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Price-Region Bids in Electricity Markets

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Abstract

Current bid formats in pool-based electricity markets are ill-equipped to accommodate the broad range of non-conventional sources of flexibility, such as demand response and interconnected heating, natural gas and water infrastructure networks. To address this issue, this paper introduces the novel *price-region bid* format to be used in both forward electricity markets and financial right auctions. We show that price-region bids are able to accommodate a broad range of techno-economic characteristics, including complex spatial and temporal couplings, and facilitate market access to non-conventional flexibility providers. We then show that this new bid format is compatible with existing market structures, and satisfies desirable market properties under common assumptions. Three numerical studies are provided: two motivating examples based on a district heating utility and a cascaded hydro power plant, and a case study based on an integrated power and heat system. These studies illustrate the inability of existing bid formats to accommodate flexible resources, and show how price-region bids overcome this shortcoming.

Keywords:
OR in energy, electricity markets, power system flexibility, price-region bids

1. Introduction

Growing concerns about climate change, as well as energy independence, have supported the sustained deployment of large shares of renewable sources of electricity such as wind and solar power (Chu and Majumbar 2012), whose inherent variability call for additional flexibility in the operation of power systems. In many countries, pool-based competitive markets play a central role in power system operation. As these platforms were initially conceived for systems with large shares of fuel-based power plants, traditional offering and bidding formats, such as price-quantity and block bids, may prevent unconventional sources of flexibility from being correctly valued in the market. For instance, cascaded hydropower plants represent a network of spatially- and temporally-correlated flexible electricity production and storage (Piekutowski et al. 1993, Shen et al. 2015). Other energy infrastructures connected to the power system at multiple nodes, such as district heating (Chen et al. 2015, Mitridati and Taylor 2018), water (Zamzam et al. 2018, Stuhlmacher and Mathieu 2020, Santhosh et al. 2014), and natural gas (Byeon and Van Hentenryck 2020, Ordoudis et al. 2020)

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networks provide a source of untapped flexibility and virtual energy storage. Distributed energy resources, large-scale virtual power plants (Naughton et al. 2020), and electricity end-users, such as networks of data centers (Zhang et al. 2020), and residential consumers (Callaway and Hiskens 2011, Anjos and Gómez 2017), are playing a central role as flexibility providers. Yet, current bid formats are unable to accurately aggregate and represent the spatial and temporal correlations of the flexibility of these market participants. Indeed, De Vivero-Serrano et al. (2019) show that simple price-quantity bids may reduce the profitability of energy storage systems participating in electricity markets, while Sioshansi et al. (2009) find out that battery storage in the PJM operating region could be under-utilized by up to 15%. Acknowledging these issues, Ahlstrom (2018) argues for the need for a new market participation model, allowing a wider range of resources to represent their techno-economic characteristics. To that end, this paper aims to propose a novel bid format which expands the range of flexible resources that can be accommodated in *pool-based* electricity markets.

Several authors have proposed bid formats for better representing specific flexible resources. For example, De Vivero-Serrano et al. (2019) propose linked charging-discharging bid formats for energy storage systems, which are shown to facilitate their participation and increase their profitability in electricity markets. Su and Kirschen (2009) propose a multi-time-period demand bid which allows for flexible time of consumption. Liu et al. (2015) propose additional bid types for deferrable, adjustable and storage-like loads, and show that a spot market which clears these bids retains efficiency and incentive compatibility. These bid formats are linear. In contrast, O'Connell et al. (2016) propose a discrete bid format in the form of asymmetric block offers and discuss its application to aggregations of thermostatically controlled loads. Other authors have suggested to treat storage aggregations as a system asset, similar to how transmission lines are treated in spot markets (Taylor 2015, Muñoz-Álvarez and Bitar 2017). In this case, the operational constraints of storage assets are directly incorporated into the spot market operator's market-clearing program.

Each one of these approaches focuses on improving the valuation in electricity markets of a specific type of flexible resources, located at a single node of the network. However, flexible interconnected infrastructures, such as district heating and natural gas networks, may feature complex operational constraints that link multiple time periods and locations. These characteristics are not well represented by the mechanisms found in existing markets and in the literature, which prevents these resources from being truly valued in the market. To address this issue, we define a new generic *price-region bid* format to be used in both forward electricity markets and financial right auctions. Price-region bids can accommodate a broad range of non-conventional flexible resources with complex techno-economic characteristics linking multiple time periods and networks locations.

This paper makes several original contributions. Firstly, the proposed price-region concept provides a general framework for market participants to exactly represent any linearly-constrained feasible region of operation and convex piecewise linear cost function. These include, but are not limited to, the representation of complex spatial and temporal couplings. To the best of our knowledge, no existing bid format can cover this range of characteristics. We show, in particular, how price-region bids overcome the limitations of price-quantity bids in representing complex flexible infrastructures. Secondly, we show that price-region bids can straightforwardly be incorporated into existing pool-based market clearing programs without conflicting with the mechanisms in place. In particular, in markets which clear special cases of price-region bids (such as price-quantity bids) and use uniform pricing, we show that price-region bids preserve important market properties: the market-clearing procedure with price-region bids ensures efficiency of the dispatch, cost recovery

for the market participants and revenue adequacy for the market operator (as in other markets based on uniform pricing, incentive compatibility holds under the assumption of perfect competition). Furthermore, the market-clearing program for price-region bids is linear, which ensures computational tractability. We further show that price-region bids generalize financial transmission and storage rights, and that revenue adequacy can be guaranteed for actors involved in both an auction for rights and a spot market. Finally, two motivating examples and one case study based on district heating systems and cascaded hydro power plants demonstrate the added value of price-region bids in a forward electricity market dispatching bids from complex flexible resources.

This paper is organized as follows. Section 2 describes the status quo in electricity markets and covers some technical preliminaries. Specifically, we describe current bid formats and the corresponding market clearing. The limitations of existing bid formats are demonstrated through two motivating examples, respectively based on a district heating network and a cascaded hydro power system. Section 3 introduces the new price-region bid format. Here we formulate the corresponding market clearing as a linear program, and define its theoretical properties. We then discuss the application of price-region bids to financial right auctions. Section 4 illustrates the workings of price-region bids through numerical examples. Finally, Section 5 gathers conclusions and perspectives for future work. The proofs of the propositions and modelling details for the case study are provided in Appendix A to Appendix C.

2. Status quo in electricity markets

We concentrate on electricity markets where forward mechanisms, e.g., day-ahead, are based on pools, such as in Europe. This section introduces relevant notations, concepts and properties in such markets. Two motivating examples then uncover some of their limitations in accommodating unconventional sources of flexibility.

2.1. Forward electricity market mechanisms

In contrast with *power exchange* markets, which explicitly model the techno-economic characteristics of market participants in their market-clearing program, *pool-based* forward electricity markets take the form of auctions, in which pre-defined bid formats implicitly embed the characteristics of market participants (also referred to as "actors") in a simplified way. These bids are defined for a set of time periods and locations, which depend on the market temporal and spatial resolution. When all bids are collected at gate closure, the market operator clears the auction, eventually obtaining the dispatch for all market participants. Settlement is then based on a payment rule.

