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CHANGE DETECTION IN SINGLE- AND MULTI-LOOK POLARIMETRIC SAR DATA

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ABSTRACT

We describe the use of P-band data from DLR's F-SAR system to change detection between three time points. Change detection on data in the covariance matrix representation is performed by means of test statistics in the complex Wishart distribution on both multi- and single-look data. We successfully detect objects including humans buried under snow in Disko Island, Greenland.

1. INTRODUCTION

In the preparation to meet future challenges in the Arctic region, the Danish Defence Acquisition and Logistics Organization (DALO) has carried out several test campaigns with the F-SAR system of the German Aerospace Center (DLR). The F-SAR offers the possibility of exploring the performance of a high resolution, fully polarimetric SAR system with five frequency bands in the range of 400 MHz to 10 GHz [1]. Here we report on use of P-band (435 MHz) data to change detection over three time points by means of an omnibus test statistic in the complex Wishart distribution [2]. Other results from this and other campaigns have previously been reported in [3].

Change detection is a very important method for many applications of remotely sensed data from space- and air-borne instruments. Especially SAR data are useful due to their all-weather capabilities, and hence planned acquisitions for change detection are normally secured. A number of studies have applied SAR data to change detection applications in single-channel SAR images applying different methods using e.g. the classical ratio detection [4,5], the Kittler-Illingworth threshold selection criterion [6–9], hidden Markov chains for thresholding [10], wavelets [11–14], linear features [15], Kullback-Leibler divergence [16], multivariate gamma distributions [17], neural networks [18], fusion of multi-similarity measures [19], and Markov random fields [20], where most of the methods are based on the classic ratio detector and improvements thereof. Also, methods for change detection using multichannel SAR data (e.g. polarimetric) have been studied using e.g. polarimetric parameters [21,22], Markov random fields for multichannel SAR data [23,24], generalized maximum likelihood test for covariance matrices [25] and the same test statistics for classification [26], partial vectors for suppression of the backscatter coefficient influence [27], the Hotelling-Lawley trace statistic [28], and a non-Wishart change detector [29], where a characteristic of the multichannel change detectors is that they are used for change detection between bi-temporal acquisitions.

When comparing several images one may apply the simple approach making pairwise comparisons. However, this approach makes it virtually impossible to control the rates of false positives (postulating a change when there actually is none) and of false negatives (missing an actual change). In general, a better approach for comparing several distributions is to perform a simultaneous test of the

hypothesis of homogeneity of the said distributions, a so-called omnibus test, see for example [30]. Here we enable this by deriving the likelihood ratio test statistic for equality of $k \geq 2$ complex variance-covariance matrices and finding an approximation for the distribution of this test statistic under the hypothesis of equality. If the conclusion of such an analysis is that the parameters in the underlying Wishart distributions are not constant, i.e., we have a non-stationary time series, then the question naturally arises, when do the changes actually occur? Here we describe a factorization of the likelihood ratio statistic into a product of test statistics that each test simpler hypotheses of homogeneity up to a certain point and that are independent if the hypothesis of total homogeneity is true. We show how this may be used in setting up a change detector for solving the said problem.

The methods described here were first presented in [2]. Further descriptions of visualization and software in this context are given [31,32].

2. POLARIMETRIC SAR

A fully polarimetric SAR measures the 2 by 2 complex so-called scattering matrix at each resolution cell on the ground. The scattering matrix relates the incident and the scattered electric fields, [33]. If S_{rt} denotes the complex scattering amplitude for receive and transmit polarization ($r, t \in \{h, v\}$ for horizontal and vertical polarization), then reciprocity, which normally applies to natural targets, gives $S_{hv} = S_{vh}$ (in the backscattering direction using the backscattering alignment convention, [33]) and S_{hv} is replaced by $(S_{hv} + S_{vh})/\sqrt{2}$. The scattering matrix is often represented by the three-component complex target vector $\mathbf{s} = [S_{hh} \ S_{hv} \ S_{vv}]^T$ where the superscript T denotes the matrix transpose.

