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Publication date:
2020

Document Version
Publisher's PDF, also known as Version of record

DTU Management
Department of Technology, Management and Economics

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PhD Thesis

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August 2020

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By
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Division:	Transport division

Published by: DTU, Department of Technology, Management and Economics, Akademivej, Building 358, DK-2800 Kongens Lyngby, Denmark.
www.man.dtu.dk

Preface

This PhD thesis entitled *Modelling passenger behaviour in mixed schedule- and frequency-based public transport systems* is submitted to meet the requirements for obtaining a PhD degree at the Department of Technology, Management and Economics, DTU Management, Technical University of Denmark. The PhD project was supervised by Professor Otto Anker Nielsen and co-supervised by Associate Professor Thomas Kjær Rasmussen, both from DTU Management. The thesis is paper-based and consists of the chapters listed in the tables of content, including separate chapters for each of the following papers:

Paper 1: M. Eltved, O. A. Nielsen, and T. K. Rasmussen (2019). "An assignment model for public transport networks with both schedule- and frequency-based services". In: *EURO Journal on Transportation and Logistics* 8, pp. 769–793. DOI: 10.1007/s13676-019-00147-4.

Paper 2: C. B. Gardner, S. D. Nielsen, M. Eltved, T. K. Rasmussen, O. A. Nielsen, and B. F. Nielsen (2020). "Conditional passenger travel time distributions in mixed schedule- and frequency-based public transport networks using Markov chains". Under review at Transportation Research Part B: Methodological.

Paper 3: M. Eltved, O. A. Nielsen, and T. K. Rasmussen (2018). "The influence of frequency on route choice in mixed schedule- and frequency-based public transport systems - The case of the Greater Copenhagen Area". In: *Proceedings of the 14th Conference on Advanced Systems in Public Transport (CASPT2018)*. Brisbane, Australia. URL: http://www.caspt.org/wp-content/uploads/2018/10/Papers/CASPT_2018_paper_81.pdf.

Paper 4: O. A. Nielsen, M. Eltved, M. K. Anderson, and C. G. Prato (2020). "Relevance of detailed transfer attributes in route choice models for public transport passengers". Re-submitted after second round of review to Transportation Research Part A: Policy and Practice.

Paper 5: M. Eltved, N. Breyer, J. Blafoss, and O. A. Nielsen (2020). "Impacts of long-term service disruptions on passenger travel behaviour: A smart card analysis from the Greater Copenhagen area". Submitted to Transportation Research Part C: Emerging Technologies.

Paper 6: M. Eltved, N. Christoffer, and P. Lemaitre (2020). "Estimation of transfer walking time distribution in multimodal public transport systems based on smart card data". Submitted to Transportation Research Part C: Emerging Technologies.

Paper 7: M. Eltved, H. N. Koutsopoulos, N. H. M. Wilson, K. Tuncel, and Z. Ma (2020). "A note on unusual path choice behavior caused by congestion in metro systems". Working paper.

Acknowledgements

First of all, I want to thank my supervisors Professor Otto Anker Nielsen and Associate Professor Thomas Kjær Rasmussen for encouraging me to pursue the PhD endeavor. Thank you for all the great discussions on topics related to PhD project, and remember, the transport system consists of more than Motorring 3 and what goes on near Allerød :)

I want to acknowledge the financial support for my PhD provided by the IPTOP (Integrated Public Transport Optimisation and Planning) Project granted by Innovation Fund Denmark (grant 4109-00005B). I presented several times on the partner meetings and want to thank all of the partners for the interesting discussions.

I also want to thank my colleagues at the Transport Division at DTU Management. I have really enjoyed the atmosphere in the building and co-arranging some of our social events at the division. A special thanks goes to my office mate Jesper B. Ingvardson for his guidance on PhD related questions, advice on (air)-travel planning and for the company at several conferences around the world. Mads Paulsen deserves a special thanks for always being willing to answer questions on coding and LaTeX and our memorable times at conferences around the world. Also, thanks to Niklas Christoffer Petersen and Philip Lemaitre for great co-work on the paper on walking times at transfers. At last, thanks to Nils Breyer, who visited the division, and although the stay was disrupted due to COVID-19, we managed to collaborate intensely and finish our paper through Zoom.

In the spring of 2019 I spent 3 months at Northeastern University and the MIT Transit Lab. This stay (and visits both before and after) was very fruitful for my research, and I want to thank Professor Haris N. Koutsopoulos and Professor Emeritus Nigel Wilson for the opportunity to visit the Lab. I am thankful to the group at the MIT Transit Lab for their hospitality and for the always inspiring weekly Friday meetings. A special thanks goes to Kerem Tuncel for working with me on the project of reverse routing. In July 2018 I visited Professor Carlo Prato at the University of Queensland, Brisbane, Australia, with my supervisors and other colleagues. Thanks for your hospitality during our stay and for being our tour guide at campus and around the state.

I am deeply appreciative to have my loving and caring family, Mette, Anders, Mom and Dad. I know my interest in transport systems can sometimes be a little overwhelming, but rest assure, this is not the last time you hear about my excitement for public transport. Also, I want to acknowledge my friends, Steffen and Martin, for always being ready to listen to my ups and downs. Finally, I owe much love and mental presence to Stine, whom I probably wouldn't have met, if it wasn't for this PhD project. Thanks for encouraging me through the tough times, and most importantly for carrying our little baby girl, whom I look very much forward to welcoming in the fall.

Morten Eltved, August 2020.

Summary

Transport systems in metropolitan areas are on both the road and public transport side challenged on providing sufficient capacity for the increasing mobility needs. An increasing number of hours is wasted in congestion on the roads, and good public transport service is needed to provide sufficient capacity in the transport system. Public transport is not only seen as a way of increasing the mobility in metropolitan areas, but also as one of the important contributors to the transition for more sustainable mobility in urban areas. The public transport systems must thus be an attractive alternative to taking the car to attract more passengers, and facilitate the transition for a more sustainable transport system.

The public transport systems are often complex with a mix of lines, where passengers for some services rely on a published detailed timetable (schedule-based lines), while they for other high frequency lines rely on the headway between consecutive services on the line (frequency-based lines). Due to the complexity of these systems, advanced models are needed to analyse the level of service provided to the passengers given the timetable of the network. Furthermore, the inputs for these models require analysis of various aspects of passenger travel behaviour based on reliable data sources.

This PhD thesis concerns several aspects within modelling of public transport systems with a passenger oriented perspective. The thesis is split into three main parts; Part I, *Assignment models for mixed schedule- and frequency-based public transport systems*, presents novel methodological approaches for determining the level of service for passengers in public transport networks with both schedule- and frequency-based services; Part II, *Route choice models for mixed schedule- and frequency-based public transport systems*, focuses on passengers' route choice preferences from origin to destination in these complex networks, based on revealed passenger route choice surveys. The third and final part of the thesis, *Studies on public transport passenger behaviour based on smart card data*, covers three analyses of passenger travel behaviour based on smart card data.

Part I of the thesis covers the difficult task of assigning (predicting) passengers to routes from origin to destination in order to evaluate the level of service provided to the passengers. Specifically, two studies focus on the combination of schedule- and frequency-based services in public transport networks and how to assess the travel times on the attractive routes in such a network. *The first paper* develops a novel methodology to assign passengers in a mixed schedule- and frequency-based network. First, choice-sets with different possible routes are generated based on a heuristic, which requires that the passenger can reach the destination within a certain threshold using the specific route. A subsequent step distributes the passengers across the alternatives using a discrete choice model. The resulting flow distributions across alternatives are stable regarding the specification of a line as either schedule- or frequency-based. Compared to other models, this allows

the modeller to make fewer assumptions on the actual schedule of a line, and eases the evaluation when several timetable scenarios need to be compared.

The second paper proposes a method that uses Markov chains to identify the travel time distributions for different route choice alternatives, when stochastic running times of a line due to delays are taken into account. Given a set of attractive lines for passengers travelling from origin to destination, the methodology calculates the travel time distribution for different combinations of lines and thereby alternative routes through the network. Both schedule- and frequency-based lines can be part of the input to the model. By using Markov chains the probability of reaching a connecting service, can be analytically described, whereby the use of traditional demanding simulation models can be avoided. Several detailed analyses can be derived based on the resulting travel time distributions, which can become an important tool for timetable planners.

While *Part I* of the thesis covers the evaluation of the level of service offered to the passengers, *Part II* covers the input to these models by investigating the route choice preferences of the passengers. Travellers evaluate the attractiveness of a route based on several features such as travel time and number of transfers, but the specific challenge concerning how passengers trade off for example routes with a high travel time vs. routes with lower travel time which include more transfers persist. These trade-offs are investigated in two papers using a dataset covering self-reported trips using public transport in the Greater Copenhagen area. In both papers a discrete choice model is the basis for the extraction of the passenger route choice preferences, and this is achieved by comparing the observed routes with a large set of alternative routes the passenger could have chosen.

The third paper investigates the trade-offs passengers have to make when choosing between alternatives with different waiting times and in-vehicle times. Waiting time for schedule-based services, such as regional trains and local busses, are estimated separately from frequency-based services (metro and high-frequency busses) and this shows, that passengers have a higher nuisance for waiting for frequency-based services compared to waiting for schedule-based services with a known timetable. However, in general lower waiting times for frequency-based services makes the decision between alternatives with schedule- or frequency-based services almost the same. If the differences in parameters of waiting time are not accounted for in assignment models, there is a risk of creating a biased flow estimation, which can eventually lead to wrong conclusions in feasibility studies. The paper also investigates whether the marginal dis-utility of in-vehicle time varies across and within each sub-mode, i.e. metro, bus and trains. It is shown, that the marginal dis-utility for metro considerably increases for longer trips whereas the marginal dis-utility decreases for in-vehicle time in trains.

The fourth paper focuses on the choice of transfer location in passenger route choice. The paper reviews existing literature on transfer attributes which affects passenger route choice, and selects three attributes found to be important for passenger route choice. The analysis shows that passengers prefer routes, which includes a shop available at any of the transfer stations visited during the trip, thus indicating the preference for being able to do smaller grocery shopping en-route. Passengers also prefer escalators over regular stairs, and prefer that transfers should be easy to navigate through. Using these attributes, it is possible to disentangle the transfer penalty for stations with different characteristics. The best possible transfer thereby has a penalty equivalent to spending 5.4 minutes extra in a bus, whereas the worst possible transfer is comparable to spending 12.1 minutes in a bus. The results have important policy implications for evaluation of different station designs and how the resulting passenger flows will be, if stations are upgraded or redesigned. Such investments can turn out to be more cost effective than track upgrades or other improvements of the railway, while still providing a better level-of-service to passengers.

The final part of the thesis, *Part III*, covers three analyses based on smart card data. Smart card data is available in rich numbers from automatic fare collection systems and is becoming an increasingly important tool for analysing passengers' travel behaviour. This thesis uses smart card data with different degrees of detail, and the studies span from analysing the individual mobility to more aggregated analysis, where smart card data covers the heterogeneity of passenger behaviour.

The fifth paper investigates individual mobility over a long time period based on data from the Danish smart card, *Rejsekort*. The study analyses travel behaviour before, during and after a three month track closure on a suburban rail line in the Greater Copenhagen area, where replacement busses served the line resulting in significantly increased travel times. Passengers are clustered based on their travel behaviour before and after the track closure. A similar track section is used as comparison to the changes in ridership at the suspended track section, as the individual passenger travel behaviour changes considerably over time due to changes in individual employment and general seasonal trends. By comparing the changes in travel behaviour for passengers travelling frequently before the disruption on either the affected or reference line, no apparent difference is seen for the period after normal operations resumed. However, the total ridership on the affected line decreased compared to the reference line, and a comparison of the changes in passenger travel for the different groups, suggests that the deficit is a result of less attraction of new passengers on the affected line. By analysing the daily travel patterns for the group who commuted on the affected line before the disruption, it is found that 17% of the passengers almost entirely stopped using public transport during the disruption, but returned to a regular usage of public transport after the normal operations resumed. This indicates, that at least some passengers favor public transport and are not forced to use the public transport system.

Data from *Rejsekort* is also used in *the sixth paper*, but on a more detailed level. The paper fuses smart card data and automatic vehicle location data to estimate the walking time used from alighting a bus until the passenger taps in at a train platform. This walking time is essential to know, as it is used in timetabling and synchronisation of busses and trains. Using the raw observed times from the data fusion leads to significantly over-estimated walking times, as some passengers are doing activities during their transfer. Therefore, a *hierarchical Bayesian mixture model* is used to isolate the passengers doing activities during the transfer from the passengers walking directly. The results show that the model is able to accurately replicate the observed walking times and estimate the walking time necessary to walk from bus stop to train platform. The study establishes a more data-driven procedure for estimation of walking times at transfers, and is applied to 129 stations in the Eastern part of Denmark. Tests show that the share of passengers doing activities during their transfers increases with the number of shops available near the transfer station.

Whereas the two preceding papers focus on the use of data from *Rejsekort*, *the seventh and final paper* utilises an extensive smart card dataset from Hong Kong. The number of passengers in the Metro in Hong Kong exceeds the available capacity during the peak hours, and the paper describes and analyses unusual path choice behaviour that stems from the excessive crowding. Under the excessive crowding situations passengers can be observed to do *reverse routing*, namely choose to transfer at a station further down a line in order to travel backwards and pass the station where passengers would usually transfer in uncrowded conditions. Such *reverse routing* can increase the travel time reliability and also increase the chance for the passenger to get a seat or better standing position in the train. However, based on the analysis, no final conclusions can be made on the share of passengers using this option of reverse routing. However, the results indicate that passengers travelling furthest after transferring have a slightly different behaviour, which could stem from a higher degree of reverse routing. It can also be substantiated by the finding in the *third paper*, that the marginal value of time is increasing for passengers using the metro. A short paragraph in the paper also considers whether such unusual route choices are occurring in the Danish Metro, but based on analysis of data from *Rejsekort*, this can quickly be ruled out to be the case.

In summary, this PhD thesis has contributed to i) new methodologies to assign passengers to routes for detailed and analytical evaluation of the level of service provided to passengers in mixed schedule- and frequency-based public transport system, ii) revealing and quantifying of the significant dis-utility of transfers in public transport route choice in combination with detailed analysis of the important characteristics of station attributes, and iii) develop two novel methods using data from *Rejsekort* for analysing both long-term travel behaviour and walking times at transfers, and iv) investigate the effects of crowding on passenger path choice in congested metro systems. Overall the thesis covers a broad span of public transport modelling and contributes to already existing knowledge in the domain. Several new methodologies are developed, especially on the use of smart card data, and these can be used for further research within the domain.

Resumé (Danish summary)

Transportsystemer i storbyområder er under pres som følge af stigende behov for mobilitet, og for både vejtrafikken og den kollektive transport er kapaciteten ved at være opbrugt. Mange timer spildes hver dag i trængslen på byernes veje, og såfremt transportsystemet skal kunne følge med mobilitetsudviklingen, kræver det, at den kollektive transport bidrager med væsentlig kapacitet til at flytte mennesker fra A til B. Den kollektive transport er, udover gang og cykling, derudover også en væsentlig bidragsyder til den grønne omstilling af transportsystemerne i storbyområderne, og det er derfor vigtigt med modeller og analyser, der kan benyttes til at gøre den kollektive transport mere attraktiv.

Kollektive transportsystemer er often komplekse med en kombination af linjer med en relativ lav frekvens og faste afgangsminuttal (køreplansbaserede linjer såsom tog og lokale busser), samt linjer med høj frekvens (frekvensbaserede linjer såsom metro og A-busser), hvor kun tiden mellem afgange publiceres til passagererne. På grund af systemernes kompleksitet kræver det avancerede modeller for at kunne evaluere, hvilket serviceniveau en given køreplan giver passagererne, da der skal tages højde for skift mellem linjetyper og disses attraktivitet. Desuden kræver sådanne modeller detaljerede input fra pålidelige datakilder, der beskriver passagerernes adfærd på de forskellige delkomponenter af den samlede kollektive rejse. Denne ph.d.-afhandling omhandler adskillige aspekter inden for modellering af kollektiv transport set fra et passagerperspektiv. Afhandlingen er inddelt i tre dele: *Del 1* præsenterer nyskabende rutevalgsmodeller, der bruges til at evaluere serviceniveauet baseret på netværk med både frekvens- og køreplansbaserede linjer. *Del 2* fokuserer på rutevalgspræferencer for passagererne fra dør til dør i sådanne komplekse netværk på basis af rapporterede ture med kollektiv transport. *Den tredje og sidste del* indeholder tre forskellige analyser af passagerers rejseadfærd baseret på data fra Rejsekortet, samt et lignende automatisk billetsystem fra Hong Kong.

Den første del af afhandlingen dækker de udfordringer der er, når serviceniveauet for passagerernes rejse fra A til B skal evalueres. To studier og artikler fokuserer på kombinationen af frekvens- og køreplansbaserede linjer i ét kollektiv transportnetværk og hvordan rejsetider for attraktive ruter vurderes heri.

Den første artikel udvikler en ny metode, hvorved evalueringen af hvilke ruter passagererne vælger fra A til B, kan beskrives på en mere adfærdsmæssig korrekt måde. Først genereres de mulige ruter fra A til B i et net med kombinerede køreplaner vha. en heuristik, hvor det sikres at passagerer ved et valg af en inkluderet rute også ankommer til destinationen indenfor en rimelig tidsramme. I et efterfølgende skridt fordeles passagerer i mellem de inkluderede alternative ruter vha. en diskret valgmodel. Resultaterne fra modellen viser sig at være stabile uagtet om en linje defineres med en eksakt køreplan med minuttal eller blot dens frekvens. En af modellens fordele er, sammenlignet med andre modeller, at trafikplanlæggerne kan nøjes med færre antagelser om den faktiske køreplan for en linje og derved gøre det lettere at evaluere og sammenligne adskillige køreplansscenarier.

Den anden artikel foreslår en metode, der benytter Markovkæder til at identificere rejsetidsfordelinger for forskellige rutevalgsalternativer, mens der samtidig tages højde for stokastiske køretider som følge af forsinkelser. Givet relevante alternativer kan metoden udregne rejsetidsfordelingen for rejsende fra A til B for forskellige kombinationer af linjer og således også ruter gennem netværket. Både køreplans- og frekvensbaserede linjer kan bruges som input til modellen. Ved at benytte Markovkæder, kan sandsynlighederne for at nå et skift beskrives analytisk, hvorved traditionelt krævende simulationsmodeller kan undgås. Flere detaljerede analyser kan udledes baseret på de resulterende rejsetidsfordelinger, hvilket kan blive et vigtigt værktøj for køreplanlæggere.

Mens *Del 1* af afhandlingen afdækker nye metoder til at vurdere serviceniveauet af en given køreplan, omhandler *Del 2* inputtet til disse modeller ved at undersøge passagerernes rutevalgspræferencer. Rejsende evaluerer en rutes attraktivitet baseret på adskillige komponenter såsom rejsetid og antal skift, men det vigtige er hvordan f.eks. et ekstra skift mellem transportmidler vægtes i forhold til en evt. kortere samlet rejsetid. To forskellige artikler i afhandlingen beskriver passagerernes præferencer baseret på data indsamlet i Transportvaneundersøgelsen (TU), hvor passagerer har rapporteret hvilken rute de har valgt i kollektiv transport i Hovedstadsområdet. I begge artikler findes præferencerne ved brug af diskrete valgmodeller, hvor sandsynligheden for den valgte rute maksimeres ud fra delkomponenterne af den valgte rute og de alternative ruter.

Den tredje artikel omhandler således, hvordan passagerer vælger i mellem frekvens- og køreplansbaserede linjer. Separate parametre estimeres for ventetid for hhv. frekvens- og køreplansbaserede linjer, og dette viser, at passagererne har en højere gene ved at vente på frekvensbaserede linjer ift. køreplansbaserede linjer. Dette kan muligvis forklares ved frygten for at vente på en bus der aldrig kommer (fordi ingen fast køreplan kendes), men opvejes til dels ved at ventetiden for frekvensbaserede linjer generelt er kortere end for køreplansbaserede linjer. De separate parametre er vigtige at inkludere i rutevalgsmodeller, da det ellers risikeres at skabe skævheder i fordelinger mellem frekvens- og køreplansbaserede linjer. Sådan skævheder kan i sidste ende lede til fejlagtige konklusioner i cost-benefit analyser. Artiklen undersøger ligeledes hvordan passagerernes præferencer for tid i de enkelte kollektive transportmidler ændrer sig, i forhold til hvor lang tid der tilbringes i køretøjet. Her findes det, at der er en meget lav marginal negativ nytte for at tilbringe kort tid i f.eks. metro, men at længere tids ophold i et metrotog marginalt opleves væsentligt værre. Dette er i kontrast til tiden i et regionaltog, hvor kort tid i toget har en marginalt stor negativ nytte, mens den marginale negative nytte falder væsentligt jo længere tid passageren er i toget.

Den fjerde artikel har fokus på hvordan passagerer vælger ruter givet hvilke stationstyper der besøges undervejs. Artiklen gør i første omgang rede for, hvilke stationskarakteristika som udenlandske studier fundet har en stor påvirkning på passagerernes rutevalg. Ud af disse bliver tre karakteristika udvalgt, som formodes at have indvirkning på danske passagerers rutevalg. Analysen viser, at rutevalgssandsynligheden påvirkes positivt af, hvorvidt der på en af skiftestationerne er en lille butik eller lignende, samt hvorvidt der er rulletrapper, der kan lette gangturen i skiftet. Ydermere er det også signifikant, at passagerer fravælger stationer, hvor det er svært at orientere sig i skiftet. Det er således muligt at estimere separate skiftestraffe for stationer med forskellige karakteristika. Det bedst mulige skifte har således en straf svarende til 5,4 minutter i bus, hvorimod det værste er sammenlignligt med 12,1 minutter i bus. Disse resultater er vigtige, da de kan bruges til at kvantificere effekten på passagerflows af stationsopgraderinger eller nye designs. Sådanne mindre ændringer kan vise sig at være mere omkostningseffektive end sporopgraderinger eller andre forbedringer af jernbanenettet, og samtidig give en bedre oplevelse til passagererne.

Den sidste del af afhandlingen, *Del 3*, omhandler tre analyser baseret på rejsekortdata samt et system som ligner Rejsekortet i Hong Kong. Rejsekortdata dækker efterhånden flere og flere rejser i den kollektive transport, og datamængden er både god i forhold til analyser af passagerernes rejsemønstre over længere perioder samt for detaljerede studier af enkeltdele af rejserne. Desuden kan forskelle i passagerernes adfærd analyseres med en høj detaljegrad.

Den femte artikel omhandler sporarbejdet på S-togsbanen mellem Valby og Frederikssund i sommeren 2018. Rejsekortdata benyttes til at analysere passagerernes rejsemønstre både før og efter den tre måneder lange lukning af banen, hvor togbusser servicerede linjen med tilhørende forlængelser af rejsetiderne. Passagererne inddeles i forskellige grupper baseret på deres rejsemønstre før og efter sporarbejdet. Da passagerenes rejsemønstre ofte ændrer sig selvom der ikke er sporarbejder, sammenlignes ændringerne på banen til Frederikssund med en tilsvarende bane til Køge, som ikke havde nogen større sporarbejder. Der ses ingen større forskel mellem banerne i ændringen fra forår til efterår for passagerne med en høj rejsefrekvens i foråret. Dog sker der en nedgang i passagertallet på Frederikssundsbanen efter sporarbejdet, når der sammenlignes med banen til Køge. Dette skyldes til dels, at der ikke tiltrækkes lige så mange frekvente rejsende til Frederikssundsbanen henover sommer. Ved at analysere de daglige rejsemønstre for passagerer der pendlede på Frederikssundsbanen før sporarbejdet, kan det konkluderes at 17% næsten stoppede med at benytte kollektiv transport under sporarbejdet, men returnerede til et frekvent rejsemønster efter sporarbejdet. Dette indikerer, at nogle passagerer faktisk tilvælger kollektiv transport, selvom de ikke er tvunget til det.

Den sjette artikel benytter også data fra Rejsekortet, men i stedet for at fokusere på passagerernes rejsemønstre, fokuseres der på at estimere gangtiderne, der er nødvendige for skift mellem bus og tog. Denne tid er vigtig at kende, så gode korrespondancer mellem bus og tog kan skabe et endnu mere attraktivt kollektivt transportsystem. Ved at kombinere data fra Rejsekortet med data fra GPS-lokationer for busserne, så kan tiden, fra bussen ankom til passageren tjekkede ind på perronen, relativt simpelt beregnes. Dog er de rå data behæftet med store bias, da passagerer i nogle tilfælde venter i ventesalen før de går ned på perronen eller ligefrem benytter muligheden for at shoppe i løbet af skiftet. Derfor benyttes en maskinlæringsmodel til at klassificere de passagerer, som gik direkte ned til perronen i forhold til de passagerer, som foretog sig noget andet under skiftet. Dermed kan der opnås relativt sikre estimationer af gangtidsfordelingen for de skiftende passagerer, som kan benyttes i stedet for tidskrævende manuelle processer med at definere gangtiden fra et busstoppested til perronen. Modellen benyttes til at estimere gangtidsfordelinger for 129 stationer i Østdanmark, og resultaterne heraf viser, at der er en større andel af passagerer med aktivtet, når der findes flere butikker i stationsområdet.

Den sidste artikel i afhandlingen, baserer sig på data fra metroen i Hong Kong, som i myldretidsperioderne har så mange passagerer, at det væsentligt overskrider den kapacitet der er i systemet. Én specifik situation, som udløses af den enorme trængsel i systemet, analyseres i artiklen. I situationen vælger nogle passagerer at køre forbi den station, hvor de under ikke trængselspåvirkede omstændigheder ville skifte. Derefter skifter de på en anden station længere nede af linjen og kører tilbage forbi den normale skiftestation. Dette kaldes 'Reverse routing' og sker udelukkende fordi passagerer kan risikere, at de ikke kan komme med de første 3-4 tog der afgår fra stationen på grund af trængsel. Der findes i artiklen ingen endelig konklusion på omfanget af denne 'Reverse routing. Et interessant resultat er dog, at de passagerer som skal rejse længst efter skiftet har en væsentlig anden adfærd, end dem som kun skal rejse kort efter skiftet. Dette kunne indikere, at nogle af disse passagerer, som skal langt efter skiftet, vælger at benytte sig af 'Reverse routing' for at opnå en højere sandsynlighed for en siddeplads eller blot et bedre sted at stå. Dette kan også hænge sammen med de stigende marginale tidsværdier, som blev fundet for metropassagerer i *den tredje artikel*. Artiklen belyser også kort, hvorvidt der ses lignende eksempler på specielle rutevalg i den danske Metro, men ud fra rejsekortdata kan det hurtigt konkluderes, at trængselsniveauet ikke er højt nok i Metroen, til at dette sker.

Sammenfattende bidrager denne ph.d.-afhandling til fire hovedpunkter: i) udvikling af nye og detaljerede modeller for evalueringen af serviceniveauet af forskellige køreplansscenarier med frekvens- og køreplansbaserede linjer, ii) afdækning og kvantificering af den signifikante negative betydning af skift for rejser med den kollektive transport, samt analyser af hvilke stationskarakteristika der kan sænke denne negative påvirkning, iii) at vise hvorledes den enorme mængde af data fra Rejsekortet kan benyttes til at analysere passageradfærd over tid, samt gangtider på skiftestation, og iv) at analysere trængslens effekt på rutevalg i metronetværk. Samlet set dækker afhandlingen bredt modelleringen og analyser af passageradfærd i kollektiv transport, og bidrager med yderligere viden til den allerede eksisterende literatur. Flere nye metoder er udviklet i ph.d.'en, især ift. brugen af Rejsekortdata, og dette kan forhåbentlig benyttes som springbræt i fremtidige studier med fokus på kollektiv transport.

Contents

1 Introduction

This chapter is an introduction to the PhD thesis. The chapter first outlines the overall motivation in Section 1.1 and is followed by the aim of the thesis in Section 1.2, which divides the thesis into three parts. Each part presents the aim and main contributions of the part and associated papers. Finally, the chapter outlines the remainder of the thesis in Section 1.3.

1.1 Background

The transport sector accounts for 23 % of the green house gas emissions. To lower this share, United Nations are trying to lead a change through the sustainable development goals, away from car usage to more use of sustainable transport modes, i.e. walking, biking and public transport (United Nations, 2020). The goal is to lower the emissions from the transport sector, and the current COVID-19 crisis has shown that a reduction is possible. For example, local emissions were lowered with around 25 % in the city of Copenhagen due to, primarily, less car traffic (Ellermann and Hertel, 2020).

In metropolitan areas like Greater Copenhagen the problem of emissions is not the only one. According to The Government Commision on Congestion (2013) the number of hours wasted in car traffic will increase over the coming years, and lead to an extra 18.3 million hours per year spent in traffic in 2025 due to congestion. Another report by Region Hovedstaden (2019) estimates, that the number of trips for all modes will increase by 20 % in 2035 compared to the current number of trips. If these trips are mainly taken by car, this will lead to major congestion in the road network, which can not be expanded due to land use restrictions. Thus, the strategy for the future transport system in Greater Copenhagen is focused on an increase in the use of sustainable transport modes and especially a more effective and attractive public transport system (Region Hovedstaden, 2019). The development of a more attractive public transport system in Greater Copenhagen requires more knowledge on passenger behaviour, which is the overall topic of this PhD project.

Public transport has historically been a relatively data-poor sector, where counts of passengers were manual and with very little knowledge on the full trip chain from origin to destination (Kurauchi and Schmöcker, 2017). But with the development of automated fare collection systems based on smart cards (AFC), automatic vehicle location data (AVL) and automatic passenger count systems (APC), the public transport sector has in recent years become one of the most data-rich sectors. Especially the implementation of smart card systems has enabled public transport agencies to gain more detailed information on the actual travel patterns of the passengers, due to the continuous stream of data generated (Pelletier et al., 2011; Faroqi et al., 2018). Smart card data has in the recent years been used to examine travel behaviour for several purposes, for example origin to destination (OD) matrix estimation (Alsger et al., 2016), modelling passenger waiting times (Ingvardson et al., 2018), and analysis of year-to-year changes for passengers in public transport (Briand et al., 2017).

While the literature on the use of smart card data to analyse passenger route choice behaviour is growing (see e.g. Jánošíková et al. (2014), Shelat et al. (2019) or Zhao et al. (2017)), it is difficult to get a complete picture on the full journey from door to door, as smart card data only records the movements inside the public transport system. Smart card data also typically lacks information about the individual socio-demographic profile of passengers and the purpose of the trip. As such, other data sources for including the access and egress to the system are needed. This can for example be data from online travel surveys, as described in Anderson (2013), which provides more details on access and egress to the transport system as well as the trip purpose. Such data can reveal passenger route choice preferences with a high degree of detail, however, only relatively small data samples can be collected compared to data from smart cards.

Having knowledge of the route choice preferences of passengers is important, but to evaluate a given timetable and for predicting the impact of potential infrastructure investments, the preferences need to be used as an input to passenger assignment models. Such models are a central part of feasibility studies of public transport investments and are also used to assess the effects of timetable changes. The timetables and public transport networks in metropolitan areas are typically a mix of schedule-based services (e.g. local busses and regional trains) with detailed timetables and frequency-based services (e.g. metro and high-frequency busses) where only the headway between two vehicles on a line is known by the passenger (Ingvardson et al., 2018). Although such mixed systems have existed for a long time, the assignment models have only considered either schedule-based or frequency-based services in one model (Liu et al., 2010). The traffic planner analysing future timetables of a system is therefore required to make a choice between a schedule-based or a frequency-based approach (Gentile and Noekel, 2016). A choice of a frequency-based model can be problematic, as connections between services are typically not described explicitly in these models, while on the other hand a choice of a schedule-based model requires the planner to make several decisions on specific departure times for all lines.

This PhD thesis contributes to the development and application of assignment models for networks with co-existing schedule- and frequency-based services, and analyses the route choice preferences of passengers based on a dataset with a high level of details of the chosen routes. In addition, the thesis develops and applies methodologies using smart card data for studying i) the impacts of long-term service disruptions on passenger travel behaviour, ii) the walking times at transfer locations, and iii) the impact of crowding on passenger route choice in congested metro systems.

1.2 Aim and main contributions

The main ambition of this thesis is to develop and apply methodologies for better understanding and representing passengers' travel behaviour in public transport systems in metropolitan areas. The thesis is split into three parts. The first part focuses on development of assignment methodologies for analysis of the level-of-service provided to the passengers. The second part focuses on estimation of public transport passengers' route choice preferences, which are an important input for assignment models. The final part focuses on using the increasingly available data from smart card systems for different types of analysis of actual revealed behaviour of the passengers. The work is presented across seven papers categorised into one of the three research areas, that each constitutes a part of the thesis:

I. Assignment models for mixed schedule- and frequency-based public transport systems

II. Route choice models for mixed schedule- and frequency-based public transport systems

III. Studies on public transport passenger behaviour based on smart card data

The motivation and aim of each part and their associated papers are presented in the following subsections.

1.2.1 Assignment models for mixed schedule- and frequency-based public transport systems

Part I of the thesis focuses on development of new assignment models for public transport systems with co-existing schedule- and frequency-based services. Such models are needed for assessing which routes passengers choose in the public transport network. The combination of having both schedule- and frequency-based services in one system requires advanced modelling techniques for handling transfers between the two types of services. The advanced models are especially needed to avoid that the traffic planner must decide between a schedule-based or frequency-based model representation, when in fact the network is a mix of both types of services (Gentile and Noekel, 2016).

The first study in this part, *An assignment model for public transport networks with both schedule- and frequency-based services*, published in *EURO Journal on Transportation and Logistics* 8, 2019 (Paper 1) proposes a novel assignment methodology for mixed schedule- and frequency-based public transport systems. The assignment of passengers follow a two-step procedure: firstly, a choice set is generated using a modification of the event-dominance algorithm developed by Florian (1999), which generates alternative routes through the network where all alternative routes can reach the destination within a time threshold compared to the fastest possible route; secondly, passengers are distributed across alternatives using a logit-based discrete choice model given the utility of the alternatives (Train, 2002). The model is a first approach for integrated modelling of both schedule- and frequency-based lines in a unified model. Delays of schedule-based services are not taken into account, however, the headway for frequency-based services follows a binomial distribution. The main contribution of the paper is the inclusion of the probabilities of different arrival times to the destination, accounting for the risk of missing transfers from frequency- to schedule-based services.

The second paper, *Conditional passenger travel time distributions in mixed schedule- and frequency-based public transport networks using Markov chains*, under review at *Transportation Research Part B: Methodological* (Paper 2), develops a methodology for calculation of the distribution of travel time between origins and destinations in public transport networks with both schedule- and frequency-based services present. Usually, calculations of such travel time distributions for any public transport network require simulation models, which can keep track of different realisations of delays for busses and trains. However, with the proposed approach these distributions are obtained analytically and therefore with a higher degree of stability in the results. Few studies have previously used Markov chains for assigning passengers to different routes, and those who have are only considering frequency-based networks (Bell et al., 2002; Kurauchi, Bell, et al., 2003; Schmöcker et al., 2008). The developed methodology, however, takes as input both schedule- and frequency-based lines where both have stochastic travel times between stops. Markov chains are used for keeping track of the probability of reaching a connection at transfer stations. Efficient matrix calculations are used to calculate the travel time distribution from origin to destination, and more importantly, the model also enables calculations of conditional travel times based on the use of specific lines and combination of lines. As such, the novel methodology can avoid time consuming simulations. It contributes to the existing literature by providing a tool for detailed route choice and travel time analyses mixed schedule- and frequency-based public transport systems.

1.2.2 Route choice models for mixed schedule- and frequency-based public transport systems

Part II examines revealed route choices of passengers in public transport reported by passengers in a continuously collected travel survey in Denmark (Center for Transport Analytics DTU, 2020). The reported routes, taken by passengers in the public transport network, include detailed information on origin, destination, lines used, and partial information on transfer locations. The dataset used in this part of the thesis is a sub-sample of the routes matched in Anderson (2013). The dataset includes around 5,000 trips, which all have origins and destinations in the Greater Copenhagen area. For both studies, a logit-based discrete choice model is used for estimating the route choice preferences of the passengers (Train, 2002).

The first paper in this part, *The influence of frequency on route choice in mixed schedule- and frequency-based public transport systems - The case of the Greater Copenhagen Area*, in *Proceedings of the 14th Conference on Advanced Systems in Public Transport (CASPT2018)* (Paper 3), aims at discovering differences in waiting preferences for either schedule- or frequency-based services, and also testing different specifications of the marginal utility of in-vehicle times. The idea for testing differences in waiting time preferences stems from estimations in Ingvardson et al. (2018), where it was found that passengers to a certain extent time their arrival to schedule-based services, while they arrive randomly for frequency-based services. The aim is thus to estimate separate parameters for the two "types" of waiting times, to test if, for example, the uncertainty of not knowing the exact departure time of frequency-based services affect the passengers' preferences. To correct for the large difference in headway of the two types of services, a variable covering the hidden waiting time of the trip (i.e. time between the passenger can take the same alternative route) is included in the model. For testing possible differences in marginal utility (due to changes in marginal value of time), Box-Cox variables are introduced for each sub-mode. The paper contributes to the vast amount of route choice preference studies in the literature by detailing some of the important aspects of waiting time and in-vehicle time, which have not previously been estimated in such detail using a large dataset of observed door-to-door routes.

The second paper utilising the detailed dataset is, *Relevance of detailed transfer attributes in route choice models for public transport passengers*, re-submitted after second round of review to *Transportation Research Part A: Policy and Practice* (Paper 4). The paper firstly reviews relevant attributes of transfer stations, which can affect passengers' route choice. These attributes are then rated according to how they can be measured and whether they are relevant for passengers in a Danish context. Three transfer station characteristics are selected and included in the detailed route choice model; i) availability of escalators, ii) availability of a shop, and finally, iii) ease of wayfinding. As outlined in Iseki and Taylor (2009), not all transfers are weighted equally, and as such a general transfer penalty should be enriched with knowledge on the type of transfer. The study contributes to the limited number of studies focusing on station characteristics in route choice analysis, such as Raveau et al. (2011) and Garcia-Martinez et al. (2018). The study is the first to use such an extensive and detailed dataset of revealed passenger route choice for disentangling the general transfer penalty, which can be used to further enhance for example the assignment models presented in Part I.

1.2.3 Studies on public transport passenger behaviour based on smart card data

The final part, Part III, consists of three studies, which have in common that they are all based on data from smart cards (AFC data). Smart card data have been used for many purposes (Pelletier et al., 2011), but this part presents two innovative use cases of smart card data along with a route choice analysis based on smart card data.

The first paper, *Impacts of long-term service disruptions on passenger travel behaviour: A smart card analysis from the Greater Copenhagen area*, submitted to *Transportation Research Part C: Emerging Technologies* (Paper 5) presents an innovative approach to measure the impact of long-term planned disruptions on passenger travel behaviour. To isolate the effect of a three month track closure of a suburban train line in the Greater Copenhagen, passengers on the disrupted track segment are compared to passengers on a comparable track section, which did not have any major disruptions. Data from the Danish smart card - *Rejsekort* - is used for the study, which enables a comprehensive longitudinal analysis. The passengers are clustered based on three travel characteristics using k-means clustering; share of active weeks in the period, the number of active days during active weeks, and the share of trips taking place during weekends. A subsequent analysis, based on hierarchical clustering for active travel days for the most regular passengers, is used to analyse the specific changes for these passengers during and after the disruption. The proposed approach overcomes typical issues of analyses of the effect of long-term disruptions by explicitly considering behavioural changes of passengers over time. Normally, such studies focus on the overall changes in travel demand (Yap et al., 2018; Nazem et al., 2018) or use small before and after surveys (Zhu et al., 2017). However, this neglects that individual passenger travel behaviour in public transport changes significantly over longer time spans (Deschaintres et al., 2019; Egu and Bonnel, 2020).

The second paper, *Estimation of transfer walking time distribution in multimodal public transport systems based on smart card data*, submitted to *Transportation Research Part C: Emerging Technologies* (Paper 6), takes advantage of the high degree of details when passengers use *Rejsekort*, in order to analyse the necessary walking times needed at transfers. Compared to most other systems, it is unique in the way that passengers must tap-in at each boarding location and must tap-out at the final destination. As such, it is relatively simple to fuse transactions in smart card data with AVL data from the busses, and thereby extract the time from when the passenger arrived to the station and subsequently tapped-in at the platform (where validators are placed on stations). However, the raw data also includes a share of passengers who may be doing an activity during the transfer such as shopping, buying coffee, etc. To isolate the time needed for walking, a *hierarchical Bayesian mixture model* is estimated for 129 stations and even more combinations of bus stops and validators (platforms). It includes one distribution for passengers walking directly and another distribution for passengers having an activity during the transfer. The paper is the first to analyse the walking times in such detail based on smart card data, where no information of whether a passenger did an activity is present. The paper contributes to the literature with an innovative approach for the estimation of the needed walking times, known to vary significantly due to passengers choosing different paths through the station (Daamen, Bovy, et al., 2006), general passenger walking speed heterogeneity (Daamen and Hoogendoorn, 2006), and distance walked (Du et al., 2009).

The final paper in this part and the thesis, *A note on unusual path choice behavior caused by congestion in metro systems*, Working paper (Paper 7), regards the effects of crowding on route choice. Crowding is known to affect passengers' route choice in congested metro systems (Kim et al., 2015; Zhang et al., 2018), and in the particular case study in the paper, the Metro in Hong Kong (MTR), passengers experience excessive crowding during peak hours. The paper investigates the specific route choice problem, *reverse routing*. This behaviour occurs when passengers who need to transfer between two lines choose to stay in the train and pass the normal transfer station. The passenger then transfers at a station further down the line, before returning and bypassing the normal transfer station. The concept is related to the behaviour of *travelling backwards* (Tirachini et al., 2016; Yu et al., 2020), where passengers board a train in the "wrong" direction and *turn back* at a station further down the line, to again bypass the origin station. However, in the case of *reverse routing*, there is no additional transfer for the passengers doing so, and this makes it more attractive to do in cases with severe crowding at the normal transfer station. The paper uses a passenger-to-train assignment model based on the exit time at the destination to isolate the time from origin to departure from the normal transfer station. This enables the use of a multiple linear regression model for comparing the travel time of passengers, which can indicate who are using the option of reverse routing. The study also briefly covers the crowding situation in the Copenhagen Metro, Denmark, by using data from *Rejsekort*, to shed light on whether unusual path choice is also occurring in Denmark. No previous studies have focused on analysis of the unusual *reverse routing* path choice behaviour. While the problem is very specific to few congested networks, e.g. the metro systems in Hong Kong, Beijing and Santiago, the results contribute to knowledge on differences in passenger travel behaviour for excessively crowded metro systems.

1.3 Outline

With the motivation and aim for the PhD thesis defined, the remainder of the thesis includes the papers for the three parts and a concluding chapter summarising the main findings. Part I contains Chapters 2-3, which presents novel methodologies for analysing the level of service in public transport in Papers 1 and 2. Part II contains Chapters 4-5 with a focus on analysing passenger route choice preferences based on observed routes from door to door (Papers 3 and 4). The final part, Part III, including Papers 5, 6 and 7, presents new and innovative uses of smart card data and is divided in Chapters 6-8. Finally, Chapter 9 sums up the findings and outlines the potential policy impacts of the developed methodologies and outcomes of the analyses.

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I

Assignment models for mixed schedule- and frequency-based public transport systems

2 Paper 1: An assignment model for public transport networks with both schedule- and frequency-based services

The following pages contain the article:

M. Eltved, O. A. Nielsen, and T. K. Rasmussen (2019). "An assignment model for public transport networks with both schedule- and frequency-based services". In: *EURO Journal on Transportation and Logistics* 8, pp. 769–793. DOI: 10.1007/s13676-019-00147-4.

Please cite accordingly.

The paper, written solely during the PhD studies, is based on the master thesis, *Unified Schedule- and Frequency-based Public Transport Assignment*, written by the PhD student at the Technical University of Denmark. The work has been presented twice at conferences, respectively at *Trafikdage (Annual Meeting on Transport in Denmark) 2017*, Aalborg, Denmark in August 2017, and at the *7th Symposium of the European Association for Research in Transportation (hEART 2018)* in Athens, Greece in September 2018.

EURO J Transp Logist (2019) 8:769–793
https://doi.org/10.1007/s13676-019-00147-4

RESEARCH PAPER

An assignment model for public transport networks with both schedule- and frequency-based services

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Received: 31 December 2018 / Accepted: 23 September 2019 / Published online: 23 October 2019

Abstract
This paper presents an assignment modeling framework for public transport networks with co-existing schedule- and frequency-based services. The paper develops, applies and discusses a joint model, which aims at representing the behavior of passengers as realistically as possible. The model consists of a choice set generation phase followed by a multinomial logit route choice model and assignment of flow to the generated alternatives. The choice set generation uses an event dominance principle to exclude alternatives with costs above a certain cost threshold. Furthermore, a heuristic for aggregating overlapping lines is proposed. The results from applying the model to a case study in the Greater Copenhagen Area show that the level of service obtained in the unified network model of mixed services is placed between the level of service for strictly schedule-based and strictly frequency-based networks. The results also show that providing timetable information to the passengers improve their utility function as compared to only providing information on frequencies.

Keywords Public transport · Route choice · Path-based assignment · Frequency based · Schedule based · Event dominance · Static versus dynamic

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1 Introduction

Urban public transport networks are by their nature often complex with many lines and alternative routes through the network. When passengers make their route choice in a public transport network, they seldom depart at the minute they desire, but must wait until the first possible departure on the considered line. But how do passengers plan their route, if they have a combination of high- and low-frequency services, where some routes from a passenger perspective are without a specific timetable and running as a frequency-based (FB) service, while other services run schedule-based (SB) with runs at specific times? Ingvardson et al. (2018) showed that the distribution of passenger arrivals to the first station on their trip differs significantly for schedule- and frequency-based services with completely random arrivals to frequency-based services and more timed arrivals for schedule-based services. This behavior should be taken into account when considering the assignment of passengers in networks with co-existing schedule- and frequency-based services which are found in most metropolitan regions in Europe. For example, the network in the Greater Copenhagen area has a mix of frequency-based services such as high-frequency buses and metro lines and schedule-based services such as the suburban trains and local bus lines.

In the field of public transport assignment models, there have been two main ways to represent the supply in the model, frequency based or schedule based. The supply side started as being described with frequencies, implying that the passenger would assume that a line would run with a given headway, but without knowing the exact timetables (Nökel and Wekeck 2007). This allows the path searches to be made in a static model with no time dimension. Later, the models developed into schedule-based models where each run on a line is included with the times when it passes different stops on the route (see, for example, Wilson and Nuzzolo (2009) for examples of schedule-based models). The schedule-based models allowed the modeler to better represent coordination between lines and thereby describing the passengers' route choice in a more detailed way (see Liu et al. (2010) for a comprehensive overview of different modeling techniques for public transport assignment).

The advantage of a schedule-based model is that coordination between lines is well defined. Representing this realistically is particularly important in low-frequency systems, as waiting times between corresponding lines might be a significant part of the cost of a route. But schedule-based models require more data input and more calculation time because the model should run for several possible departure times. The frequency-based models are on the other hand much simpler and require less input and calculation time. Frequency-based models work well with high-frequency services, where the passenger might not consult the public timetable (if it is available), but if there are low-frequency services in the network, it is very difficult to estimate the transfer times between two lines, and thereby, if a route including this transfer is attractive.

As mentioned in Gentile and Noekel (2016, chap. 6), there exists currently no framework to include these two different ways of representing the supply in the same model. They propose to take a decision on whether the network is better described with schedules or frequencies and choose the best option, since no simple way of making a combined model is available. The proposal if choosing the frequency-based approach is that one should assume truncated waiting time distributions for schedule-based services. But this is an assumption which requires many decisions on how the truncated distributions should be defined and for which services the passenger behavior should be treated as schedule-based behavior. Another proposal in Gentile and Noekel (2016, chap. 6) is to rely on a schedule-based network and incorporate passenger cost functions for frequency-based lines, while still keeping a completely schedule-based network. This is a possible solution for modeling the current system, but when evaluating changes in the public transport network, it requires the modeler to take decisions on the exact departure times of the line which might influence the transfer times to corresponding lines (Cascetta and Coppola 2016). A unified model will be able to relieve some of the work for the modeler and possibly results in more stable forecasts, as the level of service is not relying so heavily on how the modeler defines the specific runs of the public transport services.

This paper proposes a new approach for solving the assignment problem in mixed schedule- and frequency-based public transport networks. The remainder of the paper is organized in the following way: Sect. 2 describes the developed assignment framework; Sect. 3 describes the case study network and presents the results of different tests of the framework; Sect. 4 discusses the results and some further improvements that could be made; and finally, Sect. 5 concludes the main findings of the paper.

2 Assignment framework

This section first lists some general assumptions of the model and describes the different transfer possibilities in a mixed schedule- and frequency-based network. Then, the network structure used in the model is presented followed by the two phases of the model: first, a choice set generation phase, where different alternative routes through the network are generated; second, a flow allocation phase to assign flow to the generated alternatives. Both of these phases are described in detail in this section and followed by a description of a heuristic for considering frequency aggregation of overlapping services. The notation used in the model is summarized in Table 1.

2.1 General assumptions

The framework is governed by a set of general assumptions, which make it possible to assign the passengers on a mixed schedule- and frequency-based network. The assumptions are stated and explained below. Some of the assumptions can be changed if necessary, and some of these possible changes are discussed in Sect. 4.

Table 1 Table of notation

Symbol	Description
q	Hidden waiting time before passenger leave the origin and go to the first stop
a	Access/egress time between respective origin and first stop and last stop and destination
s	In-vehicle time for S-train
b	In-vehicle time for bus
o	Walking time between stops
m	Transfer penalty
p	Success parameter describing the waiting time for FB services
r	A specific route from origin to destination following the same lines and stop sequence
n	Individual passenger
R_n	Bundle of routes in choice set for passenger n
δ	Threshold parameter
PDT	Preferred departure time
$e \in E$	Events (physical movements) in the network graph
$s(e)$	Start time of an event e
$t(e)$	Time duration of an event e (either a, s, b or o)
$T(e)$	End time of an event e
$w(e)$	Waiting time for an event e
$P_t(e)$	Probability of reaching event e when transferring
$P_r(e)$	Probability of reaching specific event e at the transfer on route r
$P_a(e)$	Probability of actualizing the specific event e on route r
$P_u(e)$	Probability that event e can be reached on route r
$h(e)$	Headway of line (only FB services)
$c(e)$	Cost of event e
$C(e)$	Cumulative cost at the end of event e following a specific sequence of events until event e
$C(r)$	Total cost of route r

1. The public transport network is given as input to the model including exact runs and departures for schedule-based services and the headways for frequency-based services in whole minutes. No coordination between frequency-based lines and any other line is assumed.
2. The model considers a day with normal operations and is, therefore, not applicable for days with major incidents in the network. For this reason, the headways for frequency-based services are assumed to be constant. Schedule-based services are considered as being deterministic and no delays are assumed for these.
3. The framework is developed to distribute a group of passengers across alternatives between an origin and a destination, and the paths are predetermined at the origin and no real-time information on services is assumed.
4. The flow of passengers at a given preferred departure time (PDT) is considered being exogenous.

5. The public transport network is considered to be cyclic around the day, so a late departure at the end of the day can be completed into the next day. This means that all relations will have a viable path if a viable path exists throughout the day.