2.2. From bids to market clearing

We index bids by $s \in \{1, ..., S\}$, locations by $n \in \{1, ..., N\}$, and market time periods by $k \in \{1, ..., K\}$. We let $q_{snk} \in \mathbb{R}$ denote the amount of energy associated with bid s, location n and time period k. When q_{snk} is positive, it represents an *injection* of energy at location n and time k. When it is negative, it represents a *withdrawal* of energy from location n at time k.

Definition 1. The **injection profile** associated with a bid s is defined as a vector $\mathbf{q}_s \in \mathbb{R}^{NK}$ containing variables $q_{snk},\ n \in \{1, ..., N\},\ k \in \{1, ..., K\}$.

Definition 2. A **bid** s is characterized by a pair $(\mathcal{F}_s, \mathcal{C}_s)$. Set $\mathcal{F}_s$ describes all injection profiles $\mathbf{q}_s$ which are feasible for the actor placing the bid. The function $\mathcal{C}_s : \mathcal{F}_s \to \mathbb{R}$ describes the cost associated with each feasible injection profile $\mathbf{q}_s \in \mathcal{F}_s$.

To streamline our discussion, we use the term "cost" to refer to both positive and negative values. When positive, $\mathcal{C}_s(\mathbf{q}_s)$ represents the compensation an actor is willing to receive for a certain profile $\mathbf{q}_s$. When negative, $-\mathcal{C}_s(\mathbf{q}_s)$ represents the amount of money an actor is willing to pay for a certain profile $\mathbf{q}_s$.

Based on those definitions, at gate closure the market operator clears the market by solving a generic social-welfare maximization problem as

$$\min_{\mathbf{q}} \quad \sum_{s=1}^{S} \mathcal{C}_s(\mathbf{q}_s) \tag{1a}$$

$$\text{s.t. } \mathbf{q}_s \in \mathcal{F}_s \quad , \ \forall s \in \{1, ..., S\}, \tag{1b}$$

$$\sum_{s=1}^{S} q_{snk} = 0 \quad , \ \forall n \in \{1, ..., N\}, \ k \in \{1, ..., K\}. \tag{1c}$$

Constraints (1b) ensure that the injection profiles $\mathbf{q}_s$ are feasible for all bidders. Constraints (1c) ensure that energy injections and withdrawals are balanced in each price area. The vector $\mathbf{q}$ is a vector of variables whose elements are q_{snk} for all s, n, k.

Transmission constraints are implicitly represented in (1a)-(1c). The transmission network is represented by adding a new component to the bid set, where the bid has a feasible region that consists of all feasible flows. In particular, the transmission network can be modeled as a feasible region of operation, $\mathcal{F}_s$, and a zero cost function, $\mathcal{C}_s$, with q_{snk} the net energy flowing from location n to neighbouring locations during time period k. These locations may refer to nodes in an electricity network, or to larger zones, depending on the spatial resolution in the market program. Other system assets, such as storage, can also be represented in (1a)-(1c) as a special case of a bid $(\mathcal{F}_s, \mathcal{C}_s)$ with $\mathcal{C}_s(\mathbf{q}_s) = 0, \ \forall \mathbf{q}_s \in \mathcal{F}_s$. In this paper, we assume that system assets are modeled in this way for concision.

For an optimal solution to the market-clearing problem to exist, the set of feasible solutions must be compact and non-empty, and the objective function must be continuous. These conditions can be guaranteed, for example, by enforcing individual bids to have compact feasible regions and continuous cost functions, and to admit an empty injection profile as a feasible dispatch, i.e., $[0, ..., 0]^\top \in \mathcal{F}_s, \ \forall s$. Uniqueness of the optimal dispatch and payments to market participants are also important concerns, which can be guaranteed under conditions on the cost functions; see, e.g., Krebs et al. (2018). When the uniqueness conditions are not met, markets implement rules to choose among multiple solutions; see, e.g., Feng et al. (2012) and Alguacil et al. (2013).

2.3. Settlement and market properties

Let $\mathbf{q}^\star$ denote an *optimal dispatch*, i.e., an optimal solution to (1a)-(1c). Similarly, $q^\star_{snk}$ are the elements of $\mathbf{q}^\star$, and vectors $\mathbf{q}^\star_s$ are the instructions from the market operator to each market participant based on the optimal dispatch. After solving the market-clearing problem, the market operator is responsible for the settlement process, i.e., for the determination of payment and revenues for the quantities dispatched. Different market designs rely on different payment rules, e.g., uniform pricing, pay-as-bid, or Vickrey-Clarke-Groves. Competitive markets are often designed to

have the following four properties (Arrow and Debreu 1954, Schweppe et al. 1988, Boucher and Smeers 2001):

Property 1 (**Efficiency**). *Based on the information from bid parameters:*
(i) social welfare is maximized, and
(ii) no market participant has incentives to deviate unilaterally from the optimal dispatch.

Property 2 (**Cost recovery**). *Market participants recover the costs announced in their bids.*

Property 3 (**Incentive compatibility**). *Market participants are incentivized to bid truthfully.*

Property 4 (**Revenue adequacy**). *All transactions prescribed can be settled without incurring a shortfall for the market operator.*

We remark that with a finite number of market participants, Properties 1-4 cannot be necessarily met altogether without further assumptions (Hurwicz 1972, Myerson and Satterthwaite 1983).

The paper assumes uniform pricing, i.e., prices are determined based on the marginal value of electricity at each location and time. The injection and withdrawal locations for which each bid can be placed are referred to as *price areas*. These price areas may refer to nodes in an electricity network, or to larger zones, depending on the market's spatial resolution.[1] Markets using uniform pricing require in particular the assumption of perfect competition, i.e., all market participants are price-taker, to meet Properties 1 and 3 (Wilson 1977, Hobbs et al. 2004, Bose and Low 2018).

For the remainder of this paper, let λ_{nk} denote the Lagrange multiplier of constraint (1c) for each n, k. Let $\boldsymbol{\lambda} \in \mathbb{R}^{NK}$ be a vertical vector containing variables $\lambda_{nk},\ n \in \{1, ..., N\},\ k \in \{1, ..., K\}$, so that λ_{nk} is the $(k + K(n-1))^{\text{th}}$ element in $\boldsymbol{\lambda}$. The values $\lambda^\star_{nk}$ in vector $\boldsymbol{\lambda}^\star$ are used as uniform prices, i.e., the price at which transactions are settled in each price area n and time period k. An actor who places a bid s is thus paid $\boldsymbol{\lambda}^{\star\top}\mathbf{q}^\star_s \in \mathbb{R}$ by the market operator for the prescribed profile $\mathbf{q}^\star_s$. If this amount is negative, the actor pays $|\boldsymbol{\lambda}^{\star\top}\mathbf{q}^\star_s|$ for this profile.

2.4. Existing bid formats

Forward electricity markets generally rely on the standard *price-quantity* bids. This bid format and its limitations are described in the following.

