator in the Netherlands, the crew schedules for the annual plan are initially constructed and they are modified six times a year if there are specific changes in the timetable and vehicle schedule for a particular day (Huisman (2007)). Some authors (e.g. Ibarra-Rojas et al. (2015)) have, however, placed the vehicle and crew scheduling problems at the operational planning stage. Furthermore, an additional planning stage called *control* is included to describe the real-time control strategies and disruption management of transport systems.

There has been an increased focus on integrating two or more planning problems in recent years. At the tactical planning stage, vehicle and crew scheduling are the primary drivers of cost and, as mentioned earlier, the crew cost is known to dominate the vehicle cost in most countries. Simultaneously handling the vehicle and crew scheduling aspects potentially leads to further cost reductions and efficiency gains for bus transport systems (see e.g. Huisman et al. (2005)). Another emerging area of research in the public bus industry is the scheduling of electric buses. They are known to have limited driving ranges and long recharging times. This make them less flexible than conventional diesel buses (Transport and Environment (2018)). In this paper, we review 78 articles that are categorized into 1) vehicle, 2) crew, 3) integrated vehicle and crew and 4) electric vehicle scheduling problems as shown in Figure 2. We focus on the additional operational constraints that are considered in the literature for scheduling electric buses. Furthermore, this paper focuses on the literature for integrating vehicle and crew scheduling problems. The integrated scheduling problem is more complex to formulate and requires tremendous computational effort to solve. The increased computational complexity of the integrated approach is one of the main reasons for bus



**Figure 2:** Overview of articles on 1) vehicle, 2) crew, 3) integrated vehicle and crew and 4) electric vehicle scheduling problems in the public bus industry that are considered in this paper.

companies to adopt a sequential approach. This paper has been primarily motivated by real-world problems of bus companies and hence, we aim to give an overview of the methods in the literature for practitioners. In summary, a detailed literature review on 1) the electric vehicle scheduling problem and 2) the integrated vehicle and crew scheduling problem that could provide valuable insights to the OR community and the public bus industry are considered as the main contributions of this paper. Additionally, we point to interesting directions for future research.

The remainder of this paper is organized as follows. Section 2 discusses the vehicle scheduling problem with the focus on electric buses and the solution methods to solve the problem. In Section 3, the crew scheduling problem is discussed and an overview of the solution methods in the literature is given. Section 4 gives an overview of the solution approaches proposed in the literature to integrate the vehicle and crew scheduling problems. Finally, Section 5 addresses future research directions and concludes the paper.

## 2. The Vehicle Scheduling Problem

### 2.1. The Single Depot Vehicle Scheduling Problem

Given a bus depot, a set of timetabled trips with departure and arrival times and travel times between all pairs of bus stops, the objective of the single depot vehicle scheduling problem (SDVSP) is to find a minimum cost schedule in which each trip is assigned to a vehicle. Each starts and ends at the depot and performs a feasible sequence of trips. Such a sequence is referred to as a *block*. Each block often starts with an empty move, i.e. a move without passengers, from the depot and ends with an empty move to the depot. Additionally, empty moves are placed between trips that do not end and start at the same bus stop. These empty moves are often referred to as *deadheads*. The cost of a block typically includes a fixed cost and a variable cost that is based on the total distance, in kilometers (km), covered by the vehicle during the day. The SDVSP is known to be solvable in polynomial time (Lenstra and Kan (1981)). It has been formulated as a linear assignment problem, a transportation problem, a minimum-cost flow problem, a quasi-assignment problem and a matching problem. For a detailed overview of models for the SDVSP, see Daduna and Paixão (1995) and Bunte and Kliewer (2009).

#### *Quasi-assignment Model*

In this section, the formulation of the SDVSP as a quasi-assignment problem (QAP) is described. Let  $T$  be the set of timetabled trips. The vehicle scheduling network is defined as a directed graph

$G = (V, A)$ , where  $V$  denotes the set of vertices and  $A$  denotes the set of arcs. Each vertex  $v \in V$  represents a trip and an arc  $(i, j) \in A$  indicates that trip  $j$  can be immediately covered by a vehicle after performing trip  $i$ . A deadhead is placed on the arc  $(i, j)$  if the arrival bus stop of trip  $i$  is not the same as the departure bus stop of trip  $j$ . Additionally, source  $o \in V$  and sink  $s \in V$  vertices are created that represent the depot. An arc from  $o$  denotes the first pull-out deadhead from the depot and an arc to  $s$  denotes the last pull-in deadhead to the depot of a vehicle. A path from  $o$  to  $s$  represents a block.

Let  $c_{ij}$  be the cost of arc  $(i, j) \in A$ . The binary decision variable  $y_{ij}$  indicates if a vehicle covers trip  $j$  directly after trip  $i$  or not. The QAP model of the SDVSP was given by Paixão and Branco (1987) and Freling et al. (2001) and the mathematical model is as follows:

$$\text{Minimize } \sum_{(i,j) \in A} c_{ij} \cdot y_{ij} \quad (1)$$

subject to,

$$\sum_{j:(i,j) \in A} y_{ij} = 1 \quad \forall i \in T \quad (2)$$

$$\sum_{i:(i,j) \in A} y_{ij} = 1 \quad \forall j \in T \quad (3)$$

$$y_{ij} \in \{0, 1\} \quad \forall (i, j) \in A \quad (4)$$

The objective of the SDVSP, given by (1), is to minimize the cost of vehicle schedule. Constraints (2) and (3) ensure that each trip is assigned to exactly one predecessor and one successor. These constraints preserve a totally unimodular structure for the constraint and hence, the binary conditions on the variables are often relaxed to  $y_{ij} \geq 0$  (Freling et al. (2001)). An auction algorithm for the quasi-assignment problem is proposed by Freling et al. (2001). Their research was primarily motivated for their work on integration of vehicle and crew scheduling (Freling et al. (2003)), where the SDVSP is solved many times to find a solution. The authors test the developed algorithm on instances from bus companies in the Netherlands (RET) and Portugal (CARRIS) that contain up to 1,328 timetabled trips. The algorithm is extremely fast and the instances could be solved in few seconds.

## 2.2. The Multiple Depot Vehicle Scheduling Problem

The multiple depot vehicle scheduling problem (MDVSP) is an extension of the SDVSP, where multiple bus depots are present in the city network. A vehicle schedule must start and end at the same depot and the number of vehicles available at each depot is restricted. The MDVSP is known to be an  $\mathcal{NP}$ -hard problem (Bertossi et al. (1987)). Carpaneto et al. (1989) are the first authors to propose an exact method for the MDVSP. The authors describe a mixed integer programming (MIP) formulation based on an assignment formulation with additional path oriented flow conservation constraints and devise a branch-and-bound (B&B) algorithm to solve it. The literature on the MDVSP is abundant (see the surveys of Desrosiers et al. (1995), Desaulniers and Hickman (2007) and Bunte and Kliewer (2009)). The MDVSP has commonly been formulated as a multi-commodity flow problem (MCF) or a set partitioning problem (SPP).

### *Multicommodity Flow Model*

The MCF model of the MDVSP is described in Bodin et al. (1983) and Ribeiro and Soumis (1994). Let  $K$  be the set of bus depots and  $v_k$  be the maximum number of vehicles available at depot  $k \in K$ .  $G^k = (V^k, A^k)$  denotes the vehicle scheduling network of depot  $k \in K$  and  $c_{ij}^k$  denotes the cost of arc  $(i, j) \in A^k$ . Decision variable  $y_{ij}^k$  indicates if a vehicle from depot  $k \in K$  covers trip  $j$  immediately after trip  $i$  or not. The MCF model is as follows:

$$\text{Minimize } \sum_{k \in K} \sum_{(i,j) \in A^k} c_{ij}^k \cdot y_{ij}^k \quad (5)$$

subject to,

$$\sum_{k \in K} \sum_{j: (i,j) \in A^k} y_{ij}^k = 1 \quad \forall i \in T \quad (6)$$

$$\sum_{j: (j,i) \in A^k} y_{ji}^k - \sum_{j: (i,j) \in A^k} y_{ij}^k = 0 \quad \forall i \in V^k \setminus \{o^k, s^k\}, k \in K \quad (7)$$

$$\sum_{j: (o^k, j) \in A^k} y_{oj}^k \leq v_k \quad \forall k \in K \quad (8)$$

$$y_{ij}^k \in \{0, 1\} \quad \forall (i, j) \in A, k \in K \quad (9)$$

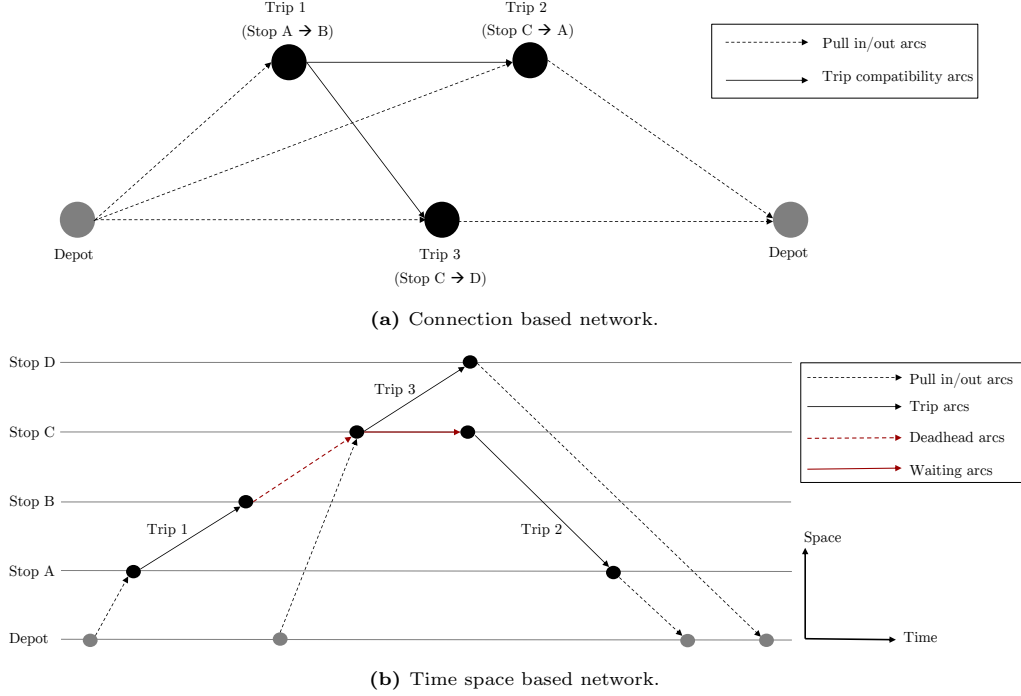
Constraints (6) ensure that each trip is covered exactly once. Flow conservation and depot capacity constraints are given by (7) and (8) respectively.

Forbes et al. (1994) solve the MCF model using a B&B algorithm. Löbel (1998) solve the linear programming (LP) relaxation of the MCF model by column generation method. The authors propose a new technique that is based on Lagrangian relaxations of the MCF model. The method is called Lagrangian pricing that generates arc variables for the master problem of the column generation method. Typically, the underlying network of the MCF model is a connection-based network, where the vertices in the network represent the trips and a pair of trips is connected by an arc if they are compatible with respect to time and space. However, Kliewer et al. (2006) propose a MCF model that is based on a time-space network structure. In the time-space network, each vertex corresponds to a arrival/departure time and arrival/departure bus stop of the trip. The network avoids the drawback of explicit consideration of all possible connections between compatible trips. Figure 3 shows an example to differentiate the connection and time space based networks. Kliewer et al. (2006) apply an aggregation procedure for reducing the number of deadhead arcs without losing any feasible vehicle schedule. The authors used a commercial MIP solver to solve the resulting MCF model. Recently, Kulkarni et al. (2018) propose a new MCF formulation, known as an inventory formulation, to model the MDVSP. In the inventory formulation network, only the arrival times and arrival locations of trips are denoted as vertices. Each compatible pair of trips is connected by a so-called inventory arc. The authors apply a column generation based heuristics to the inventory formulation.

Several heuristic solution approaches have been proposed in the OR literature to solve the MDVSP. One of the first heuristics to be successfully used in practice is the so-called concurrent scheduler that is proposed by Bodin et al. (1978). The concurrent scheduler is designed to be a “greedy” heuristic that considers trips in increasing order of departure time and assigns a trip to an existing vehicle based on minimum deadhead time. If a feasible assignment of a trip to an existing vehicle does not exist, then a new vehicle is created, and the trip is assigned to the new vehicle. Bertossi et al. (1987) propose a Lagrangian heuristic in which the trip covering constraints (6) are relaxed. Lamatsch (1992) develop a Lagrangian heuristic where the flow conservation constraints (7) are relaxed instead. Mesquita and Paixao (1992) present a single-commodity flow (SCF) model with assignment variables for the MDVSP. The set of assignment variables is used to assign a trip  $i \in T$  to depot  $k \in K$ . The authors also propose a heuristic based on Lagrangian relaxation. Dell’Amico et al. (1993) propose a polynomial time heuristic algorithm that guarantees the use of the minimum number of vehicles. The authors first solve a sequence of shortest path problems to build a good quality solution before applying different refinement procedures to improve the solution.

