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Modelling the nacelle wake of a horizontal-axis wind turbine under different yaw conditions

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Abstract

Recently, actuator line model become popular in studying wind-turbine wakes. However, existing models ignore or inaccurately describe nacelle effects, which have been shown to pose significantly impact on wakes. To address the physics underlying here, we develop the actuator line model with large-eddy simulation by introducing a new anisotropic body-force projection model. We validate the new model against a field experiment and the validation indicates that the new anisotropic model can predict the wake more precise than the existing isotropic model. Furthermore, we extend the study to wake characteristics under various yaw conditions. The results show that the thrust component normal to the flow direction creates a skewed wake behind the turbine, which in turn promotes the wake transition from the two-peak profile to the one-peak profile. The wake skew exacerbates the instability of the tip vortex and causes the wake region to narrow. At small yaw angles, the nacelle vortex radially diffuses and blends with the tip vortex in the far wake. At large yaw angles, the nacelle vortex intercepts the tip vortex in the near wake due to the different spatial distribution of thrust. It is concluded that the nacelle significantly affects wind-turbine wakes especially during yaw condition.

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1. Introduction

The wake structure is considered as one of the most important wind turbine aerodynamics characteristics [1]. Recently, with the ever rapid growing speed of wind turbine scale, the flow around a utility wind turbine can reach high Reynolds numbers of $Re \sim O(10^6)$ [2, 3, 4], resulting in prohibitively large computational resources required for a high-fidelity large-eddy simulation (LES) based on body-fitted grid. To address the physics cost-effectively, researchers and technology developers determined that a hybrid approach combining high fidelity and efficiency must be developed such as actuator disk model (ADM) [5, 6], actuator line model (ALM) [7, 8], actuator surface model (ASM) [9], and some reduced-order methods [10, 11, 12]. In the ADM, the wind turbine is simplified to a disk, which makes it a low computational cost but poor physical fidelity. The ASM can achieve better physical fidelity on a finer grid, but it increases the computational cost. Striking a balance of the high physical fidelity and low computational cost, the coupling of LES and ALM is a common method for studying wind-turbine wake structures, first proposed by Sørensen et al. [7]. Krogstad et al. [13, 14] and Shen et al. [8] performed simulations using the ALM, and compared the results with experiment. Our team has recently conducted some research on the body force distribution methods on the blade and tower, which also helps to improve the numerical accuracy of the ALM for wake simulation [15]. These works showed that the ALM can be an efficient alternative with high precision to predict wake and loads of a turbine.

For vertical axis turbines, three-dimensional effects and arm effects influence the wake development [16]; for horizontal axis turbines, nacelle effects influence the wake development. Nevertheless, the standard ALM ignored the nacelle structure, which has recently been found to significantly impact the wind-turbine wakes. Figure 1 schematically shows a typical wake distribution of a wind turbine. After the wind turbine, due to the mixing effect of the blade tip vortex, a shear layer including tip vortices is formed between the airflow and the low-velocity region [17, 18, 19]. The shear layer expands downstream, diffuses inwards, merges with the nacelle wake and forms an interaction region. The interaction between the tip shear layer and the nacelle wake will induce the wake to meander [17], affecting the velocity and
turbulent kinetic energy distribution in the far wake. Further, Foti et al. [20] found that the wake meandering occurs in the wake of wind turbines with different scales ranging from wind tunnel scale to utility scale. Santoni et al. [21] found that when the nacelle is not modeled, the unphysical high-speed zone behind the rotor center persists for more than 3 times the rotor diameter and increases with increasing TSR. Foti et al. [19] showed that nacelle modeling methods strongly influences the prediction accuracy of wind farm power. Based on different experimental results, Mittal et al. [22] and Li et al. [2, 23] concluded that ignoring the nacelle effect greatly reduce the accuracy of the near-wake velocity field respectively. To address this problem, some researchers try to describe the nacelle effect. El Kasmi et al. [24] developed the ADM with the RANS method, and processed the nacelle wake using an isotropic three-dimensional Gaussian distribution function. Based on their works, Wu et al. [25] improved the ADM and coupled it with the LES, but still handled the nacelle wake in the same way. Aitken et al. [26] and Sarlak et al. [27] used the similar methods to study the influence of incoming flow, model parameters, etc. on wind-turbine wakes. This method is, however, still deviates from the experimental test results due to the inaccurate representation of the nacelle shape. Therefore, a better alternative for nacelle modeling is desired.

The wake structure of wind turbine has been extensively studied under the condition of axial inflow, especially on the stability of the tip vortices, the dominant flow structures and wake meandering [28, 29, 30, 31, 32, 33]. However, in the actual wind farm, wind turbines are often operated in yaw conditions. Krogstad et al. [34] mainly studied the power performance of the wind turbine and the wake characteristics under different yaw angles and TSRs. They observed that the wake was narrowed and skewed towards one
side as the yaw angle increased. Sørensen et al. [35] and Qian et al. [36] studied the velocity profiles and aerodynamic performance in axial and yawed flow conditions, and analyzed the nacelle effect and wake characteristics. moreover, Munters et al. [37] reported that a suitable control strategy can be used for the wake of an upstream yawed wind turbines to bypass the downstream wind turbine, which can increase total power generation of the wind farm. Fleming et al. [38, 39] tested the effects of different control methods on power production, including yaw-based, tilt-based and blade-pitch-based approaches. Despite these emerging applications, accurately predicting the losses of lateral velocity and axial velocity in the wake [40, 41] and the skewness of wind turbine wake remains a challenge [42]. Therefore, research on the wake of yawed wind turbines is imperative to further improve the output power of wind farm.

