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Effect of superimposed compressive stresses on rolling contact fatigue initiation at hard and soft inclusions

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Abstract

A semi-analytical framework is introduced for the evaluation of favorable residual stresses for delaying rolling contact fatigue initiation. Subsurface stress histories are applied to a micro-scale model accounting for isolated inclusions of different types and geometries. Micro-scale von Mises stresses are calculated based on Eshelby's method and considered as an estimator of crack initiation due to plastic deformation. The most critical cases in terms of micro-scale plasticity are identified and the effect of macro-scale compressive residual stresses is considered. Finally, optimized residual stresses are determined that minimize the maximum attained micro-scale von Mises stress at different depths.

Keywords: Rolling contact fatigue, Non-metallic inclusions, Residual stresses, Fatigue damage initiation, Eshelby's method

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1. Introduction

Rolling element bearings, widely used to support rotary elements in modern machinery, mainly suffer from two failure mechanisms corresponding to surface originated pitting and subsurface initiated spalling [1, 2]. While surface initiated pitting is normally attributed to poor lubrication conditions [3], well-lubricated bearings with smooth contact surfaces typically fail due to subsurface initiated rolling contact fatigue with cracks emanating from non-metallic inclusions [1, 4]. With continuously emerging applications in

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9 new types of machinery, new environments and evolving bearing steel tech-
10 nologies, research for accurately predicting and extending bearing lifetime is
11 highly important.

12 Contributions to a better understanding, quantification and improvement
13 of rolling contact fatigue include i) experimental work for determining the
14 fatigue performance of bearing steels [5, 6], ii) lifetime models at the com-
15 ponent level [7, 8], iii) phenomenological fatigue models at the homogenized
16 material level [9, 10], and iv) micro-mechanical models for damage evolution
17 typically in the vicinity of inclusions [11, 12].

18 Despite the big progress in understanding the fatigue failure of bearings,
19 there are still often contradicting theories and interpretations of experimental
20 data. This partial lack of consensus [7] indicates that a full understanding of
21 the links between all phenomena observed in the aforestated research areas
22 still has to be reached. The main difficulties for reaching a full understanding
23 of rolling contact fatigue include:

- 24 • complex spatial and temporal variations of the subsurface stress state,
- 25 • varying material microstructure with material depth [13] and time [14],
- 26 • stochastic occurrence of inclusions of different types, and
- 27 • subsurface stress histories offset by an unknown reference stress state.

28 Especially with regard to the latter point, depth dependent residual and
29 hoop stresses of significant magnitude compared to the material strength are
30 not uncommon [13]. Residual stresses typically arise from phase transfor-
31 mations during heat treatment or operation but they can also be induced
32 or relaxed by plastic deformation during manufacturing or running-in of a
33 bearing [15, 16]. While any reported monotonic positive or negative effect of
34 residual stresses on fatigue performance can only be valid within a certain
35 range of values [17, 18, 19], there are also theoretical models capturing a
36 non-monotonic and hence more realistic dependence [9, 20, 21, 22, 16].

37 An often disputed aspect in rolling contact fatigue is the existence of a
38 fatigue endurance limit and its relevance for lifetime calculations [7]. Never-
39 theless, based on the proposals by Ioannides and Harris [23, 24], the concept
40 of a fatigue stress limit, defined as a stress level below which no material
41 deterioration is expected, is now part of widely used bearing life models [8].
42 Assuming the existence of such a limit, micro-mechanical models have been
43 developed to determine how this limit would be affected by inclusions, based
44 for example on Eshelby's method [21, 25] or finite element analysis [26]. In

any case, irrespective of the existence of a material fatigue limit, a better understanding of the early phase of crack development is certainly relevant for improving bearing life models.

Regarding fatigue crack initiation, cyclic plasticity both at the macro- and micro-scale is often seen as a contributing factor to damage accumulation [14]. While cyclic plasticity observed at the macro-scale is mainly related to transformation of the softer retained austenite to martensite, it is stress concentrations near material inhomogeneities that lead to localized micro-scale plastic deformations. The latter case is concerned here, with the present work dealing with micro-scale plasticity in the vicinity of non-metallic inclusions. Although the density and size of inclusions in bearing steels can be reduced to a very large extent by appropriate steelmaking practices [27, 28], their occurrence cannot be excluded entirely. Even for quite clean steels, inclusions of different forms, sizes and orientations occur stochastically leading stress concentration sites at the micro-scale. In that sense, a practical approach is to simply consider the worst case of inclusion type and shape for a given steel.

