Risk-averse optimization of disaster relief facility location and vehicle routing under stochastic demand

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Risk-averse optimization of disaster relief facility location and vehicle routing under stochastic demand

ABSTRACT
Disasters such as fires, earthquakes, and floods cause severe casualties and enormous economic losses. One effective method to reduce these losses is to construct a disaster relief network to deliver disaster supplies as quickly as possible. This method requires solutions to the following problems. 1) Given the established distribution centers, which center(s) should be open after a disaster? 2) Given a set of vehicles, how should these vehicles be assigned to each open distribution center? 3) How can the vehicles be routed from the open distribution center(s) to demand points as efficiently as possible? 4) How many supplies can be delivered to each demand point on the condition that the relief allocation plan is made a priority before the actual demands are realized? This study proposes a model for risk-averse optimization of disaster relief facility location and vehicle routing under stochastic demand that solves both problems simultaneously.

The novel contribution of this study is its presentation of a new model that includes conditional value at risk with regret (CVaR-R)—defined as the expected regret of worst-case scenarios—as a risk measure that considers both the reliability and unreliability aspects of demand variability in the disaster relief facility location and vehicle routing problem. Two objectives are proposed: the CVaR-R of the waiting time and the CVaR-R of the system cost. Due to the nonlinear capacity constraints for vehicles and distribution centers, the proposed problem is formulated as a bi-objective mixed-integer nonlinear programming model and is solved with a hybrid genetic algorithm that integrates a genetic algorithm to determine the satisfactory solution for each demand scenario and a non-dominated sorting genetic algorithm II to obtain the non-dominated Pareto solution that considers all demand scenarios. Moreover, the Nash bargaining solution is introduced to capture the decision-maker’s interests of the two objectives. Numerical examples demonstrate the trade-off between the waiting time and system cost and the effects of various parameters, including the confidence level and distance parameter, on the solution. It is found that the Pareto solutions are distributed unevenly on the Pareto frontier due to the difference in the number of the distribution centers opened. The Pareto frontier and Nash bargaining solution change along with the confidence level and distance parameter, respectively.

Keywords: Disaster relief location-routing problem; Conditional value at risk with regret; Bi-objective optimization; Nash bargaining solution; Hybrid genetic algorithm
1 Introduction

Large-scale disasters such as fires, earthquakes, and floods cause severe casualties and enormous economic losses each year. Although various types of hazard pre-warning systems have been established throughout the world, it is still impossible to predict exactly when and where a disaster will occur. It is thus critical to take action to minimize the damage and losses after a disaster. More importantly, such actions should be carried out swiftly because the likelihood of survival in a disaster area declines significantly after 72 hours (Benson et al., 1996). One of the most effective actions would be to construct a disaster relief network ahead of time that can be used to deliver relief supplies to affected areas as soon as possible. The problems in designing the disaster relief network and routing relief supplies are included in disaster relief logistics.

Disaster relief logistics contains two important subproblems: to design the relief network at the strategic level, known as the facility location problem (FLP), and to determine the routes of relief supplies at the operational level, known as the vehicle routing problem (VRP). These two problems have been addressed both separately and simultaneously. For example, Akgün et al. (2015), An et al. (2015), and Hu and Dong (2019) focused on the FLP, whereas Vitoriano et al. (2011), Huang et al. (2012), Özdamar and Demir (2012), and Meng et al. (2015) focused on the VRP. The simultaneous consideration of both subproblems is categorized as the location routing problem (LRP), which integrates both the strategic and operational decisions. Prodhon and Prins (2014) and Drexel and Schneider (2015) published comprehensive reviews of the LRP. It is worthy of note that Salhi and Rand (1989) established that solving the LPR is superior to solving the FLP and VRP separately in the sense that it could avoid the generation of suboptimal solutions. Therefore, this study aims to develop a model to solve the LRP in the context of disaster relief logistics.

Two crucial characteristics of disaster relief logistics cannot be overlooked. The first is uncertainty, such as that regarding demand, travel time, and the state of infrastructure. Demand uncertainty is among the most important factors because a supply shortage leads to ineffective rescue and incremental casualties (Bozorgi-Amiri et al., 2012). Correspondingly, demand
uncertainty has attracted the most attention in the humanitarian logistics literature (Liberatore et al., 2013). These uncertainties affect the consequences of a decision-maker’s decisions, which depend not only on the decision’s quality per se as measured by the expected outcome but also on the decision-maker’s attitudes and tolerance for the deviation between the expected outcome and the realized outcome. It is therefore necessary for researchers to account for the decision-maker’s risk attitudes when developing a modeling framework to assist in decision-making. The second characteristic associated with disaster relief logistics is that multiple criteria should be taken into consideration. It is not only monetary cost, but also criteria such as time efficiency, fairness, and reliability, that are critical to disaster relief logistics.

In light of the preceding characteristics, the following literature review focuses on three aspects: 1) methods to address uncertainties in the disaster relief LRP; 2) the application of various risk measures, such as value at risk (VaR) and conditional value at risk (CVaR), in the location and routing problems; and 3) multi-objective models and algorithms developed for the disaster relief LRP.

1) Methods to address uncertainties in the disaster relief LRP

The prevailing methods include two-stage stochastic programming and robust optimization. Both capture uncertainty via various scenarios. The scenario probability is known in stochastic programming, but it may not be known in robust programming. The key literature is summarized in rows 1 through 4 in Table 1. Two-stage stochastic programming was used by Ahmadi et al. (2015) and by Bozorgi-Amiri and Khorsi (2016). Ahmadi et al. (2015) determined the routes in the first stage and the locations of distribution centers in the second stage, whereas Bozorgi-Amiri and Khorsi (2016) solved the FLP in the first stage and the VRP in the second stage. Both models minimize the expected value of performance measures (e.g., cost, travel time) in all scenarios. The robust optimization method was used by Caunhye et al. (2016) and by Chang et al. (2017) to optimize the performance measure of the worst-case scenario. When comparing the two methods, decision-makers are typically assumed to be risk-neutral in two-stage stochastic programming but pessimistic in robust optimization, which implies that the results of the robust optimization method are rather conservative. Nevertheless,
neither stochastic programming nor robust optimization captures the variability of uncertainty in a comprehensive manner (Lu et al., 2018). This motivates the development of risk measures, such as VaR or CVaR, that can achieve a balance between stochastic programming and robust optimization methods.

2) Application of different risk measures in the location and routing problems

Rows 5 through 11 in Table 1 summarize the applications of VaR and CVaR to solve the location and routing problems. In supply chain management, Ahmadi-Javid and Seddighi (2013) and Yu and Zhang (2018) applied CVaR to solve the FLP and the LRP, respectively. In the transportation of hazardous materials, Toumazis and Kwon (2013), Toumazis et al. (2013), and Faghih-Roohi et al. (2016) determined the optimal routes for transportation of hazardous materials between a given origin-destination pair based on VaR and CVaR. In disaster relief logistics, Noyan (2012) initially incorporated CVaR into the objective function when solving a location-allocation problem, which provided more robust solutions than the risk-neutral approach. Bai et al. (2018) and Elçi and Noyan (2018) proposed a risk-averse optimization model based on VaR for location-allocation design. As for FLP, Hu et al. (2016) used CVaR to explore the effects of the aversion degree on decision-making.

The VaR and CVaR adopted in these studies focused on the cost or consequence in the uncertainty scenario and used the cost or consequence directly as a risk metric. In contrast, Chen et al. (2006) proposed a risk measure known as conditional value at risk with regret (CVaR-R), which takes regret as the risk metric for the FLP. Their paper refers to the CVaR-R as $\alpha$—reliable mean-excess regret. The regret is defined as the difference between the objective value (e.g., cost) given by an implemented solution and the optimal objective value in the same scenario. The advantage of CVaR-R over CVaR is that it captures the decision-maker’s anticipation of regret and helps him or her to make more rational choices (Janis and Mann, 1977). To the best of the authors’ knowledge, no existing study considers CVaR-R in the solution of the disaster relief LRP.