### *Set Partitioning Model*

Ribeiro and Soumis (1994) formulated the MDVSP as a set partitioning problem (SPP) with side constraints. A block is defined to be the schedule of a vehicle and  $B$  denotes the set of all



**Figure 3:** An example to illustrate the difference between a) connection and b) time space based networks.

feasible blocks. The cost of a block  $b \in B$  is denoted  $c_b$ . Binary matrix  $A^1$  is defined, where  $a_{tb}^1$  is equal to 1 if block  $b \in B$  covers trip  $t \in T$  and 0 otherwise. Binary matrix  $A^2$  is defined, where  $a_{kb}^2$  is equal to 1 if block  $b \in B$  belongs to depot  $k \in K$  and 0 otherwise. As previously defined  $v_k$  is the maximum number of vehicles available at depot  $k \in K$ . Binary variable  $y_b$  indicates if block  $b \in B$  is selected as part of the schedule or not. This results in the following model:

$$\text{Minimize } \sum_{b \in B} c_b \cdot y_b \quad (10)$$

subject to,

$$\sum_{b \in B} a_{tb}^1 \cdot y_b = 1 \quad \forall t \in T \quad (11)$$

$$\sum_{b \in B} a_{kb}^2 \cdot y_b \leq v_k \quad \forall k \in K \quad (12)$$

$$y_b \in \{0, 1\} \quad \forall b \in B \quad (13)$$

The objective function, given by (10), is to minimize the total cost. Set partitioning constraints (11) impose that each trip is covered by exactly one vehicle, and constraints (12) ensure that the number of vehicles available per depot is restricted.

Ribeiro and Soumis (1994) propose column generation for solving the LP relaxation of the model (10) - (13), which is the restricted master problem (RMP). A subproblem is defined for every depot that is formulated as a shortest path problem and the authors solve it by dynamic programming. The authors propose a branch-and-price (B&P) method and use depth-first search as the branching strategy. Hadjar et al. (2006) propose a B&B algorithm that combines column generation, variable fixing and cutting planes. Oukil et al. (2007) present a stabilized column generation approach for the MDVSP, which efficiently handles highly degenerate instances. Pepin et al. (2009) compare the performance of five different heuristics for the MDVSP, namely, a truncated branch-and-cut



method, a Lagrangian heuristic, a truncated column generation method, a large neighborhood search (LNS) heuristic and a tabu search (TS) heuristic. In the heuristic branch-and-cut method, a commercial MIP solver is used to solve the MCF formulation of the MDVSP and is terminated when an integer solution is found. The Lagrangian heuristic is similar to that of Lamatsch (1992). In the heuristic version of the column generation method, an early termination criterion is used where the column generation process is halted if the RMP objective value remains unchanged or shows marginal improvement for a certain number of iterations. To find an integer solution, a depth-first branching strategy without backtracking is used and variables  $y_b$  in the RMP that take up fractional values greater than or equal to 0.7 are rounded up to 1 at each node of the B&B tree. For the LNS heuristic, the authors propose to destroy part of the current solution at each iteration and reoptimize using the column generation heuristic. Laurent and Hao (2009) propose an iterated local search (ILS) algorithm for the MDVSP. Marín Moreno et al. (2019) propose a matheuristic that is a combination of a genetic algorithm (GA) and a commercial MIP solver. The authors use the SCF model with assignment variables that was presented by Mesquita and Paixao (1992).

One practical extension of the MDVSP is the multiple vehicle types vehicle scheduling problem (MVT-VSP). The MVT-VSP considers different vehicle types such as standard, double-decker and articulated buses that have different seating capacity, fixed and operational costs. In-vehicle overcrowding or under-utilization of the seating capacity are more likely to occur in a homogeneous fleet system. The application of multiple vehicle types improves the reliability of operating under fluctuating passenger demand and reducing operational cost (Ceder (2011)). The number of buses of each vehicle type is limited and the problem is  $\mathcal{NP}$ -hard for the single depot case (Lenstra and Kan (1981)). Bodin et al. (1983) formulate the MVT-VSP as a MCF model for the single depot case. Let  $H$  be the set of vehicle types and a network is created for each vehicle type  $h \in H$  in the MVT-VSP. For the multiple depot case, a network is created for each depot-vehicle type combination (see e.g. Kliewer et al. (2006) and Gintner et al. (2005)). In practice, the MVT-VSP also includes timetabled trip-vehicle type restrictions, which imply that each trip can be serviced by only a subset of vehicle types (Kliewer et al. (2006) and Ceder (2011)).

Costa et al. (1995) present a MCF and a SPP formulation for the MVT-VSP. The authors find the LP solution for both the formulations and if the solution is not integer then a heuristic procedure is applied to transform the solution into a feasible solution. The authors apply a column generation procedure to find the LP solution of the SPP model. Gintner et al. (2005) consider solving large multiple-depot multiple-vehicle type scheduling problem for real-world applications. The authors apply the time-space network model that was proposed by Kliewer et al. (2006). The authors propose a heuristic approach called as fix-and-optimize that involves fixing some sequences of trips prior to solving the large problem with a commercial MIP solver. Ceder (2011) develops an optimization framework to study the trade-off between service level and cost. Each timetabled trip has a minimum service level requirement that includes characteristics such as degree of comfort, seat availability and other operational features. The authors show that if all the trips are covered by the most luxurious vehicle type with the highest cost then the resulting solution has lower number of buses with a very high cost. However, this solution is used as lower bound on the fleet size by a heuristic procedure that searches for the best solution with minimal cost and satisfies the minimum service requirements for all trips.

Table 1 gives an overview of the literature on the MDVSP. The solution methods used for solving the MDVSP can be categorized into four methods:

1. mixed integer programming (MIP) methods that involve application of B&B methods or a commercial MIP solver such as CPLEX to obtain optimal solutions,
2. column generation (CG) approaches,
3. metaheuristics (MH) and
4. heuristics (H) such as Lagrangian heuristics or a specialized heuristic procedure for the MDVSP.

Notably, Löbel (1998) and Kliewer et al. (2006) succeeded in solving real-world instances from Germany to optimality involving up to 8,563 and 7,068 timetabled trips respectively. The variable fixing heuristic proposed by Gintner et al. (2005) was tested on large instances from Germany

that involved up to 11,062 timetabled trips and 55 depot-vehicle type combinations. The exact approach took 651 minutes to solve the largest instance to optimality. However, the variable fixing heuristic provided a solution with an optimality gap of 0.004% in 303 minutes. Pepin et al. (2009) tested the five heuristics on randomly generated instances with up to 1,500 trips and four depots. It was concluded that the column generation heuristic produces the best quality solutions when sufficient computational time is available (maximum 2,300 seconds) and the LNS combined with column generation heuristic is the best alternative to obtain faster solutions without significantly compromising solution quality.

Authors	Model	Solution Method				Dataset			Remarks
		MIP	CG	MH	H	$ K * H $	$ T $	Test	
Bertossi et al. (1987)	MCF				•	3	50	Random	Lagrangian heuristics
Lamatsch (1992)	MCF				•	2	215	Real-world	Lagrangian heuristics
Dell'Amico et al. (1993)					•	10	1,000	Random	
Forbes et al. (1994)	MCF	•				3	600	Random	B&B algorithm
Ribeiro and Soumis (1994)	SPP		•			6	300	Random	
Costa et al. (1995)	MCF, SPP		•		•	3	63	Real-world	
Löbel (1998)	MCF		•			49	24,906	Germany	Lagrangian pricing
Kliwer et al. (2006)	MCF	•				5	7,068	Germany	Time-space network
Hadjar et al. (2006)	SPP		•			7	2,100	Canada	
Gintner et al. (2008)	MCF				•	55	11,062	Germany	Variable fixing
Pepin et al. (2009)	MCF, SPP		•	•	•	8	1,500	Random	Lagrangian heuristics, TS, LNS
Laurent and Hao (2009)					•	8	1,500	Random	ILS
Kulkarni et al. (2018)	MCF		•			16	3,000	Random	Inventory formulation
Marín Moreno et al. (2019)	SCF				•	2	719	Colombia	GA

**Table 1:** Overview of the literature on the MDVSP. **Model:** MCF-multi-commodity flow problem, SCF-single-commodity flow problem and SPP-set partitioning problem. **Solution method:** MIP-mixed integer programming methods, CG-column generation, MH-metaheuristics and H-heuristics. **Dataset:**  $|K|*|H|$ -number of depot-vehicle type combinations,  $|T|$ -number of timetabled trips and Test-random or real-world instances. Other abbreviations: B&B-branch-and-bound, TS-tabu search, LNS-large neighborhood search, ILS-iterated local search and GA-genetic algorithm.

### 2.3. The Electric Vehicle Scheduling Problem

Cities around the world are moving to alternative-fuel vehicles such as electric, hydrogen-gas and bio-fuel based vehicles in order to create a fossil-free bus transport system (Adler and Mirchandani (2017) and Xylia et al. (2017)). Particularly, the electric bus technologies have been gaining popularity in recent years (Li (2016)). The use of electric buses requires special charging facilities which have to be accommodated into the current infrastructure. The different charging technologies are:

1. slow plug-in chargers installed at bus depots,
2. fast plug-in or pantograph chargers installed at terminals of bus lines or at bus stops,
3. overhead contact lines or inductive (wireless) chargers that are used to recharge buses during driving, and
4. battery swapping.

See Li (2016), Chen et al. (2018) and Häll et al. (2019) for a detailed description of the different charging technologies. The installation cost and the charging power in kilowatts (kW) of the different charging technologies are known to vary. Depot plug-in chargers have a low installation cost and a low charging power, whereas pantograph chargers have a high installation cost and a high charging power. Chen et al. (2018) investigate the cost competitiveness of different types of charging infrastructure such as charging stations at terminals, charging lanes (inductive chargers) and battery swapping. The authors also determine the optimal size of the electric bus fleet as well as their battery capacity, which is measured in terms of kilowatt-hour (kWh). Electric buses with large battery packages are known to be more expensive, but have a longer driving range. Therefore, Chen et al. (2018) aim to minimize the total costs, which consists of the infrastructure

cost (i.e. deploying cost of charging facilities ) and the investment cost in the bus fleet. The total investment cost and the operational cost within a defined time period is referred to as total cost of ownership (TCO) (Pihlatie et al. (2014) and Rogge et al. (2018)). Pelletier et al. (2019) present an electric bus fleet transition problem that determines bus replacement plans for transport companies to meet their electrification targets in a cost-effective way. Given different electric bus types and charger types, the problem considers investment decisions such as the number of buses per bus type and the number of chargers per charger type to purchase during the years 2020-2050. The battery capacity of buses varies from 110 to 650 kWh. A 110 kWh electric bus is estimated to have a driving range of 90 km and a 650 kWh electric bus has a driving range of approximately 370 km. The authors assume that it takes approximately 13 hours to fully recharge a 650 kWh electric bus with a 50 kW depot charger. Xylia et al. (2017) primarily study the strategic problem of optimizing the distribution of the charging infrastructure for the electric buses in the city network. Similarly, Kunith et al. (2017) aim to determine the cost-effective placement of chargers and the battery capacity of buses. In this paper, we focus on the electric vehicle scheduling problem (E-VSP), which is an extension of the VSP. The E-VSP is concerned with assigning electric buses to a set of timetabled trips while satisfying their driving range and recharging requirements. It is a tactical problem where the charging infrastructure and the battery capacity of electric buses for different types are given. The objective of the E-VSP is to minimize the total operational cost that is comprised of fixed cost per vehicle and variable cost, which includes energy cost per km (see e.g. Li (2013) and Adler and Mirchandani (2017)). Table 2 categorizes the different problems related to the electric bus technology and the categories are:

1. investment of charging infrastructure - deciding on the charging facility (such as plug-in, battery swapping) to deploy and/or the number of chargers to purchase.
2. placement of charging infrastructure - cost effective placement of chargers in depots, terminals or bus stops in the city network.
3. investment of bus fleet - the number of vehicles to purchase and their battery capacities.
4. electric vehicle scheduling - assigning electric buses to a set of timetabled trips.

Table 2 also shows the different charging infrastructures used in the literature. The plug-in chargers, pantograph chargers and overhead contact lines are conductive chargers while the wireless charger is the inductive charger. Some authors that primarily tackle the E-VSP, utilize the charging stations located in the city network for recharging the vehicles and do not indicate the specific charging technology. It is assumed that a charging station has plug-in or pantograph chargers installed. Depot charging and terminal charging are the two most common facilities used in the literature.