2.2 Network structure

The network is a dynamic graph with edges $e \in E$, which is dynamically built up along the graph search. The events e in the algorithm are meant as spatial movements in the network, which in this paper are the following:

- Access or egress network on connectors
- Ride on line (schedule or frequency based)
- Walking at transfers between stops

The time of an event $t(e)$ is deterministic for all of the events since no delays are assumed in the model. For all movements in vehicles, a concept similar to the route section method used in de Cea and Fernández (1993) is proposed. de Cea and Fernández (1993) establish route sections between two nodes on a line, which are not necessarily consecutive, to allow aggregation of common lines between stops. The purpose of doing this in de Cea and Fernández (1993) is to model congestion in public transport and model the waiting times for the aggregated lines and not just the individual lines. This paper uses the term "direct arcs", which are also route sections between nodes which are not necessarily consecutive on a line. Direct arcs are created from a stop to all following stops on the line for both schedule- and frequency-based services as shown in Fig. 1. This allows for a simpler route generation, as a route is then defined as a set of direct arcs, where origin and destination are, respectively, the first and final node, and all other nodes in the route are points where the passenger transfers between walking and riding edges or directly between

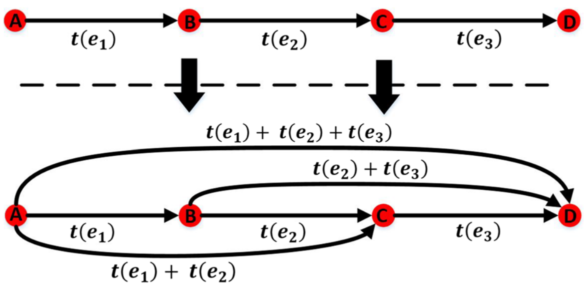


Fig. 1 Description of unfolding of direct arcs from one stop to all following stops on a single line

Table 2 Route cost parameters used in the modeling

Parameter	Symbol	Estimate	Unit
Hidden wait (Zone wait)	β_q	− 0.12	min.
Access/egress	β_a	− 0.21	min.
In-vehicle S-train	β_s	− 0.15	min.
In-vehicle bus	β_b	− 0.19	min.
Waiting	β_w	− 0.20	min.
Walking	β_o	− 0.21	min.
Transfer penalty	β_m	− 1.05	nb. of transfers

riding edges. The direct arcs also have another important usage, as they in Sect. 2.6 are used for aggregating common lines.

The costs of the network arcs are not measured in the unit of time solely. Many studies have shown that passengers do not weight the different time components in a trip in the same way (see, i.e., Anderson et al. (2014) for an overview of some of the estimated parameters in the literature). This paper applies the parameters used in the Danish National Transport Model[1] in the choice set generation and subsequent flow allocation, and Table 2 lists the route choice parameters for leisure trips from the Danish National Transport Model. These parameters are close to the parameters estimated in Anderson et al. (2014, Table 5), where a path size correction logit model was estimated. This paper does not include the PSC factor in the allocation of flow, but it is considered that the parameters in Table 2 will be sufficient to prove the ideas behind a unified assignment model.

The cost of an event $c(e)$ is the sum of all the possible components for that event multiplied by the parameter for the given component, i.e.,

$$c(e) = q * \beta_q + a * \beta_a + s * \beta_s + b * \beta_b + w * \beta_w + o * \beta_o + m * \beta_m \tag{1}$$

2.3 Definition of waiting times and transfer probabilities

When considering a public transport network with both schedule- and frequency-based services, there is a need to define the waiting times and the different transfer probabilities between the service types. Given that the model should only be used for strategic planning and no real-time information is included, a choice was made to rely on a discretization of the time unit to integers, since timetables for schedule-based services are usually discretized to whole minutes. As passengers using frequency-based services are not aware of the specific departure time (and thereby arrival time) of a specific line, the waiting time for frequency-based services must be defined. The waiting time of a frequency-based service is naturally depending on the headway of a line, and in the literature, exponential distributions and Poisson distributions have been used to describe

[1] The parameters are based on parameters from the Danish National Transport Model (http://www.landstrafikmodellen.dk/), which have been estimated to fit the counts in the National Transport Model, where the case study area in Sect. 3 is a subnetwork of the full national network.

Table 3 Waiting times for transfers between schedule- and frequency-based services

Transfer from/to	(A) SB line	(B) FB line
(A) SB line	The waiting time is deterministic (Case 1)	Waiting time follows statistical distribution (Case 2)
(B) FB line	A probability to catch next service and for this an associated waiting time (Case 3)	Waiting time follows statistical distribution (Case 4)

the headways between vehicles and thereby also the passenger waiting times for frequency-based services (Chriqui and Robillard 1975; Schmöcker et al. 2013). However, both of these distributions do not have an upper bound and is, therefore, not applicable to describe a system with constant headways. The assumption of constant headways with the passenger unaware of the specific departure time of a frequency-based service would naturally lead to the use of a discrete uniform distribution to describe the waiting time for these services. However, as will be discussed further down in this subsection, the use of discrete uniform distributions is not possible, and therefore, a choice was made to describe the waiting time for a frequency-based service by a binomial distribution. The binomial distribution takes two inputs: the maximum number of trials and the probability of success. In the case when describing waiting times for frequency-based services, the parameters of the binomial distribution is then the headway of the line and a parameter describing the probability of success, i.e., the mean waiting time, such that $w(e) \sim B(h(e), p)$, where B denotes the binomial distribution, i.e., the probability of waiting f minutes is

$$P\Big(w(e) = f\Big) = \binom{h(e)}{f} p^f (1-p)^{h(e)-f}, \tag{2}$$

where $h(e)$ is the headway of the frequency-based line and p is the parameter controlling the distribution of the waiting time. Since the passengers are unaware of the specific departure time of the frequency-based service, the parameter p is set to 0.5, as the mean of the binomial distribution is $h(e) * p$ and thereby half the headway as also shown in Ingvardson et al. (2018).

Given that the system is assumed to be completely reliable, meaning that schedule-based services run according to their timetable and frequency-based services run with constant headways, four waiting time scenarios are identified for transfers between schedule- and frequency-based services, as listed in Table 3. Considering these waiting time scenarios, we can write up the waiting times for the different cases in the following way:

$$w(e_k) = \begin{cases} s(e_k) - T(e_{k-1}), & \text{cases 1 \& 3} \\ B(h(e_k), p), & \text{cases 2 \& 4} \end{cases} \tag{3}$$

where $T(e_k)$ describes the arrival time to a node using the waiting time and duration of the event and is defined as follows:

$$T(e_k) = T(e_{k-1}) + w(e_k) + t(e_k) \tag{4}$$

Given this arrival time, the probability of reaching an event when transferring from another line $P_t(e_k)$ is given as

$$P_t(e_k) = P\big(T(e_{k-1}) \leq s(e_k)\big) \tag{5}$$

where $s(e_k)$ is the departure time of the service. This results in a situation, where transfers between schedule-based services (case 1) are certain as long as the departure time is the same or later as the arrival time of the previous schedule-based event. The probability of reaching a frequency-based event from a previous event, no matter if the previous event is schedule or frequency based (cases 2 & 4), is always 1. However, the waiting time for the frequency-based service is not known. The arrival time $T(e_k)$ of an event e_k in case 3 is given by Eq. 4, where the waiting time follows the binomial distribution. In case 4, with transfers between consecutive frequency-based services, the arrival time $T(e_k)$ of event e_k is not only dependent of the waiting time $w(e_k)$ but also the waiting time for the previous event $w(e_{k-1})$. To determine the arrival time $T(e_k)$ of event e_k, it is necessary to sum the waiting times of both events. The sum of distributions is also known as a convolution (Olds 1952). A route can follow several consecutive frequency-based lines, and it is, therefore, necessary to use the convolution of the waiting times. The choice of binomial distribution to describe the waiting times stems from the need for the convolution of several distributions. Convolutions of more than two discrete uniform distributions do not have any analytic closed-form expression; however, the convolution of multiple binomial distributions has a closed form. When several consecutive frequency-based lines are followed on a route, the arrival time at the end of event e_k can, therefore, be described as follows:

$$\begin{aligned} T(e_k) &\sim T(e_{k-j-1}) + \sum_{l=0}^{j} w(e_{k-l}) + \sum_{l=0}^{j} t(e_{k-l}) \\ &= T(e_{k-j-1}) + \sum_{l=0}^{j} B(h(e_{k-l}), p) + \sum_{l=0}^{j} t(e_{k-l}) \\ &= T(e_{k-j-1}) + B(\sum_{l=0}^{j} h(e_{k-l}), p) + \sum_{l=0}^{j} t(e_{k-l}), \quad 0 < p < 1 \quad h(e_{k-l}) = 1, 2, \ldots \end{aligned} \tag{6}$$

where j describes the number of consecutive frequency-based lines taken prior to event e_k.

The final case of the transfer scenarios is for transfers from a frequency- to a schedule-based service (case 3). In this case, the passenger is, as mentioned above, unaware of the exact arrival time of the frequency-based service, and as such, there is a probability to catch the first departure of the schedule-based line $P_t(e_{k1})$ (hereafter called a candidate) and for this an associated transfer time. If the first possible run on the schedule-based line can not be reached with certainty, multiple runs of the schedule-based line must be taken into account when determining the expected waiting time for the schedule-based line at the transfer. To describe this probability of reaching a specific candidate i of the schedule-based line, we use the symbol

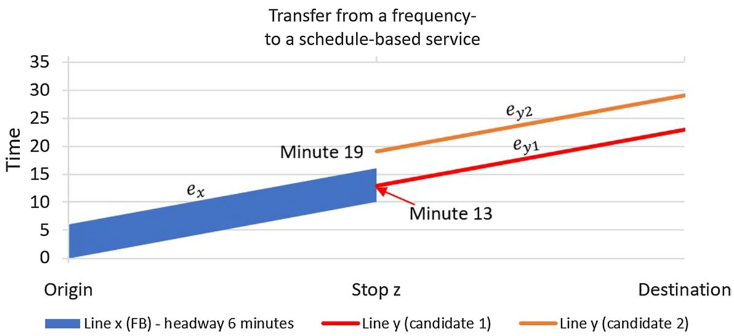


Fig. 2 Example of path with a transfer from a frequency-based service to a schedule-based service

$P_r(e_{ki})$, where the first candidate have $P_r(e_{k1}) = P_t(e_{k1})$, while the second candidate e_{k2} have $P_r(e_{k2}) = P_t(e_{k2}) - P_r(e_{k1})$ and so forth for the remaining candidates until a candidate has a $P_t(e_{k1}) = 1$

In Fig. 2, an example of a case 3 transfer from a frequency-based service (line x) to a schedule-based service (line y) is shown. Line x is a frequency-based service with a headway of six minutes and line y is a schedule-based service with departures from stop z in minutes 13 and 19. The probability of reaching the departure at minute 13 is then the same as the probability of waiting 3 minutes or less at the origin, i.e.,

$$P_t(e_{y1}) = P(T(e_x) \leq s(e_{y1}) = P(w(e_x) \leq 3) = \sum_{f=0}^{3} \binom{6}{f} 0.5^f (1 - 0.5)^{6-f} = 0.66 \tag{7}$$

The probability of reaching the second candidate of line y is 100% since $P(T(e_x) \leq s(e_{y2}) = 1$, but since this departure is only relevant if the first candidate of line y is missed, the probability of boarding the second candidate when using route r from origin to destination is $P_r(e_{y2}) = P_t(e_{y2}) - P_r(e_{y1}) = 0.34$.

A route is defined as a specific sequence following the same stops and the same lines. The cost of a route is defined by all the possible candidates and the probability of actually ending up being on a specific candidate on route r. The probability of actually ending up on a specific candidate on a route $P_a(e)$ is given by the probability of the previous event multiplied by the probability of reaching a specific candidate at the transfer, i.e.,

$$P_a(e_k) = P_a(e_{k-1}) * P_r(e_k) \tag{8}$$

The total cost of a route ($C(r)$) is then defined by the probability of actualizing a specific event on a route and the cost of the specific event.

$$C(r) = \sum_{e \in E_r} P_a(e) * c(e) \tag{9}$$

As an example of the calculation of the total cost of a route, the total cost of the example route in Fig. 2 is then:

$$C(r) = \sum_{e \in E_r} P_a(e) * c(e) = 1.00 * c(e_x) + 0.66 * c(e_{y1}) + 0.34 * c(e_{y2}) \tag{10}$$

2.4 Choice set generation phase

In previous studies by Friedrich et al. (2001) and Hoogendoorn-Lanser et al. (2007), Branch and Bound techniques were used to generate a choice set in a SB network. They incorporated different behavioral constraints to limit the choice set and generate realistic alternatives. For example, Hoogendoorn-Lanser et al. (2007) developed criteria that feasible routes should fulfill, and routes were excluded from the choice set if failing to fulfill one of these. Obvious criteria included departure after arrival time at a node, and that routes should not include cycles. After this, criteria on time, space and money were used to prune out non-reasonable alternatives. The idea of applying different criteria/rules to fulfill is used as base in the following algorithm to generate a proper choice set in a network with mixed schedule- and frequency-based services. Moreover, the idea is to build upon the event dominance principle in Florian (1999) and Florian (2004) with the introduction of a relaxation of the event dominance by considering a threshold for including nonoptimal paths. The event dominance algorithm is, in its original implementation, used to find the shortest path in a schedule-based network (see Nielsen and Frederiksen (2006) for a description of implementation and optimization of the original event dominance algorithm). It thus finds only the best path in terms of cost of a route, but can, however, with a slight modification identify additional routes if the event dominance at each node is relaxed. Algorithm 1 describes the overall concept of the event dominance algorithm, where events are pruned at each node.

```
while an event at end node has not been checked do
    Find cheapest event in heap which has not been checked
    for all possible event types going from the node considered do
        Check if event can be inserted based on criteria on both time and cost compared to the events already
          at the tonode;
        if event is not dominated then
            Insert event &
            Check if any already existing events at node is dominated and remove dominated events
    end
end
```

The criterion used in the original event dominance algorithm is shown in Eq. 11. An earlier arriving event is denoted e_1 compared to the later arriving event e_2. $C(e)$ denotes the cumulative cost of following a specific route and arriving with event e. $T(e)$ is for schedule-based services defined as the planned arrival time to a stop, while $T(e)$ for frequency-based services is defined as the mean arrival time. It is important to note that all events are only checked against later events that arrive at the same stop. Events which fulfill this criterion will always be inserted in the algorithm proposed below, ensuring that the optimal path is always found. Furthermore, this paper proposes and applies the criterion in Eq. 12. In this, the strict event dominance principle is relaxed, and it allows nonoptimal events to be included in the event heap. The threshold parameter δ defines the threshold and is set to 20% in this paper meaning that events which have a cost 20% higher at a stop are discarded. As mentioned in the beginning of the section, there are cases where it is not 100% certain to catch the first departure on a schedule-based line when transferring from a frequency-based line. In these cases, all runs (candidates) of the schedule-based lines are investigated until one of the runs is 100% certain, i.e., departs after a full headway of the frequency-based service.

$$C(e_1) + (T(e_2) - T(e_1)) * \beta_w \leq C(e_2) \tag{11}$$

$$(C(e_1) + C(e_1) * \delta) \leq C(e_2) \tag{12}$$

In Algorithm 2, a more detailed pseudocode for the full algorithm with the new criterion is shown. In the algorithm, it is only events which are certain, i.e., $P_u(e) = 1$ which can dominate other events at a node. P_u denotes the probability that an event could be realized on the route, i.e., a multiplication of P_t of all the events used to arrive with event e. Events which are uncertain can thereby only be dominated, but will not be used for pruning other events at the same stop. e^{fat} describes the first possible arrival time at a specific node with a specific event. For schedule-based services, this is equal to the scheduled arrival time, while for frequency-based services it is the first possible arrival time.

```
while the cost of any unchecked event is lower than the lowest cost (+ threshold δ) of an event
  where P_u(e) == 1 at destination do
    Find cheapest event (e_i) in heap which has not been checked
    for all possible events (e_d) do
        if e_d^fromnode == e_i^tonode ∧ e_d^starttime ≥ e_i^fat ∧ e_d^line ≠ e_i^line then
            dominated = 0;
            for all events in the heap (e_k) where e_k^tonode == e_d^tonode ∧ P_u(e_k) == 1 do
                if T(e_k) ≤ T(e_d) ∧
                (C(e_k) + (T(e_d) − T(e_k)) * β_w) ≤ C(e_d) ∧
                (C(e_k) + C(e_k) * δ) ≤ C(e_d)
                 then
                    dominated = 1;
            end
            if dominated == 0 then
                Insert e_d into event heap
                if P_u(e_d) == 1 then
                    for all events in the event heap (e_k) where
                     e_k^tonode == e_d^tonode ∧ P_u(e_k) == 1 do
                        if T(e_d) ≤ T(e_k) ∧
                        (C(e_d) + (T(e_k) − T(e_d)) * β_w) ≤ C(e_k) ∧
                        (C(e_d) + C(e_d) * δ) ≤ C(e_k) then
                            Remove e_k from event heap;
                    end
    end
    Mark e_i as checked;
end
```

Having established the feasible events to be included in the network graph, the routes are compiled by events which use the same lines and stop sequence, as exemplified in Fig. 2. Note that if some candidate event on the route is not within the overall threshold, the full route alternative is not included in the choice set. Having identified the unique feasible routes (in terms of line- and stop sequences), the choice set R_n is then composed of this set of routes which are all within the threshold with certainty.

2.5 Flow allocation phase

The allocation of flow to the generated alternatives in the choice set is based on the multinomial logit discrete choice model (Train 2002). The utility of a route r is described with a deterministic term and an error term, and the utility that each decision maker n associates to an alternative r in a choice set R_n is given as:

$$U_{rn} = V_{rn} + \epsilon_{rn} \tag{13}$$

where V_{rn} is the deterministic part of the utility and the error term ϵ_{rn} describes the random part of the utility. The total deterministic utility of a route V_{rn} is the same as the cost $C(r)$ of route r with a negative sign in front, which indicates that all components of the trip is a cost to the traveler. The choice probability of route r by traveler n is given by Eq. 14:

$$P_{rn} = \frac{\exp(V_{rn})}{\sum_{j \in R_n} \exp(V_{jn})}. \tag{14}$$

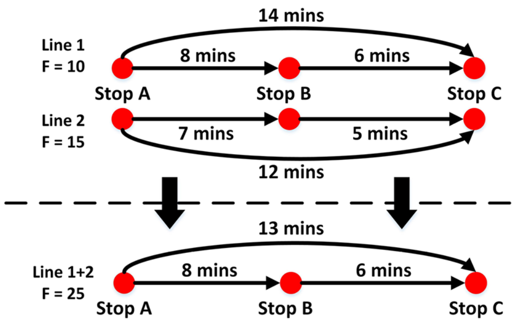
2.6 Frequency aggregation of overlapping lines

Passengers can possibly choose between a number of different lines in corridors with multiple lines serving the same stops (Nielsen 2000). From a passenger perspective, these lines might be observed as common lines and then chooses between either of the lines (Chriqui and Robillard 1975). This paper proposes a heuristic solution accommodating this, by combining lines which are serving the same stops and which have similar driving times [as seen in Nielsen (2000)]. The aggregation is done on stop level, meaning that the direct arcs from a stop to all stops still to be served by the line are created. If any of these direct arcs are similar on stop level between the two lines, then they are aggregated. The specific rules are, that arcs are aggregated if:

(i) the lines belong to the same submode, i.e., Suburban train or bus, and
(ii) the relative difference in driving time is less than 15% or the absolute difference is less than or equal to two minutes.

The frequency of the aggregated line is then the sum of the frequencies of the lines aggregated, and the driving time is a weighted average by the number of departures/h. The aggregated line is always described as a frequency-based line, even though the original lines are schedule based. An example of frequency aggregation of two lines with frequencies of, respectively, 10 and 15 departures/h is shown in Fig. 3. As the driving times for the two lines are very similar, the lines can

Fig. 3 Example of frequency aggregation



be aggregated to a single line serving the stops with a frequency of 25 departures/h. The number of passengers on the aggregated lines are distributed across the specific lines proportionally to the frequencies of the respective lines. The proposed heuristic for frequency aggregation is tested in Sect. 3.5.

3 Results

This section presents the results of applying the proposed methodology to a case study. The case study covers a part of the Greater Copenhagen Area public transport network. First, the case study is presented followed by the results of tests on five different network configurations. The first configuration is the actual network involving a combination of frequency- and schedule-based services (unified network). The second and third configurations are, respectively, a strictly schedule- or strictly frequency-based representation of the network. The last two tests (configurations) involve a network representation where certain schedule-based lines are represented as frequency based and a representation that allows evaluating the frequency aggregation heuristic. Lastly, the results of all tests are compared and discussed.

3.1 Case study network

The case study consists of a specific travel relation in the Greater Copenhagen Area in Denmark. The relation is from the Technical University of Denmark, located in the outskirts of Copenhagen, to Brønshøj, which is located in the northern part of Copenhagen. The travel relation is not frequently used in real life, however, the relation has no obvious shortest path and no paths without a transfer. The relevant part of the real-life network includes a mix of schedule- and frequency-based lines (with most schedule-based lines) with a total of 21 lines (see Fig. 4). The headways of the different lines are presented in "Appendix 1". The example is quite similar to the one studied in Nielsen (2000), where it was shown, that passengers in this relation choose a number of distinct routes.

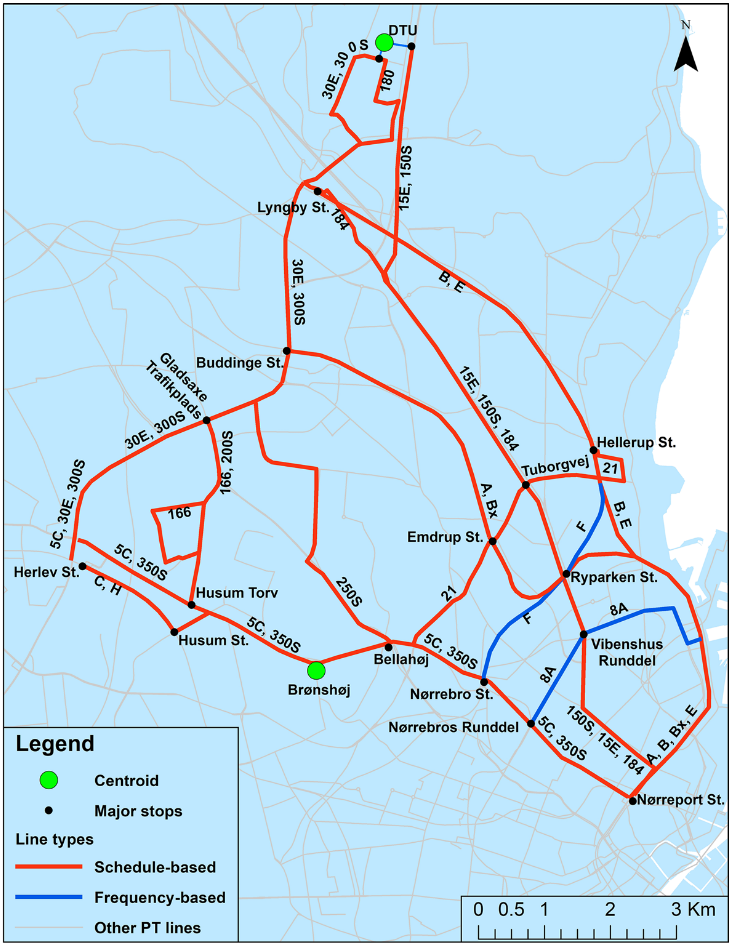


Fig. 4 Overview of network with lines and type of line

3.2 Unified network

In Fig. 5, the route shares for a launch of passengers at a preferred departure time at 7.30 a.m. are shown. The main alternative is to take line 300S to Herlev and transfer to line 5C, inducing arrival time at 8:19 a.m. The two lines are perfectly coordinated at the interchange in Herlev, where the passengers have no waiting time when transferring. There are paths which arrive earlier (8:15), but because these paths require two transfers, they are less attractive than the path via Herlev St. In total, the choice set consists of the eight different alternatives outlined in Table 4, including an 3-transfer alternative via Lyngby Station, Hellerup St. and Nørrebro St. However, this path is still attractive because it primarily involves

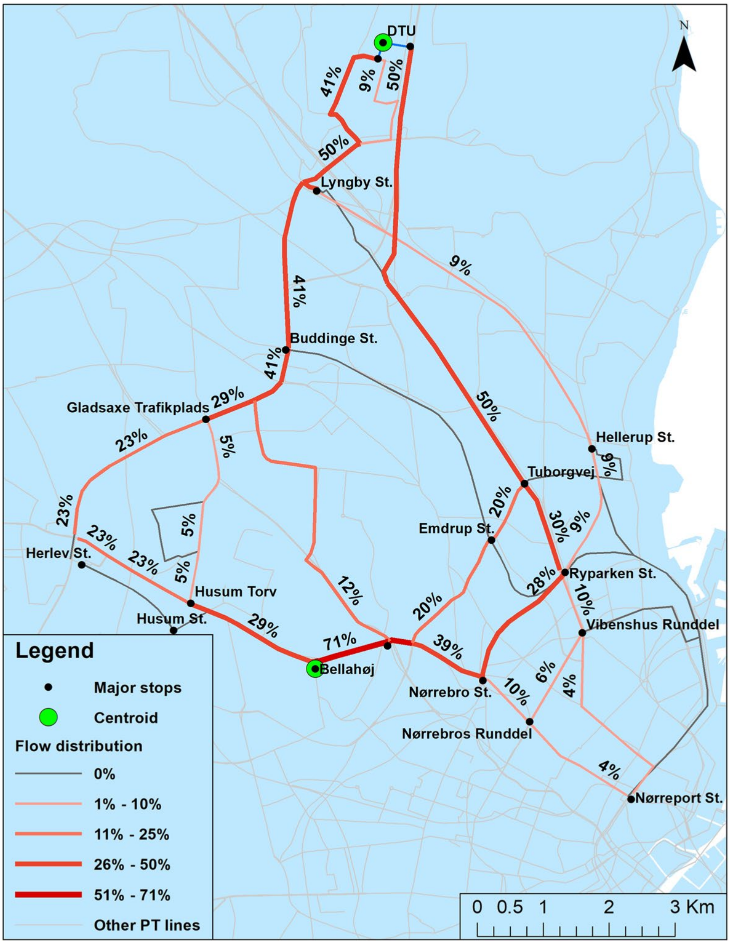
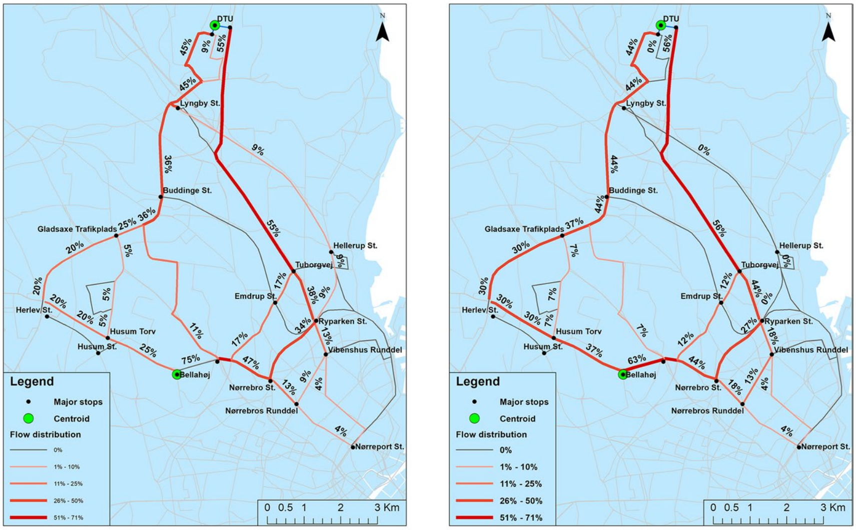


Fig. 5 Flow resulting from assignment on the unified network

Table 4 Route shares for paths with maximum 20% higher cost—departure 7:30 a.m. in the unified network (∗ indicates paths which are grouped by two candidate paths)

Path sequence	Mean arrival time	Utility	Probability
300S → 5C	08:19	− 10.50	23%
150S → 21 → 5C	08:15	− 10.65	20%
150S → F → 350S∗	08:15	− 10.71	19%
30E → 250S → 5C	08:19	− 11.15	12%
180 → B → F → 350S∗	08:15	− 11.44	9%
150S → 8A → 5C∗	08:21	− 11.85	6%
30E → 200S → 350S	08:23	− 11.96	5%
15E → 350S	08:30	− 12.17	4%
Log-sum			− 9.05



(a) Schedule-based **(b)** Frequency-based

Fig. 6 Flow allocation for strictly schedule-based (left) and frequency-based (right) scenarios

S-train which has in-vehicle time parameter around 20% lower than the parameter for bus. Other alternatives, for example, the alternative via Nørreport St. with only one transfer is assigned 4 % probability even though it is a large spatial detour, since travel speed of the included lines is high and the alternative only has one transfer. It is important to note that the number of generated alternatives and which alternatives that are generated are depending on the preferred departure time. So the 4 % assigned probability to the alternative via Nørreport St. can change, if another preferred departure time is considered. For the preferred departure time at 7.30 a.m. the average arrival time to the destination (weighed by the probability of taking each path) is 8:18 a.m. and the log-sum is − 9.05. The log-sum can be used to compare with the other examples on this network.

Using a threshold of 20% induces, in general, all relevant paths to be generated for the case example. It is, however, important to note, that no paths are 100% spatially overlapping. This means that the path of taking line 150S to Ryparken St. and going with the F line to Nørrebro St. and then taking line 350S is included, while the alternative where the trip would end with line 5C instead of line 350S is not included. Leaving out this latter alternative is probably not realistic, as it seems a relevant path; If a passenger doesn't catch the first departing line 350S (transfer from the frequency-based F line), the passenger would probably consider taking line 5C instead if it departs before the next bus on line 350S. This problem is further described in the example with frequency aggregation.

3.3 Strictly schedule- or frequency-based network representations

Two additional tests are performed to compare the results of the unified network to the results when using strictly schedule- or frequency-based representations of the network. Firstly, the network is transformed into a strictly schedule-based network, where the frequency-based lines are allocated explicit runs. Afterward, all lines in the network are only described by their frequencies. In Fig. 6, the flow allocation for both of the configurations is shown. For the schedule-based representation, the flow in the different corridors is only very slightly different than for the unified network. The flows differ more for the frequency-based assignment, which assigns more flow to the alternatives via Herlev and via Ryparken St. The log-sums for the two alternative configurations are on each side of the log-sum for the unified network, with a log-sum of − 8.90 for the strictly schedule-based network and − 9.76 for the strictly frequency-based network. The cost in the frequency-based network is thus around 8% higher than for the unified network, since the waiting times are now always half of the headway.

3.4 Change of some schedule-based services to frequency-based services

As a test of how the framework reacts to changes in the network, three lines are changed from schedule-based services to frequency-based services. These lines are 5C, 150S and 350S, which runs with, respectively, 4, 5 and 6 minutes headway. They are changed so that they run with the same frequency as in their schedule-based representation, but without explicit departure times. In contrast to the original network, it is now possible to have paths using only frequency-based services in the choice set. Also, the passengers do not have any option to arrive in Brønshøj using a schedule-based service, since line 5C and 350S are the only lines serving Brønshøj. The assignment on the modified network leads to slightly different paths in terms of the line level, and one new path (via Herlev St. and Husum St.) has replaced the path via Tuborgvej and Hulgårds Plads. The reason that the path via Tuborgvej and Hulgårds Plads using line 21 is excluded, is because the probability of catching the first candidate on line 21 from line 150S is 97% and the second candidate is not within the threshold of 20%. The shares between the paths are similar to the assignment in the unified network, with the probability of the alternative using line 21 spread across the other alternatives. The log-sum is − 9.35, and is thereby worse than the unified network, but better than the strictly frequency-based network configuration. The worse log-sum is primarily the result of some well-coordinated transfers not being as good as they were with schedule-based services. The difference in route shares between the existing network and the new more frequency-based network can be seen in Fig. 7.

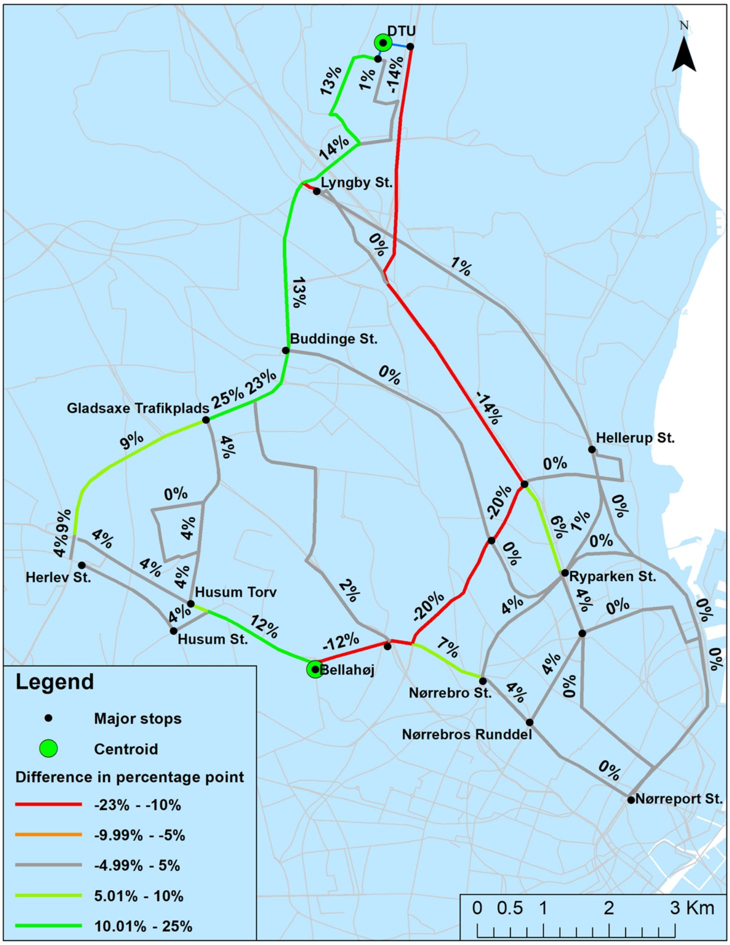


Fig. 7 Difference in flow allocation for scenario with more frequency-based lines (unified as base)

3.5 Frequency aggregation of network

In the existing (unified) network there exist a few corridors, where travelers could possibly perceive two parallel lines as the same and therefore just board the first departing line. This is the case for line 5C and 350S and for line 150S and 15E. Aggregating these and running the assignment results in the flow differences compared to the existing (unified) network shown in Fig. 8. The alternative with line 300S and line 5C via Herlev is the most attractive alternative as it is also the case in the existing network. The cost for the path is the same for the two examples, but the other paths in the frequency aggregated network have higher costs due to higher waiting times at transfers, and the choice probability

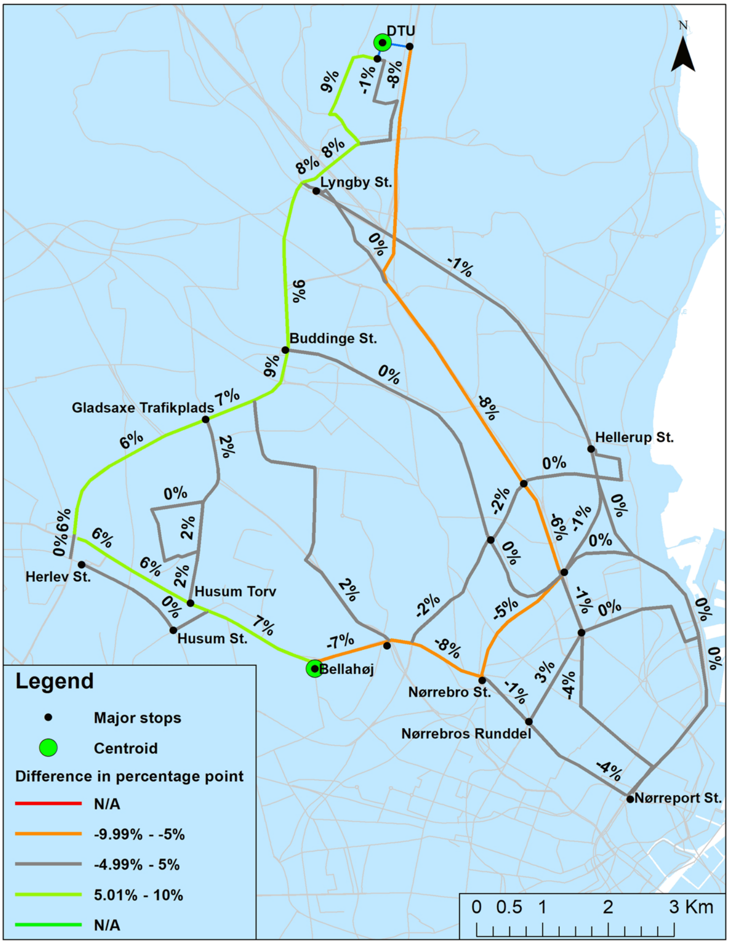


Fig. 8 Difference in flow allocation for frequency aggregated scenario (unified as base)

for the alternative via Herlev is, therefore, higher. Moreover, the alternative via Nørreport St. is not included in the choice set for the frequency aggregated network. This is because it has a cost which is 20.5% higher than the best alternative and it is, therefore, just outside the threshold. Overall, the flows are generally not different from the original flows, but the higher costs for alternatives going south at the beginning of the trip gives a slight change in the flow distribution. The log-sum is -9.26 and thereby closer to the result of the unified network than the frequency-based scenario.

Table 5 Overview of path probabilities and log-sum for each scenario

	Unified model	Schedule based	Frequency based	More frequency-based lines	Frequency aggregated scenario
DTU—Herlev—Brønshøj	23%	20%	30%	27%	29%
DTU—Tuborgvej—Hulgårds Plads—Brønshøj	20%	17%	12%	–	18%
DTU—Ryparken—Nørrebro—Brønshøj	19%	25%	27%	22%	15%
DTU—Budding—Bellahøj—Brønshøj	12%	11%	7%	14%	14%
DTU—Lyngby—Ryparken— Nørrebro— Brønshøj	9%	9%	–	10%	8%
DTU—Vibenshus Runddel— Nørrebros Runddel—Brønshøj	6%	9%	13%	10%	9%
DTU—Gladsaxe Trafikplads—Husum Torv—Brønshøj	5%	5%	7%	9%	7%
DTU—Nørreport—Brønshøj	4%	4%	5%	4%	–
DTU—Herlev—Husum St.—Brønshøj	–	–	–	4%	–
Log-sum	− 9.05	− 8.90	− 9.76	− 9.35	− 9.26
Relative change to unified network	–	+ 1.7%	− 7.8%	− 3.3%	− 2.3%

3.6 Comparison of overall cost of the different scenarios

In Table 5 an overview of the choice probabilities and log-sums for each scenario is shown. The lowest cost in terms of the log-sum is for the strictly schedule-based scenario, while the highest cost is for the strictly frequency-based scenario. This result is as expected as the schedule-based assignment can take the coordination between services into account, while the frequency-based assignment applies an average coordination between lines. The average cost of the unified network configuration lies between the strictly schedule-based and strictly frequency-based solutions. This seems reasonable, as passengers would have to include some buffer in the planning of their route when taking frequency-based services into account in their route choice. For the final two configurations, the costs are higher than for the unified network configuration, but significantly lower than the strictly frequency-based scenario. This indicates that a strictly (traditional) frequency-based assignment will estimate the level of service with higher costs than the other scenarios. When comparing the flow across alternatives, some alternatives perform better in some configurations, which is due to the coordination between lines when assuming the lines to be schedule-based.

4 Discussion

The proposed framework is able to generate a reasonable choice set and the flow assignment process results in a fair split across the alternatives and thereby handle the combination of both schedule- and frequency-based services in a network. The inclusion of both schedule- and frequency-based services in one model can, for example, allow for different arrival distributions to the first stop, which was shown in Ingvardson et al. (2018) to differ significantly between schedule- and frequency-based services, and thereby give a more behaviorally realistic assignment. The model can, however, be improved further and calibrated to better replicate and predict the flows in the network. This section presents some of the possible improvements that can be made.

Firstly, the model assumes no delays on any of the services, and hence is not able to capture how reliability affects passengers' route choice. However, with the current choice set generation algorithm, it is possible to represent delays as it allows for headway distributions being higher than the deterministic headway of a line. But, it is not possible to assume, for example, exponential headway, as the unbounded tail of the distribution would imply that there will then always be a risk, that the passenger will not arrive in the destination inside the threshold. In a normal performing network with no major disruptions, the passenger will not wait forever though, and the restriction to bounded distributions, therefore, seems behaviorally realistic. In this paper, binomial distributions were used to describe the headway of frequency-based services for the sake of being able to have an analytic solution to the choice set generation. If different services had different types of headway distributions such as beta, Johnson or uniform distributions, the computation of the probability of boarding the first candidate, when transferring from a frequency-based service

Table 6 Waiting times for transfers between schedule- and frequency-based services with some schedule-based services modeled as frequency-based services

Transfer from/to	(A) SB line	(B) SB line simplified to FB	(C) FB line
(A) SB line	The waiting time is deterministic	Waiting time is deterministic in “real life”, but must be assumed to follow a statistical distribution	Waiting time follows statistical distribution
(B) SB line simplified to FB	There is a probability to catch the line and for this a statistical distribution for waiting time	Waiting time is deterministic in “real life”, but must be assumed to follow a statistical distribution	Waiting time follows statistical distribution
(C) FB line	There is a probability to catch the line and for this a statistical distribution for waiting time	Waiting time follows statistical distribution	Waiting time follows statistical distribution

to a schedule-based service, would require numerical integration of the probability. However, if the model is only evaluated at a discrete-time resolution, the numerical integration will not be computationally heavy.

Secondly, the framework is flexible in terms of how different lines can be modeled. Table 6 gives an overview of how different transfers could be described, if some schedule-based lines are simplified to frequency-based lines. This could, for example, be useful for strategic planning of public transport networks, as it allows the modeler to simplify some lines and thereby reduce the need to check if all schedule-based lines correspond well at transfer points.

Thirdly, the framework in its current form does not allow modeling of in-vehicle congestion, denied boarding or seat availability. The introduction of these factors requires some sort of iterative scheme, where the capacity of runs and services is updated and new choice sets are generated in each iteration. The choice sets in each iteration would then depend on seat availability, in-vehicle congestion and if it is possible to board a service, and the iterative scheme would need some converge criteria to ensure that the model stops when it is converged.

Fourthly, the choice of a multinomial logit model governs the independence of irrelevant alternatives property. It thereby disregards previous findings showing that passengers tend to value overlapping lines, as it gives more possibilities in case of delays in the network (Anderson et al. 2014). In the current framework, it is possible to calculate a path-size factor or another measure of route correlation that could adjust the choice probabilities of overlapping lines[2] and thereby, possibly, give higher behavioral realism. The fit of the model should be tested against observations of real flows in the network.

[2] See Prato (2009) for an overview of different discrete choice models which adjust choice probabilities based on route correlation

Finally, the assumption that the threshold is strictly a percentage of the best alternative should be investigated further. In the case described in Sect. 3.4, one of the alternatives is disregarded in the choice set, because there is a 3% probability that the passenger will not arrive within the given threshold. The prospect theory developed by Kahneman and Tversky (1979), however, emphasizes that decision makers over-weights the smaller probabilities, which would indicate, that it is reasonable to have a strict cut-off for alternatives not being within the threshold. Another point related to the threshold is the challenge inherent in estimating the threshold to fit the behavior of travelers best possible. Watling et al. (2018) develop an assignment model based on a bounded choice model for car networks, which consistently facilitates a strict cut-off of flow allocation at a certain bound to the optimal route. They motivate their approach (partly) by empirical evidence from car users showing that 95% of all observed routes are within 20% of the cost of the optimal route. A similar analysis could also be made for public transport trips, to reveal a reasonable threshold to use in the algorithm, for example, based on data from automatic fare collection systems.

5 Conclusion

This paper has presented a novel framework for modeling the passenger assignment problem in public transport networks with co-existing schedule- and frequency-based services. The framework first generates a choice set based on a variant of the event dominance principle, where also suboptimal paths are allowed in the choice set. The different paths can be combined from different candidates, if transfers are made from a frequency-based to a schedule-based service and the first run of the schedule-based line can not be reached with certainty. A threshold controls that not all possible paths are included in the choice set, but that only reasonable routes are included. A flow allocation using a multinomial logit model distributes the flow across the different alternatives and does so without taking the overlap of different alternatives into account.

The methodology proposed was applied to different configurations of a case study network covering a part of the Greater Copenhagen Area. These tests showed that the level of service in the unified network configuration lies in between network configurations with strictly schedule- or frequency-based services, with the strictly frequency-based network representation having the worst level of service due to higher waiting times. The choice sets for all scenarios seem reasonable, and with the frequency aggregated network configuration the resulting level of service is close to the level of service in the unified network configuration.

Future research should focus on the application of the model to large-scale networks including the calibration of the assignment parameters and comparisons of the generated choice sets to observed routes from, e.g., smart card data.

Appendix 1: line frequencies in unified network

See Table 7.

Table 7 Timetable information

Service types:
1. Bus
2. S-train

Line	Service type	FB/SB	Headway
300S	1	SB	12
30E	1	SB	10
180	1	SB	5
150S	1	SB	5
15E	1	SB	10
184	1	SB	20
161	1	SB	30
350S	1	SB	5
5C	1	SB	4
B	2	SB	10
E	2	SB	10
F	2	FB	5
A	2	SB	10
Bx	2	SB	20
21	1	SB	10
8A	1	FB	6
C	2	SB	10
H	2	SB	20
250S	1	SB	10
200S	1	SB	10
166	1	SB	20

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3 Paper 2: Conditional passenger travel time distributions in mixed schedule- and frequency-based public transport networks using Markov chains

The following pages contain the article:

C. B. Gardner, S. D. Nielsen, M. Eltved, T. K. Rasmussen, O. A. Nielsen, and B. F. Nielsen (2020). "Conditional passenger travel time distributions in mixed schedule- and frequency-based public transport networks using Markov chains". Under review at Transportation Research Part B: Methodological.

Please cite accordingly.

The paper builds upon the bachelor thesis by Clara Brimnes Gardner and Sara Dorthea Nielsen, *Probabilistic Route Choice Models*, written at the Technical University of Denmark and co-advised by the PhD student.

Conditional passenger travel time distributions in mixed schedule- and frequency-based public transport networks using Markov chains

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Abstract

Calculation of passenger travel times in public transport networks is important for the evaluation of the level of service provided to passengers. The passenger travel times are deterministic for punctual and uncongested networks, but have random fluctuations when including delays and crowding. Hence, advanced methods are needed to calculate the passenger travel time distribution between a given origin and destination. This paper presents a novel approach for calculating the travel time distribution from origin to destination based on vehicle delays and possible missed connections in a mixed schedule- and frequency-based public transport network. Markov chains are used to model the network, making the travel time from the origin to the destination phase-type distributed. The approach is flexible with regards to the specification of vehicle travel times and provides the distribution of passenger travel times without any need for simulation. Additionally, it facilitates detailed analyses of passenger travel times conditional on the usage of specific line segments or stops. The merits are demonstrated using a real-life case study from Copenhagen.

Keywords: Markov chains, phase-type distributions, public transport, travel time distribution

1. Introduction

Modelling passenger travel times in urban public transport systems is not straightforward. Typically, the network structure is complex with multiple overlapping lines and a mix of frequency-based (FB) and schedule-based (SB) services. Furthermore, delays occur on a daily basis and it is essential to capture realistically the impacts of network delays on realised passenger travel times, including their distribution.

Estimation of the level of service provided to passengers in a public transport network has been dealt with using two main representations of the network, i.e. frequency-based or schedule-based (Liu et al., 2010). In the early stage of public transport assignment models, the evaluation of level-of-service for passengers was based on service frequency (headway) in the network using frequency-based assignment models (Spiess and Florian, 1989; Nguyen and Pallottino, 1988; Nielsen, 2000). The frequency-based models entail assumptions on the passenger route choice strategy, allowing selection from a subset of the lines in the network called the attractive line set. When the network is based on frequencies, it is not possible to calculate detailed waiting times at transfers between specific vehicle

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Preprint submitted to Transportation Research Part B *July 24, 2020*

runs in the network. However, the concept of hyperpaths introduced in Spiess and Florian (1989) and Nguyen and Pallottino (1988) allowed for day-to-day supply variations, with more accurate travel times obtained by allowing passengers to choose different travel strategies (Nökel and Wekeck, 2009; Schmöcker et al., 2013; Oliker and Bekhor, 2018).

With the public transport network by nature being time-dependent and based on specific runs of a service, much research has also been focused on exploring the travel times in network representations, where each run on a line has specific arrival and departure times at stops. These models, usually known as either schedule-based or run-based models, allow for modelling detailed trajectories through the network. Early models focused on fixed networks with no delays for vehicles (Tong and Richardson, 1984). Later, delays and supply variations were included by specifying day-to-day dynamics and supply variations (Nuzzolo et al., 2001; Tong and Wong, 1999; Landex and Nielsen, 2006; Nielsen et al., 2008; Hamdouch et al., 2014). Recent works on unreliable schedule-based networks include the work by Khani (2019) who proposed an online shortest path algorithm to find the optimal assignment of passengers and showed that passengers were primarily assigned to the most reliable paths.

In the past, bridging the gap between frequency- and schedule-based models has been discussed to allow a more realistic representation of the actual network structure in urban regions, where some services run with a given headway while other low-frequency services have a fixed published schedule (Gentile and Noekel, 2016, chap. 6). Recently, Eltved et al. (2019) proposed a method attempting to capture the effects of mixed schedule- and frequency-based networks on passenger route choice, since it has been shown that passengers have varying preferences for waiting to board a schedule- or frequency-based service (Eltved et al., 2018). Moreover, a framework was proposed for finding the flow between origin and destination based on a choice set generation step using a modified version of the event-dominance principle proposed in Florian (1999) and a subsequent assignment of flows to the choice set using a discrete choice model. However, Eltved et al. (2019) do not consider effects of stochastic vehicle travel times originating from e.g. delays in the road network or higher dwell times due to demand variations, both of which are considered in this paper.

A relatively small branch of research has focused on the use of Markov chains and Markov decision processes for modelling travel times in public transport networks. Teklu et al. (2007) use Markov chains to model the day-to-day variation in the costs of each link. Costs are observed at the end of each day, and a learning filter is applied to the observations to determine the costs on the following day. This method does not use the Markov chains directly in the modelling of passenger route choices, but as a method of updating the network costs.

For network loading, Markov chains have mostly been used to model congested networks with a high failure-to-board probability. In Bell et al. (2002), a discrete Markov chain is used, where each stop is represented by multiple nodes, one for each service leaving the stop. Links between the stops are represented with transition-probabilities that specify the probability of moving from one stop to another. The possibility of not boarding a service is modelled by letting the row-sum be less than one, implying a possibility of exiting the network without being allocated to another state. This idea of modelling failure-to-board by means of Markov chains is considered further in Kurauchi et al. (2003) and Schmöcker et al. (2008). In Kurauchi et al. (2003), failure nodes and failure links are introduced to represent the probability of not boarding a service. Passengers not boarding a service due to capacity limits are sent to their destination by the failure links. In Schmöcker et al. (2008) a time interval is included, allowing variation of the congestion over time. Furthermore, passengers who have not reached the destination can be reassigned in the next interval allowing them to reach the destination. This includes passengers failing to board a vehicle.

A related topic regarding the application of Markov chains in public transport modelling is the use of a Markov Decision Problem (MDP) for modelling adaptive route choice by passengers. Rambha et al. (2016) introduce the concept of MDP's for adaptive route choice modelling, focusing on how to reduce the dimensionality of finding the optimal route choice by using pre-processing steps. Nuzzolo and Comi (2019) build upon the work in Rambha et al. (2016) and present a run-based strategy model which takes into account the real-time information that can be given to the passengers, and further reduces the state space by only considering the state space for the traveller decisions and not for the buses.

