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Tunneling cracks in arbitrary oriented off-axis lamina

Lars P. Mikkelsen · Simon J. Klitgaard ·
Christian F. Niordson · Bent F. Sørensen

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Abstract The steady-state energy release rate for tunneling cracks under mixed-mode loading is determined using finite element analyses. The balanced and symmetric laminate layup $[0/\theta/0/-\theta]_s$ is investigated, where the tunneling crack is located parallel to the fiber direction of the central off-axis oriented layer. It is found that for the steady-state situation, a simple energy balance calculation of the released energy of the separated crack surfaces for a fully developed crack gives the same value as the average value of a detailed J-integral analysis of the crack front. Furthermore, the crack front mode-mixity, obtained by the same energy balance calculation, was found to give a good prediction of the average mode-mixity found from a detailed stress intensity calculation along the crack-tip. Based on this simplified energy balance approach, the energy release rate is determined for all angles $\theta \in]0; 90]^\circ$ for orthotropic elastic properties ranging from typical low modulus glass fiber reinforced polymers to high modulus carbon fiber reinforced polymers. The predicted results can be used to investigate the influence of the layup angles on static and fatigue tunnel crack evolution in composite materials used within, e.g. automotive, aerospace, and wind energy applications.

Corresponding Author:

Lars P. Mikkelsen
DTU Wind Energy
Technical University of Denmark
Frederiksborgvej 399
4000 Roskilde, Denmark
Tel.: +45 21325406
E-mail: lapm@dtu.dk

Simon J. Klitgaard
Technical University of Denmark
2800 Lyngby, Denmark

Christian F. Niordson
DTU Mechanical Engineering,
2800 Lyngby, Denmark

Bent F. Sørensen
DTU Wind Energy
4000 Roskilde, Denmark

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1 Introduction

Damage initiation in fatigue loaded fiber reinforced polymers with secondary oriented fibers is often initiated by so-called tunneling cracks growing along the secondary fibers (Reifsnider and Jamison, 1982; Tong et al., 1997; Jespersen et al., 2018). Tunneling cracks will often penetrate into the load-carrying laminas and induce fibre damage leading to the final failure of the composite (Jamison et al., 1984; Zangenberg et al., 2013; Jespersen and Mikkelsen, 2017; Mikkelsen, 2020). In other cases, crack deflection can occur at the interface to the neighboring unidirectional (UD) ply, causing the formation of delamination cracks (Talreja, 1981). For high cycle fatigue applications, such fatigue damage initiations need to be controlled, experimentally validated, and, if possible, suppressed. An example of such a high cycle fatigue application is load-carrying laminates in wind turbine blades, which will experience hundreds of millions of load cycles throughout its lifetime. Experimentally, this can be addressed measuring tunneling crack growth rates supplemented by numerical finite element predictions of the crack-tip loading as performed by e.g. Quaresimin et al. (2014). Crack-tip finite element simulations, which in the off-axis case, is getting complicated since crack growth depends on the variation of both the energy release rate and the mode-mixity along the crack front where the crack front shape and thus the local energy release rate and the local mode-mixity are unknown. To address this in the present work, an energy balance introduced by Quaresimin et al. (2014), will be validated using detailed 3D finite element simulations.

A tunneling crack is assumed to initiate from a flaw in the off-axis ply or from free edges (Tong, 2000). Once a crack has formed, it will propagate both in the width direction and the thickness direction towards the adjacent plies' interfaces. Having reached the interfaces, the subsequent crack propagation occurs only in the transverse direction parallel to the fibre direction. When the tunneling crack tip front is far away from the edges, a steady-state situation is attained. It is then possible to calculate the average energy release rate along the crack front by energy an balance (Beuth, 1992; Ho and Suo, 1993). The approach uses the crack opening displacement profile far behind the crack tip and the stress state far ahead of the crack tip (Ho and Suo, 1993).

More precisely, for a symmetrically tunneling crack of height $2h$, see figure 1, the steady-state energy release rate is obtained as (Ho and Suo, 1993)

$$\hat{G}_{ss} = \frac{1}{2h} \int_0^h \sigma_{22}(z) \delta_n(z) dz \quad (1)$$

where $\sigma_{22}(z)$ is the in-plane normal stress perpendicular to the crack plane far ahead of the crack, $\delta_n(z)$ is the crack opening profile far behind the crack front and $\hat{(\cdot)}$ denotes terms found from the energy balance approach. The benefit of the energy balance approach is that the crack front (for which $\hat{G}_{ss}$ is found) does not need to be part of the numerical model and for a cross-ply laminate, the predictions can simply be based on a 2D model as indicated by the green plane in figure 1.

Fig. 1: A part of a cross-ply laminate with a 90-degree tunneling crack where the green plane indicates the 2D model used by Ho and Suo (1993)

The relevant in-plane normal stress can be evaluated for a laminate without any cracks, and may suitably be found based on classical laminate theory representing an infinitely large plate. For a symmetric laminate subjected to tensile loading, the stresses are constant through the thickness in the un-cracked lamina, and the expression for the steady-state energy release rate can be written as

$$\hat{G}_{ss} = \frac{1}{2} \sigma_{22} \bar{\delta}_n \quad (2)$$

where $\bar{\delta}_n$ denotes the average value of the normal opening $\delta(z)$.

For a tunneling crack in a 90° ply, the crack tip is subjected to pure Mode I. Hence, the crack front shape will, adjust such that the local energy release rate is constant along the crack front and equal to the Mode I fracture energy, $G(z) = G_{IC}$.

An analysis of free surface channeling cracks with different crack front shapes, Nakamura and Kamath (1992) has shown that the stress state only differs within a distance of a few lamina thicknesses, $2h$, away from the crack front. Consequently, the stress state well ahead of the crack will be independent of the actual crack front shape. Hence, the calculation of the energy release rate by the energy balance approach does not require detailed modeling of a correct shaped crack front.

Quaresimin et al. (2014) proposed a generalization of the energy balance approach for calculating the steady-state energy release rate and the related mode-mixity for a general off-axis case. In Quaresimin et al. (2014), experimentally measured fatigue crack-growth rates in a glass-fiber composite (which later in this paper will be denoted GlassFRP1) was evaluated using this energy balance approach for two different off-axis layup angles of 50° and 60°. Quaresimin et al. (2014), compared for those specific cases, the average energy release using the energy balance method with the mean value of the J-integral along the crack front and found a good agreement. However, the accuracy of the mode-mixity was not addressed, and the validation of the energy balance method was only established

for those two particular cases. An extensive validation is the purpose of this paper, where numerical comparisons of full 3D crack front analyses of the energy release rate and the mode-mixity with the energy balance based predictions are established.