Definition 3. A **price-quantity bid** s is defined as a set of parameters $(\underline{Q}_s, \overline{Q}_s, P_s, N_s, K_s)$, where $\underline{Q}_s$ and $\overline{Q}_s$ denote lower and upper bounds for energy injection/withdrawal in price area N_s and time period K_s, while P_s denotes a bid price, i.e.,

- the feasible region $\mathcal{F}_s$ of injection profiles $\mathbf{q}_s$ is given by
$$\mathcal{F}_s = \left\{ \mathbf{q}_s \mid\ q_{sN_sK_s} \in [\underline{Q}_s, \overline{Q}_s], q_{snk} = 0,\ \forall (n,k) \neq (N_s, K_s) \right\}, \tag{2a}$$
- the cost associated with a feasible profile $\mathbf{q}_s$ is given by
$$\mathcal{C}_s(\mathbf{q}_s) = P_s \cdot q_{sN_sK_s}. \tag{2b}$$

[1] When transmission network nodes are used as price areas, uniform prices are often referred to as *locational marginal prices* or *nodal prices*.

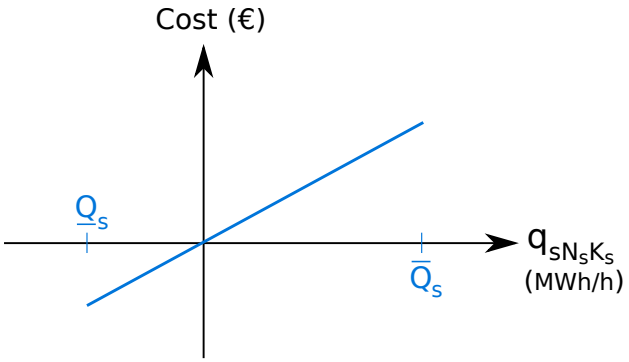


Figure 1: Example of cost function in a single price-quantity bid

Figure 1 shows an example of cost function which can be represented by a single price-quantity bid. Multiple price-quantity bids can be placed simultaneously by an actor so as to cover multiple price areas and/or time periods, as well as to represent convex piecewise linear cost functions (Anderson and Philpott 2002, Neame et al. 2003). However, sets of price-quantity bids can only describe box feasible regions (see Definition 4) and additively separable cost functions (see Definition 5). Figure 2 shows an example of a single bid that includes injections or withdrawals of power at two different nodes and/or hours, i.e., $(q_{sN_s^1K_s^1}, q_{sN_s^2K_s^2})$, with $N_s^1 \neq N_s^2$ and/or $K_s^1 \neq K_s^2$. Four price-quantity bids are placed (one for each plane section). Altogether, they describe a box feasible region (Figure 2, black surface) and an additively separable, convex, piecewise linear cost function (Figure 2, coloured surface).

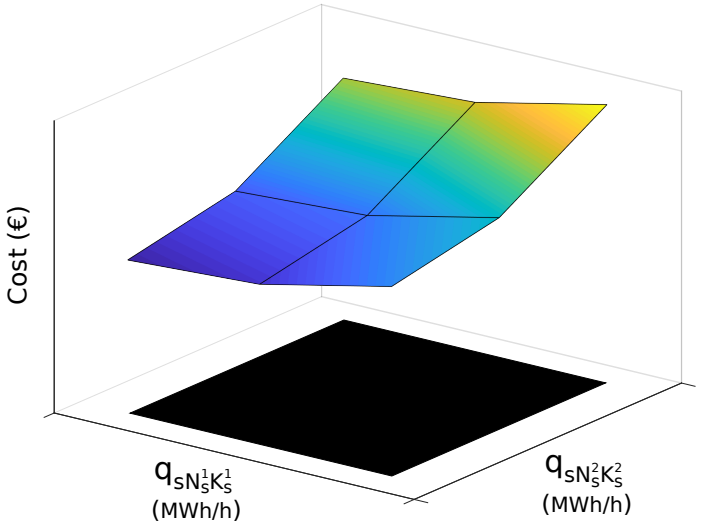


Figure 2: Cost function $\mathcal{C}_s : \mathcal{F}_s \to \mathbb{R}$ (coloured surface) and feasible region $\mathcal{F}_s$ (black surface) for a given set of four price-quantity bids. The black surface is the projection of the coloured surface on the $(q_{sN_s^1K_s^1}, q_{sN_s^2K_s^2})$ plane. The hue of the coloured surface carries the same information as the vertical axis.

Definition 4. A feasible region $\mathcal{F}_s$ is a **box** if there exists $(\underline{Q}_{snk}, \overline{Q}_{snk})$ for all n, k so that

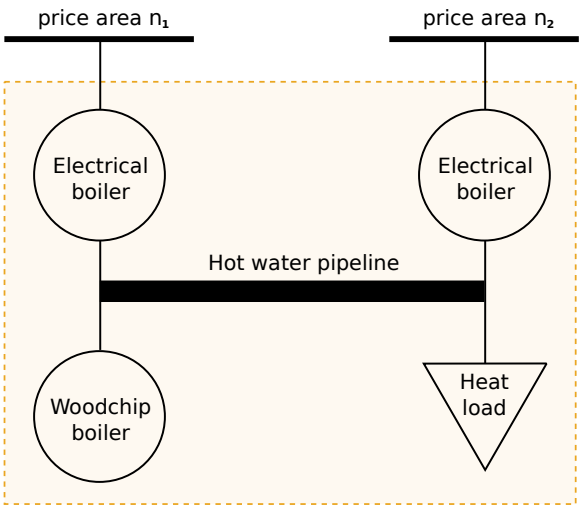


Figure 3: Simple district heating infrastructure connected to two price areas in a power system.

$$\mathcal{F}_s = \left\{ \mathbf{q}_s,\ \underline{Q}_s \leq q_{snk} \leq \overline{Q}_s\ \forall n, k \right\}.$$

Definition 5. A cost function $\mathcal{C}_s$ is **additively separable** in its arguments $\mathbf{q}_s$ if for any $n \in \{1, .., N\}$ and $k \in \{1, ..., K\}$ there exists a function $\mathcal{C}_{snk}$ so that $\mathcal{C}_s(\mathbf{q}_s) = \sum_{n,k} \mathcal{C}_{snk}(q_{snk})$.

Price-quantity bids are used by a broad range of actors, such as flexible conventional generators with piecewise linear cost functions, renewable generators such as wind power producers, load-serving entities, or storage owners who based their offering strategies on price forecasts. Note, however, that some markets accept piecewise linear cost functions without referring to them as multiple price-quantity bids; see, e.g., *price-dependent hourly orders* in the Nord Pool day-ahead market (Nord Pool 2020).

Some forward electricity markets, mostly in Europe (Dourbois et al. 2018), also accept *block bids*, which allow the coupling of injection quantities over multiple time periods, and the expression of a discontinuous feasible region. The specific constraints allowed in a block bid depend on specific design choices; see, e.g., *curtailable block orders* and *flexi orders* in the Nord Pool day-ahead market (Nord Pool 2020). Because of their discontinuous characteristics, block bids are particularly suited to actors with a discontinuous feasible region of operation or a discontinuous cost function, e.g., due to unit commitment constraints and costs.