In this paper, we propose a model based on the use of Markov chains. We (i) use a combined schedule- and frequency-based public transport network, (ii) use phase-type distributions for flexible modelling of the vehicle travel times and (iii) consistently calculate the travel time distributions conditional on the usage a specific lines in the set of attractive lines between origin and destination. The idea is to have a flexible approach to the specification of vehicle travel times and to provide the passenger travel time distributions without a need for simulation. We define the state space of the Markov chain such that states not only represent stops, but also vehicles. Furthermore, we include a time dimension in the states, making it possible for the model to handle both schedule-based and frequency-based services. Vehicle delays can be represented by the model, which makes it possible to calculate the travel times for a specific route of the passenger by taking in to account possible missed connections at transfers. The time to absorption in such a Markov chain is said to follow a discrete phase-type distribution. Phase-type distributions are well-known as a tool in other fields for modelling different phenomena. This class of distributions has many benefits such as flexibility and closure under a number of operations such as addition. Applications include the risk theory in Bladt (2005) and the health-care systems in Fackrell (2007). The model outputs the route choice probabilities and distributions for the travel times, which are also random due to the inclusion of random delays. Furthermore, the model can be used to find conditional travel time distributions based on the stops used in a route.

The paper is organized as follows: The model is described in Section 2 and applied to an example network in Section 3. In Section 4 formulas for extracting results from the model are presented, and Section 5 presents the results from the example network. Finally, the perspectives of using Markov chains for modelling travel times in mixed schedule- and frequency-based networks are provided in Section 6, and Section 7 concludes the paper.

2. Traffic assignment by a discrete Markov chain model

Before introducing its application to public transport modelling, we briefly introduce some of the important properties of discrete Markov chains. A discrete-time and integer-valued stochastic process, $X = \{X_n; n \in \mathbb{N}\}$, is a countable collection of random variables that take integer values. A discrete Markov chain is a stochastic process which satisfies the Markov property (Pinsky and Karlin, 2011, p. 79)

$$\Pr(X_{n+1} = j \mid X_0 = i_0, \ldots, X_{n-1} = i_{n-1}, X_n = i) = \Pr(X_{n+1} = j \mid X_n = i). \tag{1}$$

If the probabilities in (1) do not depend on n such that $\Pr(X_{n+1} = j \mid X_n = i) = \Pr(X_1 = j \mid X_0 = i)$, the Markov chain is said to be time-homogeneous, then $\Pr(X_{n+1} = j \mid X_n = i) = p_{ij}$. Let N be the dimension of the state space, and let $\boldsymbol{\alpha}_0$ be a row vector of probabilities of dimension N describing the initial distribution of the stochastic process. A finite time homogeneous Markov chain can then be described by the transition matrix $\mathbf{P} = \{p_{ij}\}$ and $\boldsymbol{\alpha}_0$.

In the following subsections, a discrete time-homogeneous Markov chain model for journeys is presented. The Markov chain models the movement of a single passenger through the network. First a suitable state space is defined, after which the transition probabilities, p_{ij}, in the Markov chain are described.

2.1. State space

The state space consists of a number of transient states representing the journey, and one absorbing state representing the destination, D. In this, the state space of the transient states, $E_{\text{transient}}$, is described.

The description of a state is multivariate by nature as both time and location are needed. The scheduled services induce a bound t_{max} on the range of possible values of the time variable, t. In our example, the time is measured in minutes making $t_{max} = 60$ a suitable bound. We denote the set of possible time values by

$$T = \{1, \ldots t_{max}\}.$$

Let S be the set of services for the trip. Let S_F be the frequency-based services and let S_S be the schedule-based services. Then

$$S = S_F \cup S_S.$$

Furthermore let W denote the set of stops for the trip. The set of possible locations for the passenger, L, during the trip is then

$$L = W \cup S.$$

The location of the modelled passenger is hereafter described by the location parameter $l \in L$.

To allow the time spent at each location to follow various probability distributions – for example negative binomial – a third auxiliary parameter, phase, is introduced. The phase parameter, r, is used to divide the locations into different segments. The space of the phase parameter is location-specific, and it is therefore denoted $R_k = \{1, \ldots, r_k\}$.

Combining these parameters the transient state space, $E_{transient}$, becomes

$$E_{transient} = \bigcup_{k \in L} T \times R_k.$$

A transient state in the transition matrix is classified by three indexes. Thus (k, i, t) refer to the state where $L_n = k$, $R_n = i$, $T_n = t$. The use of both location and phase in the notation can be considered redundant, as each phase belongs to a location, but both parameters are included to ease readability.

The full state space of the Markov chain, X, becomes

$$E = E_{transient} \cup D$$

It is beneficial to arrange the state space in lexicographical order, such that the index of the time parameter is changed first, then the index of the phase parameter, and finally the index of the location.

2.2. Transitions between transient States

We will now define the transient part of the transition matrix for X. The one-step transitions on the defined state space can be written in the general form

$$\Pr\left(L_{n+1} = l, R_{n+1} = j, T_{n+1} = u \mid L_n = k, R_n = i, T_n = t\right) = p_{(k,i,t),(l,j,u)}, \tag{2}$$

where $\sum_{lju \in E} p_{(k,i,t),(l,j,u)} = 1$ and $p_{(k,i,t),(l,j,u)} \geq 0$.

A number of restrictions are imposed on (2). Transitions between the majority of the states are not possible, resulting in a sparse transition matrix. As every transition corresponds to one time unit (minutes in our example) being spent, $p_{(k,i,t),(l,j,u)}$ is only non-zero when $u = t+1 \mod t_{max}$. This means that $\mathbf{P}$ can be described as $\sum_{l \in L} r_l$ sub-matrices, $\mathbf{M}_{(k,i),(l,j)}$, with dimension $t_{max} \times t_{max}$, and with non-zero elements only in the superdiagonal and in the lower left corner.

$$\mathbf{M}_{(k,i),(l,j)} = \begin{bmatrix} 0 & p_{(k,i,1),(l,j,2)} & 0 & 0 & \cdots & 0 \\ 0 & 0 & p_{(k,i,2),(l,j,3)} & 0 & \cdots & 0 \\ 0 & 0 & 0 & p_{(k,i,3),(l,j,4)} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & p_{(k,i,t_{max}-1),(l,j,t_{max})} \\ p_{(k,i,t_{max}),(l,j,1)} & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

In the following, the non-zero transitions are described in terms of these sub-matrices.

Transitions between states representing the same location $L_n = L_{n+1} = k$, are collected in a larger sub-matrix denoted $\mathbf{T}_{kk}$. Let r_k be the number of phases associated with location k. Then the dimension of $\mathbf{T}_{kk}$ is $(r_k \cdot t_{max}) \times$

$(r_k \cdot t_{max})$ and in terms of the sub-matrices, $\mathbf{M}_{(k,i),(l,j)}$, it can be written as

$$\mathbf{T}_{kk} = \begin{bmatrix} \mathbf{M}_{(k,1),(k,1)} & \mathbf{M}_{(k,1),(k,2)} & \cdots & \mathbf{M}_{(k,1)(k,r_k)} \\ \mathbf{M}_{(k,2),(k,1)} & \mathbf{M}_{(k,2),(k,2)} & \cdots & \mathbf{M}_{(k,2),(k,r_k)} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{M}_{(k,r_k),(k,1)} & \mathbf{M}_{(k,r_k),(k,2)} & \cdots & \mathbf{M}_{(k,r_k),(k,r_k)} \end{bmatrix}.$$

No further restrictions are imposed on transition probabilities between states representing the same location.

Transitions between states representing different locations, $L_n = k$, $L_{n+1} = l$ are collected in a sub-matrix denoted $\mathbf{T}_{kl}$. Let r_l be the number of states associated with location l. The dimension of $\mathbf{T}_{kl}$ is then $(r_k \cdot t_{max}) \times (r_l \cdot t_{max})$. In terms of the submatrices, $\mathbf{M}_{(k,i),(l,j)}$ it can be expressed as

$$\mathbf{T}_{kl} = \begin{bmatrix} \mathbf{M}_{(k,1),(l,1)} & \mathbf{M}_{(k,1),(l,2)} & \cdots & \mathbf{M}_{(k,1),(l,r_l)} \\ \mathbf{M}_{(k,2),(l,1)} & \mathbf{M}_{(k,2),(l,2)} & \cdots & \mathbf{M}_{(k,2),(l,r_l)} \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{M}_{(k,r_k),(l,1)} & \mathbf{M}_{(k,r_k),(l,2)} & \cdots & \mathbf{M}_{(k,r_k),(l,r_l)} \end{bmatrix}.$$

The matrix $\mathbf{T}_{kl}$ can only contain non-zero elements in cases where the location l follows immediately after the location k. If $k \in S$, l follows immediately after if and only if l represents the arrival stop of k. If on the other hand $k \in W$, l follows immediately after if and only if l is a service departing from k. Furthermore, if $k \in W$ and $l \in S_S$ is a service departing from k, only elements representing the times where l is scheduled to depart from k can be non-zero. Let T_l^* be the set of departure times of the schedule-based service. Then the elements of $\mathbf{M}_{(k,i),(l,j)}$, $p_{(k,i,t),(l,j,t+1)}$ are only non-zero for $t \in T_l^*$.

To satisfy $\sum_{lju \in E} p_{(k,i,t),(l,j,u)} = 1$, $\mathbf{T}_{kk}$ and the (possibly multiple) $\mathbf{T}_{kl}$ have to be chosen such that the row sum is one

$$\sum_{l \in L} \mathbf{T}_{kl}\mathbf{e} = \mathbf{e} \tag{3}$$

where $\mathbf{e}$ is a column vector of appropriate dimension with all entries equal to one.

2.3. Destination

The destination is treated as one state. The transitions to the destination follow the restrictions described in Section 2.2 replacing the state (l, j, u) with D. Transitions from the destination are not possible.

2.4. Initial distribution

To properly describe the Markov model, an initial distribution, $\boldsymbol{\alpha}$,should be defined. The initial distribution should satisfy

$$\boldsymbol{\alpha}\mathbf{e} = 1, \quad \alpha_i \geq 0.$$

The entries in the initial distribution are equal to the probability of the passenger starting in the corresponding state. Therefore the entries of $\boldsymbol{\alpha}$ should only be non-negative for states representing the starting location(s) of the passenger.

3. Example network

In this section, the model proposed in Section 2 is applied to the example network shown in Figure 1. A schematic version is shown in Figure 2. The network models a trip from DTU Lyngby Campus (abbreviation: DTU) to Copenhagen Airport (abbreviation: CPH). The network pictured in Figures 1 and 2 should be considered as the set of attractive lines. The attributes of the six lines considered are shown in Table 1.

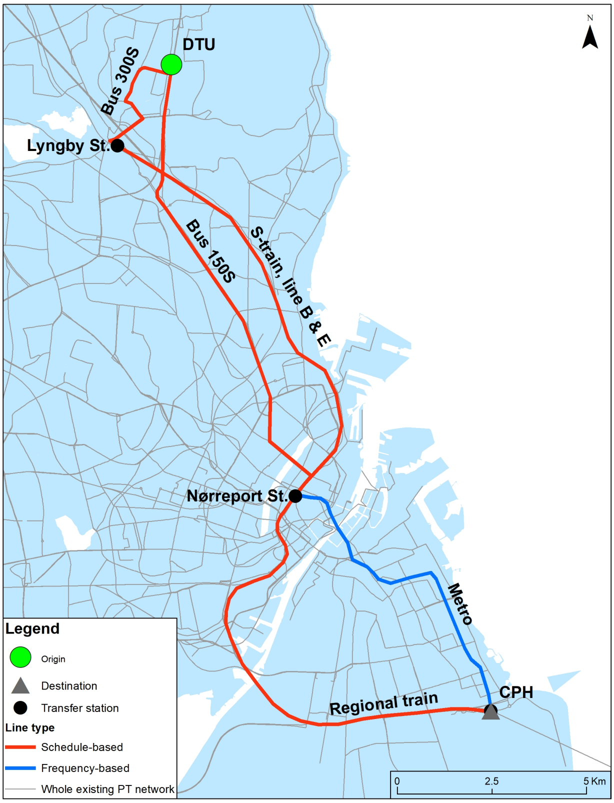


Figure 1: *The example network shown on a map of the Copenhagen area. Figure based on map in Eltved et al. (2019).*

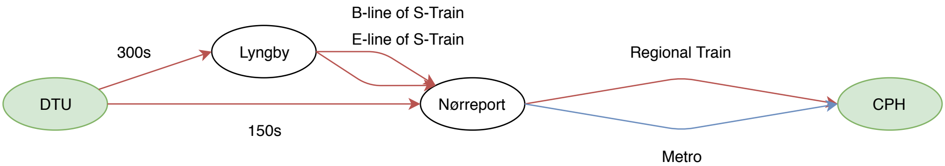


Figure 2: *Schematic version of the example network shown in Figure 1.*

Name	**Type**	**Departure Stop**	**Arrival Stop**	**Departure Times**	**Average Headway**	**Abbreviation**
Bus 150S	SB	DTU	Nørreport St.	04, 14, 24, 34, 44, 54	-	150S
Bus 300S	SB	DTU	Lyngby St.	02, 22, 42	-	300S
S-Train, line E	SB	Lyngby St.	Nørreport St.	00, 10, 20, 30, 40, 50	-	E
S-Train, line B	SB	Lyngby St.	Nørreport St.	05, 15, 25, 35, 45, 55	-	B
Regional train	SB	Nørreport St.	CPH	00, 20, 40	-	R
Metro	FB	Nørreport St.	CPH	-	6	M

Table 1: *Attributes for the set of attractive lines.*

The transition matrix, $\mathbf{P}$, then takes the following form. Notice that D, L, N and C are used as abbreviations for DTU, Lyngby St., Nørreport St. and CPH, respectively.

$$\mathbf{P} = \begin{bmatrix} \mathbf{T}_{\mathrm{D,D}} & \mathbf{T}_{\mathrm{D,150S}} & \mathbf{T}_{\mathrm{D,300S}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 \\ 0 & \mathbf{T}_{\mathrm{150S,150S}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_{\mathrm{150S,N}} & \mathbf{0} & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{T}_{\mathrm{300S,300S}} & \mathbf{T}_{\mathrm{300S,L}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_{\mathrm{L,L}} & \mathbf{T}_{\mathrm{L,B}} & \mathbf{T}_{\mathrm{L,E}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_{\mathrm{B,B}} & \mathbf{0} & \mathbf{T}_{\mathrm{B,N}} & \mathbf{0} & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_{\mathrm{E,E}} & \mathbf{T}_{\mathrm{E,N}} & \mathbf{0} & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_{\mathrm{N,N}} & \mathbf{T}_{\mathrm{N,M}} & \mathbf{T}_{\mathrm{N,R}} & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_{\mathrm{M,M}} & \mathbf{0} & \mathbf{T}_{\mathrm{M,C}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}_{\mathrm{R,R}} & \mathbf{T}_{\mathrm{R,C}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

3.1. Modelling the service times and waiting times

First we consider transitions from stops to departing schedule-based services – i.e. $\mathbf{T}_{\mathrm{D,150S}}$, $\mathbf{T}_{\mathrm{D,300S}}$, $\mathbf{T}_{\mathrm{L,B}}$, $\mathbf{T}_{\mathrm{L,E}}$ and $\mathbf{T}_{\mathrm{N,R}}$. In the example network, the transitions from stops to a departing schedule-based service are deterministic. Only one phase is used to model stops where only schedule-based services depart and, at times where the schedule-based services depart, the probability of transitioning to the first phase of the next location is 1. This corresponds to an assumption of the passenger taking the first available service. Therefore, when l is a schedule-based service departing from k, $\mathbf{T}_{kl}$ takes the form

$$\mathbf{T}_{kl} = \begin{bmatrix} \mathbf{M}_{(k,1),(l,1)} & 0 & 0 & \cdots & 0 \end{bmatrix},$$

with

$$p_{(k,1,t)(l,1,t+1)} = \begin{cases} 0 \text{ if } t \notin T_l^* \\ 1 \text{ if } t \in T_l^* \end{cases}. \tag{5}$$

This choice corresponds to the initial distribution $\boldsymbol{\alpha}_l$ being concentrated on the first phase, and thus deterministic. As only one phase is used to model the stop k,

$$\mathbf{T}_{kk} = \mathbf{M}_{(k,1),(k,1)}.$$

The entries of $\mathbf{T}_{kk}$ are found from Equation (3).

The service times in the example network are random, following phase-type distributions with representations $(\boldsymbol{\alpha}_k, \mathbf{T}_{kk})$, where $\mathbf{T}_{kk}$ is chosen such that the runtime of service k is distributed according to the negative binomial distribution. In terms of the sub-matrices, $\mathbf{M}_{(k,i),(k,j)}$ described in Section 2.2 this means that all the non-zero entries in $\mathbf{M}_{(k,i),(k,i)}$ will equal $1 - q_k$ and all the non-zero entries in $\mathbf{M}_{(k,i),(k,i+1)}$ will equal q_k. The rest of the sub-matrices in $\mathbf{T}_{kk}$ will be zero-matrices. Thus

$$\mathbf{T}_{kk} = \begin{bmatrix} \mathbf{M}_{(k,1)(k,1)} & \mathbf{M}_{(k,1)(k,2)} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{(k,2)(k,2)} & \mathbf{M}_{(k,2)(k,3)} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_{(k,3),(k,3)} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{M}_{(k,r_k-1)(k,r_k-1)} & \mathbf{M}_{(k,r_k-1)(k,r_k)} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{M}_{(k,r_k),(k,r_k)} \end{bmatrix}$$

The number of phases, r_k, and the value of q_k are location dependent as specified in Table 2. The matrix $\mathbf{T}_{kl}$ is only non-zero when l is the arrival stop of k. In agreement with Equation (3) it takes the following form

$$\mathbf{T}_{kl} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{M}_{(k,r_k),(l,1)} & \mathbf{M}_{(k,r_k),(l,2)} & \cdots & \mathbf{M}_{(k,r_k),(l,r_l)} \end{bmatrix}.$$

The non-zero entries of $\mathbf{M}_{(k,r_k),(l,j)}$ are set equal to $\frac{q_k}{r_l}$, which corresponds to the initial distribution for l, $\boldsymbol{\alpha}_l$ being uniform over the phases.

The waiting time distribution for the frequency-based service Metro is chosen to be negative binomial. Notice that the choice of $q_k = 1$ results in deterministic transitions to the next state. However, as the schedule-based Regional train service also departs from the same stop, Nørreport St., modifications to $\mathbf{T}_{\mathrm{N}}$, $\mathbf{T}_{\mathrm{N,R}}$ and $\mathbf{T}_{\mathrm{N,M}}$ have to be made such that Equation (3) is still satisfied. Modifications are only needed for $t \in T_R^*$. For $i < r_N$ transitions are either to another state representing Nørreport St., or to the Regional train, and the latter is preferred.

$$
\begin{aligned}
p_{(\mathrm{N},i,t),(\mathrm{N},j,t+1)} &= 0 \text{ if } t \in T_R^*,\ i < r_N, \\
p_{(\mathrm{N},i,t),(\mathrm{R},1,t+1)} &= 1 \text{ if } t \in T_R^*,\ i < r_N.
\end{aligned}
$$

For $i = r_N$ transitions to another state representing Nørreport St., to the Regional train and to the Metro are possible, and the two latter are preferred.

$$
\begin{aligned}
p_{(\mathrm{N},r_N,t),(\mathrm{N},r_N,t+1)} &= 0 && \text{if } t \in t_R^*, \\
p_{(\mathrm{N},r_N,t),(\mathrm{M},1,t+1)} &= \frac{1}{2} q_N && \text{if } t \in t_R^*, \\
p_{(\mathrm{N},r_N,t),(\mathrm{R},1,t+1)} &= 1 - \frac{1}{2} q_N && \text{if } t \in t_R^*.
\end{aligned}
$$

Service Name	$\mathbf{r}_k$	$\mathbf{q_k}$	**Initial Distribution**	**Average Time Spent**
Runtime of Bus 150S	24	0.9	First state	26.7
Runtime of Bus 300S	9	0.8	First state	11.3
Runtime of S-train, line E	15	0.875	First state	17.1
Runtime of S-train, line B	18	0.875	First state	20.6
Runtime of Regional train	18	0.875	First state	20.6
Waiting time for Metro	6	1	Uniform	3.5
Runtime of Metro	15	0.85	First state	17.6

Table 2: *Specifications for the distributions of the different runtimes of services and for the waiting time for the Metro, the number of phases, r_k, used to model the location, and the probability of moving to the next phase, q_k. The initial distribution is the initial distribution over the phases of the location.*

3.2. Modelling the initial distribution

The initial (arrival) distribution for the model, $\boldsymbol{\alpha}$, specifies the starting time and location for the modelled passenger. In the example network the passenger always begins the journey at the DTU stop, and thus $\boldsymbol{\alpha}$ is only non-zero at states representing this stop. The initial distribution therefore models the arrival time of the passenger at the DTU stop. Possible strategies for the arrival of the passenger might be:

- Random arrival, not taking timetable into account
- Arrival to minimize waiting time for 150S only

- Arrival to minimize waiting time for 300S only

As suggested in Ingvardson et al. (2018), a uniform distribution is used to model the random arrival times, and a beta distribution is used to model the timed arrival times. The headway (Table 1) for 150S is 10 minutes, and the headway for 300S is 20 minutes. Using the shape parameters in Ingvardson et al. (2018), the shape parameters for the normalized waiting times (in terms of headway) are defined as

$$a_{150S} = 0.36 \qquad b_{150S} = 3.39$$
$$a_{300S} = 0.27 \qquad b_{300S} = 4.57$$

By discretizing the beta-distribution and changing back to the minute-domain, two initial distributions are obtained. Figure 3 shows the probability density functions (pdf's).

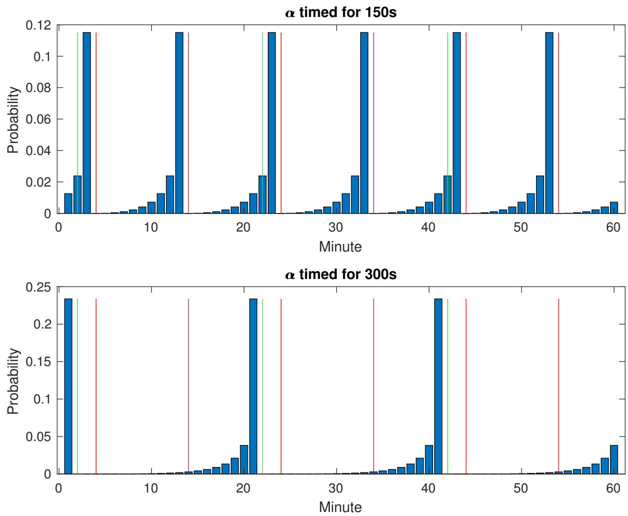


Figure 3: *The two initial distributions of arrivals timed for 150S and 300S. The red lines indicate an arrival of 150S and the green lines indicate an arrival of 300S.*

4. Analysis of the travel time distribution

In this section formulas for the distribution and expectation of the travel time are derived using phase-type distributions. Hereafter it is described how to invoke conditioning in phase-type distributions, which allows us to extract properties of conditional travel times.

4.1. Discrete phase-type distributions

As described in Section 2, the Markov chain, $X = \{X_n; n \in \mathbb{N}\}$, consists of a number of transient states and one absorbing state. The transient state space consists of states where the probability of ever returning to that state is less than one. Once entered, the absorbing state cannot be left. The travel time, τ, then equals the time until absorption in the Markov chain, which follows a discrete phase-type distribution (Bladt and Nielsen, 2017, p. 29).

Let X be defined on $r+1$ states. Assume for simplicity that the states $\{1, \ldots, r\}$ are transient, while state $r+1$ is the absorbing destination state. The transition matrix can then be written as:

$$\mathbf{P}_X = \begin{bmatrix} \mathbf{T} & \mathbf{t} \\ \mathbf{0} & 1 \end{bmatrix}, \tag{6}$$

where $\mathbf{T}$ is an $r \times r$ dimensional subtransition matrix with transitions between the transient states, and $\mathbf{t}$ is an r-dimensional column vector containing the absorption probabilities. Let $\boldsymbol{\alpha}$ be the r-dimensional vector specifying the initial distribution of X on the transient states. Then τ is discrete phase-type distributed with parameters $\boldsymbol{\alpha}$ and $\mathbf{T}$, $\tau \sim PH(\boldsymbol{\alpha}, \mathbf{T})$. The probability density function, the cumulative distributive function and the expectation are given as (Bladt and Nielsen, 2017, p. 30):

$$f_\tau(n) = \boldsymbol{\alpha}\mathbf{T}^{n-1}\mathbf{t}, \tag{7}$$

$$F_\tau(n) = 1 - \boldsymbol{\alpha}\mathbf{T}^{n}\mathbf{e}, \tag{8}$$

$$\mathbb{E}(\tau) = \boldsymbol{\alpha}\left(\mathbf{I} - \mathbf{T}\right)^{-1}\mathbf{e} = \boldsymbol{\alpha}\mathbf{U}\mathbf{e} = \mu. \tag{9}$$

The matrix $\mathbf{U} = (\mathbf{I} - \mathbf{T})^{-1} = \{u_{ij}\}$ is called the green matrix, and $\{u_{ij}\}$ is the expected time spent in state j given that the process starts in state i.

4.2. Conditioning in discrete phase-type distributions

Conditional travel times – for instance travel times depending on the usage of a certain service – can be calculated by conditioning in the phase-type distribution. A phase-type distribution with an underlying Markov chain with multiple absorbing states is then considered, as this makes it possible to distinguish between the absorbing states. By making appropriate changes to the Markov chain, X, this theory can be used to condition on the visit to any state. The continuous case is treated in Andersen et al. (2000). In the theorem we will need the probability vector, $\boldsymbol{\pi} = \mu^{-1}\boldsymbol{\alpha}(\mathbf{I} - \mathbf{T})^{-1}$. When dealing with phase type distributions with $\boldsymbol{\alpha}$ fixed, one can without loss of generality assume that all elements of $\boldsymbol{\pi}$ are positive, as zero elements correspond to states that can never be visited. In our modelling framework, however, some choices of $\boldsymbol{\alpha}$ might lead to zero elements of $\boldsymbol{\pi}$ and it would be cumbersome to adjust the model to ensure strict positivity of $\boldsymbol{\pi}$ in each individual case. We define the operator $\Delta(\cdot)$ as the operator that given the vector $\mathbf{v}$ returns a diagonal matrix with the elements of $\mathbf{v}$ in the diagonal. Furthermore, we will define the operator $\Delta^{-1*}(\cdot)$

$$\Delta^{-1*}(\mathbf{v})_{(i,i)} = \begin{cases} v_i^{-1} & \text{if } v_i \neq 0 \\ 0 & \text{if } v_i = 0 \end{cases}.$$

Using these operators and $\boldsymbol{\pi}$ we define the matrix $\mathbf{V} = \Delta^{-1*}(\boldsymbol{\pi})\mathbf{T}'\Delta(\boldsymbol{\pi})$.

Theorem

Consider a discrete Markov Chain, Z, with r transient states, and m absorbing states. Now let $\boldsymbol{\alpha}$ be the initial distribution on the transient states, and let the transition matrix, $\mathbf{P}_Z$, be given as

$$\mathbf{P}_Z = \begin{bmatrix} \mathbf{T} & \mathbf{T}^0 \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \tag{10}$$

where $\mathbf{T}^0$ is an $r \times m$ sub-transition matrix containing the absorption probabilities. Let τ denote the time until absorption in some state. Then $\tau \sim \text{PH}(\boldsymbol{\alpha}, \mathbf{T})$. Let now $\mathbf{t}_j$ denote the j'th column of $\mathbf{T}^0$, and let $Z_\tau = r + j$ denote the event that Z absorbs in state $r + j$. Then

$$\Pr[Z_\tau = r + j] = \boldsymbol{\alpha}(\mathbf{I} - \mathbf{T})^{-1}\mathbf{t}_j. \tag{11}$$

The conditional distribution of τ on absorption into state $r + j$ is PH. A representation is $(\hat{\boldsymbol{\alpha}}_j, \hat{\mathbf{T}}_j)$ with

$$\begin{aligned} \hat{\boldsymbol{\alpha}}_{\boldsymbol{j}} &= \frac{\boldsymbol{\alpha}\Delta^{-1*}(\mathbf{h}_j)}{\boldsymbol{\alpha}\Delta^{-1*}(\mathbf{h}_j)\mathbf{e}} \\ \hat{\mathbf{T}}_j &= \Delta(\mathbf{h}_j)\mathbf{T}\Delta^{-1*}(\mathbf{h}_j) \\ \hat{\mathbf{t}}_{\mathbf{j}} &= \frac{\Delta(\mathbf{h}_j)\mathbf{t}_j}{\nu\mathbf{e}'\Delta(\boldsymbol{\pi})\mathbf{t}_j}, \end{aligned} \tag{12}$$

where

$$\begin{aligned} \boldsymbol{\beta}_j &= \frac{\mathbf{t}_j'\Delta(\boldsymbol{\pi})}{\mathbf{t}_j'\Delta(\boldsymbol{\pi})\mathbf{e}}, \\ \nu &= \boldsymbol{\beta}(\mathbf{I} - \mathbf{V})^{-1}\mathbf{e}, \\ \boldsymbol{\phi} &= \nu^{-1}\boldsymbol{\beta}(\mathbf{I} - \mathbf{V})^{-1}, \\ \mathbf{h}_j &= \Delta(\boldsymbol{\pi})\Delta^{-1*}(\boldsymbol{\phi}) \end{aligned}$$

Proof of theorem

To prove that $(\hat{\boldsymbol{\alpha}}_{\boldsymbol{j}}, \hat{\mathbf{T}})$ is indeed a representation of the conditional distribution, we prove that

- $(\hat{\boldsymbol{\alpha}}_{\boldsymbol{j}}, \hat{\mathbf{T}})$ is a phase-type representation and calculate the expression for $\hat{\mathbf{t}}_j$
- The probability generating function of $\Pr[\tau = x | Z = r + j]$ is equal to the probability generating function of a PH $(\hat{\boldsymbol{\alpha}}_j, \hat{\boldsymbol{T}}_j)$ distributed variable

To show that $(\hat{\boldsymbol{\alpha}}_{\boldsymbol{j}}, \hat{\mathbf{T}})$ is PH, we show that $\hat{\boldsymbol{\alpha}}_{\boldsymbol{j}}$ is a probability vector, that $\hat{\boldsymbol{\alpha}}_{\boldsymbol{j}}\mathbf{e} = 1$, that $\alpha_i \geq 0$, and that the entries of $\hat{\mathbf{T}}_j$ are non-negative, $\hat{\mathbf{T}}_j \geq \mathbf{0}$, with row sums bounded by 1.

$\hat{\boldsymbol{\alpha}}_{\boldsymbol{j}}\mathbf{e} = 1$ and $\hat{\boldsymbol{\alpha}}_{\boldsymbol{j}} \geq \mathbf{0}$ follows directly from the definition of $\hat{\boldsymbol{\alpha}}_{\boldsymbol{j}}$

$$\hat{\boldsymbol{\alpha}}_{\boldsymbol{j}}\mathbf{e} = \frac{\boldsymbol{\alpha}\Delta^{-1*}(\mathbf{h}_j)}{\boldsymbol{\alpha}\Delta^{-1*}(\mathbf{h}_j)\mathbf{e}}\mathbf{e} = \frac{\boldsymbol{\alpha}\Delta^{-1*}(\mathbf{h}_j)\mathbf{e}}{\boldsymbol{\alpha}\Delta^{-1*}(\mathbf{h}_j)\mathbf{e}} = 1.$$

The expression for $\hat{\mathbf{t}}_j$ can be found by noting that $\hat{\mathbf{t}}_j = (\mathbf{I} - \hat{\mathbf{T}}_j)\mathbf{e}$.

$$\begin{aligned}
\hat{\mathbf{t}}_j &= (\mathbf{I} - \hat{\mathbf{T}}_j)\mathbf{e} \\
&= (\mathbf{I} - \Delta(\mathbf{h}_j)\mathbf{T}\Delta^{-1}(\mathbf{h}_j))\mathbf{e} \\
&= (\mathbf{I} - \Delta^{-1*}(\boldsymbol{\phi})\Delta(\boldsymbol{\pi})\mathbf{T}\Delta^{-1*}(\boldsymbol{\pi})\Delta(\boldsymbol{\phi}))\mathbf{e} \\
&= \Delta^{-1*}(\boldsymbol{\phi})(\mathbf{I} - \left(\Delta^{-1}(\boldsymbol{\pi})\mathbf{T}'\Delta(\boldsymbol{\pi})\right)')\Delta(\boldsymbol{\phi})\mathbf{e} \\
&= \Delta^{-1*}(\boldsymbol{\phi})(\mathbf{I} - \mathbf{V}')\boldsymbol{\phi}' \\
&= \Delta^{-1*}(\boldsymbol{\phi})(\mathbf{I} - \mathbf{V}')(\boldsymbol{\beta}(\mathbf{I} - \mathbf{V})^{-1}\nu^{-1})' \\
&= \Delta^{-1*}(\boldsymbol{\phi})(\mathbf{I} - \mathbf{V}')(\mathbf{I} - \mathbf{V}')^{-1}\boldsymbol{\beta}_j\nu^{-1} \\
&= \Delta^{-1*}(\boldsymbol{\phi})\boldsymbol{\beta}_j\nu^{-1} \\
&= \frac{\Delta^{-1*}(\boldsymbol{\phi})\Delta(\boldsymbol{\pi})\mathbf{t}_j}{\nu\mathbf{e}'\Delta(\boldsymbol{\pi})\mathbf{t}_j} \\
&= \frac{\Delta(\mathbf{h}_j)\mathbf{t}_j}{\nu\mathbf{e}'\Delta(\boldsymbol{\pi})\mathbf{t}_j}.
\end{aligned}$$

By Bladt and Nielsen (2017)[p. 34] the probability generating function of a PH distribution with representation $(\boldsymbol{\alpha}, \mathbf{T})$ is

$$H(z) = \mathbb{E}\left[z^\tau\right] = z\boldsymbol{\alpha}(\mathbf{I} - z\mathbf{T})^{-1}\mathbf{t}\mathbf{e}. \tag{13}$$

Conditioning on absorption into state $r + j$ the probability generating function becomes

$$H(z \mid Z_\tau = r + j) = \frac{\mathbb{E}\left[z^\tau, \mathbf{1}\{Z_\tau = r + j\}\right]}{\Pr\left[Z_\tau = r + j\right]} = \frac{z\boldsymbol{\alpha}(\mathbf{I} - z\mathbf{T})^{-1}\mathbf{t}_j}{\boldsymbol{\alpha}(\mathbf{I} - \mathbf{T})^{-1}\mathbf{t}_j} = (\mu_j)^{-1}z\boldsymbol{\alpha}(\mathbf{I} - z\mathbf{T})^{-1}\mathbf{t}_j \tag{14}$$

We now algebraically manipulate with Equation (14), so it appears in the same form as Equation (13).

$$
\begin{aligned}
H(z \mid Z_\tau = r + j) &= (\mu_j)^{-1} z\boldsymbol{\alpha}(\mathbf{I} - z\mathbf{T})^{-1}\mathbf{t}_j \\
&= (\mu_j)^{-1} z\boldsymbol{\alpha}\left(I - z\Delta^{-1}(\mathbf{h}_j)\Delta(\mathbf{h}_j)\mathbf{T}\Delta^{-1}(\mathbf{h}_j)\Delta(\mathbf{h}_j)\right)^{-1}\mathbf{t}_j \\
&= (\mu_j)^{-1} z\boldsymbol{\alpha}\Delta^{-1}(\mathbf{h}_j)(\mathbf{I} - z\Delta(\mathbf{h}_j)\mathbf{T}\Delta^{-1}(\mathbf{h}_j))^{-1}\Delta(\mathbf{h}_j)\mathbf{t}_j \\
&= (\mu_j)^{-1} z c_1 \hat{\boldsymbol{\alpha}}_j(\mathbf{I} - z\hat{\mathbf{T}})^{-1} c_2 \hat{\mathbf{t}}_j
\end{aligned}
$$

By definition $(\mu_j)^{-1}c_1c_2 = 1$ which finalizes the proof. The calculations are verified in appendix A. This finishes the proof as

$$H(z \mid Z_\tau = r + j) = z\hat{\boldsymbol{\alpha}}(\mathbf{I} - z\hat{\mathbf{T}})^{-1}\hat{\mathbf{t}}_j\mathbf{e}.$$

4.3. Extracting results from the model

As described in Section 4.1 the travel time, τ, follows a discrete phase-type distribution. When the model has been formulated and a proper transition matrix $\mathbf{P}$ has been created, $\mathbf{T}$ can be found by excluding the row and column in $\mathbf{P}$ representing the destination. Likewise $\mathbf{t}$ can be found by extracting the column in $\mathbf{P}$ representing the destination. When an initial distribution $\boldsymbol{\alpha}$ has been defined, the pdf, cdf and expectation of τ can be found using (7), (8) and (9).

The probability of visiting one location can be found by collapsing the states representing this location into one absorbing state. The probability of visiting this location then equals the probability of absorption into this state, which can be found using (11).

The mean time spent at a location can be found by using the Green matrix, $\mathbf{U} = (\mathbf{I} - \mathbf{T})^{-1}$. Let K be the indices of the states representing the location k. The mean time spent at a location is

$$\mathbb{E}\left[\text{Time spent in location k}\right] = \sum_i \sum_{j \in K} \alpha_i u_{ij}.$$

Notice that not visiting the location is a possibility. The mean time spent in the location, given a visit to the location, is found as

$$\mathbb{E}\left[\text{Time spent in location k} \mid \text{Visiting location k}\right] = \frac{\sum_i \sum_{j \in K} \alpha_i u_{ij}}{\Pr\left[\text{Visiting location k}\right]}$$

Finally, the distribution of τ conditioned on visiting a certain location can be found by modifying the Markov chain such that journeys through this location absorb into a special absorbing state. The conditional distribution of τ can then be found using the representation given in Equation (12).

5. Results from example network

This section presents key statistics of the (conditional) travel time distributions and route choice probabilities of the example network shown in Section 3. The section presents several detailed analyses, which would in most other cases only be available through simulation approaches.

5.1. Impact of arrival distribution to first stop

At first the model is analyzed using the different arrival distributions to the first stop. Table 3 shows the expected travel times for the different $\boldsymbol{\alpha}$ presented in Section 3.2, and Figure 4 shows the distributions of the travel time. Passengers timing their arrival to 150S experience a lower travel time of around 4 minutes compared to passengers arriving randomly to the first stop, while the passengers timing the arrival to 300S have a higher expected travel time than random arriving passengers. From the distributions, it is noticeable that the passengers timing their arrival to 300S experience a bi-modal distribution due to the probability of catching either line B or E at Lyngby St. It is also worth noting the quite large variance in travel time when arriving uniformly to the first stop, where the passenger can experience a travel time on a wide bane of 40 minutes and up to approximately 70 minutes. This example shows one of the strengths of the model, as planners can now analyse in detail the travel time distributions of passengers and not only aim at lowering the average passenger travel time and/or individual vehicle travel time variation.

$\boldsymbol{\alpha}$	**Expected Travel Time**
Uniform	55.65
Timed for 150S	51.40
Timed for 300S	58.01

Table 3: *Expected travel times for different initial distributions,* $\boldsymbol{\alpha}$*.*

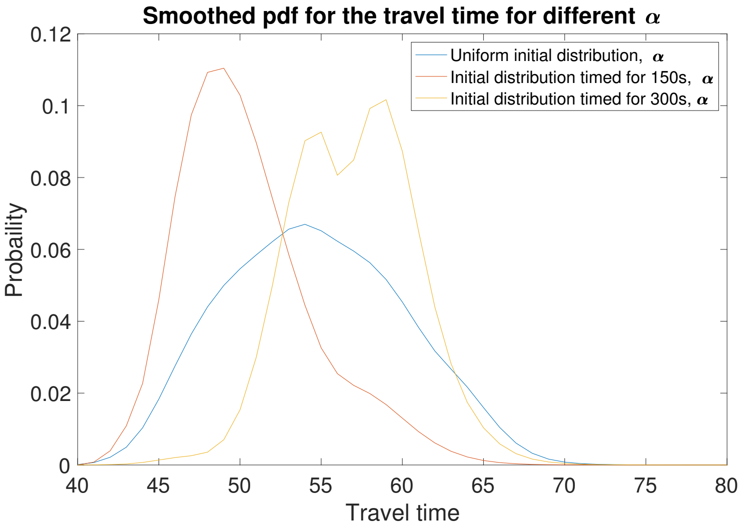


Figure 4: *The densities of the travel time for a uniform and two peaked initial distribution where the passenger arrives one minute before the departure of 150S or 300S. Smoothed refers to the plotting technique where the area under the curve is equal to 1.*

The model also allows for assessing the probability of choosing each service which can be determined by using Equation (11). Table 4 shows the probabilities of using each service for the three initial distributions. This shows that passengers timing their arrival to a specific line at the first stop largely end up on this line, with respectively 85 % on 150S when timing the arrival to this line and 98 % on 300S when timing for this line. The timing to the first stop does not influence which of the S-train lines (B or E) are boarded, since the distribution between these two is only determined by the arrival of 300S to Lyngby St. The distribution between the Metro and Regional train is more dependent on how the passengers time their arrival to the first stop, but in all scenarios with most passengers boarding the Metro.

Service	**Uniform α**	**α Timed for 150S**	**α Timed for 300S**
Bus 150S	0.60	0.85	0.02
Bus 300S	0.40	0.15	0.98
S-Train, line E	0.08	0.03	0.20
S-Train, line B	0.32	0.12	0.78
Metro	0.74	0.82	0.64
Regional train	0.26	0.18	0.36

Table 4: *The probabilities of choosing each service.*

Table 5 shows the expected time spent waiting at the transfer stations, Lyngby St. and Nørreport St.. Notice that the passenger always arrives to Lyngby St. by Bus 300S, and therefore the waiting time at transfers on Lyngby St. does not change with the initial distribution. The waiting time at Nørreport St. is in all scenarios around three minutes, which is less than if the passenger only chooses to wait for the Metro with a headway of 6 minutes.

Stop	**Uniform $\boldsymbol{\alpha}$**	**$\boldsymbol{\alpha}$ Timed for 150S**	**$\boldsymbol{\alpha}$ Timed for 300S**
Nørreport St.	2.96	3.10	2.92
Lyngby St.	2.79	2.79	2.79

Table 5: *The expected transfer time at Nørreport St. and Lyngby St. for the different choices of* $\boldsymbol{\alpha}$.

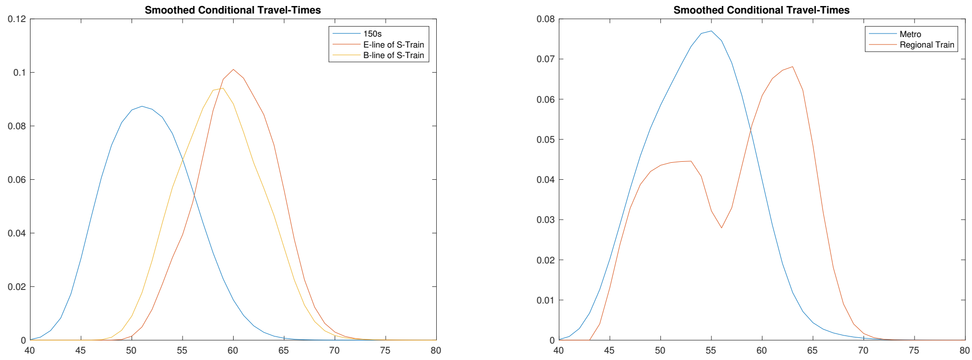
5.2. Impact of route choice

The impact of the route choice is investigated by extracting conditional results from the model. Table 6 shows the expected travel times for the use of the different services for the uniform $\boldsymbol{\alpha}$, and some considerable differences are seen. The results show that passengers boarding 150S have a lower expected travel time compared to passengers boarding 300S. Furthermore, the passengers boarding the Metro at Nørreport St. also have a lower expected travel time compared to passengers boarding the Regional train, which is intuitive due to the lower runtime of the Metro (15 minutes) compared to that of the Regional train (18 minutes).

Choice of Service	**Expected travel time**
Bus 150S	52.72
Bus 300S	60.05
S-Train, line E	61.34
S-Train, line B	59.72
Metro	54.75
Regional train	58.14

Table 6: *Expected travel time when using each of the services.*

Figure 5 shows the probability density function of the travel time conditioning on the use of specific services. Note the two peaks in the distribution for the regional train in Figure 5b, which are the results of some missed connections to the Regional train.



(a) *Conditional travel times for the three route choices from DTU to Nørreport St.*

(b) *Conditional travel times for the two route choices from Nørreport St. to CPH.*

Figure 5: *Conditional travel times.*

5.3. Modifying the example network

To investigate the effect of whether lines are represented using a frequency- or schedule-based representation, the example network is now modified in two ways:

- Modification 1: The Metro is changed to a schedule-based service
- Modification 2: Bus 150S is changed to a frequency-based service

Both modifications are made such that the average headway does not change. Table 7 shows the expected travel times for the uniform initial arrival distribution, corresponding to $\boldsymbol{\alpha}$ having all elements equal. The modifications affect the expected travel time with a maximum of around one minute.

Expected travel time		
Original Network	**Modification 1**	**Modification 2**
55.65	55.69	54.53

Table 7: *The expected travel times for the original network, and for the two modified networks with uniform* $\boldsymbol{\alpha}$.

Table 8 shows the route choice probabilities for the uniform $\boldsymbol{\alpha}$ for the original network and two modified networks. The change of the Metro to a schedule-based service does not change the route choice probabilities for any of the services, while the change of Bus 150S to a frequency-based service changes the distribution between Bus 150S and Bus 300S slightly and thereby also the distribution between the Metro and the Regional train.

Service	Original network	Modification 1	Modification 2
150S	0.60	0.60	0.75
300S	0.40	0.40	0.25
S-Train, line E	0.08	0.08	0.05
S-Train, line B	0.32	0.32	0.20
Metro	0.74	0.74	0.78
Regional train	0.26	0.26	0.22

Table 8: *The probabilities of choosing each service for the original network and the two modified networks.*

6. Perspectives

The framework proposed in this paper is capable of modelling the distribution of travel times from origin to destination and of calculating the route choice probabilities for the alternatives in the choice set. This section discusses how the framework can be used by e.g. policy makers to make various detailed analyses of the route choices and travel time distributions. Subsequently, the section discusses scalability and elaborates on possible further extensions allowed by the flexibility of the modelling framework.

Possibilities of disaggregate analyses

A great asset of the proposed model is that it outputs both the distribution of the travel times and the probability of choosing a service. The travel time distributions for the possible routes from origin to destination reveal the variance of travel times, which has been shown to be of great annoyance for travellers in general (Carrion and Levinson, 2012). Furthermore, the travel time distributions can reveal whether some connections between services result in distributions with two or more peaks due to missed connections, especially between schedule-based services.

The possibility of extracting conditional travel times based on the chosen lines or stops allow for very detailed analyses of the travel patterns. This can help planners to understand why specific lines are attractive, for instance when travel times have both a low mean value and low variance. On the other hand it can also reveal lines that have non-optimal coordination with other services. If more origin and destination pairs are examined, an analysis will determine whether the non-robust synchronization is related to a specific line or is due to a more general network problem.

Estimation of model parameters for vehicle travel times

As described in Sections 2 and 4.1, discrete phase-type distributions are used to model in-vehicle times and headways between frequency-based services. The realism of the model outcome is thus highly dependent on the specification of these distributions. The parameters in the phase-type distributions can be estimated from real data, such as widely available Automated Vehicle Location data, by using the relatively simple EM algorithm described in Bladt and Nielsen (2017)[Ch. 13].

Complexity and scalability

Markov chains provide an efficient modelling tool, as most calculations consist of matrix multiplications and inversions. As seen in Equation (7), the density function of a discrete phase-type distribution evaluated in n requires $n-1$ matrix-vector multiplications (alternatively $\mathbf{log}_2(n)$ matrix multiplications). Similarly, the distribution function in Equation (8) requires n matrix-vector multiplications (alternatively $\mathbf{log}_2(n)$ matrix multiplications). The expectation shown in Equation (9) is found by performing an inversion. As the transition matrix is very sparse, the calculations can be done efficiently. For comparison, the transition matrix constructed for the example network has dimension 6480×6480, but only 13,323 non-zero entries. This corresponds to 0.0317% of the matrix being non-zero. The computational effectiveness of Markov chains indicates that the method should be suitable for large scale applications.

Inclusion of alternative passenger route choice strategies

For the example presented in this paper, a route choice strategy of boarding the first departing line from a given stop was implemented. This strategy is well-known from the first examples of frequency-based models (Spiess and Florian, 1989). In the last decade, the increasing availability of real-time information has driven the need for models to represent route choice in more detailed ways, e.g. through adaptive route choice strategies. Additionally, utility-based models (e.g. logit or probit-type models) have been proposed, providing a more realistic description of route choice. The flexible specification of the proposed modelling framework allows the implementation of alternative route choice strategies. For instance, the boarding probability in Equation (5) could be computed based on the expected remaining travel time or expected utility of boarding a specific line, thus inducing a logit route choice probability. Adopting such a choice strategy is similar to the approach for road networks presented in Fosgerau et al. (2013) and for schedule-based public transport networks in Rambha et al. (2016) and Nuzzolo and Comi (2019). An implementation of such a strategy could for example lower the probabilities of boarding 300S and the Regional train in the example network, since both services lead to higher travel times (see Table 6 and Figure 5).

Further extensions of the model framework

In this paper stops are represented in a relatively simple way, but the framework allows these to be modelled in more detail. First, a natural extension of the stop representation is to introduce another phase parameter to account for delays in schedule-based services arising before the passenger boards the service. In the current modelling framework the schedule-based lines run according to their planned departure times from the stops, but arrive randomly according to the delay distribution to the following stops.

Secondly, in the current formulation it is assumed that the passenger can board a service arriving immediately after having exited another service, as seen in the creation of the transition matrix $\mathbf{T}_{kl}$. It is rarely a possibility that the passenger can instantaneously move from one service to another, as both the layout of stops and crowding may delay the passenger. An extra phase parameter can be introduced to account for this. It should then only be possible to board a new service when the passenger is in a specified set of the phases (e.g. the last phase). This forces the passenger to spend a minimum amount of time at the stop.

Lastly, an important aspect of modelling route choice in public transport in metropolitan areas is the inclusion of the effect of crowding in the network. The inclusion of denied boarding for specific runs can be handled by the inclusion of extra phase-type parameters to allow the passenger to be denied boarding and to wait for the next departure.

7. Conclusion

This paper presents a framework for calculating the distribution of passenger travel times and the route choice probabilities for trips in a public transport network. The method allows both frequency-based and schedule-based services and takes vehicle delays into account. This is especially important for schedule-based services, as the impact on the passenger travel time by a missed connection between services due to delays can be considerable. The model is based on a multi-dimensional Markov chain that models both the position of the passenger and the time. Vehicles and stops are possible locations of the passengers and are thus represented by states in the Markov chain. This is an extension of earlier usage of Markov chains in transport modelling, which, however, only represented stops as states.

The time spent in each service is modelled by discrete phase-type distributions, a very flexible class of distributions. This makes it possible to model many different types of running time distributions for the vehicles. The model was demonstrated using a small real-life case study that provided many detailed outputs useful for analyses of the route choice probabilities and travel time distributions for the passengers.

The introduction of phases with a view to modelling the time spent at each location has many possible extensions, e.g. the introduction of phases at each station to represent possible crowding. Phases could also be used to model possible delays in schedule-based services before their arrival at the departure station of the passenger.