The inherent speckle in the SAR data can be reduced by spatial averaging at the expense of spatial resolution. In this so-called multi-look case (below n is the number of looks) a more appropriate representation of the backscattered signal is the covariance matrix in which the average properties of a group of resolution cells can be expressed in a single matrix formed by the outer products of the averaged target vectors. The sample covariance matrix is defined as [33]

$$\begin{aligned} \langle \mathbf{C} \rangle &= \langle \mathbf{s}(i) \mathbf{s}(i)^H \rangle \\ &= \begin{bmatrix} \langle S_{hh} S_{hh}^* \rangle & \langle S_{hh} S_{hv}^* \rangle & \langle S_{hh} S_{vv}^* \rangle \\ \langle S_{hv} S_{hh}^* \rangle & \langle S_{hv} S_{hv}^* \rangle & \langle S_{hv} S_{vv}^* \rangle \\ \langle S_{vv} S_{hh}^* \rangle & \langle S_{vv} S_{hv}^* \rangle & \langle S_{vv} S_{vv}^* \rangle \end{bmatrix} \end{aligned}$$

where $\langle \cdot \rangle$ denotes ensemble averaging, the $*$ denotes complex conjugation, and the superscript H denotes the complex conjugate transpose. Reciprocity results in a covariance matrix with rank 3. $n\langle \mathbf{C} \rangle$ follows a complex Wishart distribution.

3. TEST STATISTICS AND THEIR DISTRIBUTIONS

The two subsections below briefly recapitulate the main results from [2] on the test statistics Q and R_j , and the corresponding probabilities P (probability of change) for finding smaller values of $-2\rho \ln Q$ and $-2\rho_j \ln R_j$ relating to change detection in a series of $k \geq 2$ covariance matrix representation multi-look SAR data with all (including the off-diagonal) elements in the covariance matrix, see [33].

The expressions for the test statistics and the corresponding probabilities are valid also for block-diagonal covariance data, e.g., diagonal-only data which works with single-look data also, but the values of the number of degrees of freedom and parameters ρ , ω_2 , ρ_j , and ω_{2j} change as given in [34].

3.1. Test for Equality of Several Complex Covariance Matrices

For the logarithm of the likelihood ratio test statistic Q for testing whether a series of $k \geq 2$ complex variance-covariance matrices Σ_i are equal, i.e., $\Sigma_1 = \Sigma_2 = \dots = \Sigma_k$ we get (for the real case see [35]; for the case with two complex matrices see [25, 36])

$$\ln Q = n\{pk \ln k + \sum_{i=1}^k \ln |\mathbf{X}_i| - k \ln |\sum_{i=1}^k \mathbf{X}_i|\}.$$

Here $|\cdot|$ denotes the determinant, n is the equivalent number of looks, the $\mathbf{X}_i$ (and the Σ_i) are p by p ($p = 3$ for full pol data, $p = 2$ for dual pol data, and $p = 1$ for single channel power data), and the $\mathbf{X}_i = n\hat{\Sigma}_i = n\langle \mathbf{C} \rangle_i$ follow the complex Wishart distribution, i.e., $\mathbf{X}_i \sim W_C(p, n, \Sigma_i)$.

Because Q tests for equality in all matrices simultaneously, it is termed an omnibus test statistic. $Q = 1$ for equality.

For full and dual pol data ($p = 3$ and $p = 2$) with all (including off-diagonal) elements in the covariance matrix and with

$$\begin{aligned} f &= (k-1)p^2 \\ \rho &= 1 - \frac{p(2p^2-1)}{6f} \left(\frac{k}{n} - \frac{1}{nk} \right) \\ \omega_2 &= -\frac{f}{4} \left(1 - \frac{1}{\rho} \right)^2 + \frac{p^2(p^2-1)}{24\rho^2} \left(\frac{k}{n^2} - \frac{1}{n^2k^2} \right) \end{aligned}$$

the probability of finding a smaller value of $-2\rho \ln Q$ is

$$\begin{aligned} P\{-2\rho \ln Q \leq z\} &\simeq P\{\chi^2(f) \leq z\} + \\ \omega_2 [P\{\chi^2(f+4) \leq z\} - P\{\chi^2(f) \leq z\}], \end{aligned}$$

where $z = -2\rho \ln q$ and q is a particular realization (the observed value) of the stochastic variable Q .