In order to have a high accuracy understanding of the nacelle effect on wind turbine wake, especially its effect under yaw conditions, we develop a new actuator line model by introducing an anisotropic Gaussian distribution model in Section 2. We validate it against a field experiment carried out in China, which is described in Section 3. By using the improved ALM-LES method, we extensively study a full-scale wind turbine wake under the field experiment condition and various other yaw conditions in Section 4. Finally, we present our conclusions in Section 5.

2. Numerical Modeling

In this work, the LES is used to simulate the turbulence, and the ALM is used to model the wind turbine as we used in the previous work such as [7, 8, 2]. Therefore, some description will be re-stated here while neglecting details. Interested readers can refer to those papers. The grid-filtered continuity and Navier–Stokes (N-S) equations for the incompressible LES are as follows:

\[
\frac{\partial \hat{u}_i}{\partial x_i} = 0
\]

(1)

\[
\frac{\partial \hat{u}_i}{\partial t} + \frac{\partial}{\partial x_j} (\hat{u}_i \hat{u}_j) = -\frac{1}{\rho} \frac{\partial \hat{p}}{\partial x_i} + \frac{\mu}{\rho} \frac{\partial^2 \hat{u}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} + \frac{f_i}{\rho}
\]

(2)

where the overbar denotes a grid-scale filtering operation. From left to right, the terms in the N-S equation represent transient, convective, pressure, molecular viscosity stress, subgrid-scale stress, and the forces exerted...
by the wind turbine, respectively. The effects of the unresolved small scales appear in the subgrid-scale stress tensor, \( \tau_{ij} = u_i u_j - u_i u_j \). To capture the helical tip and hub vortices in the wake of wind turbine, the LES with a localized dynamic kinetic energy sub-grid scale (SGS) model [43, 44, 45] is used to model the turbulence. The dynamic kinetic energy equation accounts for the history and non-local effects, having the potential to benefit complex flows with non-equilibrium turbulence. The relationship between subgrid-scale turbulent kinetic energy \( (k_{sgs}) \) and \( \tau_{ij} \) is

\[
\tau_{ij} = -2\nu_{sgs} \bar{S}_{ij}, \quad k_{sgs} = \frac{1}{2} \sum_i \bar{\tau}_{ii}, \quad \nu_{sgs} = C_k k_{sgs}^{1/2} \Delta \tag{3}
\]

The \( k_{sgs} \) is obtained by solving the following equation:

\[
\frac{\partial k_{sgs}}{\partial t} + \nabla \cdot (k_{sgs} \bar{u}_i) = \Pi_{k_{sgs}} + \nabla \cdot \left[ (\nu + C_k k_{sgs}^{1/2} \Delta) \nabla k_{sgs} \right] - C_* k_{sgs}^{3/2} / \Delta \tag{4}
\]

The dissipation of sub-grid scale turbulent kinetic energy, \( \Pi_{k_{sgs}} \), dynamic parameter, \( C_k \), and dissipation coefficient, \( C_* \) are obtained from the following equations based on the test filtering. In the test-scale field, for a variable \( \phi \), \( \hat{\phi} \) represents applying the test filter to \( \phi \).

\[
\Pi_{k_{sgs}} = 2C_k \Delta k_{sgs}^{1/2} \bar{S}_{ij} \bar{S}_{ij}, \quad \bar{S}_{ij} = \frac{1}{2} \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \tag{5}
\]

\[
C_k = \frac{L_{ij} \sigma_{ij}}{2 \sigma_{ij} \sigma_{ij}}, \quad \sigma_{ij} = -\hat{\Delta} k_{test}^{1/2} \hat{S}_{ij}, \quad k_{test} = \frac{1}{2} \left( \hat{u}_k \hat{u}_k - \hat{u}_k \hat{u}_k \right) = \frac{1}{2} L_{kk} \tag{6}
\]

\[
C_* = \frac{\hat{\Delta}}{k_{test}^{3/2}} (\mu + \mu_{sgs}) \left( \frac{\partial \hat{u}_i}{\partial x_j} \frac{\partial \hat{u}_j}{\partial x_i} - \frac{\partial \hat{u}_i}{\partial x_j} \frac{\partial \hat{u}_j}{\partial x_i} \right) \tag{7}
\]

where the dynamic Leonard stresses, \( L_{ij} \), is described as follows

\[
L_{ij} = -2C_k \hat{\Delta} k_{sgs}^{1/2} \hat{S}_{ij} + \frac{2}{3} \delta_{ij} k_{test} \tag{8}
\]
2.1. Rotor Modeling

The wind turbine blades are replaced by two or three rotating actuator lines, depending on the number of blades. Multiple actuator points are arranged on the actuator line (as shown in Fig. 2a). The body force is calculated by the airfoils’ lift and drag characteristics and the local Reynolds number at each actuator point (as shown in Fig. 2b).