2.5. Challenges for non-conventional flexibility providers

This forward electricity market mechanism, and in particular the existing bid formats, has been developed in the context of energy systems with fully controllable thermal generators as main source of operational flexibility. However, these limited bid formats may hinder the ability for non-conventional flexibility providers, such as complex energy or water infrastructures connected to the power system, to participate in electricity markets. The following introduces two motivating examples to illustrate these challenges.

2.5.1. Motivating example 1: district heating utility

Consider a district heating utility participating in a forward electricity market with an hourly time resolution. Let the index s denote its bid. Let the indices $\{n_1, n_2\}$ denote two price areas where the heating utility is connected to the power system. Let the indices $\{k_1, k_2\}$ denote two hours the heating utility is placing a bid for. The infrastructure is illustrated in Figure 3 and includes

- Two electrical boilers, both with an energy conversion efficiency of 100% (for concision) and a capacity of 5 MW, respectively connected to price areas n_1 and n_2.
- A woodchip boiler with a capacity of 3 MW and a fuel cost of 100 €/MWh, connected to price area n_1.
- A heat load, which expects to receive 5 MWh of heat energy in hour k_2. The load can be involuntarily curtailed and the value of lost load is 300 €/MWh.
- A hot water pipeline, which can transport heat energy from price area n_1 to price area n_2 with a delay of one hour, and without losses.

Let $\mathbf{q}_s$ denote the electricity injection profile of the district heating utility from price areas $\{n_1, n_2\}$ over two subsequent hours $\{k_1, k_2\}$. We use the term *injection* for consistency with our notation, but as the interface with the power system consists here solely of boilers, the profile will consists of values lower than or equal to zero. Let x_s denote a state variable representing the woodchip boiler heat output in hour k_1. The operational constraints of the heating utility can be written as

$$x_s - q_{sn_1k_1} - q_{sn_2k_2} \leq 5, \tag{3a}$$

$$q_{sn_1k_2} = 0, \quad q_{sn_2k_1} = 0, \tag{3b}$$

$$-5 \leq q_{sn_1k_1} \leq 0, \quad -5 \leq q_{sn_2k_2} \leq 0, \quad 0 \leq x_s \leq 3. \tag{3c}$$

The heat load in hour k_2 can consume heat produced in the same hour by the electrical boiler at n_2, as well as heat produced in the previous hour by boilers at n_1 and transported in the pipeline. Equation (3a) states that the heat energy consumed by the load is bounded by its capacity, but can be lower if the load is curtailed. Equations (3b) state that the heat production of the electrical boilers at other times must be zero as there is no load to dissipate it. Equations (3c) enforce the three respective production limits of boilers.

For a given electricity injection profile, the heating utility is expected to minimize its total operational cost. Let $c^\star(\mathbf{q}_s)$ denote the minimum operational cost with a profile $\mathbf{q}_s$. With a value of lost load of VOLL = 300 €/MWh and a woodchip fuel cost of C^w = 100 €/MWh, the operational cost is given by

$$c^\star(\mathbf{q}_s) = \min_{x_s} \Big\{ (5 + q_{sn_1k_1} + q_{sn_2k_2} - x_s)\text{VOLL} + x_s C^w, \text{ s.t. (3a)-(3c)} \Big\}. \tag{4}$$

The heating utility is willing to pay for electrical energy according to the opportunity cost of not withdrawing this quantity. Let $\text{WTP}(\mathbf{q}_s)$ denote the willingness to pay for the injection profile

and let $\mathbf{0}_4 = [0\ 0\ 0\ 0]^\top$. We have

$$\begin{aligned}
\mathrm{WTP}(\mathbf{q_s}) &= c^\star(\mathbf{0_4}) - c^\star(\mathbf{q_s}) && \text{(5a)}\\
&= \max_{x_s}\{300(-q_{sn_1k_1} - q_{sn_2k_2}) + 200x_s - 600,\ \text{s.t. (3a)-(3c)}\} && \text{(5b)}\\
&= \begin{cases} 300(-q_{sn_1k_1} - q_{sn_2k_2}),\ \text{if } (-q_{sn_1k_1} - q_{sn_2k_2}) \leq 2, \\ 100(-q_{sn_1k_1} - q_{sn_2k_2}) + 400,\ \text{otherwise.} \end{cases} && \text{(5c)}
\end{aligned}$$

Intermediate steps are provided in Appendix A. Given expression (5c), Figure 4 displays the marginal and total willingness to pay for electrical energy as a function of the total energy withdrawn over the two hours, i.e., $-\ q_{sn_1k_1} - q_{sn_2k_2}$ (recall that $q_{sn_1k_2} = q_{sn_2k_1} = 0$). The left-hand figure indicates that the heating utility is not willing to pay for electricity at higher prices than the value of lost load (300 €/MWh). The electrical boiler has enough capacity to cover the entire load, but if electricity prices are higher than the woodchip fuel cost (100 €/MWh), it may be more economical to use the woodchip boiler to cover part of the load. Only up to 60% of the load can be covered by the woodchip boiler, due to limited capacity. If electricity prices are lower than 100 €/MWh, the heating utility is ready to substitute some of the woodchip-based energy with electrical energy.

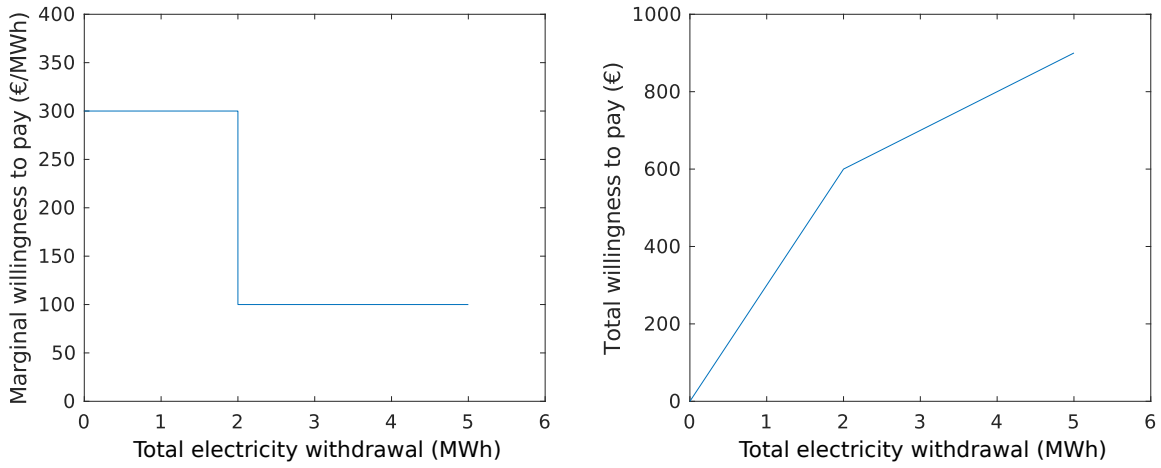


Figure 4: Marginal and total willingness to pay.

