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EML webinar overview: Topology Optimization - Status and Perspectives

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Abstract

Since its introduction by Bendsøe and Kikuchi in (1988) [1], topology optimization has developed from a purely academic discipline to the preferred tool for light-weighting structures in automotive, aerospace and other weight conscious industries. Topology optimization solves mechanical and multiphysics design problems allowing the ultimate design freedom, i.e. it determines whether any point (or element) in space should be filled with material or left empty in order to optimize a given objective function while satisfying physical and geometrical constraints. The talk will give an overview of the field, a.o. demonstrated by recent giga-scale applications in airplane wing and super-long suspension bridge design. Originally, the approach focused on simple compliance minimization problems but recent works to be discussed have paved the way for solving large scale stress constraint problems with hundreds of millions of constraints as well as large scale buckling problems. We also discuss ways to reduce the CPU time for large scale problems by use of efficient multiscale approaches and knowledge of optimal microstructures. Finally, other directions including design for geometry control and manufacturability, metamaterial design and multiphysics problems will be briefly reviewed.

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Keywords: Topology optimization, inverse methods, homogenization, high performance computing, architected materials

Extended Summary with references

This extended summary gives some additional details and background as well as a list of relevant references in connection with my EML Webinar presentation "Topology Optimization - Status and Perspectives" given June 16, 2020. Topology optimization has grown to become a huge and diverse research field and to give a fair overview of all activities within the field would be a formidable task not practical for the webinar format. Hence, discussions and references are here mostly limited to or related to activities of the TopOpt group at the Technical University of Denmark, supplemented with pointers to relevant overview papers. For graphics and visualizations, readers are referred to the videos of the webinar or the referenced papers.

Background. In their ground breaking work published in (1988), Bendsøe and Kikuchi presented the so-called homogenization approach to topology optimization (TO) [1]. This approach considered knowledge of optimal or near optimal microstructures, used quite complex math, and resulted in rather blurred and non-well-defined topologies. Nevertheless, the potential was obvious and the approach, in multiple variations and modifications, has now become the preferred design tool for light-weighting and beyond. The growing popularity has been further spurred by the developments in additive manufacturing (AM) that allows manufacturing of complex structures and hence to take full advantage of the extreme design freedom allowed by TO.

The original homogenization approach was soon supplemented by simpler density approaches, so-called SIMP (Simplified Isotropic Material with Penalization) approaches [2, 3], and after a proof of the mechanical realizability of the underlying simple density interpolation schemes [4], the SIMP approach became the method of choice. The popularity of SIMP methods has also been spurred by the availability of simple Matlab codes [5, 6, 7]. Later on, a huge number of alternative approaches have appeared, including level-set, evolutionary, phase-field, a.o.. Sometimes, the differences between various approaches are rather subtle or unclear as discussed and reviewed in ref. [8].

The original density approach, where relative material density in each element constitutes a design variable, is prone to numerical artefacts like checkerboard instabilities and mesh-depedence, but extensive developments in advanced filter and regularization techniques now allow almost complete geometry control ranging from imposing of length-scale over robustness to manufacturing variations to control of interface thicknesses and more. Things are still developing rapidly however, a.o. to accommodate different AM constraints. For a review of regularization methods up to 2016, readers are referred to ref. [9].

By now, TO software in various forms is available in all major numerical modelling packages as well as most CAD software and AM simulation software. The TopOpt group has also developed a number of free Apps, where users on smart phones or tablets can perform interactive topology optimization - see Appstore: TopOpt, TopOpt3d, TopOpt Shape a.o.. Some of these Apps are also available or will be made available on the Android platform (see Google Play).