3) Multi-objective models and algorithms for the disaster relief LRP

In disaster relief logistics, several studies have formulated LRP as a multi-objective
optimization problem. The considered objectives include the total travel time; the total distribution cost, which consists of the setup cost of depots and travel cost; route reliability; unsatisfied demand; and carbon emissions (Wang et al., 2014; Bozorgi-Amiri and Khors, 2016; Vahdani et al., 2018; Shen et al., 2019). The methods for solving multi-objective models can be classified into two categories. In the first, a multi-objective model is converted into a single-objective model with various techniques, such as the $\varepsilon$-constraint method (Bozorgi-Amiri and Khors, 2016) or the weighted sum method (Shen et al., 2019). The other model uses multi-objective evolutionary algorithms, which commonly adopt the non-dominated sorting genetic algorithm II (NSGA-II) (Wang et al., 2014; Vahdani et al., 2018). The NSGA-II has been shown to be superior to other algorithms, such as multi-objective particle swarm optimization (Asadi et al., 2018; Rabbani et al., 2018) and the multi-objective evolutionary algorithm based on decomposition (Mahmoudsoltani et al., 2018), in most evaluation criteria, including solution quality and computation time. Hence, this study adopted NSGA-II to solve the proposed multi-objective optimization model. The solution obtained by a multi-objective optimization model is usually a set of Pareto solutions, but it is still difficult for the decision-maker to select a solution from the Pareto frontier. To mitigate this difficulty, this study initially regarded the bi-objective LRP as a bargaining game between two players and introduced the concept of the Nash bargaining solution (NBS) to help the manager determine the final solution.

The existing literature reviews indicate that, in disaster relief logistics, the demand stochasticity and decision-maker’s risk attitude have not been addressed to solve the LRP and that, in the location and routing problem, no study has proposed a bi-objective CVaR-R model. Therefore, this study seeks to fill this research gap by developing a bi-objective risk-averse optimization model for the disaster relief LRP with stochastic demand. To address uncertainty, a set of demand scenarios with occurrence probability is generated in light of the stochastic programming approach. The CVaR-R is used as the risk measure, which can both achieve a balance between the stochastic programming and robust optimization methods and capture the decision-maker’s risk attitude. Simultaneous consideration of two vital objectives in the disaster relief logistics—cost and time—results in a bi-objective optimization model that is
solved with a proposed hybrid NSGA-II algorithm.

To sum up, the contributions of this paper are as follows.

(1) An integrated LRP with stochastic demand is formulated to handle strategic and operational issues in disaster relief at the same time because traditional methods that solve FLP and VRP separately may lead to a suboptimal solution.

(2) A risk measure called CVaR-R is proposed to manage the risk caused by stochastic demand, which considers both the reliability and unreliability aspects of demand variability. To the best of the authors’ knowledge, this study represents the first attempt to solve disaster relief LRP based on CVaR-R.

(3) A bi-objective model is developed, and the trade-off between CVaR-R of the waiting time and that of system cost is illustrated. Meanwhile, the NBS is introduced to help the decision-maker select the final solution from the Pareto frontier according to a distance function.

(4) A hybrid genetic algorithm (GA) is devised, which integrates a CPLEX/GA to determine an optimal/satisfactory solution for each demand scenario for small or medium-size instances and NSGA-II to find the Pareto solution of the bi-objective risk-averse model under stochastic demand. This hybrid GA is devised to demonstrate the properties of the proposed risk-averse approach. Sensitivity analysis is conducted to examine the effects of confidence level and distance parameter, which provides managerial insights for managers.
<table>
<thead>
<tr>
<th>Row</th>
<th>Reference</th>
<th>Scope</th>
<th>Problem</th>
<th>Uncertainty</th>
<th>Objectives</th>
<th>Methodology</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ahmadi et al. (2015)</td>
<td>Disaster logistics</td>
<td>LRP</td>
<td>Travel time</td>
<td>Min penalty cost of unsatisfied demand, fixed costs of opening local depots, expected total distribution time, expected cost of violations of standard relief time</td>
<td>Two-stage stochastic programming</td>
</tr>
<tr>
<td>2</td>
<td>Bozorgi-Amiri and Khorsi (2016)</td>
<td>Disaster logistics</td>
<td>LRP</td>
<td>Demand, travel time, and procuring and travel costs</td>
<td>Min expected maximum amount of shortages among demand points in all periods; Min expected total travel time; Min total pre-disaster and expected post-disaster costs</td>
<td>Two-stage stochastic programming</td>
</tr>
<tr>
<td>3</td>
<td>Caunhye et al. (2016)</td>
<td>Disaster logistics</td>
<td>LRP</td>
<td>Demand and state of infrastructure</td>
<td>Min weighted sum of total setup cost of warehouses and worst-case optimal response time among all scenarios</td>
<td>Two-stage robust optimization</td>
</tr>
<tr>
<td>4</td>
<td>Chang et al. (2017)</td>
<td>Disaster logistics</td>
<td>LRP</td>
<td>Demand and travel time</td>
<td>Min distribution cost; Max worst path satisfaction rates; Max worst path transport capacities</td>
<td>Robust optimization</td>
</tr>
<tr>
<td>5</td>
<td>Chen et al.</td>
<td>Supply</td>
<td>FLP</td>
<td>Demand, distances between</td>
<td>Min CVaR-R of (distance × weight)</td>
<td>CVaR-R</td>
</tr>
<tr>
<td>Year</td>
<td>Authors</td>
<td>Domain</td>
<td>Problem Description</td>
<td>Optimization Objective</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>-------------------------------</td>
<td>---------------------------------------------</td>
<td>-------------------------------------------------------------------------------------</td>
<td>-----------------------------------------------------------------------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2006</td>
<td>Yu and Zhang</td>
<td>Supply chain</td>
<td>FLP Facility disruption</td>
<td>Min CVaR of the sum of fixed costs, transportation costs, and penalty costs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2018</td>
<td>Ahmadi-Javid and Seddighi</td>
<td>Supply chain</td>
<td>LRP Capacity of distribution centers and the number of available vehicles</td>
<td>Min CVaR of the sum of the opening cost of distribution centers, route costs, and production and distribution disruption cost</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>Toumazis and Kwon</td>
<td>Hazardous materials</td>
<td>Routing Accident probability and consequences</td>
<td>Min CVaR of travel cost</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>Toumazis et al.</td>
<td>Hazardous materials</td>
<td>Routing Accident consequences</td>
<td>Min VaR of accident consequences; Min CVaR of accident consequences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2016</td>
<td>Faghih-Roohi et al.</td>
<td>Hazardous materials</td>
<td>Routing Accident probability and consequences</td>
<td>Min CVaR of accident consequences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2016</td>
<td>Hu et al.</td>
<td>Disaster logistics</td>
<td>FLP Reliability of road networks</td>
<td>CVaR of penalty for death, waiting time, and shortages</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The remainder of this paper is organized as follows. Section 2 starts with a description of the problem and an introduction of the bi-objective formulation under fixed demand. The two proposed CVaR-R objectives are then defined and the bi-objective formulation is extended to consider stochastic demand. Section 3 devises a solution algorithm to solve the bi-objective risk-averse optimization problem. Section 4 includes numerical studies to demonstrate the properties of the proposed model and algorithm. Finally, Section 5 concludes the paper.

2 Problem Description and Formulation

2.1 Problem description

This paper considers a disaster relief distribution system that includes distribution centers (DCs), demand points, and vehicles, as shown in Fig. 1. The following decisions shall be made: 1) given established DCs, which center(s) should be opened after a disaster (because not all DCs are necessarily open considering opening costs and demands); 2) given a set of vehicles, how these vehicles should be assigned to each open DC; 3) how these vehicles should be assigned from DCs to demand points as efficiently as possible; and 4) what quantity of goods should be delivered at each demand point on the condition that the relief allocation plan is made before the actual demand is revealed.