A similar extension to the VSP is the vehicle scheduling problem with route constraints (VSP-RC), where a maximum route time constraint is present to ensure that the total time a vehicle is away from the depot is no more than a specified threshold. Bodin et al. (1983) show that any resource constrained VSP is  $\mathcal{NP}$ -hard. Haghani and Banihashemi (2002) propose an exact approach for solving the multiple-depot VSP-RC. The approach iteratively solves the MCF to optimality and adds the violated route time constraints to the model. The authors also propose heuristic procedures that considers some of the steps from the exact approach. Wang and Shen (2007) consider refueling time constraints for the VSP-RC with the focus on electric vehicles. The maximum range of an electric vehicle is set to 420 minutes and the recharging time is 180 minutes. The authors propose an ant colony optimization (ACO) procedure to solve the VSP-RC with refueling time constraints.

Li (2013) addresses the single-depot VSP for electric buses with battery swapping or fast charging at given battery stations. The authors assume that there exists one battery service station located at the depot and only a certain number of vehicles can be serviced at a time. Additionally, the battery service time is assumed to be 10 minutes and the maximum driving range of the electric buses is 150 km. The authors present an arc formulation of the problem that consists of maximum distance before recharging or battery renewal constraints. The arc model is solved using a commercial MIP solver. The authors also reformulate the problem as a SPP or a path-based model. The SPP model is solved by a column generation method and a variable fixing strategy is used to find integer solutions. Adler and Mirchandani (2017) present an alternative-fuel MDVSP, where

Authors	Problem				Charging infrastructure
	Investment of charging infrastructure	Placement of charging infrastructure	Investment of bus fleet	Electric vehicle scheduling	
Li (2013)				•	Fast-charging and battery swapping
Wen et al. (2016)				•	Charging stations
Adler and Mirchandani (2017)				•	Charging stations
Van Kooten Niekerk et al. (2017)				•	Charging stations
Kunith et al. (2017)		•	•		Fast-charging
Xylia et al. (2017)	•	•	•		Conductive and inductive chargers
Chen et al. (2018)	•		•		Charging stations, inductive chargers and battery swapping
Rogge et al. (2018)	•		•	•	Depot charging
Pelletier et al. (2019)	•		•		Depot plug-in, fast plug-in pantograph and wireless chargers
Yao et al. (2020)			•	•	Depot charging
Liu and Ceder (2020)	•			•	Fast-charging

**Table 2:** Categorization of the different problems related to electric bus technology.

other alternative-fuel vehicles such as hydrogen-gas vehicles and biofuel based vehicles that have limited driving ranges are considered. The authors assume that the buses have a range of 120 km before needing to be refueled and the refueling time is considered to be 10 minutes. An exact B&P algorithm and a heuristic that is based on a concurrent scheduler algorithm (Bodin et al. (1978)) are proposed to solve the problem. Reuer et al. (2015) consider a mixed fleet composed of conventional diesel and electric buses. The authors extend the time-space network approach proposed by Kliewer et al. (2012) to solve the standard VSP and develop an algorithm that identifies when a vehicle needs to be recharged. In this manner, the authors estimate a bound on the maximum number of electric buses in a mixed fleet. The authors also devise heuristic procedures based on the ideas of Adler and Mirchandani (2017) to find feasible solutions for the E-VSP that only uses electric buses. The authors set the battery capacity of buses to 120 kWh and the energy consumption of buses to 1 kWh per km on timetabled trips and 0.8 kWh per km on deadheads. The charging stations are at terminals that are visited frequently and the charging time is considered to be 10 minutes. Wen et al. (2016) propose an adaptive large neighborhood search (ALNS) heuristic for solving the E-VSP. The driving range of the electric buses is set to 150 km and the authors assume that it takes two hours to fully recharge a vehicle. However, the authors allow the vehicles to be partially recharged as well and the recharging time is assumed to be a linear function, which is proportional to the amount of battery charged. Van Kooten Niekerk et al. (2017) state that the price of electricity significantly varies over the day and, in practice, the cost is dependent on the time when the electricity is taken from the grid. The charging stations are most likely to be at the depots or terminals of lines and each charging station has a certain space capacity that determines the number of vehicles that can be charged simultaneously. The charging station also has an energy capacity, and larger capacities imply that the electric vehicles can be charged faster. The electric buses have a battery capacity of 244 kWh and an energy consumption of 1.2 kWh per km. The charging speed is considered to be 2.0 kWh per minute. The authors propose two models to solve the E-VSP. The first model is a MIP model with continuous variables for battery charge. For every trip, an extra variable is assigned that keeps track of the charge at the start of a trip. The model considers only linear charging behaviour of the batteries and a constant price of electricity during the day. The second model allows for non-linear charging behaviour of the batteries and takes the actual electricity prices during the day into account. The second model is also reformulated as a SPP so that it can be solved by a column generation method. The authors describe three solution methods for the second model; namely, a MIP solver and column generation heuristics that are based on LP and Lagrangian relaxations.

Rogge et al. (2018) focus on strategic and tactical challenges in electric bus planning and aim to minimize the TCO of electric vehicle fleets. For a given set of timetabled trips and vehicle types, the problem determines the vehicle schedule to serve all trips and the number of vehicles to buy per vehicle type. The charging infrastructure is considered to be installed at the depot and the problem also focuses on the number of chargers to buy per depot. Two vehicle types are considered; one with a battery capacity of 90 kWh and another with a battery capacity of 380 kWh. Additionally, the energy consumption of the different vehicle types vary; the vehicle type with smaller capacity uses 0.5 kWh per km and the large vehicle type uses 0.9 kWh per km. A GA in combination with MIP formulation is proposed by the authors to solve the problem. Yao et al. (2020) also consider the E-VSP with multiple vehicle types that differ in driving range and recharging duration. The authors consider two vehicle types; one that has a driving range of 170 km and the other has a driving range of 120 km. Depot charging is considered and the recharging duration is considered to be 51 and 30 minutes for vehicle type 1 and 2, respectively. The authors also propose a GA to minimize the total cost, which includes the purchasing cost of electric buses and chargers, and the operating costs of deadheads and timetabled trips. Liu and Ceder (2020) present a bi-objective MIP model for the E-VSP; the first objective is to minimize the total number of electric vehicles required and the second objective is to minimize the number of chargers required. The battery capacity of the buses is 100 kWh. The chargers are located at the terminals and the charging power is assumed to be 50 kW. The authors use full and partial charging strategies and consider a non-linear battery charging behaviour. The authors propose a two-stage construction-and-optimization solution procedure and an adjusted max-flow solution method.

Perumal et al. (2020a) consider the integrated electric vehicle and crew scheduling problem which will be discussed in Section 4. The authors consider the driving range of the bus to be 120 km. Depot charging is considered and the recharging duration of buses is set to two hours. For solving the E-VSP, the authors implement the B&P heuristics that fixes variables that have fractional values greater than or equal to a threshold value (in this case 0.8) to one at each node of the B&B tree. If there are no such variables, then the variable with a fractional value closest to one is selected and fixed to one. The B&B tree is explored in a depth-first manner without backtracking. This procedure has been commonly used to solve the VSP and its extensions (Pepin et al. (2009), Li (2013) and Van Kooten Niekerk et al. (2017)).

Table 3 gives an overview of the constraints considered in the E-VSP. The limited driving range of the buses and the recharging duration are the most critical constraints in the E-VSP. Furthermore, Table 4 gives an overview of the literature on the E-VSP. Li (2013) compares the performance of an arc model that is solved by a MIP solver and an heuristic column generation method. For the large instances with 947 timetabled trips, the LP relaxation of the arc model is not solved to optimality by the MIP solver in 12 hours. The column generation based method provided solutions that have an average optimality gap of 7%, and the average computation time was 72 hours. Adler and Mirchandani (2017) test an exact B&P algorithm only on subsets of the original data, which contained 4,373 timetabled trips. The subsets of the data had up to 72 trips, eight refuelling stations and four depots. The B&P algorithm took between two and 12 hours of computation time to solve the small instances. In comparison, the heuristic that is based on the concurrent scheduler algorithm took less than a second, but the average optimality gap was 11.80%. Van Kooten Niekerk et al. (2017) use MIP models to solve only the small instances that had up to 241 timetabled trips, and column generation based methods are used to solve the larger instances.

### 3. The Crew Scheduling Problem

The CSP in the bus industry is also referred to as the driver scheduling problem (DSP) in the literature and is similar to the crew problems that arise in other industries such as airline and railway. However, in contrast to the bus industry, the crew cost for railway companies does not account for a relatively high share of the total operational cost (Heil et al. (2019)). Therefore, the cost structure does not necessitate the need for an integrated planning approach and hence, the integration of

Authors	Multiple depots	Multiple vehicle types	Number of chargers	Recharging duration	Driving range of buses	Non-linear battery behaviour	Partial charging
Haghani and Banihashemi (2002)	•					•	
Wang and Shen (2007)	•			•	•		
Li (2013)			•	•	•		
Reuer et al. (2015)		•		•	•		
Wen et al. (2016)	•			•	•		•
Adler and Mirchandani (2017)	•			•	•		
Van Kooten Niekerk et al. (2017)			•	•	•	•	•
Rogge et al. (2018)		•		•	•		
Yao et al. (2020)	•	•		•	•		
Liu and Ceder (2020)	•			•	•	•	•
Perumal et al. (2020a)				•	•		

**Table 3:** Overview of the constraints in the E-VSP.

Authors	Solution Method				Number of recharging stations	Dataset			Remark
	MIP	CG	MH	H		K	T	Test	
Haghani and Banihashemi (2002)				•	4	5,650	USA		
Wang and Shen (2007)			•		3	3	261	Random	ACO
Li (2013)	•	•			1	1	947	USA	Compares MIP model and CG
Reuer et al. (2015)				•	1	26	10,702	Real-world	
Adler and Mirchandani (2017)		•		•	8	4	4,373	USA	
Wen et al. (2016)			•		16	8	500	Random	ALNS
Van Kooten Niekerk et al. (2017)	•	•			4	1	543	Belgium	Compares MIP model and CG
Rogge et al. (2018)			•		1	1	200	Germany, Denmark	GA
Yao et al. (2020)			•		2	2	931	China	GA
Liu and Ceder (2020)				•	3	3	272	Singapore	
Perumal et al. (2020a)		•			1	1	1,109	Denmark, Sweden	

**Table 4:** Overview of the literature on the E-VSP. **Solution method:** MIP-mixed integer programming methods, CG-column generation, MH-metaheuristics and H-heuristics. **Dataset:** |K|-number of depots, |T|-number of timetabled trips and Test-random or real-world instances. Other abbreviations: ACO-ant colony optimization, GA-genetic algorithm and ALNS-adaptive large neighborhood search.

vehicle and crew scheduling problems in railway operations has not been comprehensively studied. For a detailed literature survey on the railway crew scheduling problem, see Heil et al. (2019).

The CSP is the third step in the tactical planning stage (see Figure 1), where a set of bus trips is given. The set of bus trips or tasks includes the set of timetabled trips and deadheads performed by the vehicles. As mentioned earlier, the schedule of a driver for a day is known as a duty. The feasibility of a duty is influenced by various labor rules and regulations that govern the working conditions of the drivers. A cost is associated with each duty and, in most cases, the wages paid to the driver are used as an estimate for the cost. Given a set of bus trips, the CSP is concerned with finding an optimal set of duties that covers all bus trips with minimal cost and satisfies all the labor regulations.

### 3.1. Labor Regulations

A public bus company is often associated with different labor unions that impose varying rules and regulations. Most regulations are concerned with the working period of the drivers and ensure that the drivers receive a sufficient number of breaks. The following are the most common regulations that are found in the literature and apply to most bus companies:

- **Maximum duration**

Duration of a duty is defined as the period of time between the start and end of a driver’s duty. The duration of a driver’s duty can never exceed a certain limit (see e.g. Desrochers and Soumis (1989), Fores et al. (2002) and Yunes et al. (2005)). In most cases, there are also limitations on the total driving time. For example, Fores et al. (2002) considers the maximum duration of a duty to be 12 hours and the maximum driving time to be nine hours.

- **Minimum break duration**

This specifies that breaks must be at least a certain duration. Fores et al. (2002) and Yunes et al. (2005) set the minimum break duration as 30 minutes. In some cases, there are also restrictions on the maximum duration of the break. For example, Chen (2013) considers the maximum duration of a break to be 150 minutes.

- **Maximum time without break**

The duration between breaks cannot exceed a certain limit. This rule ensures that a driver does not drive a bus for a prolonged period without a break. Fores et al. (2002) considers the maximum duration without a break to be five hours.