In addition to outputting the route choice probabilities, the model also finds the distributions of the travel times and not just the mean values. This is an advantage as it allows the modeller to analyze all possible travel times. The model can also find the conditional distributions of travel times, allowing the modeller to focus on the effects of one particular service.

Declarations of interest: none

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Appendix A: Verification of conditioning result

We verify that $(\mu_j)^{-1}c_1c_2 = 1$

$$\begin{aligned}
(\mu_j)^{-1}c_1c_2 &= \frac{\boldsymbol{\alpha}\Delta^{-1}(\mathbf{h})\mathbf{e}\nu\mathbf{e}'\Delta(\boldsymbol{\pi})\mathbf{t}_j}{\boldsymbol{\alpha}(\mathbf{I}-\mathbf{T})^{-1}\mathbf{j}_j} \\
&= \frac{\boldsymbol{\alpha}\Delta^{-1}(\mathbf{h}_j)\mathbf{e}\nu\mathbf{e}'\Delta(\boldsymbol{\pi})\mathbf{t}_j}{\boldsymbol{\alpha}(\mathbf{I}-\Delta^{-1}(\mathbf{h}_j)\Delta(\mathbf{h}_j)\mathbf{T}\Delta^{-1}(\mathbf{h}_j)\Delta(\mathbf{h}_j))^{-1}\mathbf{t}_j} \\
&= \frac{\boldsymbol{\alpha}\Delta^{-1}(\mathbf{h}_j)\mathbf{e}\nu\mathbf{e}'\Delta(\boldsymbol{\pi})\mathbf{t}_j}{\boldsymbol{\alpha}\Delta^{-1}(\mathbf{h}_j)(\mathbf{I}-\Delta(\mathbf{h}_j)\mathbf{T}\Delta^{-1}(\mathbf{h}_j))^{-1}\Delta(\mathbf{h}_j)\mathbf{t}_j} \\
&= \frac{\boldsymbol{\alpha}\Delta^{-1}(\mathbf{h}_j)\mathbf{e}\nu\mathbf{e}'\Delta(\boldsymbol{\pi})\mathbf{t}_j}{\boldsymbol{\alpha}\Delta^{-1}(\mathbf{h}_j)(\mathbf{I}-\Delta^{-1}(\boldsymbol{\phi})\Delta(\boldsymbol{\pi})\mathbf{T}\Delta^{-1}(\boldsymbol{\pi})\Delta(\boldsymbol{\phi}))^{-1}\Delta(\mathbf{h}_j)\mathbf{t}_j} \\
&= \frac{\boldsymbol{\alpha}\Delta^{-1}(\mathbf{h}_j)\mathbf{e}\nu\mathbf{e}'\Delta(\boldsymbol{\pi})\mathbf{t}_j}{\boldsymbol{\alpha}\Delta^{-1}(\mathbf{h}_j)\Delta^{-1}(\boldsymbol{\phi})(\mathbf{I}-\Delta(\boldsymbol{\pi})\mathbf{T}\Delta^{-1}(\boldsymbol{\pi}))^{-1}\Delta(\boldsymbol{\phi})\Delta(\mathbf{h}_j)\mathbf{t}_j} \\
&= \frac{\boldsymbol{\alpha}\Delta^{-1}(\mathbf{h}_j)\mathbf{e}\nu\mathbf{e}'\Delta(\boldsymbol{\pi})\mathbf{t}_j}{\boldsymbol{\alpha}\Delta^{-1}(\mathbf{h}_j)\Delta^{-1}(\boldsymbol{\phi})(\mathbf{I}-\Delta(\boldsymbol{\pi})\mathbf{T}\Delta^{-1}(\boldsymbol{\pi}))^{-1}\Delta(\boldsymbol{\pi})\mathbf{t}_j}
\end{aligned}$$

To finish the proof we need to show the equality

$$\Delta^{-1}(\boldsymbol{\phi})(\mathbf{I}-\Delta(\boldsymbol{\pi})\mathbf{T}\Delta^{-1}(\boldsymbol{\pi}))^{-1} = \mathbf{e}\mathbf{e}'\nu.$$

This is done by manipulating with the expression for ν

$$\begin{aligned}
&\nu = 1\cdot\nu\cdot 1 \\
\Rightarrow&\nu = \boldsymbol{\beta}\mathbf{e}\nu\mathbf{e}'\Delta(\boldsymbol{\phi})\mathbf{e} \\
\Rightarrow&\boldsymbol{\beta}(\mathbf{I}-\mathbf{V})^{-1}\mathbf{e} = \boldsymbol{\beta}\mathbf{e}\nu\mathbf{e}'\Delta(\boldsymbol{\phi})\mathbf{e} \\
\Rightarrow&(\mathbf{I}-\mathbf{V})^{-1} = \mathbf{e}\nu\mathbf{e}'\Delta(\boldsymbol{\phi}) \\
\Rightarrow&(\mathbf{I}-\Delta^{-1}(\boldsymbol{\pi})\mathbf{T}'\Delta(\boldsymbol{\pi}))^{-1} = \mathbf{e}\nu\mathbf{e}'\Delta(\boldsymbol{\phi}) \\
\Rightarrow&\left((\mathbf{I}-\Delta^{-1}(\boldsymbol{\pi})\mathbf{T}'\Delta(\boldsymbol{\pi}))^{-1}\right)' = (\mathbf{e}\nu\mathbf{e}'\Delta(\boldsymbol{\phi}))' \\
\Rightarrow&(\mathbf{I}-\Delta(\boldsymbol{\pi})\mathbf{T}\Delta^{-1}(\boldsymbol{\pi}))^{-1} = \Delta(\boldsymbol{\phi})\mathbf{e}\nu\mathbf{e}' \\
\Rightarrow&\Delta^{-1}(\boldsymbol{\phi})(\mathbf{I}-\Delta(\boldsymbol{\pi})\mathbf{T}\Delta^{-1}(\boldsymbol{\pi}))^{-1} = \mathbf{e}\mathbf{e}'\nu.
\end{aligned}$$

II

Route choice models for mixed schedule- and frequency-based public transport systems

4 Paper 3: The influence of frequency on route choice in mixed schedule- and frequency-based public transport systems - The case of the Greater Copenhagen Area

The following pages contain the article:

M. Eltved, O. A. Nielsen, and T. K. Rasmussen (2018). "The influence of frequency on route choice in mixed schedule- and frequency-based public transport systems - The case of the Greater Copenhagen Area". In: *Proceedings of the 14th Conference on Advanced Systems in Public Transport (CASPT2018)*. Brisbane, Australia. URL: http://www.caspt.org/wp-content/uploads/2018/10/Papers/CASPT_2018_paper_81.pdf.

Please cite accordingly.

The paper was presented at the *14th Conference on Advanced Systems in Public Transport (CASPT2018)*, Brisbane, Australia in July 2018.

The influence of frequency on route choice in mixed schedule- and frequency-based public transport systems – The case of the Greater Copenhagen Area

Morten Eltved · Otto Anker Nielsen · Thomas Kjær Rasmussen

Abstract

Understanding and analysing passengers' route choice preferences is critical to realistically predict the level of service for the passengers', when timetables change or new infrastructure is build. This paper argues and presents evidence on the influence of frequency of public transport services and whether published timetables are schedule- or frequency-based when describing passengers' route choice in mixed schedule- and frequency-based public transport systems. The study is based on a revealed preference survey with 5,121 reported trips in the Greater Copenhagen Area. Given the observed trips and a corresponding large choice set with alternative routes, passenger preferences are revealed using the well-known Multinomial Logit model.

Utilising recently published research on how passengers time their arrival to the first stop, the paper shows how to estimate passengers' preferences for avoiding waiting at the first stop. The analysis also shows that passengers prefer high frequency routes. This is shown by considering the highest headway in any leg of a trip, as well as by introducing a variable capturing passengers' higher preference for frequency-based compared to schedule-based services. On the other hand it is shown, that passengers prefer waiting for a schedule-based service compared to a frequency-based service when transferring, implying that passengers want to be certain about the time they need to wait when transferring. Finally, the paper examines the transformation of the in-vehicle time components according to a Box-Cox transformation, and highlights the varying trade-offs between in-vehicle times of different vehicles at different travel time levels.

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1 Introduction

The public transport system in most metropolitan areas, including the Greater Copenhagen Area, is a mix of high and low frequency services where the published timetable for some lines is schedule-based (SB), while being frequency-based (FB) for others. The passengers are therefore often in a situation where the route choice includes options where both SB and FB services are viable alternatives relevant to consider. The combination of the two types of services, independent of public transport mode, leads to a more complex route choice, where the frequency of services are important for various reasons, e.g. in relation to transfers.

In the literature, the utility function of passenger's route choice primarily includes the quantifiable time components (in-vehicle, access/egress time, waiting time at transfers and walking time), a transfer penalty and in relevant cases the ticket price (Gentile and Noekel, 2016, chap. 4). Some applications also include the other factors such as in-vehicle crowding, level changes at transfers and other attributes at transfer stations as well as topological characteristics for the spatial dimension of a trip (Raveau et al., 2014). Implicitly, the impact of frequency on route choice has typically been captured by the hidden waiting time and waiting time in a linear way. However, as it was shown in Anderson et al. (2014) the inclusion of headway of the trip proved to provide a significantly better model fit.

This paper utilises a large disaggregate dataset of observed behaviour in the Greater Copenhagen Area. The results demonstrate, that the model fit to observed behaviour can be improved further by (i) separating waiting for SB and FB services at transfers; (ii) using new knowledge identified in (Ingvardson et al., 2018) about passengers' waiting time at the first station to enrich the detailed dataset; (iii) relating passengers' route choice to the published timetables to identify preferences for FB and SB services by including a dummy for FB services; and (iv) analysing trade-offs between in-vehicle times at different travel time levels.

The paper is structured as follows; Section 2 introduces the dataset used in the study and the methodology for estimating passenger preferences, Section 3 presents the results, followed by a discussion and conclusion of the study in section 4.

2 Data foundation and methodology

The dataset used in this study consists of 5,121 observed routes made by public transport in the Greater Copenhagen Area. The data were collected in the years 2009-2011 as part of the Danish National Transport Survey (Center for Transport Analytics DTU, 2017). The observed routes were matched to a SB representation of the public transport network as described in Anderson and Rasmussen (2010). A choice set of alternatives corresponding to each observed route was generated using a simulation-based choice set generation method described in Rasmussen et al. (2016). The final choice sets consist of between 18-200 alternatives for each observation with an average of 128 unique alternatives per observed route.

2.1 Description of network

The public transport network in the Greater Copenhagen Area is a mix of SB and FB services. The FB services are found in the most densely populated areas of Copenhagen, consisting primarily of the metro operating with a headway of 2-4 minutes during peak hour, and the "A-buses" (high frequency buses), with headways between 3-8 minutes during peak hours. All other buses, regional trains and suburban trains (S-tog) in the Greater Copenhagen Area operate with published schedules with headways between 5-90 minutes. The figures below show examples of the published timetable for a FB and a SB bus line. In particular, note in the case of line 3A and 150S, the SB bus line 150S runs with a higher frequency than the bus line 3A.

3A

mandag - fredag: Nordhavn st.	Trianglen	Elmegade	Åboulevard	Vesterbrogade	Enghave st. (Enghavevej)	Sydhavn st.	Kgs. Enghave (Valbyparken)
4.50	4.55	5.00	5.03	5.09	5.13	5.15	5.21
5.10	5.15	5.20	5.23	5.29	5.33	5.35	5.41
5.30	5.35	5.40	5.43	5.49	5.53	5.55	6.01
5.40	5.45	5.50	5.53	5.59	6.03	6.05	6.11
5.50	5.55	6.00	6.03	6.09	6.13	6.15	6.21
6.00	6.05	6.10	6.13	6.19	6.23	6.25	6.31
6.09	6.14	6.19	6.22	6.28	6.32	6.34	6.40
6.19	6.24	6.29	6.32	6.38	6.42	6.44	6.50
6.28	6.33	6.39	6.42	6.48	6.52	6.54	7.00
6.36	6.42	6.48	6.51	6.57	7.01	7.03	7.10
6.44	6.51	6.57	7.00	7.07	7.11	7.13	7.20
6.52	6.59	7.05	7.09	7.16	7.20	7.22	7.29
Interval:				6-8 min			
17.15	17.22	17.29	17.33	17.40	17.45	17.47	17.53
17.23	17.30	17.37	17.41	17.48	17.52	17.54	18.00

150S

mandag - fredag: Kokkedal St.	Fredensborg Rådhus	Bolbrovej	Hovedgaden	Gl. Holte (Mariehøjvej)	Nærum St.	Klampenborgvej	Ryparken St.	Hans Knudsens Plads	Nørreport St.
5.25	5.30	5.33	5.37	—	5.48	5.54	6.04	6.05	6.14
5.46	5.51	5.54	5.58	—	6.09	6.15	6.25	6.26	6.36
6.04	6.09	6.12	6.16	—	6.27	6.33	6.43	6.44	6.54
6.14	6.19	6.22	6.26	—	6.37	6.43	6.53	6.54	7.05
6.24	6.29	6.32	6.36	—	6.47	6.53	7.03	7.04	7.15
6.33	6.38	6.41	6.45	—	6.57	7.03	7.14	7.15	7.26
—	—	—	—	—	7.05	7.11	7.22	7.23	7.35
6.45	6.50	6.53	6.58	—	7.10	7.16	7.27	7.28	7.40
—	—	—	—	7.06	7.16	7.22	7.33	7.34	7.46
6.55	7.00	7.03	7.08	—	7.21	7.27	7.38	7.39	7.51
—	—	—	—	—	7.26	7.32	7.43	7.44	7.56
7.05	7.11	7.14	7.19	—	7.32	7.38	7.50	7.51	8.03
—	—	—	—	7.27	7.37	7.43	7.55	7.56	8.08
7.15	7.21	7.24	7.29	—	7.43	7.49	8.01	8.02	8.14
—	—	—	—	—	7.49	7.55	8.07	8.08	8.20
7.25	7.31	7.34	7.39	—	7.55	8.01	8.13	8.14	8.26
—	—	—	—	7.50	8.00	8.06	8.18	8.19	8.31
7.35	7.41	7.44	7.49	—	8.05	8.11	8.23	8.24	8.36
—	—	—	—	—	8.10	8.16	8.28	8.29	8.41
7.45	7.51	7.54	7.59	—	8.15	8.21	8.33	8.34	8.46
—	—	—	—	8.10	8.20	8.26	8.38	8.39	8.51
7.55	8.01	8.04	8.09	—	8.25	8.31	8.42	8.43	8.55
—	—	—	—	—	8.32	8.38	8.49	8.50	9.02

Figure 1 - Example of a frequency-based bus (3A between 7am-5pm)
and a schedule-based service (150S) (Movia, 2011)

Figure 2 gives an overview of the public transport system in the Greater Copenhagen Area, when dividing the network into SB (rail and most busses) and FB lines (metro and some busses) respectively. The map clearly shows the concentration of FB services in the central part of Copenhagen.

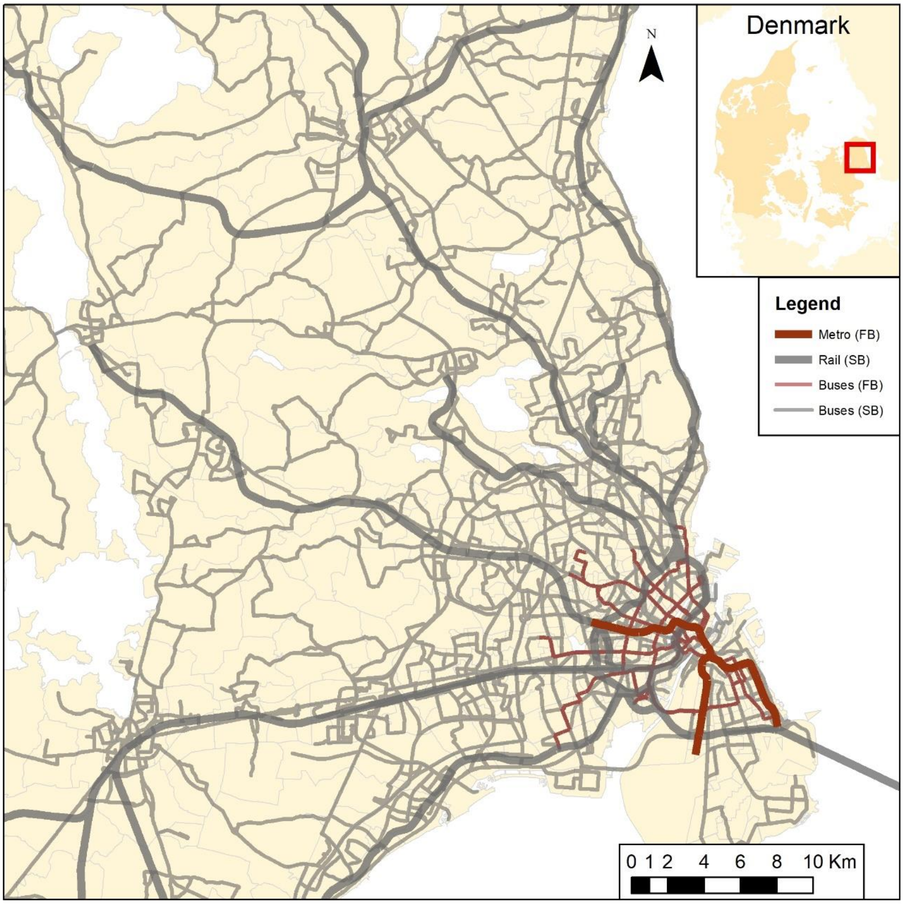

Figure 2 - Schedule-based (SB) and frequency-based (FB) services in the Greater Copenhagen Area

2.2 Observed routes

The observed routes are distributed across the whole case-study area, and have a large variation in terms of the components (Table 1). All variables have large standard deviations, which for most of the variables, e.g. sub mode specific in-vehicle times and waiting time at transfers, is due to the many routes which have not used a specific sub mode or made any transfers, i.e. not having any waiting times at transfers. The most used sub modes in the data are buses and S-trains which are also the primary services covering the case-study area.

All variables, except the headway variables and waiting time at the first stop, are directly extracted from the matched observed routes. The headway of each leg in the trip is found by the minimum amount of the time to the previous and next departure (run) of the same line between the same stops. The highest headway of the trip is defined as the highest headway of the legs in the trip.

Table 1 - Trip characteristics for observed routes

Trip component	Mean	Std. dev.
Total travel time	36.47	20.82
In-vehicle time total	20.21	13.76
In-vehicle time bus	8.25	11.08
In-vehicle time SB bus	6.14	10.33
In-vehicle time FB bus	2.11	5.91
In-vehicle time metro	1.39	3.56
In-vehicle time S-train	7.44	11.18
In-vehicle time local train	0.60	4.00
In-vehicle time regional train	2.53	8.43
Nb. of transfers	0.48	0.64
Waiting time at transfers	2.52	6.10
Waiting time at first stop	3.85	2.64
Walking time	0.97	1.58
Access/egress	12.78	9.36
Headway of first leg	11.88	16.29
Highest headway in trip	14.32	17.42
Include frequency-based service (dummy)	0.34	0.47
Total number of observations	5,121	

The waiting time at first stop is derived from the headway of the first leg, and whether the first leg is a SB or FB service. The distinction between SB and FB services is made because a recent study from Ingvardson et al. (2018) showed that passengers who know the exact planned departure time of a run come partially planned to the first stop thereby minimizing the waiting time at the first stop. The study by Ingvardson et al. (2018) only covered rail services, but the assumption of this present work is that the calculated waiting times also applies for bus services. The waiting time at the first stop for FB services is given by half of the headway, as passengers are assumed to arrive completely random to these services, which was also shown to be true in the study. For SB services the passengers arrive more timed the longer the headway is as shown in Figure 3. The waiting time at the first stop (F) increases with the headway, and is found by the following formula:

$$F = \begin{cases} 0.5 * H & if\ L = FB \\ 0.5181 * \exp(H) * H & if\ L = SB \end{cases}$$

, where L is the first leg in the trip and H is the headway of the first leg in the trip.

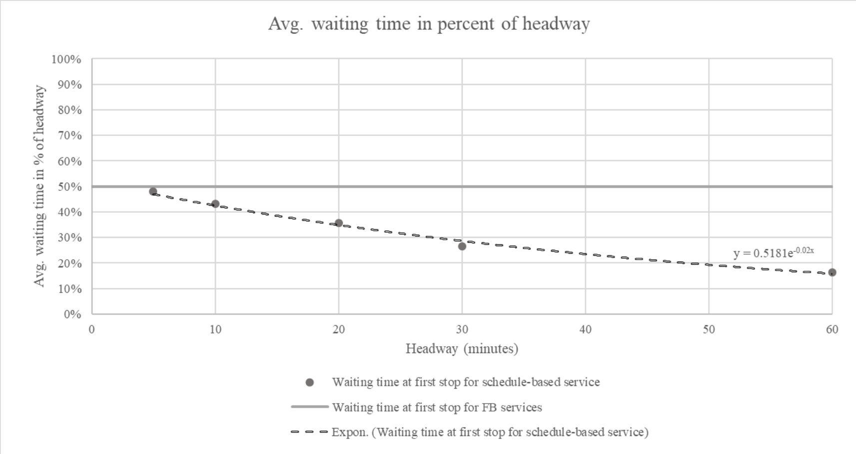

Figure 3 -Average waiting time at first stop in percent of headway for respectively SB and FB services

Figure 4 illustrates the total travel time and in-vehicle time for the observed trips. As seen in the cumulative distribution function for the total travel time, the dataset include a wide range of travel times, with most observations having around 20 to 40 minutes total travel time. Around 15% of the 5,121 trips last for more than one hour, and 27 observed routes have a travel time exceeding two hours. The total in-vehicle time varies between very short trips with only a few minutes of in-vehicle time to in-vehicle times of more than an hour. Most trips include between 15-30 minutes in-vehicle time, which is also reflected in the total travel time for most trips being 20-40 minutes.

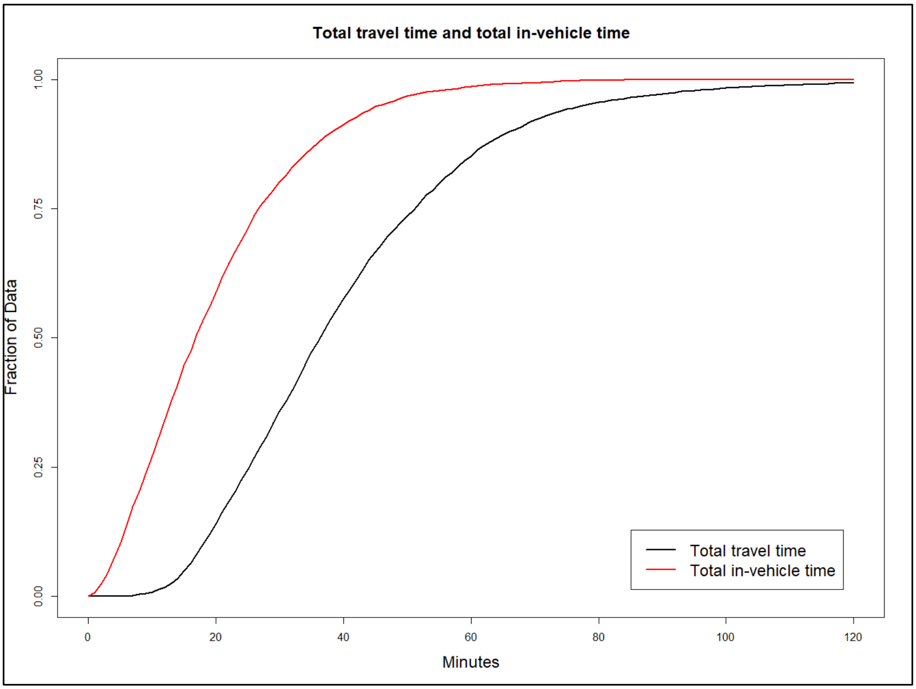

Figure 4 - Cumulative distribution function for total travel time and total in-vehicle time for observed trips. (27 observations have a total travel time higher than 120 minutes – max 232 minutes)

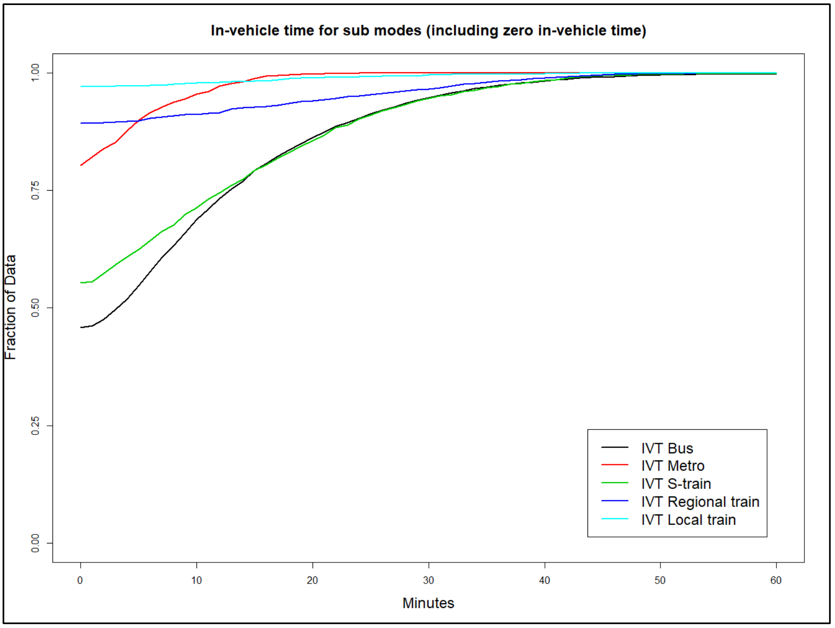

Figure 5 -Cumulative distribution functions of sub mode in-vehicle times of observed trips including trips that did not use the specific sub mode (zeros)

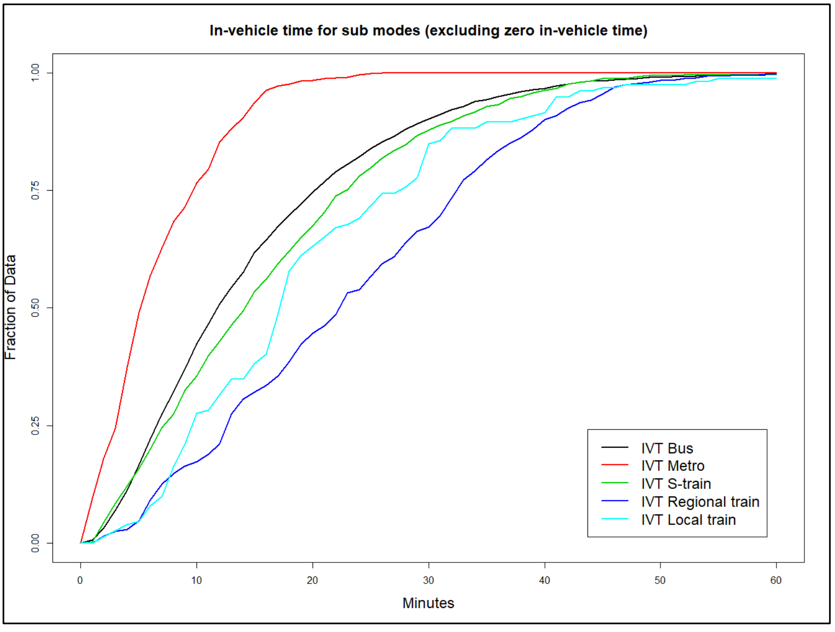

Figure 6 - Cumulative distribution functions for in-vehicle times of observed trips excluding trips that did not use the specific sub mode (zeros).
Few observations for local train result in a less smooth curve than for other sub modes.

6

Figure 5 and Figure 6 illustrate the distribution of the different variables related to the in-vehicle times of sub modes of the observed routes. Figure 5 shows the distributions when also including trips where the passenger did not use the specific sub mode, while Figure 6 shows the distributions only including trips where passengers used the sub mode. More than 50% of the trips use a bus and almost half of the trips use the S-trains, while only around 20% use the metro, and the regional and local trains are used even less frequently than the metro. When removing all trips not including a specific sub mode, the plots in Figure 6 shows a wide range of in-vehicle times for S-train, buses and regional train, while the in-vehicle times for metro use is significantly shorter. This is due to shorter lines which serve more trips centred in the inner areas of Copenhagen, which would be expected to have shorter trips.

Figure 7 shows the cumulative distribution for the other component of the trips. The access/egress times varies between a few minutes and 30 minutes, where the waiting times at the first stop is centred from two to five minutes. The walking and waiting times at stops is proportional to the number of transfers in the trips, where more than half of the trips are single legged. The final variable is the highest headway of the trip, where most trips have headways of 10 minutes or below, while few have headways higher than thirty minutes.

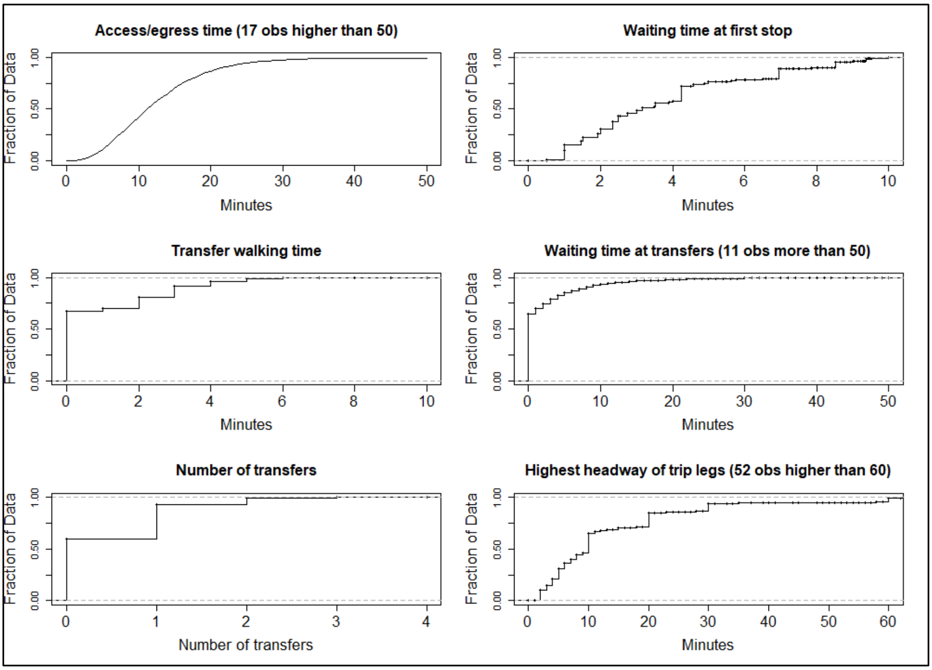

Figure 7 - Cumulative distribution functions for trip components other than IVT of observed trips

2.3 Methodology

Various multinomial logit models are estimated to reveal the route choice preferences of travellers (Train, 2002). The utility U_{kn} of an alternative k in the choice set C_n for each observed route n is described with the following utility specification:

$$U_{kn} = V_{kn} + \epsilon_{kn} \qquad \forall K \in C_n$$

, where, V_{kn} is the deterministic part of the utility and ϵ_{kn} is the random utility assumed to be gumbel distributed.

The deterministic part of the utility V_{kn} is specified as:

$$V_{kn} = \sum_{m} \beta_{IVT,m} IVT_{mkn} + \sum_{c} \beta_{t,c} t_{ckn} + \sum_{q} \beta_{y,q} y_{qkn}$$

, where IVT_{mkn} is the in-vehicle time for component m, t_{ckn} is the time component c not related to in-vehicle time (e.g., waiting, walking, access/egress and headway) and y_{qkn} is component q not related to time (e.g., transfer penalties and dummy variable for trips including FB services). The choice probability of route k for observation n is given as:

$$P_{kn} = \frac{\exp(V_{kn})}{\sum_{l \in C_n} \exp(V_{ln})}$$

The estimations made in the analysis for this paper build on the work made in Anderson et al. (2014), but exclude the path size correction factor, as the factor proved not to be significant when adding multiple new variables. The focus of the estimation procedure was to test the hypothesis concerning waiting time preference when transferring to either a SB or FB service; estimating first waiting time correctly; check for passenger preferences by including a dummy variable for trips including a FB service; and finally to estimate Box-Cox transformations for in-vehicle times. The Box-Cox transformations estimated follow the formula given below:

$$x(\lambda) = \frac{x^{\lambda} - 1}{\lambda}$$

where λ is the transformation parameter to be estimated (Box and Cox, 1964), and x is the variable, which is transformed.

3 Results

This section presents the results of estimations of models including different variables, which describe passengers' route choice preferences in the Greater Copenhagen Area. In total more than 100 different specifications were tested to achieve the best model fit. This section presents the base model followed by the elaborate model; including the waiting time at first stop, highest headway of trip, split between waiting for SB and FB services, and dummy for whether the route includes a FB service.

3.1 Base specification

Table 2 show the estimates of the base specification including the variables access/egress, in-vehicle time of the sub modes, waiting times at transfers, walking time at transfers and transfer penalty. The results show that passengers prefer S-train, local trains and, especially metro compared to bus and regional trains. The higher disutility of regional trains compared to bus use is not as expected, but could be a result of few good viable alternatives to trips using regional trains, because there are typically no other services running in the same corridors as the trains. The access and egress time is, as expected a higher disutility than being inside a vehicle.

From the model it appears that walking and waiting time at transfers is preferred compared to in-vehicle time, but this is due to the transfer penalty, and it is therefore important to note that waiting and walking time at transfers have to be seen in the context of the high transfer penalty. The transfer penalty is equivalent to approximately 9 minutes of in-vehicle time in bus, but is lower for work related trips and higher for leisure trips. For work trips this could be due to passengers primarily trying to minimize the total travel time, while leisure trips avoid transfers to a higher extent possibly because they are not as familiar with the transfer options and the certainty of reaching a connecting service.

Table 2 - Estimates (robust t-test) for model with base specification and rates of substitution scaled to bus in-vehicle time

	Model estimates			**Rates of substitution (to bus IVT)**		
	Trip purpose			**Trip purpose**		
Parameter	**All**	**Work**	**Leisure**	**All**	**Work**	**Leisure**
In-vehicle times						
Bus	-0.190 (-32.84)	-0.216 (-26.47)	-0.163 (-21.61)	1.00	1.00	1.00
Metro	-0.066 (-7.37)	-0.086 (-6.54)	-0.044 (-3.74)	0.35	0.40	0.27
S-train	-0.146 (-22.46)	-0.170 (-17.23)	-0.117 (-14.60)	0.77	0.79	0.72
Regional train	-0.200 (-20.57	-0.215 (-16.61)	-0.183 (-10.87)	1.05	1.00	1.12
Local train	-0.150 (-9.63)	-0.189 (-8.80)	-0.117 (-4.50)	0.79	0.88	0.72
Other time components						
Access/egress	-0.352 (-30.74)	-0.375 (-27.38)	-0.329 (-18.24)	1.85	1.74	2.02
Waiting time at transfers	-0.034 (-13.43)	-0.036 (-15.02)	-0.030 (-6.72)	0.18	0.17	0.18
Walking time at transfers	-0.087 (-7.45)	-0.083 (-5.48)	-0.098 (-5.27)	0.46	0.38	0.60
Other components						
Transfer penalty	-1.750 (-30.71)	-1.740 (-24.22)	-1.800 (-19.65)	9.21	8.06	11.04
Number of observations	5,121	2,667	2,454			
Null log-likelihood	-24,722	-13,063	-11,659			
Final log-likelihood	-12,592	-6,229	-6,327			
Adjusted rho square	0.490	0.523	0.457			

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3.2 Elaborate specification

Taking outset in the base specification, various alternative specifications including additional variables were estimated. This process led to the model specification and parameters presented in Table 3. The log-likelihood of this specification is significantly better than the base specification (-12,075 vs. -12,592), and all parameters are significant and with the expected sign. Looking at the rates of substitution, the results show that waiting time at the first stop is preferred compared to bus in-vehicle time. However, this is most likely due to the inclusion of the highest headway of the trip, because many trips only have one leg and the waiting time at the first stop depends on the headway and service type (SB/FB) of the first leg. When considering the highest headway of the trip, the reduction of one minute in bus in-vehicle time is equivalent to a reduction of the headway of 7 minutes. This headway should also be reflected in a lower waiting time at the first stop, so the reduction in utility would be greater than just the contribution from the headway parameter.

The distinction between waiting for a FB vs. SB service gives significantly different parameter estimates as waiting for FB services is four times worse than waiting for SB services. It is important to note, that the interval covered by FB waiting time is between 0 to 14 minutes, while waiting times for SB services extend into more than an hour. Moreover, tests using piecewise linear parameters and Box-Cox transformations for the split waiting times showed that waiting time for FB services is in all cases worse than waiting for SB services. The difference in the parameter estimates could be due to the higher uncertainty of waiting time for a FB service, as the passenger does not know when the next service will depart exactly. For SB services the waiting time is more certain, as it is given from the explicit timetable, and this could influence the passenger's route choice because they are more certain on when they will arrive at their destination.

The specification also includes a dummy describing whether the route includes a FB service. The parameter estimate of 0.545 indicates that passengers prefer routes with FB services compared routes without FB services. The positive parameter could be explained by the fact, that routes including FB services are typically high frequent routes, which gives a security for the passenger to not be significantly delayed if the first departure is missed.

Table 3 - Model estimates (robust t-test) for model with elaborate specification with linear parameters and rates of substitution scaled to bus in-vehicle time

	Model estimates			Rates of substitution (to bus IVT)		
	Trip purpose			Trip purpose		
Parameter	**All**	**Work**	**Leisure**	**All**	**Work**	**Leisure**
In-vehicle times						
Bus	-0.180 (-29.84)	-0.205 (-24.67)	-0.154 (-19.1)	1.00	1.00	1.00
Metro	-0.096 (-9.29)	-0.117 (-7.92)	-0.072 (-5.23)	0.53	0.57	0.47
S-train	-0.152 (-22.00)	-0.176 (-16.77)	-0.124 (-14.53)	0.84	0.86	0.81
Regional train	-0.209 (-19.83)	-0.221 (-16.00)	-0.196 (-10.30)	1.16	1.08	1.27
Local train	-0.156 (-10.45)	-0.193 (-9.90)	-0.126 (-4.40)	0.87	0.94	0.82
Other time components						
Access/egress	-0.377 (-29.79)	-0.405 (-26.48)	-0.351 (-17.83)	2.09	1.98	2.28
Headway	-0.026 (-8.61)	-0.028 (-6.39)	-0.025 (-5.80)	0.14	0.14	0.16
Waiting time at first stop	-0.100 (-5.31)	-0.118 (-5.05)	-0.085 (-2.79)	0.56	0.58	0.55
Transfer waiting time for SB service	-0.027 (-11.43)	-0.029 (-12.92)	-0.023 (-5.64)	0.15	0.14	0.15
Transfer waiting time for FB service	-0.116 (-12.36)	-0.128 (-9.35)	-0.105 (-8.16)	0.64	0.62	0.68
Walking time at transfers	-0.098 (-8.18)	-0.095 (-6.00)	-0.106 (-5.74)	0.54	0.46	0.69
Other components						
Transfer penalty	-1.820 (-29.68)	-1.830 (-23.54)	-1.850 (-18.98)	10.11	8.93	12.01
Trip include FB service	0.545 (8.17)	0.576 (5.75)	0.521 (5.75)	-3.03	-2.81	-3.38
Number of observations	5,121	2,667	2,454			
Null log-likelihood	-24,722	-13,063	-11,659			
Final log-likelihood	-12,075	-5,944	-6,089			
Adjusted rho square	0.511	0.544	0.477			

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3.3 Elaborate specification with Box-Cox transformations of in-vehicle time

As shown in the previous subsections passengers have different preferences for the individual sub modes. To test whether the marginal utility of each of the variables change depending on time spent in the vehicles, a specification identical to the one described in section 3.2 is estimated, however using Box-Cox transformations of all variables related to in-vehicle times. The resulting parameters of the estimation on the full dataset are for all non in-vehicle time parameters almost identical to the elaborate model presented in Table 3 and all parameters remain significant. The log-likelihood is improved from -12,075 in the linear elaborate model to -11,901 in the elaborate model with Box-Cox transformations. Figure 8 shows how the marginal utility decreases over time for regional train, while for S-train and metro the marginal utility increases. The high marginal increase in utility for in-vehicle time in metro ($\lambda = 2.02$) could be a result of the few seats in the metro and thereby simulating a standing penalty, which is mostly in place for longer trips of more than 10 minutes. For the in-vehicle time in S-trains the marginal increase is less than for metro ($\lambda = 1.47$), but with no tables at the seats and a high load on the trains in peak hours, the marginal increase for longer in-vehicle times seem behaviourally correct. The marginal decrease in utility for regional train ($\lambda = 0.70$) is as expected, as passengers in regional trains can use the time more efficient with tables at the seats and a general higher comfort level. Figure 8 only shows the utility on the central 95% of the observations, but from the figure it is clear, that for trips longer than an hour, the regional train is preferred. In-vehicle time for bus is almost linear ($\lambda = 0.95$) and could be explained by a high disutility for shorter trips, where a seat might not be available, and a lower disutility for longer trips, where a seat will often become available on the bus at some point.

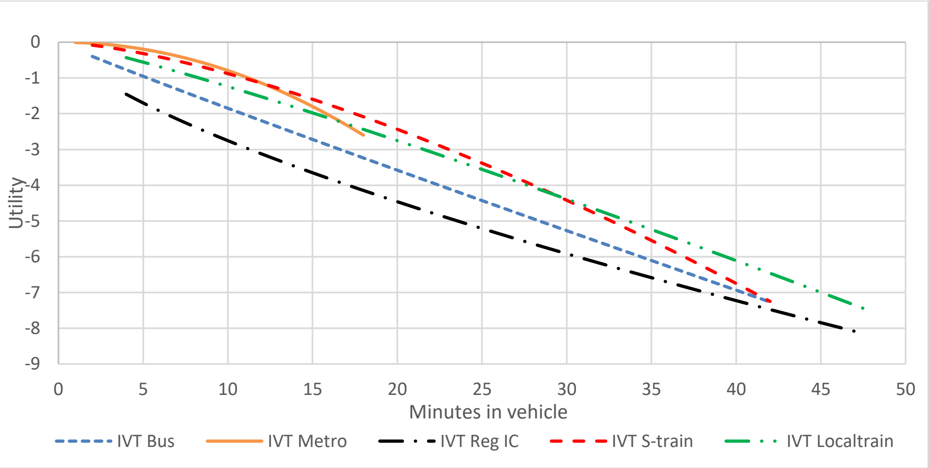

Figure 8 - Utility of in-vehicle time for model with elaborate specification with Box-Cox transformation of the variables related to in-vehicle time.
Curves only shown for the central 95% of observations for each variable (zeros excluded)

4 Discussion and conclusion

This section discusses and concludes on the findings presented in section 3 and how these findings can be used for further improving public transport assignment models.

4.1 Dealing with departure time choice within route choice models

An important aspect of public transport route choice is when the passenger can depart from the origin, and in most cases even more important when the passenger can arrive to the destination. For FB networks (and FB assignment models) this aspect is not as important, as departures are possible at all times. In SB networks (and SB assignment) the possible departure and arrival times are crucial, because passengers want to time their arrival to for example work or leisure activities. As the departure times are discrete in time, it might be, that passengers need to arrive earlier or later than the preferred arrival time. This time between preferred and actual arrival time is called hidden waiting time. Departure time choice is a well-established research field (see Thorhauge (2015, chap. 1) for a comprehensive list of previous studies), and for this study it would be relevant to include, as many alternatives have a departure time which differs significantly (more than 10 minutes) from the reported departure time. It has not been possible to estimate a parameter for this hidden waiting time, because the observed route will always be the best on this variable, making it impossible to estimate the parameter. Future research will focus on how to deal with this issue by for example fixing the parameter according to the total in-vehicle time based on stated preference surveys.

4.2 Implications of findings for public transport traffic assignment models

The findings of this paper can be used to model the route choice of public transport users in traffic assignment models at a higher level of detail. The difference in preferences for SB and FB services underlines the need to focus on creating an assignment model that can take both types of services into consideration. Modellers today are faced with only the choice between either a FB or SB assignment model to model a certain area, but if the model could represent realistically both FB and SB services there might be a potential benefit in the ability to better replicate passenger choices in public transport.

4.3 Conclusion

The estimations using real-life observed route choice data collected in a complex multi-modal public transport network have provided an insight into the impact of several factors that affect passengers' route choice in mixed SB and FB public transport systems. Findings related to preferences for waiting time for FB and SB services show that passengers prefer to be more certain about their waiting time. On the other hand, the positive parameter for whether a route includes a FB service shows that passengers prefer highly frequent services, which FB services typically are when compared to SB services. This indicates that passengers' value having many possible departures, which is in line with the preference for routes with lower headways. Finally, the paper showed, that the preferences for in-vehicle time in sub modes change according to how much time is spent in a specific sub mode.

Acknowledgements: This research was supported by the IPTOP project (Integrated Public Transport Optimization and Planning) funded by Innovation Fund Denmark.

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5 Paper 4: Relevance of detailed transfer attributes in route choice models for public transport passengers

The following pages contain the article:

O. A. Nielsen, M. Eltved, M. K. Anderson, and C. G. Prato (2020). "Relevance of detailed transfer attributes in route choice models for public transport passengers". Re-submitted after second round of review to Transportation Research Part A: Policy and Practice.

Please cite accordingly.

The paper was presented at *Trafikdage (Annual Meeting on Transport in Denmark) 2019*, Aalborg, Denmark, August 2019.

Relevance of detailed transfer attributes in route choice models for public transport passengers

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ABSTRACT

Given the aim of increasing public transport patronage, it is important to understand how passengers perceive different trip characteristics. Most of the existing studies about public transport demand and route choice assigned a higher value of time to transfers than in-vehicle time and used a general transfer penalty to capture an average increase in the travel disutility because of the amount of transfers. However, it is likely that there are nuances to the transfer behaviour depending on specific transfer conditions that existing models do not capture and hence it is difficult to evaluate measures aimed at improving transfers to make public transport more attractive.

This study presents a route choice model for the multimodal public transport network in the Greater Copenhagen Region where a variety of transfer attributes were explicitly considered within a unified model framework. The model was estimated on an extensive dataset of 4,810 observed routes that made it possible to evaluate the rates of substitution of transfer related attributes. The model results revealed that travellers do consider attributes for transfers such as ease of wayfinding, presence of shops and escalators at stations when choosing routes in the public transport network and this influences the attractiveness of the respective routes with a quite large range of the transfer penalty from 5.4 minutes compared to bus in-vehicle time for the best possible transfer to 12.1 minutes for the worst. Furthermore, the model results revealed some differences in the preferences for transfer attributes across passengers. This suggest a quite large potential for improving transfers and hence public transport patronage focusing on the attributes of the transfers.

KEYWORDS

Multimodal, Public Transport, Route Choice, Transfer Penalty, Transfer Attributes

1 INTRODUCTION

The attractiveness of public transport depends certainly on the services offered by the public transport agencies, but also on terminals and transfer conditions (Cascetta and Cartení, 2014). Public transport agencies may choose between a number of different suggestions to improve the terminals and thus create more attractive transfers. Supporting informed decisions requires the understanding of travellers' preferences and route choice behaviour to be able to predict traffic flows and passenger benefits under different scenarios.

In a public transport network, travellers perceive time differently according to how time is spent. It is for example well known that most people prefer travelling by train rather than bus (Nielsen, 2000; Fosgerau et al, 2007; Varela et al., 2018), as factors such as comfort and reliability are perceived as inherent mainly to travelling by train. It is also well known that a transfer in a public transport system not only adds disutility to the trip proportionally to the time spent on the transfer, but also includes a fixed disutility for each additional transfer on the trip, also known as transfer penalty. Previously, many public transport route choice models and value of time studies have revealed quite large transfer penalties varying between 5 and 20 minutes of in-vehicle time (Van der Waard, 1988, Vrtic & Axhausen, 2003, Bovy & Hoogendoorn-Lanser, 2005, Nielsen, 2004, Nielsen & Frederiksen, 2006), where the penalties added a fixed disutility regardless of the characteristics of the transfer stations or terminals.

In recent years, different studies have focused on exploring the differences of the experienced penalty of transferring by considering transfer related characteristics. Iseki and Taylor (2009) described the importance of modelling transfer penalties and suggested that three factors are key contributors: (i) operational factors that influence the waiting time at the terminal, (ii) physical facilities at the terminal such as safety and security measures as well as facilities to provide comfort such as correct signing and shelters, and (iii) factors relating to passengers' familiarity with the network. For unimodal (metro) networks, Raveau et al. (2011), Guo and Wilson (2011) and Raveau et al. (2014) estimated the effect of attributes such as escalator presence, differences in platform levels and ramp lengths, as well as network knowledge of the passengers. For multimodal networks, Chowdhury and Ceder (2013) and Chowdhury et al. (2014) looked at the effect of attributes such as transfer information, real-time display and security measures; Navarrete and Ortúzar (2013) estimated the effect of intermodal transfers (bus/metro) and escalator presence at transfers; Schakenbos et al. (2016) took a more aggregate approach defining different typical transfer stations based on shop availability; Anderson et al. (2017) modelled the effect of intermodal transfers; and Garcia-Martinez et al. (2018) modelled intermodal transfers, real-time information and difference in levels when transferring.

All the multimodal studies were based on surveys, including stated preference (SP) surveys, while most unimodal studies were based on observed trips. For the unimodal studies, Gou and Wilson (2011) mentioned that it was difficult to obtain detailed data about the transfer attributes, and Raveau et al. (2011) mentioned that a limitation of their study was the unimodality and hence the inability to describe the full journey, which in many cases consisted of combinations of legs with bus, train and/or metro. The limitation of unimodality has been approached in the multimodal studies (Chowdhury and Ceder, 2013, Chowdhury et al., 2014, Navarrete and Ortúzar, 2013, Schakenbos et al., 2016, Anderson et al., 2017, Garcia-Martinez et al., 2018), but none of these studies have tackled the issue of obtaining detailed data about the transfer attributes, as they have used SP surveys to estimate their effect.

The aim of this study is thus to analyse in detail the components of transfer penalties by modelling observed route choices in a multimodal network and collecting relevant transfer attributes of the respective transfer terminals. The analysis was performed on the multimodal public transport network of the Greater Copenhagen Region, which is served by metro, three different train services (local, regional and suburban) and bus services. The analysis focused on revealed preferences by modelling 4,810 actual route choices that were collected and map-matched for all trips with both start and end in the Greater Copenhagen Region (for details, see Anderson, 2013).

The relevance of the transfer attributes to the route choices of travellers was investigated via the estimation of route choice models with different formulations to capture the effects of transfer attributes on passenger preferences. Choice sets were generated via a doubly stochastic generation method (Nielsen, 2004; Rasmussen et al., 2016) that produced up to 200 alternative routes to each observed route. Most importantly, the estimation of route choice models considered several specifications given the exploratory nature of the study and the absence of reference values for the sensitivity to transfer attributes. The applications of the findings are as follows: (i) they can be applied to suggest effective improvements to existing transfer stations with the aim of decreasing the disutility of public transport trips, (ii) they can be applied to provide design guidelines to new transfer stations, and (iii) they can improve route choice models for public transport enabling them to evaluate overall passenger effects of improved public transport terminals.

The remainder of this paper is structured as follows. Section 2 introduces the transfer attributes and their measurement. Section 3 provides the description of the methodology and Section 4 presents the case-study. Section 5 illustrates the results prior to the last section discussing the results and giving recommendations about the conclusions from the findings of the study.