3.2. Test for Equality of First $j \leq k$ Matrices

If the above $\ln Q$ test statistic shows that we cannot reject the hypothesis $\Sigma_1 = \Sigma_2 = \dots = \Sigma_k$, there is no change over the time span covered by the data. If we can reject the hypothesis, there is change at some time point. To find that time point, we test whether the first j ($1 < j \leq k$) complex variance-covariance matrices Σ_i are equal, i.e., given that $\Sigma_1 = \Sigma_2 = \dots = \Sigma_{j-1}$, then the logarithm of the likelihood ratio test statistic R_j for testing $\Sigma_j = \Sigma_1$ is

$$\begin{aligned} \ln R_j &= n\{p(j \ln j - (j-1) \ln(j-1)) + \\ &\quad (j-1) \ln |\sum_{i=1}^{j-1} \mathbf{X}_i| + \ln |\mathbf{X}_j| - j \ln |\sum_{i=1}^j \mathbf{X}_i|\}. \end{aligned}$$

Furthermore, the R_j constitute a factorization of Q , $Q = \prod_{j=2}^k R_j$ or $\ln Q = \sum_{j=2}^k \ln R_j$.

For full and dual pol data ($p = 2$ and $p = 3$) with all (including off-diagonal) elements in the covariance matrix and with

$$\begin{aligned} g &= p^2 \\ \rho_j &= 1 - \frac{2p^2-1}{6pn} \left(1 + \frac{1}{j(j-1)} \right) \\ \omega_{2j} &= -\frac{p^2}{4} \left(1 - \frac{1}{\rho_j} \right)^2 + \frac{p^2(p^2-1)}{24n^2\rho_j^2} \left(1 + \frac{2j-1}{j^2(j-1)^2} \right) \end{aligned}$$

the probability of finding a smaller value of $-2\rho_j \ln R_j$ is

$$\begin{aligned} P\{-2\rho_j \ln R_j \leq z\} &\simeq P\{\chi^2(g) \leq z\} + \\ \omega_{2j} [P\{\chi^2(g+4) \leq z\} - P\{\chi^2(g) \leq z\}], \end{aligned}$$

where $z = -2\rho_j \ln r_j$ and r_j is a particular realization (the observed value) of the stochastic variable R_j .

In the block-diagonal case, i.e., for azimuthal symmetry and diagonal-only data, the expression for R_j is the same as here but the parameters in the distribution of $-2\rho_j \ln R_j$ are different, see [34].

3.3. The Loewner Order

The above test statistics Q and R_j are all symmetric in the matrices $\mathbf{X}_i$ (and $\mathbf{X}_j$). This means that they are insensitive to the direction of change.

For scalar quantities it is easy to establish whether one quantity is larger than another, for example, we can check whether the difference between them is positive, negative or zero. For matrices this is a more complicated matter. The Loewner (or Löwner) order provides a partial ordering of matrices [32, 37, 38]. Here in our context it gives a direction of change: does the radar response $\mathbf{X}$ at time point one and $\mathbf{Y}$ at time point two increase or decrease (or does it possibly change structure or nature) between the two time points? To establish the Loewner order we calculate the definiteness of the difference $\mathbf{X}_i - \mathbf{X}_{i+1}$. If $\mathbf{X}_i - \mathbf{X}_{i+1}$ is positive definite, we write $\mathbf{X}_{i+1} <_L \mathbf{X}_i$, if $\mathbf{X}_i - \mathbf{X}_{i+1}$ is negative definite, we write $\mathbf{X}_i <_L \mathbf{X}_{i+1}$.

4. AN EXAMPLE, DISKO ISLAND, GREENLAND

A test area near Qeqertarsuaq (Godhavn), Disko Island, Greenland provides opportunities for collecting SAR image data of buried test objects including humans hidden in snow caves, dog sleds and snow scooters (as well as test setups on the ice and snow cover of the Disko Island with tent and corner reflectors). Here, we work with two versions of the same data, one is single-look, another is multi-look (ENL = 3), both polarimetric F-SAR P-band (435 MHz) data near a (blue) cottage, see Figure 1. The data are acquired on 13 May 2015 (flightID 16) at three time points so k above is 3

1. passID 05 starting at 16:44:53 (t_1),
2. passID 06 starting at 16:55:43 (t_2),
3. passID 07 starting at 17:07:00 (t_3).