- **Maximum number of pieces of work**

A driver may cover only a few consecutive bus trips before the driver takes a break or is relieved of duty. A *piece of work* is a feasible sequence of consecutive trips of a single bus that can be covered by a driver (Freling et al. (2003) and Kliever et al. (2012)). A duty is typically composed of pieces of work that are separated by breaks. In practice, a duty consists of two-three pieces of work. Desrochers and Soumis (1989) set the maximum number of pieces of work as three and Freling et al. (2003) considers a maximum of two pieces of work. In most cases, the maximum duration of a piece of work is regarded as the maximum time without break. For example, Desrochers and Soumis (1989) consider the maximum duration of a piece of work to be six hours and Chen (2013) considers it to be four hours. A duty with two pieces of work essentially denotes one bus or vehicle change for the driver. Some authors (see e.g. Portugal et al. (2009)) have also used a rule to restrict the number of vehicle changes that a driver can make during his/her duty.

- **Multiple duty types**

A duty can be categorized into one of several types depending on its characteristics. The most common categorization is based on the starting times of the duties during the day such as an early or a late duty (see e.g. Freling et al. (2003) and Li et al. (2015)). Therefore, the CSP considers multiple duty types and the rules often vary for each duty type.

- **Multiple driver depots**

Some authors (see e.g. Boschetti et al. (2004), Abbink et al. (2005) and Perumal et al. (2020b)) have considered multiple driver depots, where drivers are allowed to start their respective duties from any of the given depots. However, a driver is required to end his/her duty at the same depot where the duty was started.

In some cases, some of the above rules such as the maximum duration and maximum time without break have been considered as a “soft” rule that is allowed to be violated (see e.g. Desrochers and Soumis (1989) and Portugal et al. (2009)). However, an additional cost, such as an overtime rate, is usually included to discourage this. The various break rules enforce drivers to travel between bus stops and depots in the city network to take a break or start/end their respective duties. The travel or deadheading activities of the drivers are essential for creating feasible duties. Some examples include travel by foot (Wren et al. (2003)) and taxis (Abbink et al. (2011) and Potthoff et al. (2010)). Perumal et al. (2019) introduced the CSP with staff cars, where drivers could use company-owned cars as part of their travel activities. However, in most case, only the travel times between specific locations are taken into account for scheduling the drivers and the mode of transport is not considered (see e.g. Smith and Wren (1988), Desrochers and Soumis (1989) and Boschetti et al. (2004)).

### 3.2. Mathematical Model

Common formulations of the CSP are based on the SPP or the set covering problem (SCP), where the formulation is used as a duty selection module with the selected duties covering all bus trips at minimum cost (Ibarra-Rojas et al. (2015)). Let  $S$  be the set of all bus trips that includes the timetabled trips and the deadheads performed by the vehicles. Let  $D$  be the set of all feasible

duties and the cost of a duty  $d \in D$  is represented as  $c_d$ . Binary matrix  $A^3$  is defined, where  $a_{sd}^3$  is equal to 1 if duty  $d \in D$  covers bus trip  $s \in S$  and is 0 otherwise. Let  $L$  be the set of duty types and the CSP includes constraints on the allowed number of duties per duty type (see e.g. Desrochers and Soumis (1989) and Perumal et al. (2020b)). Let  $w_l$  be the maximum number of duties of duty type  $l \in L$ . Binary matrix  $A^4$  is defined, where  $a_{ld}^4$  is equal to 1 if duty  $d \in D$  is of duty type  $l \in L$ . Binary variables  $x_d$  indicate if duty  $d \in D$  is selected as part of the schedule or not. The SPP model of the CSP is given as:

$$\text{Minimize } \sum_{d \in D} c_d \cdot x_d \quad (14)$$

subject to,

$$\sum_{d \in D} a_{sd}^3 \cdot x_d = 1 \quad \forall s \in S \quad (15)$$

$$\sum_{d \in D} a_{ld}^4 \cdot x_d \leq w_l \quad \forall l \in L \quad (16)$$

$$x_d \in \{0, 1\} \quad \forall d \in D \quad (17)$$

The objective of the CSP, given by (14), is to minimize the total cost of duties. Constraints (15) ensure that each bus trip is covered by exactly one duty. In a SCP model, the equality signs in (15) are replaced by “ $\geq$ ” signs, which indicate that drivers are allowed to travel in the city network as a passenger on the bus. Constraints (16) ensure that maximum allowed number of duties per duty type is not violated. Lourenço et al. (2001) consider multiple objective functions to the problem such as minimizing number of duties and minimizing number of duties with only one piece of work. The authors also consider minimizing the number of vehicle changes since a change of the driver responsible for a vehicle can cause disruptions to a company’s operation.

To find the optimal solution, all the feasible duties have to be considered in the SCP formulation. However, the formulation is intractable by exhaustive enumeration techniques because of the large number of feasible duties for all reasonably sized instances (200 bus trips or more). Some authors, e.g. Smith and Wren (1988) and Wren et al. (2003), heuristically generate a feasible subset of duties for the SCP and solve it by a specialized B&B algorithm or a commercial MIP solver. Wren et al. (2003) focus more on combining theory and practice to create a user-friendly and flexible system. Similarly, Portugal et al. (2009) present SPP based models that are solved by a MIP solver and aim to implement the models as part of a planning system. Some authors consider solving the SCP by metaheuristic procedures such as genetic algorithms (GA) and tabu search (TS) (Lourenço et al. (2001) and Li and Kwan (2003)). Caprara et al. (1999) present a Lagrangian-based heuristic for solving the SCP. The algorithm was designed to solve large-scale crew SCP instances arising from crew scheduling in an Italian railway company.

Desrochers and Soumis (1989) were the first authors to propose a column generation approach, more precisely a B&P method, for the CSP. The subproblem of the column generation method is modelled as a shortest path problem with resource constraints (SPPRC) that is solved by a dynamic programming approach. The authors use a branching rule that is similar to the one developed by Ryan and Foster (1981). The branching rule identifies a pair of bus trips  $(s_1, s_2)$  for which  $0 < \sum_{d \in D(s_1, s_2)} x_d < 1$ , where  $D(s_1, s_2)$  is the set of all duties that cover trips  $s_1$  and  $s_2$  simultaneously. Thereafter, an effective constraint branching is implemented that creates the following two branches:

$$\text{1-branch: } \sum_{d \in D(s_1, s_2)} x_d \geq 1 \quad (18)$$

$$\text{0-branch: } \sum_{d \in D(s_1, s_2)} x_d \leq 0 \quad (19)$$



Solving large instances of the CSP by column generation approaches have been reported in the literature as being computationally expensive due to the need to solve SPPRC at every iteration (Wren et al. (2003), Yunes et al. (2005) and Ibarra-Rojas et al. (2015)). Yunes et al. (2005) compare a constraint programming (CP) approach and a dynamic programming technique, which was suggested by Desrochers and Soumis (1989), to solve the SPPRC. It was reported that solving the SPPRC by CP in a column generation setting outperforms that of the dynamic programming in terms of computation time. Steinzen (2007) and Kliewer et al. (2012) tackle the CSP as part of the integrated VSP and CSP (VCSP) and propose acceleration and heuristic procedures for solving the SPPRC by dynamic programming. Freling et al. (2003) and Huisman et al. (2005) use the pieces of work to model the subproblem of the CSP for solving the VCSP. A piece of work can be viewed as a partial duty and the authors enumerate all pieces of work for a given vehicle schedule.

Fores et al. (2002) do not solve the SPPRC to generate duties, but create a pregenerated duty set with a large number of feasible duties (around 1.5 million duties for the largest instance with 1,500 bus trips) and then apply column generation to evaluate the pregenerated feasible duties at each iteration. Chen (2013) propose a similar approach, but solve the SPPRC when there are no more duties in the pregenerated duty set that could improve the solution objective. Several metaheuristic approaches have been proposed in the literature to solve the subproblem. Mauri and Lorena (2007) devise a GA to solve the SPPRC. Similarly, Dos Santos and Mateus (2009) initially use a GA-based approach to solve the SPPRC and then later shift to an exact method for ensuring optimality. Li et al. (2015) generate all feasible variables for a given instance and several heuristics (local search, swap heuristic and greedy based heuristic) are devised to select a subset of feasible variables at each iteration of the column generation framework. For an instance with 500 trips, the number of feasible duties was found to be around 8.3 million. Kecskeméti and Bilics (2013) present a hybrid of column generation and evolutionary algorithm (EA). The EA is used to solve large-scale set covering problems and it is re-run after a number of new columns are generated to find a feasible solution quickly. Boschetti et al. (2004) propose a method that is based on Lagrangian relaxation and column generation. This method has commonly been used for solving the CSP in the railway industry (e.g. Abbink et al. (2005) and Abbink et al. (2011)) and for solving the CSP in the context of the VCSP (see e.g. Freling et al. (2003)), which will be briefly described in Section 4. Perumal et al. (2019, 2020b) tackle the CSP with staff cars, which requires additional constraints in the mathematical model. The authors propose an ALNS that utilizes a commercial MIP solver. Additionally, the authors explore heuristic approaches based on column generation for solving the CSP with staff cars.

Table 5 gives an overview of the literature on the CSP. The solution methods used for solving the CSP can be categorized into three methods: 1) column generation (CG) approaches, 2) metaheuristics (MH) that are primarily devised to solve large-scale SCP and 3) heuristics (H) that focus on generating a large subset of feasible duties for a SCP model, which is then solved by a commercial MIP solver. In conclusion, most of the research carried out on the CSP has been motivated by real-world applications where bus companies have to handle many labor regulations. Furthermore, most of the solution methods proposed in the literature have been developed to be part of commercial decision support tools (see e.g. Desrochers and Soumis (1989), Lourenço et al. (2001), Fores et al. (2002), Wren et al. (2003) and Portugal et al. (2009)).

#### 4. The Integrated Vehicle and Crew Scheduling Problem

Given a set of timetabled trips, the integrated vehicle and crew scheduling problem (VCSP) aims to find a minimum cost schedule for the vehicles and the crews such that both the vehicle and crew schedules are feasible and mutually compatible. The crew schedule has to be feasible with respect to the labor regulations that were briefly discussed in Section 3.1. In addition to assigning the timetabled trips to a vehicle and a driver, any deadheads in the vehicle schedule need to be assigned a driver. Furthermore, in most cases, a *continuous attendance* is required, i.e. there is always a driver present when the vehicle is outside the depot (Freling et al. (2003) and Huisman et al. (2005)). The solution methods in the literature for tackling the VCSP fall into one of the following three categories (Freling et al. (2003)):

Authors	Solution Method			Dataset		Remark
	CG	MH	H	S	Test	
Smith and Wren (1988)			•	309	UK	B&B
Desrochers and Soumis (1989)	•			235	USA and UK	
Lourenço et al. (2001)		•		348	Portugal	TS and GA
Fores et al. (2002)	•			1,500	UK	
Wren et al. (2003)			•	1,274	UK	B&B
Li and Kwan (2003)		•		1,873	UK	GA
Boschetti et al. (2004)	•			580	Real-world	
Yunes et al. (2005)	•			210	Brazil	CP
Mauri and Lorena (2007)	•			500	Real-world	GA
Dos Santos and Mateus (2009)	•			101	Brazil	GA
Chen (2013)	•			701	China	
Kecskeméti and Bilics (2013)	•	•		366	Real-world	EA
Li et al. (2015)	•			265	China	Hyper-heuristics
Perumal et al. (2019, 2020b)	•	•		1,926	Northern Europe	ALNS

**Table 5:** Overview of literature on the CSP. **Solution method:** CG-column generation, MH-metaheuristics and H-heuristics. **Dataset:** |S|-number of bus trips and Test-random or real-world instances. Other abbreviations: B&B-branch-and-bound, TS-tabu search, GA-genetic algorithm, CP-constrained programming, CP-constrained programming, CP-constrained programming, CP-constrained programming, EA-evolutionary algorithm and ALNS-adaptive large neighborhood search.

1. Inclusion of vehicle considerations in the crew scheduling problem and the vehicle scheduling is carried out afterwards (crew first - vehicle second).
2. Inclusion of crew considerations in the vehicle scheduling problem and the crew scheduling is carried out afterwards (vehicle first - crew second).
3. Complete integration of vehicle and crew scheduling

The first and second categories are recognized as partial integration methods of the vehicle and crew scheduling problems. Ball et al. (1983) proposes a heuristic procedure that emphasizes on the crew scheduling aspects and the vehicle schedule is derived from the crew solution. Similar heuristic procedures that fall under the first category are proposed by Falkner and Ryan (1987), and Patrikalakis and Xerocostas (1992). Scott (1985) proposes a heuristic procedure of the second category to determine the vehicle schedules that take the crew cost into account. Similarly, Darby-Dowman (1988) consider the crew scheduling aspects during the determination of the vehicle schedules in an integrated decision support system. One interesting approach that belongs to neither of the partial integration categories is proposed by Gintner et al. (2008). The authors consider a set of optimal vehicle schedules to find the best crew schedule. To the best of our knowledge, there has not been any partial integration methods proposed in the OR literature in recent years and the focus has been more on the third category which corresponds to the complete integration methods.