2 SELECTION AND DEFINITION OF TRANSFER ATTRIBUTES

As aforementioned, several studies in the past decade have focused on unraveling passenger preferences while considering the characteristics of the transfer terminals. Table 1 summarizes the transfer attributes considered in these previous studies and differentiates whether the attributes have been estimated or just mentioned as possibly considered in the route choice. All the attributes in table 1 can potentially be relevant to the passengers' route choices, but the data availability and the variable definition possibility for modelling purposes can be critical. The process of selecting the most important variables for passengers' route choice and the possible definition of the variables is described below.

Table 1. Transfer attributes in previous studies (attributes with an **X** have been estimated, while attributes with an (x) have only been mentioned)

Study	**Level changes**	**Escalators**	**Ramp length**	**Shelter**	**Real-time information**	**Ease of wayfinding**	**Security**	**Shopping availability**	**Apperance**	**Seats at platform**	**Safety**
Anderson (2013)								(x)			
Chowdhury and Ceder (2013)					**X**	**X**	**X**				
Chowdhury et al. (2014)				**X**		**X**				(x)	
Garcia-Martinez et al. (2018)	**X**				**X**						
Gou and Wilson (2011)	**X**	**X**	**X**								
Iseki and Taylor (2009)	(x)	(x)		(x)	(x)	(x)	(x)		(x)		(x)
Navarrete and Ortúzar (2013)		**X**				**X**					
Raveau et al. (2011)	**X**	**X**									
Raveau et al. (2014)	**X**	**X**									
Schakenbos et al. (2016)								**X**			

2.1 Selection and definition of transfer attributes to be estimated

Since the data for this paper is from the period 2009-2011, real-time information was not included in the long-list of variables to consider and also ramp length was disregarded as it would almost only be present for transfers to and from metro services in the Greater Copenhagen network. With the aim of finding the most suitable attributes to consider for the estimations, the remaining eight attributes from table 1 were then evaluated in further detail with regard to four criteria: validity, reliability, measurability, and data availability (Dyrberg & Christensen, 2015).

The *validity criterion* defined how well an attribute measures what it is supposed to measure. In this case, how much impact the attribute has on the transfer. The *reliability criterion* evaluated how objective the measure of the attribute can be: ideally, two independent measures of the same attribute under the same circumstances would give the same result, but this might be a problem when measuring more qualitative attributes. The *measurability criterion* assessed how easy to measure each of the eight attributes are. The *data availability* checked how demanding the data collection would be for each attribute (Joumard et al., 2010).

Table 2 shows the long-list of eight attributes along with the evaluation over each criterion. Shelters, shops and level changes were rated as "high" on all four criteria and were all included in the short-list of attributes to collect data for. Seat availability at a transfer performed rather well in all criteria except for validity, due to the fact that travellers do not always use seats. Furthermore, this attribute would probably be correlated with the attribute representing shelters. Occupancy scored well in several criteria without making it to the short-list, mainly due to poor data availability. Ease of wayfinding was rated as "high" or "medium" for all criteria, and despite the fact that this attribute has some issues especially on the reliability criterion, it was considered to be highly valid and was therefore included in the short-list. The attributes appearance and safety were both disregarded as they were rated as “low” in terms of validity and measurability. Security was also disregarded as it would be difficult to measure considering the “medium” validity and measurability.

Data were then collected for the four attributes shelters, shop availability, level changes and ease of wayfinding. However, during the estimation of the route choice models the shelter attribute was found to be non-significant regardless of the model specification and hence the attribute is not discussed further. The data collection and specification for shop availability, level changes and ease of wayfinding follows below.

Table 2. An overview of how each attribute has been rated according to the four criteria

	Validity	Reliability	Measurability	Data availability
Appearance	Low	Low	Low	Low
Seats	Medium	High	High	Low
Safety	Low	Low	Low	Medium
Security	Medium	Medium	Medium	Medium
Shop availability	High	High	High	High
Level changes	High	High	High	High
Shelters	High	High	High	High
Ease of wayfinding	High	Medium	Medium	Medium

Shop availability

Three levels of shops were considered to capture shop availability at a given transfer: (i) no shop; (ii) a kiosk; (iii) several shops. *No shop* was defined as a transfer without access to any kind of shop. This is often the case for bus-bus transfers away from stations. *A kiosk* was defined as a transfer with access to a small shop where it is possible to buy snacks, drinks, magazines, tickets etc. Many suburban train (S-train) stations have at least a kiosk. *Several shops* were registered when there is more than a kiosk, for example a grocery store, a bakery or any other type of shops. Some stations have a shopping mall right next to the station and transfers here are registered with several shops, since there is direct access to the shopping mall from the station.

Ease of wayfinding

Ease of wayfinding describes how easy it is to find the direction from the stop where the traveller arrives to the stop where the next transport mode is departing. Notably, the information level can affect the perceived and actual transfer walk time (Iseki and Taylor, 2009). As a longer walk exposes the travellers to more situations where they risk taking the wrong turn or getting lost, the two parameters are considered somewhat correlated. The ease of wayfinding was divided into four categories, where each category was defined as a dummy since a transfer could only be within one of the four categories: (i) easy; (ii) low difficulty; (iii) moderate difficulty; (iv) difficult.

Easy was defined as a transfer where it is straightforward to find the direction between the arriving and the departing stop, as the information level is good and the departing stop can be found intuitively. An example could be a transfer from one train to another where the two trains arrive at and depart from the same platform.

Low difficulty described transfers where it takes more than a few seconds to find the direction for the next departing stop, but then it is still quite simple to find the departing stop given the information signs. The distance from the arriving stop to the departing stop is often relatively short because it is somewhat correlated to the number of times a traveller risks getting lost.

Moderate difficulty indicated transfers where it is more difficult to find the right direction towards the next departing stop and, once the direction is found, there are still risks of getting lost on the way. Moderate difficulty often includes transfers where the distance between the arriving stop and the next departing stop requires walking for several minutes, consequently increasing the risk of getting lost. Another possible transfer with moderate difficulty is when finding the direction corresponds to low difficulty but, once the departing area is reached, it is confusing to understand where the specific line is departing from. An example is finding the correct bus stop at a larger terminal with many buses, finding the right platform for a specific train when there are many platforms to choose from, or other similar situations.

Very difficult defined transfers where the information level is low and/or the travellers find it very difficult to find the direction when transferring from an arriving stop to the next departing stop, and it may be confusing where the specific line is departing from. The traveller risks getting lost more than once after locating the departing stop, or facing many options when it comes to finding the correct way. Only few transfers in the case study presented below were characterized as being very difficult.

Level changes

The number of level changes per transfer corresponded to the number of times a traveller has to ascend or descend stairs. Furthermore, it recorded whether escalator assistance was available for the ascend or descend, and if so the escalators were assumed to be used. The case study area does not include stations with more than approximately three storeys vertical difference and hence the vertical height differences was captured by counting the number of ascending and descending stairs and escalators. The number of ascends and descends at transfer stations were summed for the whole alternative route, and since a general transfer penalty is included in the models, the number of ascends and descends with and without escalators describes the difference to transfers without level changes.

3 ROUTE CHOICE MODELS

Passenger preferences were estimated within a discrete choice modelling framework after generating a choice set for each of the observed routes and retrieving information about trip components and attributes of the transfer stations.

3.1 Model formulation

A random utility model assuming that each traveller n maximises his or her utility by choosing route i among a possible set C_n of routes is assumed. The deterministic part V_{ni} of the random utility U_{ni} for each route is formulated as a linear-in-parameter function (Ben-Akiva & Lerman, 1985).

In a multi-modal public transport network, travellers choose their route among a number of alternatives that might overlap to some extent. Accordingly, the probability was formulated according to a Path Size Correction (PSC) Logit model by considering the length of the common links between routes (Bovy et al., 2008)

$$P_{ni} = \frac{\exp(V_{ni} + \beta_{PSC} \cdot PSC_i)}{\sum_{l \in C_n} \exp(V_{nl} + \beta_{PSC} \cdot PSC_l)}$$

where PSC_i is the Path Size Correction factor for route i, and β_{PSC} is the parameter to be estimated. PSC_i is given by the following expression (Prato, 2009):

$$PSC_i = -\sum_{a \in \Gamma_i} \left(\frac{L_a}{L_i} ln \sum_{l \epsilon C_n} \delta_{al} \right)$$

where L_a is the length in minutes of link a between stops, L_i is the length in minutes of route i, Γ_i is the set of links belonging to route i, δ_{al} is the link and route incidence dummy equal to one if route i uses link a and zero otherwise. The Path Size Correction factor varies from $-\infty$ to 0, where 0 represents a completely independent route.

Heterogeneity across travellers was considered with the estimation of a Mixed PSC Logit model. The density distribution for each of the parameters were considered as being either normally or log-normally distributed, and the probability P_{ni} of traveller n choosing alternative route i within the choice set C_n was expressed as:

$$P_{ni} = \int \frac{\exp(V_{ni})}{\sum_{l \in C_n} \exp(V_{nl})} f(\beta) d\theta$$

where the probability is integrated over the distributions of the θ's, which can be either entered in the model as non-distributed, lognormal distributed or normal distributed parameters. All models were estimated using PythonBiogeme (Bierlaire, 2016) and the probabilities of the Mixed Logit models were simulated with 500 draws for the final models.

To test whether the extra transfer related variables and allowing heterogeneity across travellers give a significant better model fit the likelihood ratio test (LRT) is used. The test statistic is chi-squared distributed and takes into account the number of restricted parameters in the restricted model compared to the unrestricted model.

$$LRT = -2\left|\mathcal{L}_R(\hat{\theta}) - \mathcal{L}_U(\hat{\theta})\right| \sim X^2_{DOF}$$

The following subsection introduces the model specification of the base model, which is a restricted model of the model including transfer related variables.

3.2 Model specification

It is well-known from the literature that time and cost are very important for public transport travellers. Previous studies have included various time components, namely in-vehicle time, access/egress time, and transferring time (waiting/walking), as the main descriptors of the passengers' route choices together with a general transfer penalty (van der Waard, 1988, Nielsen, 2000, Bovy and Hoogendoorn-Lanser, 2005, Tørset, 2005, Eluru et al., 2012). Also fare and/or frequency of the lines are commonly used parameters in the models (Vrtic and Axhausen, 2003, Abrantes and Wardman, 2011, Navarrete and Ortúzar, 2013, Schakenbos et al., 2016).

In this study, the first and last part of a trip using public transport is always by walking or bicycle when considering the trip at the address level. The access and egress time used to the public transport network can be a considerable part of the total trip and is perceived as a disutility which the travellers seek to minimize and can be modelled in great detail as seen in for example Park et al. (2015). For this study, the level of information about the access and egress is only based on the time it took and whether it was made by walking or cycling. The access and egress times were therefore calculated via a simple regression model that takes into account that longer access/egress trips are typically made by bike compared to shorter access and egress legs (Anderson, 2013).

The in-vehicle travel time is also a factor that the passengers try to minimize, and the literature indicates that they do not perceive travel time in different public transport sub-modes to be the same. For example, many studies (Nielsen, 2000, Anderson et al., 2017, Varela et al. 2018) suggested that travellers prefer trains to buses (more than can be explained by the higher frequencies, faster travel time, etc.).

In public transport network representations, there is in all systems a hidden waiting time that captures the fact that passengers cannot always time their departure to the first stop or the arrival to the destination at their preferred time. To capture this in the model, the least frequent service in the route is used as an indicator of the hidden waiting time for the route. This variable is defined as half of the headway of the least frequent service of the route, which was also found to be a good indication of the hidden waiting time in Anderson (2013).

Although fares and prices have been considered relevant in many studies, this is not a parameter to include for the case-study considered. The fare structure in the case-study area is zone-based and the price of the trip is based on the furthest away zone visited during the trip. This will in almost all cases, with very few exceptions, be the destination zone, and hence the price for the trip will be the same no matter which route is taken. Also, there are no price difference for the specific sub-modes used or extra costs associated to transferring between services. Hence, it was decided not to include the price of the trip in the analysis.

In section 5 the results of a model including these well-known descriptors will be used as a baseline model, which is then extended with the selected transfer attributes from section 2. The chosen specifications of these new attributes are presented in section 5.2.

4 CASE-STUDY

4.1 The multimodal public transport network

The study analysed the multi-modal public transport network of the Greater Copenhagen Region that includes metro, S-trains, local trains, regional trains and busses. The main train corridors in the Greater Copenhagen Region are radial, going out from Copenhagen. Only one circular train line is operated and this is located rather close to the Copenhagen centre and high frequent express busses serve the circular roads further away. The S-trains are operated with 5-10 minutes headways, the metro with 1½-3 minutes, and the regional trains have headways varying from 20 to 120 minutes. Busses near the Copenhagen Central Business District (CBD) have headways of 3-10 minutes and busses in the outskirts of 20-60 minutes. The Region has a population of about 2 million people and is the most densely populated area in Denmark.

The network structure is complex and a public transport trip between two points often has several competing route alternatives consisting of different transport sub-modes, transfer terminals, in-vehicle times, transfer times, etc. The network database originates from the Danish National Model (NTM) (also described in Anderson et al., 2017). This is a schedule-based network consisting of 369 public transport lines with a total of 18,487 daily runs, 5,021 stop groups (292 train stations and 4,729 bus stops) with 1,718 between-mode transfer options (multi-modal transfers). The stop groups are represented by nodes and consists of closely located stops served by one or several lines of the same type, for example two bus stops at each side of a two-way road. The transfers in NTM are described by transfer edges between the stop groups with a transfer walking time depending on the length of the edge and transfer waiting time in the final node. For this study, the transfers were considered in more details as described in section 4.3.

4.2 Revealed preference survey

The observed routes were collected by Anderson (2013) as part of the Danish Travel Survey. The travel survey collected information about actual trips from a representative sample of the Danish population between 10-84 years. The respondents were asked to describe in details all their trips conducted at a specific day with both private and public transport modes. Anderson (2013) map-matched the details of the observed trips to the GIS network described above by identifying the actual public transport lines, bus stops, train stations and schedules used by the traveller. As described in Anderson (2013), not all of the observations were possible to match to timetable. Since the observations was matched to a planned timetable for a representative day, some observations were also discarded since the reported and matched times did not correspond, which could be due to timetable changes on the specific day of the observation or large delays in the network. The dataset used in this study consists of 4,810 observed trips and routes in the public transport network. The purpose of the observed trips was also collected and the trips were divided into two main purposes: 2,553 work related trips (commute and work trips) and 2,257 leisure trips.

The observed trips are mainly using the radial fingers of the network, with some trips using the different combinations of lines crossing the city. As seen in Figure 1, the trips are expectedly distributed across the day, with the work related trips following the typical pattern of outbound trips in the morning and homebound trips in the afternoon. The leisure related trips are primarily taking place between 10am and 6pm, but also with some trips departing in the evening hours.

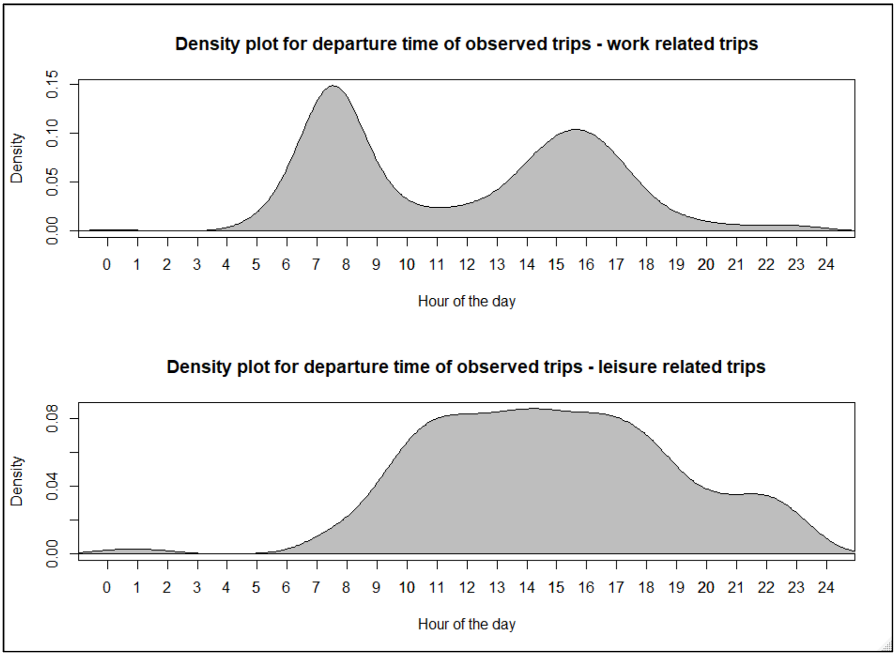

Figure 1 - Density plots for the departure times of respectively work and leisure related trips

Most of the observed trips are direct trips with no transfers (58 %). The rest of the trips include transfers and 34.7 % of the total number of trips have a single transfer, 6.7 % of the trips have two transfers and 0.6 % have three transfers. Only a single observation has four transfers. The headways in the model area are, as outlined above, in general low, which also results in low waiting times per transfer. The average waiting time for observed transfers is around 7 minutes and few observations have an average waiting time per transfer of more than 10 minutes as seen in Figure 2. Most observations have low average waiting times per transfer of less than five minutes and many of the observations have zero minute waiting times giving a perfect coordination in the correspondence between services.

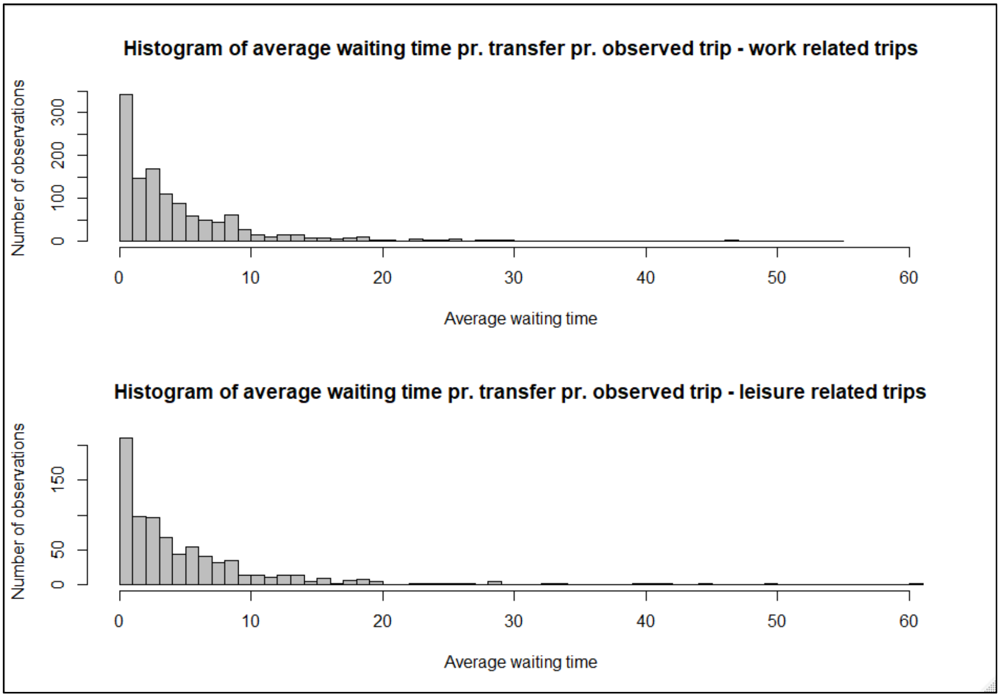

Figure 2 - Histogram of average waiting time per transfer per observation for the two trip purposes

Information about the non-chosen alternatives for each traveller was also needed in order to estimate the route choice models. The method used is based on repeated searches for the shortest path, where impedances are randomly drawn from a distribution in a doubly stochastic generation function (Nielsen, 2004). The choice set in Anderson (2013) was generated with 200 iterations after which the routes that were not unique or coincided with the actual chosen route were removed from the choice set, providing choice sets with 18-200 alternative routes for the respective observed routes. Rasmussen et al. (2016) investigated the robustness of this choice set generation procedure for model estimation purposes for the same case study. The reader is referred to the two mentioned studies for further details of the generated choice sets as these have been reused in the present study. However, it should be mentioned that for the purpose of this study further tests were conducted to check the removal of some irrelevant alternatives. This was, however, not found to significantly improve the model fits, which is in line with the finding in Rasmussen et al. (2016), that a large choice set is superior to a choice set with some relevant alternatives missing.

4.3 Data collection of transfer attributes

Information about all stations in the Greater Copenhagen Region were collected in order to assign the correct attributes to each station and thereby determine the design of each transfer. The stations were divided into two groups, where the first included the 20 largest transfer stations with the attached bus stops and the second included the remaining stations and bus stops. The 20 largest transfer stations cover 65% of all transfers in the observed data and each of these have been examined for very detailed information about every possible transfer. For the remaining 35% of the transfers a more general method was used to determine the transfer attributes. The stops for each sub-mode were divided into several categories explaining the transfer attributes for the specific stop: bus (two groups), metro (four groups), S-train (eight groups), regional/intercity train (seven groups), local train (two groups). The data was collected by using Google Streetview, personal knowledge of the network and visits to some of the stations. All the variables included in the final model are summarised in table 3, which includes descriptive statistics about the number of routes which include the specific variable.

From the collected data on transfer attributes, it is also possible to investigate the correlation between specific transfer attributes on the transfers in the network, which is important to investigate as mentioned in Hoogendorn-Lanser et al. (2006). Figure 3 shows the correlation between the collected transfer attributes on the different transfers in the network. Many of the correlations are intuitive, for example that the number of ascends and descends is positively correlated since many transfers involve a footbridge or tunnel to connect the stops and hence both an ascend and descend. The positive correlation between the easy wayfinding and respectively no level changes and no shops at the transfer is also intuitive, but importantly these correlations are not critically high being respectively 0.31 and 0.24. The negative correlations between the variable for no level changes and the variables for stairs and escalators is the result of many of the transfers having one of the attributes, but all others are then non-existing for the other variables and hence there is a negative correlation with the "no level change" variable. For the shopping variables, a similar pattern appear, where the transfers with several shops does not just have one shop, and hence there is a high negative correlation between these variables. It is noticeable that the very difficult wayfinding is not significantly correlated with any of the stairs and escalator variables, which indicates that the definition of the very difficult transfers is able to distinguish itself from just reflecting ascends and descends.

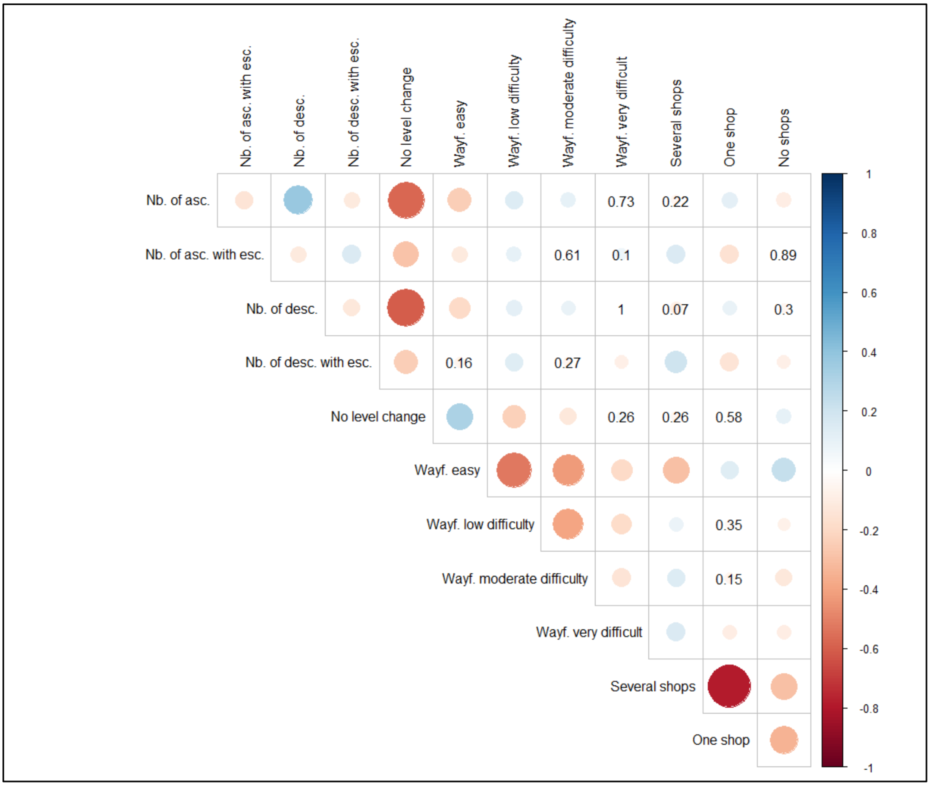

Figure 3 - Correlation between the transfer attributes for the transfers in the network (color indicates correlation and non-significant correlations are marked with the associated p-value)

Table 3. Descriptive statistics for the observed routes

Parameters	Work Mean (For obs. incl att.)	Work Std. dev. (For obs. incl att.)	Work Obs. incl. attribute	Leisure Mean (For obs. incl att.)	Leisure Std. dev. (For obs. incl att.)	Leisure Obs. incl. attribute
Time components						
Bus (min.)	15.81	10.80	1,278	14.96	11.34	1,229
Local train (min.)	19.27	10.81	83	20.99	13.80	69
Metro (min.)	7.16	4.80	541	6.96	5.10	445
Reg. train (min.)	23.66	12.44	355	23.48	13.60	192
S-train (min.)	17.15	12.44	1,317	16.31	12.05	937
Access (min.)	6.97	6.11	2.553	5.97	5.43	2,257
Egress (min.)	7.00	6.48	2,553	6.37	6.90	2,257
Transfer attributes						
Walking time	2.97	1.32	1,043	2.91	1.35	620
Waiting time	6.81	7.51	1,069	7.69	9.79	687
Number of transfers	1.20	0.44	1,228	1.18	0.42	795
Ease of wayfinding transfers						
Easy	1.05	0.22	484	1.09	0.28	267
Little difficulty	1.07	0.26	685	1.08	0.29	444
Moderate difficulty	1.03	0.16	195	1.02	0.15	135
Difficult	1.00	0.00	31	1.00	0.00	33
Shop level						
Shop av. at any transfer	1.00	0.00	1,106	1.00	0.00	686
Level changes						
Ascending stairs at transfers	1.14	0.36	456	1.13	0.37	267
Descending stairs at transfers	1.13	0.34	478	1.13	0.34	289
Ascending escalators at transfers	1.28	0.45	259	1.33	0.48	158
Descending escalators at transfers	1.36	0.50	250	1.39	0.50	140
Overall measures						
Half of highest headway in trip	6.87	9.20	2,553	7.23	7.87	2,257
Total trip time	40.22	19.61	2,553	34.31	21.74	2,257
Crow flies distance	13.00	10.47	2,553	9.77	10.11	2,257
Number of observations		2,553			2,257	

5 RESULTS

This section first presents a base model where none of the transfer attribute variables are included. This is followed by the presentation of the model with the best specification of transfer related attributes. Finally, a model that allows for heterogeneity in passenger preferences, based on the model with the best specification of transfer attributes, is presented.

5.1 Base model

Table 4 shows a base model similar to Anderson et al. (2017), estimated with reasonable sizes and signs for in-vehicle time (IVT), number of transfers, waiting and walking time at transfers.

Both the base model and the model with transfer related attributes included were estimated with a path-size factor. However, since the model with transfer related variables yielded a non-significant path size correction term (PSC), only the results of the models without the PSC-term are presented to ease comparisons between the models.. The PSC would normally correct for overlapping routes in a way where utilities for overlapping routes are reduced. However, some studies of public transport have showed negative estimates for path-size terms (Hoogendoorn-Lanser & Bovy, 2007, Anderson et.al. 2017), most likely because this corresponds to having more opportunities to reach their destination from their origin. Given the inherent risk of delays and irregularity in public transport networks, travellers might simply value the availability of a large number of en-route alternative options over the uniqueness of the route (Anderson et.al. 2017). The non-significance of the PSC in the present study may indicate a balance between normal correction of overlapping routes by the inclusion of transfer related variables.

For both travel purposes, the transfer waiting time rate of substitution is low; however, waiting time at transfers is always complemented by a transfer penalty, which is equivalent to roughly 8-9 minutes of bus in-vehicle time depending on the purpose of the trip. The low estimate can also to some degree be affected by the highest headway of services in the alternative, since a route with a high hidden waiting time can suit the passenger well, and possibly make the passenger disregard more frequent alternatives, where the passenger might need to walk further at the access and egress part.

For the in-vehicle time parameters, the different parameters are slightly different from each other. Trips using regional train also in most cases take shorter time compared to busses due to the higher speeds of the regional and intercity trains and this might affect the parameter estimate. Regional trains had also quite some punctuality problems in the period of the survey, which might explain the higher disutility for in-vehicle time for regional and intercity trains for the leisure related trips. The reason for the very low rate of substitution for the metro in-vehicle time might be explained by metro trips being relatively shorter journeys compared to e.g. regional trains and that the correction for highest frequency in the route does not fully cover the very high frequency of the metro, since trips using only the metro have low hidden waiting times.

Table 4. Estimated parameters and values scaled to bus in-vehicle time for the base models

	Work		Leisure		Rate of substitution (to bus IVT)	
Parameters	Coef.	*Rob. t-test*	Coef.	*Rob. t-test*	Work	Leisure
In-vehicle time						
Bus	-0.313	*-20.80*	-0.254	*-22.71*	1.00	1.00
Local train	-0.274	*-9.49*	-0.258	*-9.12*	0.88	1.02
Metro	-0.139	*-6.84*	-0.082	*-4.13*	0.44	0.32
Reg. and intercity train	-0.281	*-12.72*	-0.299	*-10.61*	0.90	1.18
S-train	-0.234	*-15.93*	-0.184	*-14.05*	0.75	0.72
Transfer components						
Transfer penalty	-2.480	*-18.75*	-2.320	*-19.88*	7.92	9.13
Transfer waiting time	-0.048	*-12.69*	-0.042	*-9.36*	0.15	0.17
Transfer walking time	-0.217	*-8.25*	-0.178	*-7.75*	0.69	0.70
Other components						
Access time	-0.488	*-18.14*	-0.441	*-23.90*	1.56	1.74
Egress time	-0.418	*-17.53*	-0.364	*-16.57*	1.34	1.43
Half of highest headway in trip	-0.120	*-8.48*	-0.114	*-11.15*	0.38	0.45
No. of est. parameters:	11		11			
Number of observations:	2,553		2,257			
Null log-likelihood:	-12,589		-10,765			
Final log-likelihood:	-2,993		-3,489			
Adjusted rho-square:	0.761		0.675			

5.2 Model with transfer attributes

Dyrberg & Christensen (2015) tested several specifications of the different transfer attributes defined in section 2 and found the most suitable representation given significance of the parameters, signs of parameters and overall model. These specifications were further tested for this paper and the different specifications tested are described below with Table 5 presenting the final MNL estimations for both work and leisure trips. A comparison with the restricted base model using a likelihood-ratio test shows that the model fit is significantly improved for both trip purposes by introducing the transfer attributes to the model, however with the highest degree of impact of the transfer attributes in the model for work related trips.

The parameters capturing **ease of wayfinding** were tested in four different specifications. Recalling that the ease of wayfinding was assigned a value from easy to difficult for each transfer, it was tested whether ease of wayfinding could be described by just one number: the *sum* of all the levels encountered (i.e a sum of the levels, when assigning the values 1-4 to the levels), the *maximum* (worst) transfer or the *average* of the levels. However, none of these definitions proved useful and thus the method of counting the individual levels of encountered transfers was found to give the best fit. During tests it proved to be of a high importance to include a general transfer penalty for each transfer encountered, so the different levels of ease of wayfinding were more distinct. The reference level to find differences between the levels was set to "easy". During tests it was found that there was no significant difference between the "little" and "medium" levels and thus they were combined. The negative parameters for the more difficult ease of wayfinding show that passengers prefer stops and stations with easy wayfinding. However, these parameters are significant only for work related trips, which could be explained by the fact that leisure passengers can be assumed to have less information about the available alternative routes and to have lower value of time, so that the ease of wayfinding does not play a crucial role in the route choices for this group of passengers. For the work related trips, the most difficult transfer stations have a much higher disutility compared to the stations with easier wayfinding. Our empirical results are in line with the hypothesis by Iseki and Taylor (2009) that the level of information has an influence on the perceived walk time.

Similar to the tests of the ease of wayfinding attribute it was tested whether a *sum, maximum, average* or *sum of individual levels* gave the best representation of the **shopping availability.** The tests showed that it did not matter for the passengers which types of shops or how many shops they encountered on the transfers on the route, but only whether at any transfer station there was a shopping possibility. The shopping parameter estimate is not highly significant for work related trips, but the positive estimate shows that passengers prefer routes where transfer stations offer some kind of shopping opportunity, whether this is a kiosk or a larger shop. Since the parameter is less significant for the passenger with a leisure related trip purpose, this indicates that shopping availability does not influence the route choice of leisure passengers because of their assumed lower knowledge of the network, but also that commuters find it attractive to have the opportunity of doing smaller grocery shopping en-route to and from work.

When testing the different specifications of the **level changes** parameters, only the sum of the escalators encountered at transfer terminals proved to be significant, leaving the model to describe the number of escalators encountered at transfers. **Escalators** at transfers are preferred by passengers for both trip purposes and the parameter estimate is significant for both purposes. The positive effect of escalator presence is in line with previous findings by Raveau et al. (2011), Guo and Wilson (2011) and Raveau et al. (2014). Escalators reduce the disutility of a trip by about one minute of bus in-vehicle time. The reason that escalators are experienced positively by the passengers could be explained by the fact that it reduces the effort of walking. Also, since the public transport system in the Greater Copenhagen Area does not experience excessive crowding, the escalators will in most cases move the passenger faster through the transfer station compared to stairs or long walkways.

The parameters for **waiting times at transfers** show a clear difference in terms of the significance and the estimates of small and higher waiting times. A total waiting time below 10 minutes is not significant, while the estimate for total waiting time over 10 minutes is significant. The non-significant parameter for low waiting times can be explained by the transfer penalties, which covers the annoyance of having a transfer in the route.

Table 5 also presents the rates of substitution with in-vehicle time by bus as the reference. For the in-vehicle parameters, the change from the base model is small and metro is still the preferred mode when compared to the other sub-modes (everything else being equal). The transfer penalty is still equivalent to roughly 8-9 minutes of bus in-vehicle time for both trip purposes. However, when different transfer attributes are included, a transfer can now be more or less convenient depending on the facilities at the transfer point. The transfer penalties can thus be dissected into several parameters that explain the different preferences for the different transfers, and the transfer penalty can range between 5.4 minutes of bus in-vehicle time for the best transfer (station with easy wayfinding, shopping available and two escalators) to 12.1 minutes for the worst (station with difficult wayfinding and no escalators or shops). Given that the transfer penalties account for up to 12.1 minutes of bus in-vehicle time, this can also be reflected by the insignificance of the small waiting times. It is important to mention, as shown in section 4.2, that very few observations include average waiting times above 10 minutes per transfer and hence these results show that passengers dislike routes with many (long) transfers.

Table 5. Estimated parameter coefficients (robust t-tests) and values scaled to bus in-vehicle time for extended model with transfer attributes

	Work		**Leisure**		**Rate of substitution (to bus IVT)**	
Parameters	Coef.	*Rob. t-test*	Coef.	*Rob. t-test*	Work	Leisure
In-vehicle time						
Bus	-0.309	*-20.51*	-0.251	*-22.42*	1.00	1.00
Local train	-0.272	*-9.47*	-0.259	*-9.04*	0.88	1.03
Metro	-0.144	*-7.10*	-0.087	*-4.30*	0.47	0.35
Reg. and intercity train	-0.288	*-12.72*	-0.300	*-10.55*	0.93	1.20
S-train	-0.238	*-16.17*	-0.185	*-14.07*	0.77	0.74
Transfer components						
Transfer penalty	-2.600	*-15.86*	-2.380	*-15.24*	8.41	9.48
Transfer waiting time 0-10 min.	-0.005	*-0.38**	-0.023	*-1.71**	0.01	0.09
Transfer waiting time +10 min.	-0.068	*-8.42*	-0.047	*-6.35*	0.22	0.19
Transfer walking time	-0.219	*-7.67*	-0.193	*-7.81*	0.71	0.77
Shop available at any transfer	0.176	*1.32**	0.111	*0.88**	-0.57	-0.44
Ease of wayfinding – Lit./Mod.	-0.285	*-2.20*	-0.165	*-1.27**	0.92	0.66
Ease of wayfinding - Difficult	-1.130	*-3.70*	-0.127	*-0.46**	3.66	0.51
Escalators at transfer points	0.384	*4.83*	0.267	*2.89*	-1.24	-1.06
Other components						
Access time	-0.484	*-17.95*	-0.440	*-23.87*	1.57	1.75
Egress time	-0.420	*-17.00*	-0.365	*-16.44*	1.36	1.45
Half of highest headway in trip	-0.119	*-8.35*	-0.113	*-11.08*	0.39	0.45
No. of est. parameters:	16		16			
Number of observations:	2,553		2,257			
Null log-likelihood:	-12,589		-10,765			
Final log-likelihood:	-2,965		-3,482			
LRT - to base model:	56.4	(p=0.00)	13.4	(p=0.02)		
Adjusted rho-square:	0.763		0.675			

*Parameter estimate not significantly different from zero at a 90% confidence level

5.3 Model capturing heterogeneity in passenger preferences

The estimation of an MNL model with the additional transfer variables showed that the different transfers can have a different impact on the transfer penalty perceived by the passengers. A Mixed Logit model was estimated based on the final MNL model to investigate possible heterogeneity in how passengers perceive the penalties. Initial models were run with only one parameter mixed at a time in order to assess whether a parameter should be included as a distributed parameter, and a final model was estimated where all parameters with a significant distribution were included until all the distributed parameters left in the model were significant. All time-related variables and the general transfer penalty were tested with log-normal distributions, while for the additional transfer variables it was tested if either log-normally or normally distributed parameters led to the best model fit. In the test of the single parameters it was found that the heterogeneity in perception of ease of wayfinding was better captured by normal distributions, while both shopping availability and escalators were perceived with heterogeneity better represented by positive log-normal distributions.

Table 6 shows the final Mixed Logit models for work related trips while Table 7 shows the final model for leisure related trips, where likelihood-ratio tests show that the added mixed variables significantly improves the model fit compared to the MNL model with transfer related variables. The in-vehicle time related variables only have significant distributions for some of these and it differs between the trip purposes, with only the parameter for in-vehicle time for regional train distributed for both purposes. The access and egress parameters have significant distributions for both purposes with similar standard deviations for both access and egress.

The general transfer penalty has a significant distribution with high standard deviation especially for the leisure trips, meaning leisure passengers perceive the penalty of transferring quite differently. The waiting and walking times did not show any significant distribution parameter for either trip purpose. Regarding the transfer attributes only the parameters for shopping availability for work related trips and number of escalators for leisure related trips proved to have significant distributions. The shopping availability for work related trips has a significant distribution, while the mean of the distribution is not highly significant. The ease of wayfinding is still insignificant in the model for leisure related trips, while they are significant in the model for work related trips and with an even larger disutility for stations with very difficult wayfinding.

With the aim of comparing the rates of substitution for different parameters with respect to the bus in-vehicle time, Monte Carlo simulations with 1 million draws from the distributions were performed and 95% confidence levels were calculated as shown in Table 6 and 7. The rate of substitution between the different in-vehicle time parameters are in general within expectations and the intervals are in general largest in the model for leisure related trips time and reflects that especially these passengers do have different preferences for the different sub-modes. The confidence intervals for access and egress rate of substitution is in general higher than the rate of substitution in the MNL model, but still with access being the most critical part of the access and egress to stops. The hidden waiting time is highly distributed leading to a large span in the rate of substitutions, with some passengers finding the possibility of departing frequently very important.

The rate of substitution for the transfer penalty is between 3.5 and 29.6 minutes of bus in-vehicle time, with a higher standard deviation for leisure passengers compared to passengers travelling for work related trip purposes. This suggests that the leisure passengers are quite a heterogeneous group of passengers, with possibly heterogeneous spatial patterns and time constraints throughout the day. The importance of walking time at transfers is now closer to bus in-vehicle time, while the importance of waiting time is still suppressed by the transfer penalties covering the annoyance of transferring. The distribution for shopping available at transfers for work related trips show that this is equal to 0.1 to 3.2 minutes of bus in-vehicle time while approximately the same range is the case for escalators at transfer for leisure related trips.

Table 6 - Mixed Logit estimates, means and standard deviation for the log-normal distribution, rates of substitution (w.r.t. bus in-vehicle time) and [95 % confidence intervals] – work related trips

	Work		**Parameters in equivalent normal**		**Rate of substitution (to bus IVT)**	
Parameters	Coef.	*Rob. t-test*	Mean	*Std. dev*	Mean	*95% confidence interval*
In-vehicle time						
Bus (μ)	-0.711	*-9.19*	-(0.51)		1.00	[1.00-1.00]
Bus (σ)	0.307	*5.91*		*0.16*		
Local train	-0.399	*-7.70*			0.85	[0.44-1.48]
Metro	-0.198	*-6.08*			0.42	[0.22-0.74]
Reg. and intercity train (μ)	-0.945	*-9.55*	-(0.41)		0.88	[0.33-1.91]
Reg. and intercity train (σ)	0.329	*4.44*		*0.14*		
S-train	-0.355	*-12.64*			0.76	[0.40-1.32]
Transfer components						
Transfer penalty (μ)	1.430	*15.40*	-(4.42)		9.44	[3.49-20.74]
Transfer penalty (σ)	0.335	*5.56*		*1.52*		
Transfer waiting time 0-10 min.	-0.008	*-0.38**			0.02	[0.01-0.03]
Transfer waiting time +10 min.	-0.109	*-6.30*			0.23	[0.12-0.41]
Transfer walking time	-0.387	*-8.20*			0.83	[0.43-1.44]
Shop available at any transfer (μ)	-1.580	*-1.44**	0.33		-0.71	[-3.16-(-0.06)]
Shop available at any transfer (σ)	0.984	*2.98*		*0.43*		
Ease of wayfinding – Lit./Mod.	-0.377	*-1.98*			0.8	[0.42-1.40]
Ease of wayfinding - Difficult	-1.840	*-3.76*			3.93	[2.05-6.85]
Escalators at transfer points	0.591	*4.33*			-1.26	[-2.20-(-0.66)]
Other components						
Access (μ)	-0.149	*-2.05*	-(0.93)		1.99	[0.66-4.66]
Access (σ)	0.392	*7.27*		*0.38*		
Egress (μ)	-0.276	*-3.48*	-(0.82)		1.75	[0.58-4.10]
Egress (σ)	0.390	*7.39*		*0.33*		
Half of highest headway in trip (μ)	-1.450	*-12.70*	-(0.71)		1.52	[0.02-9.48]
Half of highest headway in trip (σ)	1.490	*42.63*		*2.04*		
No. of est. parameters:	23					
Number of observations:	2,553					
Null log-likelihood:	-12,589					
Final log-likelihood:	-2,596					
LRT - to transfer model:	737.5	(p=0.00)				
Adjusted rho-square:	0.792					

*Parameter estimate not significantly different from zero at a 90% confidence level
-() for means indicate negative log-normal distributions. Estimate for shop availability at any transfer is the only positive log-normal distribution.

Table 7 - Mixed Logit estimates, means and standard deviation for the log-normal distribution, rates of substitution (w.r.t. bus in-vehicle time) and [95 % confidence intervals] – leisure related trips

	Work		**Parameters in equivalent normal**		**Rate of substitution (to bus IVT)**	
Parameters	Coef.	*Rob. t-test*	Mean	*Std. dev*	Mean	*95% confidence interval*
In-vehicle time						
Bus	-0.412	*-14.31*			1.00	[1.00-1.00]
Local train	-0.236	*-5.27*			0.57	[0.57-0.57]
Metro (μ)	-3.600	*-4.65*	-(0.25)		0.60	[0.00-4.04]
Metro (σ)	2.100	*5.38*		*2.23*		
Reg. and intercity train (μ)	-0.944	*-8.44*	-(0.49)		1.18	[0.25-3.53]
Reg. and intercity train (σ)	0.674	*6.19*		*0.37*		
S-train (μ)	-1.590	*-12.89*	-(0.27)		0.67	[0.11-2.23]
S-train (σ)	0.768	*12.56*		*0.25*		
Transfer components						
Transfer penalty (μ)	1.540	*12.64*	-(5.26)		12.76	[4.33-29.57]
Transfer penalty (σ)	0.490	*2.66*		*2.74*		
Transfer waiting time 0-10 min.	-0.024	*-1.10**			0.06	[0.06-0.06]
Transfer waiting time +10 min.	-0.096	*-5.86*			0.23	[0.23-0.23]
Transfer walking time	-0.400	*-7.57*			0.97	[0.97-0.97]
Shop available at any transfer	0.490	*1.97*			-1.19	[-1.19-(-1.19)]
Ease of wayfinding – Lit./Mod.	-0.337	*-1.51**			0.82	[0.82-0.82]
Ease of wayfinding - Difficult	0.127	*0.27**			-0.31	[-0.31-(-0.31)]
Escalators at transfer points (μ)	-0.700	*-1.95*	0.57		-1.39	[-3.43-(-0.42)]
Escalators at transfer points (σ)	0.533	*3.03*		*0.33*		
Other components						
Access (μ)	-0.138	*-1.60**	-(1.00)		2.42	[0.76-5.89]
Access (σ)	0.523	*3.84*		*0.56*		
Egress (μ)	-0.278	*-3.56*	-(2.05)		2.11	[0.65-5.15]
Egress (σ)	0.526	*12.02*		*0.49*		
Half of highest headway in trip (μ)	-1.410	*-13.11*	-(0.40)		0.98	[0.08-4.2]
Half of highest headway in trip (σ)	1.000	*10.92*		*0.53*		
No. of est. parameters:	24					
Number of observations:	2,257					
Null log-likelihood:	-12,589					
Final log-likelihood:	-3,017					
LRT - to transfer model:	929.6	(p=0.00)				
Adjusted rho-square:	0.717					

*Parameter estimate not significantly different from zero at a 90% confidence level

-() for means indicate negative log-normal distributions. Estimate for escalators at transfer points is the only positive log-normal distribution.

6 DISCUSSION AND CONCLUSIONS

This study has analysed how passengers consider attributes for transfers in public transport such as ease of wayfinding, presence of shops and escalators. We proposed different ways of defining and measuring these variables, and based on initial testing in Dyrberg & Christensen (2015) and further model tests, concluded on the variable definitions. We then presented route choice models for the multimodal public transport network in the Greater Copenhagen Region, where these transfer attributes were included in a unified model framework. The models were estimated on an extensive dataset of 4,810 observed routes. We believe that this is the first time that such an extensive dataset of observed routes has been used to estimate a route choice model that includes a variety of transfer related attributes that can be used for explanatory as well as predictive purposes.

The main overall conclusion is that it was possible to disentangle transfer penalties and values of time for transfers into the sub-components mentioned above, and to significantly estimate different parameters for this. In the specific case, this was used to improve the route choice modelling of passengers in the Greater Copenhagen Region and hence to make it possible to analyse policies to improve public transport terminals. While studies in the literature have provided ranges of fixed transfer penalties from 5 to 20 minutes for different cases, this paper disentangled the value for one transfer, ranging from 5.4 minutes for the best possible transfer to 12.1 minutes for the worst possible transfer.

Although it is difficult to compare the impedance of individual parameters defining the route choice preferences across studies, the range of the values for transfer related parameters in this study are in line with fixed values of transfer penalties and values of walking and waiting times in other studies. We therefore propose that they can be a guideline for more studies to disentangle transfer penalties in other cities, as the specific values and ratios between parameters may depend on the case context. The three additional transfer variables included in this study, namely presence of shops, ease of wayfinding and escalators at transfer points are easily measurable and more detailed data can be applied if for example the distance to shops at transfers is available in other datasets. Although different weather conditions in different countries can affect the magnitude of the impact on passengers' route choices, we expect that the measures are transferable to other cities in the world.

The large heterogeneity found in the preferences of different transfer attributes suggest that there might be other factors, which influence the transfer penalty. This could for example be different socio-economic factors, which influence the perceived importance of shopping availability or the comfort of having escalators at stations. The large heterogeneity also indicates that further research is needed to refine the definition of the attributes and to explain the differences in passengers' route choice preferences. An interesting line of research to explain more on transfer related variables could be including waiting time and boarding strategies at transfer stations (see for example Nassir et. al. (2019) or Schmöcker et. al. (2013). Analysing the problem using a sequential choice strategy could possibly allow for more detailed descriptions of the choice of different transfer stations.

Most politicians are focused on investments that improve the level of service of public transport operations, for example travel time savings or increase of frequencies, which require massive investments in infrastructure and rolling stocks, and which often only improve travel times or waiting times with few minutes. The study presented here suggests that improved transfers may be perceived by passengers to be of at least the same order of magnitude as such projects, whereas they are often much cheaper in terms of investments. We therefore recommend that more detailed route choice models and analyses are used when prioritising among investments in public transport.

7 ACKNOWLEDGEMENTS

The Danish Innovation Fund is acknowledged for funding through the IPTOP (Integrated Public Transport Optimisation and Planning) project. The authors are very thankful for the work done by Mette Dyrberg and Charlotte Christensen for the data collection and testing of variable configurations.

Declaration of interests: none

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[1] Modelling of public transport; Can existing transport be developed such that they become better at analysing public transport.

III

Studies on public transport passenger behaviour based on smart card data

6 Paper 5: Impacts of long-term service disruptions on passenger travel behaviour: A smart card analysis from the Greater Copenhagen area

The following pages contain the article:

M. Eltved, N. Breyer, J. Blafoss, and O. A. Nielsen (2020). "Impacts of long-term service disruptions on passenger travel behaviour: A smart card analysis from the Greater Copenhagen area". Submitted to Transportation Research Part C: Emerging Technologies.

Please cite accordingly.

The paper was presented at the 6^{th} *International Symposium Use of Public Transit Automated Data for Planning and Operations (TransitData2020)* held online 11-13 August 2020.

Impacts of long-term service disruptions on passenger travel behaviour: A smart card analysis from the Greater Copenhagen area

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Abstract

Disruptions in public transport are a major source of frustration for passengers and result in lower public transport usage. Previous studies on the effect of disruptions on passenger travel behaviour have mainly focused on shorter disruptions while the few studies on impacts of long-term disruptions have had limited focus on individual passenger behaviour. This paper fills the gap in research by proposing a novel methodology, based on smart card data, for analysing the impacts of long-term planned disruptions on passenger travel behaviour. Passengers are classified into clusters based on their travel behaviour and activity before and after the disruption using a k-means clustering algorithm, dividing passengers into eight groups. The method is applied on a 3-month closure of a rail line in the Greater Copenhagen area. The results showed no considerable difference between the passengers affected by the disruption compared to those passengers on a comparable segment not affected by track closures, hence suggesting that most passengers returned after the disruption. However, results indicate that new passengers are not attracted to the affected lines, thus resulting in a decrease in ridership on the disrupted line. The proposed methodology enables explicit analysis of the impact of disruptions on diverse passengers segments while the specific results are useful for public transport agencies when planning long-term maintenance projects.

Keywords: Public transport, Planned disruptions, Individual mobility, Smart Card, Travel behaviour, Passenger segmentation

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Preprint submitted to Transportation Research Part C *August 31, 2020*

1. Introduction

Disruptions in public transport are a major cause of passenger dissatisfaction (van Lierop et al., 2018) and can result in decreased public transport usage (Nazem et al., 2018). They have large impacts on passengers who rely on using the public transport network, especially in places where public transport constitute a large part of total travel, e.g. in metropolitan areas. But even in areas where public transport only constitutes a small share of total traffic, disruptions in the public transport network can have significant impacts also on road traffic congestion (Lo and Hall, 2006; Spyropoulou, 2020). This makes it important to not only consider impacts resulting from disruptions in the planning process, but also to identify detailed behavioural reactions from passengers resulting from such disruptions.