In contrast to the Mode I, 90-degree tunneling crack case analyzed by Beuth (1992) and Ho and Suo (1993), the crack growth-rate of an off-axis crack will depend on both the crack-tip energy release rate $G_{ss}(z)$ and the mode-mixity parameter $\psi(z)$. In order to obtain static crack-growth, the requirement of onset of cracking $G(z) = G_C(\psi)$ needs to be full-filled in all points along the unknown shaped crack front. For cyclic loading, the crack front shape is still unknown, but now all points progress with the same crack-growth rate. Quaresimin et al. (2014) addressed this by the use of a thickness averaged steady-state energy release rate $\bar{G}_{ss}$ and a thickness averaged mode-mixity parameter $\bar{\psi}$. In the present work, all terms denoted by $(\bar{})$ will indicate an averaged term defined by

$$(\bar{}) = \frac{1}{h} \int_0^h () dz = \frac{1}{2h} \int_{-h}^h () dz, \quad (3)$$

and evaluated numerically using the trapezoidal rule.

The averaged energy release rate and mode-mixity parameters can be calculated from either a crack-tip simulation or by a generalization of the energy approach, Eqs. (1)-(2). It will be shown that a benefit of using the energy approach is that it is similar to the green plane of the cross-ply laminate in figure 1 also for the off-axis case in that it is not necessary to include the actual crack front in the simulation. Furthermore, a good agreement between the two methods will be demonstrated. Using the energy balance approach, the steady-state energy release rate, $\bar{G}_{ss}$, and the mode-mixity, $\bar{\psi}$ for the full off-axis range, $\theta \in [0; 90]^\circ$, will be determined for parameters covering conventional glass and carbon fiber reinforced polymer composites.

Although linear elastic fracture mechanics serves as foundation and limitation of the investigations in this paper, non-linear material behavior is often observed due to the extensive shear deformation of the resin between the fibers. Nevertheless, as shown for e.g. the material system studied by Varna et al. (1999), non-linearity will not dominate at strains levels below 0.4-0.5% relevant for many high cycle fatigue applications such as load-carrying laminates in wind turbine blades.

2 Problem description

2.1 Laminate with an off-axis tunneling crack

The energy release rate for an isolated off-axis tunneling crack is investigated under steady-state growth conditions. The layup investigated is a symmetric and balanced layup $[0/\theta/0/-\theta]_s$ loaded in uniaxial tension as shown in figure 2a. The plate has the overall length L , width W , and thickness $2H$. The off-axis crack is located in the central layer of the composite layup in a direction parallel to the fiber direction θ . Since each ply has the thickness h , the off-axis crack has the thickness $2h$. Due to symmetry, only half of the $2H$ thick plate is modeled with a symmetry boundary condition along the symmetry plane ($z = 0$), as shown in

Fig. 2: The plate with the tunneling crack under investigation in the present study. The global x, y, z coordinate and the local coordinate systems x_1, x_2, x_3 are indicated.

figure 2b. The plate is elongated by a prescribed uniform displacement along the plate end resulting in the axial force N_x .

In the present paper, we operate with two coordinate systems, a global x, y, z coordinate system, and a local, ply coordinate system, x_1, x_2, x_3 . In the global coordinate system, x is the loading direction, y is the in-plane width direction, and z is the out-of-plane direction. For the local coordinate system, x_1 is parallel to the fibre direction in the central layer, and x_2 is the in-plane coordinate perpendicular to the fibre direction. The global z and the local x_3 axes coincide.

Table 1: Materials system used

Material unit	E_1 [GPa]	$E_2 = E_3$ [GPa]	$\nu_{12} = \nu_{13}$ [-]	ν_{23} [-]	$G_{12} = G_{13}$ [GPa]	G_{23} [GPa]	λ [-]	ρ [-]
GlassFRP1	30.6	8.62	0.29	0.33	3.25	2.90	0.282	2.34
GlassFRP2	41.2	10.3	0.26	0.41	3.79	3.66	0.250	2.59
CarbonFRP1	126	7.56	0.26	0.40	3.69	2.70	0.060	4.12
CarbonFRP2	266	5.49	0.27	0.40	3.54	2.37	0.021	5.36

The layup is modelled as homogeneous orthotropic materials with material properties representing typical glass and carbon fiber composites. Four material cases are investigated as specified in Table 1. Here, E_1 is the Young's modulus in the fibre direction (the x_1 -direction) while E_2 and E_3 are the Young's moduli in the transverse in-plane, x_2 , and out-of-plane, x_3 , directions, respectively. The in-plane shear modulus is denoted G_{12} , while G_{13} and G_{23} are the out-of-plane shear moduli. Correspondingly, ν_{12} is the major in-plane Poisson ratio, while ν_{13} and ν_{23} are the out-of-plane Poisson ratios. We take the common assumption of transversal isotropi around the fibre direction so that $E_2 = E_3$, $\nu_{12} = \nu_{13}$, $G_{12} = G_{13}$, and $G_{23} = E_2/(2(1 + \nu_{23}))$. The four material systems are denoted GlassFRP1, GlassFRP2, CarbonFRP1 and CarbonFRP2. The properties of the GlassFRP1 is identical to the material system investigated by Quaresimin et al. (2014). The two non-dimensional parameters λ and ρ ,

$$\lambda = \frac{E_2}{E_1} \quad \rho = \frac{\sqrt{E_1 E_2}}{2G_{12}} - \sqrt{\nu_{12} \nu_{21}} \quad (4)$$

are of central importance, as it has been shown by Suo et al. (1991) that for a 2-dimensional orthotropic linear elastic material with prescribed tractions, the stress field depends only on these. Residual stresses are not included in the present analysis.

In the following, the results for the stresses, σ_{22} , τ_{12} , for the crack-openings, δ_n , δ_t , for the J-integral and energy release rates, J and G , will be presented in non-dimensional form as follows:

$$\begin{aligned} \sigma_{22}^* &= \frac{\sigma_{22}}{\sigma_{xx}} ; \quad \tau_{12}^* = \frac{\tau_{12}}{\sigma_{xx}} ; \quad \delta_n^* = \frac{\delta_n}{2h\varepsilon_{xx}} ; \delta_t^* = \frac{\delta_t}{2h\varepsilon_{xx}} ; \\ J^* &= \frac{J}{2h\varepsilon_{xx}\sigma_{xx}} \text{ and } G^* = \frac{G}{2h\varepsilon_{xx}\sigma_{xx}} \end{aligned} \quad (5)$$

The normalizations are introduced in order to make the solutions independent of load and ply thickness. The normalization parameters ε_{xx} and σ_{xx} are the local axial strain and stress in the off-axis layer obtained from a classical laminate theory. At the same time, h is the lamina thickness, which except in figure 14 is given by $h = H/4$. The normalization factor for the J-integral and the energy release rates, J , G agrees with the normalization factor $\sigma_{xx}^2 2h/\bar{E}_x$ used by Ho and Suo (1993); Hutchinson and Suo (1992) for the $\theta = 90^\circ$ case. Ho and Suo (1993) and Hutchinson and Suo (1992) used a normalization parameter that contains h instead of $2h$ as the total thickness of the tunneling crack in their case is h .