The following assumptions are made for the model. A1) The network includes only the demand points that can be visited through the traffic network and ignores the areas that require other special means of transportation. A2) The travel time through the network is given and deterministic, because this paper only addresses the stochasticity on the demand side. A3) Each vehicle has a limited capacity. A4) Each vehicle begins at the DC to which it belongs and returns to the same DC when completing delivery tasks to demand points. A5) Each demand point is only visited once. A6) The capacity of each DC is limited. A7) The goods stored at each DC and required at the demand points are assumed to be homogeneous. A8) A set of scenarios and corresponding probabilities that represent various possible demand realizations are given in the planning phase.

In the following subsections, we introduce the key notations used throughout this paper, followed by the formulation for the LRP under fixed demand. The proposed risk measure,
CVaR-R, is then depicted, and the deterministic model is extended to consider the proposed measures. Finally, the method used to obtain the NBS is defined.

![Illustration of a disaster relief distribution system](image)

**Fig. 1.** Illustration of a disaster relief distribution system

### 2.2 Notations

#### Sets

<table>
<thead>
<tr>
<th>Set</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>Set of candidate distribution centers (DCs)</td>
</tr>
<tr>
<td>$C_2$</td>
<td>Set of demand points</td>
</tr>
<tr>
<td>$C$</td>
<td>Set of nodes in the disaster relief distribution system, $C = C_1 \cup C_2$</td>
</tr>
<tr>
<td>$L$</td>
<td>Set of scenarios</td>
</tr>
</tbody>
</table>

#### Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>Number of vehicles</td>
</tr>
<tr>
<td>$SC_i$</td>
<td>Setup cost for candidate DC $i$</td>
</tr>
<tr>
<td>$Cap_{i,\text{center}}$</td>
<td>Capacity of DC $i$</td>
</tr>
<tr>
<td>$Len_{ab}$</td>
<td>Distance between nodes $a$ and $b$</td>
</tr>
<tr>
<td>$T_{ab}$</td>
<td>Travel time between nodes $a$ and $b$</td>
</tr>
<tr>
<td>$UC$</td>
<td>Travel cost per kilometer</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>FC&lt;sub&gt;k&lt;/sub&gt;</td>
<td>Fixed operation cost of vehicle ( k )</td>
</tr>
<tr>
<td>Cap&lt;sub&gt;k&lt;/sub&gt;&lt;sup&gt;veh&lt;/sup&gt;</td>
<td>Capacity of vehicle ( k )</td>
</tr>
<tr>
<td>( D_j )</td>
<td>Exact demand required at demand point ( j )</td>
</tr>
<tr>
<td>( \bar{D}_j )</td>
<td>Minimum and maximum demand required at demand point ( j )</td>
</tr>
<tr>
<td>LT&lt;sub&gt;j&lt;/sub&gt;</td>
<td>Latest arrival time at demand point ( j )</td>
</tr>
<tr>
<td>( \rho^\text{demand} )</td>
<td>Penalty for unsatisfied demand</td>
</tr>
<tr>
<td>( \rho^\text{supply} )</td>
<td>Penalty for excess supply</td>
</tr>
<tr>
<td>( \Pr_l )</td>
<td>Occurrence probability of scenario ( l )</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Desired confidence level</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Distance parameter used to identify the Nash bargaining solution</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Weighting parameters associated with the waiting time objective, ( 0 \leq \gamma \leq 1 )</td>
</tr>
</tbody>
</table>

**Variables and functions of variables**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_i )</td>
<td>( x_i = 1 ), if DC ( i ) is selected to open; otherwise ( x_i = 0 )</td>
</tr>
<tr>
<td>( y_{ik} )</td>
<td>( y_{ik} = 1 ), if vehicle ( k ) is assigned to DC ( i ); otherwise ( y_{ik} = 0 )</td>
</tr>
<tr>
<td>( z_{abk} )</td>
<td>( z_{abk} = 1 ), if node ( a ) precedes node ( b ) in the route of vehicle ( k ); otherwise ( z_{abk} = 0 )</td>
</tr>
<tr>
<td>( q_j )</td>
<td>Quantity of goods delivered at demand point ( j )</td>
</tr>
<tr>
<td>( t_{ak} )</td>
<td>Arrival time of vehicle ( k ) at demand point ( a )</td>
</tr>
<tr>
<td>( \varsigma_{aijk}, \vartheta_{aijk} )</td>
<td>Auxiliary variables used to linearize nonlinear constraints</td>
</tr>
<tr>
<td>( r^\text{time}_i(s), r^\text{cost}_i(s) )</td>
<td>Regret of waiting time (delay) and system cost (cost overrun) of scenario ( l ) given by solution ( s )</td>
</tr>
<tr>
<td>( w_i(s), h_i(s) )</td>
<td>Total waiting time and system cost of scenario ( l ) given by solution ( s )</td>
</tr>
<tr>
<td>( f^\text{time}<em>\alpha, f^\text{cost}</em>\alpha )</td>
<td>CVaR-R of waiting time and system cost</td>
</tr>
</tbody>
</table>
2.3 Formulation under deterministic demand

When demand vector \( \mathbf{D} = (D_j, \forall j \in C_2) \) is fixed and given, the disaster relief LRP can be formulated as the following bi-objective mixed-integer nonlinear programming problem:

Model \( P_1 \):

\[
\min_{x, y, z, q} z_1(\mathbf{D}) = \{w, h\} \tag{1}
\]

s.t.

\[
\sum_{a \in C} \sum_{j \in C_2} z_{ajk} q_j \leq Cap_{ik}^{\text{veh}}, k = 1, 2, \ldots, K \tag{2}
\]

\[
\sum_{a \in C} \sum_{j \in C_2} \sum_{k=1}^K z_{ajk} y_{ik} q_j \leq Cap_{ik}^{\text{center}}, \forall i \in C_1 \tag{3}
\]

\[
D_j \leq q_j \leq \overline{D}_j, \forall j \in C_2 \tag{4}
\]

\[
t_{ak} + T_{aj} - (1 - z_{ajk}) M \leq t_{jk} \leq LT_{j}, \forall a \in C, j \in C_2, k = 1, 2, \ldots, K \tag{5}
\]

\[
\sum_{k=1}^K y_{ik} - x_i \geq 0, \forall i \in C_1 \tag{6}
\]

\[
y_{ik} \leq x_i, \forall i \in C_1, k = 1, 2, \ldots, K \tag{7}
\]

\[
y_{ik} = \sum_{j \in C_2} z_{ajk}, \forall i \in C_1, k = 1, 2, \ldots, K \tag{8}
\]

\[
\sum_{k=1}^K z_{akb} = 0, \forall a, b \in C_1 \tag{9}
\]

\[
\sum_{a \in C} z_{bak} - \sum_{a \in C} z_{abk} = 0, \forall b \in C, k = 1, 2, \ldots, K \tag{10}
\]

\[
\sum_{a \in C} \sum_{k=1}^K z_{ajk} = 1, \forall j \in C_2 \tag{11}
\]

\[
\sum_{i \in C_1} y_{ik} \leq 1, k = 1, 2, \ldots, K \tag{12}
\]

\[
x_i \in \{0, 1\}, \forall i \in C_1 \tag{13}
\]

\[
y_{ik} \in \{0, 1\}, \forall i \in C_1, k = 1, 2, \ldots, K \tag{14}
\]
\[ z_{abk} = \{0,1\}, \forall a, b \in C, k = 1, 2, \ldots, K \] (15)

where the two objectives in Eq.(1) are defined by

\[ w = \sum_{j \in C_2} \sum_{k=1}^{K} f_{jk} \] (16)

\[ h = \sum_{i \in C_1} SC_i x_i + UC \sum_{i \in C} \sum_{b \in C} \sum_{k=1}^{K} \text{Len}_{ab} z_{abk} + \sum_{i \in C_1} \sum_{k=1}^{K} FC_i \eta_{ik} \]

\[ + P_{\text{demand}} \sum_{j \in C_2} \max\{D_j - q_j, 0\} + P_{\text{supply}} \sum_{j \in C_2} \max\{q_j - D_j, 0\} \] (17)