The willingness to pay of the heating utility can be expressed through a negative cost function $\mathcal{C}_s = -\mathrm{WTP}$. Both functions are defined over a domain corresponding to the feasible region of injection profiles, denoted by $\mathcal{F}_s$. This region is characterized by equation (6) as

$$\mathbf{q}_s \in \mathcal{F}_s \quad \Leftrightarrow \quad \left\{\exists\ x_s \in \mathbb{R} \text{ s.t. } (\mathbf{q}_s, x_s) \text{ feasible to (3a)-(3c)}\right\} \tag{6}$$

$$\Leftrightarrow \quad \begin{cases} q_{sn_1k_1} \geq 0, \quad q_{sn_2k_2} \geq 0 & \text{(7a)}\\ q_{sn_1k_2} = 0, \quad q_{sn_2k_1} = 0, & \text{(7b)}\\ q_{sn_1k_1} + q_{sn_2k_2} \leq 5. & \text{(7c)} \end{cases}$$

Constraints (7a)-(7c) do not form a box feasible region, and the willingness-to-pay function (5c) is not additively separable. Therefore, price-quantity bids cannot accurately represent these characteristics. See Figures 5 and 6 for an example of inner approximation.

Existing block bids do allow the coupling of injection quantities over time, but typically not in different price areas. Moreover, the feasible region of block bids is generally constrained to a point (all-or-nothing) or a line (curtailable bid with a fixed profile).

To the best of our knowledge, no bid format in existing electricity markets or in the literature allows the coupling of injection quantities in different price areas, nor allows the definition of linearly-constrained feasible regions and non-additively separable convex piecewise linear cost functions. In order to improve the representation of flexible resources and their efficient utilization in electricity markets, this paper proposes a novel *price-region bid* format, and shows that it can exactly represent these characteristics. This contribution is not only relevant for district heating infrastructures such as in this example, but for any flexible resource that can be described by a linear model coupling time periods and/or price areas, such as cascaded hydro power plants (see next motivating example, Section 2.5.2), water or gas supply infrastructures, battery storage systems, flexible load aggregations, networks of data centers, etc. A case study in Section 4 suggests that, in a market with large shares of renewable energy, improving the representation of flexible resources increases social welfare.

2.5.2. Motivating example 2: cascaded hydro power plants

Consider a system of two cascaded hydro power plants participating in a forward electricity market with an hourly time resolution. Let index s denote the bid of the hydro power system operator. Let indices $\{n_1, n_2\}$ denote two price areas where the hydro power plants are respectively connected to the power system. Let indices $\{k_1, k_2\}$ denote two hours the operator is placing a bid for. The hydro power system is depicted in Figure 7 and includes

- A reservoir in price area n_1, with a water volume $V^0_{n_1}$ (m^3) at the beginning of the bidding horizon. The discharge from the reservoir is converted to electricity with a factor ρ_{n_1} ($\mathrm{MWh/m}^3$), and is limited by lower and upper bounds $\underline{D}_{n_1}$ and $\overline{D}_{n_1}$ (m^3/h). We assume no inflow or spillage during the bidding horizon. We also assume that water volume bounds are sufficiently loose to be irrelevant for the bidding horizon. The water volume in the reservoir at the end of the bidding horizon has a value W_{n_1}, based on a forecast of future water value.

- A reservoir in price area n_2, whose inflow corresponds to the water discharged from the reservoir in price area n_1. Parameters $V^0_{n_2}$, ρ_{n_2}, $\underline{D}_{n_2}$, $\overline{D}_{n_2}$ and W_{n_2} are analogously defined. This reservoir however has tight volume bounds $\underline{V}_{n_2}$ and $\overline{V}_{n_2}$. As with the first reservoir, spillage is assumed insignificant.

This cascaded hydro power plant model is analogous to models from standard references on power system operations such as Wood et al. (2013).

Let $\mathbf{q}_s$ denote the electricity injection profile of the hydro power plants from price areas $\{n_1, n_2\}$ over hours $\{k_1, k_2\}$. The operational constraints of the cascaded hydro power system can be written

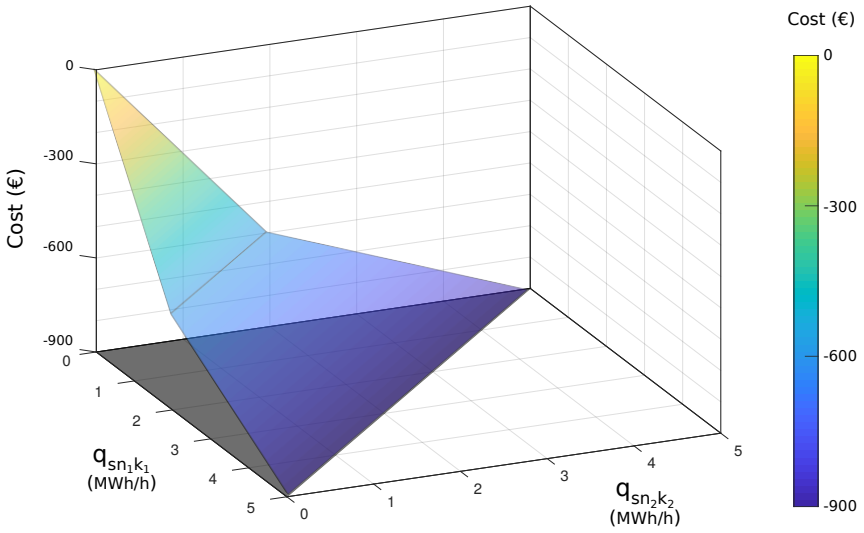


Figure 5: Cost function $\mathcal{C}_s = -\text{WTP}$ (transparent coloured surface) and feasible region $\mathcal{F}_s$ (grey triangular surface) of the district heating utility. The grey surface is the projection of the coloured surface on the $(q_{sn_1k_1}, q_{sn_2k_2})$ plane. The hue of the coloured surface carries the same information as the vertical axis.

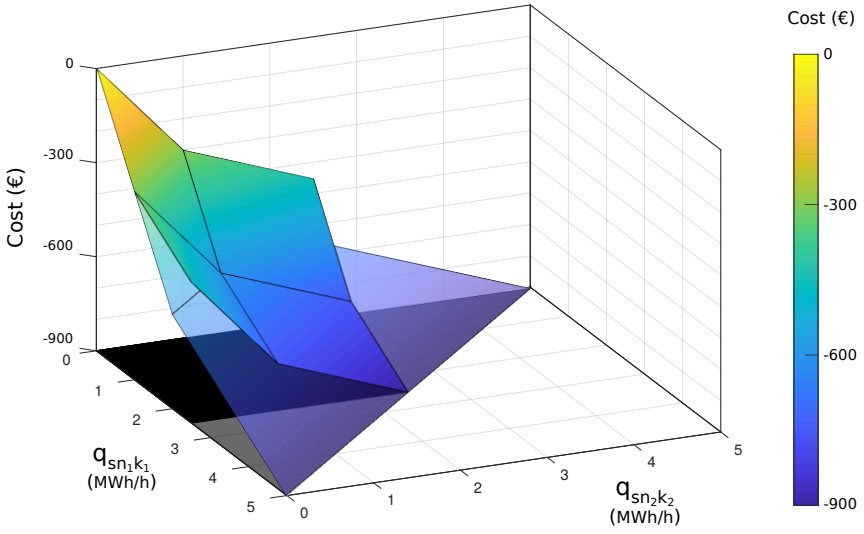


Figure 6: Inner approximation of $(\mathcal{C}_s, \mathcal{F}_s)$ using price-quantity bids, as an additively separable piecewise linear cost function (solid coloured surface) defined over a box feasible region (black rectangular surface). The black surface is the projection of the solid coloured surface on the $(q_{sn_1k_1}, q_{sn_2k_2})$ plane. The hue of the coloured surface carries the same information as the vertical axis. The transparent coloured surface and the grey surface are the original characteristics (see Figure 5).

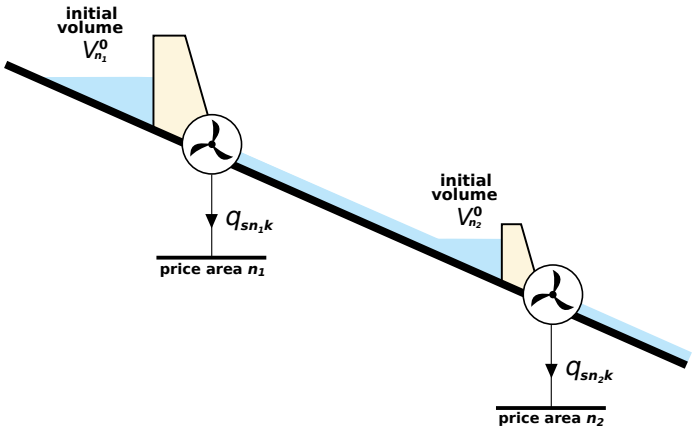


Figure 7: Simple cascaded hydro power infrastructure connected to two price areas in a power system.

as

$$\underline{D}_n \leq \frac{q_{snk}}{\rho_n} \leq \overline{D}_n, \quad \forall\, n \in \{n_1, n_2\},\ k \in \{k_1, k_2\}, \tag{8a}$$

$$\underline{V}_{n_2} \leq V^0_{n_2} + \frac{q_{sn_1k_1}}{\rho_{n_1}} - \frac{q_{sn_2k_1}}{\rho_{n_2}} \leq \overline{V}_{n_2}, \tag{8b}$$

$$\underline{V}_{n_2} \leq V^0_{n_2} + \frac{q_{sn_1k_1} + q_{sn_1k_2}}{\rho_{n_1}} - \frac{q_{sn_2k_1} + q_{sn_2k_2}}{\rho_{n_2}} \leq \overline{V}_{n_2}. \tag{8c}$$

Equation (8a) enforces the discharge constraints, while equations (8b) and (8c) represent the volume constraints in the lower reservoir. In this motivating example, constraints do not include state variables, and the feasible region of injection profiles $\mathcal{F}_s$ is directly given by: $\mathbf{q}_s \in \mathcal{F}_s \Leftrightarrow$ (8a)-(8c). In a more complex model, state variables could be used for example to represent voluntary water spillage.

The cost of electricity production from the hydro power plants relates to the loss of stored water during the bidding horizon. It is written as

$$\mathcal{C}_s(\mathbf{q}_s) = \frac{W_{n_1}\left(q_{sn_1k_1} + q_{sn_1k_2}\right)}{\rho_{n_1}} + \frac{W_{n_2}\left(q_{sn_2k_1} + q_{sn_2k_2} - q_{sn_1k_1} - q_{sn_1k_2}\right)}{\rho_{n_2}}. \tag{9}$$

As in the district heating example, the feasible region $\mathcal{F}_s$ is not a box region, and therefore, it cannot be accurately represented by price-quantity bids. See Figure 8 for a numerical example.

3. Price-region bids

In this section, we define the novel price-region bid format and analyze its properties. We consider the clearing of price-region bids in a forward electricity market, and discuss their implementation as financial rights.

3.1. Price-region bid format

Recall that an injection profile is described by a vector $\mathbf{q}_s \in \mathbb{R}^{NK}$. Let $\mathbf{x}_s = [x_{s1}, ..., x_{sL_s}] \in \mathbb{R}^{L_s}$ be a vector of state variables $x_{sl},\ l \in \{1, ..., L_s\}$, associated with bid $s \in \{1, ..., S\}$. Let $\mathbf{A}_s$ be a matrix of real numbers with $NK + L_s + 1$ columns for each s. Let J_s denote the number of rows in $\mathbf{A}_s$. Let $\mathbf{B}_s$ be a horizontal vector of $NK + L_s$ real numbers. Let $\mathbf{0}$ be a vector of NK zeros.

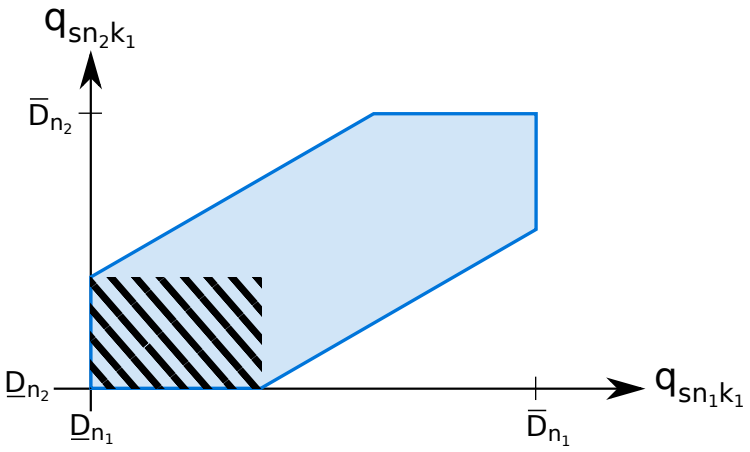


Figure 8: Feasible region of the electricity injection from the hydro power plants in the first hour of the bidding horizon step (solid blue area), and inner approximation as a box region for price-quantity bids (black striped area). In this illustration we assume $q_{sn_1k_2} = q_{sn_2k_2} = 0$ for allowing a two-dimensional representation. Constraints that define the blue area are therefore (8a) for horizontal/vertical edges and (8b) for oblique edges.

Definition 6. A **price-region bid** s is defined as a pair $(\mathbf{A}_s, \mathbf{B}_s)$, so that

- the feasible region $\mathcal{F}_s$ of injection profiles $\mathbf{q}_s$ is given by

$$\mathcal{F}_s = \left\{ \mathbf{q}_s \;\middle|\; \exists\, \mathbf{x}_s \in \mathbb{R}^{L_s},\ \text{s.t. } \mathbf{A}_s \begin{bmatrix} 1 \\ \mathbf{q}_s \\ \mathbf{x}_s \end{bmatrix} \leq 0 \right\}, \tag{10a}$$

- the cost associated with a feasible profile $\mathbf{q}_s$ is given by

$$\mathcal{C}_s(\mathbf{q}_s) = \min_{\mathbf{x}_s} \left\{ \mathbf{B}_s \begin{bmatrix} \mathbf{q}_s \\ \mathbf{x}_s \end{bmatrix},\ \text{s.t. } \mathbf{A}_s \begin{bmatrix} 1 \\ \mathbf{q}_s \\ \mathbf{x}_s \end{bmatrix} \leq 0 \right\} - V_s^0, \tag{10b}$$

where V_s^0 represents the cost of operation when there is no injection or withdrawal of electric power.