3 Theory - tunnelling cracks in orthotropic materials

3.1 Mechanics of cracks in orthotropic materials

For an orthotropic linear elastic material, the relationship between the Mode I and Mode II stress intensity factors, K_I and K_{II} and the Mode I and Mode II energy release rates, G_I and G_{II} , is (Bao et al., 1992)

$$\begin{Bmatrix} G_I \\ G_{II} \end{Bmatrix} = \left(\frac{1 + \rho}{2E_1 E_2} \right)^{1/2} \begin{Bmatrix} \lambda^{-1/4} K_I^2 \\ \lambda^{+1/4} K_{II}^2 \end{Bmatrix}. \quad (6)$$

The linear elastic fracture mechanics mode-mixity is defined as the phase angle of the stress intensity factors,

$$\psi = \arctan\left(\frac{K_{II}}{K_I}\right) = \arctan\left(\lambda^{-1/4} \sqrt{\frac{G_{II}}{G_I}}\right). \quad (7)$$

3.2 Analysis of an off-axis tunneling crack

The analysis of the more general case with $\theta \in]0; 90]^\circ$ is described in the following and details the derivation by Quaresimin et al. (2014). The stress state at the tunneling crack has a varying mode-mixity depending on the position along the crack front. Since the fracture energy is usually mode-mixity dependent, $G_c = G_c(\psi)$ (Hutchinson and Suo, 1992), the energy dissipation at the crack front may vary across the thickness of the layer. Still, under steady-state cracking, the crack front will adjust itself so that for all points across the layer, the local energy release rate will be identical to the fracture energy for the associated mode-mixity. Consequently, in the steady-state situation, energy dissipation at the crack front will remain the same (independent of crack length) since the crack front retains the same shape and local mode-mixity distribution.

Consider an advancement Δa of the crack front under steady-state conditions. Although the crack front is curved, the crack advance (in the x_1 -direction) will be the same along the entire front. The crack front thus maintains its shape, and the crack front moves across the width of the specimen in a self-similar fashion. During steady-state crack growth, the crack front is remote from edges such that a zone exists far ahead of the crack front where the stress field is uniform (i.e., independent of x_1 position). Likewise, a zone exists far behind the crack front in which the crack opening profile is independent of crack advance. The stress and displacement fields near the crack front are invariant, in the sense that they translate along with the crack front in a self-similar manner. For this steady-state situation, a tunneling crack advance by the distance Δa , corresponds exactly to remove a slice of the laminate of width Δa far ahead of the crack (uniform stress and therefore independent of the x_1 -position) and inserting a slice of a laminate of width Δa far behind the crack front. For fixed applied displacements, the change in the potential energy for the half of the symmetric problem between the two states (before and after crack advance) can be determined as the work required to close the crack and re-establish the stress state far ahead of the crack:

$$\Delta U = \Delta U_I + \Delta U_{II} \quad (8)$$

where

$$\Delta U_I = \frac{\Delta a}{2} \int_0^h \sigma_{22} \delta_n(z) dz \quad (9)$$

and

$$\Delta U_{II} = \frac{\Delta a}{2} \int_0^h \tau_{12} \delta_t(z) dz. \quad (10)$$

Here, δ_n , δ_t are the normal and tangential crack openings, respectively. The work for the stresses to close the crack is precisely identical to the reverse problem - the energy released for the advancing crack, or more precisely, the total energy release rate integrated across the layer thickness multiplied by the crack advance Δa ,

$$\Delta U = \Delta a \int_0^h G_{ss}(z) dz. \quad (11)$$

From the expressions (8) and (11), we can now define an average steady-state energy release rate

$$\hat{G}_{ss} = \frac{\Delta U}{h \Delta a} = \frac{1}{2} \sigma_{22} \bar{\delta}_n + \frac{1}{2} \tau_{12} \bar{\delta}_t, \quad (12)$$

where the two terms, $\bar{\delta}_n$, $\bar{\delta}_t$, in (12) are the average values, found from equation (3), of the normal and the tangential opening displacements of the tunneling crack surfaces far behind the crack tip.

In the following, we will split the steady-state energy release rate into two parts, a Mode I part and a Mode II part. Although it cannot be established rigorously, we will base the partitioning of the energy release rates on the two terms of the energy balance method. Hence, we define

$$\hat{G}_I = \frac{1}{2} \sigma_{22} \bar{\delta}_n \quad (13)$$

and

$$\hat{G}_{II} = \frac{1}{2} \tau_{12} \bar{\delta}_t, \quad (14)$$

respectively, so that

$$\hat{G}_{ss} = \hat{G}_I + \hat{G}_{II}. \quad (15)$$

Inspired by equation (7), we can define a nominal mode-mixity parameter by

$$\hat{\psi} = \arctan \left(\lambda^{-1/4} \sqrt{\frac{\hat{G}_{II}}{\hat{G}_I}} \right). \quad (16)$$

3.3 Finite element models

The tunneling crack problem will be analyzed using finite element models with the discretizations shown in figure 3. In figure 3a, a top-view of the mesh used for the J-integral and stress intensity factor determination is shown. The model represents a plate with the dimension $L \times W = 140h \times 60h$ consisting of around 6 millions 8-node brick elements with a focused homogeneous mesh at the crack tip with the element dimensions $L_{el} \times W_{el} \times h_{el} = 0.03h \times 0.03h \times 0.008h$. A version with a focused mesh in the middle of the half-width crack and $L = 170h$ has been used for the energy balance method for the half-width case. Figure 3b shows the full-width crack plate with the dimension of $L \times W = 140h \times 40h$ and a mesh of around 3.5 millions 8-node brick elements with a focused homogeneous mesh at the center region of the tunnel-crack with the element dimensions $L_{el} \times W_{el} \times h_{el} = 0.04h \times 0.04h \times 0.01h$. Figure 3c shows the model used for the small angle ($\theta < 45^\circ$) cases with dimensions of $L \times W = 180h \times 220h$ with 5 millions elements again with a focused homogeneous mesh in the center part of the tunnel-crack with the element dimensions $L_{el} \times W_{el} \times h_{el} = 0.04h \times 0.04h \times 0.01h$. The numerical models simulated in the commercial finite element code Abaqus (2019) representing the different versions described above can be downloaded from (Mikkelsen et al., 2020).

The mesh refinement is based on a convergence study ensuring that halving the linear element size results in less than 0.2% change in the calculated energy release rate. The stress intensity factors extracted along the straight crack front for the nine validation cases (which will be described later) have been a little more challenging, and a more focused mesh has been used in some of these cases. These cases are also available from (Mikkelsen et al., 2020). Figure 4, shows the focused mesh of the region used for evaluating the J-integral, the stress intensity, and the energy balance. The solutions from the linear finite element model using Abaqus standard (Abaqus, 2019) is obtained in 1-4 hours using 200 CPUs on a Linux cluster.