Figure 2b shows a cross-sectional element at radius \( r \) along the blade. \( \Omega \) denotes the angular velocity. \( V_R \) and \( U_0 \) represent the local tangential and axial velocities at the actuator points, respectively, which are interpolated from the velocities at the neighbor grid. The local relative velocity to the rotating blade is determined as

\[
V_{rel} = \sqrt{U_0^2 + (\Omega r - V_R)^2}
\]  

The local angle of attack is defined as \( \alpha = \phi - \beta \), where \( \beta \) denotes the local pitch angle and \( \phi = \arctan(U_0 / (\Omega r - V_R)) \) is the angle between the rotor plane and \( V_{rel} \). For each actuator points on the actuator line, once the local angle of attack and the local relative velocity are determined, the turbine induced force per spanwise length is given by the following equation [7, 8]:

\[
\vec{f}_{2D} = \frac{1}{2}\rho V_{rel}^2 c (C_L \vec{e}_L + C_D \vec{e}_D) \cdot \vec{F}_{tip}
\]

where \( C_L = C_L(\alpha, Re_c) \) and \( C_D = C_D(\alpha, Re_c) \) are the lift coefficient and the drag coefficient, respectively. \( c \) is the chord length, and \( \vec{e}_L \) and \( \vec{e}_D \) are the
unit vectors in the directions of the lift and drag, respectively. $F_{\text{tip}}$ is the tip loss, which is obtained by the following equation [46]:

$$F_{\text{tip}} = \frac{2}{\pi} \arccos \left[ \exp \left( -\frac{N_b (R - r)}{2r \sin \phi} \right) \right]$$  \hspace{1cm} (11)

where $N_b$ is the number of the turbine blades and $R$ is the blade tip radius.

The body forces $\vec{f}$ exerted by the turbine is given by the following Gaussian projection [7, 8]:

$$\vec{f} = \vec{f}_{2D} \cdot \eta_\varepsilon, \quad \eta_\varepsilon = \frac{1}{\varepsilon^3 \pi^{3/2}} \exp \left( -\frac{d^2}{\varepsilon^2} \right)$$  \hspace{1cm} (12)

where $d$ is the distance between grid points and actuator points, and $\varepsilon$ is the Gaussian width. There are many studies about the value of the Gaussian width [2, 8, 31, 47, 48]. Some researchers recommend that $\varepsilon \approx 2\Delta r$ [47], and some researchers recommend that $\varepsilon \approx 1.76\Delta r$ [48]. These suggestions are based on the relationship between $\varepsilon$ and the numerical results of wind-turbine power to choose a value of $\varepsilon$. However, Martínez-Tossas [49] found that when the Gaussian width is less than $2\Delta r$, it will cause numerical oscillation, so the lower bound of $\varepsilon$ in the LES is recommended to be $2\Delta r$. In our work, referring to the works of Shen et al. [8], Sørensen et al. [31], Churchfield et al. [47] and our previous work [2], the Gaussian width is set to $2\Delta r$. It is worth noting that when there are enough mesh cells in the blade region, the mesh step is much smaller than the blade chord length, then the Gaussian width has a relationship with the blade chord length. Although the increase of mesh cells sacrifices computational efficiency, it will increase the numerical accuracy of wake simulation, which is a compromise between computational efficiency and accuracy. Those who are interested can refer to this work [50].

### 2.2. Nacelle Modeling

To improve the numerical accuracy of the near wake, the nacelle effect is modeled using an anisotropic Gaussian distribution model. Similar to the actuator line model for the rotor, the body force calculated by the nacelle model is still loaded into the momentum equation. The nacelle is simplified to a blunt-headed cylinder, and the body force at one actuator point, $f_{\text{nac}}$, is:

$$f_{\text{nac}} = \frac{1}{2} \rho U_{\text{nac}}^2 C_{D,\text{nac}} A_{\text{nac}}$$  \hspace{1cm} (13)
where $U_{nac}$ is the axial velocity in front of the nacelle, $C_{D,nac}$ is the drag coefficient of the cylinder, which is set according to experimental and numerical results of El Kasmi et al. [24] and Aitken et al. [26], and $A_{nac}$ is the projected area of the nacelle in its rotational plane.

![Isotropic Gaussian model](image1)

![Anisotropic Gaussian model](image2)

Figure 3: Schematic of body force distributions. a) Isotropic Gaussian model. b) Anisotropic Gaussian model.