For a properly loaded bearing, it is in general expected that even if the yield limit of the material is initially exceeded locally near an inhomogeneity, the amount of near inclusion plasticity per load cycle will eventually decay to zero. A numerical simulation of such an elastic shakedown process after the initial exceeding of the material yield limit is rather complex and computationally expensive. However, predicting a conservative load range for which the material yield limit is not exceeded in the first place is rather simple. This fact has been exploited in [29] in order to predict a load limit, claimed to represent a fatigue endurance limit, which ensures that micro-scale plasticity does not occur in any phase of the bearing operation, not even initially.

Moreover, since the onset of plasticity is governed by the maximum von Mises stress attained during a complete overrolling, it is expected that any superimposed stresses at the component level, such as residual or hoop stresses, will affect the estimated fatigue endurance limit. This effect has clearly been demonstrated in [29] and is further pursued in the present work in order to determine optimal residual stresses as a function of the depth for different inclusion types and shapes. A simple semi-analytical method is proposed for determining a macro-scale residual stress that minimizes the maximum micro-scale von Mises at any time instant and any point around a subsurface inclusion of a given type and shape.

As input for the method, macroscopic subsurface stress histories at dif-

ferent depths below a Hertzian line contact are extracted from a plane-strain finite element model. These are subsequently applied as far-field stresses for a 3D micro-scale model accounting for isolated inclusions of different geometries and orientations. Micro-scale stress histories around each considered inclusion are calculated efficiently by Eshelby's method. Finally, adopting the micro-scale von Mises stress as the optimization criterion for delaying micro-scale plasticity and thereby also damage accumulation, optimal compressive residual stresses added to the axial and circumferential stress components are determined as a function of depth for different types of inclusions.

2. Analytical and numerical methods

In the first part of this section, the analytical method employed to determine the micro-scale stress field, is explained. Then, a numerical procedure is presented, for obtaining optimal initial residual stresses that delay the onset of micro-scale plasticity as a function of depth from the surface.

2.1. Micro-mechanical model

In a typical roller bearing, subsurface stress histories at different depths y into the bearing raceway can easily be computed for any given nominal Hertzian pressure P_H in the roller-raceway contact. Assuming for instance, a frictionless line contact, a plane-strain finite element model with a theoretical parabolic Hertzian contact load can be used for extracting all relevant stress profiles at different depths as illustrated in Figure 1.

Such macroscopic stresses, obtained based on the assumption of an elastic and homogeneous material, can then be applied as far-field stresses to a micro-scale model accounting for the presence of non-metallic inclusions. This is a valid approach as long as the size of the assumed micro-scale inclusions is small enough compared to stress gradients at the macro-scale and the distance to the surface. In the present work, the four stress components included in Figure 1 are used for defining a 3D stress state transferred from the 2D plane-strain macro-scale model to the 3D micro-scale model. In both models, the coordinates x , y , and z are respectively parallel to the circumferential, radial and axial directions of a bearing raceway. For a periodic load, a material point at a certain depth below the surface will repeatedly experience the four extracted macroscopic stress histories during every loading cycle.

Assuming a fully bonded elastic ellipsoidal inclusion inside an infinite elastic matrix at the micro-scale, Eshelby's method [30, 31] is extremely

Figure 1: Determination of macroscopic stress histories: (a) Schematic view of the 2D plane strain finite element model, (b) Sub-surface stresses at depth $y = -0.54b$.

efficient, compared to numerical methods, in providing the local stress field around the inclusion [21]. In particular, the implementation of Eshelby's method by Meng et al. [32] is the one used in this study to relate the far-field macroscopic stresses to local stresses around non-metallic inclusions.

Let $\sigma_{kl}(y, t)$ be the far-field stress tensor depending on the depth y and time t , extracted from the aforementioned macro-scale plane-strain finite element model. Then, assuming an ellipsoidal inclusion at a specific depth y , the idea is to use Eshelby's method to find the corresponding stress field $\tilde{\sigma}_{ij}(P, y, t)$ covering all points P around the inclusion at the micro-scale. Under the linearity assumption of Eshelby's theory, the far-field stress state σ_{kl} can be converted to the microscopic one $\tilde{\sigma}_{ij}$ at each specific point P around the inclusion with the help of a stress concentration tensor $T_{ijkl}^{(P)}$ defined such that

$$\tilde{\sigma}_{ij}(P, y, t) = T_{ijkl}^{(P)} \sigma_{kl}(y, t). \quad (1)$$

In this linear mapping, the fourth order tensor $T_{ijkl}^{(P)}$ is independent of time t and depth y , and it only depends on the shape of the inclusion and the considered location P around it. For a given inclusion shape, it is therefore sufficient to employ Eshelby's method with only six unit load cases corresponding to each independent component of σ_{kl} , in order to evaluate all 36 independent components of the stress concentration tensor $T_{ijkl}^{(P)}$ for any point P , based on Eq. (1). In the plane strain case considered here, only four