With these meshes, the energy release rate of an off-axis tunneling crack is analyzed using the following three approaches:

1. A local J-integral value, $J(z)$, evaluated at points along a straight crack front of the tunneling crack.
2. A local value of the stress intensity factors, $K_I(z); K_{II}(z)$, evaluated along a straight crack front of the tunneling crack.
3. The potential energy balance, $\Delta U_I; \Delta U_{II}$, calculated from the far-field stress state and the crack opening in a region far behind the crack front

Figure 5 illustrates the normal $\delta_n(z)$ and tangential $\delta_t(z)$ crack opening displacements of points (finite element nodes) along the crack opening as well as the crack-tip values of stress intensity factors and J-integral values extracted from the finite element model. As shown in the figure, the crack front is modeled as being straight. This assumption will be discussed later in the paper.

Approach 1 utilizes a built-in evaluation of the J-integral from the commercial finite element code Abaqus, which is based on a domain integral formulated by Shih, Moran, and Nakamura (1986). From the J-value along the crack front, a mean J-integral value is calculated as the mean of the mean values for the J-integral for

Fig. 3: Top view of finite element meshes where the red line identify the predefined crack.

the contours 4-7 around the crack tip; each averaged across the ply thickness, h .

$$\bar{J} = \frac{1}{4} \sum_{k=4}^7 \bar{J}^k \text{ with } \bar{J}^k = \frac{1}{h} \int_0^h J(z) dz \quad (17)$$

where k indicate the contour number with $k = 1$ closest to the crack tip.

Fig. 4: Zoomed-in side-view of finite element mesh where the red line identify the predefined crack. Dashed line indicate the interface to the 0° ply. For the stress-intensity crack-tip simulations this will be separated from the predefined crack with a layer of 1-3 elements.

Approach 2 calculates the local stress intensity factors along the crack front using a built-in function in Abaqus. This functionality can be used to determine the stress intensity factors locally for an anisotropic material using an interaction integral method as proposed by Shih and Asaro (1988). For more details on the implementation, see the documentation Abaqus (2019). The stress intensity factors can not be calculated along an interface between two dissimilar materials (Abaqus, 2019). Therefore it has been necessary to add a very thin layer of $0.01h$ below the tunneling crack in order to separate the crack-front in the off-axis layer from the 0° layer. By comparing the J-integral for models with and without this separating layer, it has been found that this layer will influence the averaged J-integral value by less than 1%.

Approach 3 is the generalized energy balance method presented in eq. (12) as first proposed by Quaresimin et al. (2014). The method uses the far-field stress state calculated using a classical laminate theory and the crack opening profile extracted from the deformed mesh.

The first two approaches require that the FE-model includes the crack-tip as shown for the half-crack in figure 6a using the finite element mesh shown in figure 3a. In figure 6, the parameter a denotes the length of the tunneling crack. The third method (the energy balance) does not require a crack-tip in the model. Therefore, the energy balance calculations can be made using the more simple case shown in figure 6b using the meshes shown in figure 3b and c. The three approaches are described in detail in the following section.

Results from all three approaches will be compared for nine cases in total, namely, the three materials GlassFRP1, CarbonFRP1 and CarbonFRP1 for the three layups $\theta = 45^\circ$, $\theta = 60^\circ$ and $\theta = 75^\circ$. In this part of the study, the material case GlassFRP2 is omitted due to its similarity to the predictions to GlassFRP1. A sufficiently large plate geometry is needed in order to obtain plate size independent

Fig. 5: Sketch of the concept for crack-tip analysis (straight crack front) and crack opening displacement analysis behind the crack tip.

results. This is obtained using a plate length of $L/h = 170$ ($L/h = 140$ for the crack-tip analysis) together with a plate width on $W/h = 60$ for the half-width crack case and $L/h = 140$ together with $W/h = 40$ for the full-width crack case. The solutions are judged to be independent of L and W when they differ by less than 0.1% when doubling the length or width.

3.4 The energy balance approach

In order to evaluate the energy release rate using equation (12), the stress-state far in front of the crack is needed. Figure 7 shows the shear and transverse normal stresses found in the local material coordinate system ($x_1 - x_2$) in the central off-axis layer for laminate $[0/\theta/0/-\theta]_s$ without a tunneling crack. As expected, a perfect match is found between stress components calculated through the classical laminate theory and a linear finite element model of the laminate. In figure 7, the stresses are normalized by the local axial stress σ_{xx} acting in the same off-axis layers. For all four material systems, compressive transverse normal stresses ($\sigma_{22} < 0$) for small off-axis angles and tensile normal stresses ($\sigma_{22} > 0$) for larger off-axis angles are observed. Hence, for small off-axis angles, a tunneling crack is expected to experience contact between the crack faces accompanied by compressive normal stresses. For larger angles, the crack-opening normal stress (σ_{22}) is positive and increases monotonically reaching a maximum value $\sigma_{22}/\sigma_{xx} = 1$ for $\theta = 90^\circ$. The shear stress τ_{12} is always positive and exhibits a maximum in the range $\theta \in [50; 60]^\circ$ depending on the material properties. As expected for symmetry reasons, τ_{12} vanishes both for $\theta = 0^\circ$ and $\theta = 90^\circ$. Quaresimin et al. (2014) studied the two cases $\theta = 50^\circ$ and $\theta = 60^\circ$ for the material GlassFRP1.

(a) Half-width crack

(b) Full-width crack

Fig. 6: Geometries used for the determination of the energy release rate.

In the following, the GlassFRP1 and CarbonFRP1 layup $[0/\theta/0/-\theta]_s$ with $\theta = 45^\circ$ will be used as two reference cases. For these two reference cases, the figure 7 shows that

$$\begin{aligned} \text{GlassFRP1: } \sigma_{22}^* &= 0.2847, \quad \tau_{12}^* = 0.4327 \\ \text{CarbonFRP1: } \sigma_{22}^* &= 0.0852, \quad \tau_{12}^* = 0.3932 \end{aligned} \quad (18)$$

where $()^*$ indicates a normalized component following (5). These values will be used in the calculation of the normalized energy release rate for both the two cases.

Figure 8 shows the crack opening displacements of the crack surface far behind the crack tip for the two reference cases predicted by the FE-model for the full-width crack. Despite a small positive value of the normal stress $\sigma_{22}^* = 0.0852$ for CarbonFRP1, the normal crack opening displacement δ_n is negative in the lower part of the crack. A negative normal opening (crack face penetration) is physically

Fig. 7: The normalized stress components σ_{22}/σ_{xx} and τ_{12}/σ_{xx} in the off-axis layer versus the off-axis angle θ found using a classical laminate theory.

unrealistic. This crack face penetration is an outcome of not taking into account the contact between the crack surfaces in the finite element simulations. In the following, those unrealistic negative crack openings will be set artificially to zero, as shown in the bottom part of the CarbonFRP1 case in figure 8. An average normal and tangential crack opening displacement are - for the data points shown in figure 8 with $\theta = 45^\circ$ - found to be

$$\begin{aligned} \text{GlassFRP1: } \bar{\delta}_n^* &= 0.4822, \quad \bar{\delta}_t^* = 1.0164 \\ \text{CarbonFRP1: } \bar{\delta}_n^* &= 0.1081, \quad \bar{\delta}_t^* = 1.1696 \end{aligned} \quad (19)$$

Substituting the values for the GlassFRP1 from (18) and (19) into into the energy balance relations (13) and (14) gives an estimate for the average value of the energy release rate

$$\hat{G}_I^* = \frac{1}{2} \sigma_{22}^* \bar{\delta}_n^* = \frac{1}{2} \cdot 0.2847 \cdot 0.4822 = 0.0686 \quad (20)$$

$$\hat{G}_{II}^* = \frac{1}{2} \tau_{12}^* \bar{\delta}_t^* = \frac{1}{2} \cdot 0.4327 \cdot 1.0164 = 0.2199 \quad (21)$$

which substituted into (15) and (16), gives an estimate for the steady-state energy release rate and the mode-mixity

$$\text{GlassFRP1: } \hat{G}_{ss}^* = 0.289 \text{ and } \hat{\psi} = 67.9^\circ. \quad (22)$$

Fig. 8: Normal, δ_n , and tangential, δ_t , crack opening displacements across the layer for the GlassFRP1 and CarbonFRP1 case with $\theta = 45^\circ$.

Doing the same for the CarbonFRP1 case gives

$$\text{CarbonFRP1: } \hat{G}_{ss}^* = 0.235 \text{ and } \hat{\psi} = 86.0^\circ. \quad (23)$$

3.5 Crack-tip simulations

In order to investigate how well the above calculated energy release rates and mode-mixities following the energy balance methods predicts the actual energy release rates calculated along the crack front using the J-integral or stress intensity factors a comparison is made in the following.

Figure 9 shows the J-integral evaluated along the tunneling crack tip (approach 1) with a straight crack front orthogonal to the tunneling crack growth direction for the GlassFRP1 case. From Figure 9, it can be seen that except for the first 2-3 contours, the J-integral values for the different contours coincide as shown in the figure as coinciding solid curves. This is an expected result due to the path independence of the J-integral. In the following, the mean values of the J-integral will be found from the contours number 4-7, as shown in equation (17). For the GlassFRP1 reference case, the $\hat{G}_{ss}^* = 0.289$ value calculated in (22) using the energy balance method is found to differ less than 1% from the mean J-integral value $\bar{J}^* = 0.291$ found from the results in figure 9.

In addition to the local J-integral (approach 1), also the stress intensity factors, $K_I(z)$, $K_{II}(z)$, (approach 2) and thereby the local energy release rates, $G_I(z)$,

Fig. 9: J-integral values for different integration contours are shown as a function of position along a straight crack front for the GlassFRP1 case with $\theta = 45^\circ$

$G_{II}(z)$, along the crack front using equations (6) can be determined. Figure 10 shows the calculated local energy release rates for the same case with the off-axis direction given by $\theta = 45^\circ$ as used in Figure 9 for the J-integral. Only the coinciding curves from contour 4-7 are shown. The average values have been calculated similarly to equation (17) for the average of the J-integral. The average energy release rate found along the crack front $\bar{G}_{ss} = \bar{G}_I + \bar{G}_{II}$ in figure 10 was found to $\bar{G}_{ss}^* = 0.302$. This value differs 4% from the J-integral obtained using the same mesh along the same crack front.

The averaged J-integral value $\bar{J}$ and the averaged energy release rate $\bar{G}_{ss}$ should be equal to each other for a sufficiently refined mesh. Nevertheless, for a given mesh, the J-integral method is most likely a more robust than the stress intensity calculation. Therefore, in the following the J-integral will be used for validating the energy balance based $\hat{G}_{ss}$ estimate.

In addition to the energy release rate, we would also like to extract the average mode-mixity along the crack front in order to compare that with the mode-mixity prediction based on the energy balance method. The mode-mixity can not be calculated from the J-integral. Here, we need to be able to separate the two contributions in the form $K_I(z)$, $K_{II}(z)$ or $G_I(z)$, $G_{II}(z)$ from which the mode-mixity can be calculated using equation (7). A total of 9 cases is shown in figure 11. The three materials: GlassFRP1, CarbonFRP1 and CarbonFRP2 are employed for the three layup angles $\theta = 45^\circ$, $\theta = 60^\circ$ and $\theta = 75^\circ$. For the $\theta = 75^\circ$ case, the Abaqus built-in function for extracting the local stress intensity factors along the crack front was not able to calculate values along the entire crack front. The missing part shown as the dotted part of the three curves for $\theta = 75^\circ$ in figure 11 was interpolated, assuming a linear relation between $J(z)$ and $G(z)$.

Fig. 10: The local energy release rates, G_I , G_{II} and $G_{ss} = G_I + G_{II}$ is shown as a function of position along a straight crack front for the GlassFRP1 case with $\theta = 45^\circ$

For the specific case with a straight crack front, the local mode-mixity is found to be rather constant along the whole crack front, as shown in figure 11. The largest deviation from the average value was found near the bottom of the layer ($z/(2h) = -0.5$), where the tunneling crack tip stress field is likely to be influenced by the crack-tip along the interface to the neighboring UD layer. The mean value of the mode mixities, $\bar{\psi}$, was for the 9 cases found in a range of $\bar{\psi} \in [20; 89]^\circ$. For the two reference cases GlassFRP1 and CarbonFRP1 for $\theta = 45^\circ$, a mode-mixity prediction on $\bar{\psi} = 67.9^\circ$ and $\bar{\psi} = 86.0^\circ$, respectively, was obtained using the energy balance method. The corresponding averaged mode-mixities found from the stress intensity factors in figure 11 are $\bar{\psi} = 66.9^\circ$ and $\bar{\psi} = 84.9^\circ$ for the two reference cases, respectively. The difference between the mode-mixity values is thus only around 2% despite a larger difference on the $\bar{G}_I$ and $\bar{G}_{II}$ values on 10% and 3%, respectively.

In summary, the energy balance method is found to give a good prediction of the average energy release rate and mode-mixity along a straight crack front for the two reference cases.

$$\bar{G}_I \approx \hat{G}_I ; \bar{G}_{II} \approx \hat{G}_{II} ; \bar{G}_{ss} \approx \bar{J} \approx \hat{G}_{ss} \text{ and } \bar{\psi} \approx \hat{\psi}. \quad (24)$$

It will later be demonstrated that this agreement is achieved for all the nine cases presented in figure 11 covering the mode-mixities $\bar{\psi} \in [20; 89]^\circ$.

Fig. 11: The local mode-mixity as a function of position along a straight crack front for nine different mode-mixities in the range for 20° to 89° . The dotted part of the curves are interpolated based on the calculated solid part of the lines

4 Results

4.1 Energy balance values along the crack

In figure 12, the solid and dotted curves show the average energy release rates, $\hat{G}_I$, $\hat{G}_{II}$, and $\hat{G}_{ss}$ calculated using the energy balance (approach 3), along the crack path for the full- and half-width case, respectively. The figure shows the curves for the reference GlassFRP1 case with $\theta = 45^\circ$. The average energy release rate is calculated along the tunneling crack starting at the laminate edge approaching either the crack-tip for a half-width crack case or the opposite plate edge for the full-width case. Except from near the edge and the crack-front, a rather constant value is found for the average energy release rates, confirming the anticipated steady-state nature of the tunneling crack problem. Note that for the half-width crack, $\hat{G}_I$ is lower than the steady-state value for $x_1/a < 0.75$ as the crack opening displacement near the crack front is less than the steady-state opening further away. For all the cases analyzed in the following the steady-state energy release rate values will be taken at $x_1/a = 0.5$ i.e., at the midpoint position between the edge and crack tip for the half-width crack case and in the midpoint of the plate for the full-width crack case.

The average values obtained from crack-tip simulations are shown in figure 12 as circle-marks for the averaged J-integral and star-marks for the averaged energy release rate calculated from the local stress intensity factors. Note that, as the J-integral value can not be separated into a Mode I and Mode II contribution, there is only shown one circle mark in figure 12. Even though the crack-front values have

Fig. 12: The variation of the energy balance $\hat{G}_I$, $\hat{G}_{II}$ and $\hat{G}_{ss} = \hat{G}_I + \hat{G}_{II}$ along the crack analysed for the half-width ($a/h \approx 42$) and full-width crack ($a/h \approx 57$) for GlassFRP1 with $\theta = 45^\circ$. The energy balance is compared with the mean values of the J-integral and the energy release rate G calculated from the stress intensity factors of the half-width case along the crack front. Even though the crack front values have been calculated at the crack front, $x_1 = a$, the values have been shown at the crack mid-point, $x_1/a = 0.5$, for easing the comparison. The J-integral value can not be separated into a Mode I and Mode II contribution and is therefore only shown as one circle mark.

been calculated at the crack-front, $x_1 = a$, the values have been marked on the figure at the tunnel crack mid-point, $x_1/a = 0.5$, easing the comparison with the mid-point values from the energy balance approach. The difference between the non-coinciding mean values of the J-integral and the energy release rates achieved along the crack-front can be seen from the non-coinciding black star and circle marks in figure 12. Also, the two contributions $\hat{G}_I$ and $\hat{G}_{II}$ can be seen in figure 12 to be slightly above the prediction based on the energy balance method.

4.2 Effects of plate geometry

Figure 13 shows the effect of plate width, W , on the energy release rate for the GlassFRP1 material case with the balanced and symmetric layup $[0/\theta/0/-\theta]_s$ for $\theta = 45^\circ$. For the half-width crack problem, the J-integral energy release rate is highest for small values of W/h decreasing to a constant width-independent value

Fig. 13: The energy release rate shown as a function of the plate width W/h for the GlassFRP1 case with $\theta = 45^\circ$ for $L/h = 140$.

for larger values of W/h . A constant energy release rate (the steady-state value) is obtained for $W = 40h$. Figure 13 shows that all three procedures are approaching the same energy release rate value for larger values of W/h . For smaller W/h , the crack tip interacts with the free edges, and hence, there will be no uniform stress field ahead of the crack tip and no uniform crack opening displacements in the crack wake. It is therefore not surprising that $\hat{G}$ obtained by the energy balance approach is imprecise for $W/h < 40$.

Regarding the plate length, a corresponding investigation has shown that a constant energy release rate value is obtained for $L = 140h$, which is the minimum laminate length used in Figure 13.

4.3 Effects of laminate thickness

Figure 14 shows the dependence of the energy release rate on the ratio between the laminate thickness, $2H$, and the lamina thickness, h . In the figure, the laminate thickness is increased by increasing the number of uni-directional (UD) plies next to the tunneling crack, i.e., for laminate lay-ups of the form $[0_n/\theta/0_n/-\theta]_s$, where n is the number of UD plies. It is seen from figure 14 that the energy release rate decreases with increasing laminate thickness, approaching an asymptotic value for large values of H/h . For the reference case with $[0/\theta/0/-\theta]_s$ ($H/h = 4$), the energy release rate can be seen to be around 15% higher than for the case with a vanishing thickness of the off-axis ply compared with the UD ply. Both the uniform

Fig. 14: The dependence of the steady-state energy release rate on the ratio between the laminate thickness and the lamina thicknesses for the GlassFRP1 case with $\theta = 45^\circ$ for a sufficiently large plate with $W = 90h$ and $L = 170h$

ply thickness case $[0/\theta/0/-\theta]_s$ as well as the large thickness difference is of high practical relevance. Increasing H/h can also be realized by decreasing h with H fixed, this corresponds to a decreasing ply thickness. The latter represents tunnel cracking occurring in the off-axis backing layers present in non-crimp fabrics used in the wind turbine industry. Here, the thickness of the backing bundles is typically significantly smaller than the UD plies (see e.g., Jespersen et al. (2018)).

4.4 Effects of crack spacing on the energy release rate

Tunneling cracks in off-axis oriented lamina will typically occur as multiple cracking (see e.g. Quaresimin et al. (2014); Jespersen et al. (2018)). Hence, it is vital to determine how closely spaced neighboring partly or fully developed cracks can be without significant interaction, that is, without influencing the energy release rate and thus changing the crack growth behavior of the individual cracks. Figure 15 illustrates the problem of a crack growing between existing cracks at a distance ℓ . For closely spaced cracks the stresses far ahead of the crack-tip differs from the far-field stress and varies as a function of position through thickness $\sigma_{22}(z)$, $\tau_{12}(z)$.

In the following, this critical distance is determined for the GlassFRP1 case with $\theta = 45^\circ$. Figure 16 shows the energy release rate of the crack tip of a half-width tunneling crack as a function of crack spacing, ℓ . The energy release rate is calculated using both the J -integral and the energy balance approach. For small ℓ ,

Fig. 15: Sketch of the investigated multiple crack case

Fig. 16: Influence of the distance, ℓ , to neighboring cracks on G_{ss} for the GlassFRP1 case with $\theta = 45^\circ$

the steady-state energy release rate is lower than the steady-state energy release rate of an isolated tunneling crack. Compared with $\bar{J}$ and $\hat{G}_{ss}$ for the case of isolated cracks it can be seen that the presence of neighboring cracks lowers the energy release rate with less than 1% for $\ell/h > 10$ and with less than 10% for $\ell/h > 5$. Note that the energy balance must be calculated based on the actual stress state between the two surrounding cracks, as indicated in figure 15 and not based on the far-field stress state.