#### 4.1. Mathematical Model

In this section, the mathematical model presented by Friberg and Haase (1999) for the complete integration of vehicle and crew scheduling is described. The authors present a formulation of the VCSP that combines the approaches of Desrochers and Soumis (1989) and Ribeiro and Soumis (1994) for solving the CSP and VSP, respectively. The VCSP is formulated as a SPP with additional constraints that link the crew and vehicle schedules. A linking constraint ensures that a deadhead is covered by a crew member only if it is covered by a vehicle. Table 6 gives the descriptions of the notations in the VCSP.

The mathematical model for the VCSP is as follows:

$$\text{Minimize } \sum_{b \in B} c_b^1 \cdot y_b + \sum_{d \in D} c_d^2 \cdot x_d \quad (20)$$

Notation	Description
$T$	Set of timetabled trips.
$F$	Set of all deadheads.
$B$	Set of all blocks.
$D$	Set of all feasible duties.
$c_b^1$	Cost of block $b \in B$ .
$c_d^2$	Cost of duty $d \in D$ .
$A^1$	Binary matrix, where $a_{tb}^1$ is 1 if block $b \in B$ covers trip $t \in T$ and is 0 otherwise.
$A^3$	Binary matrix, where $a_{td}^3$ is 1 if duty $d \in D$ covers trip $t \in T$ and is 0 otherwise.
$A^5$	Binary matrix, where $a_{fb}^5$ is 1 if block $b \in B$ contains deadhead $f \in F$ and is 0 otherwise.
$A^6$	Binary matrix, where $a_{fd}^6$ is 1 if duty $d \in D$ contains deadhead $f \in F$ and is 0 otherwise.
$y_b$	A binary variable that indicates if block $b \in B$ is selected as part of the schedule or not.
$x_d$	A binary variable that indicates if duty $d \in D$ is selected as part of the schedule or not.

**Table 6:** Descriptions of the notations in the VCSP.

subject to,

$$\sum_{b \in B} a_{tb}^1 \cdot y_b = 1 \quad \forall t \in T \quad (21)$$

$$\sum_{d \in D} a_{td}^3 \cdot x_d = 1 \quad \forall t \in T \quad (22)$$

$$\sum_{d \in D} a_{fd}^6 \cdot x_d - \sum_{b \in B} a_{fb}^5 \cdot y_b = 0 \quad \forall f \in F \quad (23)$$

$$y_b \in \{0, 1\} \quad \forall b \in B \quad (24)$$

$$x_d \in \{0, 1\} \quad \forall d \in D \quad (25)$$

The objective of the VCSP, given by (20), is to minimize the total cost of blocks and duties. Constraints (21) and (22) ensure that each timetabled trip is assigned to a vehicle and a driver respectively. Constraints (23) are the deadhead linking constraints.

Friberg and Haase (1999) propose the first exact algorithm for the single depot case of the VCSP that uses the mathematical model (20)-(25). A B&P method for obtaining optimal solutions is proposed and a column generation procedure is performed to generate both vehicle and crew variables. Haase et al. (2001) present another SPP model with side constraints that only involves crew variables for the single depot VCSP. Inclusion of vehicle cost and the side constraints ensure that an overall optimal solution is found after deriving a compatible vehicle schedule. The authors also propose an exact B&P approach for solving the single depot case of the VCSP and use the Ryan and Foster (1981) for attaining integer solutions. To solve larger instances, the authors develop a heuristic procedure that explores the B&B tree using a depth-first procedure without backtracking. Additionally, several acceleration strategies such as omission of redundant constraints in the master problem, dynamic generation of bus count constraints and substitution of partitioning constraints are utilized to speed up the solution process. Freling et al. (2003) present a mathematical formulation for the single-depot VCSP that is a combination of the QAP formulation (described in Section 2.1) for the VSP and the SPP/SCP formulation for the CSP. The authors propose a column generation procedure based on Lagrangian relaxation. Column generation is generally applied in the context of LP; however, the authors state that the number of constraints in the VCSP model make the LP relaxation an unrealistic option. Lagrangian relaxation is used to relax the constraints related to the crew in the VCSP model and Lagrangian multipliers are associated with each of the relaxed constraint. The column generation procedure only involves generating crew variables and the Lagrangian subproblem involving the single depot VSP is solved using the auction algorithm

(Freling et al. (2001)). The authors use subgradient optimization to solve the Lagrangian dual problem approximately. Furthermore, the columns that are generated to compute the lower bound are used to construct a feasible solution either by applying the heuristics of Caprara et al. (1999) or by using a commercial MIP solver.

For the multiple depot VCSP, the mathematical formulation presented in the literature involves the MCF formulation (described in Section 2.2) for the VSP and the SPP/SCP formulation for the CSP (Huisman et al. (2005), Borndörfer et al. (2008), Mesquita and Paias (2008) and Steinzen et al. (2010)). Gaffi and Nonato (1999) introduce the multiple depot vehicle and crew scheduling problem (MD-VCSP). The mathematical formulation is similar to that of Freling et al. (2003), and the authors propose a solution approach that is based on Lagrangian relaxation with column generation. The authors assume that a driver is assigned to one vehicle for the whole planning period and all pieces of work start and end at a depot. This assumption is considered for a particular application, and does not hold in general for most applications. It has been argued in the literature that this assumption simplifies the problem and makes it computationally more tractable (Huisman et al. (2005) and Steinzen et al. (2010)). Huisman et al. (2005) extend the work of Freling et al. (2003) and the solution approach is applied to the MD-VCSP. Borndörfer et al. (2008) propose a similar method to that of Freling et al. (2003) and Huisman et al. (2005) to integrate the vehicle and driver scheduling problems. However, the authors use bundle techniques for the solution of the Lagrangian relaxations. The authors state that the advantages of the bundle method are that it provides high quality bounds and automatically generates primal information. Steinzen et al. (2010) also use column generation in combination with Lagrangian relaxation. However, the authors use a time-space network to represent the underlying network of the vehicle scheduling problem.

Mesquita and Paias (2008) solve the LP relaxation of the MD-VCSP model using column generation. If the resulting solution is not integer, then the authors use a B&B procedure over the set of feasible crew duties that was generated while solving the LP relaxation to obtain a feasible integer solution. Mesquita et al. (2009) extend the work of Mesquita and Paias (2008) by proposing different branching strategies for a B&P method. The authors use the MD-VCSP model that involves the MCF formulation for the VSP and SPP/SCP formulation for the CSP. As described in Section 2.2, the vehicle scheduling network is denoted as  $G^k = (V^k, A^k)$ , where  $V^k$  is the set of vertices and  $A^k$  is the set of arcs for depot  $k \in K$ . The authors perform branching by identifying an arc  $(i, j) \in A^k$  for which the sum of vehicle variables is not integer, i.e.  $0 < \sum_{k \in K} y_{ij}^k < 1$ . The following two branches are created:

$$\text{1-branch: } \sum_{k \in K} y_{ij}^k = 1 \quad (26)$$

$$\text{0-branch: } \sum_{k \in K} y_{ij}^k = 0 \quad (27)$$

Constraints (26) ensure that the arc is covered by one vehicle, which enforces at least one crew to cover it. The authors use two branching strategies that differ in the order of considering the candidates for branching. The first strategy prefers selecting arcs that link timetabled trips. The second strategy starts with considering arcs that link a depot to a timetabled trip (pull-out arc) or vice-versa (pull-in arc). However, the authors were not able to conclude which strategy performs better. Additionally, for the instances tackled by the authors, it was sufficient to branch only over the sum of vehicle variables. The authors state that the exact B&P was only able to handle small sized instances (100 trips or less) and heuristic B&P procedure was used for the larger instances.

De Groot and Huisman (2008) apply the same procedure as Huisman et al. (2005) for solving the MD-VCSP. However, the authors discuss several approaches for splitting large instances of MD-VCSP into smaller ones in order to apply an integrated approach within a reasonable computation time without significantly compromising the quality of the solutions. The splitting procedure involves assigning each trip to a depot. Laurent and Hao (2008) present a constraint programming based model for the VCSP and propose a greedy randomized adaptive search procedure (GRASP) for solving it. Similarly, De Leone et al. (2011) propose a GRASP algorithm for solving the VCSP. Horváth and Kis (2019) use the time-space network proposed by Steinzen et al. (2010), and the

VCSP model involves the SPP formulation for the crew with side variables and constraints to ensure that a valid vehicle schedule can be derived from any feasible integer solution. The authors propose an exact B&P procedure and two branching strategies are used. One strategy is based on the Ryan and Foster (1981) branching rule and the other is based on assigning trips to depots. Additionally, the authors use a commercial MIP solver with the current column set to attain feasible integer solutions at certain times during the B&P procedure. Himmich et al. (2020) present the model proposed by Haase et al. (2001). The authors propose a primal adjacency-based algorithm and a multidirectional dynamic programming to efficiently solve the SPPRC in the subproblem of the column generation procedure. This framework is known as a primal column generation (PCG) framework. Compared to the standard column generation method that uses dynamic programming to solve the SPPRC, the proposed method was able to reduce the time of solving the subproblem by factors up to seven. Perumal et al. (2020a) introduce the integrated electric vehicle and crew scheduling problem (E-VCSP), which considers the driving range and recharging times of the electric buses. The mathematical model of the E-VCSP involves the SPP formulations for the vehicle and the crew with additional linking constraints. The authors propose an ALNS heuristic that utilizes B&P heuristic methods to solve the E-VCSP.

Table 7 gives an overview of the literature on the VCSP. The solution methods used for solving the VCSP can be categorized into two methods: 1) column generation (CG) approaches that are either based on LP relaxation or Lagrangian relaxation and 2) metaheuristics (MH). Some authors such as Friberg and Haase (1999) and Haase et al. (2001) have proposed exact B&P methods; however, almost all column generation approaches are based on heuristics for solving large instances of VCSP.

Authors	Model		Solution Method		Dataset			Remark
	Vehicle Scheduling	Crew Scheduling	CG	MH	K	T	Test	
Friberg and Haase (1999)	SPP	SPP	•		1	30	Random	B&P
Haase et al. (2001)	Side constraints	SPP	•		1	250	Random	B&P
Freling et al. (2003)	QAP	SPP/SCP	•		1	238	Netherlands	Lagrangian relaxation
Huisman et al. (2005)	MCF	SPP/SCP	•		4	653	Netherlands	Lagrangian relaxation
Borndörfer et al. (2008)	MCF	SPP	•		3	1,414	Germany	Lagrangian relaxation
Laurent and Hao (2008)	Constraint programming			•	1	249	Real-world	GRASP
Mesquita and Paias (2008)	MCF	SPP/SCP	•		4	400	Random	LP relaxation
Mesquita et al. (2009)	MCF	SPP/SCP	•		4	238	Portugal	B&P
Steinzen et al. (2010)	MCF	SPP	•		4	640	Random	Time-space network
De Leone et al. (2011)				•	2	400	Random	GRASP
Horváth and Kis (2019)	Side variables and constraints	SPP	•		4	100	Random	B&P
Himmich et al. (2020)	Side constraints	SPP	•		1	240	Random	PCG
Perumal et al. (2020a)	SPP	SPP	•	•	1	1,109	Denmark, Sweden	Electric buses, ALNS

**Table 7:** Overview of literature on the VCSP. **Model:** SPP/SCP-set partitioning/covering problem, QAP-quasi-assignment problem and MCF-multi-commodity flow problem. **Solution method:** CG-column generation (based on linear programming relaxation or Lagrangian relaxation) and MH-metaheuristics. **Dataset:** |K|-number of depots, |T|-number of timetabled trips and Test-random or real-world instances. Other abbreviations: B&P-branch-and-price, LP-linear programming, PCG-primal column generation, GRASP-greedy randomized adaptive search procedure and ALNS-adaptive large neighborhood search.

Freling et al. (2003) are the first authors to make a comparison between the integrated approach and the traditional sequential approach where the VSP is solved first and then the CSP. The primary objective was to minimize the sum of vehicles and drivers used in the schedule. The proposed integrated approach provided savings of up to one driver when compared to the sequential approach. Similarly, Huisman et al. (2005) showed that the integrated approach has a significant impact when compared to the traditional sequential approach; for an instance with 220 trips, the integrated approach provided a solution with 10 fewer drivers than that of the sequential approach. Borndörfer et al. (2008) used an objective function that is a mix of fixed and variable vehicle cost, fixed cost and paid time of duties and various penalties related to operational requirements of the CSP. For the largest instance, the integrated approach provided an improvement of 3.69% in the objective value when compared to that of the sequential approach. Perumal et al. (2020a)

showed that an improvement of up to 4.37% can be achieved by integrating electric vehicle and crew scheduling. The authors considered only the depot charging facility in their study of the E-VCSP and briefly discussed analyzing other charging systems as future directions of research.