138 out of the six macro-scale components are active and the number of relevant
139 tensor components is actually reduced to 24. The described procedure can
140 be applied as a pre-processing step, in order to precompute and store a set
141 of tensors $T_{ijkl}^{(P)}$ for multiple points P around the inclusion. Later, in studies
142 possibly involving multiple load cases and depths y , the stress state at any
143 of the pre-treated points P can easily be recovered by direct use of Eq. (1)
144 at a minimal computational cost.

145 2.2. Determination of optimal residual stresses

146 The basic idea behind the optimization presented below is to delay the on-
147 set of plasticity at all points P in the vicinity of an inclusion of a given shape.
148 Ideally, the definition of a fatigue stress limit criterion would only require to
149 ensure elastic shakedown, but avoiding the plastic deformation completely
150 can also be seen as a conservative approach. In a purely elastic model, it is
151 of course not possible to differentiate between possible steady state conditions
152 such as elastic or plastic shakedown and ratcheting. However, the linear elas-
153 ticity based maximum von Mises stress throughout a complete loading period
154 is a valid criterion for avoiding micro-scale plasticity entirely and at the same
155 time a conservative constraint for excluding cyclic plasticity. Minimizing the
156 maximum elastic von Mises stress experienced by any point around a given
157 inclusion through time is therefore chosen here as the optimization objective.

158 As recognized for example in [20] and [29], the presence of residual stresses
159 will affect the von Mises stress history at a material point at the macro-
160 scale. This dependence actually carries over through Eq. (1) to the von
161 Mises stresses at the micro-scale [21]. In fact, even a change in the hydrostatic
162 stress at the macro-scale can affect the micro-scale von Mises stresses because
163 the transformation according to Eq. (1) does not in general preserve the
164 hydrostatic or deviatoric nature of a given stress state.

165 First, the macro-scale stress history $\sigma_{kl}(y, t)$ must be modified according
166 to the presence of macro-residual stresses. Although for point contact bear-
167 ings, subsurface radial residual stresses can reportedly be significant [14],
168 neglecting the radial residual stress component in the central part of line
169 contacts is a reasonable simplification [33]. Moreover, for an initial resid-
170 ual stress state imposed through heat treatment, the circumferential and
171 axial components will be of similar magnitude. Hence, one rather common
172 scenario of superimposed residual stresses $\sigma_{\text{res}}(y)$ at the macro-scale can be
173 represented by the following shift in stress space

$$\hat{\sigma}_{kl}(y, t, \sigma_{\text{res}}(y)) = \sigma_{kl}(y, t) + \begin{bmatrix} \sigma_{\text{res}}(y) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_{\text{res}}(y) \end{bmatrix} \quad (2)$$

with the offset macro-scale stress $\hat{\sigma}_{kl}$ replacing the original one σ_{kl} . Other patterns of initial stress states, such as hoop stress due to mounting and residual stresses due to plastic deformation may be accounted for through modifications of Eq. (2).

For the macro-scale, the von Mises stresses $\sigma_{\text{VM}}(y, t)$ and $\hat{\sigma}_{\text{VM}}(y, t, \sigma_{\text{res}}(y))$ can be calculated either for the unmodified stress state σ_{kl} or for the modified one $\hat{\sigma}_{kl}$ that includes the superimposed residual stresses, respectively. Figure 2 shows the effect of adding a residual stress of $\sigma_{\text{res}} = -0.17P_H$ on the two affected stress components according to Eq. (2) and the von Mises stress at the macro-scale at $y = -0.54b$. It is evident that the significant reduction in the maximum attained von Mises stress can be seen as a means for suppressing or delaying the onset of plasticity.

Figure 2: Effect of a residual stress $\sigma_{\text{res}} = -0.17P_H$ on the macro-scale von Mises stresses at depth $y = -0.54b$.