All three flights are with heading 175° . Flights are performed from a nominal altitude of 11,200 feet (3,675 m). The full polarimetry P-band sub-image data below are 450 rows by 3,000 columns. Pixels are 4.0 m (range) by 1.0 m (azimuth), images are 1.8 km by 3.0 km.

Fig. 1. Overview of experimentation area around the blue cottage, Disko Island, Greenland. Two snow scooters and two dog sleds were used as stationary and moving test objects during flights. Snow caves were dug for experiments with hidden objects: two persons, a ladder made of aluminium, and two aluminium boxes forming a dihedral corner structure with mostly double-bounce scattering. From [3].

4.1. Multi-look data, ENL = 3

Figure 2 shows $S_{hv}S_{hv}^*$, $S_{hh}S_{hh}^*$ and $S_{vv}S_{vv}^*$ as RGB for the three time points. Figure 3 shows change test statistics (top) $-2\rho \ln Q$, (middle) $-2\rho_2 \ln R_2$ and (bottom) $-2\rho_3 \ln R_3$ for the same area as in Figure 2. Change at some time point occurs where values of $-2\rho \ln Q$ are high (bright pixels), change between t_1 and t_2 occurs where values of $-2\rho_2 \ln R_2$ are high, change between t_2 and t_3 occurs where values of $-2\rho_3 \ln R_3$ are high. The horizontal stripes in Figure 3 indicating change just above the boxed zoom region are related to moving dog sleds and/or snow scooters. Their appearance in the images are shifted due to a velocity component perpendicular to the flight direction and smeared due to a component parallel to the flight direction.

Figure 4 shows a zoomed version of Figure 2 centered on the blue cottage seen in Figure 1. Figure 5 shows a zoomed version of Figure 3, same zoom as in Figure 4. Some of the detected targets in the zooms (right next to the digits 1 through 4) in Figure 5 top are objects buried in the snow, namely (here we focus on these objects)

1. a person,
2. an aluminium ladder,

3. two aluminium boxes,
4. another person.

We see that all four targets change between t_1 and t_2 , and that only targets ‘1’ and ‘4’ change between t_2 and t_3 . The Loewner order (not shown) can be used to illustrate whether targets come or go at different time points, see [32, 38]. (The only target detectable at a 5% significance level is the very bright target below target ‘4’; the remaining targets including the buried objects marked 1 through 4 exhibit less significant change.)

4.2. Single-look data

To improve the spatial resolution we work on single-look data also. The complex Wishart distribution based method in this case accommodates diagonal-only data but not polarimetric data. Figure 6 shows a zoomed version of the change test statistics in this case, same zoom as in Figure 5.

5. CONCLUSIONS

Using multi-look polarimetric covariance matrix data we detect the four buried objects albeit at low significance levels and judged visually with few false alarms.

Using single-look diagonal-only data causes buried object ‘4’ to be detected at a higher significance level for $-2\rho_2 \ln R_2$ detecting change between t_1 and t_2 . Contrary to this the multi-look change detected for object ‘4’ by $-2\rho_3 \ln R_3$ between t_2 and t_3 is not detected by single-look diagonal-only data. Also, objects ‘1’ and ‘3’ are not detected. Further, because of the absence of multi-look speckle reduction, single-look detection gives more false positives and a less conspicuous spatial “togetherness” of the detected objects.

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Fig. 2. Original data $S_{hv}S_{hv}^*$, $S_{hh}S_{hh}^*$ and $S_{vv}S_{vv}^*$ as RGB, three time points, note the red box on the top image (t_1); this is the zoom area centered on the blue cottage.

Fig. 3. Test statistics, $-2\rho \ln Q$ (top), $-2\rho_2 \ln R_2$ (middle), $-2\rho_3 \ln R_3$ (bottom). The (short) bright lines above the blue cottage are dog sled or snow scooter tracks.

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Fig. 4. Original data $S_{hv}S_{hv}^*$, $S_{hh}S_{hh}^*$ and $S_{vv}S_{vv}^*$ as RGB, three time points, zoom is centered on the blue cottage of images in Figure 2.

Fig. 5. Test statistics, same zooms on the blue cottage as in Figure 4; note labels '1', '2', '3' and '4' on the $-2\rho \ln Q$ image. These labels mark the buried objects.

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Fig. 6. Test statistics for single-look diagonal-only data, zoom.

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