An extension of the VCSP is the application of time windows for the timetabled trips, where the departure and arrival times of trips can be shifted within a specified interval (Kéri and Haase (2008) and Kliewer et al. (2012)). Such an extension can be seen as a partial integration of timetabling into the VCSP that offers further flexibility for scheduling vehicles and crews. Kliewer et al. (2012) state that trip shifting enables additional break possibilities between trips for the drivers. Even with very short time windows (up to four minutes) for the timetabled trips, the authors show that enormous savings in the number of planned vehicles and drivers can be achieved.

Research studies have been carried out in the airline industry to integrate the aircraft routing and crew scheduling problem. Cordeau et al. (2001) introduce a mathematical model for the complete integration of both the scheduling problems. The model is similar to model (20)-(25). The authors propose a solution approach that combines column generation and Benders decomposition. The methodology iterates between a Benders master problem that solves the aircraft routing problem and a Benders subproblem that solves the crew scheduling problem, and both the problems are solved by column generation. A heuristic B&B method is used to compute integer solutions. In line with Cordeau et al. (2001), Mercier et al. (2005) propose to model the crew scheduling problem as the Benders master problem and the aircraft routing problem as the Benders subproblem.

## 5. Future Research Areas and Conclusion

A growing area of research is the integration of timetabling and vehicle scheduling that simultaneously minimizes travel times of passengers and operational cost of vehicles (see e.g. Ibarra-Rojas et al. (2014), Schmid and Ehmke (2015), Fonseca et al. (2018) and Desfontaines and Desaulniers (2018)). Hassold and Ceder (2014) studies the VSP with multiple vehicle types using pareto-optimal timetables. Schöbel (2017) develop algorithms to integrate line planning, timetabling and vehicle scheduling. However, to the best of our knowledge, there has not been any study that simultaneously handles timetabling and the E-VSP such that the charging activities for the electric vehicles are incorporated into the timetable. This point has also been briefly discussed in Häll et al. (2019); however, a solution approach is not proposed. Integration of timetabling and electric vehicle scheduling would be an interesting topic of research that could potentially provide more insights into efficiently operating a transportation system with electric vehicles.

The vehicle and crew schedules are usually computed several months before the actual day-of-operation. However, unforeseen events during operations such as vehicle breakdowns, weather conditions and traffic jams can severely disrupt the planned schedules. Furthermore, in some cases, planned events such as maintenance activities of the infrastructure for a certain period enforce changes to the existing timetable. Therefore, the vehicle and crew schedules may have to be modified as well according to the altered timetable. One area of research in the field of OR is the development of real-time rescheduling methods to reduce the impact of disruptions such as delays or vehicle failures. Visentini et al. (2014) state that much research has been done on the VSP, but considerations on vehicle rescheduling are still relatively unexplored. Furthermore, with its ability to guard against delays, robust planning is receiving more and more attention in the academic literature (Lusby et al. (2018)). A solution is said to be operationally robust when the effects of potential delays are minimal (Weide et al. (2010)). Huisman and Wagelmans (2006) and Weide et al. (2010) are two examples that propose real-time control strategies and robust solution approaches for the integrated vehicle and crew scheduling problem. Such approaches are found to be scarce in the OR literature and the applicability of such approaches in a real-life setting have been hardly reported. Therefore, incorporation of robustness and real-time control strategies in integrated transport planning problems is seen as a future area of research. Rescheduling aspects or considerations of robustness for scheduling of electric vehicles have not been reported in the OR literature to the best of our knowledge. Since there are many technological limitations concerning the scheduling of electric vehicles, the development of recovery methods that support the practical application of electric vehicles can be seen as a future area of research.

The VCSP can be integrated with other transport planning problems. One example is the integration with timetabling that was explored by Kliewer et al. (2012). The successor problem of the VCSP is the crew rostering problem (CRP). This has been extensively studied in the OR literature (see e.g. Caprara et al. (1998)). Another research area is the integration of the CSP and the CRP (see e.g. Borndörfer et al. (2017) and Lin et al. (2020)). To the best of our knowledge, Mesquita et al. (2013) is the only paper reported in the literature that integrates the VSP, CSP and CRP. Integration of the VCSP with other bus transport planning problems adds further computational complexity; however such an approach can further improve efficiency of transport systems.

In conclusion, this paper gives a detailed literature review of the integrated approaches for optimizing electric vehicle and crew schedules. Integration of two or more public bus transport planning problems is a growing area of research. Particularly, integrating the vehicle and crew scheduling problems leads to cost reductions for bus companies when compared to a traditional sequential approach. A brief overview of the electric bus technologies and the constraints associated with scheduling electric buses are given in this paper. Electrification of bus fleets in most cities is expected to rise. However, due to the limitations and challenges of the electric bus technologies, further adjustments have to be made to the current bus transport planning problems. Therefore, the scheduling of electric vehicles is recognized as a crucial and fast growing area of research.

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## References

- Abbink, E., Fischetti, M., Kroon, L., Timmer, G., Vromans, M., 2005. Reinventing Crew Scheduling at Netherlands Railways. *Interfaces* 35, 393–401. URL: <http://pubsonline.informs.org/doi/abs/10.1287/inte.1050.0158>, doi:10.1287/inte.1050.0158.
- Abbink, E.J.W., Albino, L., Dollevoet, T., Huisman, D., Roussado, J., Saldanha, R.L., 2011. Solving large scale crew scheduling problems in practice. *Public Transport* 3, 149–164. URL: <http://link.springer.com/10.1007/s12469-011-0045-x>, doi:10.1007/s12469-011-0045-x.
- Adler, J.D., Mirchandani, P.B., 2017. The Vehicle Scheduling Problem for Fleets with Alternative-Fuel Vehicles. *Transportation Science* 51, 441–456. URL: <http://pubsonline.informs.org/doi/10.1287/trsc.2015.0615>, doi:10.1287/trsc.2015.0615.
- Ball, M., Bodin, L., Dial, R., 1983. A Matching Based Heuristic for Scheduling Mass Transit Crews and Vehicles. *Transportation Science* 17, 4–31. URL: <http://pubsonline.informs.org/doi/abs/10.1287/trsc.17.1.4>, doi:10.1287/trsc.17.1.4.
- Bertossi, A.A., Carraresi, P., Gallo, G., 1987. On some matching problems arising in vehicle scheduling models. *Networks* 17, 271–281. URL: <http://doi.wiley.com/10.1002/net.3230170303>, doi:10.1002/net.3230170303.
- Bodin, L., Golden, B., Assad, A., Ball, M., 1983. Routing and Scheduling of Vehicles and Crew - The State of the Art. *Computers and Operations Research* 10, 63–211.
- Bodin, L., Rosenfield, D., Kydes A., 1978. UCOST: a micro approach to a transportation planning problem. *Journal of Urban Analysis* 5, 47–69.
- Borndörfer, R., Löbel, A., Weider, S., 2008. A bundle method for integrated multi-depot vehicle and duty scheduling in public transit, in: *Lecture Notes in Economics and Mathematical Systems*, pp. 3–24. doi:10.1007/978-3-540-73312-6\_1.

- Borndörfer, R., Schulz, C., Seidl, S., Weider, S., 2017. Integration of duty scheduling and rostering to increase driver satisfaction. *Public Transport* 9, 177–191. URL: <http://link.springer.com/10.1007/s12469-017-0153-3>, doi:10.1007/s12469-017-0153-3.
- Boschetti, M., Mingozzi, A., Ricciardelli, S., 2004. An Exact Algorithm for the Simplified Multiple Depot Crew Scheduling Problem. *Annals of Operations Research* 127, 177–201. URL: <http://link.springer.com/10.1023/B:ANOR.0000019089.86834.91>, doi:10.1023/B:ANOR.0000019089.86834.91.
- Bunte, S., Kliwer, N., 2009. An overview on vehicle scheduling models. *Public Transport* 1, 299–317. URL: <http://link.springer.com/10.1007/s12469-010-0018-5>, doi:10.1007/s12469-010-0018-5.
- C40 Cities, 2017. C40 Fossil Fuel Free Street Declaration. URL: <https://www.c40.org/other/fossil-fuel-free-streets-declaration>. Accessed: 23 August 2019.
- Caprara, A., Fischetti, M., Toth, P., 1999. A Heuristic Method for the Set Covering Problem. *Operations Research* 47, 730–743. doi:10.1287/opre.47.5.730.
- Caprara, A., Toth, P., Vigo, D., Fischetti, M., 1998. Modeling and Solving the Crew Rostering Problem. *Operations Research* 46, 820–830. URL: <http://pubsonline.informs.org/doi/abs/10.1287/opre.46.6.820>, doi:10.1287/opre.46.6.820.
- Carpaneto, G., Dell’amico, M., Fischetti, M., Toth, P., 1989. A branch and bound algorithm for the multiple depot vehicle scheduling problem. *Networks* 19, 531–548. doi:10.1002/net.3230190505.
- Ceder, A.A., 2011. Public-transport vehicle scheduling with multi vehicle type. *Transportation Research Part C: Emerging Technologies* 19, 485–497. doi:10.1016/j.trc.2010.07.007.
- Chen, S., 2013. An improved column generation algorithm for crew scheduling problems. *Journal of Information and Computational Science* 10. URL: <https://findit.dtu.dk/en/catalog/2369942476>.
- Chen, Z., Yin, Y., Song, Z., 2018. A cost-competitiveness analysis of charging infrastructure for electric bus operations. *Transportation Research Part C: Emerging Technologies* 93, 351–366. URL: <https://linkinghub.elsevier.com/retrieve/pii/S0968090X18308465>, doi:10.1016/j.trc.2018.06.006.
- Copenhagen Capacity, 2019. Copenhagen city buses will be electric by 2025. URL: <https://www.copcap.com/newslist/2019/copenhagen-city-buses-will-be-electric-by-2025>. Accessed: 2 November 2019.
- Cordeau, J.F., Stojković, G., Soumis, F., Desrosiers, J., 2001. Benders Decomposition for Simultaneous Aircraft Routing and Crew Scheduling. *Transportation Science* 35, 375–388. URL: <http://pubsonline.informs.org/doi/abs/10.1287/trsc.35.4.375.10432>, doi:10.1287/trsc.35.4.375.10432.
- Costa, A., Branco, I., Pinto Paixio, J.M., 1995. Vehicle Scheduling Problem with Multiple Type of Vehicles and a Single Depot, in: *Computer-Aided Transit Scheduling. Lecture Notes in Economics and Mathematical Systems*, pp. 115–129. doi:[https://doi.org/10.1007/978-3-642-57762-8\\_9](https://doi.org/10.1007/978-3-642-57762-8_9).
- Daduna, J.R., Paixão, J.M.P., 1995. Vehicle Scheduling for Public Mass Transit — An Overview, in: *Computer-Aided Transit Scheduling*, Springer Berlin Heidelberg, Berlin, Heidelberg. pp. 76–90.
- Darby-Dowman, K., 1988. Integrated decision support systems for urban transport scheduling: discussion of implementation and experience. *Computer-aided Transit Scheduling. Proceedings of the Fourth International Workshop on Computer-aided Scheduling of Public Transport* URL: <https://findit.dtu.dk/en/catalog/2301921774>.