In practical applications, bearings will normally be dimensioned such that macro-scale stresses will anyway be safely below the material yield limit. Nevertheless, even if stresses at the macro-scale are well below the yield limit, stress concentrations around inclusions can still lead to plastic deformations

190 locally. Therefore, the concept illustrated in Figure 2 for the macro-scale,
 191 becomes more relevant to apply at the micro-scale. In that manner, macro-
 192 scale residual stresses could be exploited to avoid micro-scale plasticity close
 193 to an inclusion.

194 In order to account for the possibility of plasticity at the micro-scale, a
 195 practical approach is to consider a representative cloud of points around an
 196 inclusion as illustrated conceptually in Figure 3. Knowing the Eshelby based
 197 stress concentration tensors $T_{ijkl}^{(P)}$ for any point P in the cloud, the local stress
 198 tensor can be obtained as in Eq. (1), now including the effect of macro-scale
 199 residual stresses with

$$\hat{\sigma}_{ij} = T_{ijkl}^{(P)} \hat{\sigma}_{kl} \quad (3)$$

200 which can finally lead to micro-scale von Mises stresses

$$\sigma_{\text{VM}}^{(P)}(t, \sigma_{\text{res}}) = \sqrt{\frac{(\hat{\sigma}_{11} - \hat{\sigma}_{22})^2 + (\hat{\sigma}_{22} - \hat{\sigma}_{33})^2 + (\hat{\sigma}_{33} - \hat{\sigma}_{11})^2 + 6\hat{\sigma}_{12}^2}{2}} \quad (4)$$

201 where dependence on the depth y is dropped for simplicity.

Figure 3: Representative cloud of points around a spherical inclusion.

202 The idea now is to determine an optimal value σ_{res}^* of the compressive
 203 residual stress σ_{res} , at each depth y , that will minimize the maximum local
 204 von Mises stresses throughout a loading cycle and among all points around
 205 the inclusion. This optimization can be formulated as

$$\sigma_{\text{res}}^* \leftarrow \min_{\sigma_{\text{res}}} \max_t \max_P \sigma_{\text{VM}}^{(P)}(t, \sigma_{\text{res}}). \quad (5)$$

206 This univariate minimization problem could be solved using any available
 207 single variable minimizer, but for simply demonstrating the concept, a brute
 208 force approach is also sufficient. In the present work, a range of values is
 209 simply swept for σ_{res} with fixed increments. For any probed residual stress
 210 value, the time step t and point P around the inclusion are determined,
 211 which correspond to the maximum von Mises stress, according to the inner
 212 two max operators in Eq. (5). Then, the optimum σ_{res}^* value is determined
 213 by comparing the obtained maximum von Mises stresses among all probed
 214 σ_{res} values. It should be mentioned that the process has to be repeated at
 215 different depths y from the surface, ultimately leading to an optimal residual
 216 stresses profile $\sigma_{\text{res}}^*(y)$.

217 Despite the simplicity of the proposed method, when it comes to ap-
 218 plying it to real applications, there are a few practical aspects that require
 219 consideration. Two basic assumptions behind Eq. (2) is that the loading is
 220 defined through a unique Hertzian pressure P_H , and that macro-scale resid-
 221 ual stresses remain constant. In reality, a material point in a bearing will
 222 experience a load spectrum where load peaks experienced occasionally may
 223 be considerable higher than the Hertzian pressure range that is representative
 224 for most of the lifetime. A deliberate choice of a specific load level to op-
 225 timize for is necessary before applying the method. An alternative strategy
 226 could also be to optimize for different Hertzian pressures at different sub-
 227 surface depths. Moreover, when implementing calculated optimal residual
 228 stresses in a bearing, it will possibly be necessary to compensate for effects
 229 like mounting stresses and phase transformations that may affect the actual
 230 residual stresses established after running in.

231 3. Results and discussions

232 In this section, bearing geometry and material parameters used in this
 233 study are provided, followed by maximum von Mises stress profiles calculated
 234 for different inclusions. Finally, optimal superimposed compressive residual
 235 stress profiles are reported.

236 3.1. Geometry and material information

237 An 81212 thrust bearing with the geometrical and mechanical properties
 238 given in Table 1 forms the basis for this study. At the micro-scale, two differ-
 239 ent types of inhomogeneities are investigated with aluminium oxide (Al_2O_3)

240 as a typical hard inclusion and manganese sulfide (MnS) as a typical soft in-
 241 clusion. For both types, the investigation performed considers inclusions of
 242 spherical form first, followed by inclusions of a prolate ellipsoidal form with
 243 an aspect ratio of 3 at different orientations, illustrated in Figure 4. The
 244 considered inclusions have a typical size of 5-10 μm [21, 26, 34]. However, it
 245 is necessary to mention that the stress fields calculated by Eshelby’s method
 246 are independent of the inclusion size, hence not accounting for size effects.