Eq.(16) computes the total waiting time defined by the sum of the arrival times of vehicles at the demand points (Moshref-Javadi and Lee, 2016), whereas Eq.(17) calculates the total system cost, including the setup cost to open a DC, the fixed operating cost of the vehicles, the vehicle travel cost, and penalties for both lack of and oversupply (Wang et al., 2014). The two objectives capture both customers’ and operators’ interests. Constraints (2) and (3) are the capacity constraints for vehicles and DCs, respectively. Constraint (4) restricts the quantity of goods that can be delivered at a demand point to within its minimum and maximum demand. Constraint (5) is the time window constraint, which states that the arrival time of vehicle \(k\) at node \(j\) is no earlier than the completing time at node \(a\) plus the travel time between the two nodes if vehicle \(k\) travels from node \(a\) to \(j\), and before a given latest arrival time. Constraint (6) requires that a positive number of vehicles depart a candidate DC that is selected to open, whereas constraint (7) ensures that no vehicle leaves from a DC that is not open. Constraint (8) requires that each vehicle assigned to a DC must be assigned to a demand point. Constraint (9) forbids vehicles from traveling between DCs. Constraint (10) is the flow conservation constraint. Constraint (11) requires that each demand point accepts service exactly once. Constraint (12) ensures that each vehicle is assigned to only one DC. Finally, constraints (13) to (15) are the definitional constraints for the binary decision variables.

Model \( P_1 \) is a bi-objective mixed-integer nonlinear problem because constraints (2) and (3) are nonlinear and can be linearized with the following big-M method. A similar technique can be found in Mahdavi et al. (2010) and Ahmadi et al. (2015).
Given a general nonlinear constraint \( \varepsilon \zeta \leq A \) where \( \varepsilon \) is a continuous variable, \( \zeta \) is a binary variable, and \( A \) is a parameter, we define a new continuous variable \( \delta \) as \( \delta = \varepsilon \zeta \), which satisfies the following conditions:

When \( \zeta = 1 \), then \( \delta \) is forced to be equal to \( \varepsilon \); that is to say, \( \varepsilon \leq \delta \leq \varepsilon \). Otherwise, \( \varepsilon \) can have any values as long as the absolute value of the difference between \( \delta \) and \( \varepsilon \) is bounded by a positive number \( M \), which should be sufficiently large.

Accordingly, nonlinear constraints (2) and (3) are replaced by constraints (19) through (22) and constraints (23) through (26), respectively. Meanwhile, we use the weighted sum method to convert the bi-objective model into a single objective model by introducing a weighting parameter \( \gamma \). The resulting model, \( \overline{P}_1 \), is given below; it is a single-objective mixed-integer linear programming problem that can be solved with existing commercial solvers, such as CPLEX, when the size of the problem is small.

Model \( \overline{P}_1 \):

\[
\begin{align*}
\min & \quad z_j(D) = \gamma w + (1-\gamma)h \\
\text{s.t.} & \quad \text{constraints } (4) - (15) \\
& \quad \sum_{a \in C} \sum_{j \in C_2} \xi_{ajk} \leq \text{Cap}_k^{\text{veh}}, \forall k = 1, 2, \ldots, K \\
& \quad \xi_{ajk} \leq q_j + (1 - z_{ajk}) M, \forall a \in C, j \in C_2, k = 1, 2, \ldots, K \\
& \quad q_j + (z_{ajk} - 1) M \leq \xi_{ajk}, a \in C, j \in C_2, k = 1, 2, \ldots, K \\
& \quad \xi_{ajk} \geq 0, \forall a \in C, j \in C_2, k = 1, 2, \ldots, K \\
& \quad \sum_{a \in C} \sum_{j \in C_2} \sum_{k=1}^{K} \theta_{ajk} \leq \text{Cap}_i^{\text{center}} x_i, \forall i \in C_1 \\
& \quad \theta_{ajk} \geq q_j - M (1 - y_{ak}) - M (1 - z_{ajk}), \forall a \in C, i \in C_1, j \in C_2, k = 1, 2, \ldots, K \\
& \quad \theta_{ajk} \leq q_j + M (1 - y_{ak}) + M (1 - z_{ajk}), \forall a \in C, i \in C_1, j \in C_2, k = 1, 2, \ldots, K \\
& \quad \theta_{ajk} \geq 0, \forall a \in C, i \in C_1, j \in C_2, k = 1, 2, \ldots, K
\end{align*}
\]
2.4 Formulation under stochastic demand

2.4.1 CVaR and CVaR-R

Fig. 2 illustrates the concept of CVaR. The X-axis represents a random variable $X$. The Y-axis represents the occurrence probability.

![Fig. 2. Illustration of the concept of the CVaR](image)

a. Continuous variable  

b. Discrete variable

Given a random variable, $X$, whose probability density function and cumulative distribution function are defined, respectively, by $P_X(\cdot)$ and $F_X(\cdot)$. VaR$_{\alpha}$ of $X$, defined as the $\alpha$-quantile of $X$, is expressed as Eq.(27).

$$\text{VaR}_{\alpha}(X) = \inf\{\eta \in \mathbb{R} : F_X(\eta) \geq \alpha\} \quad (27)$$

The VaR has three drawbacks. 1) The reduction in VaR could increase the unreliability aspect of risk variability because minimizing VaR may lead to a stretch of the risk value to exceed VaR (Larsen et al., 2002). 2) VaR ignores the risk in the tail (Artzner et al., 1997, 1999), and the failure to consider catastrophic events may lead to an unacceptable outcome. 3) VaR is not a coherent risk measure (Artzner et al., 1999).

The shortcomings of VaR motivated the development of CVaR. CVaR at confidence level $\alpha$ is the mean of the risk incurred by the worst-case scenarios in the tail with collective probability at $1 - \alpha$.

When $X$ is a continuous variable, the CVaR at confidence level $\alpha$ is mathematically formulated as Eq.(28):
CVaR\(_\alpha\)(X) = \( \frac{1}{1 - \alpha} \int_{\alpha}^{\infty} xP_X(x)dx \) \hspace{2cm} (28)

where \( \eta \) is the value of VaR\(_\alpha\) computed in Eq.(27). When \( X \) is a discrete variable, \( F_X() \) is a step function that may jump at the point of VaR\(_\alpha\). To ensure that the collective occurrence probability of worst-case scenarios is exactly 1 - \( \alpha \), the probability of a scenario should be split. Suppose that discrete random variable \( X \) has \( N \) possible values. Let \( X_1 < X_2 < ... < X_{N-1} < X_N \), and the corresponding occurrence probabilities are \( \text{Prob}_1, \text{Prob}_2, ..., \text{Prob}_N \). The CVaR at confidence level \( \alpha \) with discrete variable is formulated as Eq. (29) (Rockafellar and Uryasev, 2002; Yin, 2008):

\[
CVaR\alpha(X) = \frac{1}{1 - \alpha} \left[ \sum_{n=1}^{N_\alpha} \text{Prob}_n - \alpha \right] X_{N_\alpha} + \sum_{n=N_\alpha+1}^{N} \text{Prob}_n X_n \hspace{2cm} (29)
\]

where \( N_\alpha \) satisfying Eq.(30) is used to find the worst-case scenarios with collective occurrence probability at 1 - \( \alpha \).

\[
\sum_{n=1}^{N_\alpha} \text{Prob}_n \geq \alpha > \sum_{n=1}^{N_{\alpha-1}} \text{Prob}_n \hspace{2cm} (30)
\]

Rockafellar and Uryasev (2000) proved that CVaR is a coherent risk measure that is transition-equivariant, positively homogeneous, convex, and monotonic. They also noted that minimizing CVaR also leads to a lower VaR.

CVaR-R is a special case of CVaR in which the random variable is associated with the regret of an objective value, such as travel time or cost, and a detailed definition of regret is given in the next section.

### 2.4.2 CVaR-R-based LRP model

Once a disaster occurs, the decision-maker should determine and implement a location-routing plan as quickly as possible. However, without knowing the exact demand information, the deterministic demand model developed in the previous section cannot be applied directly. Although the decision-maker could estimate the expected demand values and determine a
solution accordingly, the revealed demand could differ from the estimated demand. Here, instead of considering one demand estimation, we create a set of demand scenarios, each of which occurs with a certain probability.