After obtaining the body force of the nacelle, a distribution function is required to smoothly project the body force into the flow field. Some scholars [24, 25, 26, 27] used the isotropic three-dimensional Gaussian function to project the body force exerted by the nacelle. Let $x$ be the direction of the nacelle axis and $r$ be the radial direction of the nacelle. Since this projection method is isotropic in the $x$ and $r$ directions as shown in Fig.3a, it is only suitable for the nacelle with a small cross-sectional area. When the cross-sectional area of the nacelle is large, i.e. the nacelle diameter in the $r$ direction is large, such a projection method is difficult to reflect the true shape of the nacelle. Instead, an anisotropic Gaussian function is used here to project the body force exerted by the nacelle. This projection method ensures the body force to be constant for a projection radius smaller than the nacelle radius, and to be attenuated in the radial direction for a radius larger than the nacelle radius (see Fig.3b). The anisotropic Gaussian distribution function, $\eta(r, x)$, is defined in the cylindrical coordinate system:

$$
\eta(r, x) = \begin{cases} 
(\varepsilon^2 \pi)^{-1} \exp \left[ - \left( \frac{x - x_c}{\varepsilon} \right)^2 \right], & r \leq r_{nac} \\
(\varepsilon^2 \pi)^{-1} \exp \left[ - \left( \frac{x - x_c}{\varepsilon} \right)^2 - \left( \frac{r - r_{nac}}{\varepsilon^2} \right)^2 \right], & r > r_{nac}
\end{cases}
$$

(14)
where $x$ and $r$ are the axial and radial directions, respectively, centered upon the nacelle axis. We set $\varepsilon \approx 2\Delta r$ to control the Gaussian width [2, 8, 31, 47]. $x_c$ is the axial location of each actuator point, and $r_{nac}$ is the nacelle radius.

Fig. 4 ~ Fig. 5 show the distribution of body force in the flow field based on the isotropic Gaussian model and the anisotropic Gaussian model. The red dashed boxes in Fig. 5 show the geometrical extent (radius and length) of the nacelle in the radial and streamwise directions, respectively. When an isotropic Gaussian model is used to project the body force of the nacelle, the radial size of nacelle effect zone is only about 12 mesh steps (Fig. 4a), which is less than the nacelle radius (Fig. 5a). The body force is concentrated in the region near the nacelle axis ($y = 0$), which also makes the body force in the streamwise distribution overestimated (Fig. 5b). With the anisotropic Gaussian distribution model, the radial distribution and streamwise distribution of the body force are more consistent with the nacelle geometric range.

**Figure 4:** Comparison of different Gaussian models. a) Isotropic Gaussian model. b) Anisotropic Gaussian model.

### 2.3. Wind Tunnel Validation

In order to validate the numerical accuracy of this new model, a wind tunnel experiment was selected for comparison[14], which has been widely used recently for various purposes [8, 35, 36]. The length, width and height of the wind tunnel’s test section are 12 m, 2 m, and 3 m, respectively, which have been taken into account in the simulation to account for wall effects. A 2-blade wind turbine with a diameter of 0.894 m is located 0.817 m above the floor level.

The power coefficients obtained from the new method and the wind tunnel experiment data are shown in Fig. 6a. Moreover, results obtained in previous
studies by other researchers are also plotted [14]. The darker shaded region indicates a tip speed greater than 100 m/s, while the light shaded region indicates a tip speed between 65 m/s and 100 m/s. The results show that when the TSR is less than 9, the power coefficients calculated by all the algorithms are consistent with the experimental values. For a tip speed greater than 100 m/s, there is some deviation between the numerical results and the experimental values. Since the wind turbine is a scaled model, the rotor speed is higher than that of an actual commercial turbine in order to satisfy the similarity criterion. Therefore, when its TSR is greater than 10, the tip speed exceeds 100 m/s and the compressibility effect becomes important, which may be one cause of the larger deviation. Under normal operating conditions, onshore wind turbines generally maintain a tip speed of less than 65 m/s to avoid noise pollution [51]. The comparison in Fig.6a is just to validate the numerical accuracy of the new method in the calculating the wind turbine power performance.

We also validate the new model by comparing the velocity obtained from the numerical simulation against the wind tunnel experiment data at different downstream positions (1D, 3D and 5D behind the rotor) on the horizontal plane at hub height (Fig.6b). The shaded region represents the projected diameter of the nacelle. The numerical results without nacelle model deviate the experimental results much near the nacelle. With nacelle model, the numerical results are relatively consistent with the experimental results. Since the isotropic model cannot reflect the size and shape of the nacelle, the body force is unreasonably loaded into the flow field, resulting in a large velocity overestimation in the hub region in the far wake. It is worth noting that the
radial asymmetry of the velocity distribution is shown in the experimental and numerical results, although the degree is smaller in the numerical results, which may be related to the finer geometric characteristics of the nacelle and the rotating direction of rotor. Due to the radial asymmetry in the wake, the velocity at the position of the rotor axis \((y/r_{nac} = 0)\) is used as a reference. Table 1 shows the dimensionless axial velocity deficit at \(x/D = 1, 3, 5\) on the rotor axis \((y/r_{nac} = 0)\). It can be seen that the isotropic Gaussian model is too small, resulting in a concentrated body force distribution and an overestimated axial velocity deficit. The numerical deviation between the calculated results of the anisotropic Gaussian distribution model and the experiment results is within 9%.