Table 1: Thrust bearing geometry and material parameters used in this study.

Inner diameter	Outer diameter	Number of rollers	Roller element length	Max. Hertzian pressure (P_H)	Hertzian contact half-width (b)
60 mm	95 mm	20	10.5 mm	2.3 GPa	0.22 mm

Figure 4: Schematic view of subsurface inclusions with respect to a Hertzian contact.

247 Regarding the material properties of the steel matrix, a Young’s modulus
 248 of 203 GPa and a Poisson’s ratio of 0.3 are used while the corresponding values
 249 for aluminium oxide are 375 GPa and 0.25, respectively [26]. As manganese
 250 sulfide is known to yield at stresses as low as 90 MPa at room temperature
 251 [35], plastic deformation can not simply be neglected for this inclusion type.
 252 Therefore, the lack of plasticity in the Eshelby based micro-mechanical model
 253 has to be compensated for, by determining effective elasticity parameters
 254 for this inclusion type based on a simple approximation of its elastoplastic
 255 response.

256 Under realistic loads and the actual elastic properties for manganese sul-
 257 fide of 118 GPa for Young’s modulus and 0.23 for Poisson’s ratio [26], a

258 purely elastic solution predicts equivalent stresses in the inclusion well above
 259 the yield limit of 90 MPa. In reality, severe plastic deformation is expected
 260 where the inclusion will mainly contribute to its surrounding with its bulk
 261 modulus while shear stresses on the order of the yield limit will be very low
 262 compared to shear stresses in the surrounding steel matrix. Hence, a better
 263 elastic approximation of manganese sulfide inclusions under realistic loading
 264 could be reached by maintaining the original bulk modulus while reducing
 265 the shear modulus by at least one order of magnitude.

266 The actual elasticity parameters of manganese sulfide, mentioned above,
 267 lead to the following bulk and shear moduli

$$K = \frac{E}{3(1 - 2\nu)} \simeq 73 \text{ GPa} \quad \text{and} \quad G = \frac{E}{2(1 + \nu)} \simeq 48 \text{ GPa} \quad (6)$$

Maintaining the same bulk modulus, but adopting a much lower effective shear modulus $G^* = 0.0306G$ leads to the effective elasticity parameters used for representing manganese sulfide inclusions in this study, evaluated as

$$E^* = \frac{9KG^*}{3K + G^*} \simeq 4.37 \text{ GPa} \quad \text{and} \quad \nu^* = \frac{3K - 2G^*}{2(3K + G^*)} \simeq 0.49 \quad (7)$$

268 The validity of the approximation has been confirmed by comparing displace-
 269 ments and stress fields between an elastoplastic finite element simulation
 270 with the original elasticity parameters and a purely elastic simulation with
 271 the modified elasticity parameters.

272 3.2. Von Mises stresses without superimposed compressive stresses

273 The stress concentration effect of different inclusion types is investigated
 274 first without any additional macro-scale residual stresses. While a single
 275 maximum von Mises stress over time is representative for the onset of plas-
 276 ticity at each considered depth y for the macro-scale, at the micro-scale, the
 277 entire field of von Mises stresses around a considered inclusion needs to be
 278 computed at each time step to determine a maximum over space and time.
 279 For practical purposes, micro-scale von Mises stresses are evaluated here at
 280 a large number of points around an inclusion in order to approximate the
 281 maximum. Repeating this procedure at 16 different depths from the surface
 282 up to approximately 2.5 times the contact half-width, the maximum von
 283 Mises stress profiles shown in Figures 5 and 6 are obtained for hard and soft
 284 inclusions, respectively.

Figure 5: Von Mises stresses as a function of depth for the hard inclusions.

Figure 6: Von Mises stresses as a function of depth for the soft inclusions.