Disruptions in public transport systems can be *unplanned*, e.g. due to accidents or technical failures, or *planned*, e.g. due to construction works. Most previous literature on disruptions has focused on *unplanned disruptions*, e.g. by focusing on robustness of networks (Cats, 2016), network planning during disruptions (Van Der Hurk et al., 2016), and passenger information provision (Bruglieri et al., 2015). However, *planned disruptions*, such as the closure of a station or an entire corridor, are common, especially due to the aging railway infrastructure, which needs frequent maintenance and thorough upgrading. Such disruptions often last over a long period, thus adding to the complications for passengers who need to find alternative means of transport during the disruption period while during short-term disruptions passengers can better adjust their activities. Despite the large consequences for passengers, only few studies have analysed the impacts of planned disruptions on travel behaviour (Zhu et al., 2017; Nazem et al., 2018; Yap et al., 2018). However, these previous studies have either been based on a small dataset (Zhu et al., 2017), only covering limited track closure, i.e. single stations (Nazem et al., 2018), or focusing on prediction models rather than evaluating demand effects (Yap et al., 2018).

This paper contributes to existing literature by analysing in detail the effects of a long-term closure on the travel behaviour of public transport passengers. We analyse the passenger travel behaviour before, during and after a 3-month closure of an important suburban railway line in the Greater Copenhagen area in Denmark. Using smart card data from a large-scale automated fare collection (AFC) system, we identify different types of users based on their travel behaviour before the disruption. We analyse how the different users react to the disruption and how they change travel behaviour after the disruption. The novel approach proposed in this study allows for isolating the effect of the disruption from continuous dynamic changes in passenger behaviour. This is important as the analysis of a long-term disruption requires analysing data over long time, since users change travel behaviour for many reasons, not only caused by the disruption. Excluding those is achieved by comparing the changes in passenger behaviour on the disrupted track with passengers on a comparable track section. The use of AFC data makes such analysis more feasible than if applying traditional survey methods.

The remainder of this paper starts with an overview of the existing literature related to long-term disruptions in public transport in Section 2. The methodology of analysing the temporal changes in passenger behaviour is described in Section 3. Section 4 presents the case study and the smart card data used in the study for which we present results of analysing the effects of the disruption in Section 5. Finally, Section 6 discusses the main findings of the analysis and Section 7 concludes on the possible policy implications of the study.

2. Previous studies on effects of disruptions

The implications of unplanned disruptions, e.g. strikes and system failures, have been discussed in multiple studies. Currie and Muir (2017) found lower satisfaction levels among passengers experiencing unplanned rail disruptions than other rail passengers, and lowest satisfaction among passengers on replacement buses. During unplanned rail disruptions two thirds of users used the replacement buses, and 28% chose alternative modes. The largest concern by passengers was lack of information provision. The relatively large use of buses is in line with Saxena et al. (2019), which found that passengers perceived cancelled services 3 times more onerous than service disruptions due to delays.

Several studies analysed the impacts of long-term service disruptions resulting from strikes, which results in both cancelled trips and mode shifts. A 13-day transit strike in New York City in 1966 resulted in 10% of travellers cancelling their trip (50% on the first day), 16.7% switching to carpooling, and 50% switching to their own car. This had long-term effects of permanent decrease in ridership of 2.1-2.6% after service was restored (Zhu and Levinson, 2012). In Pittsburgh, the 1976 transit strike resulted in 38% of users switching to cars (alone and car-pool) while most travellers were dropped-off by a non-commuter (e.g. spouse). In California, larger effects of the strikes in 1981 and 1986 were seen as trips were reduced by 15-20% (Ferguson, 1992). In 1995, in the Netherlands 30% switched to driving and 10% cancelled their trips (Zhu et al., 2017). Reviewing the impacts across 13 major strikes, van Excel and Rietveld (2001) found varying impacts dependent on the importance of public transport compared to other modes.

Other studies focused on factors affecting behavioural change caused by service disruptions. Nguyen-Phuoc et al. (2018) found that long-term mode shifts after service disruptions were mainly influenced by context-specific factors such as car accessibility, travel time and travel costs. This is in line with Adelé et al. (2019), which found that user expertise, car availability, perception of service recovery time, opinions on passenger information services, available transport services, time constraints, and the moment and place at which communication about the disruption is received influenced user behaviour.

Only few studies analysed the influence of planned disruptions on travel behaviour. Mojica (2008) analysed behavioural changes of rail commuters during a large scale maintenance project in Chicago. Using AFC data the study found that the majority of users continued using the train during deteriorated service conditions whereas between 8% and 11% of the passengers used the bus system as a commuting alternative. Zhu et al. (2017) analysed the impacts of 15 separate planned maintenance projects on the Washington D. C. Metrorail lines. Based on travel surveys distributed before and after the disruptions three types of behavioural changes were identified, i.e. i) same behaviour, ii) change of mode, and iii) changed departure time. 20% of the 738 respondents did not return to using the metro after service was restored, thus suggesting a critical fallout of passengers after planned service disruptions. Utilising a large-scale dataset based on smart card data from Montreal, Canada, Nazem et al. (2018) analysed travel behaviour changes due to two separate 4-month single metro station closures. The study found that demand at both affected stations was reduced several months after the end of the disruption, hence suggesting long-term impacts on travel behaviour.

3. Methodology

To understand different aspects of the impacts of a long-term disruption, we propose a method of analysis on three levels (see Figure 1). The first initial step is an analysis of the impact on total ridership. Second, we propose a passenger segmentation based on k-means clustering to identify different travel behaviours before and after the disruption with the purpose of quantifying changes in travel behaviour. In the third level, an in-depth analysis of individual reactions to the disruption is done by using hierarchical clustering to segment certain passenger groups further according to their daily travel patterns.

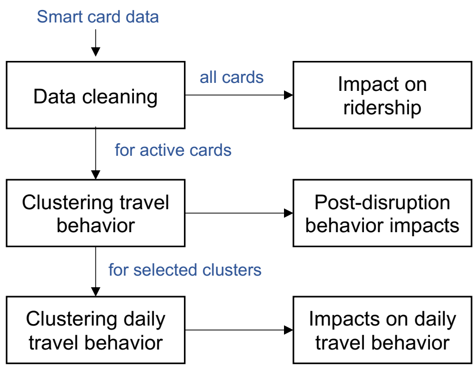


Figure 1: Overview of the method of analysing the impacts of the long-term disruption

Travel patterns for passengers are known to change significantly over time (Egu and Bonnel, 2020), even without disruptions. A challenge in analysing long-term smart card data is to distinguish between the change in ridership due to the disruption and due to regular seasonal variations. In addition, when alternative ticket types are available, passengers' use of the smart card compared to alternative ticket types might vary over time, which can lead to variations in the ratio between trips included in the smart card data and the total number of trips. Therefore, instead of directly analysing changes on the affected line, we compare the changes on the affected line to the changes on a reference line during the same period. The reference line is a line with similar characteristics as the affected line, but without any major disruptions during the analysis period.

3.1. Data cleaning and impact on ridership

The smart card data in our case study covers the entire region of East Denmark, which is much more data than what is needed to understand the impacts of a long-term disruption on a particular line. Hence, the first step is to extract the relevant data. We exclude invalid trips, e.g. those with missing tap-out information. Both the affected and the reference route are defined by a list of specific stations. We consider a trip to be using the route if at least one of the stations was used. This may be the first tap-in at the start of the trip, the final tap-out at the end of the trip, or a transfer tap-in when changing between bus and train or vice versa at one of the stations.

For the initial analysis of the impact on ridership, we count the number of passengers on the affected and the reference route. This allows for a comparison of the ridership over time on the affected route to an unaffected route thereby controlling for the general trend.

3.2. Clustering of travel behaviour and post-disruption impacts

To analyse changes in travel behaviour, we first define an analysis period consisting of three sub-periods: *pre*, a period with normal operations before the disruption, *affected*, the period of the disruption, and *post*, a period with normal operations after the disruption. For the clustering, only cards that have been active at least once before the pre-period and after the post-period are used. This ensures that the card existed and was available for travel during the whole analysis period.

To understand how the disruption impacts different passenger groups, we segment the smart cards into a number of groups using data clustering techniques based on the travel behaviour revealed in each smart card, as also proposed in several previous studies (El Mahrsi et al., 2017; Briand et al., 2017; Kieu et al., 2015). Each card must have had at least one trip on either the affected or the reference route during the pre- or post-phase to be included. While this constraint significantly reduces the number of cards included in the analysis, it is necessary as other cards cannot explain the possible implications of the disruption considering that the analysis only concerns the affected and reference route.

The resulting segmentation from clustering is specific to the data used as input. To be able to compare and ensure consistent groups, we perform one clustering of all observations at once, where each observation is a feature vector for a given card during a given period. The observations to cluster is the set of all feature vectors $V_{c,p}$ where c is a smart card and $p \in \{pre, post\}$ a period.

Each feature vector $V_{c,p}$ contains the three variables shown in Table 1. While `ShareActiveWeeks` and `ActiveDaysPerActiveWeek` together describe the regularity and intensity of travel, `ShareWeekend` describes on which days (weekend or weekday) the trips are made. While using more variables could potentially allow finding more specific behaviour, these three variables result in an easily understandable segmentation of users. We also considered using more detailed features, such as for example card specific time profiles as used by several previous studies, e.g. El Mahrsi et al. (2017). However, in the context of analysing the impacts of a long-term disruption, we did not find more complex variables such as specific time profiles on hourly level to give sufficient additional insights to motivate the loss of easily explainable clusters.

Table 1: Card features describing the travel behaviour in each period used for clustering

Variable	Domain	Description
`ShareActiveWeeks`	0-1	Share of weeks during the period with at least one trip
`ActiveDaysPerActiveWeek`	1-7	Average share of days of each active week with at least one trip
`ShareWeekend`	0-1	Share of trips during the period made on Saturdays or Sundays

We normalize all features to have mean 0 and standard deviation 1 as the variables are of different domain. To cluster the observations, we apply k-means clustering. K-means clustering has been used extensively in previous research to cluster travel behaviour from smart card data, e.g. by Ma et al. (2013), Deschaintres et al. (2019), among others. The clusters obtained using k-means clustering can be characterised by the distribution of variable values in each cluster.

The clusters assigned to each card for the pre and post period allows for analysing whether smart card users changed travel behaviour due to the disruption and is thus focusing on the change of behaviour from the pre period to the post period. Using the identified travel behaviour clusters (see Section 3.2), we can quantify how many cards with a certain behaviour in the pre phase changed behaviour after the disruption. To isolate the effects of the disruption, we compare the changes between users of the affected and the reference line. More specifically, we compare users with the majority of trips on the affected line to users with the majority of trips on the reference line. This ensures that only users that actually are associated with the respective line are included in the analysis. Furthermore, both sporadic and frequent users are included, which would not have been the case if requiring an absolute minimum number of trips.

3.3. Impacts on daily travel patterns by comparing interpersonal variability

While the passenger segmentation based on the simple and easily measurable indicators can explain the change in public transport usage in the pre and post period, a complementary methodology is deployed for analysing the passenger behaviour including the disrupted period. This methodology, originally proposed in Egu and Bonnel (2020), is used for clustering the passengers according to their travel regularity. The methodology is applied specifically for segmenting the passengers who travelled regularly on the disrupted track before the lines were closed for maintenance. Hence, the previous clustering based on the travel behaviour is used as an initial filtering of which passengers are included in this subsequent analysis.

The methodology allows for capturing the regularity of passengers travel behaviour by looking at a single variable - whether the passenger travels on a specific day or not. For each passenger the boolean vector $X_k = [x_{k1}, ..., x_{kd}, ..., x_{kn}]$ is defined, where x_{kd} is 1 if passenger k travelled on day d, and 0 otherwise.

Using the vectors for each of the passengers allows for creating a dissimilarity measure for each combination of cards. This dissimilarity, ς, is computed using the Simple Matching Distance (SMD), which, in this case, calculates whether two users have the same travel pattern, i.e. both travel on the same day and also do not travel on the same day. The calculation of the dissimilarity is then given as:

$$\varsigma_{kl} = 1 - \frac{\sum_{d=1}^{n}[x_{kd} = x_{ld}]}{n} \tag{1}$$

Where the Iverson bracket is 1 for the days where passenger k and l either both travelled or both didn't travel. Using these similarity measures, we can then compute the similarity matrix S, where the elements correspond to ς_{kl}.

To cluster passengers based on their (dis)similarities, we use a hierarchical clustering methodology (Hastie et al., 2009). Each card is clustered in a recursive process using an agglomerative approach. In each step, the Ward method is used to merge those two clusters which minimize the change in the total sum of squares (Ward, 1963). The agglomerative approach combined with the Ward method was selected, as this proved to create compact groups of passengers with similar travel patterns, without creating clusters of very few passengers.

4. Case study

In 2018, the commuter railway line linking Frederikssund to Copenhagen on the suburban rail network was subject to major maintenance works. The line, which normally has around 1.6 million monthly passengers (DSB, 2020), was closed several times during weekends and public holidays in the spring followed by a 13-week closure from June 1^{st} to August 26^{th} 2018. During the closures replacement buses were operating the line at similar service frequency, but resulting in highly increased travel times for passengers. We analyse the effects of this long-term disruption using smart card data.

4.1. Routes and analysis period

Although, the track closure of the Frederikssund line covered 35 kilometers of the track, cf. Figure 2, only passengers who travelled from or to the stations on the suburban part of the two lines are extracted for this study as outlined in Figure 2. The rationale for excluding stations closer to the city center is that passengers who travel to and from the suburban stations do not have any other parallel or alternative routes which they could use during the disrupted period. As such, these passengers were forced to use the replacement bus services if they wanted to travel during the disrupted period.

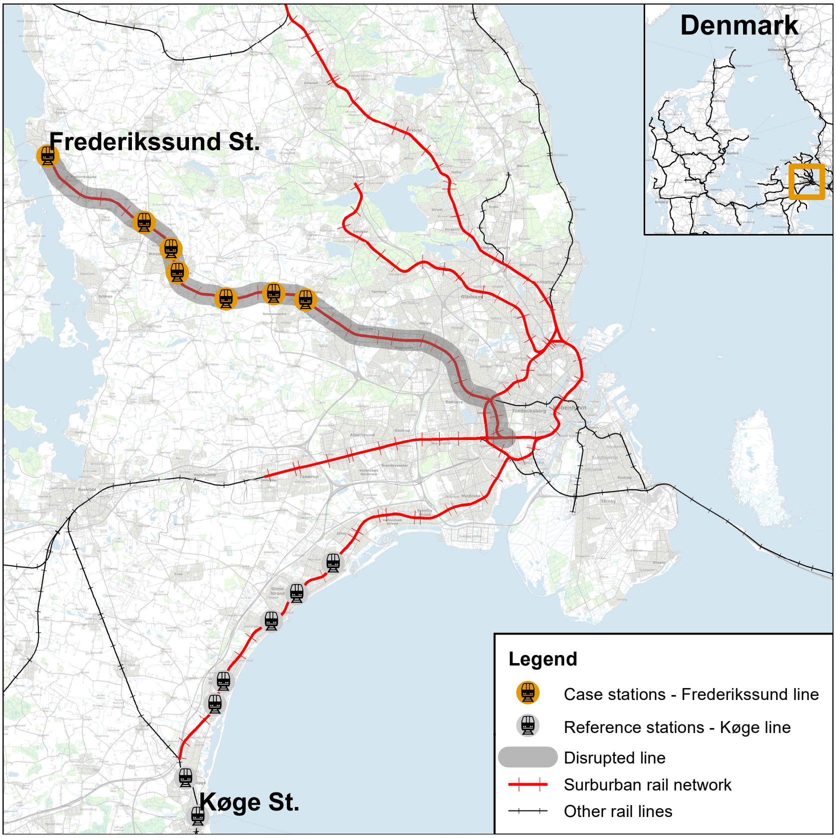


Figure 2: Overview of the closed track section and the stations included in the analysis. Background map source: GeoDanmark-data (2020)

The suburban rail network in the Greater Copenhagen area consists of radial lines, where passengers mainly travel between stations on the fingers and the city. To exclude trends not caused by the disruption, it is reasonable to compare the changes on the affected line to one of the other radial lines. We use the line going to Køge as the reference line (see Figure 2), which is comparable to the Frederikssund line and in 2017 and 2018 did not have any long-term disruptions other than a few closures on weekends.

Passengers who travel to and from the stations on the outer parts of the lines mainly travel to and from the city center, cf. Figure 3, which shows the number of trips from the case stations to other stations in the suburban rail network during the pre period. Around 25% of the trips from case stations are internal trips to other case stations while 15% of trips are to stations on the line outside the city center. The remaining approx. 60% of trips are between case stations and the city centre.

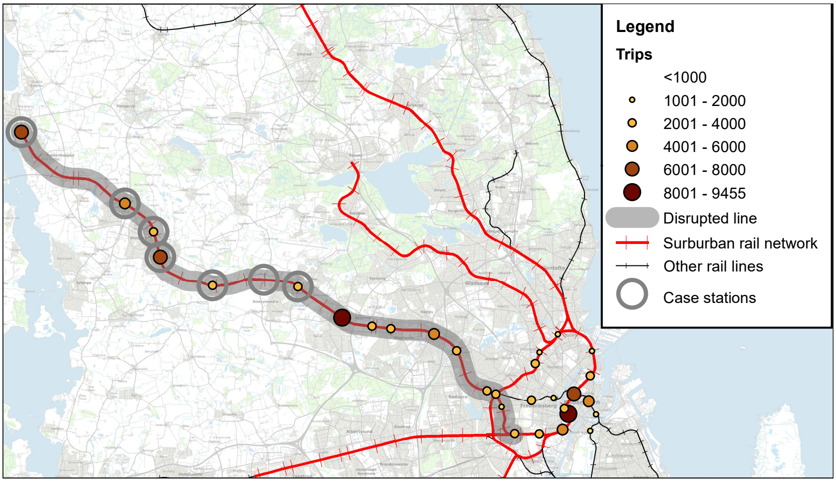


Figure 3: Most visited destinations during pre-period when passengers board at case stations. Background map source: GeoDanmark-data (2020)

The analysis period for the case study consists of the three sub-periods given in Table 2. Due to several weekend disruptions in the months directly before the three month closure, the pre period is set to a period before the first weekend closure representing a period of normal travel.

Table 2: The sub-periods of the analysis period.

Period	Start	End	Duration
Pre	2018-01-01	2018-03-25	12 weeks
Affected	2018-06-01	2018-08-26	13 weeks
Post	2018-08-27	2018-11-18	12 weeks

4.2. Smart card dataset

The smart card system in Denmark, Rejsekortet (Rejsekort, 2020), is a nationwide system where passengers are required to tap-in at the origin and at transfer locations as well as tap-out at the destination. The smart cards can be of different types. For this study only data for the personal smart cards are used, as these can only be used by a single person. This person can have other passengers with him/her tapped-in on the card, but the person, that the card was issued for, must always be present when using the card. The two other types of cards, flex and anonymous, are disregarded as they may be used by several persons. In addition, only personal cards for adults are used whereas cards for children, disabled and pensioners are excluded. This was chosen because they are expected to have less flexibility in their mode choice and because they only constitute a small percentage of the total number of personal cards. The smart card database includes such transactions for more than 1.4 million smart cards during 2018.

Many of the passengers using these cards have never travelled on neither the affected nor the reference line and are therefore not relevant for the analysis. Hence, a total of 299,231 cards with at least one trip on the affected or the reference line in 2018 are extracted and used in the analysis. To consider that smart cards can be lost and at the latest are replaced when expiring after five years of usage, we only use cards with at least one trip in the system before the pre period and after the post period. This ensures that passengers' with lost cards or card renewals do not affect the results.

5. Results

We apply the methodology for analysing the effects of a long-term disruption as presented in Section 3 to the case of the Frederikssund line closure (see Section 4). On the first level we present the impacts on the total ridership. Second, we show the results from clustering the travellers by their travel behaviour and analyse changes in travel behaviour due to the disruption. Finally, for the cards in the regular commuter cluster before the disruption we study in-depth the different reactions to the disruption.

5.1. Impact of the disruption on ridership

To understand the overall impact of the disruption, we extract all trips using one of the case stations, either as origin, destination or transfer station. This allows to compare the total ridership in terms of the number of passengers over time on the affected line and on the the reference line, cf. Figure 4, which also compares to passenger numbers reported by the operator (DSB, 2020). The difference in ridership figures between the smart card data and the operator data is likely due to the variation in the ratio of trips using the smart card fare system versus other fare systems. This deviation highlights the need to compare to a reference line representing the general trend instead of solely comparing the smart card data from different periods directly.

On the line affected by the disruption there is a deviation from the reference line already starting before the full 3-months closure. This can be explained by a number of shorter closures in April and May affecting ridership that mostly took place during weekends. Therefore, we use the first 12 weeks of 2018 where no such closure took place as the pre period representing normal travel behaviour

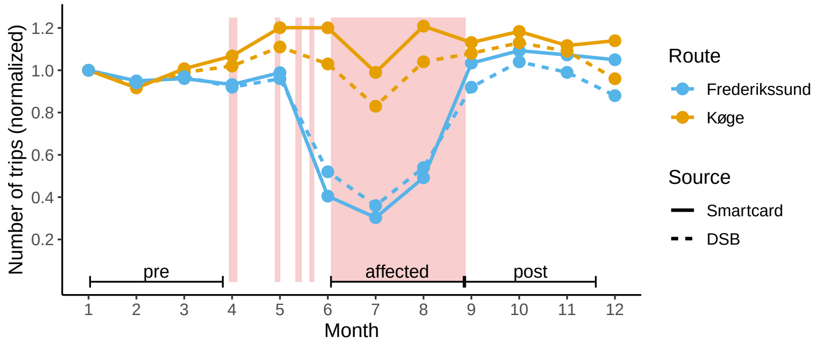


Figure 4: Number of passengers on the affected (Frederikssund) and reference line (Køge) normalized by the number of passengers in January 2018 from smart card data as well as passenger numbers as reported by the operator (DSB, 2020). The stations included differ slightly between the sources. The red shaded periods indicate period were the Frederikssund line was closed. The time periods of the analysis period are indicated at the bottom.

(see Table 2). During the long-term disruption the ridership is very low on the affected line. However, the data quality might be lower during that period as some trips might not have been recorded properly on the replacement buses. Even after the disruption the line does not quite catch up to the reference line, which could indicate long-term effects of the disruption.

5.2. Segmentation of smart card users

We cluster all cards which have been active before and after the analysis period by their travel behaviour during the pre as well as the post period according to the method described in Section 3.2. The number of clusters used is $k = 8$, as increasing the number of clusters further did only reduce the cluster-within sum of squares marginally. The cluster characteristics can be described by the variable values of its members as shown in Figure 5. We also assign a name to each cluster based on their characteristics for easier reference. The most active clusters are cluster **4** (Commuters) travelling 4-5 days every week and cluster **8** (Regular weekday users). Clusters **1**, **2** and **5** in contrast are rare users with few trips only.

The segmentation of travel behaviour during the pre period is similar on both the affected and the reference line (see Table 3), which supports the assumption that the affected line would have had a similar development as the reference line if no disruption had occurred. On both lines, the travel behaviour clusters representing low travel activity contain the majority of cards. A small number of users in the high activity clusters **4** and **8**, however, account for the majority of trips.

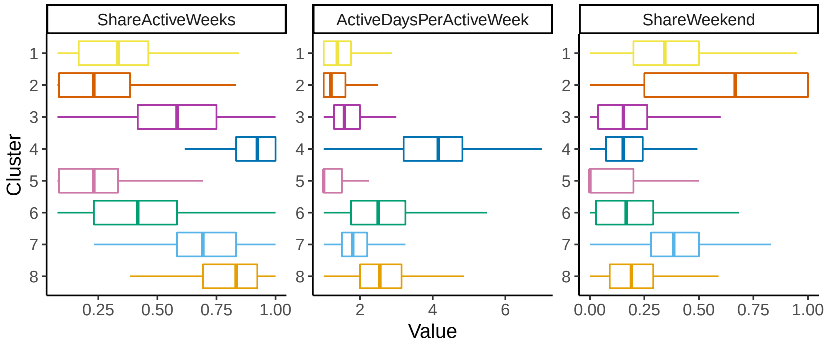


Figure 5: The characteristics of the travel behaviour clusters

Table 3: Segmentation of cards by travel behaviour cluster in the pre phase for cards with the majority of the trips on the affected line (Frederikssund) and on the reference line (Køge), including the share of cards in each cluster and each cluster's share of the total number of trips.

	Frederikssund		**Køge**	
Cluster	Card share	Trip share	Card share	Trip share
1: Sporadic weekend users	18.4%	7.0%	15.8%	5.9%
2: Rare weekend users	14.8%	2.9%	12.4%	2.4%
3: Occasional users	12.6%	13.5%	13.1%	13.8%
4: Commuters	6.3%	35.0%	5.9%	31.1%
5: Rare weekday users	30.4%	6.7%	32.6%	7.5%
6: Irregular weekday users	5.1%	5.9%	5.7%	6.4%
7: Regular weekend users	4.2%	5.7%	5.0%	6.5%
8: Regular weekday users	8.1%	23.3%	9.4%	26.3%

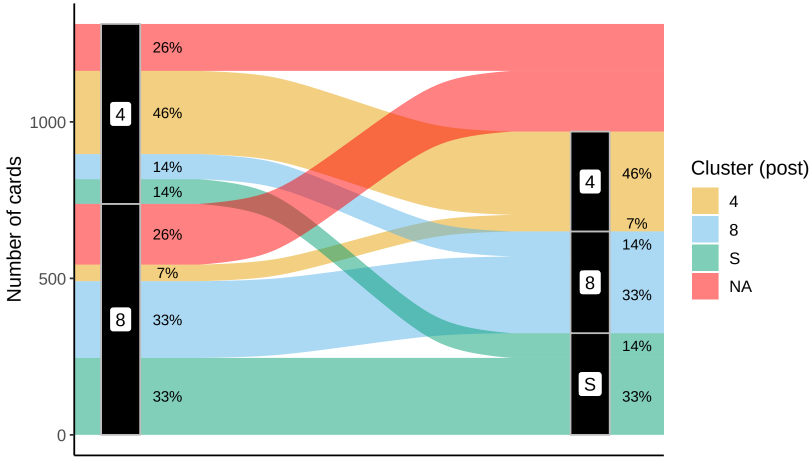
5.3. Post-disruption impacts on individual travel behaviour

To understand which user groups cause the lower passenger numbers on the line after the disruption compared to the reference line (see Figure 4), we analyse the change of travel behaviour from the pre to the post phase. Figure 6 shows how users from the most active clusters **4** (Commuters) and **8** (Regular weekday users) are changing their travel behaviour on the Frederikssund and Køge line. In general, notable behavioural changes between the pre and post period are observed, even on the reference line without disruption. While the post-disruption behaviour changes of cluster **4** (Commuter) in the pre phase are similar on both routes, there is some difference for those that have been in cluster **8** (Regular weekday users). It seems that on the affected line, a smaller portion started to commute (cluster **4**) in the post phase (7% compared to 10% on the reference line), while instead more cards are switching to a more sporadic travel behaviour (33% compared to 30% on the reference line).

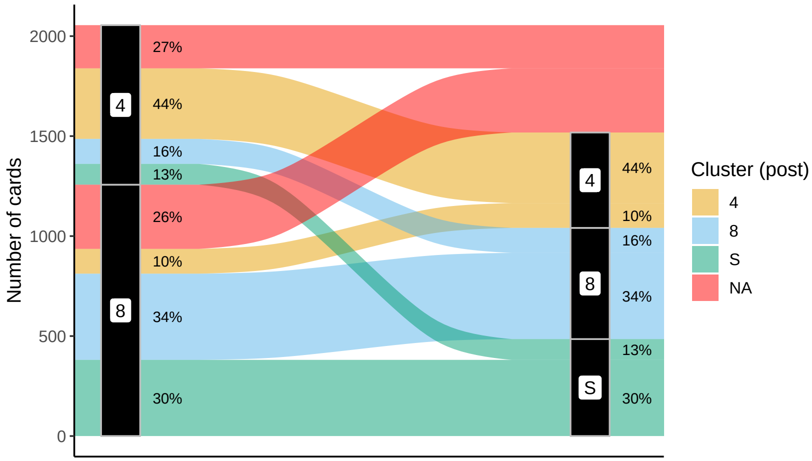
While the total number of active cards with the majority of trips on the Frederikssund line is only increasing by 1.4%, it was increasing by 5.6% for Køge from the pre to the post period (see Table 4). The change of travel behaviour is common for an individual card. On an aggregated level, however, the sizes of the clusters are not changing as much. Comparing the size of each cluster, we find that cluster **4** (Commuters) is decreasing for Frederikssund, while it increases for Køge (see Table 4). As this cluster is the most active cluster, this difference has an even larger impact on the number of trips. Most clusters seem to have been impacted negatively by the disruption as they show a lower change in size than on the Køge line. The only exceptions are cluster 8 (Regular weekday users) to some extent, and in particular cluster 7 (regular weekend users), which is a group that increased by 15.8% compared to -0.6% on the reference line. However, it should be noted that cluster 7 was the smallest of all groups in the pre period.

Table 4: Number of cards in each cluster before (Pre) and after (Post) the disruption for cards with the majority of trips on the affected and reference line.

	Frederikssund			**Køge**		
Cluster	Pre	Post	Change	Pre	Post	Change
1: Sporadic weekend users	1675	1774	5.9%	2126	2489	17.1%
2: Rare weekend users	1343	1376	2.5%	1666	1760	5.6%
3: Occasional users	1145	1211	5.8%	1761	2061	17.0%
4: Commuters	575	553	-3.8%	798	829	3.9%
5: Rare weekday users	2762	2630	-4.8%	4380	4389	0.2%
6: Irregular weekday users	462	449	-2.8%	763	700	-8.3%
7: Regular weekend users	385	446	15.8%	673	669	-0.6%
8: Regular weekday users	738	775	5.0%	1257	1280	1.8%
Total	9085	9214	1.4%	13424	14177	5.6%



(a) Cards with the majority of trips on the Frederikssund line



(b) Cards with the majority of trips on the Køge line

Figure 6: Alluvial diagram of cards in clusters **4** and **8** in the pre phase (left) to the post phase (right) for the affected and reference line. *S* groups all clusters with more sporadic use (that is all clusters except **4** and **8**). Users without cluster in the post phase have stopped travelling on the given line or did at least not have a majority of trips on the line anymore).

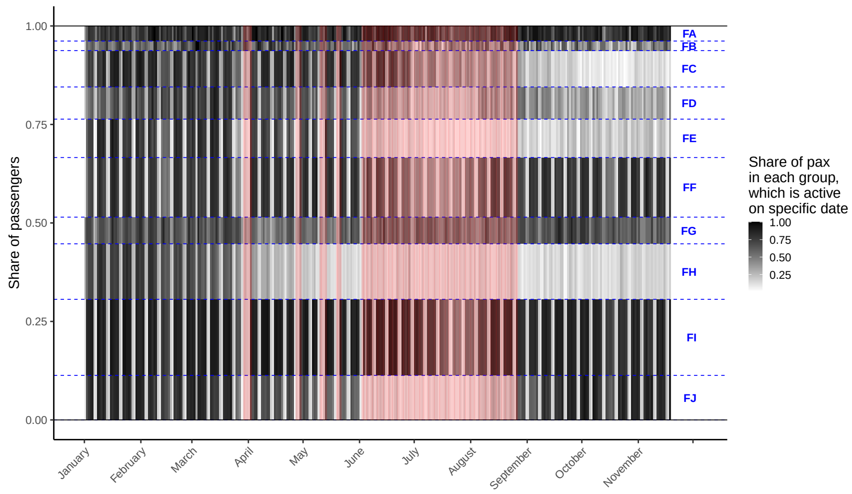
5.4. Impacts on daily travel behaviour

A majority of trips is taken by passengers in a few travel behaviour clusters (see Figure 3). These passengers are thus the main contributors to the overall travel demand, and the effect of the disruption for the most frequent passengers is analysed further below using the method described in Section 3.3. First, separate segmentations for each line are presented and followed up with a joint segmentation focusing on the differences between passengers on the two lines.

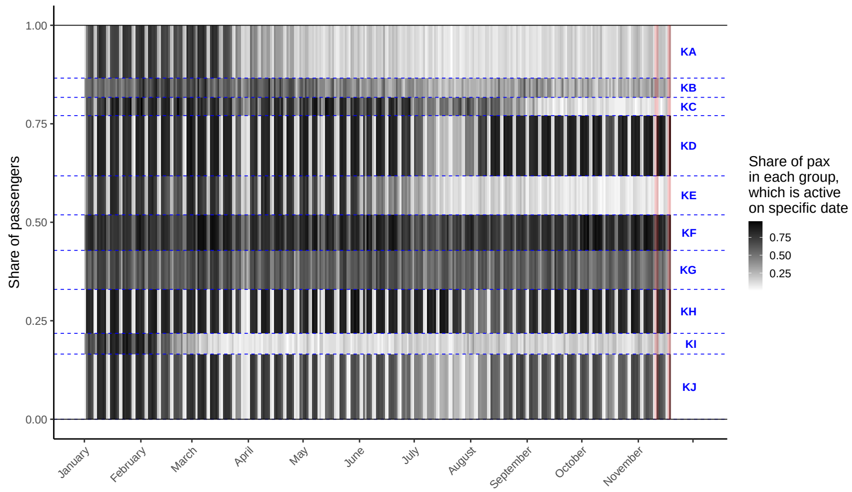
Figure 7 presents the hierarchical clustering based on daily travel activity for cards in travel behaviour cluster **4** (Commuters) during the pre-period. By testing several specifications of the number of reasonable clusters to segment the passenger, it was found, that 10 clusters represented compact groups, where the longitudinal behaviour between the groups was considerably different after the pre-period ended. As seen in Figure 7a, the passenger travel behaviour for most clusters is similar for the pre-period, which should be expected, as all of the passengers in the plot were in cluster **4** in the pre-period. One group of passengers (**FH**) have a serious decline in travel activity after the pre-period ends and never returns to regular usage of public transport. The other groups continue to travel regularly until the track closure starts. At this time, group **FJ** and **FE** stop travelling. While group **FE** never fully returns to regular usage, passengers in group **FJ** start using public transport frequently after normal operations resume. This group accounts for 17% of the passengers who commuted before the disruption started. It seems that these passengers find other modes of transport or is able to reduce the number of trips for a long period.

For the group of passengers mostly travelling on the line to Køge, two clusters (**KA** and **KI**) experience a serious decline in the number of active days during the spring. None of the passengers in these clusters return to frequent public transport usage after the summer period. Interestingly, there seems to be no major cluster of passengers who abandon the public transport system and return, although the passengers in cluster **KJ** travel less during the summer period and returns to a regular pattern in the fall.

The segmentation based on passengers who were commuters on any of the two lines during the pre period, presented in Figure 8, shows considerable differences in the distribution across the clusters for the two lines. Passengers on the Frederikssund line is over-represented in cluster **H** compared to passengers on the line to Køge. The cluster includes passengers, who are almost completely abandoning the public transport system during the summer, but returns to regular use in the fall. With 19.5% and 10.8% of passengers on the Frederikssund and Køge lines, respectively, this indicates that the long-term disruption resulted in some passengers choosing other modes of transport during the summer, which cannot be due to simply choosing to bike or being on vacation. For cluster **D**, which is characterised by passengers who are using public transport less regularly during the summer, a similar pattern as for cluster **H** is seen, where the majority of passengers in the cluster are from the line to Frederikssund. Generally, it seems that larger shares of passengers from the Køge line are placed in clusters **B,G,I**, which have abandoned the public transport system in the fall, although the differences between the two lines are not immense.



(a) Plot of the passengers, who had most of their trips on the track to Frederikssund in pre-period and who were high-frequent users (cluster 4) in this period - N = 575. Red areas indicate days where the track was closed.



(b) Plot of passengers, who had most of their trips on the line to Køge in the pre-period and who were high-frequent users (cluster 4) in this period - N = 798. Red areas indicate days where the track was closed.

Figure 7: Plot of the temporal change in activity for each group of passengers found in the hierarchical clustering.

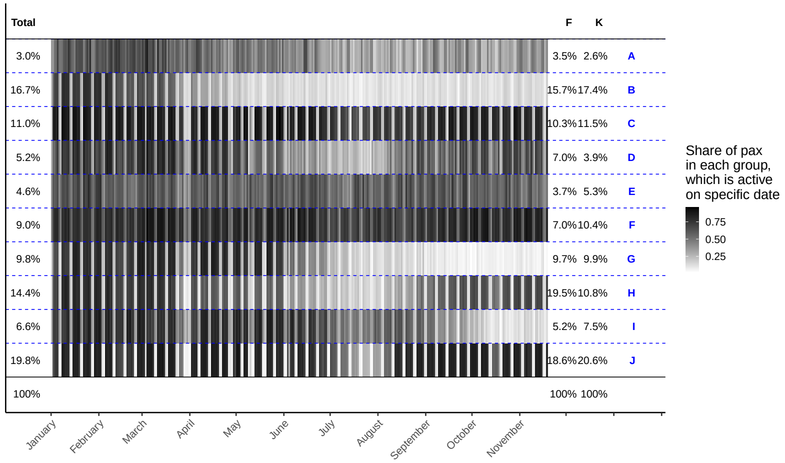


Figure 8: Daily travel activity for passengers who were commuters on either line in the pre-period - $N = 1,373$. F denotes the case line (Frederikssund) and K denotes reference line (Køge)

6. Discussion

The detailed analyses of the individual mobility, both during and resulting from the long-term disruption, have provided several interesting insights on passenger travel behaviour.

The study design has proven useful for analysing the impacts over time, which are usually only studied by before and after surveys. One drawback of using before and after surveys is that passengers change behaviour due to several reasons, even when there are no disruptions on the line. As such, it is difficult to isolate the effect of the disruption to the dynamic change in passenger travel behaviour, unless a reference line is also included in the survey. As smart card data often is available for a whole network of lines, using a reference line for comparison, as in the method we present, is a straightforward way to isolate effects that only occurred on the disrupted line. It is however important to choose a reference line that is comparable and for example does not have different seasonal patterns.

The proposed methodology allows for analysing the passenger behaviour during the disruption (within the public transport system). This makes it possible to reveal when and where passengers abandon the system. While other data sources could also identify such changes in passenger behaviour, e.g. GPS-data or other telecommunication data, these require that people are willing to be followed for a long period of time in order to analyse the effects of the disruption.

While providing important behavioural insights resulting from long-term disruptions an important limitation persist. The data used for the analysis is based on the subset of commuters and regular passengers using the smart card, which requires tap-in and tap-out. The other products for commuters at the time; *paper based monthly-pass*, *app-based monthly pass* and *smart card based monthly pass with no tap-in or tap-out*, have not been available for the study. However, data from these ticket types are difficult to analyse as passengers neither need to tap-in nor out when travelling. This makes it difficult to draw conclusions on the absolute changes in ridership for this group of passengers. Nonetheless, the results revealed a small decrease in usage among the cluster representing commuters on the affected line, whereas a small increase was observed among commuters on the reference line. This suggests that similar patterns can be expected among commuters using other ticket types.

7. Conclusion

This paper is the first to use data from individual smart cards to understand changes in passenger travel behaviour due to a long-term full closure of a rail line. The proposed method allows to quantify changes in travel regularity after the disruption as well as different reactions of travellers during and after the disruption using clustering approaches. By comparing behavioural changes to those on a reference line, it is possible to isolate the effects of the disruption from the general changes in the individual travel patterns due to seasonal variations or changes in use of the smart card fare system.

The need to compare to a reference line is apparent in the case study of a long-term closure on a commuter rail line in the Greater Copenhagen area, where we find that a change in behaviour is common for individual cards even on the reference line without any major disruption. We find no apparent differences when comparing the changes in travel behaviour and travel regularity after the disruption of those that were frequent commuters before the disruption on the affected line to those on the reference line. However, we find a -3.8% decrease of frequent commuters on the affected line after the disruption compared to a 3.9% increase on the reference line, which indicates that very few new users have been attracted to the affected line during the closure. Analysing the daily travel behaviour, we find that a noticeable share of 17% of those that were frequent commuters before the disruption on the affected line abandon the public transport system during the closure, but return as frequent commuters when normal operations resume.

The proposed smart card based method is an alternative, or complement, to using traditional travel surveys. The longitudinal structure of smart card datasets allows for revealing more detailed changes to individual passenger travel behaviour beyond the change in total demand. In future work, combining smart card data with other data sources including GPS or cellular network data could give an even more complete picture of the impacts of a long-term disruption.

Acknowledgements

The authors would like to thank Rejsekort A/S for providing data from the Danish smart card with the consent from Movia, DSB (Danish State Railways) and the Danish Metro Company (Metroselskabet I/S).

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7 Paper 6: Determining transfer times and transfer activities in multimodal public transport systems using smart card data

The following pages contain the article:

M. Eltved, N. Christoffer, and P. Lemaitre (2020). “Estimation of transfer walking time distribution in multimodal public transport systems based on smart card data”. Submitted to Transportation Research Part C: Emerging Technologies.

Please cite accordingly.

The paper was presented at the 6^{th} *International Symposium Use of Public Transit Automated Data for Planning and Operations (TransitData2020)* held online 11-13 August 2020.

Estimation of transfer walking time distribution in multimodal public transport systems based on smart card data

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Abstract

Transfers are a major contributor to travel time unreliability for journeys in public transport. Thus, connections between services in the public transport network must be reliable. To plan such reliable transfers from e.g. busses to trains, it is crucial to know the necessary walking times from stops to platforms. This paper presents an innovative approach for estimation of walking time distributions from bus stops to train platforms based on a matching of smart card data and automatic vehicle location data. The observed times from bus stop to rail platform turns out to have a large variance, due to two reasons: differences in passenger walking speeds, and passengers who are doing activities during the transfer. To account for these variations a *hierarchical Bayesian mixture model* is applied, where the time for passengers walking directly and passengers doing activities during the transfer follows separate distributions. The proposed methodology is applied to 129 stations in the Eastern part of Denmark. Results from two stations with different characteristics are presented in details along with justifications and analyses of model accuracy. The outcome of the model with distributions of the necessary walking times from bus stops to train platforms is important input for timetabling connections, and the data-driven methodology can easily be applied at scale.

Keywords: Public transport, Transfers, Walking time, Smart Card, Automatic Vehicle Location

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Preprint submitted to Transportation Research Part C *August 30, 2020*

1. Introduction

The attractiveness of public transport is defined by many parameters, but transfers between services are consistently viewed as inconvenient (Iseki and Taylor, 2009; Raveau et al., 2014; Schakenbos et al., 2016). Transfers require the passenger to alight a service, and in most cases walk to another stop to board the connecting service. When transferring between services there is a risk of a large increase in the journey time of the whole trip if a connecting service is missed (Dixit et al., 2019), and thereby decreasing the reliability of the trip, which is known to be of large nuisance to passengers (Kouwenhoven et al., 2014).

Creating good connections between services require knowledge on the time needed for passengers to walk from one stop to another (Parbo et al., 2014). This knowledge is usually determined by identifying the walkways between stops and assuming a walking speed for the passengers, or by manual surveys where passengers are followed through the station (Daamen et al., 2006). Overestimations of the necessary time to make a transfer affects passengers' waiting times when services are synchronised, and it is therefore important to get accurate estimates of the time needed to make the transfer (Xiao et al., 2016).

This paper presents a novel methodology for estimating the walking time distribution for transferring passengers from busses to train stations. The study utilises the vast available amount of automatic fare collection (AFC) data from smart cards and combines this with automatic vehicle location (AVL) data from busses. In this way it is possible to calculate the walking time for passengers from alighting at the bus stop until the passenger taps in at a validator on the platform. However, the raw data can not be used directly for estimation of the required walking time, since passengers may be doing activities during their transfers (Wahaballa et al., 2018). A *hierarchical Bayesian mixture model* with one distribution for passengers walking directly and another distribution for passengers having an activity during the transfer is estimated, to obtain accurate estimates of the walking time distribution for directly walking passengers. The method is applied to a large scale case study and results are studied in detail for two stations with different characteristics.

The novel methodology adds to existing knowledge of transferring passengers by separating passengers walking directly and passengers doing activities during the transfer, and does this using an unsupervised method. The approach is able to handle different types of transfers, where either the synchronisation of busses and trains or the number of shops near the station increases the amount of activities undertaken by passengers during the transfer. The methodology can be easily applied at scale, and thus overcomes the scalability issues of time consuming manual surveys where passengers are followed through the station.

The paper is organised in the following way; Section 2 reviews the existing studies on estimation of walking times at transfers, Section 3 outlines the methodology for estimation of walking times based on smart card data, Section 4 presents the case study used for testing the methodology and analyses of the results, Section 5 discusses the model accuracy with possible verification techniques that can be applied at scale. Finally, Section 6 concludes on the findings in the paper.

2. Literature review

Walking is a central part of using public transport, and in many cases the passenger also needs to walk due to a transfer between services. The number of trips in metropolitan areas requiring a transfer can range between anywhere from 30 % to 80 % depending on the network layout and which modes of public transport the passengers use (Guo and Wilson, 2011). For the Greater Copenhagen area, which is part of the area used for the case study presented in Section 4, the number of trips requiring at least one transfer is approximately 40 % (Anderson, 2013). Given the large number of transfers in the network, it is important to estimate the necessary walking times for these transfers.

Walking speeds are known to be heterogeneous (Fruin, 1971), even when there is nothing that constrains the walkways (Daamen and Hoogendoorn, 2006). A number of studies have spent significant efforts for obtaining walking times at different transport facilities. Young (1999) for example studies the walking speeds in airport terminals and find that moving walkways and passing obstructions in a corridor significantly impact the walking speed. For public transport stations, Chen et al. (2016) studies the walking speeds for transfer passengers in a subway passage in Beijing and finds that the speeds differ significantly between males and females and between passengers walking alone and passengers in a company, with the walking speed generally following a log-normal distribution. A similar finding on the walking speeds following a log-normal distribution is reported in Zhu et al. (2017). Kasehyani et al. (2019) studies the walking times at different times of the day and finds that these differ, but other factors such as if passengers carry luggage also affects the walking speed.

Due to the varying walking speeds, the walking times at public transport stations are also not a constant factor of the distance walked. Daamen et al. (2006) studies passenger walking times for both boarding and alighting passengers at two stations in the Netherlands and specifically investigates which paths they use to and from the platform. By following passengers from when they enter the station to the platform and vice versa, they find that passengers mainly choose the shortest path through the station. A similar methodology on following passengers to observe the walking times is used in Du et al. (2009), but with a focus on transferring passengers in Beijing. Significantly different walking times are found for passengers in the peak period and outside this period due to effects of crowding. The effect of crowding is also found to be significant in the study by Zhou et al. (2016) on walking speeds at different cross-sections of stations such as escalators, horizon passage and on the platform.

In recent years the focus has shifted from manual observations of walking times to estimations of the walking times based on smart card data. Smart card data is a valuable source for different types of analysis of passenger travel behaviour, such as travel time estimation, estimation of demand from origins to destinations and analysis of passenger route choice (Pelletier et al., 2011). The availability of the data is increasing in almost any major city and can help public transport agencies for better planning of the system and thereby for attracting more passengers to the system (Faroqi et al., 2018).

The vast majority of the studies using smart card data for estimation of walking times focus on the access and egress part of the trip from gate to platform and vice versa (Leurent and Xie, 2017; Xie and Leurent, 2017; Li et al., 2020; Singh et al., 2020), while only few studies focus on estimating the walking times at transfers (Zhu et al., 2020; Wahaballa et al., 2018), which are the times investigated in this paper. Zhu et al. (2020) estimates the walking time of transferring passengers by finding the egress speed percentile of an individual passenger compared to other passengers. This percentile is used to find passengers' walking times at transfers by again comparing to the group of transferring passengers. The model is part of a complete approach for estimation of the total travel times from origin to destination and no validation of the transfer walking times are provided, other than fitted distributions of the walking times, which is a result of a fifth-degree polynomial estimation of the total travel time. The other study with a focus on transfer times, Wahaballa et al. (2018), studies the walking and waiting times at transfers between bus and rail using smart card data. The study proposes a stochastic frontier model, which aims at estimating the waiting time at transfers, while also considering the heterogeneity in walking times as these differ between passengers. The walking times can be observed from bus to the entry-gate, and these times are used directly as the walking time from rail to bus. A clear advantage of the smart card system used in the study, when considering walking times, is that the cards also are used for shopping and thereby these passengers are removed. No numbers are provided on the share of passengers shopping during the transfer, and hence it is difficult to tell how many observations can be removed due to this information.

This information on whether a passenger is doing an activity during the transfer is not generally available in smart card systems and no studies investigating this have been found. However, Fujiyama and Cao (2016) has shed some light on this for terminal stations by studying the additional time spend at terminal stations in London before boarding the train. This can be observed, as passengers tap-in when entering the station and again near the platform. By assuming a general walking speed and a calibration for the individual paths made by the authors, they measure the additional time spend in the station. Interestingly, no correlation is found between the additional time spend and neither the total travel time or frequency of the line used. However, the additional time spent at the station is longer in the afternoon and evening compared to the morning.

3. Methodology

In this section the methodology is presented, along with preliminary requirements and data pre-processing needed prior to modelling. Figure 1 illustrates a transfer site, and the overall terminology for the proposed method. The goal is to estimate the walking time distributions for the different *path pairs* (4 shown), without explicit knowledge of passengers true walking time nor knowledge on whether or not they performed an activity during their transfer.

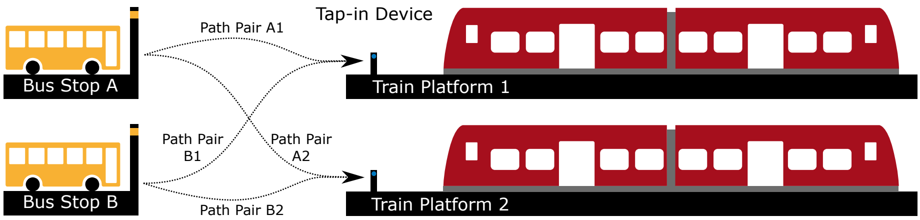


Figure 1: Overview of challenge and infrastructure setup.

We assume an AFC infrastructure, where tap-ins occurs both when boarding a bus, and when entering a train platform. We assume the tap-in devices are located at platforms so it is possible to board a train immediately after tapping in.

3.1. Data Requirements and Pre-Processing

To apply the proposed method we need to prepare a data fusion between *AVL data* and *AFC data*. The following describes this fusion of data. We generally distinguish information belonging to the k'th stop of *bus trip* j (bus AVL dataset) and information belonging to the n'th trip leg of *passenger trip* i (AFC dataset).

We assume that the following information on bus AVL data is available or can be transformed to a similar structure. For each bus trip j we assume the availability of the following information:

- $\textit{Bus Ref}_j$: A unique reference to the vehicle that was observed running *bus trip* j
- $\textit{Bus Stop Point Ref}_{j,k}$: A unique reference to k'th stop point for bus trip j which was observed arriving/departing.
- $\textit{Bus Arrival}_{j,k}$: Moment at which the vehicle was measured arriving to the k'th stop point of bus trip j.
- $\textit{Bus Departure}_{j,k}$: Moment at which the vehicle was measured departing from k'th stop point of bus trip j.

This information is standard output for most public transport AVL systems, and is included as part of the *GTFS-RT feed specification* (Google, 2020), although not all variables are considered mandatory.