Table 1: Numerical deviation between the axial velocity deficit calculated by different models and the experimental results

<table>
<thead>
<tr>
<th>(x/D)</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anisotropic Gaussian model</td>
<td>5.73</td>
<td>8.55</td>
<td>1.08</td>
</tr>
<tr>
<td>Without nacelle modeling</td>
<td>-53.92</td>
<td>-50.69</td>
<td>-42.58</td>
</tr>
<tr>
<td>Isotropic Gaussian model</td>
<td>51.81</td>
<td>-25.19</td>
<td>-39.09</td>
</tr>
</tbody>
</table>
3. Field Experiment Validation

3.1. Experimental Setup

The performance of the new model for a full-scale wind turbine under realistic atmospheric boundary layer and yaw conditions is wanted to be understood. For this purpose, a full-scale wind turbine installed in Gansu is studied by our new method against the field experiment condition. As shown in Fig. 7a, the wind turbine field experiment was carried out in the east of the Qilian mountains and the south of the Tengger Desert, Gansu, China (37.1° N, 104.0° E). About 11.5 km northwest of the measurement site, a branch of the Qilian mountains extends from west to east, while the other surrounding terrain is comparatively flat. A 2-blade 14.8 m diameter 33 kW wind turbine was setup there with various instrumentations (Fig. 7b). The wind turbine operates with a yaw angle approximately 10.6°, the mean inflow velocity is 4.2 m/s, and the mean direction is 351°. Table 2 shows the distributions of the blade initial twist angle, chord length and airfoils along the normalized rotor radius. Readers who are interested in detailed information about the wind turbine or the field experiment can refer to Li [23] by a few authors from this paper.

![Figure 7: Experimental setup in Gansu province of China. a) Satellite photographs of the measurement site. b) Field experiment system.](image)

3.2. Numerical Analysis

For the simulation, based on the experiment from Li [23], a $26R \times 8R \times 8R$ geometric domain is modeled with the wind turbine inside, where $R$ is the tip radius (Fig. 8a). In some studies, such as the works of Fleming et al. [39]
and Churchfield et al. [47], the geometric domain is set to several kilometers. This is because, on the one hand, the rotor diameter in these works is on the order of hundreds of meters; on the other hand, these works are to analyze the impact of atmospheric turbulence on wind turbines, and the scale of atmospheric turbulence is usually on the order of several hundreds of meters. When the wake of a standalone wind turbine is studied, or when the influence of large-scale turbulence structure is not considered, the geometric domain is usually set to $(15R \sim 30R) \times (4R \sim 10R) \times (4R \sim 10R)$, such as the works of Santoni et al. [21], Carrión et al. [32], Shapiro et al. [42] and Sarmast et al. [52]. Therefore, we choose the geometric domain of $26R \times 8R \times 8R$ in our work. The mean velocity of inflow is set according to the experiment, and a no-gradient outlet condition is applied. On other sides of the domain, slip boundary conditions are applied to avoid interference of different boundary geometries on the flow. The origin of the computational domain is set at the center of the wind-turbine rotation. The blockMesh utility in OpenFOAM is used to generate Cartesian meshes in the geometric domain. The initial grid points in the flow, spanwise and vertical directions of the computational domain are 130, 40, and 40, respectively, and then the mesh is refined by 5 layers. As shown in Fig.8b, the refinement region A1 is used to refine the mesh near the nacelle, and its range in the streamwise ($x$) direction is -1 m ~ 3 m, and the height is 2 m. The refinement regions A2 ~ A5 are used to refine the region near the rotor and the wake region. Along the $x$ direction, the refinement ranges of A2 ~ A5 are -3 ~ 45 m, -5 ~ 60 m, -7 ~ 75 m, and -9 ~ 90 m, respectively. Along the directions of spanwise ($y$) and vertical ($z$), the length of the refinement region A2 is 16 m, and the lengths of the regions A3, A4 and A5 are sequentially increased by 2 m. After five layers of mesh
refinement, the mesh step size in the nacelle region is 0.0469 m, and the mesh step size in the turbine region is 0.0938 m, which means that about 160 grids are distributed across the rotor (Fig.9). Finally, the total number of mesh cells is about 15 million. In addition, we did conduct a grid-independent study on the characteristics of wind-turbine loads. The results show that when the number of grids is greater than 6.5 million, the power and thrust loads change less than 1%. However, since the LES in this work uses implicit filtering, the result is always grid-dependent. As the grid spacing decreases, the resolved scale of the LES continues to increase, and the vortices that can be calculated are also fine. A grid number of 15 million is chosen in this work, which is our compromise choice based on the computational efficiency and precision.

Figure 8: Geometric discretization. a) Computational domain. b) Mesh resolution.

Figure 9: Geometric discretization around the wind turbine.

In the field experimental conditions, the rotor speed is 54.5 rpm, and the corresponding tip speed is about 42.2 m/s. In the calculation, the time step is set to 0.002 s, so the tip displacement in one time step is less than the
size of one grid. The simulation was first performed for 100 s to ensure that
the quasi-equilibrium state is reached, at which time the turbine is rotated
about 91 rotations and the air flows from the inlet through a distance of
about 2 times the total length of the computational domain. After that, the
simulation was performed for 100 s to gather data. Therefore, the analysis of
physical quantities such as axial velocity, TKE and CTKE in the following
are all based on the average value of data within 100 s. This work implements
the numerical model on the well-established pressure-implicit with splitting
of operators (PISO) algorithm [53]. All algorithms are implemented based
on the open source code OpenFOAM 4.0.