Giga-resolution. Design resolution of publicly or commercially available software is limited to a few or maybe tens of millions of elements, partly due to solver capabilities and partly due to hardware availability for average users. Large and complex structures require fine resolutions to adequately model and optimize their geometrical complexity. If the goal is to topology optimize a full airplane wing structure, say a Boing 777 wing, the resolution provided by standard software of say 5 million voxels (elements) in a regular mesh means that the smallest structural features at the wing root would be discretized by a couple of elements of size 5cm cubed, i.e. far too large to capture normal structural features in a wing of that size. To reach an acceptable level of detail, elements have to be less than one centimeter cubed, corresponding to 1 billion elements, i.e. giga-resolution. Based on a previously developed PETSc supercomputing framework [10], overcoming extensive scaling challenges, as well as gaining access to a PRACE super computer, TopOpt group members have managed to optimize a full B777 wing structure discretized by more than 1 billion

elements [11]. The results at this resolution revealed several interesting insights, like curved ribs and spars that may inspire airplane designers to improve future wing designs. In a follow up study we extended the approach to even finer discretizations and enforced periodicity in order to optimize long-span bridge girders whose design otherwise has remained the same for the last 60 years [12]. Based on this study, a simplified design involving simple curved diaphragms could be extracted that potentially may save 28% on the girder weight. By knock-on effects, this saving extends to potentially 19% weight savings for the full bridge, equivalent to 43,000 tons of CO_2. It is the hope that such work may inspire further applications and resulting reduced environmental impact in the civil engineering industry as e.g. reviewed in ref. [13].

The two mentioned giga-resolution studies have pushed the limit for what is possible to optimize using TO. However, together with a majority of other past and present topology optimization applications, they still consider simple compliance minimization and neither take strength nor stability into account. Both these effects may be included in a post-processing stage, however, to harvest true optimization potential it is desireable to include such constraints directly in the optimization process.

Stress constraints. For more than a century, it has been known that stiffness optimized, low density minimal structures also are equi-stressed if no geometrical restrictions are present [14]. However, as soon as the design domain is restricted or worse so: the design domain is concave and has reentrant corners, the stiffness optimized structure may contain fatal stress singularities. Hence, in contrast to stiffness goals, stress goals result in non-self-adjoint optimization problems. In turn, this means that intuitive "fully stressed design approaches" will fail to deliver optimal designs [15].

Including stress constraints in TO is a long studied subject with challenges regarding non-differentiability, large number of constraints and choice of appropriate density interpolation functions [16]. To avoid dealing with a number of stress constraints corresponding to number of elements (or Gauss points), stress

aggregation functions like the p-norm have been popular [17, 18]. However, finding appropriate p-values is challenging since too high values make the optimization problem highly non-linear and too low values dilute the achievable stress control [15]. To alleviate these and other issues, augmented Lagrangian approaches have been proposed recently as an efficient alternative [19, 20, 21]. Here, references [19, 20] also include consideration of robustness towards manufacturing variations, which turns out to be critical for appropriate stress-based design. The latter approach has recently been extended to the PETSc platform allowing for solving a 3D problem with a record breaking 135 million elements with 505 million stress constraints [22]. Hence, the road is paved for future applications in the giga-scale.

Buckling. Including stability constraints in TO is another topic that has been pursued for many years but still has some issues to be resolved. Including buckling constraints in TO was first pursued by Neves et al. [23] and the topic was recently reviewed and revisited by Ferrari et al. [24]. Apart from various now solved issues with non-differentiability of multiple eigenvalues and appropriate modelling of low density regions to avoid ill-convergence and artificial buckling modes, a detrimental issue for large scale buckling optimization is the agglomeration of multiple eigenvalues as optimization progresses. This is due to the forming of hierarchical structures where multiple local and global modes as well as artefact modes associated with local stress concentrations become active at the same time [24, 25]. The involvement of more and more modes requires solving for more and more eigenvalues which is prohibitively expensive. Various approaches to resolve this issue have been investigated, also for similar dynamic problems [26, 27, 28]. For the buckling problem, however, a multilevel approach where the costly eigenvalue problem is only solved at the coarsest scale [29], promises reduced CPU times corresponding to solving much cheaper multiload linear problems, again paving the way for future applications in the giga-scale.

Multiscale projection methods. The giga-scale TO problems discussed before are extremely costly. The wing study [11] required more than one million CPU hours

and the bridge study [12] was run for 85 hours on 16000 cores, corresponding to 1.4 million CPU hours. Although methods and hardware are steadily improving, such computing times are not practical in regular engineering design settings. An option to solve this problem is to resort to multi-scale approaches.

Actually, the original homogenization approach by Bendsøe and Kikuchi [1, 2] was already a multi-scale approach. However, this approach was more or less abandoned, partly due to its mathematical and practical complexity and partly due to lack of numerical and practical methods to realize its associated multi-scale microstructures.

A "resurrection" of the homogenization approach has happened lately, initiated by Pantz and Trabelsi [30] and significantly expanded and improved upon by Groen and co-workers [31, 32, 33, 34, 35] and in parallel by Geoffroy-Donders and co-workers [36, 37]. Latest developments of these so-called de-homogenization approaches, indicate time savings approaching a factor of 2000 compared to problems with similar resolutions in the PETSc framework (with insignificant degradation in mechanical performance) [38]. On-going efforts are devoted to improving this factor even further, in turn aiming at giga-scale interactive TO in the future.

It is here worth to remark that stiffness optimimal microstructures are closed-walled as implied by the optimal multiscale rank-n microstructures proved to attain the theoretical bounds [39, 40, 41, 42]. This is in contrast to the widespread use of truss lattice structures in a.o. the AM domain [43]. It can be shown [44] that optimal closed-walled microstructures are always stiffer up to a factor of 3 compared to their open truss lattice counterparts for arbitrary loadings. Realizing multiscale rank-n structures is challenging, but single scale realizations of these may approach the bounds with less than 10% error [45, 46, 33, 47].

Architected materials. TO has been applied to architected material design or so-called inverse homogenization problems from early on [48, 49, 50]. For pure stiffness objectives, optimal microstructures are by now well studied and known

[51] but lately, several works have considered optimal microstructure design for programmable non-linear responses like in auxetics [52, 53, 54] and local and global stability [25, 55]. It is the goal of on-going studies to combine non-linear local microstructure design with the buckling and de-homogenization approaches discussed above, in order to efficiently solve large scale problems with all stiffness and strength requirements appropriately taken into account.

AM integration. AM and TO were both founded in the late 1980's and have hence grown up side by side. The two technologies provide a perfect match in the sense that TO provides complex geometries that may be realized by AM. Despite its large geometric freedom, AM is still dependent on control of overhang angles and other manufacturing constraints that conveniently can be taken into account already in the TO process [56]. On the other hand, AM also offers unique possibilities by designing dedicated infill structures [57] that indirectly may provide higher buckling stability at little loss in stiffness [58, 59]. TO with AM integration is probably the most active research subject within TO these days and more of these activities are reviewed in Ref. [60].

Multiphyics. TO was originally developed for mechanical design problems, however, over the years, the approach has been applied to virtually every multiphysics problem imaginable. The essential requirement is that the associated PDE (Partial Differential Equation) can be reliably discretized and modelled using finite element, finite differece, finite volume or other volume discretization schemes. Important application areas are in fluidics and thermofluidics as recently exhaustively reviewed by Alexandersen and Andreasen [61], wave propagation problems like acoustics [62, 63], nano-photonics [64, 65], photo-voltaics [66, 67], electromagnetism and antennas [68, 69, 70] and MEMS (Micro Electro Mechanical Systems) [71, 72, 73].

AI / Machine Learning. During the discussion session there were several questions and remarks concerning machine learning and its potential for TO. Multiple research groups have been and are investigating this but we are yet to see

something that has the potential to beat the very efficient TO algorithms already established. My guess is that machine learning may become useful in solving subtasks of the optimization and/or modelling problem increasing overall efficiency, but not as a purely graphical tool where one specifies general boundary conditions and loads and expects to get the optimal structure as output after an image-based learning process.

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