In each scenario \( l \), given its demand vector \( D_l \), we can formulate Model \( \mathcal{P}_l \) for each scenario as

\[
\min_{s=[x,y,z,q]} \pi_l(D_l) = \gamma w + (1-\gamma)h
\]

s.t. constraints (4)–(15) and (19)–(26)

Any location-routing design may not be optimal for scenario \( l \), so the decision-maker may regret his or her decisions due to the performance difference between the implemented solution and the optimal solution of the realized scenario. To minimize the unexpected expectation of regret by worst-case scenarios with collective probability at \( 1 - \alpha \), we introduce the CVaR-R measure.

In our problem, given a solution \( s=[x,y,z,q] \), the CVaR-R measures are computed by the algorithm in Fig. 3. First, we obtain the minimum waiting time and the minimum system cost of each scenario. Then, in each scenario, the regret of a solution with respect to an objective is defined by the difference between the objective value obtained via the solution considered and that of the optimal (ideal) solution corresponding to the scenario (see Eq.(32) and Eq.(33)). The regret of the waiting time can be seen as delivery delay, and the regret of the system cost can be regarded as the cost overrun, defined as the expenditure exceeding the minimum cost of the realized scenario. It is straightforward that the occurrence probability of the risk corresponds to the probability of the scenario; therefore, the cumulative probability function of regret can be obtained, and the worst-case scenarios can be identified. Based on Eq.(29), the mean regret of the waiting time and the system cost of the worst \( (1 - \alpha) \) scenarios, namely, the CVaR-R of the waiting time and the CVaR-R of the system cost, are formulated as Eq.(36) and Eq.(37), respectively.
Step 1. Find the ideal point \((w_l^*, h_l^*)\) for each demand scenario \(l \in L\).

Step 1.1. Solve model \(\overline{P_l}\) with \(\gamma = 1\) to obtain the minimum waiting time \(w_l^*\).

Step 1.2. Solve model \(\overline{P_l}\) with \(\gamma = 0\) to obtain the minimum system cost \(h_l^*\).

Step 2. Compute the regret values for all scenarios via the following equations

\[
\begin{align*}
    r_l^{\text{time}}(s) &= w_l(s) - w_l^*, \quad \forall l \in L \\
    r_l^{\text{cost}}(s) &= h_l(s) - h_l^*, \quad \forall l \in L
\end{align*}
\]

where \(w_l(s)\) and \(h_l(s)\) denote the waiting time and the system cost if scenario \(l\) occurs and solution \(s\) is implemented; \(r_l^{\text{time}}(s)\) and \(r_l^{\text{cost}}(s)\) are the regrets of the waiting time (delay) and system cost (cost overrun) given by solution \(s\) under scenario \(l\).

Step 3. Identify the worst scenarios, whose collective probability of occurrence is \(1 - \alpha\).

Step 3.1. Rank and index the regret of the waiting time and the system cost given by solution \(s\) in ascending order, i.e.,

\[
    r_l^{\text{time}}(s) \leq r_{l+1}^{\text{time}}(s) \leq \cdots \leq r_L^{\text{time}}(s) \quad \text{and} \quad r_l^{\text{cost}}(s) \leq r_{l+1}^{\text{cost}}(s) \leq \cdots \leq r_L^{\text{cost}}(s).
\]

Step 3.2. Obtain the reordered set of scenario probabilities associated with the two objectives:

\[
\begin{align*}
    \Gamma^{\text{time}} &= \{Pr_1, Pr_2, \ldots, Pr_{|L|}\} \quad \text{such that the index of the probabilities satisfies} \\
    r_l^{\text{time}}(s) &\leq r_{l+1}^{\text{time}}(s), \quad \text{for } l = 1, 2, \ldots, |L| - 1, \quad \text{and} \quad \Gamma^{\text{cost}} &= \{Pr_1, Pr_2, \ldots, Pr_{|L|}\} \quad \text{such that the index of the probabilities satisfies} \\
    r_l^{\text{cost}}(s) &\leq r_{l+1}^{\text{cost}}(s), \quad \text{for } l = 1, 2, \ldots, |L| - 1.
\end{align*}
\]

Step 3.3. Find unique indices \(\tau_{\alpha}^{\text{time}}\) and \(\tau_{\alpha}^{\text{cost}}\) that satisfy the following conditions:

\[
\begin{align*}
    \sum_{l=1, Pr_l \geq \tau_{\alpha}^{\text{time}}}^{\tau_{\alpha}^{\text{time}}} Pr_l &> \sum_{l=1, Pr_l < \tau_{\alpha}^{\text{time}}} Pr_l \\
    \sum_{l=1, Pr_l \geq \tau_{\alpha}^{\text{cost}}}^{\tau_{\alpha}^{\text{cost}}} Pr_l &> \sum_{l=1, Pr_l < \tau_{\alpha}^{\text{cost}}} Pr_l
\end{align*}
\]

Step 4. Compute the CVaR-R of the waiting time and the CVaR-R of the system cost, which are formulated as
\[ \text{CVaR-R}_{\text{time}}^\alpha(s) = \frac{1}{1 - \alpha} \left[ \left( \sum_{l=1}^{L} \text{Pr}_l \cdot \eta_{\text{time}}^\alpha \right) r_{\text{time}}^\alpha(s) + \sum_{l=L+1}^{L} \text{Pr}_l r_{l\text{time}}^\alpha(s) \right] \] (36)

\[ \text{CVaR-R}_{\text{cost}}^\alpha(s) = \frac{1}{1 - \alpha} \left[ \left( \sum_{l=1}^{L} \text{Pr}_l \cdot \eta_{\text{cost}}^\alpha \right) r_{\text{cost}}^\alpha(s) + \sum_{l=L+1}^{L} \text{Pr}_l r_{l\text{cost}}^\alpha(s) \right] \] (37)

Fig. 3. Algorithm for computing CVaR-R of waiting time and CVaR-R of system cost

According to Rockafellar and Uryasev (2002), Eq.(36) and Eq.(37) can be transformed into Eq.(38) and Eq.(39).

\[ f_{\text{time}}^\alpha(s) = \text{CVaR-R}_{\text{time}}^\alpha(s) = \inf \left\{ \eta_{\text{time}}^\alpha + \frac{1}{1 - \alpha} \left[ \sum_{l=1}^{L} \text{Pr}_l \cdot \max(r_{l\text{time}}^\alpha(s) - \eta_{\text{time}}^\alpha, 0) \right] \right\} \] (38)

\[ f_{\text{cost}}^\alpha(s) = \text{CVaR-R}_{\text{cost}}^\alpha(s) = \inf \left\{ \eta_{\text{cost}}^\alpha + \frac{1}{1 - \alpha} \left[ \sum_{l=1}^{L} \text{Pr}_l \cdot \max(r_{l\text{cost}}^\alpha(s) - \eta_{\text{cost}}^\alpha, 0) \right] \right\} \] (39)

where \( \eta_{\text{time}}^\alpha \) and \( \eta_{\text{cost}}^\alpha \) are free decision variables. The minimums of Eq.(38) and Eq.(39) are obtained when \( \eta_{\text{time}}^\alpha \) and \( \eta_{\text{cost}}^\alpha \) are at the \( \alpha \)-quantile of \( r_{l\text{time}}^\alpha \) and \( r_{l\text{cost}}^\alpha \), which are the VaR-R of the waiting time and the VaR-R of the system cost, respectively (Ogryczak and Ruszczynski, 2002; Noyan, 2012). By accounting for VaR-R and worst (1 - \( \alpha \)) scenarios simultaneously, the CVaR-R measure answers two questions: 1) from the aspect of reliability, how much regret (i.e., delay, cost overrun) will result with a certain confidence level; 2) from the aspect of unreliability, how much regret will result when the worst-case scenarios occur.