Under the yaw condition, this overestimated wake is tilted to one side and
covers the center \( P_{\text{center}} \) and side \( P_{\text{side}} \) measurement points, which leads
to a large deviation in the velocity calculation at the measurement points.
The numerical deviation between the numerical results and the experimental
results at the wake measurement points \( P_{\text{center}}, P_{\text{side}} \) is reduced by introd-
ucing the anisotropic nacelle effect model. Similar to the validation above, it is
clear to see that the new model provides a much more precise results com-
pared with the models without nacelle effect (Table 3). Particularly, when
the nacelle effect is ignored, the velocity deviation of measurement points
\( P_{\text{center}} \) and \( P_{\text{side}} \) is about 30%; when considering the effect of the nacelle,
the speed deviation of all measurement points is less than 8%. Overall, the
new model can be at least 22% more precisely compare with the standard
ALM (without nacelle modeling). From velocity distribution, one can see
that ignoring the nacelle effect can lead to an overestimation of the velocity
downstream of the nacelle region (Fig.10). Table 3 also shows the comparison
of turbulence intensity under different models. Ignoring the nacelle effect will
result in an underestimation of the turbulence intensity. After the nacelle
model is introduced, the numerical accuracy can be increased by 4% ∼ 11%.

<table>
<thead>
<tr>
<th>Index</th>
<th>Axial Velocity (m/s)</th>
<th>Turbulence Intensity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( P_{\text{center}} )</td>
<td>( P_{\text{side}} )</td>
</tr>
<tr>
<td>Exp.</td>
<td>2.183</td>
<td>2.151</td>
</tr>
<tr>
<td>Without nacelle modeling</td>
<td>2.820</td>
<td>2.836</td>
</tr>
<tr>
<td>With nacelle modeling</td>
<td>2.339</td>
<td>2.278</td>
</tr>
<tr>
<td>Improvement (%)</td>
<td>22.03</td>
<td>25.94</td>
</tr>
</tbody>
</table>
Figure 10: Velocity distribution in the wind-turbine wake. a) Without nacelle modeling. b) With nacelle modeling.

The Grid Convergence Index (GCI) is used to measure the convergence for grid refinement, and this method is described in detail in Roache’s work [54]. We use 3 million (Mesh1), 6.5 million (Mesh2), and 15 million (Mesh3) grid cells respectively for grid convergence analysis. Table 4 shows the GCI index under these three grids, and uses the velocity values at two measuring points ($P_{center}$ and $P_{side}$) as key variables. The variables of $p$ and $R$ in the table represent the apparent order and convergence ratio, respectively. The results show that the numerical calculation is monotonously convergent ($0 < R < 1$), and the numerical uncertainty in the Mesh3 for $V_{center}$ and $V_{side}$ is 2.14% and 3.38%, respectively.

<table>
<thead>
<tr>
<th>Index</th>
<th>Mesh1</th>
<th>Mesh2</th>
<th>Mesh3</th>
<th>$p$</th>
<th>$R$</th>
<th>GCI_{Mesh3}</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{center}$</td>
<td>2.620</td>
<td>2.408</td>
<td>2.339</td>
<td>1.1963</td>
<td>0.3255</td>
<td>0.0214</td>
</tr>
<tr>
<td>$V_{side}$</td>
<td>2.483</td>
<td>2.341</td>
<td>2.278</td>
<td>0.8427</td>
<td>0.4437</td>
<td>0.0338</td>
</tr>
</tbody>
</table>

Figure 11 shows the spatial distribution of vorticity and TKE in the wake under a yaw angle of 10.6°. For a yawed wind turbine, the thrust of the airflow on the rotor is perpendicular to the rotation plane, that is $T_{thrust}$ in Fig.11a. The $T_{thrust}$ can be decomposed into the forward force $T_x$ and its normal force $T_y$. The reaction forces on the airflow, $-T_x$ and $-T_y$, cause the airflow to be accelerated by the wind turbine in upwind and lateral direction, thus skewing the wake toward one side. One can also see a significant asymmetry in TKE (Fig.11b) caused by the geometric asymmetry of the yawed turbine. The tip
vortex distributed on the negative half of the \( y \)-axis extends into the main flow, and the fluid in the wake region naturally exchanges momentum with the outer flow. However, in the upper half plane (positive \( y \)), the outer flow mixes with the low-speed fluid in the wake by the dynamics of the vortex itself. In addition, the tip vortices begin to oscillate at about \( x = 1.8D \), as shown in the turbulent kinetic energy field, which may be caused by the fact that the \( y \)-component of \(-T_{thrust}\) promotes the spiral instability of the tip vortex.

![Image of vorticity and turbulent kinetic energy distribution](image)

**Figure 11**: Distribution of vorticity and turbulent kinetic energy at 10.6° yaw angle. a) \( z \)-component of vorticity. b) Turbulent kinetic energy.