- De Leone, R., Festa, P., Marchitto, E., 2011. A bus driver scheduling problem: A new mathematical model and a GRASP approximate solution. *Journal of Heuristics* 17, 441–466. doi:10.1007/s10732-010-9141-3.
- De Groot, S.W., Huisman, D., 2008. Vehicle and Crew Scheduling: Solving Large Real-World Instances with an Integrated Approach, in: *Lecture Notes in Economics and Mathematical Systems*, pp. 43–56. URL: [http://link.springer.com/10.1007/978-3-540-73312-6\\_3](http://link.springer.com/10.1007/978-3-540-73312-6_3), doi:10.1007/978-3-540-73312-6\_3.
- Dell’Amico, M., Fischetti, M., Toth, P., 1993. Heuristic Algorithms for the Multiple Depot Vehicle Scheduling Problem. *Management Science* 39, 115–125. URL: <http://pubsonline.informs.org/doi/abs/10.1287/mnsc.39.1.115>, doi:10.1287/mnsc.39.1.115.
- Desaulniers, G., Hickman, M.D., 2007. Chapter 2 Public Transit. *Handbooks in Operations Research and Management Science* 14, 69–127. doi:10.1016/S0927-0507(06)14002-5.
- Desfontaines, L., Desaulniers, G., 2018. Multiple depot vehicle scheduling with controlled trip shifting. *Transportation Research Part B: Methodological* 113, 34–53. doi:10.1016/j.trb.2018.05.011.
- Desrochers, M., Soumis, F., 1989. A Column Generation Approach to the Urban Transit Crew Scheduling Problem. *Transportation Science* 23, 1–13. doi:10.1287/trsc.23.1.1.
- Desrosiers, J., Dumas, Y., Solomon, M.M., Soumis, F., 1995. Time constrained routing and scheduling, in: *Handbooks in Operations Research and Management Science*. volume 8, pp. 35–139. URL: <https://linkinghub.elsevier.com/retrieve/pii/S0927050705801069>, doi:10.1016/S0927-0507(05)80106-9.
- Dos Santos, A.G., Mateus, G.R., 2009. General hybrid column generation algorithm for crew scheduling problems using genetic algorithm, in: *2009 IEEE Congress on Evolutionary Computation, IEEE*. pp. 1799–1806. URL: <http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=4983159>, doi:10.1109/CEC.2009.4983159.
- European Union, 2018. 2050 long-term strategy. URL: [https://ec.europa.eu/clima/policies/strategies/2050\\_en](https://ec.europa.eu/clima/policies/strategies/2050_en). Accessed: 23 August 2019.
- Falkner, J., Ryan, D., 1987. A Bus Crew Scheduling System Using A Set Partitioning Model. *Asia-pacific Journal of Operational Research* 4, 39–56.
- Fonseca, J.P., van der Hurk, E., Roberti, R., Larsen, A., 2018. A matheuristic for transfer synchronization through integrated timetabling and vehicle scheduling. *Transportation Research Part B: Methodological* 109, 128–149. doi:10.1016/j.trb.2018.01.012.
- Forbes, M., Holt, J., Watts, A., 1994. An exact algorithm for multiple depot bus scheduling. *European Journal of Operational Research* 72, 115–124. URL: <https://linkinghub.elsevier.com/retrieve/pii/0377221794903344>, doi:10.1016/0377-2217(94)90334-4.
- Fores, S., Proll, L., Wren, A., 2002. TRACS II: a hybrid IP/heuristic driver scheduling system for public transport. *Journal of the Operational Research Society* 53, 1093–1100. URL: <https://www.tandfonline.com/doi/full/10.1057/palgrave.jors.2601271>, doi:10.1057/palgrave.jors.2601271.
- Freling, R., Huisman, D., Wagelmans, A.P., 2003. Models and algorithms for integration of vehicle and crew scheduling. *Journal of Scheduling* 6, 63–85. doi:10.1023/A:1022287504028.
- Freling, R., Wagelmans, A.P.M., Paixão, J.M.P., 2001. Models and Algorithms for Single-Depot Vehicle Scheduling. *Transportation Science* 35, 165–180. URL: <http://pubsonline.informs.org/doi/abs/10.1287/trsc.35.2.165.10135>, doi:10.1287/trsc.35.2.165.10135.

- Friberg, C., Haase, K., 1999. An exact algorithm for the vehicle and crew scheduling problem. *Computer-Aided Transit Scheduling* 471, 63–80.
- Gaffi, A., Nonato, M., 1999. An Integrated Approach to Ex-Urban Crew and Vehicle Scheduling. *Lecture Notes in Economics and Mathematical Systems* 471, 103–128.
- Gintner, V., Kliewer, N., Suhl, L., 2005. Solving large multiple-depot multiple-vehicle-type bus scheduling problems in practice. *OR Spectrum* 27, 507–523. URL: <http://link.springer.com/10.1007/s00291-005-0207-9>, doi:10.1007/s00291-005-0207-9.
- Gintner, V., Kliewer, N., Suhl, L., 2008. A Crew Scheduling Approach for Public Transit Enhanced with Aspects from Vehicle Scheduling, in: *Computer-aided Systems in Public Transport*. Springer Berlin Heidelberg, Berlin, Heidelberg, pp. 25–42. URL: [http://link.springer.com/10.1007/978-3-540-73312-6\\_2](http://link.springer.com/10.1007/978-3-540-73312-6_2), doi:10.1007/978-3-540-73312-6\_2.
- Haase, K., Desaulniers, G., Desrosiers, J., 2001. Simultaneous Vehicle and Crew Scheduling in Urban Mass Transit Systems. *Transportation Science* 35, 286–303. doi:10.1287/trsc.35.3.286.10153.
- Hadjar, A., Marcotte, O., Soumis, F., 2006. A Branch-and-Cut Algorithm for the Multiple Depot Vehicle Scheduling Problem. *Operations Research* 54, 130–149. URL: <http://pubsonline.informs.org/doi/abs/10.1287/opre.1050.0240>, doi:10.1287/opre.1050.0240.
- Haghani, A., Banihashemi, M., 2002. Heuristic approaches for solving large-scale bus transit vehicle scheduling problem with route time constraints. *Transportation Research Part A: Policy and Practice* 36, 309–333. URL: <https://linkinghub.elsevier.com/retrieve/pii/S0965856401000040>, doi:10.1016/S0965-8564(01)00004-0.
- Häll, C.H., Ceder, A.A., Ekström, J., Quttineh, N.H., 2019. Adjustments of public transit operations planning process for the use of electric buses. *Journal of Intelligent Transportation Systems* 23, 216–230. URL: <https://www.tandfonline.com/doi/full/10.1080/15472450.2018.1488131>, doi:10.1080/15472450.2018.1488131.
- Hassold, S., Ceder, A.A., 2014. Public transport vehicle scheduling featuring multiple vehicle types. *Transportation Research Part B: Methodological* 67, 129–143. URL: <https://linkinghub.elsevier.com/retrieve/pii/S0191261514000654>, doi:10.1016/j.trb.2014.04.009.
- Heil, J., Hoffmann, K., Buscher, U., 2019. Railway crew scheduling: Models, methods and applications. *European Journal of Operational Research* URL: <https://linkinghub.elsevier.com/retrieve/pii/S0377221719304916>, doi:10.1016/j.ejor.2019.06.016.
- Himmich, I., El Hallaoui, I., Soumis, F., 2020. Primal column generation framework for vehicle and crew scheduling problems. *Networks* 75, 291–309. URL: <https://onlinelibrary.wiley.com/doi/abs/10.1002/net.21925>, doi:10.1002/net.21925.
- Horváth, M., Kis, T., 2019. Computing strong lower and upper bounds for the integrated multiple-depot vehicle and crew scheduling problem with branch-and-price. *Central European Journal of Operations Research* 27, 39–67. doi:10.1007/s10100-017-0489-4.
- Huisman, D., 2007. A column generation approach for the rail crew re-scheduling problem. *European Journal of Operational Research* 180, 163–173. URL: <https://linkinghub.elsevier.com/retrieve/pii/S0377221706002736>, doi:10.1016/j.ejor.2006.04.026.
- Huisman, D., Freling, R., Wagelmans, A.P.M., 2005. Multiple-Depot Integrated Vehicle and Crew Scheduling. *Transportation Science* 39, 491–502. URL: <http://pubsonline.informs.org/doi/abs/10.1287/trsc.1040.0104>, doi:10.1287/trsc.1040.0104.
- Huisman, D., Wagelmans, A.P., 2006. A solution approach for dynamic vehicle and crew scheduling. *European Journal of Operational Research* 172, 453–471. URL: <https://linkinghub.elsevier.com/retrieve/pii/S0377221704008082>, doi:10.1016/j.ejor.2004.10.009.

- Ibarra-Rojas, O.J., Delgado, F., Giesen, R., Muñoz, J.C., 2015. Planning, operation, and control of bus transport systems: A literature review. *Transportation Research Part B: Methodological* 77, 38–75. URL: <http://dx.doi.org/10.1016/j.trb.2015.03.002>, doi:10.1016/j.trb.2015.03.002.
- Ibarra-Rojas, O.J., Giesen, R., Rios-Solis, Y.A., 2014. An integrated approach for timetabling and vehicle scheduling problems to analyze the trade-off between level of service and operating costs of transit networks. *Transportation Research Part B: Methodological* 70, 35–46. doi:10.1016/j.trb.2014.08.010.
- Kecskeméti, B., Bilics, A., 2013. Bus driver duty optimization using an integer programming and evolutionary hybrid algorithm. *Central European Journal of Operations Research* 21, 745–755. URL: <http://link.springer.com/10.1007/s10100-013-0292-9>, doi:10.1007/s10100-013-0292-9.
- Kéri, A., Haase, K., 2008. Simultaneous Vehicle and Crew Scheduling with Trip Shifting, in: *Operations Research Proceedings 2007*. Springer Berlin Heidelberg, Berlin, Heidelberg, pp. 467–472. URL: [http://link.springer.com/10.1007/978-3-540-77903-2\\_72](http://link.springer.com/10.1007/978-3-540-77903-2_72), doi:10.1007/978-3-540-77903-2\_72.
- Kliwer, N., Amberg, B., Amberg, B., 2012. Multiple depot vehicle and crew scheduling with time windows for scheduled trips. *Public Transport* 3, 213–244. URL: <http://link.springer.com/10.1007/s12469-011-0049-6>, doi:10.1007/s12469-011-0049-6.
- Kliwer, N., Mellouli, T., Suhl, L., 2006. A time–space network based exact optimization model for multi-depot bus scheduling. *European Journal of Operational Research* 175, 1616–1627. URL: <https://linkinghub.elsevier.com/retrieve/pii/S0377221705002274>, doi:10.1016/j.ejor.2005.02.030.
- Kulkarni, S., Krishnamoorthy, M., Ranade, A., Ernst, A.T., Patil, R., 2018. A new formulation and a column generation-based heuristic for the multiple depot vehicle scheduling problem. *Transportation Research Part B: Methodological* 118, 457–487. URL: <https://linkinghub.elsevier.com/retrieve/pii/S0191261518301115>, doi:10.1016/j.trb.2018.11.007.
- Kunith, A., Mendelevitch, R., Goehlich, D., 2017. Electrification of a city bus network—An optimization model for cost-effective placing of charging infrastructure and battery sizing of fast-charging electric bus systems. *International Journal of Sustainable Transportation* 11, 707–720. URL: <https://www.tandfonline.com/doi/full/10.1080/15568318.2017.1310962>, doi:10.1080/15568318.2017.1310962.
- Lamatsch, A., 1992. An approach to vehicle scheduling with depot capacity constraints. *Lecture Notes in Economics and Mathematical Systems* 386. URL: <https://findit.dtu.dk/en/catalog/2384427404>.
- Laurent, B., Hao, J.K., 2008. Simultaneous Vehicle and Crew Scheduling for Extra Urban Transports, in: *New Frontiers in Applied Artificial Intelligence*. Springer Berlin Heidelberg, Berlin, Heidelberg, volume 5027, pp. 466–475. URL: [http://link.springer.com/10.1007/978-3-540-69052-8\\_49](http://link.springer.com/10.1007/978-3-540-69052-8_49), doi:10.1007/978-3-540-69052-8\_49.
- Laurent, B., Hao, J.K., 2009. Iterated local search for the multiple depot vehicle scheduling problem. *Computers & Industrial Engineering* 57, 277–286. URL: <https://linkinghub.elsevier.com/retrieve/pii/S0360835208003148>, doi:10.1016/j.cie.2008.11.028.
- Lenstra, J.K., Kan, A.H.G.R., 1981. Complexity of vehicle routing and scheduling problems. *Networks* 11, 221–227. URL: <http://doi.wiley.com/10.1002/net.3230110211>, doi:10.1002/net.3230110211.