From the AFC system we assume data is available or transformable to the following form:

- $\textit{Tap In}_{i,n}$: Moment at which the passenger tapped in for the n'th time on passenger trip i.
- $\textit{Bus Ref}_{i,n}$: A unique reference to the vehicle in which the $\textit{Tap In}_{i,n}$ occurred. For tap-ins conducted on train platforms $\textit{Bus Ref}_{i,n} = \emptyset$.
- $\textit{Stop Point Ref}_{i,n}$: A unique reference to bus stop point or train station platform this tap-in was conducted at.
- $\textit{Tap Out}_i$: The final tap out time for passenger trip i, i.e. at the passengers' destination.

We further assume the availability of a function $D(x, y)$ which measures the Euclidean walking distance between bus stop point x and train stations platform y.

The matching and data fusion between bus AVL and AFC data is a two-step process where we iterate AFC entries. First step is to match the passenger boarding to the bus AVL and secondly match the passenger alighting given the constraints of the boarding match. The match of the boarding is done by searching in bus AVL entries. For the n'th trip leg in passenger trip i we identify j and k by minimizing $|\mathit{Tap\ In}_{i,n} - \mathit{Bus\ Departure}_{j,k}|$ where $\mathit{Bus\ Ref}_i = \mathit{Bus\ Ref}_j$ and $\mathit{Stop\ Point\ Ref}_{i,n} = \mathit{Stop\ Point\ Ref}_{j,k}$. We denote the result of the boarding match:

$$\mathit{Match\ Departure}_{i,n} \leftarrow (j, k)$$

We have now aligned information between bus AVL and AFC data for the boardings using tap-ins from AFC. To complete the second step we also want to match the alightings, and thus allowing the measurement of the observed walking time W^O. We need to identify the alighting stop k' prior to $\mathit{Tap\ In}_{i,n}$ and do so by minimizing $D(\mathit{Stop\ Point\ Ref}_{j,k'}, \mathit{Stop\ Point\ Ref}_{i,n})$ where $j = \mathit{Match\ Departure}^j_{i,n-1}$ and $k' > \mathit{Match\ Departure}^k_{i,n-1}$. I.e. we search for the closest alighting stop on the matched *bus trip* j on the previous trip leg $(n-1)$ of *passenger trip* i. We constrain the search to only stops visited by the bus after the boarding stop. We denote the result of alighting stop match:

$$\mathit{Match\ Arrival}_{i,n} \leftarrow (j, k')$$

The final result of the data pre-processing and matching process is a fused dataset for train tap ins (i.e. $\mathit{Bus\ Ref}_{i,n} = \emptyset$), along with the matched bus alighting of the previous trip leg. Since we wish to estimate walking time for bus to train transfers we denote each combination of bus alighting stop point and train platform as a *path pair*. We split the data into separate data sets for each train station, and for each station data set we will consider the number of unique *path pairs* as $Q \in \mathbb{N}$ with $q \in \{1, \ldots, Q\}$. We denote the i'th observed walking time on *path pair* q as $W^O_{q,i}$.

3.2. Model

To model the behaviour of walking time during a transfer, we propose a hierarchical mixture model for each station with transfers of bus stop to train stations. Each station will have Q path pairs, where the observed variable is the walking time $\boldsymbol{W}^O_q \in \mathbb{R}^N_q$ of $N_q \in \mathbb{N}$ trips along the q'th path pair. The observed walking time is assumed to originate from two types of unobserved behaviours $Z \in \{D, A\}$: (i) passengers walking directly, and (ii) passengers doing an activity during the transfer, which gives the following assumption and definitions:

Assumption 1 (Origin of walking time)**.** *It is assumed that the i'th walking time, $W^O_{q,i}$, originates from either walking directly ($Z_i = D$) or activity-based walking ($Z_i = A$), given the direct walking time, $W^D_{p,i}$, and activity walking time, $W^A_{p,i}$.*

Definition 1. *Direct walking time, $\boldsymbol{W}^D$, is assumed to stem from a transfer done by a passenger who walks directly from a bus stop to a train platform.*

Definition 2. *Activity walking time, $\boldsymbol{W}^A$, is assumed to stem from a transfer, where an activity affects the walking time, such as shopping, buying coffee, etc.*

Using the first assumption to derive equation (1), the direct and activity walking times can be inferred by applying Bayes' rule to write the posterior distribution as the walking time given the direct and activity walking time.

$$P(\boldsymbol{W}^D, \boldsymbol{W}^A, \boldsymbol{Z}|\boldsymbol{W}^O) \propto P(\boldsymbol{W}^O|\boldsymbol{W}^D, \boldsymbol{W}^A, \boldsymbol{Z})P(\boldsymbol{W}^D, \boldsymbol{W}^A, \boldsymbol{Z}) \tag{1}$$

To obtain the final model the following two assumptions are made relating to the walking time and path pairs.

Assumption 2 (Independent path pairs and trips walking time)**.** *It is assumed that the walking time of trip i is independent of the walking time of all other trips and that all path pairs are independent of all other path pairs, such that*

$$P(\boldsymbol{W}^O|\boldsymbol{W}^D, \boldsymbol{W}^A, \boldsymbol{Z}) = \prod_{q=1}^{Q} \left[\prod_{i=1}^{N_q} P(W_{q,i}|W_q^D, W_q^A, Z_{q,i}) \right] P(W_q^D, W_q^A, Z_q) \tag{2}$$

Assumption 3 (Conditional independence between walking types)**.** *The conditional probabilities of direct and activity walking time are assumed to only depend on its own given behaviour, i.e.* $P(W_q^A|Z_q) = P(W_q^A)$ *and* $P(W_q^D|Z_q) = P(W_q^D)$ *, such that*

$$\begin{aligned} P(W_q^D, W_q^A, Z_q) &\propto P(W_q^D|W_q^A, Z_q)P(W_q^A|Z_q)P(Z_q) \\ &= P(W_q^D)P(W_q^A)P(Z_q) \end{aligned} \tag{3}$$

Using equation 1 in combination with the assumption 2 and 3 relating to the path pairs and walking time, we can derive equation 4, given

$$P(\boldsymbol{W}^O|\boldsymbol{W}^D, \boldsymbol{W}^A, \boldsymbol{Z}) = \prod_{q=1}^{Q} \left[\prod_{i=1}^{N_q} P(W_{q,i}|W_q^D, W_q^A, Z_{q,i}) \right] P(W_q^D)P(W_q^A)P(Z_q) \tag{4}$$

Using the law of total probability equation, 4 can be rewritten as the probability of walking directly with $P(Z = D) = \lambda$, giving the final equation

$$P(\boldsymbol{W}^D, \boldsymbol{W}^A, \boldsymbol{\lambda}|\boldsymbol{W}^O) \propto \prod_{q=1}^{Q} \left[\prod_{i=1}^{N_q} \lambda_q P(W_i^O|W_q^D) + (1-\lambda_q)P(W_i^O|W_q^A) \right] P(W_q^D)P(W_q^A)P(\lambda_q) \tag{5}$$

In equation 5 the three latent variables $\boldsymbol{W}^D$, $\boldsymbol{W}^A$ and $\boldsymbol{\lambda}$ are assumed to be beta distributed, where $\boldsymbol{\lambda}$ is given a weakly informed prior, assuming that most passengers are walking directly.

$$\lambda_q \sim B(4,2) \qquad W_q^D \sim B(\alpha_q^D, \beta_q^D) \qquad W_q^A \sim B(\alpha_q^A, \beta_q^A)$$

The mean of walking times for directly walking passengers is assumed be below half of the maximum transfer time, in case of danish transport system 15 minutes, which is modelled by the constraint

$$\alpha_q^D \leq \beta_q^D$$

Since activities can be many things and most activities will likely increase the walking time of the passenger, the hyper-priors α^A and β^A are constrained to the range of $[2,3]$ to insure a large variance and mean between 12 and 18 minutes.

4. Case study

Our case study is conducted for the entire Eastern Denmark for November 2019. We include most train stations serviced by the national rail service provider, metro stations and some local train stations. Figure 2 shows a map of the included stations. The model was estimated on 129 stations with a total of 1,145 path pairs. Only path pairs with 100 or more observations during November were estimated, as these pairs then have an average of at least three transferring passengers pr. day. The final dataset consists of 542,713 observations, i.e. unique transfers. Each station was estimated separately by the probabilistic language STAN using NUTS sampling with four chains, each with 3,000 iterations, and a warm-up period of 2,000 iterations. Since it is not feasible to present all the results in detail, two stations have been selected for detailed analysis of the results and verification of the model assumptions.

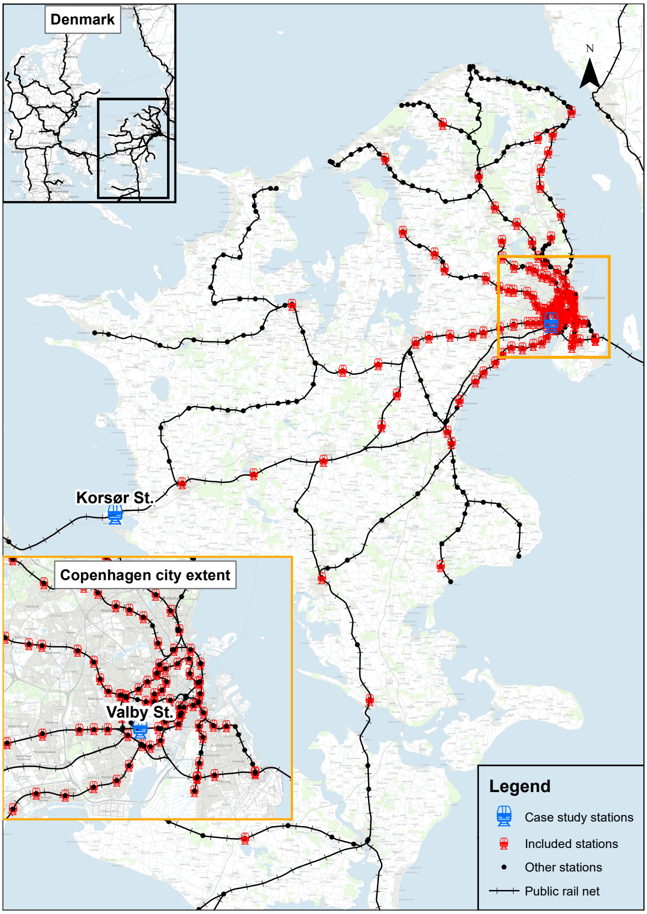


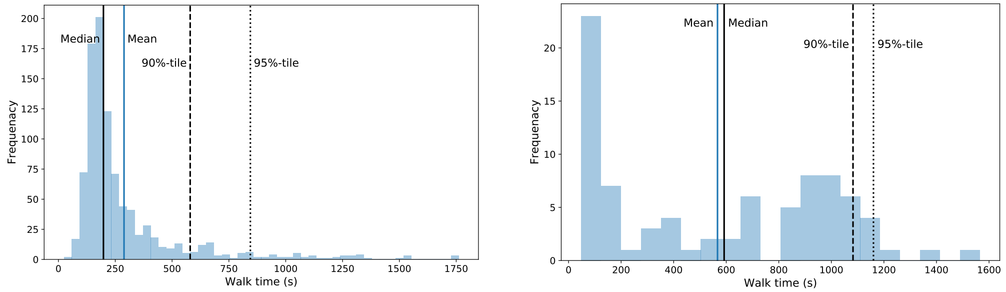
Figure 2: Overview of included stations in the analysis. Background map source: GeoDanmark-data (2020)

To illustrate and analyse the model estimations in more detail, the stations at Valby (case 1) and Korsør (case 2) will be used as examples. As a larger transfer station in the Copenhagen area, Valby Station has an expected distribution of the walking times as shown in Figure 3a, where most passengers have a relative low walking time. The layout of the station is presented in Figure 4a, where the path pairs selected for the analysis are also presented.

In contrast to Valby, Korsør is a small rural station with an abnormal observed walking time distribution with two peaks shown in Figure 3b. The first peak has the expected location of a relative low walking time, where second peak is located above the median of 10 minutes. The station layout of Korsør station is shown in Figure 4b. The station building includes a waiting hall and a convenience store.

4.1. Case station 1: Valby

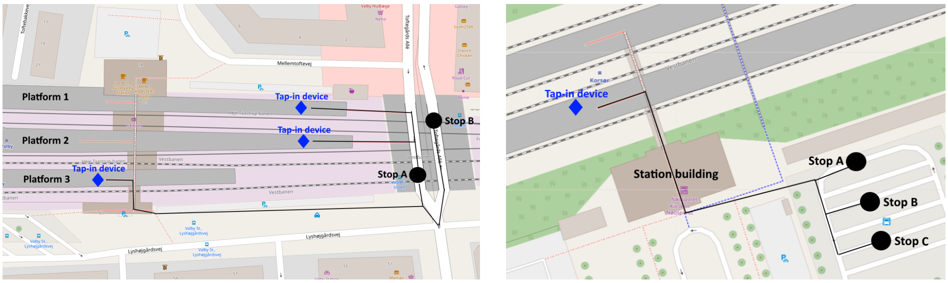
Valby has 32 different path pairs, where we have selected the results from six path pairs, which are combinations of the two bus stops and three platforms shown in Figure 4a. The six path pairs include a



(a) Observed walking times (from alighting the bus until tap-in at platform) from AFC data at Valby Station (urban station on Zealand, Denmark)

(b) Observed walking times (from alighting the bus until tap-in at platform) from AFC data at Korsør Station (rural station on Zealand, Denmark)

Figure 3: Histograms of the raw walking time observations for two stations (all path-pairs)



(a) Overview of Valby Station

(b) Overview of Korsør Station

Figure 4: Overview of station layouts for selected stations. Background source: OpenStreetMap

total of 11,875 observations, which is a subset of the 19,439 total observations at Valby Station. The four path pairs V-A1, V-A2, V-B1 and V-B2 are transfers to platforms used by suburban train services, whereas V-A3 and V-B3 are transfers to regional trains. Table 1 shows that the path pair with the largest distance V-A3 and V-B3 have the highest mean observed walking time with respectively 4 and 5 minutes. From stop B the passengers walking have to cross a pedestrian crossing to get to the different platforms, which results in a mean difference between stop A and B of 50 seconds on average.

The observed walking times are compared to the scheduled walking time, which is used in travel planners and for planning of connection. This shows that at least 4% of the passengers transferring to platform 1 and 2 are not able to make the scheduled transfer time, where in the case of V-A3 and V-B3 there are respectively 27% and 35%. If the raw walking time was to be used as an indicator for the direct walking time, the scheduled walking time for both stops to platform 3 should be increased to accommodate the higher walking times.

Table 2 presents the results of the model for both the share of passengers walking directly, the direct walking time $\hat{W}^D$, walking time for passengers with activity $\hat{W}^A$ and the predictive posterior distribution $\hat{W}$ from each stop to the three platforms. If we compare the direct walking time $\hat{W}^D$ to the scheduled walking time, there is larger share of the passengers that are able to make the transfer compared to the observed walking time. All transfers for direct walking passengers to platform 1 and 2 have less than 1% of the density above the scheduled walking time, where V-A3 has 1.35% and V-B3 has 24.85% above. For path pair V-A1 and V-A2 it is possible to reduce the scheduled walking time to 2 minutes and still have less than 1% of the density above the scheduled walking time.

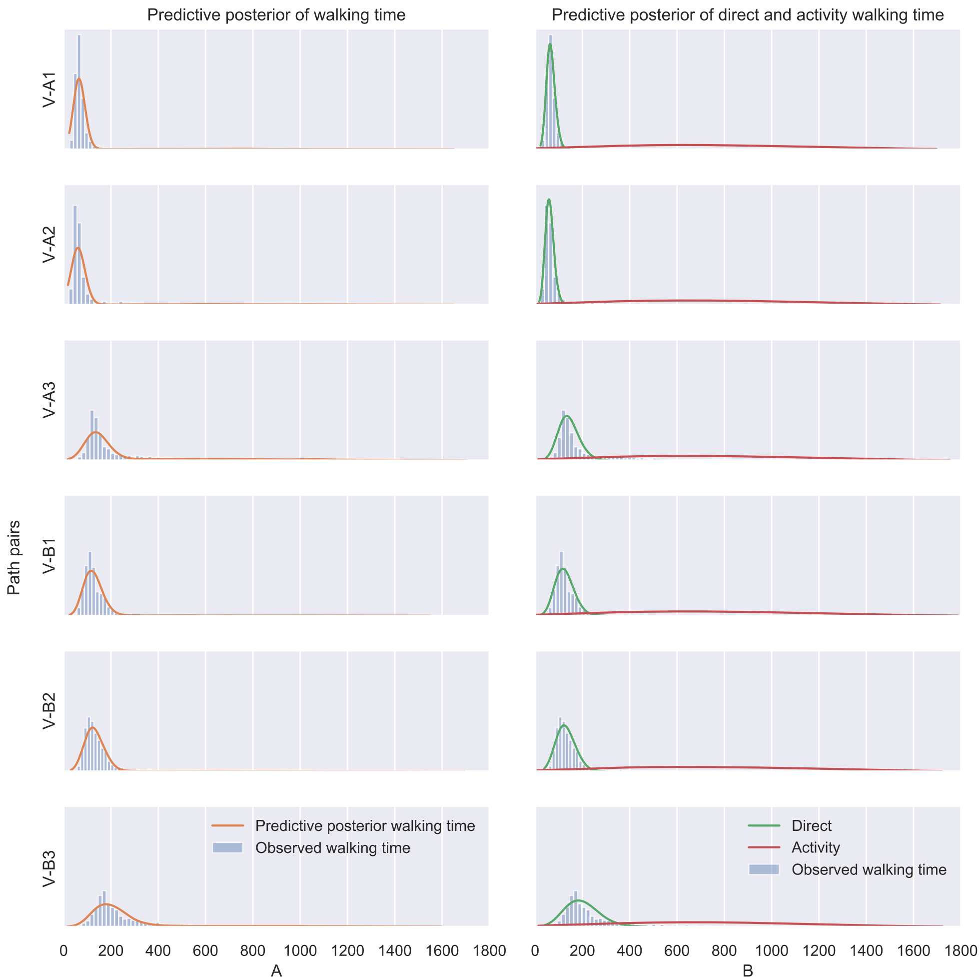
		Observed walking time					Scheduled Walking time	
Path Pair	N	Mean	Std	2.5%-tile	50%-tile	97.5%-tile	Value	Above
V-A1	2206	90.61	121.15	40.00	65.00	426.62	240	4.26%
V-A2	523	103.18	180.29	35.05	60.00	541.90	240	6.69%
V-A3	3460	244.72	235.25	82.00	149.00	981.93	240	27.57%
V-B1	2878	142.86	107.70	68.00	119.00	394.00	240	5.77%
V-B2	1153	159.08	139.79	77.00	126.00	579.00	240	7.37%
V-B3	1655	283.68	236.92	100.00	199.00	1054.95	240	35.59%

Table 1: Observed walking time and Schedule walking time of path pairs at Valby.

Parameters	ID	Mean	Sd	2.5%-tile	50%-tile	97.5%-tile	ess	$\hat{R}$	Above scheduled time
λ	V-A1	0.93	0.01	0.92	0.93	0.94	5958	1.0	-
	V-A2	0.89	0.01	0.86	0.89	0.92	7274	1.0	-
	V-A3	0.73	0.01	0.71	0.73	0.74	4574	1.0	-
	V-B1	0.95	0.00	0.94	0.95	0.96	6428	1.0	-
	V-B2	0.93	0.01	0.92	0.93	0.95	6691	1.0	-
	V-B3	0.80	0.01	0.78	0.80	0.83	5009	1.0	-
$\hat{W}^A$	V-A1	724.77	365.30	113.18	695.17	1462.60	3970	1.0	-
	V-A2	728.12	361.87	129.06	703.23	1460.16	4000	1.0	-
	V-A3	724.14	358.89	121.91	694.96	1457.00	3650	1.0	-
	V-B1	725.68	362.40	120.31	706.11	1461.97	3895	1.0	-
	V-B2	735.94	361.96	128.52	712.20	1462.73	4025	1.0	-
	V-B3	728.23	357.04	122.51	711.24	1455.79	3964	1.0	-
$\hat{W}^D$	V-A1	66.64	16.63	37.93	65.20	102.55	4089	1.0	0.00%
	V-A2	61.58	16.54	33.39	60.14	97.70	4202	1.0	0.00%
	V-A3	142.63	39.08	76.35	139.00	228.43	3643	1.0	1.35%
	V-B1	124.82	36.65	63.08	121.48	205.15	3700	1.0	0.45%
	V-B2	130.05	37.27	65.68	126.91	208.48	4083	1.0	0.43%
	V-B3	198.52	65.34	89.80	192.48	343.95	4008	1.0	24.85%
$\hat{W}$	V-A1	112.99	187.74	38.51	66.84	804.97	3834	1.0	10.72%
	V-A2	139.13	245.37	33.84	62.72	1036.08	3995	1.0	6.82%
	V-A3	301.25	324.48	78.56	155.33	1246.30	3903	1.0	25.85%
	V-B1	159.09	169.34	63.13	124.04	763.39	4119	1.0	5.65%
	V-B2	169.55	177.39	66.06	129.79	827.76	4015	1.0	6.42%
	V-B3	294.88	263.69	90.96	207.85	1162.98	3896	1.0	37.05%

Table 2: Valby - Posterior means and statistics in seconds.

Continuing to the fit of the model, we see that the direct walking time $\hat{W}^D$ aligns with the differences between path pairs described for the observed walking time. The highest walking times from both bus stops are found for passengers walking to platform 3 and the model estimates that it takes on average 1 minute longer for passengers to walk from stop B than stop A. A visual inspection of the model estimations in Figure 5A of the predictive posterior walking $\hat{W}$ shows that all path pairs have a peak at the same position as the observed walking time followed with a long tail. The peak originates from the direct walking time distribution shown in Figure 5B, where the long tail originates from the activity. The figure shows that density of the activity distribution ranges over the direct walking time distribution, which results in an underestimation of the direct walking share λ giving the smaller peak of the predictive posterior walking compared to the observed walking time. Examining the posterior predictive walking time closer in Table 2, the median values of each path pair is on average 4,7 seconds higher than observed walking time. For the upper percentiles, the estimation is notable above the observed walking time, supporting the visual inspection of the underestimation of the direct walking share. The model estimates a share of passengers walking directly λ ranging from 73% to 95%, where the two lowest λ values (73% and 80%) are estimated for path pairs to the regional train services at platform 3. Compared to the suburban rails services on platform 1 and 2, the headway is larger for the regional, making it easier for passengers to do an activity without missing their train.



(A) The predictive posterior walking time distribution is generated from the weighting of the direct walking share of activity and direct walking time. (B) The distribution of activity and direct walking time distribution without the direct walking share.

Figure 5: Valby station - Predictive posterior of walking time compared to observed walking time.

4.2. Case station 2: Korsør

Korsør has three path pairs shown in Figure 4b, which are three different bus stops to the same platform at the station. As shown in Table 3 the path pair K-C has the lowest mean observed walking time of 3.7 minutes in combination with the highest schedule walking time of 4 minutes compared to the two others path pairs schedule walking time of 3 minutes. With a lower scheduled walking time, it would be expected, that the observed mean walking time would be smaller for the path pairs K-A and K-B, but we can see from the Table 3 that the mean walking time is nearly double for both. Comparing the scheduled walking time to the observed, we see that path pair K-C has 30% of the observed walking time above the scheduled walking time, while path pairs K-A and K-B have respectively 50% and 70% above. Using the raw walking time as an indicator for the needed walking time, would thus increase the scheduled walking time significantly.

		Observed walking time					Scheduled Walking time	
Path Pair	N	Mean	Std	2.5%-tile	50%-tile	97.5%-tile	Value	Above
K-A	130	427.32	449.57	59.22	199.00	1443.10	180	50.77%
K-B	187	577.06	416.16	56.00	596.00	1327.40	180	69.52%
K-C	386	227.41	260.37	41.87	94.50	960.12	240	29.53%

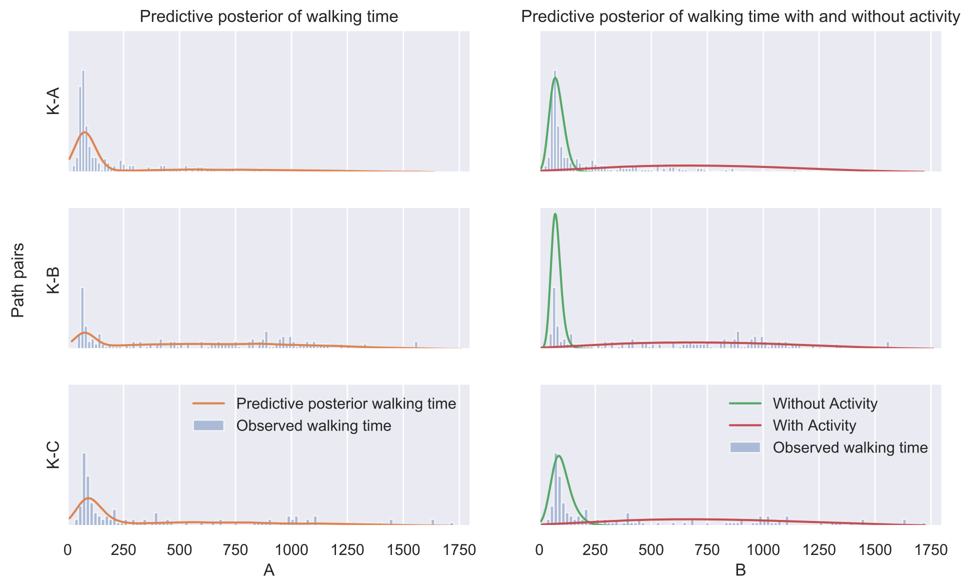
Table 3: Korsør - Observed walking time and Schedule walking time of path pairs.

The model estimates a low degree of the transfer passengers walking directly from the bus to the station, where the mean share of passengers walking directly ranges from 26% to 63%. The highest activity share is the path pair K-B, which was suspected of having an abnormal transfer pattern. If we look at the posterior predictive walking time $\hat{W}$ of the path pair K-B in Figure 6A, we see that a large part of the density is spread in the tail. At the same time, we see a significant number of the observed walking time samples are located here, thus supporting the high degree of activity. A comparison between the distribution of $\hat{W}$ in Table 4 and the observed walking time in Table 3 shows a reasonable match between the two. The fit does not seem as good as for the other case station, since the lower percentiles underestimates and the upper percentiles overestimates values. Looking at the predictive posterior of the directly and activity walking time we see separated peaks for the two distributions, but there are, as with the estimation for the other case station, areas where the density of the activity and directly walking time overlaps. This could possibly affect the models ability to separate the two distributions.

Parameters	ID	Mean	Sd	2.5%-tile	50%-tile	97.5%-tile	ess	$\hat{R}$	Above scheduled time
	K-A	0.63	0.03	0.57	0.63	0.68	2779	1.0	-
λ	K-B	0.26	0.04	0.19	0.26	0.34	2059	1.0	-
	K-C	0.52	0.06	0.41	0.52	0.63	2536	1.0	-
	K-A	730.26	359.86	119.92	709.46	1471.66	4042	1.0	-
$\hat{W}^A$	K-B	742.19	359.35	142.14	726.35	1460.54	3933	1.0	-
	K-C	755.01	366.21	143.12	728.99	1490.58	3933	1.0	-
	K-A	78.88	28.54	32.26	75.27	141.04	3906	1.0	0.35 %
$\hat{W}^D$	K-B	75.94	21.41	42.27	73.06	124.90	3743	1.0	0.18 %
	K-C	100.36	42.17	37.95	93.46	200.68	3799	1.0	0.88 %
	K-A	314.31	381.38	35.96	99.72	1307.28	4138	1.0	34.35 %
$\hat{W}$	K-B	577.97	426.36	51.14	549.11	1428.90	3900	1.0	71.90 %
	K-C	408.92	411.18	43.10	167.40	1378.21	4121	1.0	44.35 %

Table 4: Korsør - Posterior means and statistics in seconds.

If we compare the scheduled walking time to the direct walking time $\hat{W}^D$ distribution, there is less than 1% of the density above the scheduled walking time for three path pair K-A, K-B and K-C, making them



(A) The predictive posterior walking time distribution is generated from the weighting of the direct walking share of activity and direct walking time. (B) The distribution of activity and direct walking time distribution without the direct walking share.

Figure 6: Korsør station - Predictive posterior of walking time compared to observed walking time.

reasonable scheduled walking times. This indicates that the high number of the observed walking time, is due to the high degree of activity at the station.

5. Discussion

The validation of the proposed method is indeed difficult. As described in Section 3.1 we do not assume ground truth about whether passengers transferred directly is available, nor do we assume availability of their true walking time or choice of path.

As a consequence of the desire for a general and large scale applicable solution, manual validation in the form of accompanying or somehow recording passengers during their transfers in order to determine their true walking time and possible time used for activities were deemed infeasible. Such an approach would be both error-prone due to the human factor, and very time-consuming for collection of a representative sample. It can also be argued, that people might not recollect doing activities during transfers as for example used in Mosallanejad et al. (2018) for splitting trip chains into separate trips. On top of this, passengers also have difficulties in reporting reasonable walking times in surveys (Anderson, 2013). Therefore validation with classic surveys and interviews are considered insufficient and impractical.

To overcome this challenge we suggest two generalizable verification approaches that are applicable at scale: (i) *Verification using number of feasible trains*; and (ii) *Verification using shop availability data*. In the following sections we detail the two verification approaches. We recognize that the verification can be further improved for concrete cases, depending on the data available.

5.1. Verification of model results using number feasible trains

Verification using train assignment requires access to train AVL data similar to the bus AVL data described in Section 3.1. We only consider passengers who finished their journey after riding the train, i.e. $Tap\ In_{i,n}$ is the train tap in on the last trip leg, n, for passenger trip i. We assign each passenger a set of feasible trains which runs directly to the destination station based on $Tap\ In_{i,n}$ and $Tap\ Out_i$. We likewise assign each passenger a set of feasible trains based on $Bus\ Arrival_{j,k'}$ and $Tap\ Out_i$, where $(j, k') = Match\ Arrival_{i,n-1}$. The latter one corresponds to feasible trains given the passenger had absolutely no walking time at all.

Table 5 shows the number of observations decomposed by the number of feasible trains based on the two approaches for train assignment cf. above for passengers at Valby station.

	Using $Tap\ In_{i,n}$				
Using $Bus\ Arrival_{j,k'}$	1	2	3	4	**Total**
1	9,287				**9,287**
2	2,421	90			**3,323**
3	270	351	117		**738**
4	79	116	90	111	**396**
Total	**12,057**	**1,369**	**207**	**111**	**13,744**

Table 5: Decomposition of feasible trains for Valby station by approach.

The table indicates that around 85% of the passengers have only one feasible train given their $Tap\ In_{i,n}$ time and the final $Tap\ Out_i$. Most of these passengers have also only one feasible train given the $Bus\ Arrival_{j,k'}$ time. However, there are also a considerable number of passengers who have a difference in the number of feasible trains given the two criteria. Some of these passengers, especially those who have a large difference on the number of feasible trains given the two criteria, are possibly more likely to have had an activity during the transfer, as they did not board some possible trains they could have caught if they walked fast to the platform. To test how the model predicts passengers within these groups, the observations can be combined with the prediction of the model. 4,000 samples of the set of parameters in the model are used to categorise passengers into three groups:

- Directly - All sampled sets of parameters assigned the highest probability of the observation belonging to the directly walking distribution.
- Activity - All sampled sets of parameters assigned the highest probability of the observation belonging to the activity walking distribution.
- Mixed - The observation was not consistently assigned to one of the groups.

Figure 7 presents the share of passengers within each of the predicted groups belonging to the combination of each count of feasible trains. There is a noticeable difference between the distribution of passengers in the respective groups across the different combinations. For the group predicted to walk directly, around 70% of these have only one feasible train given both their tap in time and the arrival time of the bus. The shares for the group predicted to have an activity during the transfer is lower for this combination, and instead higher for the combination with two feasible trains given the bus arrival time and only one feasible train given the tap in time. The result that almost no passengers predicted to walk directly is placed in the group with three feasible trains given the bus arrival time and only one feasible train given the tap in time is reassuring, as this cluster indicates that the passenger could have possibly reached at least one train prior to the one boarded.

At Korsør station the dataset consists of 490 passengers who tapped out at the end of the train leg. Only 10 of these passengers had more than one feasible train given the bus arrival time, and hence the long observed walking times found in Section 4.2 stems from passengers who spend time at the station building

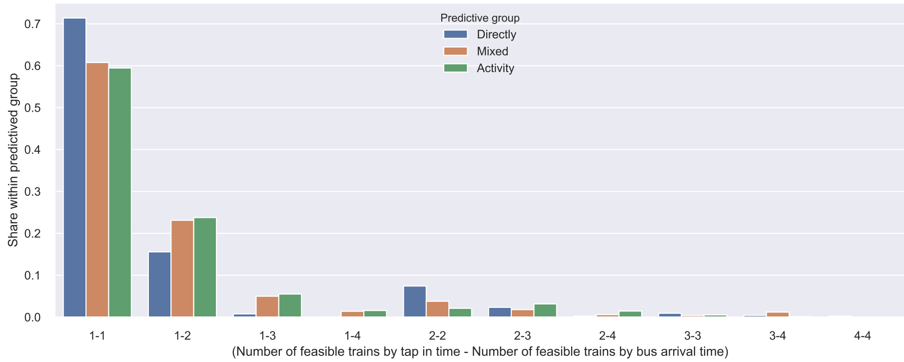


Figure 7: Distribution of passenger groups predicted to respectively walk, do an activity, or not uniquely identified, across combinations of number of feasible trains (tap in time vs. bus arrival time)

instead of walking directly to the platform. The long walking times are thus an effect of the long transfer times, due to the lack of coordination between busses and trains.

5.2. Verification using shop availability data

One of the main assumptions for passengers not walking directly during the transfer is shopping activities. In order to support this assumption and provide a weak, but scalable verification of the proposed method, the share of activity transfers $(1 - \lambda_q)$ is correlated with shop availability. Since a unique value of λ_q per path pair q is obtained, we also use this granularity for shop availability.

Data is extracted from Open Street Map (OpenStreetMap contributors, 2018) using a buffer zone around the crow flies distance of path pair q as illustrated by Figure 8. The size of the buffer zone has been fixed to 500m in this experiment. We search this buffer zone using the Open Street Maps tag features, specifically nodes containing the tag `shop`.

We denote the number of shops in the buffer zone formed from path pair q as $\mathit{Shop\ Availability}_q$, and investigate the correlation between $1 - \lambda_q$ and $\log(\mathit{Shop\ Availability}_q)$. We apply the logarithm based on an expectation that the marginal effect of extra shops will eventually have a limited effect on how many passengers will take advantage of the availability.

Figure 9 shows the relation between $1 - \lambda_q$ and $\log(\mathit{Shop\ Availability}_q)$. We see a positive correlation between the two variables. The result supports some relationship between the estimated activity share for each path pair, and the shop availability along the path pair. Although the relationship is clearly not linear ($R^2 = 0.25$), given that a high availability of shops does not guarantee a high share of passengers with activities. On the other hand, in all cases where the presented method has estimated high activity transfer share, we find a high availability of shops.

5.3. Waiting times for different passenger groups

Given the already identified feasible trains cf. Section 5.1 we extend this further to an actual train assignment by minimizing the exit time (i.e. $\mathit{Tap\ Out}_i - \mathit{Train\ Arrival}_{j,k}$). With the passenger trips assigned to trains it is possible to calculate the waiting time on the train platform. Since some trips has several feasible trains we have only focused on the trips with exactly one feasible train itinerary to limit the uncertainty of the true waiting time. Having these groups, the observed waiting and walking time can be plotted for each station as seen in Figure 10.

For Valby, the passengers predicted to the directly walking group have the lowest walking and waiting time compared to the activity group. The low walking and waiting time align with the assumption that

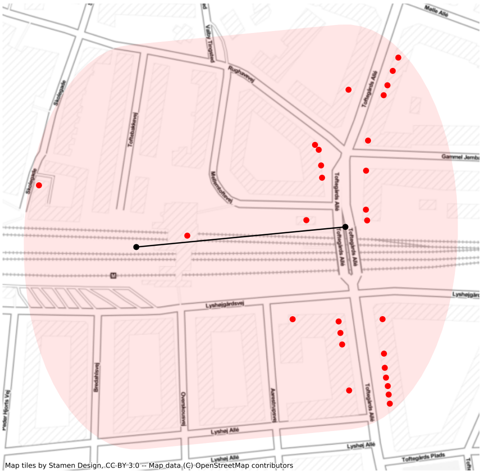


Figure 8: Example of shop availability buffer zone for Valby Station. Crow flies distance of path pair (black), Buffer zone (transparent red), Shops (red). Background source: OpenStreetMap

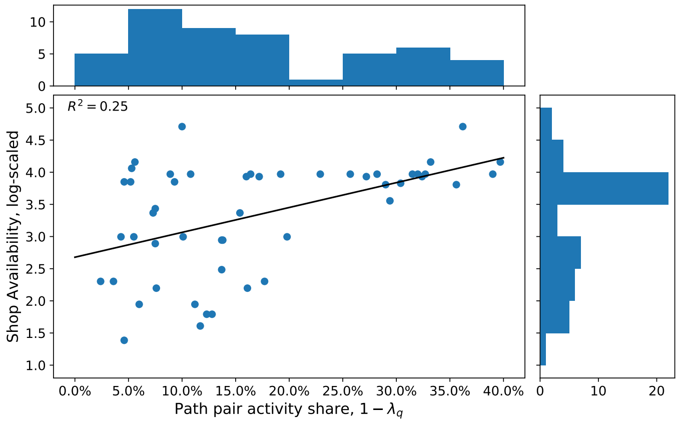


Figure 9: Results of shop availability and activity share relation. Only path-pairs with more than 2000 observations are included.

the directly walking group describes the passengers who walk directly to minimize their overall transfer time. In the case of Korsør we see the same pattern for the walking time, with lowest mean walking time for the directly walking group and highest for activity group, but the reverse pattern for the waiting time. This indicates that the bus arrival and train departures are not synchronised, especially when taking into account that the median transfer time is 14.7 minutes for Korsør compared to Valby's 6.5 minutes. The lack of synchronisation between busses and trains make it difficult to minimize the overall transfer time for the directly walking passengers, which just results in a high waiting time. This shows that it is possible for the model to separate the activity of waiting in the station building and walking directly to train platform, thereby being able to identify inefficient connections.

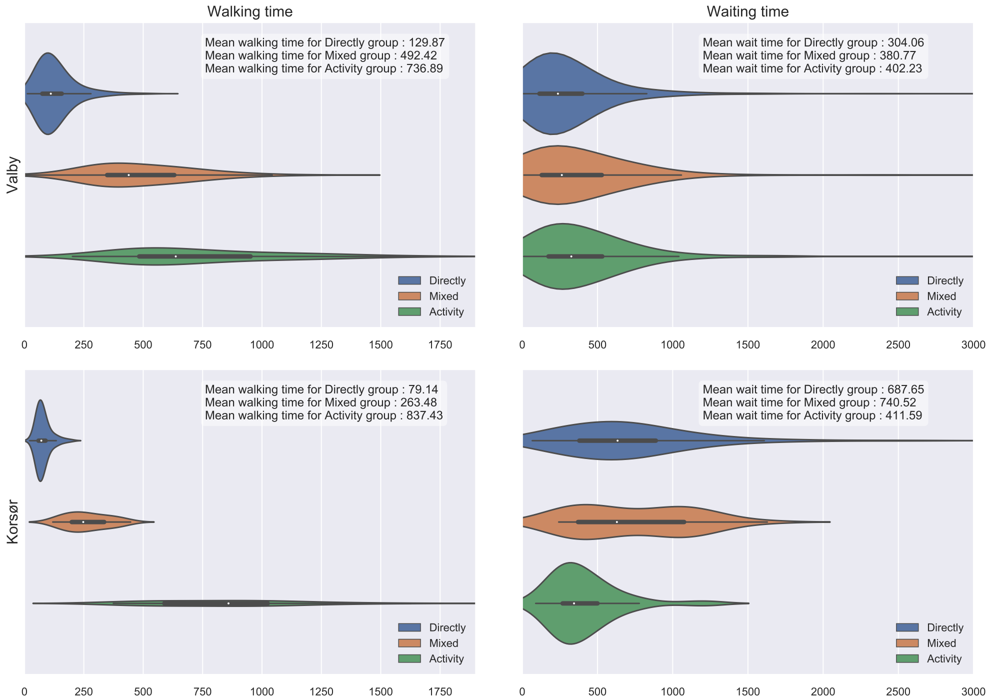


Figure 10: Observed waiting and walking time distribution for Valby and Korsør for each prediction group.

6. Conclusion

This study has presented a novel methodology for providing accurate walking time distributions at transfers from bus to train based on smart card data. The model requires AVL data from busses and smart card data where the passenger must tap-in at the train station, preferably at the platform to avoid uncertainty of possible time spent in a station building.

The proposed approach is able to reproduce the observed times between the passenger alights a bus taps in at the platform using a hierarchical Bayesian mixture model, where passengers are assumed to either walk directly to the platform or perform an activity during the transfer. The model is applied to a large-scale case study with 129 stations in the Eastern part of Denmark. Detailed investigations from two stations show that the model is able to estimate accurate walking time distributions for two types of stations: i) stations where passengers are spending extra time during the transfer due to poor synchronisation between busses and trains, and ii) stations where passengers or are doing shopping, buying coffee other short errands during the transfer.

The model can be easily applied at scale, and thus offer a more feasible methodology than manual surveys where passengers are followed through the transfer, when public transport agencies need to estimate the necessary walking time to perform transfers. The resulting distribution for walking time for the direct walking passengers can be compared to the scheduled walking time published by public transport agencies, and thereby identifying places where extra scheduled walking time is needed. In this way the agencies are able to plan more reliable connections between busses and trains.

Acknowledgements

The authors would like to thank Rail Net Denmark (BaneDanmark), Movia and DSB (Danish State Railways) for providing automatic vehicle location data for the project. Furthermore, the authors would like to thank Movia for providing data from the Danish smart card (Rejsekort) for use in the project.

Funding sources and declaration of interests

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Declarations of interest: none

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8 Paper 7: A note on unusual path choice behavior caused by congestion in metro systems

The following pages contain the article:

M. Eltved, H. N. Koutsopoulos, N. H. M. Wilson, K. Tuncel, and Z. Ma (2020). "A note on unusual path choice behavior caused by congestion in metro systems". Working paper.

Please cite accordingly.

Earlier versions of the paper was presented at the *5*th *International Symposium Use of Public Transit Automated Data for Planning and Operations (TransitData2019)*, Paris, France in August 2019, and at the *99*th *Annual meeting of the Transportation Research Board*, Washington D.C, United States in January 2020.

A note on unusual path choice behavior caused by congestion in metro systems

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Based on Conference Paper "Understanding Reverse Routing Path Choice Behaviour in Congested Metro Systems" presented in poster session at the Transportation Research Board (TRB) 99th Annual Meeting, Washington D.C., USA, January 13th-16th, 2020.

Abstract

Passenger path choice in congested metro systems is affected by crowding in vehicles which can lead to denied boardings onto trains. This paper deals with special situations where passengers choose unusual paths due to overcrowding. An analysis of a special case, in this paper referred to as reverse routing, where passengers choose to remain longer in a train and then transfer to another train, which ultimately involves reversing their direction, is presented.

The case study is a heavily crowded section of the MTR metro system in Hong Kong. Smart card data and a passenger-to-train assignment model combining automatic fare collection data and automatic vehicle location data, are used to analyze the possible underlying causes for this reverse routing behavior. It is found that passengers travelling furthest have significantly higher travel times based on the passenger-to-train assignment model. This indicates that the passengers travelling furthest behave differently than passengers travelling shorter distances, which could be due to these passengers choosing to reverse route. The analysis also examines the impact of travel experience, and shows that more experienced passengers have lower journey times than less experienced passengers. For comparison with the heavily congested metro system in Hong Kong, a brief analysis of unusual path choice behavior in the Copenhagen metro system is also presented. Using simple statistics it can be easily shown, that such unusual path choice behavior does not exist in the Copenhagen system, at least not due to overcrowding.

Finally, the paper discusses different methodologies for obtaining correct estimates of the fraction of passengers making unusual path choices, when the ground truth can not be directly obtained. The results of such estimations can help agencies evaluate new operational strategies and reduce overcrowding, ultimately benefitting the passengers.

Keywords: Transit, Metro systems, Crowding, Reverse routing, Travelling backwards

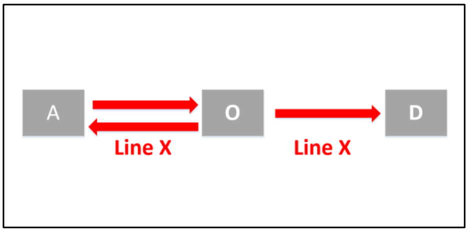
1 Introduction

Ridership in transit systems is constrained by the capacity of the system, and overcrowding in peak hours on some systems can lead to special circumstances, in which a passenger can gain an advantage by choosing a path, which under normal conditions would be a dominated alternative. This note investigates possible causes of such unusual travel behavior by first analyzing its potential causes using a case from the metro system in Hong Kong (MTR), and secondly using smart card data from the Copenhagen metro to test whether similar unusual path choice behavior also exists in this system.

It is well-known in the literature that crowding in public transport is uncomfortable for passengers (see e.g. Batarce et al., 2017, Haywood et al., 2017, Li and Hensher, 2013 or Tirachini et al., 2017). Under very crowded situations, passengers perceive in-vehicle time to be up to 2.5 times as onerous as in-vehicle time (Batarce et al., 2017). However, it is also important to note that while some passengers put a high penalty on crowding, others give it a much lower weight (Tirachini et al., 2017).

Given the large potential discomfort of in-vehicle crowding, some passengers also may react to this by changing their path choice in metro systems. Kim et al., 2015 investigated the effect of crowding on passenger path choice, and found that the increased travel time resulting from crowding can affect path choice, but, also the discomfort of crowding itself can affect path choice. This shows that passengers do not base their path choice solely on travel time, but also take into account the comfort of the trip on top of well-known parameters such as waiting time, walking time and number of transfers (see e.g. Raveau et al., 2014).

This paper concerns two unusual path choice behaviors, which would not occur in normal (i.e. uncrowded) situations - travelling backwards (TB) and reverse routing, as illustrated in Figure 1. The concept of travelling backwards involves a passenger boarding a train on a given line going in the “wrong” direction from where the passenger actually wants to go. The passenger then transfers at a “turn-back” station and boards a train going in the correct direction, and passing back through the origin station. This path choice is relevant in cases where the denied boarding rate at origin station O is high. In uncrowded situations, this would not be a reasonable path.



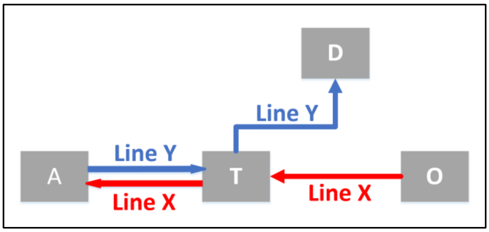


Figure 1A – Concept of travelling backwards **Figure 1B – Concept of reverse routing**

FIGURE 1 – Illustration of unusual path choice behaviors considered in the paper

The concept of reverse routing can be considered a special case of travelling backwards. However, a difference is that in the travelling backwards situation the reverse routing behavior does not add an extra transfer to the alternative path. As illustrated in Figure 1B, a passenger travelling from O to D would, under normal conditions, transfer at station T, given that the transfer at T is as convenient as the transfer at station A. Transferring at station T minimizes the in-vehicle time and in a high-frequency system, almost certainly allows the passenger to board an earlier train than if the transfer was made at station A. However, if the passenger risks being denied boarding several times at station T, or has a higher probability of obtaining a seat by transferring at station A, the passenger might choose to stay on line X until station A and transfer there, thus performing what in this paper is defined as *reverse routing*.

This paper analyses both types of unusual path choice behavior, but in two different settings. First, the concept of reverse routing is analysed using a case from the MTR metro network in Hong Kong and second, the paper briefly describes possible travelling backwards situations in the Copenhagen metro system. Finally, recommendations are made for further analysis of these problems.

The paper consists of the following sections: Section 2 introduces the existing literature on the topic of unusual path choice behavior; Section 3 describes the methodology used for analyzing different aspects of reverse routing in the MTR system; Section 4 investigates the MTR case study and presents the main results of this analysis; Section 5 briefly describes the Copenhagen metro case study, and analyses whether travelling backwards behavior can be observed in the system; Section 6 describes possible ways forward for further analysis of the travelling backwards and reverse routing situations in other systems. Finally, Section 7 concludes the paper.

2 Prior studies of unusual path choice behavior

Although, many studies have focused on evaluating the cost of crowding and investigated the effect on path choice in transit systems, very few studies have dealt with unusual path choices such as travelling backwards and reverse routing. No studies have specifically dealt with the example of reverse routing as shown in Figure 1B, although a handful of studies have considered the concept of travelling backwards. The concept of travelling backwards has been identified in the metro systems in Singapore (Chakirov & Erath, 2011, Othman et al., 2015,Tirachini et al., 2016) and Bejing (Li et al., 2017, Xu et al., 2018, Yu et al. 2020). Although the behavior can potentially be seen in many parts of a network, it is most often seen at stations near the start of a line, where passengers at the second or third station travel back to the starting station for a much higher probability of obtaining a seat.

Chakirov & Erath (2011) was the first paper to verify the unusual behavior of travelling backwards based on data from Singapore. They used estimates of waiting times calculated based on the fastest possible person through the system to find that the distribution of waiting times at stations close to the starting station was bimodal. Since no denied boarding was observed at these stations they explained this bimodal distribution by some passengers choosing to travel backwards. Based on this finding, they concluded that some passengers were in this case willing to exchange ten minutes extra in-vehicle time for a seat. Othman et al. (2015) also studied the Singapore case and focused on the development of an agent-based model to estimate the effects of crowding in the metro system. They developed a simple model to replicate the empirically observed bimodal journey time distributions, which took into account the number of stations the passenger had to travel on a given line and for how many stations the passenger travelled backwards. This improved their model and gave a more realistic estimation of the crowding levels in the system. The final study which used Singapore as the case was Tirachini et al. (2016). They specifically used the observations of passengers travelling backwards and quantified the standing multiplier as around 1.2 compared to being seated with the current crowding levels.

The Beijing studies focused on analyzing the fraction of passengers travelling backwards and also focused on cases where passengers at stations close to the start of the line travel backwards to the first station of the line. Li et al. (2017) developed a clustering methodology to group passengers based on their journey times. By comparing the results with observed travel behavior at some stations, they estimated that up to 10% of passengers on some OD pairs travelled backwards in peak hours, and that the proportion of passengers travelling backwards increased with the trip length. Xu et al. (2018) refined the methodology developed in Li et al. (2017) and developed a clustering methodology to determine if the passenger travelled backwards or not. They were able to identify specific stations on a specific line, where up to 10% of passengers travel backwards. Finally, Yu et al. (2020) developed a hierarchical Bayesian model to further investigate the problem of travelling backwards. They first split the passengers into passengers travelling normally and passengers travelling backwards, and subsequently estimated distributions for passengers boarding the respective trains. The approach is based on the assumption that the tail of bi-modal travel time distribution can be attributed mostly to passengers travelling backwards. This means that the passengers travelling backwards most likely have longer travel times than passengers being denied boarding one or two trains. They used a survey in which passengers were counted if they transferred between trains at a “turn-back” station to verify the results of the model, and used the results to optimize the passenger flow assignment. The results for the stations in the case study showed, that around 25 % of the passengers travelled backwards in peak hours.

A related topic is the passenger choice of boarding station. This has previously been investigated in Hassan et al. (2016), where elements like access time to a specific stop, the expected waiting time and possible route choices from a station were included in the analysis. In the context of unusual path choice behavior, it is related to the travelling backwards concept, where the passenger can choose to walk (or bike) to a stop further upstream on a line to have a higher probability of boarding the first possible train. The stop choice problem is not analysed in this paper, but some notes on possible ways to analyse the problem are given in Section 6.