### 4. Wake Analysis under Various Yaw Angles

To analyze the development of wind-turbine wakes under different yaw conditions, we conduct further simulations with the same full-scale turbine under various yaw conditions beyond the field experiment condition. As shown in Fig.12a, the nacelle vortex developing downstream interacts the skewed tip vortex in the wake, and the location of interaction moves upstream with increasing yaw angle (Fig.12b). At yaw angles of 20° to 30° and 40°, the interaction can be observed at approximately 3.0D, 2.2D and 1.9D, respectively. This interaction is not only caused by the fact that the nacelle vortex is radially diffused and blended with the tip vortex, but also by the
difference in the spatial distribution of local thrust on the blades and the nacelle. This is because the thrust of the blade is derived from the airfoil’s lift and drag characteristics, but the thrust of the nacelle is mainly derived from the drag characteristics of the cylinder, whose lift coefficient is almost zero. Therefore, in the modeling of the nacelle, the y-component of $-T_{thrust}$ of the nacelle on the airflow is less than the force of the blade on the airflow, resulting in the nacelle wake being less skewed and thus interacting with the tip vortex. During the interaction between the nacelle vortex and the skewed tip vortex, the initially coherent tip vortices break down into small-scale vortices, facilitating the diffusion of turbulent kinetic energy to the surroundings, which in turn promotes momentum exchange between the wake region and the outer flow. This shows the possibility of influencing wake dynamics through specially designed hub or nacelle geometry to accelerate wake recovery. For more vorticity distributions under various yaw angles, please refer to Fig.A1 in the Appendix.

Figure 12: Interaction positions of blade tip vortex and nacelle vortex under various yaw angles. a) Vorticity distribution in the wake under $\gamma = 40^\circ$. b) Vortex interaction positions.

The interaction between the tip vortex and the hub vortex also occurs under large yaw conditions. To quantitatively analyze this interaction, we performed a spectral characteristic of the velocity at a measurement point $(x/D= 2, y/D= 0, z/D= 0)$ under these four different yaw conditions. Figure 13a shows the power spectrum of the axial velocity at $40^\circ$ yaw angle. As the yaw angle increases, the tip vortex is more skewed and closer to the measurement point, which causes an increase in the power spectral density (PSD) of the velocity. Further, under the four yaw conditions, the PSD is integrated in the overall frequency domain (Fig.13b), and the latter yaw condition is sequentially increased by about 26.7%, 78.5% and 1.1%, respectively, than
the integral value of the former yaw condition. Under the 40° yaw condition, the tip vortex in the upper half plane passes through the measurement point and mixes with the nacelle vortex, which causes the PSD of the velocity to increase at about 0.42 Hz. Although the narrower wake under yaw conditions causes the local turbulent kinetic energy to rise, the staggered layout for a wind-farm can utilized by exploiting the fact that the wake region is reduced. Thus, a downstream wind turbine can be positioned to avoid the wake of the upstream wind turbine, thereby increasing overall power output of the wind farm. For more power spectra under various yaw angles, please refer to Fig.A2 in the Appendix.

Figure 13: Interaction positions of blade tip vortex and nacelle vortex under various yaw angles. a) Vorticity distribution in the wake under $\gamma = 40^\circ$. b) Vortex interaction positions.

In addition to the interaction with the nacelle vortex, the tip vortex in the upper half plane (positive $y$-axis) is mixed with the tip vortex in the lower half plane (negative $y$-axis) in the far wake under the condition of a large yaw angle, as shown in the plots of the wake velocity distribution for the four different yaw angles in Fig.14. The tip vortex in the negative half plane (negative $y$-axis) extends into the outer flow. With the momentum of the wake is sustained by the outer flow, the direction of the tip vortex is gradually aligned with that of the outer flow. Due to the difference in the direction of propagation, the tip vortex on the positive half plane is gradually skewed and mixed with the tip vortex originating on the negative half plane. The circled line in Fig.14 indicates the first downstream location at which a one-peak deficit velocity profile can be identified, which implies the mixing of tip vortices from two sides and the transition of the wake velocity pattern (from the two-peak distribution to the one-peak distribution). At the 10.6° yaw angle, the one-peak profile appears in the far wake ($x/D > 5$), while in
the cases of yaw angle above 20°, the one-peak profile appears in the near wake ($x/D < 5$). This is because, at small yaw angles, the wake dynamics is the main factor causing the wake mixing; at larger yaw angles, the difference of thrust distribution between the upper and lower parts of the tip vortices promote the wake mixing. Since the rotation directions of the two types of tip vortices are opposite (red and blue tip vortices in Fig.12), the mixed vortices are broken again, which causes the turbulent kinetic energy to spread to the surroundings and the wake region to expand.

Figure 14: Velocity distribution under various yaw angles.