The vector of state variables $\mathbf{x}_s$ associated with bid s may be used to represent internal variables and constraints of the bidder, as well as to model its piecewise linear cost function. A concrete illustration of these state variables in the case of a district heating utility will be provided in Section 3.2. Note that the physical meaning of state variables in an electricity market clearing is known by the bidder only.

Matrix $\mathbf{A}_s$ characterizes a set of linear constraints coupling the injection profile $\mathbf{q}_s$ and state variables $\mathbf{x}_s$. These constraints describe an $NK + L_s$-dimensional polytope. The NK-dimensional feasible region $\mathcal{F}_s$ is computed in (10a) by projecting that polytope on the NK dimensions of $\mathbf{q}_s$.

Vector $\mathbf{B}_s$ characterizes the cost associated to each variable, i.e., the bid prices. Equation (10b) computes an opportunity cost, i.e., the difference between (a) the cost associated with profile $\mathbf{q}_s$ given that state variables $\mathbf{x}_s$ are set to their optimal values, and (b) the cost associated with no electricity injection/withdrawal given that state variables $\mathbf{x}_s$ are set to their optimal values.

Note that the cost of operation without electric power injection or withdrawal can be formally defined as

$$V_s^0 = \min_{\mathbf{x}_s} \left\{ \mathbf{B}_s \begin{bmatrix} \mathbf{0} \\ \mathbf{x}_s \end{bmatrix}, \text{ s.t. } \mathbf{A}_s \begin{bmatrix} 1 \\ \mathbf{0} \\ \mathbf{x}_s \end{bmatrix} \leq 0 \right\}.$$

Unlike price-quantity bids, a price-region bid does not require the cost function to be additively separable or the feasible region to be a box. Figure 9 provides an example of a non-separable, convex, piecewise linear cost function defined over a linearly-constrained feasible region. Such characteristics can be described by a price-region bid where

- a single state variable x_{s1} represents the cost function of bid s, i.e., $L_s = 1$ and $\mathbf{B}_s = [0 \ldots 0\ 1]$,
- for each of the six pieces of the cost function, a row in matrix $\mathbf{A}_s$ constrains x_{s1} to be above the corresponding hyperplane, and
- for each facet of the feasible region, a row in matrix $\mathbf{A}_s$ constrains $\mathbf{q}_s$ to be within the region.

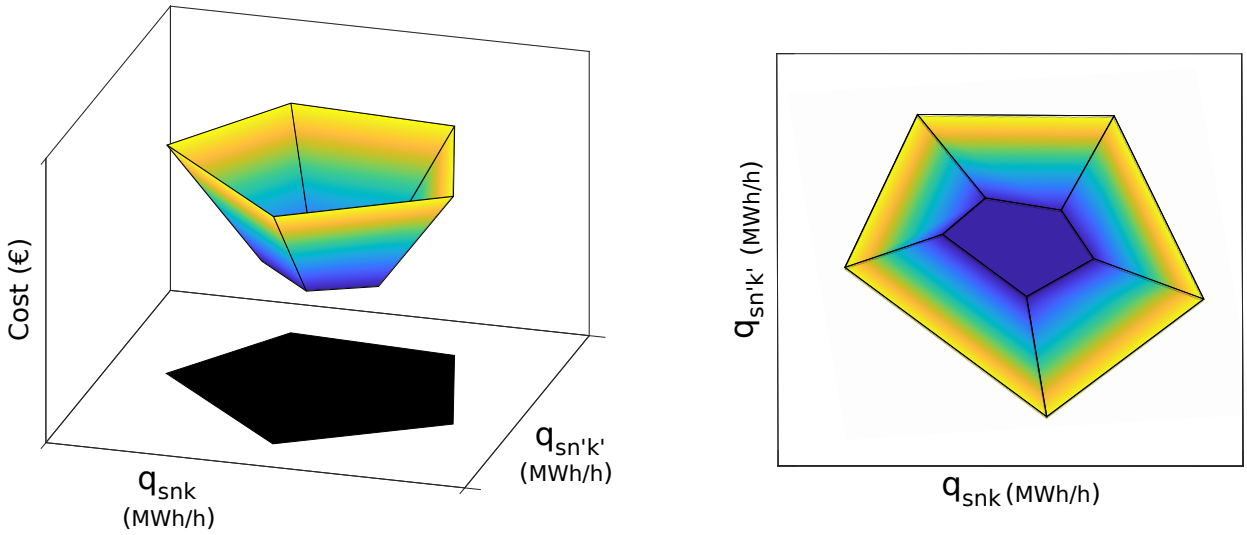


Figure 9: Left: example of a non-separable, convex, piecewise linear cost function (coloured surface) defined over a linearly-constrained feasible region (black surface). The black surface is the projection of the coloured surface on the $(q_{snk}, q_{sn'k'})$ plane. The hue of the coloured surface carries the same information as the vertical axis. Right: same figure seen from above.

Price-region bids can exactly represent any convex piecewise linear function defined over a linearly-constrained feasible region. It follows that any actor whose operations are constrained by a set of linear inequalities coupling continuous variables (including electricity injection variables as well as internal variables, such as a storage state-of-charge, temperature setpoints, etc.) across multiple time periods and locations, and whose costs of operation are a linear combination of these variables, can exactly represent its characteristics in a forward electricity market using a price-region bid. To the best of our knowledge, no existing bid format can accommodate such a broad range of techno-economic characteristics.

Using the example in Figure 9, we described how a single state variable x_{s1} can be sufficient to describe a piecewise linear cost function without assuming separability. However, a larger number of state variables may be used so as to model the internal variables of the bidder, who can thus derive a price-region bid straightforwardly from a linear operational model. This assumes no limitations on the number of constraints and variables in a bid. We discuss issues of bid dimensionality in Section 5.

We assume in this paper that state variables are continuous. Allowing the price-region bid format to include discrete state variables is a straightforward extension, though would be challenging due to the non-convex nature of the resulting market-clearing problem (see Section 5 for a discussion). Price-region bids can still be used by actors with non-linear characteristics, in which case the characteristics of the bidder are approximated. The approximation is however less restrictive than by using, for example, price-quantity bids, which we show are special cases of price-region bids in Remark 1 below.

Remark 1. Let q_{snk} be the $(k+K(n-1))^{\text{th}}$ element in $\mathbf{q}_s$. A price-quantity bid $(\underline{Q}_s, \overline{Q}_s, P_s, N_s, K_s)$, as in Definition 3, is a special case of a price-region bid $(\mathbf{A}_s, \mathbf{B}_s)$ with

$$\mathbf{A}_s = \begin{bmatrix} -\overline{Q}_s & 0 & \dots & 0 & 1 & 0 & \dots & 0 \\ \underline{Q}_s & 0 & \dots & 0 & -1 & 0 & \dots & 0 \end{bmatrix}, \quad \begin{matrix} \\ \\ \\ \\ \uparrow \\ \text{column} \\ (N_s-1)K+K_s+1 \end{matrix} \tag{11a}$$

$$\mathbf{B}_s = \begin{bmatrix} 0 & \dots & 0 & P_s & 0 & \dots & 0 \end{bmatrix}. \quad \begin{matrix} \\ \uparrow \\ \text{column} \\ (N_s-1)K+K_s \end{matrix} \tag{11b}$$

3.2. Illustrative examples

As described below, the proposed price-region bid format can accurately model various types of flexible assets connected to the power system.