Finally, the bi-objective CVaR-R-based LRP model to determine an optimal \( s = [x, y, z, q] \) that minimizes \( f_{\alpha}^{\text{time}}(s) \) and \( f_{\alpha}^{\text{cost}}(s) \) simultaneously is mathematically expressed as

Model \( P_2 \):

\[ \min_{s \in [x, y, z, q]} z_2 = \{ f_{\alpha}^{\text{time}}(s), f_{\alpha}^{\text{cost}}(s) \} \] (40)

s.t. constraints (4)–(15) and (19)–(26)

2.5 Nash bargaining solution

A Pareto frontier can be obtained by solving the model \( P_2 \) and regarding the bi-objective
disaster LRP as a bargaining game between two players. One player is represented by the people affected by the disaster who hope to receive the supplies as soon as possible, and the other is the operator, who prefers to provide the supplies to the affected people with a lower cost on the premise of satisfying the requirements from both supply and demand aspects (e.g., capacity constraints and time window constraints). The two purposes are neither completely coincident nor completely opposed, so the two players can cooperate and reach an enforceable agreement on a reasonable plan of action (Nash, 1953). Nash (1950, 1953) proved that under a set of axioms, a unique solution, called the NBS, always exists to such a bargaining game. The final solution depends on the game rules negotiated by the two players. In this section, a distance function (see Eq.(41)) to calculate the proximity between a solution on the Pareto curve and the ideal solution is introduced to represent this rule, so that the decision-maker can determine the NBS from a Pareto frontier. This distance function has been used by Zhang and Ge (2004) and by Zhong et al. (2017) to design road pricing systems.

$$d_\lambda(s) = \left[\frac{f^t_{a}(s) - \min_{s' \in \Omega} \left\{ f^t_{a}(s') \right\}}{\max_{s' \in \Omega} \left\{ f^t_{a}(s') \right\} - \min_{s' \in \Omega} \left\{ f^t_{a}(s') \right\}}\right]^\lambda + \lambda \left[\frac{f^c_{a}(s) - \min_{s' \in \Omega} \left\{ f^c_{a}(s') \right\}}{\max_{s' \in \Omega} \left\{ f^c_{a}(s') \right\} - \min_{s' \in \Omega} \left\{ f^c_{a}(s') \right\}}\right]^\lambda, \forall s \in \Omega$$

(41)

where $\Omega$ denotes the set on the Pareto frontier, and $\lambda$ is a distance parameter. Given a certain $\lambda$, the point closest to the ideal point, namely, the NBS, can be determined by

$$s^* = \arg \min_{s \in \Omega} d_\lambda(s)$$

(42)

According to Eq.(41), if $0 < \lambda < 1$, the objective with a smaller deviation plays a more important role in $d_\lambda(s)$; if $\lambda = 1$, both objectives have the same weight; if $1 < \lambda < \infty$, the objective with larger deviation has greater weight in $d_\lambda(s)$.

3 Solution Procedure

To solve the proposed model and determine an NBS requires 1) solving a single-objective LRP problem (i.e., Steps 1.1. and 1.2 in Fig. 3), and 2) obtaining the Pareto frontier to calculate an NBS (i.e., Eqs. (41) and (42)). A hybrid GA was developed in this study to address these two
problems. This hybrid algorithm uses a GA to solve the single-objective LRP problem and an
NSGA-II to obtain the Pareto frontier. The motivations for developing such a hybrid algorithm
are twofold. First, the LRP is NP-hard because it is an integration of the facility location
problem and the vehicle routing problem, both of which are NP-hard. Therefore, it is common
to devise a heuristic or metaheuristic algorithm. Existing metaheuristics for solving a single-
objective LPR include Tabu search (Tuzun and Burke, 1999), simulated annealing (Yu et al.,
2010), and GA (Ardjmand et al., 2015; Hiassat et al., 2017; Fazayeli et al., 2018; Lee, 2018).
GA is adopted because Fazayeli et al. (2018) showed that it could find the near-optimal solution
for the LRP within a reasonable time. Second, to address multi-objective optimization problems,
NSGA-II, developed by Deb et al. (2002), is a fast and efficient solution method, and in the
context of LRP, its superiority has been shown by Rabbani et al. (2018), Asadi et al. (2018),
and Mahmoudsoltani et al. (2018).

The following subsection begins with an overview of the solution algorithm, followed
by the detailed crossover and mutation operators specifically devised for our problem.

3.1 Overview of the algorithm

The overview of the algorithm is given in Fig. 4.

Step 1. Initialization
   Step 1.1. Set parameters for the GA, including population size $T$, crossover and mutation
             rates, and maximum number of iterations, $I_{\text{max}}$;
   Step 1.2. Obtain $(\mathbf{w}, \mathbf{h})$ for each scenario $l \in L$ (See. Fig. 3. Step 1)
Step 2. Generate the Pareto frontier using NSGA-II.
   Step 2.1. Set $I = 0$.
   Step 2.2. Generate initial population $P^I$. Calculate the fitness values (CVaR-R of waiting
time and CVaR-R of system cost) of each chromosome in population $P^I$ and assign a
ranking and crowding distance to each based on non-dominated sorting.
   Step 2.3. Generate offspring $Q^I$ using crossover and mutation operators.
   Step 2.4. Construct a temporary population $R^I = P^I \cup Q^I$, and trim $R^I$ to form the next
generation $P^{I+1}$ according to non-dominated sorting and elitism selection.
   Step 2.5. Check the termination criterion. If $I < I_{\text{max}}$, then set $I = I + 1$ and return to Step
2.3; otherwise, proceed to step 3.
Step 3. Obtain the Nash bargaining solution.

Fig. 4. Overview of the algorithm

Remarks:

1. To apply GA to solve our problem, the chromosome representation, solution generation, crossover, and mutation operators are specifically designed to cater to the solution structure and will be introduced in Section 3.2.

2. In Step 1.2, obtaining \( (w_i', h_i') \) requires the solution of model \( \overline{p}_i \). For a small problem, the model can be solved with existing commercial solvers such as CPLEX, which allows us to design experiments to benchmark the performance of the proposed GA, which will be given in Section 4.

3. For convenience, the GA for solving the single-objective LRP uses the same chromosome representation, crossover and mutation operators, and parameter setting as those in NSGA-II.

3.2 Operators in GA

3.2.1 Chromosome representation

Based on the decision variables considered, the representation of an individual chromosome is designed to have four substrings, as depicted in Eq.(43), where \( g_{s1} \), \( g_{s2} \), \( g_{s3} \), and \( g_{s4} \) are the mutant vectors that together represent a generated solution \( s \).

\[
\mathbf{g}_s = \left( \begin{array}{c}
g_{s1}^1, g_{s1}^2, \ldots, g_{s1}^{K_1} \\
g_{s2}^1, g_{s2}^2, \ldots, g_{s2}^{K_2} \\
g_{s3}^1, g_{s3}^2, \ldots, g_{s3}^{K_3} \\
g_{s4}^1, g_{s4}^2, \ldots, g_{s4}^{K_4} \\
\end{array} \right)
\]

(43)

\( g_{s1} \) contains \( |C_2| \) gens, which represents all demand points. The number of each gen is an integer between 1 and \( K \) that indicates which vehicle is allocated to the demand point corresponding to the gen. \( g_{s2} \) contains the same number of gens as \( g_{s1} \), and the value of each gen is an integer between 1 and \( |C_2| \). Each number between 1 and \( |C_2| \) appears only once in
$g_{s2}$ to indicate the vehicle routing among demand points. $g_{s3}$ contains $K$ gens, and the value of each gen is an integer between 1 and $|C_j|$ that denotes the starting DC for each vehicle and indicates how vehicles are allocated to DCs. From $g_{s1}$, $g_{s2}$, and $g_{s3}$, vehicle routes can be inferred. For instance, consider an example with $|C_1| = 2$, $K = 3$, and $|C_2| = 5$. The first three substrings are $g_{s1} = (1, 2, 2, 1, 2)$, $g_{s2} = (5, 3, 4, 2, 1)$, and $g_{s3} = (2, 1, 1)$. $g_{s1}$ states that only vehicles 1 and 2 are used because only numbers 1 and 2 have appeared. To deduce vehicle 1’s route, we observe $g_{s1}$ and notice that vehicle 1 appears at the 1st and 4th gen. We then check the 1st and 4th gen in $g_{s2}$. The corresponding numbers are 5 and 2, respectively. This means that vehicle 1 passes demand point 5 and then demand point 2. Finally, looking into $g_{s3}$ shows that the number in the first-gen is 2; thus vehicle 1 is assigned to DC 2. To conclude, the route of vehicle 1 is DC $2 \to 5 \to 2 \to DC_2$. Similarly, we can deduce that the route of vehicle 2 is DC $1 \to 3 \to 4 \to 1 \to DC_1$.

The last substring, $g_{s4}$, denotes the quantity of goods delivered at each demand point using binary gens. In our study, each demand value is encoded by 5 binary gens $^*$, i.e., $g_{s4}^i = (b_5^i, b_4^i, b_3^i, b_2^i, b_1^i)$; thus, the total length of $g_{s4}$ is $5 \times |C_2|$. The decoding method is presented in Eq.(44).

\[
q_j = D_j + \left( \sum_{i=1}^{5} b_i 2^{-i} \right) \frac{D_j - D_j}{2^5 - 1} \tag{44}
\]

### 3.2.2 Crossover

Crossover is used to produce two offspring by selecting and mating two chromosomes, known as parents, from the population. In this study, the tournament selection is used to select parents based on fitness values. Two crossover strategies are adopted: two-point crossover and partially

---

* The number depends on the upper and lower bounds of the demand values.
mapped crossover (Goldberg and Lingle, 1985). The former applies to substrings \( g_{s1}, g_{s3}, \)
and \( g_{s4}, \) and the latter applies to substring \( g_{s2}. \) The two crossovers are illustrated in Fig. 5.

### Two-point crossover

| Before crossover: | Parent 1: 1 || 2 2 | 1 || 2 1 |
|                  | Parent 2: 2 || 2 1 2 || 1 |
| After crossover: | Offspring 1: 1 || 2 1 2 || 2 1 |
|                  | Offspring 2: 2 || 2 2 1 || 1 |

### Partially mapped crossover

| Before crossover: | Parent 1: 1 || 2 3 || 4 5 |
|                  | Parent 2: 5 || 4 1 || 3 2 |
| Swap:            | Temporary offspring 1: 1 || 4 1 || 4 5 |
|                  | Temporary offspring 2: 5 || 2 3 || 3 2 |
| Map:             | 4\(\rightarrow\)2, 1\(\rightarrow\)3 |
| Switch:          | Offspring 1: 3 || 4 1 || 2 5 |
|                  | Offspring 2: 5 || 2 3 || 1 4 |

**Fig. 5.** Crossover operations

### Swap mutation

| Before mutation: | 1 2 2 1 2 |
| After mutation:  | 1 1 2 2 |

### Random resetting

| Before mutation: | 2 1 1 |
| After mutation:  | 1 1 3 |

### Inversion mutation

| Before mutation: | 5 4 3 2 1 |
| After mutation:  | 5 2 3 4 1 |

**Fig. 6.** Mutation operations

### 3.2.3 Mutation

Three mutation strategies—swap, random resetting, and inversion—are developed and applied
to different substrings. The swap mutation is applied to substring $g_{s1}$. The random resetting mutation is for substring $g_{s3}$. For substrings $g_{s2}$ and $g_{s4}$, the inversion mutation is used. The mutation operators are illustrated in Fig. 6.

3.2.4 Selection

The procedures to locate the Pareto frontier and select survival populations follow those described by Deb et al. (2002), in which a non-dominated ranking and crowding distance are used. Interested readers are referred to this paper for more detail.

4 Numerical Study

Because no benchmark has been established for the proposed LRP, we constructed a network based on Solomon’s VRPTW benchmark test RC208. We randomly selected 3 points as the candidate DCs and another 20 points as demand points. The coordinates in our network are the same as those in RC208. Ten vehicles are available in the logistics system. The time window of each demand point and those of the 10 demand scenarios are generated randomly. The information about candidate distribution centers, vehicles, and demand points are listed in Tables 2, 3, and 4, respectively. Fig. 7 shows the geographic distribution of the network’s DCs and demand points. Numerical tests are designed to benchmark the performance of the developed GA and to demonstrate the solution property of the proposed CVaR-R model and the effects of different parameters on the solution.
Fig. 7. Locations of candidate DCs and potential demand points

Table 2 Candidate distribution center attributes

<table>
<thead>
<tr>
<th>Candidate DC</th>
<th>Coordinate X/km</th>
<th>Coordinate Y/km</th>
<th>Capacity (#items)</th>
<th>Setup cost (CNY)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>42</td>
<td>60</td>
<td>500</td>
<td>20000</td>
</tr>
<tr>
<td>B</td>
<td>72</td>
<td>35</td>
<td>400</td>
<td>10000</td>
</tr>
<tr>
<td>C</td>
<td>35</td>
<td>30</td>
<td>600</td>
<td>30000</td>
</tr>
</tbody>
</table>

Table 3 Vehicle attributes

<table>
<thead>
<tr>
<th>Vehicle No.</th>
<th>Capacity</th>
<th>Fixed cost (CNY)</th>
<th>Cost per unit length (CNY/km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>100</td>
<td>400</td>
<td>9</td>
</tr>
<tr>
<td>6-10</td>
<td>150</td>
<td>500</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 4 Information about demand points

<table>
<thead>
<tr>
<th>Demand point</th>
<th>Coordinate X/km</th>
<th>Coordinate Y/km</th>
<th>Demand in each scenario</th>
<th>Latest arrival time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>85</td>
<td>S1 30  S2 8  S3 23  S4 35  S5 35  S6 22  S7 33  S8 19  S9 34  S10 33</td>
<td>126</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
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<td>----</td>
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<tr>
<td></td>
<td>5</td>
<td>42</td>
<td>38</td>
<td>95</td>
</tr>
<tr>
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<td>45</td>
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<td>5</td>
<td>35</td>
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<td>28</td>
<td>35</td>
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<tr>
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<td>22</td>
<td>29</td>
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<td>20</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>32</td>
<td>8</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>31</td>
<td>14</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>31</td>
<td>36</td>
<td>35</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>36</td>
<td>35</td>
<td>32</td>
<td>32</td>
</tr>
</tbody>
</table>

4.1 Benchmark results with CPLEX

The solution algorithm in Fig. 4 relies on identifying the ideal point for each scenario by solving the problem $P_i$ with different weights (i.e., $\gamma = 0$ and $\gamma = 1$). For a small network, the problem can be solved with off-the-shelf software, such as CPLEX. As the network size increases, the problem cannot be solved. Indeed, our preliminary test showed that when the number of demand points exceeded 11, CPLEX could not solve the problem on a personal computer due to insufficient memory, which justifies the necessity of adopting a metaheuristic algorithm.
Table 5 Comparison between CPLEX and GA in solving model $P_1$

(a) Results

<table>
<thead>
<tr>
<th>Scenario</th>
<th>CPLEX</th>
<th>GA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Waiting time $(w)$ (set $\gamma = 1$)</td>
<td>System cost $(h)$ (set $\gamma = 0$)</td>
</tr>
<tr>
<td></td>
<td>Optimal objective value</td>
<td>CPU time (seconds)</td>
</tr>
<tr>
<td>S1</td>
<td>144</td>
<td>4389</td>
</tr>
<tr>
<td>S2</td>
<td>144</td>
<td>2488</td>
</tr>
<tr>
<td>S3</td>
<td>144</td>
<td>2413</td>
</tr>
<tr>
<td>S4</td>
<td>144</td>
<td>3648</td>
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<tr>
<td>S5</td>
<td>144</td>
<td>3504</td>
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<tr>
<td>S6</td>
<td>144</td>
<td>5473</td>
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<td>S7</td>
<td>144</td>
<td>10340</td>
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<td>S8</td>
<td>144</td>
<td>4401</td>
</tr>
<tr>
<td>S9</td>
<td>144</td>
<td>6401</td>
</tr>
<tr>
<td>S10</td>
<td>144</td>
<td>10126</td>
</tr>
</tbody>
</table>