285   Apart from the spherical form, the graphs include two selected orienta-
 286   tions of ellipsoidal inclusions, with the ellipsoid major axis either parallel
 287   to the rolling direction (0°) or perpendicular to the raceway surface (90°).
 288   According to the results of Figure 5 for hard inclusions, the case of inclusions
 289   elongated in the direction perpendicular to the surface leads to the highest
 290   von Mises value and is therefore more prone to micro-scale plasticity. On

the contrary for soft inclusions, Figure 6 shows that ellipsoidal inclusions elongated along the rolling direction lead to higher von Mises stresses. It is seen that there is a considerably stronger stress concentration effect of soft inclusions on the maximum von Mises stresses when compared to hard inclusions. Hence, accounting for the very limited capacity of manganese sulfide to withstand shear loads due to its low yield limit, this type of inclusion appears to be more critical for micro-scale plasticity in the surrounding steel matrix. The often experimentally observed initiation of cracks around hard inclusions must therefore be attributed to debonding between inclusion and matrix. In particular, hard inclusions can either debond from the surrounding matrix during manufacturing, due to thermal expansion mismatch or severe deformation, or during the initial operation, due to the applied load. In both cases, the debonded hard inclusions lead to even stronger stress concentration governing damage initiation [27, 36]. Nevertheless, results for fully bonded hard inclusions are still provided in the present work, similar to other studies from the literature [21], in order to put the presented effect of soft inclusions better into perspective.

Table 2 summarizes all maximum von Mises stresses relative to the applied Hertzian pressure, obtained for the aforementioned inclusion types and forms and additionally for different orientations between 0° and 90° . The table also provides the relative increase in the attained von Mises stresses compared to the maximum value at the macro-scale, i.e. without the effect of any inclusion. Compared to the macro-scale maximum von Mises stress, the presence of a hard spherical inclusion leads to a maximum micro-scale von Mises stress increased by a factor of 1.31. For a hard ellipsoidal inclusion of aspect ratio equal to 3, the corresponding stress increase factor varies from 1.29 to 1.55 for the different orientations. The generally higher von Mises stress values observed for soft inclusions are reflected in the corresponding inclusion factors ranging from 1.49 to 2.22.

In reality the amount, shape, size, and orientation of different kinds of inclusions depend very much on the steel making process [34]. However, it is well accepted that hard aluminium oxide inclusions are typically close to a spherical shape while manganese sulfide inclusions are often found as stringers of ellipsoidal form [27, 34]. Therefore, the following part of the paper focuses on these realistic geometries for each inclusion type, instead of simply adopting the most critical cases from Table 2.

Table 2: Maximum von Mises stresses for different inclusion types and inclusion factors with respect to the reference value $\sigma_{\text{VM}}^{\text{max}}/P_H = 0.569$ in the absence of any inclusion.

		Spherical	Ellipsoidal				
			0°	30°	45°	60°	90°
Hard	$\sigma_{\text{VM}}^{\text{max}}/P_H$	0.748	0.735	0.745	0.802	0.846	0.882
	Inclusion factor	1.31	1.29	1.31	1.41	1.49	1.55
Soft	$\sigma_{\text{VM}}^{\text{max}}/P_H$	1.095	1.261	1.211	1.149	1.067	0.849
	Inclusion factor	1.92	2.22	2.13	2.02	1.88	1.49

3.3. Optimal residual stress profiles

The procedure presented in section 2.2 is now applied to some selected cases of inclusion types and geometries in order to determine an optimal residual stress profile $\sigma_{\text{res}}^*(y)$ that will minimize the attained von Mises stresses. For this purpose, σ_{res} is varied in the range of compressive stresses from -800 MPa ($-0.35P_H$) to 0 MPa in fixed increments of 5 MPa ($0.00217P_H$).

Figure 7 illustrates this optimization process through an example of a soft 0° ellipsoidal inclusion at three arbitrary depths. The horizontal axis of the diagram corresponds to the probed range of compressive residual stresses σ_{res} , appearing in the outer min operator in Eq. (5), while the vertical axis shows the result of the two inner max operators. A positive effect of the superimposed residual stresses up to an optimal level around $-0.30P_H$ is observed while there is a clear negative effect upon increasing compressive residual stresses beyond the detected optimum. The minima of the three plotted curves correspond to the sought optimal residual stresses σ_{res}^* at the corresponding depths y . For better illustrating the above-mentioned non-monotonic effect of compressive residual stresses, the horizontal axis is extended up to $-0.7P_H$, beyond the limit of $-0.35P_H$, which is otherwise used for extracting the results in the remaining part of the paper.

Figure 8 shows the original and optimized micro-scale von Mises stresses for hard spherical inclusions as well as the obtained optimal residual stress profile $\sigma_{\text{res}}^*(y)$. A significant reduction in the maximum von Mises stresses throughout the investigated depths can be observed.