In this work, the position where the average wake velocity is 98% of the outflow velocity is regarded as the wake boundary. The distance between the upper and lower wake boundary in Fig.14 is the wake width. The wake center is defined as the velocity extreme point position of the wake center area in Fig.14. Figure 15 shows the variation of the wake center and wake width in the streamwise direction ($x/D = 0 \sim 9$). The skew of the wake centerline and the increase of the wake width occur under all yaw conditions. In Fig.15a, the location of $x/D = 0$ and 1 is closer to the wind turbine, and the development of wake is not sufficient, so the difference of the wake center under different yaw angles is small. At the location of $x/D = 2, 3$ and 5, the wake center position changes sharply between 30° and 40° yaw angles, between 20° and 30° yaw angles, and between 10° and 20° yaw angles, respectively. This corresponds to the transition of the wake velocity pattern in Fig.14. It can be seen that under yaw conditions, the skew wake promotes the wake mixing and the transition of the wake velocity pattern. As the yaw angle increases, the reduction of the wake width at various streamwise
locations is also significant (Fig.15b). When the yaw angle is increased by 10°, the dimensionless wake width decreases by about 10% to 20%. The power performance of the wind turbine will decrease under yaw conditions, but the skew of the wake centerline and the reduction of the wake width have potential utilization value for wind farm layout.

Figure 15: Distribution of wake center and wake width under various yaw angles. a) Wake center. b) Wake width.

5. Summary and Discussion

Since the isotropic distribution model has the same distribution characteristics of the body force in all directions, it is impossible to ensure that the body force is constantly projected inside the nacelle, which will cause excessive concentration of the body force and a large axial velocity deviation. In this work, a new anisotropic body-force projection method is proposed to model the nacelle effect, which can take into account more geometrical features of nacelle as compared to the standard actuator line model. Based on the field experiment in Gansu, the anisotropic nacelle model can improve the prediction accuracy of time-averaged velocity by about 22% over the traditional model, and increase the prediction accuracy of turbulence intensity by 4% to 7%.

When the wind turbine is operated under yaw conditions, due to the reaction force of the thrust, a part of the tip vortex develops toward the wake region, and the other part of the tip vortex develops into the outer flow. This part of the tip vortex gradually adapts its direction to that of the outer flow, therefore narrowing the wake region. As the yaw angle increases, the
wake transition from the two-peak profile to the one-peak profile is advanced, which is related to the diffusion and mixing of the wake structures. Each 10° increase in yaw angle reduces the wake width by 10% to 20%. These narrowing of the wake width and the skew of the wake centerline may be exploited in layout optimization of a wind farm. For example, by setting the optimal yaw angle for the wind turbine to maximize the sum of the power of the upstream wind turbine and the downstream wind turbine. Under the conditions of large yaw angles, the mixing of the nacelle vortex and the tip vortex is observed and attributed mainly to the difference in the spatial distribution of the local thrust on the blades and the nacelle. The radial diffusion motion of the nacelle vortex and its blending with the tip vortex promote the breakdown of the tip vortex and turbulence dissipation, which is beneficial to the wake recovery.

The results in this work clearly show the interaction between the nacelle wake and wind turbine wakes, and this effect will be more obvious under yaw conditions. Therefore, we suggest to consider the nacelle effect to more accurately simulate the unsteady wake in the LES. The interaction between the nacelle wake and wind turbine wakes also has potential applications, such as using a different geometry design of the nacelle or hub to accelerate the wake recovery. In addition, due to the Reynolds number effect of the wind tunnel, we used a field experiment to validate the numerical accuracy of the nacelle model. Limited by the experimental conditions, the number of measurement points in the field experiment is difficult to be as large as the number of measurement points in the wind tunnel experiment. Therefore, in our future work, we will further carry out detailed field experiments to study the influence of the nacelle and inboard blade section on the velocity distribution and meandering effect in the wake.

Appendix: Wake Structures under Various Yaw Angles

At yaw angles of 20° to 30° and 40°, the vorticity distribution in the wake and the power spectrum characteristics of the wind turbine at the measurement point are shown in Fig.A1 and Fig.A2, respectively. The breaking position of the blade tip vortex is advanced from 3.0D to 2.2D and 1.9D. The power spectrum of the axial fluctuation velocity at the wake measurement point also gradually increased, enhancing the local fluctuation characteristics and promoting the breaking of the tip vortex.
Figure A1: Vorticity distribution in the wake under various yaw angles. a) $\gamma = 20^\circ$. b) $\gamma = 30^\circ$. c) $\gamma = 40^\circ$. 

Journal Pre-proof
Figure A2: Power spectra of axial velocity at the measurement point under various yaw angles. a) $\gamma = 20^\circ$. b) $\gamma = 30^\circ$. c) $\gamma = 40^\circ$.

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Highlights

Modelling the nacelle wake of a horizontal-axis wind turbine under different yaw conditions

Zhiteng Gao, Ye Li, Tongguang Wang, Wenzhong Shen, Xiaobo Zheng, Stefan Pröbsting, Deshun Li, Rennian Li

- Nacelle wake affects the wind-turbine wakes especially under yaw conditions.
- A new anisotropic model is introduced to simulate the nacelle wake.
- A field test with yaw condition is used to validate the new model.
- The wake characteristics under different yaw conditions are extensively analyzed.
Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: