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Mode-based Beam and Connection Analysis of Frames

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ABSTRACT

Major steel structures are commonly constructed as frames or trusses. The connections between its beams and columns have a crucial influence on the global behaviour. However, a detailed assessment of the joints is often neglected with rough simplifications, which may lead to poor results. On the other hand, if performing a detailed assessment it is a time-consuming task requiring special expertise. Consequently, the idea stressed in this paper is a novel approach on how to analyse thin-walled steel frames including a detailed assessment of both the joint and beam mechanics.
The approach is based on a transformation from standard degrees of freedom into a displacement-based mode formulation. Furthermore, this transformation reduces the total number of degrees of freedom to be computed for advanced joints and beam elements. All this is achieved by adopting an advanced beam element formulation as well as introducing the idea of a 'joint element' formulated by finite shell elements. One of the main advantages of this idea is a general formulation of the joint, which makes it valid for many beam-to-column configurations. Furthermore, beam members can either be closed or open sections due to the adopted advanced generalised beam theory. Furthermore, a detailed analysis of the joint element itself is also achieved by this approach. Finally, an increased efficiency is achieved due to reduction of the total amount of degrees of freedom of the advanced model, and a new in-depth knowledge of the joint behaviour is gained from this approach as well, due to the displacement-based mode formulation.

Keywords: Connections, thin-walled structures, steel frames, mode-based formulation

1 INTRODUCTION

Steel frames and trusses are commonly used in larger buildings and structures such as power plants, bridges, or in the offshore industry. The structural systems have been used and improved for centuries in line with scientific developments and findings. Here, especially beam theories have been developed and refined. From around 1940-50, thin-walled beam theories started to emerge as the influences from warping was realized, Vlasov (1). Later, the assessment of joints able to include the warping effect became relevant in scientific communities (2, 3). However, through time, more advanced thin-walled beam theories have developed. These includes the warping effect, but also in plane distortional deformation patterns, both implemented in these new theories. An example is the "Generalized Beam Theory" (GBT) or the "constrained Finite Strip Method" (cFSM). The cFSM was developed by Ádány and Schafer (4) being a beam theory using displacement-based modes that includes higher order distortional effects. To explain, a displacement mode relies on a cross sectional displacement field with an associated sinusoidal axial variation. On the other hand, the GBT was at first developed by Schardt (5, 6), widespread by Davis and co-workers (7, 8), after which a lot of further development was carried out by the group around Camotim, (9, 10). A signature to the GBT is its potential of doing a displacement mode decomposition, i.e. an arbitrary deformation is subdivided into a set of well known displacement modes that for example could be pure bending, torsion, and so on. However, the GBT is based on a finite element formulation requiring an axial assembly of elements to assess a

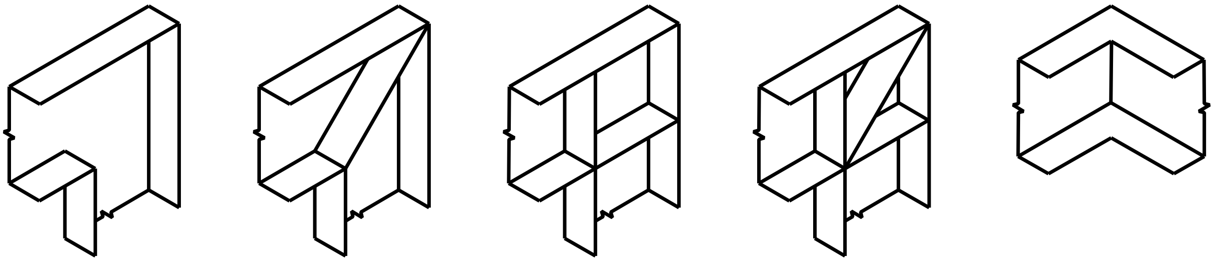

Fig. 1. Frame corners; a) unstiffened with web continuity; b) diagonally stiffened; c) box stiffened; d) multi stiffened (box and diagonal); e) unstiffened with flange continuity

single beam member. Furthermore, the axial variation of a finite GBT-element is interpolated by Hermite cubic functions.
Despite the development of advanced beam theories, the analysis of frames is difficult due to the complexity associated with joining non-aligned components. Nonetheless, Basaglia and co-workers have put an effort into developing a detailed approach within the framework of GBT, (11–13). They present an advanced formulation connecting non-aligned members. This has been done by means of introducing a 'joint element'. This joint element is formulated on global beam end displacements as well as constraint equations depending on the shape of the adjacent beam cross sections as well as the joint geometry, which could be different kinds of stiffening plates, see *Fig. 1*. Consequently, in order to formulate the constraint equations a prior assessment of the joint element is required.

2 THE IDEA

In general, the idea is to analyse frames based on beam displacement modes. Beams and columns are modelled using an advanced beam element formulation. Beam element degrees of freedom are transformed into a mode-based space where a degree of freedom corresponds to an entire displacement mode of the beam element. Connections between beam elements are analysed by introducing the idea of a 'joint element'. A joint element is an assembly of shell finite elements. The nodal displacement degrees of freedom on the connected faces are transformed into mode-based degrees of freedom governed by the adjacent beam elements. As a result, the frame components are expressed in a mode-based formulation yielding the opportunity of regulating the number of included modal degrees of freedom. Furthermore, techniques such as substructureing and static condensation may be utilized in order to increase efficiency.

2.1 Beam element

Beams and columns within a frame shall be modelled as beam elements adopting the advanced beam theory presented by the authors, (14, 15). Hence, a beam element is based on a linear elastic analysis and support members having a thin-walled cross section. Regardless of the beam length only one beam element is introduced.
At first, a beam cross section is subdivided into wall elements, see *Fig. 2*. Its two nodes having six degrees of freedom each governing the wall element deformations. With traditional shape functions, the displacements of a wall element is described based on its degrees of freedom. Secondly, a system of equilibrium equations are deduced from a linear elastic strain energy formulation of the cross section stiffness. Having the equilibrium equations, it is possible to compute a set of modal cross sectional displacement fields. These modal fields are determined as eigenvectors of a general eigenvalue problem formed on the basis of the equilibrium equations. The associated eigenvalues indicates either a polynomial or exponential variation of the cross sectional displacement field in the axial direction. Hence, the beam element stiffness matrix is computed using the displacement fields as interpolation functions. As a result, an advanced beam element is formulated having degrees of freedom at the end cross sections.

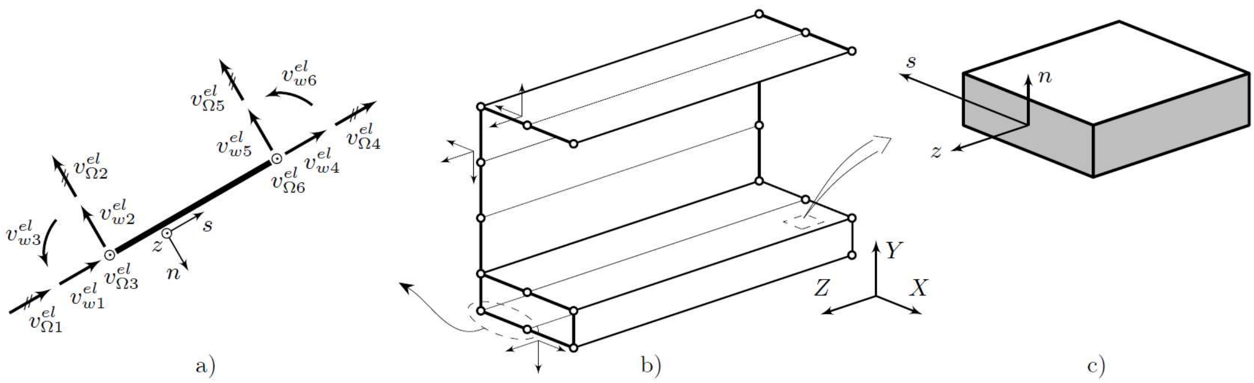

Fig. 2. Beam element; a) Wall element and its local degrees of freedom; b) A beam element with discretised cross section; c) Wall cut-out illustrating local coordinate system

2.2 Joint element

A joint geometry is discretised by standard shell finite elements and assembled in a 'joint element'. Consequently, a joint element allows a detailed displacement behaviour, but a main drawback is the high number of degrees of freedom incorporated in the element due to the amount of finite elements.

2.3 Mode space

The following section presents a proposal on how to increase efficiency by reducing the number of degrees of freedom and simultaneously increase the level of information that it is possible to gain from a frame analysis. The task is solved by introducing a number of beam related deformation modes as degrees of freedom instead of the standard degrees of freedom. To do so, we introduce a mode space. Hence, instead of displacements governed by individual degrees of freedom, the mode space approach handles a whole set of degrees of freedom as a single 'modal degree of freedom'.

As stated in *Sec. 2.1*, the beam element is governed by its degrees of freedom located at its ends. The mode space formulation is achieved by using the displacement fields found in the cross section analysis. Hence, a number of modes are judiciously selected to form the beam mode space. A single mode degree of freedom corresponds to a cross sectional displacement field with a particular axial variation. Consequently, the standard degrees of freedom are transformed into a set of beam modes.

The joint element formulation is different. The displacements related to this joint element is highly governed by the chosen displacement modes selected in the adjacent beam elements. Therefore, the degrees of freedom are grouped into two. Those being at a beam interface and those being internal degrees of freedom. At first, the degrees of freedom at a connected face are transformed into a set of displacement modes governed by adjacent beam elements. To clarify, the allowable displacements of the nodes at the interface are limited to the cross sectional displacement fields included in the selected modes of the adjacent beam. Then, either the internal degrees of freedom could be eliminated using static condensation, or they could be kept for further modal analysis and reduction. This could for example be an analysis finding deformation modes, which can be in a second order buckling analysis.

2.4 Example

A brief example is included here to illustrate the idea as we have currently implemented it. Let us analyse an L-shaped frame constructed by two lipped open channel sections being joined together similar to the configuration in *Fig. 1b*, i.e. the joint is stiffened by a diagonal plate. The frame is illustrated in *Fig. 3a*. At the end cross sections, we restrain the nodes at the web centre points against translation in all three directions (point A and C). The node at point B is restrained against translation in the *X*-direction. An idealized presentation of the frame is shown in *Fig. 3b*. The upper and lower flanges at the end cross section near point A are exposed to a positive and negative unit displacement in the *X*-direction, respectively. This yields a twisting displacement as illustrated in *Fig. 3c* showing how torsion and warping is transmitted through the diagonally stiffened joint of the frame.