3 Methodology for analysis of reverse routing passenger behavior in metro systems

For studying the behavior of reverse routing passengers in Hong Kong two data sources are available: automatic fare collection (AFC) data with tap-in and tap-out information and automatic vehicle location (AVL) data with train departure and arrival times at stations. Passengers in most closed metro systems only tap-in at the origin and tap-out at the destination and no information on the transfer stations is recorded. The idea for analyzing the potential factors affecting the reverse routing behavior is therefore to use passenger-to-train assignment models to identify which trains passengers boarded, thus eliminating some uncertainty in the journey times from tap-in to tap-out time. The focus is on determining the train that passengers boarded on the second legs of their trips. Returning to the sketch in Figure 1B, this means that the time which is analysed is the time from tap-in at station O to when the passenger leaves station T. The time spent on line Y between station T and D can be eliminated from the journey time since the passenger is assigned to a specific train with the departure time from station T known. Below, the passenger-to-train assignment methodology is described in further detail.

Passenger-to train assignment

The passenger to train assignment utilizes the egress time at the destination station. It is assumed that each group of passengers who board different trains have the same egress time distribution. Thus the egress times of passengers tapping-out in different time intervals are assumed to be generated from the same distribution that is specific to the destination platform. Based on this assumption, a sample of egress times can be acquired by looking at the passengers who have a single feasible train (given the tap-in and tap-out time) in their feasible itineraries and tapped-in on the same line. However, this is a biased sample since the passengers who have a single feasible train have an egress time that is smaller than the headway between their boarded train and the next train. Therefore, some correction for this bias is necessary. This correction is made using a truncated distribution to represent the observed egress times (Zhu et al., 2017). Given that the headway experienced by each passenger serves as the upper bound for their egress time, the egress time distribution can be written as a truncated random variable as follows;

$$f(t^e | t^e < H) = \frac{g(t^e)}{F(H)} \tag{1}$$

where t^e is the egress time, H is the headway, $f(t^e)$ is the probability density function associated with the egress time and $F(H)$ is the cumulative distribution function associated with the egress time. Also, $g(t^e) = f(t^e)$ for all $t^e < H$ and $g(t^e) = 0$ for other values. Using this formulation, any continuous probability distribution can be fitted to the observed egress times using the following likelihood function;

$$L = \prod_i f(t_i^e | t_i^e < H_i) \tag{2}$$

where t_i^e is the egress time for i[th] passenger. Based on the corrected egress time distribution, we can evaluate all the possible egress time values for a passenger. Then, it is trivial to assign each passenger to the train with the highest probability within her feasible train set. A posterior probability can be calculated for each passenger and each feasible train using the possible egress times;

$$P_{ij} = \frac{f(t_{ij}^e)}{\sum_k f(t_{ik}^e)} \tag{3}$$

where P_{ij} is the probability of passenger i boarding train j and t_{ij}^e is the egress time associated with passenger i, if that passenger boarded train j. Thus $f(t_{ij}^e)$ is the pdf of observing that egress time value. For the purposes of this study, a lognormal distribution is used to represent the egress time distribution, since it has been used to represent walking times (Zhu et al., 2017). In Figure 2 an example of the passenger-to-train assignment is shown. The egress time distribution is modelled in a previous step and, based on this distribution, the most likely train the passenger boarded on line Y is train Y3. With this knowledge the departure time from station T can be found using the AVL data and the time from tap-in to departure from station T denoted τ is defined by:

$$\tau = t_T - t_O \tag{4}$$

The time τ does not define whether the passenger transferred at station T or station A, but can indicate whether some passengers spend more time than others, given that they departed on the same train on line Y.

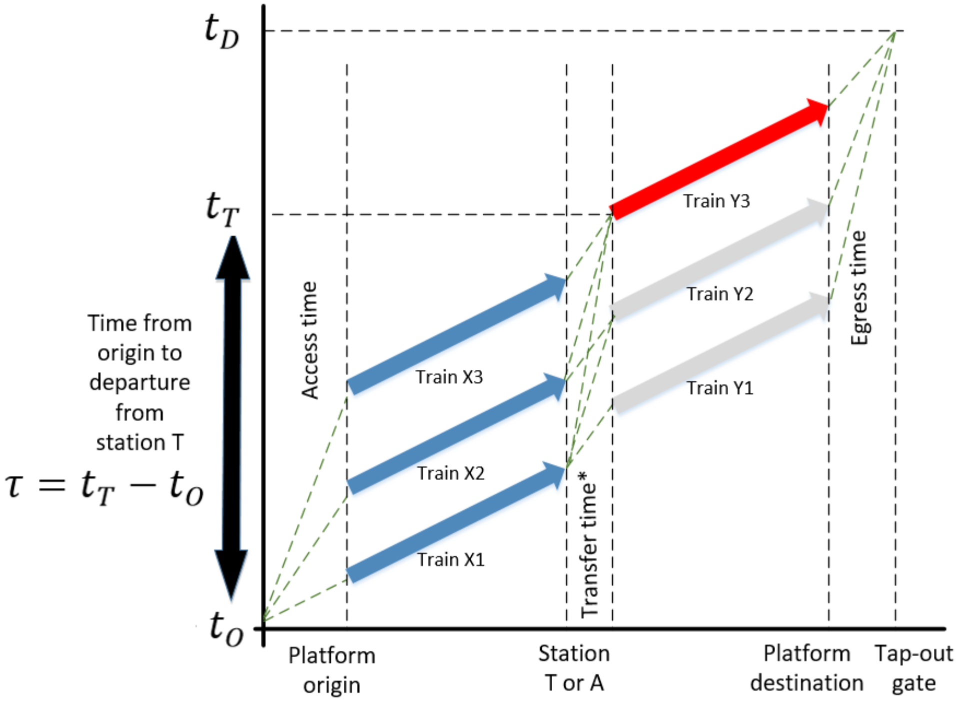


FIGURE 2 - Passenger to train assignment

Model with factors affecting the journey time τ

Given the journey times τ it is possible to analyze several factors leading to different behavior in terms of reverse routing, using a multiple linear regression model. The dependent variable is the journey time τ, which is explained by the following function:

$$\tau_{o,d,k,x} \sim \beta_{base} + \beta_d + \beta_o + \beta_k + \beta_x, d \in D, o \in O, k \in K, x \in X \tag{5}$$

where $d \in D$ are the possible destinations, $o \in O$ are the possible origins, $k \in K$ is a specific 15-minute timeinterval and $x \in X$ is the travel experience in different categories. In order not to clutter the notation for the journey time $\tau_{o,d,k,x}$, the subscripts are omitted, i.e. τ. Note that each of the β's in this way characterizes separate parameters for each *origin*, *destination*, *timeinterval* and *experience*, respectively.

In previous studies on reverse routing in metro systems, one of the clear findings is that passengers travelling furthest are most likely to travel backwards (e.g. Tiranchini et al. 2016, Othman et al., 2015 and Li et al., 2017). This hypothesis is tested by using the destination stations as explanatory variables for the journey time from tap-in until departure from station T. The origins are included in the model, as passengers naturally have higher journey times for trips to more remote stations, and the variable for time interval is included to explain the extra travel time imposed from crowding in the peak hours.

Variables on passenger experience are also included, since Kim et al. (2014) showed that passengers with more experience chose a specific metro car to minimize the walking distance at the destination station. In the case of reverse routing a hypothesis is that passengers with more experience have lower journey times as they are able to observe the current conditions, and choose whether to transfer at the normal transfer station T or to reverse route through station A.

4 Case study on reverse routing in Hong Kong

The MTR system in Hong Kong has almost 5 million daily passengers (MTR, 2019) and some sections experience severe congestion in peak hours. The specific case study concerns two major stations in the central part of Hong Kong, station 1 and 2 in Figure 3 below. Passengers travelling from stations 27-30 on the blue line must transfer at either station 1 or 2 to reach stations 3-17 on the red line.

FIGURE 3 - Case study network

Overcrowding at station 2 leads to some passengers being denied boarding for one (or more) trains. In a 2017 MTR survey, denied boarding was observed for all Red line passengers, no matter if they were transferring or entering at station 2, see Table 1. This showed, that no passengers in the peak period between 18:15 and 18:45 were able to catch the first possible train and only around 10% were able to board the second train. Most passengers were able to board the third train, but some passengers were only able to board the 4th possible train.

	1st train	2nd train	3rd train	4th train
18:00-18:15	27%	61%	12%	0%
18:15-18:30	0%	7%	**80%**	**14%**
18:30-18:45	0%	10%	**65%**	**25%**
18:45-19:00	7%	20%	63%	10%

TABLE 1 – Denied boarding at station 2 based on a manual survey on January 10th 2017 (MTR, 2017)

In a survey carried out in 2012 and analyzed in Li (2014) approximately 30,000 MTR passengers across the whole system were asked about their route choice. The survey revealed that around 8% of the passengers in the evening peak period going from the blue line to the red line transferred at station 1, whereas all passengers outside the peak period transferred at station 2. This indicates that denied boarding and overcrowding leads to a different behavior for some passengers. Since 2012 the number of passengers in the system increased by 14% until 2017 when a new line opened terminating at station 2, adding more congestion to this already crowded station (MTR, 2019). The transfer at station 2 is cross-platform whereas passengers at station 1 have to walk up one flight of stairs (or use escalators to ascend two levels and descend one level) to transfer to the red line. The additional train travel time to station 1 is around 3 minutes (1.5 minutes in each direction). The headways on both the blue and red line are between 90-120 seconds.

Data description and passenger-to-train assignment

To analyze the factors influencing reverse routing behavior data from three weekdays, 21st-23rd, March 2017 (Tuesday-Thursday), was used. These days were selected, as they had very regular headways, thus eliminating some uncertainty on the in-vehicle travel time. A decision was made to limit the sample to adult passengers, as other passenger groups, such as pensioners, might have more heterogenous travel behavior. The passenger-to-train assignment model was used to assign passengers to specific trains on the red line and outliers, those with a journey time (τ) greater then three standard deviations from the mean for passengers on a given OD pair and a specific train, were removed. Outlier detection with both two and three standard deviations were tested, and it was found that the resulting estimates of the model did not differ significantly.

Passengers who most likely departed station 2 between 17:30 and 19:30 based on the passenger to train assignment are included in the analysis, since the behavior of reverse routing is mainly observed for this time period (see table 1 above). This results in a sample of 37,050 trips of which 698 trips are removed as outliers. In Figure 4, the probability of the most likely train, a passenger is assigned to, is shown grouped by the destination station. Passengers with destination stations 4, 5, 16 and 17 can be assigned to a single train with very high confidence. For passengers going to station 3 or 6 the assignment probability is somewhat lower, but still 50 % of the passengers are assigned to the most likely train with more than 96 % confidence. Passengers assigned to the most likely train with at least 80 % confidence are included in the further analysis, which reduces the sample size to 32,451 observations.

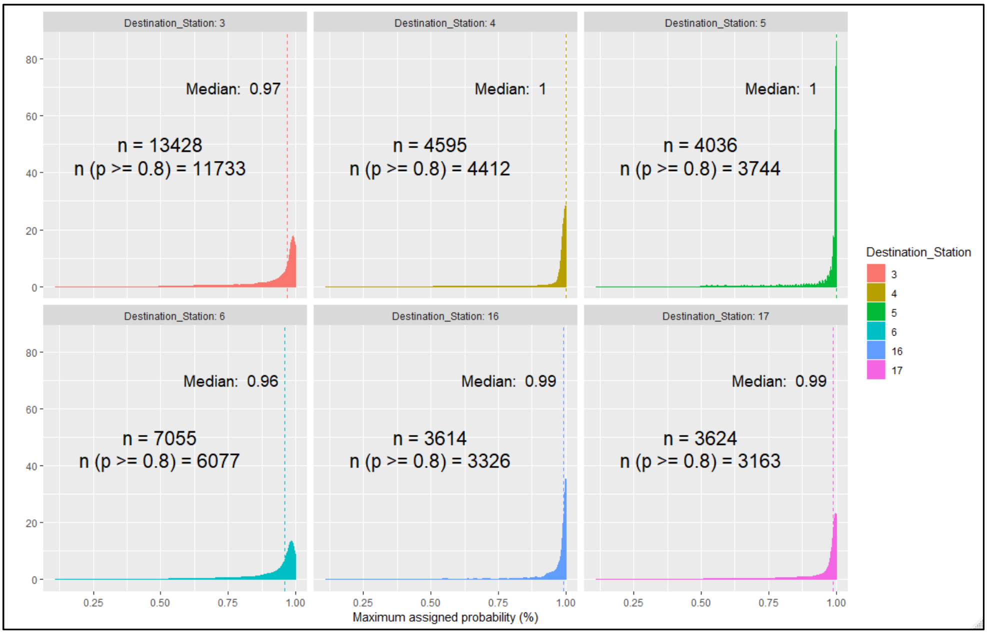


FIGURE 4 – Kernel density plot of the distribution of assignment percentage for most likely train

Naturally, the stations in the analysis have different numbers of boardings and alightings. In Table 2 the numbers of passengers from each origin to each destination are presented. The largest origins (by far) are stations 27 and 28, while the distribution across destination stations is more evenly spread, but with most passengers alighting at station 3.

Origin \ Destination	3	4	5	6	16	17	Total
27	5,196	2,183	1,894	2,886	1,701	1,673	15,533
28	4,927	1,585	1,299	2,292	1,137	1,027	12,267
29	530	227	176	339	164	187	1,623
30	1,076	417	375	560	324	276	3,028
Total	11,729	4,412	3,744	6,077	3,326	3,163	32,451

TABLE 2 – Origin - destination matrix

Since the passengers are assigned to a single train, the journey time τ from each origin to departure at station 2 is similar. In Figure 5 the journey time distributions from each of the four origins to departure at station 2 are shown. As seen from the plots, the journey time distributions are not bi-modal, but there is a large variance in the travel times for all origins.

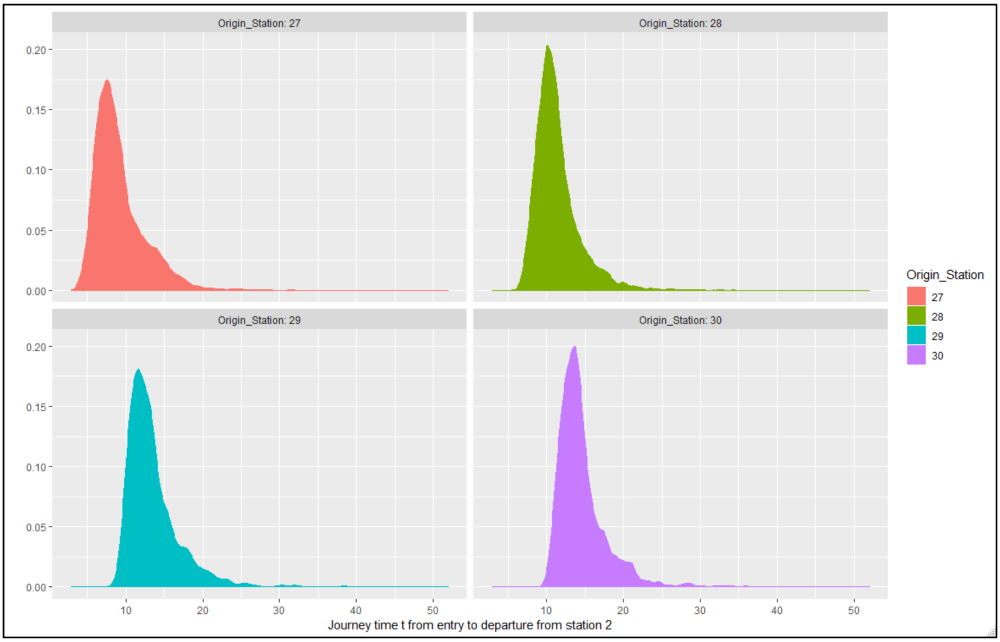


FIGURE 5 – Travel time distribution from each origin station to departure from station 2

When looking in more detail at the travel time distributions for each time interval, as shown in Figure 6, it comes apparent, that severe crowding is seen at the peak of the peak. The travel time in the shoulders of the peak period is around 7.5 minutes from station 27 to departure from station 2, while the mean travel time in the most congested 15 minutes from 18:30-18:45 is 11.6 minutes - a difference of 4 minutes in mean travel time.

When plotting the journey times τ by the origin – destination pair, as shown in Figure 7, there is a tendency that passengers travelling furthest on the red line (i.e. second leg of the trip) have longer travel times from origin to departure from station 2. The journey times for passengers going to station 3 are higher than for passengers going to station 4 and 5, but rarely higher than the times for passengers going to station 16 and 17.

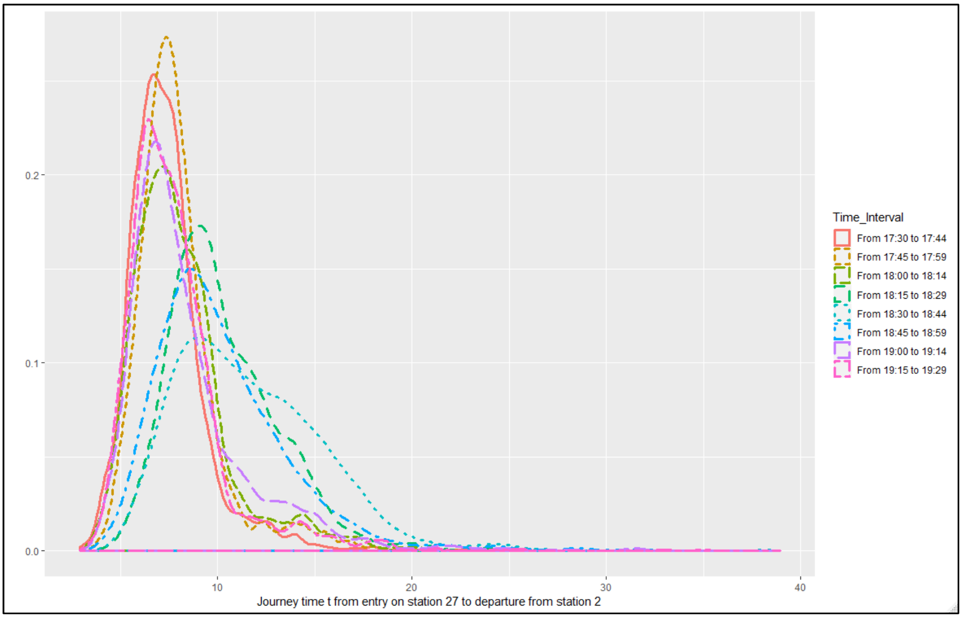


FIGURE 6 – Travel time distribution for passengers from station 27 to departure from station 2 in time intervals

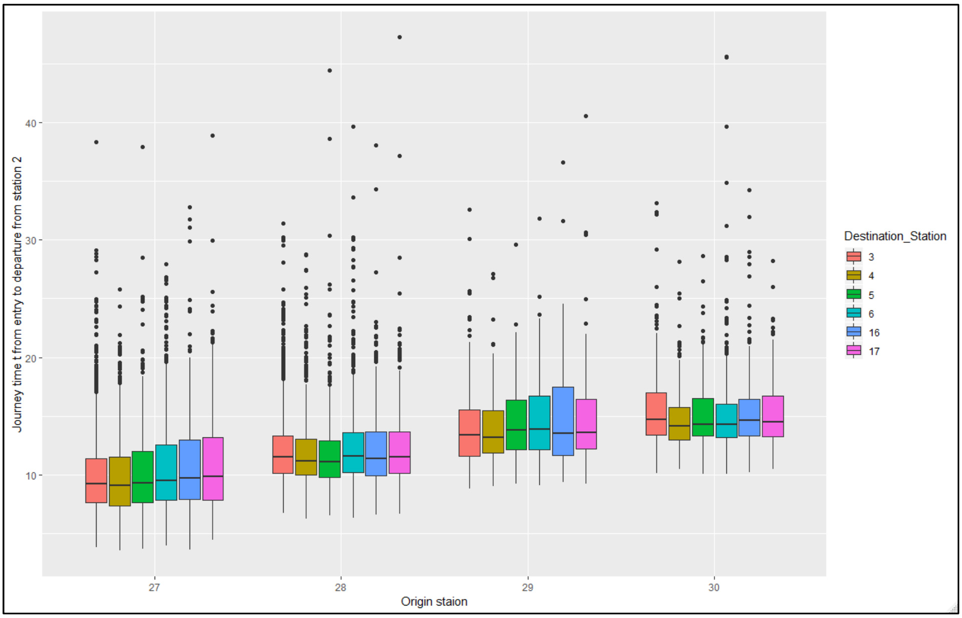


FIGURE 7 – Boxplot of travel times from origin to departure of station 2 (between 18.00-19.00) by destination

As mentioned in Section 3, previous studies have shown that passenger experience has an important effect on path choice and travel time. In this study smart card data from March 2017 was available, and by testing different specifications of the variable describing passenger experience, it was found that intervals of <5, 5-9, 10-19 and >=20 trips in March 2017 from the blue line to the red line between 17:00 and 20:00 resulted in the best fit. Figure 8 clearly shows that passengers with more experience have lower journey times than less experienced passengers for a specific OD pair.

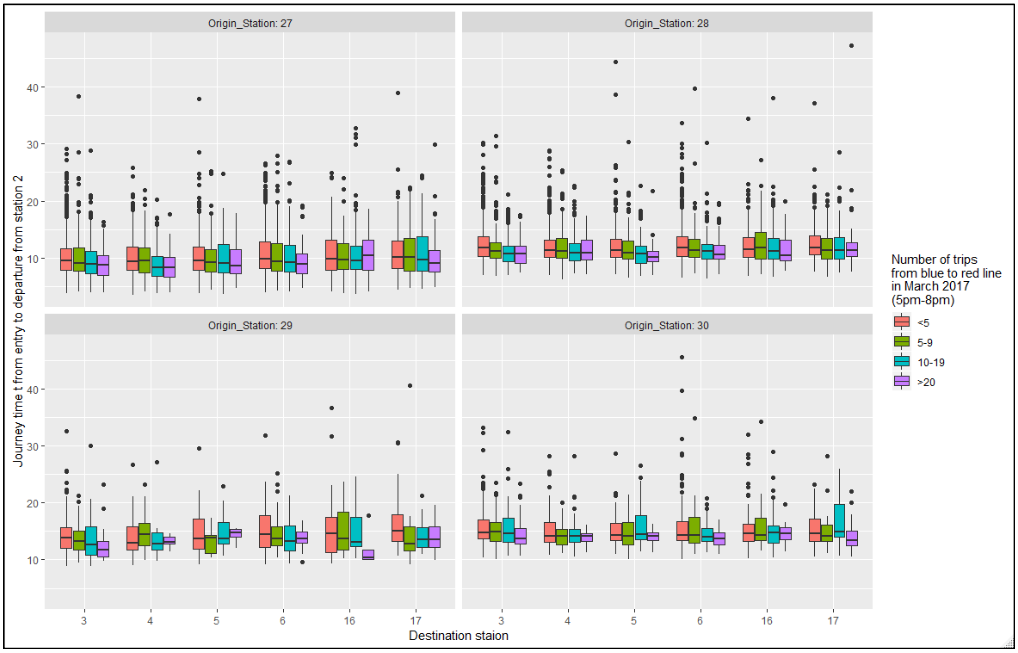


FIGURE 8 - Boxplot of travel time from origin to departure from station 2 (between 18.00-19.00) by destination and travel experience

Results of model for factors affecting the journey time τ

Table 3 below shows the final model explaining the journey time τ from origin to departure from station 2. The intercept of 7.66 minutes represents the journey time for a passenger from station 27 to destination station 3 (closest origin and destination to the transfer stations), between 17:30-17:44 and with less than five trips on this route in March 2017. The estimates for the origin stations are reflecting the approximately 2 minutes between two consecutive stations on the blue line. The estimate for station 30 is only around 1.25 minutes higher than for station 29, but this is due to a much shorter access distance from the fare gates to the platform at station 30 than at the other stations.

The variables indicating the different time periods show significant differences between the eight 15-minute intervals. The most congested time period is from 18:30-18:44, where passengers spend almost four minutes extra compared to the reference level from 17:30-17:44. The estimation of the differences between time intervals does not add information on whether reverse routing is more likely in a given time period, but helps correct for the extra congestion in the system, so that the parameters for destinations and travel experience are unaffected by the additional congestion.

Given that the journey time τ is explaining the time from tap-in to departure from station 2 there should intuitively be little difference in the journey times for passengers going to different destinations. However, as seen in Table 3, the estimates for destinations 6, 16 and 17 are significantly different from the reference level of destination 3. The estimates for stations 4 and 5 are not significantly different from that for station 3. For passengers going to station 6 the journey times are approximately 20 seconds longer, for station 16 the journey times are around half a minute higher than for station 3, and the journey times for passengers going to station 17 are 40 seconds higher than for station 3. This indicates, that passengers travelling further on the second leg have a different behavior than passengers only traveling a few minutes on the red line. This could indicate that some of these passengers are reverse routing via station 1, which could be due to a preference for a seat, or getting a better standing position in the train as also indicated in Tiranchini et al. (2016).

When investigating the parameters for travel experience there is a clear tendency, similar to the boxplots in Figure 8, that passengers with more experience have lower journey times. This is consistent with the findings in Kim et al. (2014), where passengers with more experience chose metro cars which minimize walking distance and thereby their journey time. A very experienced passenger on the route from the blue line to the red line saves around one minute compared to inexperienced passengers. As only data from March 2017 was available for the analysis, it was not possible to check whether passengers also travelled many times in other months, and thereby could be classified as commuters. However, more detailed clusterings of different passenger groups could give more insight into which types of passengers are most effective at minimizing their journey times.

The adjusted R^2 of the model is 0.35, meaning that the model only explains a portion of the variance in passenger travel times. Some of the remaining variation is due to the unknown access times from gate entry to the platform and that it is not possible to assign passengers to a specific train on the blue line. Tests were carried out on whether the findings of longer journey times with longer travel on the red line was due to any correlations not accounted for in the model. Models where the dates were included showed that all three days had similar travel times, and that the difference for the destination stations were similar. Also, a model where each specific train on a specific day was a variable in the model showed that this did not increase the explanatory power of the model, nor did it change the difference on the destination stations. This indicates that passengers travelling furthest on the red line probably do have different behavior than other passengers.

TABLE 3 - Results of Multiple Linear Regression Model for Journey Time τ
Sig. levels: *0.05, **0.01, ***0.001

Parameter	Estimate (minutes)	T-value
Base (intercept)	7.66	123.31***
Origin Station 27	-	Ref. Level
Origin Station 28	2.15	58.62***
Origin Station 29	4.32	55.49***
Origin Station 30	5.38	90.82***
Destination Station 3	-	Ref. Level
Destination Station 4	-0.03	-0.52
Destination Station 5	0.09	1.55
Destination Station 6	0.35	7.45***
Destination Station 16	0.46	7.90***
Destination Station 17	0.68	11.25***
Time interval 17:30-17:44	-	Ref. Level
Time interval 17:45-17:59	0.56	7.66***
Time interval 18:00-18:14	0.68	9.62***
Time interval 18:15-18:29	2.47	34.91***
Time interval 18:30-18:44	3.78	54.73***
Time interval 18:45-18:59	2.57	36.89***
Time interval 19:00-19:14	1.00	13.83***
Time interval 19:15-19:30	0.51	7.07***
Less than 5 trips in month	-	Ref. level
5-9 trips in month	-0.45	-9.66***
10-19 trips in month	-0.61	-13.75***
More than 20 trips in month	-0.93	-12.88***
Number of observations		**32,451**
Adj. R-Squared		**0.35**

5 A small case study of the Copenhagen metro

While the metro in Hong Kong has severe problems with congestion, this is not the case for the Copenhagen metro. However, the Danish Transport Authority already reported passengers being denied boarding in a 2012 analysis, although this was only true in very short time periods and on the central section (Trafikstyrelsen, 2012). The metro has since been increasing service to keep up with the increasing demand and is attempting to limit the occurrence of denied boarding to the central section as shown in Figure 9 (Metro, 2018).

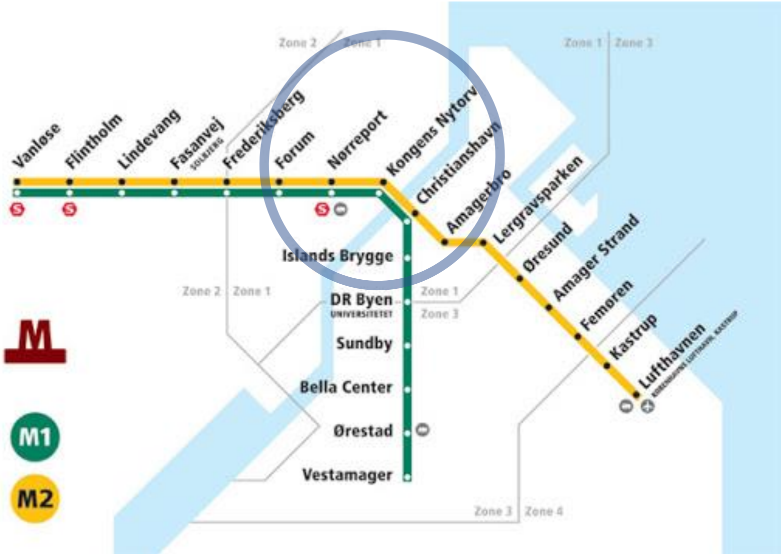


FIGURE 9 – Map of the metro in Copenhagen (before opening of the metrocity circle line)
Source: Metroselskabet

Since the Danish smart card system requires passengers to tap-in at all transfers, it is possible to test whether any backwards travel occurs at stations with possible denied boarding. Data from the Danish smart card system from November 2019 was used to test whether passengers travelled backwards at any time. The stations analysed were Forum, Christianshavn and Amagerbro, which are known to have large numbers of boarding passengers. For each station, a query selecting all passengers who tapped in at the station and since visited Nørreport Station (the central station on the metro network, see Figure 9) were used to find passengers who might have travelled backwards. For these passengers, a check was made on whether they had tapped-in at the first possible "turn-back" station. In Table 4 the total number of passengers travelling backwards is shown for each of these stations. Although the shares show a negligibly small percentage of passengers travelling backwards, it is clear from the split across timebands that these passengers are not doing it due to overcrowding. Rather, it could be passengers accidently taking the train in the wrong direction or passengers who had a short errand at the "turn-back" station and then combining their trips. Through this short analysis it has been shown, that although there might be some denied boarding at some stations in the peak of the peak, there are few, if any passengers who are travelling backwards.

TABLE 4 – Results of analysis of travelling backwards observations on Copenhagen, Denmark

Forum		Christianshavn		Amagerbro	
Direct to Nørreport	21,731	Direct to Nørreport	42,012	Direct to Nørreport	37,341
via Frederiksberg	53	via Amagerbro	40	via Lergravsparken	20
		via Islands Brygge	30		
TB share	0.24%	TB share	0.17%	TB share	0.05%
TB pax in timebands		**TB pax in timebands**		**TB pax in timebands**	
From 0.00-7.00	4	From 0.00-7.00	2	From 0.00-7.00	2
From 7.00-9.00	2	From 7.00-9.00	5	From 7.00-9.00	8
From 9.00-15.00	29	From 9.00-15.00	31	From 9.00-15.00	6
From 15.00-17.00	10	From 15.00-17.00	6	From 15.00-17.00	3
From 17.00-0.00	8	From 17.00-0.00	26	From 17.00-0.00	1

6 Potential approaches for identifying the extent of unusual path choice behavior

This section discuss possible ways to validate the extent of unusual path choice behavior. As described in the literature review, there are already some studies on the *problem of travelling backwards*, which were used as a basis for the models developed for the analysis. For most of the studies the validation data are surveys, where the number of passengers transferring between trains in opposite directions at “turn back” stations are counted. This information is valuable for model development, as the model parameters used to find the share of passengers travelling backwards can then be calibrated. However, there is some uncertainty in this validation technique, as it is not known whether the passengers travelling backwards travelled one, two or three stations before turning back. A way to overcome this uncertainty would be to use Bluetooth or Wifi tracking at stations, to follow (anonymized) persons. This would facilitate large-scale samples, and give a clear indication of the behavior of travelers, e.g. whether passengers really travel three stations back before turning back, as for example found in Yu et al. (2020).

For the *problem of reverse routing*, a less data-driven approach than the one presented above, could also be using surveys to estimate the shares of passengers who reverse route. As mentioned in the case study, a survey was conducted in 2012 (Li, 2014) in the Hong Kong MTR, which showed that around 8% of passengers in the evening period were reverse routing. However, the data sample for the specific case of reverse routing at stations 1 and 2 was small (around 120 for the evening peak period), since the survey covered the entire MTR system. If a more dedicated survey was distributed, either through a web-based questionnaire or a survey on the platforms, a more precise estimate could be used for validating models estimating the fraction of passengers who reverse route. Such a model could be similar to the one presented in Yu et al. (2020) where several Gaussian distributions combined, reveal the share of passengers travelling backwards. However, the problem of using such an approach for the case of reverse routing is that passengers can very well catch an earlier train by reverse routing via station 1 than transferring at station 2. The extra travel time (3 minutes) plus additional walking time at station 1 (estimated from station layout maps to be between 30 seconds and two minutes) can be lower than being denied boarding several times at station 2, where the headway is approximately 90 seconds.

An additional approach for analyzing the *problem of reverse routing*, could be to fuse train load data (for example from automatic train weighing systems) and information from smart cards. Since the smart card system covers a very large proportion of the trips in the system, combining these with a naïve assumption that all passengers transfer at the first possible transfer, and comparing this to the estimated count between station 1 and 2, could reveal a large difference. This could indicate that some passengers are actually reverse routing. Such knowledge could further be used to develop more realistic route choice models, for example including extra parameters for discomfort accounting for denied boarding in a logit-based route choice model (see e.g. Raveau et al. (2014)). The information could also be used in the estimations of hierarchical Bayesian models as shown in Yu et al. (2020). If results of the fraction of reverse routing passengers could be obtained, this could provide valuable information to the operations management team of the MTR, since this could lead to different operational strategies by, for example, dispatching empty trains from station 1 to station 2. This could relieve the pressure on station 2 and encourage more passengers to transfer there. However, this might not lead to better operations, as the dwell time on station 2 could increase, since more passengers then need to board at this station. Since station 1 is the terminal station on the red line, the dwell times at this station are not as critical for operations and the optimal operating strategy might well be to have some passengers reverse routing and in this way cause more congestion on the segment between stations 1 and 2.

Finally, an interesting research avenue for the *problem of stop choice* in metro networks could be to test whether passengers in peak hours choose stations further upstream on a line, to have a better chance of getting a seat or lowering their probability of being denied boarding. The problem in using smart card data for this purpose is that the origin and destination of a trip are unknown: only the place where the passenger enters and exits the system is known. So more detailed data on this problem is needed, which could for example come from surveys. In the Danish National Travel Survey (Transportvaneundersøgelsen, 2020) detailed information on trips in public transport is collected; origin, destination, stations visited, lines used etc. This information could be used to test whether passengers in peak hours access other stations than in non-peak hours. A limitation of the Danish survey, however, is that persons only report on one day of travel, making it difficult to find a pattern. However, if the survey was collected through an app, for example TravelVu (2020), persons are usually tracked over several days, and this could give more precise estimates of whether the stop choice is different in peak hours. Although a survey through an app could potentially show some differences in behavior between peak and non-peak hours, it is likely that the congestion in the metro system would have to be higher than in the Danish system. However, in the case of Hong Kong or other major systems where passengers risk being denied boarding several trains, the stop choice may well be affected by the crowding situation.

7 Conclusion

This paper investigated unusual path choices, where passengers choose different routes due to congestion in metro networks. The paper analyses possible causes and indication for reverse routing behavior by applying a passenger-to-train assignment model to the second leg of the trip. It is then possible to calculate the time from tap-in to departure from the transfer station which would be used under uncrowded conditions.

The results show that passengers who travel further on the second leg of the trip spend significantly more time from tap-in to departure from the normal transfer station, when accounting for the time from the origin and the longer travel times during different time intervals in the evening peak period. The extra time spent may be largely due to the effects of reverse routing passengers. The analysis also shows that passengers who travel more during a month have significantly lower travel times compared to passengers who travel less frequently. This indicates that passengers' decisions are heavily influenced by their previous experience and knowledge, and that passengers with extensive travel experience can assess the current crowding level and possibly choose whether, or not, to reverse route.

A small case study in Denmark showed that travelling backwards in the Copenhagen metro is rare, since the crowding conditions do not often result in passengers being denied boarding.

Finally, the paper discusses possible approaches to detect and predict the fraction of passengers with unusual path choice behavior. The fractions can e.g. be relevant to the operations management team, as different operational strategies, such as dispatching empty trains, could potentially relieve congestion in some areas of the metro, but might lead to worse overall performance of the network.

8 ACKNOWLEDGMENTS

MTR is acknowledged for their support and for providing automatic fare collection and automatic vehicle location data for the Hong Kong case study. Rejsekort A/S is acknowledged for providing smart card data for the analysis of the Danish case study.

9 AUTHOR CONTRIBUTIONS

The authors confirm contribution to the paper as follows: all authors contributed to the study conception and design. M. Eltved and K. Tuncel prepared the analysis in the paper and all authors contributed to the interpretation of the results. M. Eltved prepared the manuscript.

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9 Conclusions

This PhD thesis presents several novel modelling frameworks for analysing passenger travel behaviour in mixed schedule- and frequency-based public transport systems. While each of the presented papers include separate conclusions, Sections 9.1-9.3 concludes on the main findings in this thesis. Section 9.4 discusses the potential policy implications of the developed methodologies and results. Finally, Section 9.5 outlines possible future research based on the findings in this thesis.

9.1 Assignment models for mixed schedule- and frequency-based public transport systems

Part I of the thesis, constituting Paper 1 and Paper 2, develops novel methodologies for assignment of passenger flows in mixed schedule- and frequency-based public transport systems. As modellers typically have to choose between either a schedule-based or frequency-based model design (Gentile and Noekel, 2016), the proposed models bring flexibility for the modellers to choose the most suitable representation of each line in a joint modelling framework.

The development of a complete passenger assignment model for mixed schedule- and frequency-based networks in Paper 1, demonstrate that it is possible to generate reasonable choice sets, which vary both in terms of arrival times, but also in the spatial dimension when routing through the network. Generating a diverse set of alternatives is important, as passengers' route choice preferences have been shown to be heterogeneous (Paper 4). The model allows to assign passengers to different routes through the network, which is more behaviourally realistic than an all-or-nothing assignment procedure. The case study tests of the model, where representation of lines are changed between schedule- and frequency-based, show that the frequency-based representation results in higher travel times and costs to the passengers. This is due to the coordination of schedule-based lines, which results in lower waiting times than half of the headway, as assumed in the frequency-based representation. As the assignment of passengers to different routes is based on a discrete choice model, where the utility of an alternative is based on the generalised cost, the model can be calibrated to fit observed route choices by changing the weights of the individual components of the trip.

While Paper 1 assumes a regular public transport network with no delays, Paper 2 incorporates vehicle delays and their effect on connections between services. By the use of Markov chains to model transfer probabilities, the model calculates analytically (no simulation required) the travel time distribution from origin to destination. The use of efficient matrix operations allows detailed analysis of travel time distributions including travel time distributions conditional on the usage of specific lines. In a case study the change of frequency-based services to a schedule-based representation shows that this only induces minor differences in the travel time distributions. The introduction of phases to model the time spent at each location has many possible extensions, e.g. the introduction of phases at each station to represent possible crowding or differences in the walking times as found in Paper 6. Phases could also be used to model possible delays in schedule-based services before their arrival at the departure station of the passenger, and introduce more dynamic routing decision for passengers than simply boarding the first arriving vehicle at a given stop.

9.2 Route choice models for mixed schedule- and frequency-based public transport systems

Part II of the thesis, constituting Paper 3 and Paper 4, investigates passenger route choice preferences based on reported door-to-door trips. The papers partially solve the problem of correcting the choice probabilities for spatially overlapping alternatives by adding more easily interpretable variables to the models. Such corrections have typically been solved by introducing path-size corrections (Prato, 2009). The path-size factors generally lowers the choice probabilities of overlapping alternatives, but in public transport they have also been found to increase the choice probabilities with the interpretation that passengers value a large number of en-route alternative options (Anderson et al., 2014). However, the extended models seem to capture some of these effects by adding the more easily interpretable variables.

The main conclusions in Paper 3 is that frequency of services and in-vehicle time are two factors which explain a large part of passengers' route choice preferences. The frequency affects the waiting time for services, and lower waiting time parameters are found for schedule-based services compared to frequency-based services. However, it is important to note, that passengers usually wait longer for schedule-based services, as they generally have higher headways than frequency-based services. The finding of difference in parameters is important, as it can be used in assignment models to find more accurate flows than if a single parameter is used for waiting time. The final model includes a binary variable showing that passengers prefer frequency-based services, although the interpretation and potential use in assignment models is difficult, as this variable does not take into account differences in the total travel time or other modes used during the trip. The introduction of Box-Cox variables for the different in-vehicle times explains to some degree which modes are preferred depending on the length of the trip. The results show that passengers have a low inconvenience for short metro trips, but with a rapidly increasing nuisance for longer metro trips. For in-vehicle time in regional trains this is different, as the marginal dis-utility of longer times in the train is decreasing compared to shorter trips using regional trains.

While Paper 3 focus on the in-vehicle times and the effect of frequencies on passenger route choice preferences, Paper 4 focus on disentangling the transfer penalty and increasing the knowledge on the effect of different characteristics of transfer stations. The results show that passengers are indeed taking the characteristics of transfer stations into account when choosing their route in the public transport system. It is found that escalators increase the probability of choosing a specific transfer station, which can possibly be explained by the comfort of these and that they move passengers faster to the connecting service. Having a shop at any of the transfer stations on the route also increases the choice probability of this, and as shown in Paper 6 passengers are indeed spending extra time at transfers in this case, which could be due to shopping, buying coffee etc. Furthermore, it is found that transfers, which are difficult to navigate through, decrease the probability of passengers using such a transfer station. While typical estimates of the general transfer penalty ranges from 5 to 20 minutes, this paper is able to disentangle the transfer penalty for stations with different characteristics. As such, the penalty ranges from 5.4 minutes for the best possible transfer to 12.1 minutes for the worst possible transfer. These results are important as they can improve the fit of current public transport assignment models and moreover can be used to improve the transfer experience for passenger, and ultimately make public transport more attractive.

9.3 Studies on public transport passenger behaviour based on smart card data

Part III of the thesis, constituting Paper 5, Paper 6 and Paper 7, presents three papers with new approaches on how to model passenger behaviour using smart card data. The papers show some key benefits of having a large number of observations, but also the importance of being able to record passengers' travel behaviour over time.

Paper 5 presents an analysis of passengers' travel behaviour before, during and after a track closure which lasted almost three months. The results underline the importance of detecting the dynamic changes to passengers' travel behaviour, when the goal is to isolate the effect of the service disruption on passengers' travel behaviour. When comparing to the reference line it is found, that there is no apparent impact on the travel behaviour after the disruption for passengers who travelled frequently before the disruption. However, it is shown that around 17% of the frequent travellers on the disrupted line almost entirely stopped using public transport during the disruption, but returned when normal operations were resumed. Although no considerable differences for the passengers already using the affected line compared to passengers on the reference line were found, there is a deficit in the number of frequent travellers after the disruption, which to some degree can explain the overall decrease in ridership after the disruption. The proposed smart card based method to investigate the impact of long-term service disruption is an alternative, or complementary, approach to traditional surveys, which is able to isolate the effects of the disruption from the ongoing dynamic changes in passengers' travel behaviour.

While Paper 5 makes use of the longitudinal and panel structure of *Rejsekort*, Paper 6 takes advantage of the massive number of observations collected in the system. The proposed methodology allows for separating directly walking passengers and passengers with activities during the transfer for stations with different characteristics. Firstly, the model estimates accurate walking time distributions from bus stop to train platform at stations where the passenger activities stem from passengers shopping or doing other errands during the transfer. Secondly, the model is able to detect poor synchronisations between busses and trains, where passengers are waiting in the station building instead of walking directly to the platform. The validation of the proposed model is indeed difficult, as this would require manual surveys with inspectors following transferring passengers or tracking systems which are more intrusive on passengers' privacy. To overcome this, two verification techniques are proposed based on i) passenger-to-train assignment and tests of whether the model predicts passengers with many missed trains as passengers with activities during the transfer, and ii) the relation between number of shops at a transfer station and the share of passengers predicted to do activities during the transfer. Although the tests can not validate the predicted distributions, as the ground truth walking time is unknown, the tests indicate that the model does indeed provide reasonable estimates of the necessary walking times at transfers.

The final paper, Paper 7, investigates unusual path choices in congested metro systems. The *reverse routing* behaviour in the specific part of the metro system in Hong Kong is investigated using passenger-to-train assignment and a multiple linear regression model. The results of the model shows an excessively extra amount of travel time needed during the evening peak hours, where passengers are denied boarding two to three trains when transferring at the normal transfer station. The analysis shows that passengers travelling furthest on the second leg have an increased travel time to the departure from the normal transfer station, which exceeds the travel times for passengers only travelling short distances on the second leg. This extra time spent for passengers travelling further on the second leg could possibly indicate that some of these passengers are *reverse routing*. Doing *reverse routing* can in the cases where passengers are denied boarding several times at the normal transfer station be an optimal route choice, both in terms of travel time and also if the passenger wants a chance of getting a seat or a better standing position in the train. The main part of the research project was carried out just months before the protests in Hong Kong began in the middle of 2019 (BBC, 2019). This has certainly changed the travel patterns in the metro system along with the changes due to COVID-19. Since the work on the presented paper was not finalised before the protests began, it has not been possible to do any types of surveys or other counts to verify the extent of the *reverse routing* behaviour. However, the paper suggests several further investigations that could be performed, including suggestions on the use of train load data or counts using Bluetooth technology. The paper also briefly investigates possible *travelling backwards* behaviour in the Danish metro. However, using simple statistics it can be shown, that such behaviour is non-existing.

9.4 Policy implications

While the conclusions of the different parts span across several topics within the modelling of passenger travel behaviour, there are some general takeaways for the implications on future policies and design of mixed schedule- and frequency-based public transport systems.

Firstly, the thesis has contributed to the design of models for evaluating passenger route choice in greater detail. The novel assignment methodology presented in Paper 1 has already been used as inspiration to the public transport assignment model in COMPASS (Copenhagen Greater Area Model for Passenger Transport - Kjems et al. (2019)), the new state-of-the-art activity-based transport model for Greater Copenhagen. As such, this newly developed model has contributed to ease the tasks needed for traffic modellers to run several timetable scenarios, as they can now choose to let some lines have a frequency-based representation. When combining the newly developed models with the results from the analyses on route choice preferences in Paper 3 and Paper 4, it is possible to gain more accurate estimations on the level-of-service provided to the passengers by the public transport network. The studies have confirmed the significant negative impact of transfers on the behaviour of public transport passengers, and hence the number of transfers is an important measure when new timetables or networks are implemented. The results in Paper 4, which suggest that not all transfers are weighted equally, are in this regard especially important for the evaluation of the effect of improved public transport terminals. Such improvement to stations can be more cost effective than building new infrastructure, while still resulting in better level-of-service for the passengers.

Secondly, the study on walking time estimations can have a considerable impact on public transport planning in Denmark. The current practice in Denmark of assessing the necessary walking times, when transferring between bus and train, is based on GIS-analysis of the Euclidean distance between stop points and an added extra time buffer accounting for possible delays. The proposed approach based on smart card data facilitates more accurate estimations than just using simple Euclidean distances, as these may differ from the actual walking paths. This is especially the case for transfers between bus and underground metro stations, where the vertical distance is difficult to account for. Since the approach outputs distributions of walking times, these can also be used to enhance online route planning apps, such as the Danish travel planner *Rejseplanen*. While *Rejseplanen* is already capable of handling whether passengers set their walking speed to "slow", "fast" or "normal", this is only used for the access/egress part of the trip. Combining passengers' selection of their walking speed and the estimated walking time distributions enables better route suggestions, as the current point estimates does not take into account the probability of a passenger reaching a suggested connection. The proposed approach is expected to be used as part of a larger project between DSB (rail operator) and Movia (bus agency). This project will investigate the synchronisation for busses and trains at rural stations, and the methodology will play a key role in the estimation of whether passengers can reach a planned connection or not.

A final implication of this thesis is based on the study on effects of long-term disruptions on passengers travel behaviour. The study not only provides valuable insights on the effect of the specific track closure, but more importantly it shows the value of being able to follow the dynamic changes in passenger travel behaviour over time. Although the study only includes adult passengers with a personal smart card, it is possible to segment the passengers in eight distinct groups based on easily measurable indicators of travel regularity. As also shown in Deschaintres et al. (2019) and Egu and Bonnel (2020), passenger travel behaviour in public transport changes significantly during the year. New passengers enter the system while others leave the system and possibly return. The study on the impacts of the track closure shows that it is not sufficient to solely analyse the effects before and after the closure, without taking into account the significant changes to passenger behaviour over time. As such, the methodology provides a much clearer distinction between normal changes over time and those actually related to the severe level-of-service degradation during the affected period.

9.5 Future research

Although, this thesis has made several contributions to the existing literature on passenger travel behaviour in mixed schedule- and frequency-based systems, there are still plenty of research needed for making public transport more attractive. Most of the papers have outlined directions for future research, but a few general directions based on the findings in this thesis are worth highlighting.

Firstly, there are much further research to be done in the area of route choice modelling, especially considering the vast available data from smart card and the detailed data from travel surveys. As *Rejsekort* includes both origin, destination and transfer stations, this is a treasure chest filled with detailed information on how passengers choose routes within public transport systems. Smart card data has been used in several studies on route choice, see e.g. Raveau et al. (2014) and K. M. Kim et al. (2015), but can also be used to see if passengers choose different routes over time J. Kim et al. (2017).

However, smart card data does not include information on access and egress, which, from a passenger perspective, is also an important part of using public transport. Luckily, the route choice data from the Danish National Travel survey includes this information (Center for Transport Analytics DTU, 2020). An interesting line of research would thus be to combine these datasets for route choice estimations, which could add to an emerging research area combining big-data sources with data sources which includes more socio-demographic information (see e.g. Zhang et al. (2018) who combine revealed preference and smart card data for route choice estimations).

Secondly, this thesis has shown the potential of passenger segmentation based on smart card data. Smart card data comes in large numbers and there are several use cases for this as outlined in Pelletier et al. (2011) and Faroqi et al. (2018). Especially in the area of passenger segmentation, the data is relatively simple to analyse. Passengers can easily be tracked over time and thereby creating a large longitudinal survey, which is otherwise not simple to observe in any other survey formats. For example, the temporal pattern of trips within specific days and spatial patterns can shed more light on the purpose of trips and thereby understanding of which type of passengers are using public transport (Egu and Bonnel, 2020; Zhao et al., 2017). Analysis on passenger segmentation can help public transport agencies understanding their customers better, and use this for creating a more attractive public transport system, which better serves the mobility needs of people living in metropolitan areas.

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This PhD thesis contributes to several topics concerning modelling and analysis of passenger behaviour in metropolitan public transport systems.

Firstly, the thesis develops novel methodologies for evaluating the flows in public transport systems with co-existing schedule- and frequency-based services. Secondly, the thesis investigates the route choice preferences in terms of the penalties for transferring and waiting. Thirdly, the thesis presents a study of the impact of long-term service disruption on passenger travel behaviour based on smart card data. Finally, smart card data is also used in two innovative studies for analysing walking times and how crowding affects passenger route choice, respectively.

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