A consequence of this result is that superposition of the obtained residual stress profiles will lead to a significant reduction of micro-scale plastic deformation during the running-in phase of the bearing operation. To the degree that micro-scale plasticity affects fatigue performance through nucle-

Figure 7: Effect of superimposed compressive residual stresses on the maximum von Mises stress around a soft 0° ellipsoidal inclusion at arbitrary depths y .

Figure 8: Optimal residual stress profiles along the depth direction and their effect on maximum von Mises stresses for a hard spherical inclusion.

354 ation of near inclusion cracks, the expected improvement will carry over to
 355 the component fatigue life.

356 It should be kept in mind that von Mises stress results presented so far are
 357 maximum values over the entire load cycle, as well as over space, covering a
 358 large number of discrete points around the inclusion. Moreover, the presented

359 von Mises stress curves may attain their maximum at different depths y from
 360 the raceway surface. Figure 9 illustrates the occurrence of the maximum von
 361 Mises stress for a hard spherical inclusion without and with residual stresses.
 362 In the former case, the maximum von Mises stress occurs approximately at
 363 a depth of $y = -0.73b$ while in the latter case the corresponding maximum
 364 occurs around $y = -0.54b$. Therefore, the stress histories shown on the left
 365 and right hand side of Figure 9, apart from the offsetting of σ_{xx} and σ_{zz}
 366 by σ_{res} , they also contain some additional differences due to the different
 367 considered depths.

Figure 9: Spatial and temporal position of maximum micro-scale von Mises stresses for hard spherical inclusions, without and with residual stresses.

368 During a complete load cycle, the maximum von Mises stress is attained
 369 approximately when the σ_{yy} component reaches its extremum for the case
 370 without any residual stresses. On the contrary, after adding the optimal
 371 residual stress, the corresponding maximum occurs approximately when the
 372 orthogonal shear stress component σ_{xy} is at its peak. The plots at the bottom
 373 of the figure show the spatial distribution of the maximum von Mises stress

close to the inclusion interface, attained over a complete load cycle. For the case without residual stresses, micro-scale plasticity is expected to initiate at the four locations marked on the left subfigure. On the other hand, with the added residual stress, the largest von Mises stresses occur at the poles of the inclusion although very similar equivalent stress levels are found along the whole meridian of the inclusion, in this case.

Corresponding results are now presented for soft inclusions of ellipsoidal shape. Figure 10 shows the optimal residual stresses obtained for soft ellipsoidal inclusions at 0° and 90° and its effect on the maximum micro-scale von Mises stresses. For soft inclusions elongated along the rolling direction, a considerable reduction is achieved over the complete range of depths. For the less critical case of soft 90° ellipsoidal inclusions though, especially at very shallow depths, it is not possible to obtain any significant improvement.

Of course as mentioned earlier, inclusions of different forms and orientations occur in real materials stochastically. In that sense, optimizing for the most critical inclusion shape seems to be a logical option, but it is also interesting to cross-check the effect of the optimal residual stress profile obtained for one specific inclusion geometry on inclusions of different geometries. Adopting the optimal residual stress obtained for the most critical case of the soft 0° ellipsoidal inclusion and cross-checking its effect on the behavior of a 90° ellipsoidal inclusion leads to the results shown in Figure 11. The superimposed compressive residual stresses still reduce the maximum value of von Mises stresses at larger depths to some extent, but they have a significant negative effect at depths smaller than $0.8b$. In this case, a compromise between the two optimal residual stress profiles reported in Figure 10 seems to be a more appropriate choice.

Figure 12 illustrates the occurrence of the maximum von Mises stress in space and time for the initially most critical soft ellipsoidal inclusion type with a 0° orientation. In this case, both before and after adding compressive residual stresses, the maximum von Mises stress occurs approximately at the same depth around $y = -0.54b$. It can also be deduced from the figure that the location of the maximum von Mises stress remains the same, i.e. on the equator of the inclusion in the middle along the ellipsoid's long axis. Nevertheless, this maximum occurs at different time instants during the load cycle as can be seen in the figure. It should be noted though that both before and after the superposition of residual stresses, von Mises stress values very close to the detected maximum are reached all along the equator of the inclusion. Hence, for this soft ellipsoidal inclusion, plastic deformation is

Figure 10: Optimal residual stress profiles along the depth direction and their effect on maximum von Mises stress for soft ellipsoidal inclusions oriented at 0° (a) and 90° (b).

probably to occur within an annular region lying parallel to the raceway surface, i.e. in the xz plane, irrespective of the presence of a macro-scale residual stress.