- Li, H., Wang, Y., Li, S., Li, S., 2015. A Column Generation Based Hyper-Heuristic to the Bus Driver Scheduling Problem. *Discrete Dynamics in Nature and Society* 2015, 1–10. URL: <http://www.hindawi.com/journals/ddns/2015/638104/>, doi:10.1155/2015/638104.
- Li, J., Kwan, R., 2003. A fuzzy genetic algorithm for driver scheduling. *European Journal Of Operational Research* 147, 334–344. URL: <http://linkinghub.elsevier.com/retrieve/pii/S0377221702005647>, doi:10.1016/S0377-2217(02)00564-7.
- Li, J.Q., 2013. Transit Bus Scheduling with Limited Energy. *Transportation Science* 48, 521–539. doi:10.1287/trsc.2013.0468.
- Li, J.Q., 2016. Battery-electric transit bus developments and operations: A review. *International Journal of Sustainable Transportation* 10, 157–169. doi:10.1080/15568318.2013.872737.
- Lin, D.Y., Juan, C.J., Chang, C.C., 2020. A Branch-and-Price-and-Cut Algorithm for the Integrated Scheduling and Rostering Problem of Bus Drivers. *Journal of Advanced Transportation* doi:10.1155/2020/3153201.
- Liu, T., Ceder, A., 2020. Battery-electric transit vehicle scheduling with optimal number of stationary chargers. *Transportation Research Part C: Emerging Technologies* 114, 118–139. URL: <https://linkinghub.elsevier.com/retrieve/pii/S0968090X19304061>, doi:10.1016/j.trc.2020.02.009.
- Löbel, A., 1998. Vehicle Scheduling in Public Transit and Lagrangean Pricing. *Management Science* 44, 1637–1649. URL: <http://pubsonline.informs.org/doi/abs/10.1287/mnsc.44.12.1637>, doi:10.1287/mnsc.44.12.1637.
- Lourenço, H.R., Paixão, J.P., Portugal, R., 2001. Multiobjective Metaheuristics for the Bus Driver Scheduling Problem. *Transportation Science* 35, 331–343. URL: <http://pubsonline.informs.org/doi/abs/10.1287/trsc.35.3.331.10147>, doi:10.1287/trsc.35.3.331.10147.
- Lusby, R.M., Larsen, J., Bull, S., 2018. A survey on robustness in railway planning. *European Journal of Operational Research* 266, 1–15. URL: <https://linkinghub.elsevier.com/retrieve/pii/S0377221717306719>, doi:10.1016/j.ejor.2017.07.044.
- Marín Moreno, C.A., Escobar Falcón, L.M., Bolaños, R.I., Subramanian, A., Escobar Zuluaga, A.H., Echeverri, M.G., 2019. A hybrid algorithm for the multi-depot vehicle scheduling problem arising in public transportation. *International Journal of Industrial Engineering Computations* 10, 361–374. doi:10.5267/j.ijiec.2019.2.002.
- Mauri, G., Lorena, L., 2007. A new hybrid heuristic for driver scheduling. *International Journal of Hybrid Intelligent Systems* 4, 39–47. URL: <http://www.medra.org/servlet/aliasResolver?alias=iospress&doi=10.3233/HIS-2007-4105>, doi:10.3233/HIS-2007-4105.
- Mercier, A., Cordeau, J.F., Soumis, F., 2005. A computational study of Benders decomposition for the integrated aircraft routing and crew scheduling problem. *Computers & Operations Research* 32, 1451–1476. URL: <http://linkinghub.elsevier.com/retrieve/pii/S0305054803003447>, doi:10.1016/j.cor.2003.11.013.
- Mesquita, M., Moz, M., Paias, A., Pato, M., 2013. A decomposition approach for the integrated vehicle-crew-roster problem with days-off pattern. *European Journal of Operational Research* 229, 318–331. URL: <https://linkinghub.elsevier.com/retrieve/pii/S037722171300204X>, doi:10.1016/j.ejor.2013.02.055.
- Mesquita, M., Paias, A., 2008. Set partitioning/covering-based approaches for the integrated vehicle and crew scheduling problem. *Computers & Operations Research* 35, 1562–1575. URL: <https://linkinghub.elsevier.com/retrieve/pii/S0305054806002139>, doi:10.1016/j.cor.2006.09.001.

- Mesquita, M., Paias, A., Respício, A., 2009. Branching approaches for integrated vehicle and crew scheduling. *Public Transport* 1, 21–37. URL: <http://link.springer.com/10.1007/s12469-008-0005-2>, doi:10.1007/s12469-008-0005-2.
- Mesquita, M., Paixao, J., 1992. Multiple Depot Vehicle Scheduling Problem - A New Heuristic Based on Quasi-Assignment Algorithms. *Lecture Notes in Economics and Mathematical Systems* 386, 167–180.
- Oukil, A., Amor, H.B., Desrosiers, J., El Gueddari, H., 2007. Stabilized column generation for highly degenerate multiple-depot vehicle scheduling problems. *Computers & Operations Research* 34, 817–834. URL: <https://linkinghub.elsevier.com/retrieve/pii/S0305054805001590>, doi:10.1016/j.cor.2005.05.011.
- Paixão, J.M.P., Branco, I.M., 1987. A quasi-assignment algorithm for bus scheduling. *Networks* 17, 249–269. URL: <http://doi.wiley.com/10.1002/net.3230170302>, doi:10.1002/net.3230170302.
- Patrikalakis, I., Xerocostas, D., 1992. A New Decomposition Scheme of the Urban Public Transport Scheduling Problem. *Lecture Notes in Economics and Mathematical Systems* 386, 407–425.
- Pelletier, S., Jabali, O., Mendoza, J.E., Laporte, G., 2019. The electric bus fleet transition problem. *Transportation Research Part C: Emerging Technologies* 109, 174–193. URL: <https://linkinghub.elsevier.com/retrieve/pii/S0968090X1930868X>, doi:10.1016/j.trc.2019.10.012.
- Pepin, A.S., Desaulniers, G., Hertz, A., Huisman, D., 2009. A comparison of five heuristics for the multiple depot vehicle scheduling problem. *Journal of Scheduling* 12, 17–30. doi:10.1007/s10951-008-0072-x.
- Perumal, S.S., Dollevoet, T., Huisman, D., Lusby, R.M., Larsen, J., Riis, M., 2020a. Solution Approaches for Vehicle and Crew Scheduling with Electric Buses. Technical Report. Econometric Institute Research Papers. URL: <https://repub.eur.nl/pub/123963>.
- Perumal, S.S., Larsen, J., Lusby, R.M., Riis, M., Christensen, T.R., 2020b. A column generation approach for the driver scheduling problem with staff cars. Technical Report. Technical University of Denmark. URL: <https://findit.dtu.dk/en/catalog/2525192042>.
- Perumal, S.S., Larsen, J., Lusby, R.M., Riis, M., Sørensen, K.S., 2019. A matheuristic for the driver scheduling problem with staff cars. *European Journal of Operational Research* 275, 280–294. URL: <https://linkinghub.elsevier.com/retrieve/pii/S0377221718309366>, doi:10.1016/j.ejor.2018.11.011.
- Pihlatie, M., Kukkonen, S., Halmeaho, T., Karvonen, V., Nylund, N.O., 2014. Fully electric city buses - The viable option, in: 2014 IEEE International Electric Vehicle Conference (IEVC), IEEE. pp. 1–8. URL: <https://ieeexplore.ieee.org/document/7056145/>, doi:10.1109/IEVC.2014.7056145.
- Portugal, R., Lourenço, H.R., Paixão, J.P., 2009. Driver scheduling problem modelling. *Public Transport* 1, 103–120. URL: <http://link.springer.com/10.1007/s12469-008-0007-0>, doi:10.1007/s12469-008-0007-0.
- Potthoff, D., Huisman, D., Desaulniers, G., 2010. Column Generation with Dynamic Duty Selection for Railway Crew Rescheduling. *Transportation Science* 44, 493–505. URL: <http://pubsonline.informs.org/doi/abs/10.1287/trsc.1100.0322>, doi:10.1287/trsc.1100.0322.
- Reuer, J., Kliewer, N., Wolbeck, L., 2015. The Electric Vehicle Scheduling Problem A study on time-space network based and heuristic solution approaches, in: Conference on advanced systems in public transport and transit data (CASPT), Rotterdam. URL: <http://www.rotterdam2015.caspt.org/proceedings/paper93.pdf>.

- Ribeiro, C.C., Soumis, F., 1994. A Column Generation Approach to the Multiple-Depot Vehicle Scheduling Problem. *Operations Research* 42, 41–52. doi:10.1287/opre.42.1.41.
- Rogge, M., van der Hurk, E., Larsen, A., Sauer, D.U., 2018. Electric bus fleet size and mix problem with optimization of charging infrastructure. *Applied Energy* 211, 282–295. URL: <https://linkinghub.elsevier.com/retrieve/pii/S0306261917316355>, doi:10.1016/j.apenergy.2017.11.051.
- Ryan, D.M., Foster, B.A., 1981. An integer programming approach to scheduling. *Computer Scheduling of Public Transport. Urban Passenger Vehicle and Crew Scheduling. Proceedings of an International Workshop* URL: <https://findit.dtu.dk/en/catalog/2300620374>.
- Schmid, V., Ehmke, J.F., 2015. Integrated timetabling and vehicle scheduling with balanced departure times. *OR Spectrum* 37, 903–928. URL: <http://link.springer.com/10.1007/s00291-015-0398-7>, doi:10.1007/s00291-015-0398-7.
- Schöbel, A., 2012. Line planning in public transportation: Models and methods. *OR Spectrum* 34, 491–510. doi:10.1007/s00291-011-0251-6.
- Schöbel, A., 2017. An eigenmodel for iterative line planning, timetabling and vehicle scheduling in public transportation. *Transportation Research Part C: Emerging Technologies* 74, 348–365. doi:10.1016/j.trc.2016.11.018.
- Scott, D., 1985. A large linear programming approach to the public transport scheduling and cost model, in: J.M. Rousseau (Ed.), *Computer Scheduling of Public Transport 2*. Amsterdam, North Holland, pp. 473–491.
- Smith, B.M., Wren, A., 1988. A bus crew scheduling system using a set covering formulation. *Transportation Research Part A: General* 22, 97–108. URL: <http://linkinghub.elsevier.com/retrieve/pii/0191260788900222>, doi:10.1016/0191-2607(88)90022-2.
- Steinzen, I., 2007. *Topics in Integrated Vehicle and Crew Scheduling in Public Transport*. Ph.D. thesis. University of Paderborn, Germany. URL: <http://digital.ub.uni-paderborn.de/hsmig/content/titleinfo/4835>.
- Steinzen, I., Gintner, V., Suhl, L., Kliwer, N., 2010. A Time-Space Network Approach for the Integrated Vehicle- and Crew-Scheduling Problem with Multiple Depots. *Transportation Science* 44, 367–382. URL: <http://pubsonline.informs.org/doi/abs/10.1287/trsc.1090.0304>, doi:10.1287/trsc.1090.0304.
- Transport and Environment, 2018. Electric buses arrive on time. URL: <https://www.transportenvironment.org/publications/electric-buses-arrive-time>. Accessed: 23 August 2019.
- United Nations, 2018. *World Urbanization Prospects*. URL: <https://population.un.org/wup/Publications/>. Accessed: 13 November 2019.
- United Nations Climate Change, 2015. *The Paris Agreement*. URL: <https://unfccc.int/process-and-meetings/the-paris-agreement/the-paris-agreement>. Accessed: 23 August 2019.
- Van Kooten Niekerk, M.E., van den Akker, J.M., Hoogeveen, J.A., 2017. Scheduling electric vehicles. *Public Transport* 9, 155–176. doi:10.1007/s12469-017-0164-0.
- Visentini, M.S., Borenstein, D., Li, J.Q., Mirchandani, P.B., 2014. Review of real-time vehicle schedule recovery methods in transportation services. *Journal of Scheduling* 17, 541–567. URL: <http://link.springer.com/10.1007/s10951-013-0339-8>, doi:10.1007/s10951-013-0339-8.

- Wang, H., Shen, J., 2007. Heuristic approaches for solving transit vehicle scheduling problem with route and fueling time constraints. *Applied Mathematics and Computation* 190, 1237–1249. URL: <https://linkinghub.elsevier.com/retrieve/pii/S0096300307001543>, doi:10.1016/j.amc.2007.02.141.
- Weide, O., Ryan, D., Ehrgott, M., 2010. An iterative approach to robust and integrated aircraft routing and crew scheduling. *Computers & Operations Research* 37, 833–844. URL: <http://linkinghub.elsevier.com/retrieve/pii/S0305054809000963>, doi:10.1016/j.cor.2009.03.024.
- Wen, M., Linde, E., Ropke, S., Mirchandani, P., Larsen, A., 2016. An adaptive large neighborhood search heuristic for the Electric Vehicle Scheduling Problem. *Computers & Operations Research* 76, 73–83. URL: <https://linkinghub.elsevier.com/retrieve/pii/S0305054816301460>, doi:10.1016/j.cor.2016.06.013.
- Wren, A., Fores, S., Kwan, A., Kwan, R., Parker, M., Proll, L., 2003. A flexible system for scheduling drivers. *Journal of Scheduling* 6, 437–455. doi:10.1023/A:1024854522373.
- Xylia, M., Leduc, S., Patrizio, P., Kraxner, F., Silveira, S., 2017. Locating charging infrastructure for electric buses in Stockholm. *Transportation Research Part C: Emerging Technologies* 78, 183–200. URL: <https://linkinghub.elsevier.com/retrieve/pii/S0968090X17300700>, doi:10.1016/j.trc.2017.03.005.
- Yao, E., Liu, T., Lu, T., Yang, Y., 2020. Optimization of electric vehicle scheduling with multiple vehicle types in public transport. *Sustainable Cities and Society* 52, 101862. URL: <https://linkinghub.elsevier.com/retrieve/pii/S2210670719318098>, doi:10.1016/j.scs.2019.101862.
- Yunes, T.H., Moura, A.V., de Souza, C.C., 2005. Hybrid Column Generation Approaches for Urban Transit Crew Management Problems. *Transportation Science* 39, 273–288. URL: <http://pubsonline.informs.org/doi/abs/10.1287/trsc.1030.0078>, doi:10.1287/trsc.1030.0078.