Fig. 17: The energy release rate G_I , G_{II} and $G_{ss} = G_I + G_{II}$ are shown for the glass fiber composite GlassFRP1, calculated using the energy balance approach (lines) and the crack tip analysis of the half-width problems (symbols). The dotted lines indicate regions where it due to crack-surface penetrations has been necessary to adjust the negative normal crack opening, δ_n to zero.

4.5 Effect of the off-axis crack orientation

Figure 17 shows the dependence of $\hat{G}_I$, $\hat{G}_{II}$ and $\hat{G}_{ss} = \hat{G}_I + \hat{G}_{II}$ on the layup angle θ for the GlassFRP1 case for a laminate with a length and a width given to be minimum $W/h = 40$ and $L/h = 140$. The energy release rate is found from the crack opening profile in the middle of the full-width crack configuration, and the layup $[0/\theta/0/-\theta]_s$ is used. For small angles below $\theta < 45^\circ$, the model analyzed is modified slightly such that the full-width crack ends at the ends where the loading is applied ($x = 0$ and $x = L$) instead of at the sides of the plate ($y = 0$ and $y = W$). Furthermore, a slightly larger plate is analyzed with $W/h = 220$ and $L/h = 180$, see figure 3c. For an intermediate angle of $\theta = 45^\circ$, it has been validated that predictions from the two models, figure 3b and c, coincide. As stated earlier, for smaller angles, there will be an unphysical normal penetration of parts of the two crack surfaces. The energy balance was calculated by setting those negative normal openings values, $\delta_n(z)$, equal to zero. Data-points for which this has been done are shown as dotted lines in figure 17 and the following figures.

The shape of the curves in figure 17 for $\hat{G}_I$ and $\hat{G}_{II}$ can be seen to be somewhat similar with the laminate stress state, σ_{22} and τ_{12} , respectively, shown in figure 7. It is seen in figure 17, that $\hat{G}_{ss}$ increases monotonically with an increasing layup

Fig. 18: Energy release rate G_I and G_{II} for all four material cases, calculated using the energy balance method (lines) and crack tip analysis (symbols).

angle θ . The contribution goes from a purely $\hat{G}_{II}$ for $\theta \leq 30^\circ$, to a pure $\hat{G}_I$, for θ approaching 90° . In addition to the curves, the mean values found along the crack tip for the three cases $\theta = (45^\circ, 60^\circ, 75^\circ)$ are also shown on figure 17. Good agreement is found between $\hat{G}_{ss}$ and $\bar{J}$ with less than 1% difference, while the difference between $\hat{G}_{ss}$ and G_{ss} was been found to be up to 8% and the difference between $\hat{G}_I$ and $\bar{G}_I$ is below 11% while the difference between $\hat{G}_{II}$ and $\bar{G}_{II}$ is under 3%.

4.6 Effect of the material properties

As an alternative to make a large parametric study of the individual stiffness parameters in the orthotropic material, we have chosen to investigate the influence of elastic properties on the steady-state energy release rate for the four material cases defined in Table 1. These materials represent the frequently used glass- and carbon fiber-based material used in structural applications such as in the automotive, aerospace, and wind turbine industry.

In figure 18, $\hat{G}_I$ and $\hat{G}_{II}$ are shown for the four different materials. It can be seen that for stiffness properties going from a low modulus glass fiber composite (GlassFRP1) to a high modulus carbon fiber composite (CarbonFRP2), the dependence of the energy release rate to the layup angle θ is somewhat similar. Nevertheless, it is observed that the higher modulus carbon fiber composite has a

Fig. 19: The total energy release rate $\hat{G}_{ss}$ and mode-mixity $\hat{\psi}$ for the 4 material cases. Lines are results obtained using energy balance method; symbols are obtained using crack tip analysis.

lower normalized $\hat{G}_I$ contribution and a slightly higher $\hat{G}_{II}$ contribution than that of the lower modulus glass fibre composites. From the dotted part of the curves for the carbon fibre composites, it can be seen that there are predicted crack surface penetrations ($\delta_n(z) < 0$) from the finite element simulations for layup angles up to 55° . Nevertheless, similar to figure 8 for CarbonFRP1 for $\theta = 45^\circ$, this crack penetration is found to be limited to the very low part of the crack surfaces close to the interface to the UD-ply.

The mean values from the nine crack tip analyses for the cases also analyzed in figure 11, are shown as marks in matching colors on figure 18. For all cases, the energy approach is found to approximate the mean values along the crack-tip reasonably well, even though some discrepancy is found, especially for the G_I values when $\theta = 75^\circ$.

The total energy release rate, $\hat{G}_{ss}$, and average mode-mixity is shown in figure 19 where the average mode-mixity parameter $\hat{\psi}$, defined from (7), is calculated using equation (16). The mode-mixity $\hat{\psi}$ is 90° for small values of θ ($\theta < 25^\circ$ for the glass fibre composites; $\theta < 40^\circ$ for the carbon fibre composites), and decreases to 0° for $\theta = 90^\circ$. The correlation between $\hat{\psi}$ and θ is rather different for the various materials.

The glass fibre composites, which have the lowest orthotropy ($\lambda \approx 0.3$; $\rho \approx 2$), experience dominating Mode II ($\hat{\psi} = 90^\circ$) for a smaller off-axis angle range ($0 < \theta < 30^\circ$) than the carbon fibre composites ($0 < \theta < 45^\circ$), which have a higher orthotropy ($\lambda \approx 0.05$; $\rho \approx 5$). For larger values of θ , the difference in $\hat{\psi}$ between

the two glass fibre composites is very small. In contrast, the results for the carbon fibre composites indicate that increasing orthotropy (decreasing λ and increasing ρ) increases $\hat{\psi}$ for a given θ .

The nine cases from figure 18 are also represented in figure 19. For all nine cases, the energy balance methods are found to predict both the energy release rate and the mode-mixity rather precisely with a difference between $\bar{J}$ and $\hat{G}_{ss}$ of less than 1% and with a difference between $\bar{\psi}$ and $\hat{\psi}$ of less than 5%.

5 Discussion

The energy balance approach proposed by Ho and Suo (1993) for transverse tunnel cracks has been generalized to the analysis of off-axis tunnel cracks. A good agreement has been found between the average value of $\bar{G}_I$, $\bar{G}_{II}$ and $\bar{G}_{ss}$ at the crack tip with the energy balance components $\hat{G}_I$, $\hat{G}_{II}$ and $\hat{G}_{ss}$ calculated from crack opening displacements behind the crack tip. For a straight crack front, some variation of the mode-mixity $\psi(z)$ is found along the crack front. Despite that, a good agreement is found with the averaged mode-mixity $\bar{\psi} \approx \hat{\psi}$ obtained based on the energy balance approach behind the crack tip.

Material systems covering the range from a low-stiffness glass-fiber composite to a stiff carbon fiber composite have been studied. With the established results for $\hat{G}_{ss}$ and $\hat{\psi}$, it is possible to design a fatigue test series for extracting the crack growth rate of tunneling cracks in off-axis layers. Even though the analyses are linear elastic, the results can be used in fatigue studies similar to the method established by Quaresimin et al. (2014). For fatigue loading, tunneling cracks are considered as the initial damage mechanism (Jamison et al. (1984); Jespersen et al. (2018); Reifsnider and Jamison (1982); Talreja (1981); Zangenberg et al. (2013)). As such, tunnel cracks influence the development of other fatigue damages mechanisms such as fiber failure or delamination between the plies later during fatigue loading.

5.1 On crack face closure

The present linear FE-model does not model crack face contact. Omitting crack face contact in the analysis can result in negative normal crack opening displacements (crack face penetration), which must be considered unphysical. More rigorous modeling could include contacting surfaces and friction, but such non-linear analyses would require a considerably longer computation time. Appropriately modeling contact may influence the remaining crack opening shape slightly both in terms of normal δ_n and tangential δ_t openings. However, as this is only the case for small layup angles, the effect may reasonably be neglected.

5.2 Thin off-axis plies

The present study shows that for very thin off-axis plies (h/H approaching 0), such as backing bundles in non-crimp fabrics, the normalized energy release rate will only be slightly lower as presented in figure 14. The results suggest that the

most efficient way of decreasing the energy release rate is to decrease the thickness of the backing plies, h (see figure 14) and to select low off-axis angles (see figure 19).

5.3 The simplified mode-mixity calculation

It is encouraging that there is a good agreement between results obtained by the crack tip analysis ($\bar{J}$ and $\bar{G}$) energy account method from the half-width crack case that includes a straight crack tip and results obtained by the energy balance approach of the full-width crack case that does not include a crack-tip as shown in figure 12. This suggests that the use of the proposed energy balance approach on a full-width model (avoiding a crack tip) can give both the average energy release rate and the average mode-mixity. Then, the finite element model would not need to include the crack tip of the tunneling crack, and this enables a more simple mesh and thus a much faster analysis of tunneling crack problems. Furthermore, this will also open up the possibility of analyzing the off-axis tunnel crack as a two-dimensional problem, an approach that will be investigated in a future study.

5.4 Straight crack-tip simulations

The energy balance method is found to predict the mean value of the steady-state energy release rate, $\bar{G}_{ss}$, and mode-mixity, $\bar{\psi}$, with 5% accuracy for the nine cases investigated. The energy balancing method does not depend on the actual shape of the crack-front, and it is not even necessary to include the crack-front in the numerical model; only the opening displacements of the fully-developed tunneling crack is needed. On the other hand, the local energy release rate, $G_{ss}(z)$, and the mode-mixity, $\psi(z)$ will depend on the actually crack-tip shape. The question is whether also the mean values $\bar{G}_{ss}$ and $\bar{\psi}$ will depend on the shape of the crack-front. Simulations performed by Nakamura and Kamath (1992) indicate that this will not be the case for $\bar{G}_{ss}$, but to our knowledge, an investigation of a possible dependence of the average mode-mixity $\bar{\psi}$ on the crack-front shape has not been done. As mentioned earlier, for 90-degree layup, the cracking is pure Mode I (Ho and Suo, 1993), and the real crack front will adjust itself so that the energy release rate equals the Mode I fracture energy G_{Ic} .

The situation for tunneling cracks in off-axis laminas is more complicated. As shown in Fig. 11, the mode-mixity varies as a function of position across the off-axis layer for a straight crack front, $\psi = \psi(z)$. It is likely, that for real tunneling cracks in the off-axis lamina, the crack front is curved. The mode-mixity is likely to vary across the thickness of the off-axis lamina, and the crack front shape will adjust itself so that the energy release rate at a given position would be equal to the fracture energy of the mode-mixity at that position.

The mode mixity-dependency of the fracture energy $G_c(\psi)$ can be obtained independently by mixed mode fracture experiments, e.g., Sørensen and Jacobsen (2009). Such data could be used as input data in a more accurate analysis (e.g., a 3D FE model) to determine the shape of the crack front. Then, the problem would be to adjust the crack front shape until the criterion $G(z) = G_c(z)$, where $G_c(z)$ is $G(\psi)$. This process most likely would require an iterative approach. Thus, this

Fig. 20: The ratio between the maximum and the mean crack opening in the normal, δ_n , and the transverse direction, δ_t , for the 4 material cases.

accurate approach would be much more time consuming and might not be well suited for engineering problems.

The approach of using average values of energy release rate and mode-mixity proposed by Quaresimin et al. (2014) requires only a model with a full-width crack. This is a substantial reduction in the complexity in comparison with the more accurate 3D modeling of a curved crack front. The observation that the mode-mixity does not vary strongly with z for a straight crack-front, suggests that using the average mode-mixity could be sufficiently accurate for engineering purposes.

5.5 Experimental determined energy release rate

In all the numerical results presented, the energy release rates calculated by the energy balance approach is found from the stress-state in the un-cracked lamina and the average crack opening displacements, $\bar{\delta}_n$, $\bar{\delta}_t$, through equations (13-15). A similar approach could also be used by extracting the energy release rate from a physical experiment. Nevertheless, in such an experiment, it will be difficult to measure the crack opening profile in the steady-state region away from the edge and crack front. Instead, it is experimental more feasible to measure the maximum crack-opening. Figure 20 shows the relation between the average and the maximum crack opening as a function of the layup angle θ for the four materials analyzed. For all cases, the aspect ratio for the tangential opening displacement δ_t is found to

be in the very narrow range of $\bar{\delta}_t/\delta_t^{max} \in [0.65; 0.69]$ while for the normal opening it is slightly larger with $\bar{\delta}_n/\delta_n^{max} \in [0.57; 0.66]$ for the cases with non-zero normal openings. For a tunnel-crack, the maximum value of the tangential opening will be difficult to measure. On the other hand, the normal opening displacement can be measured using X-ray tomography in combination with a staining procedure (see e.g., Wang et al. (2018)), here reported for a loaded sample. For a channel-crack along a free surface, both the δ_n^{max} and δ_t^{max} will be experimental attainable with high precision using e.g., scanning electron microscopy during an in-situ test. Values of the stress state and crack opening displacement at the onset of cracking could then be used for calculating the fracture energies.

6 Conclusions

The steady-state energy release rate and the mode-mixity were determined for off-axis orientations covering $\theta \in [0; 90]^\circ$ of a balanced and symmetric $[0/\theta/0/-\theta]_s$ laminate. The results were determined for structural relevant glass, and carbon fiber reinforced composites with a ply stiffness of 30 GPa for the glass-fiber-reinforced lamina and 270 GPa for the carbon fiber reinforced lamina. The analysis, which can replace a crack-tip stress intensity factor analysis, is a generalization of the Ho and Suo (1993) energy balance approach to off-axis tunneling cracks. Through 9 validation cases, the accuracy of the prediction is found to be better than 1% for the energy release rate and 5% for the mode-mixity. The results obtained in the present study can be used to design laminates for off-axis crack-growth experiments during static and cyclic loading.

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