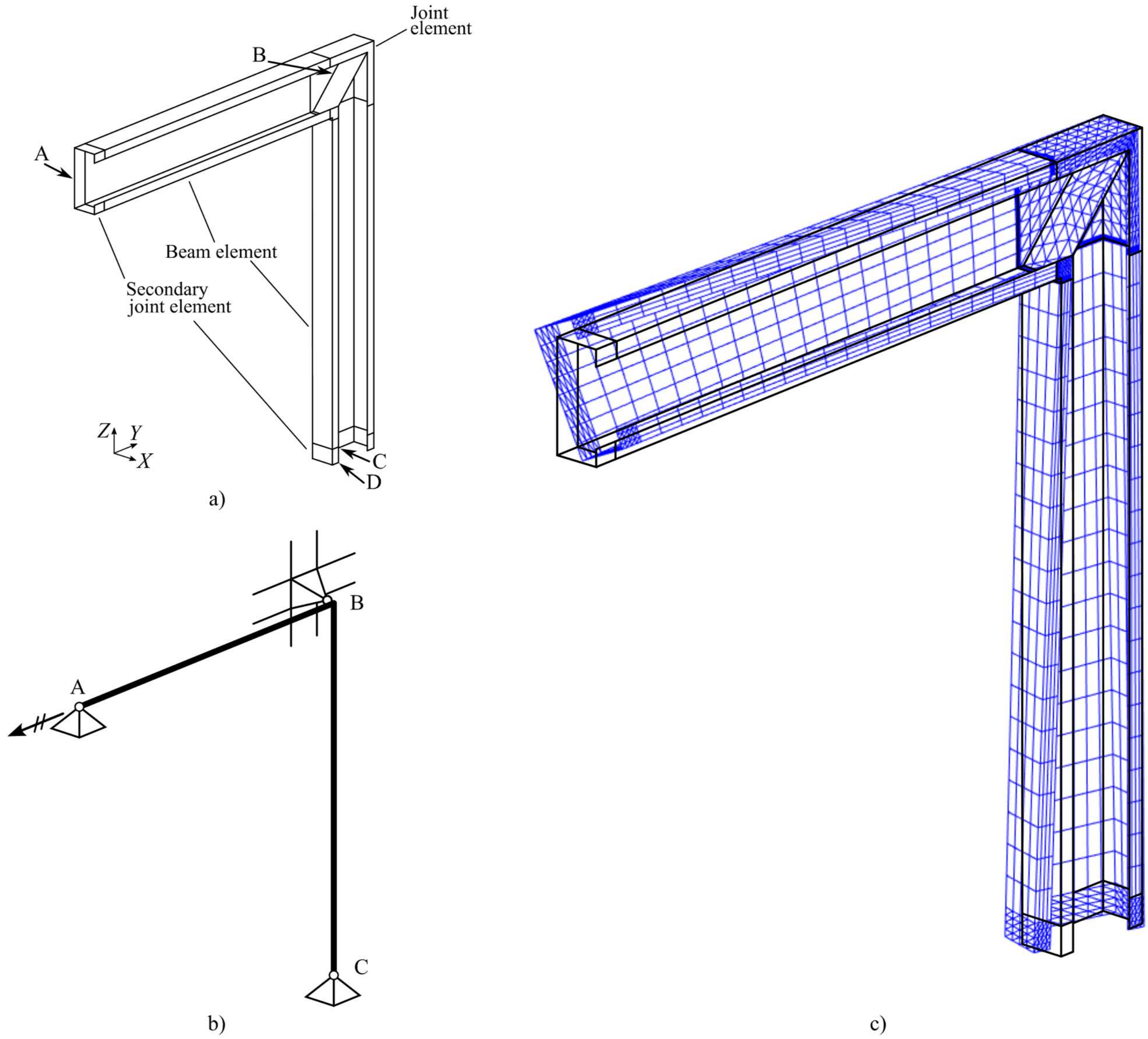

Fig. 3. Example with L-shaped frame; a) Frame set-up; b) Idealized static system and boundary conditions; c) Deformed shape including all beam modes (scaled 2.5 times)

The bar-diagrams in *Fig. 4* show the intensities of the different beam modes. It is seen that only a few modes are activated. Now, the same set-up is assessed, however, only including the first 50 out of 300 beam modes in the two elements. Comparing the displacement at point D (at beam lip-end), we achieve similar results with the largest relative deviation at –1.8%. An increase in computational speed is also obtained.

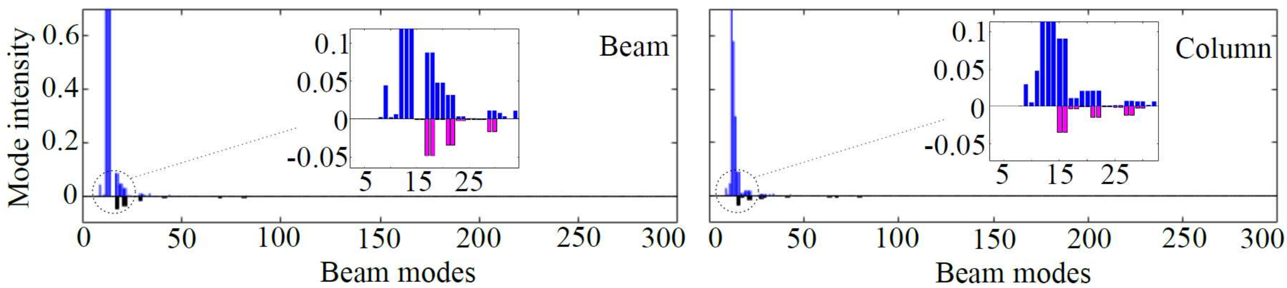

Fig. 4. Beam mode intensities regarding the beam and column members in the L-frame

3 DISCUSSION

In research it is common to try to include mechanical properties of a joint in frame and truss structures by introducing a 'joint element' in a global finite element environment. An example is the approach given by Basaglia et al. (11) introducing a joint element within the framework of GBT that includes higher order effects such as distortional displacements. Another example is the technique presented by Shayan & Rasmussen (16). Here, non-aligned finite beam elements are assembled with rigid links and springs. The finite elements used are Vlasov beam elements including a seventh degree of freedom representing the warping effect. Consequently, these approaches allows a more refined transmission of displacements through the joint, for example warping. However, a prior assessment of a joint is required in order to establish necessary constraint equations. Thus, the idea presented here gives an approach that stands out in the way that no prior special assessment of the joint is required other than building a standard finite element model of it. Beams and columns are modelled using the advanced beam element formulation previously presented by the authors (14, 15). In addition, a joint is modelled with a discretization of finite shell elements assembled in a joint element. Hence, the idea presented here should be seen as a complete approach to assess steel frames.

The main idea presented within this paper is that of a displacement-based mode formulation instead of the use of standard nodal displacement degrees of freedom. To clarify, beam element degrees of freedom at the end cross sections are transformed into a set of selected displacement modes. These modes are computed through the beam element formulation. Now, the displacement modes are used as new modal degrees of freedom. If the included modes are judiciously chosen, an efficient reduction in degrees of freedom is achieved through this procedure. A similar method is used by Abambres et al. (17) with regards to beam elements in the framework of GBT.

Having the beam element expressed in terms of a few number of displacement modes the interface at the joint element shall be transformed as well. This is done having a transformation of the standard degrees of freedom at the interface, into beam end cross section displacement fields. The remaining degrees of freedom within the joint element are either condensed or kept for further assessment. For example for determination of buckling modes within the joint element.

Since both beam elements and joint elements are presented in a mode-based formulation a frame assessment will result in a novel in-sight on how deformations may be transmitted through a joint. This will be done with a mode decomposition of the individual elements revealing what kind of displacements the structure is exposed to. Furthermore, from the mode decomposition it may be an easy task to assess optimization that is performed either on the entire structural system or on individual components.

4 ACKNOWLEDGEMENT

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