(b) Optimal solutions

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Waiting time $(w)$ (set $\gamma = 1$)</th>
<th>System cost $(h)$ (set $\gamma = 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. of DCs selected</td>
<td>No. of vehicles utilized</td>
</tr>
<tr>
<td>S1 - S6</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>S7</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>S8 - S10</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

To examine the performance of the proposed GA, we extracted a small network from the network presented in Fig. 7 and conducted experiments to compare the results between GA and CPLEX. The network includes 3 candidate DCs, 10 demand points (demand points 1 to 10), and 4 vehicles. Ten demand scenarios were simulated. All tests were run on a computer with an Intel Core i5-8250U 1.60 GHz CPU and 8 GB RAM. The CPLEX version was 12.1, and C++ API was used. The GA was coded in Matlab R2019a, and its parameters were set as follows: maximum iteration $I_{\text{max}} = 2000$, and the crossover and mutation rates were set as 0.8 and 0.2, respectively. The results are presented in Table 5.
Table 5 (a) shows that the GA successfully obtains the same optimal solution as CPLEX in all scenarios but with a significantly shorter computation time, which indicates that the developed GA can find the exact solution in a small network. Meanwhile, Table 5 (a) shows that in all scenarios, the total waiting time was the same but the system cost varied because the demand stochasticity in our model does not affect the objective of minimizing the waiting time when $\gamma = 1$, whereas it affects the demand penalty term in the objective of minimizing the total system cost when $\gamma = 0$.

Table 5 (b) lists the optimal number of DCs and vehicles selected. It shows that when the objective is to minimize the waiting time, more DCs and vehicles will be selected because a shorter waiting time can be achieved if more vehicles start simultaneously from the DCs that are near the demand points. In contrast, when the objective is to minimize the total system cost, fewer DCs and vehicles are selected, mainly due to the high setup cost for DCs.

4.2 Effect of $\alpha$ on the Pareto frontier

The confidence level $\alpha$ is predetermined by the decision-maker. This experiment was designed to examine the effect of $\alpha$ on the Pareto frontier. The Pareto frontiers associated with confidence levels $\alpha = 90\%$, $\alpha = 70\%$, and $\alpha = 50\%$ are plotted in Fig. 8, from which the following conclusions are made.

1) A trade-off is required between the two proposed CVaR-R objectives. Increasing the number of open DCs leads to a reduction in the CVaR-R of the waiting time at the cost of increasing the CVaR-R of the system cost.

2) The Pareto solutions are distributed unevenly on the Pareto frontier regardless of the value of $\alpha$ due to the difference in the numbers of opening DCs. For instance, the solutions in areas G1, G2, and G3 choose three DCs, two DCs, and one DC, respectively.

3) The magnitude of the changes in the two objectives varies when a new DC is open. A comparison of solutions in areas G2 and G3 shows that when DC B is open, the increase in the CVaR-R of the system cost is minor, but the CVaR-R of the waiting time is reduced by almost half. This is because DC A is far from the demand points in the upper side of Fig. 7. If all vehicles start from DC A, the CVaR-R of the waiting time will be very long. Once DC
B is open, the demand points distant from DC A can be allocated to it. Therefore, the CVaR-R of the waiting time can be shortened significantly. In contrast, a comparison of solutions in areas G1 and G2 shows that opening DC C has significant effects on both the CVaR-R of the system cost and the CVaR-R of the waiting time.

4) The value of $\alpha$ plays an important role in the final decisions of the location-routing design. The Pareto frontier moves upward as the value of $\alpha$ increases because a smaller value of $\alpha$ means less confidence that the worst risk value will exceed the VaR-R. This conclusion is consistent with the discovery of Faghih-Roohi et al. (2016) in the context of the routing and scheduling of hazardous material transportation networks.

Fig. 8. Pareto frontiers of the medium-sized network with different confidence level $\alpha$

4.3 Effect of $\lambda$ on NBS

To balance the interests between the CVaR-R of the waiting time and the CVaR-R of the system
cost, this study proposed to find the NBS based on its proximity to the ideal solution captured by parameter $\lambda$. Taking the Pareto frontier with $\alpha = 90\%$ (the blue dots in Fig. 8) as an example, the effects of $\lambda$ are demonstrated in Fig. 9. In the test, the value of $\lambda$ increases from 1 to 5 in steps of 0.1. The NBS varies with the value of $\lambda$ in a stepwise manner, and there are only two steps in the figure, which indicates that only three nodes on the Pareto frontier are selected as NBSs. These three points and their corresponding values of $\lambda$ are marked in Fig. 8. For example, when $0.4 \leq \lambda \leq 3.7$, the CVaR-R values of the waiting time and the system cost are 36 and 86057, respectively. The solution states that if the worst-case scenarios in the distribution tail occur, the expected values for total delay and cost overrun are 36 min and 86,057 Chinese yuan (CNY), respectively. The resulting opened DC and vehicle routes are presented in Fig. 10, and the actual quantity of goods delivered to each demand point are listed in Table 6.

Fig. 9. Optimal NBS with different values of $\lambda$
Table 6 Quantity of goods delivered to each demand point of NBS when $0.4 \leq \lambda \leq 3.7$

<table>
<thead>
<tr>
<th>Demand point</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount of delivered goods (item)</td>
<td>33</td>
<td>25</td>
<td>30</td>
<td>30</td>
<td>27</td>
<td>27</td>
<td>32</td>
<td>23</td>
<td>32</td>
<td>22</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Demand point</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount of delivered goods (item)</td>
<td>24</td>
<td>18</td>
<td>23</td>
<td>29</td>
<td>23</td>
<td>30</td>
<td>12</td>
<td>22</td>
<td>19</td>
<td>31</td>
</tr>
</tbody>
</table>

Fig. 10. Facility location and vehicle routes of NBS when $0.4 \leq \lambda \leq 3.7$

5 Conclusions

In disaster relief logistics, the demand stochasticity and decision-maker’s risk attitude have not been addressed to solve the LRP, and in the location and routing problem, no existing studies propose a bi-objective CVaR-R model.

In this study, a risk-averse approach was developed to construct an integrated location and vehicle routing plan for the disaster relief distribution problem with attributes of multiple capacitated depots, multiple capacitated vehicles, and hard time windows under stochastic demand. Although previous studies examined the integrated location and vehicle routing
problem under different sources of stochasticity, little effort has been made to solve the disaster relief LRP based on CVaR-R. This research gap is filled in this study. Two CVaR-R measures were proposed, the CVaR-R of the waiting time and the CVaR-R of the system cost. As a result, the risk-averse optimization problem was formulated as a bi-objective mixed-integer nonlinear programming model. To help the decision-maker determine the final solution from the Pareto frontier, this study adopted the concept of the NBS. The problem was solved via a hybrid GA, in which the NSGA-II was adopted to obtain the Pareto frontier of the two CVaR-R objectives. Because the computation of a CVaR objective requires knowledge of the ideal solution associated with the objective in each scenario, a single objective GA was embedded to obtain such a solution. Three case studies were carried out to illustrate the performance of the solution method and to demonstrate the properties of the proposed model. It was shown that the proposed GA can determine the exact solution in a small network as CPLEX with a significantly shorter computation time, and the hybrid GA can solve the bi-objective model in a large network.

The following conclusions are worth noting. 1) A trade-off exists between the two proposed CVaR-R objectives (e.g., the CVaR-R of the waiting time and the CVaR-R of the system cost). 2) The Pareto solutions are distributed unevenly on the Pareto frontiers according to the open DCs, regardless of the value of $\alpha$. 3) A larger value of $\alpha$ leads to more conservative Pareto solutions. 4) Few nodes on the Pareto frontier can be selected as NBSs. On the one hand, this means that not all solutions on the Pareto frontier are promising to decision-makers. On the other hand, it implies that decision-makers with different interests could reach the same decision. In practice, it means that when multiple stakeholders with different interests are involved in the decision-making process, it is possible that a solution exists that satisfies all stakeholders.

**Acknowledgment**

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**References**