The presented results for both hard and soft inclusions have demonstrated that in most cases an optimal level of superimposed equibiaxial compressive residual stresses can be found in terms of avoiding initial micro-scale plasticity. As illustrated in Figure 7 for example, compressive stresses beyond the optimum point will lead to a reduced gain or even to a deterioration compared to the initial state. Hence, to the degree that initial micro-scale

Figure 11: The effect of superimposed residual stresses optimized for soft 0° ellipsoidal inclusions on the von Mises stresses for soft 90° ellipsoidal inclusions.

Figure 12: Spatial and temporal position of maximum micro-scale von Mises stresses for soft 0° ellipsoidal inclusions, without and with residual stresses.

421 plasticity can be linked to posterior fatigue behavior through nucleation of
 422 near inclusion micro-cracks, this outcome suggests that excessive compressive

423 residual stresses can also compromise fatigue resistance.

424   Apart from the two types of inclusions investigated here, void imperfec-
425 tions may also be relevant for bearing steels. This case is omitted here as
426 it leads to results similar to the case of soft inclusions. For hard inclusions,
427 the results presented should be used with care since in reality as mentioned
428 earlier, hard inclusions can be initially debonded from the matrix, leading to
429 an entirely different behavior. In this case, a void model might be a better
430 approximation of the defect site rather than a fully bonded inclusion.

431   As for the parabolic Hertzian load assumed in the present investigation,
432 it can be seen as a practical compromise in the interest of simplicity as
433 compared to a more realistic elastohydrodynamic pressure distribution. It is
434 expected though that adoption of a more realistic load, possibly also with
435 tangential traction components, will not affect the main conclusions of the
436 presented investigations significantly.

437   Last, it is important to note the significant practical constraints regarding
438 the through depth residual stress profiles that are feasible to implement. Even
439 if tailored surface hardening processes are used for approximating a desired
440 residual stress profile, overall equilibrium requires that compressive residual
441 stresses close to the surface are in balance with tensile residual stresses at
442 larger depths. In order to keep the introduced tensile residual stresses at
443 larger depths as low as possible, it is hence appropriate to aim at the com-
444 pressive residual stresses, proposed here, only up to a certain depth, e.g.
445 $y = -b$, and allow a gradual decrease at larger depths. Accounting for the
446 stochastic occurrence of inclusions of different shapes and orientations in the
447 same type of component might also lead to necessary compromises. Based
448 on the observation made in connection to Figure 11, it seems reasonable for
449 instance to aim at an average between the two optimal residual stress profiles
450 reported in Figure 10 for different inclusion orientations.

451 4. Conclusions

452   In this study, a semi-analytical method has been proposed for calculating
453 favorable compressive residual stress profiles, aiming to suppress initial micro-
454 scale plasticity around non-metallic inclusions in rolling contact fatigue. To
455 the degree that micro-scale plasticity can be linked to the subsequent fa-
456 tigue process, the superimposed residual stresses are expected to also have a
457 positive effect on fatigue performance.

458 According to the analysis of hard and soft inclusions of different geome-
459 tries in the absence of macro-scale residual stresses, soft inclusions appear to
460 be more critical with regard to causing micro-scale plasticity in their vicini-
461 ty. Soft ellipsoidal inclusions elongated parallel to the rolling direction are
462 the most critical among the investigated geometries. On the contrary, for
463 hard inclusions, it is radially elongated ellipsoidal inclusions that lead to the
464 highest von Mises stresses.

465 In most of the investigated cases, optimal compressive residual stresses
466 could be obtained that lead to a remarkable reduction in the maximum von
467 Mises stresses close to the considered inclusions. This is seen as a potential
468 mechanism for improving the rolling contact fatigue performance of a compo-
469 nent by avoiding crack nucleation due to initial plastic deformation. It should
470 be noted though that this result also suggests that an excessive level of com-
471 pressive residual stress can have a negative effect on fatigue performance,
472 through the same mechanism.

473 An analysis of the distribution of von Mises stresses in space and its
474 evolution through time has shown that macro-scale residual stresses can affect
475 the occurrence of micro-scale plasticity both in space and time. In general,
476 the subsurface depth of maximum micro-scale von Mises stresses changes
477 after the addition of the optimal residual stress profiles. Moreover, in several
478 cases, the site of the maximum von Mises stress around an inclusion is altered
479 and its occurrence during a loading cycle is shifted.

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