3.2.1. Application to motivating example 1 based on a district heating utility

Let the district heating utility described in Section 2.5.1 place a price-region bid $(\mathbf{A}_s, \mathbf{B}_s)$ defined as

$$\mathbf{A}_s = \begin{bmatrix} \text{-5} & \text{-1} & 0 & 0 & \text{-1} & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \text{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \text{-1} & 0 & 0 \\ \text{-5} & \text{-1} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \text{-5} & 0 & 0 & 0 & \text{-1} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \text{-3} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \text{-1} \end{bmatrix}, \quad \mathbf{B}_s^\top = \begin{bmatrix} 300 \\ 0 \\ 0 \\ 300 \\ \text{-200} \end{bmatrix}. \tag{12}$$

Now assume that $\mathbf{q}_s = [q_{sn_1k_1},\ q_{sn_1k_2},\ q_{sn_2k_1},\ q_{sn_2k_2}]^\top$ and $\mathbf{x}_s = [x_s]$. With matrix $\mathbf{A}_s$ defined above, the feasible region of the bid (see equation (10a) in Definition 6) exactly represents the feasible region of the heating utility (see equation (6) in Section 2.5.1). With vector $\mathbf{B}_s$ defined above, the cost function of the bid (see equation (10b) in Definition 6) exactly characterizes the

willingness-to-pay of the district heating utility (see equation (5b) in Section 2.5.1). Thus, the price-region bid $(\mathbf{A}_s, \mathbf{B}_s)$ exactly represents the physical and economic characteristics of the district heating utility.

3.2.2. Application to motivating example 2 based on cascaded hydro power plants

Similarly to the example above, now with $\mathbf{q}_s = [q_{sn_1k_1},\ q_{sn_1k_2},\ q_{sn_2k_1},\ q_{sn_2k_2}]^\top$ and $\mathbf{x}_s$ an empty vector, the characteristics of the cascaded hydro power system described in Section 2.5.2 can be captured by a price-region bid $(\mathbf{A}_s, \mathbf{B}_s)$ defined by

$$\mathbf{A}_s = \begin{bmatrix} \underline{D}_{n_1} & -\rho_{n_1}^{-1} & 0 & 0 & 0 \\ -\overline{D}_{n_1} & \rho_{n_1}^{-1} & 0 & 0 & 0 \\ \underline{D}_{n_1} & 0 & -\rho_{n_1}^{-1} & 0 & 0 \\ -\overline{D}_{n_1} & 0 & \rho_{n_1}^{-1} & 0 & 0 \\ \underline{D}_{n_2} & 0 & 0 & -\rho_{n_2}^{-1} & 0 \\ -\overline{D}_{n_2} & 0 & 0 & \rho_{n_2}^{-1} & 0 \\ \underline{D}_{n_2} & 0 & 0 & 0 & -\rho_{n_2}^{-1} \\ -\overline{D}_{n_2} & 0 & 0 & 0 & \rho_{n_2}^{-1} \\ \underline{V}_{n_2} - V_{n_2}^0 & -\rho_{n_1}^{-1} & 0 & \rho_{n_2}^{-1} & 0 \\ V_{n_2}^0 - \underline{V}_{n_2} & \rho_{n_1}^{-1} & 0 & -\rho_{n_2}^{-1} & 0 \\ \underline{V}_{n_2} - V_{n_2}^0 & -\rho_{n_1}^{-1} & -\rho_{n_1}^{-1} & \rho_{n_2}^{-1} & \rho_{n_2}^{-1} \\ V_{n_2}^0 - \underline{V}_{n_2} & \rho_{n_1}^{-1} & \rho_{n_1}^{-1} & -\rho_{n_2}^{-1} & -\rho_{n_2}^{-1} \end{bmatrix},$$

$$\mathbf{B}_s^\top = \begin{bmatrix} \frac{W_{n_1}}{\rho_{n_1}} - \frac{W_{n_2}}{\rho_{n_2}} \\ \frac{W_{n_1}}{\rho_{n_1}} - \frac{W_{n_2}}{\rho_{n_2}} \\ \frac{W_{n_2}}{\rho_{n_2}} \\ \frac{W_{n_2}}{\rho_{n_2}} \end{bmatrix}. \tag{13}$$

Additionally to these non-conventional flexibility providers, price-region bids can be used to model traditional power system assets, such as transmission networks and energy storage.

3.2.3. Application to the transmission network

Let the price areas covered by a market be connected by an alternating current (AC) transmission network. Let β_{nm} denote the susceptance of a transmission line between areas n and m, in Ω^{-1}, and T_{nm} the transmission capacity in MW. Let $\beta_{nm} = T_{nm} = 0$ when there is no transmission line between n and m. Let $\bar{s}$ be an index associated with the entire transmission network, and let $x_{\bar{s}nk}$ denote the voltage angle in n at time k, and $\mathbf{x}_{\bar{s}}$ a vector which contains all voltage angle variables. Let $q_{\bar{s}nk}$ (in MWh/h) denote the net energy flowing into or out of area n during time period k. Using a lossless linearized power flow model, the operational constraints of the transmission network are given by

$$q_{\bar{s}nk} = \sum_{m=1}^{N} \beta_{nm}(x_{\bar{s}mk} - x_{\bar{s}nk}) \quad ,\ \forall n \in \{1, ..., N\},\ k \in \{1, ..., K\}, \tag{14a}$$

$$\beta_{nm}(x_{\bar{s}nk} - x_{\bar{s}mk}) \leq T_{nm} \quad ,\ \forall n, m \in \{1, ..., N\},\ k \in \{1, ..., K\}, \tag{14b}$$

$$x_{\bar{s}1k} = 0 \quad ,\ \forall k \in \{1, ..., K\}. \tag{14c}$$

These constraints can be written as a bid $(\mathcal{F}_{\bar{s}}, \mathcal{C}_{\bar{s}})$, with

$$\mathcal{F}_{\bar{s}} = \left\{ \mathbf{q}_{\bar{s}} \;\middle|\; \exists\, \mathbf{x}_{\bar{s}} \in \mathbb{R}^N \text{ s.t. } (\mathbf{q}_{\bar{s}}, \mathbf{x}_{\bar{s}}) \text{ feasible to (14a)-(14c)} \right\},\ \mathcal{C}_{\bar{s}}(\mathbf{q}_{\bar{s}}) = 0.$$

We can equivalently write this as a price-region bid parametrized by $(\mathbf{A}_{\bar{s}}, \mathbf{B}_{\bar{s}})$ as

$$\mathbf{A}_{\bar{s}} \begin{bmatrix} 1 \\ \mathbf{q}_{\bar{s}} \\ \mathbf{x}_{\bar{s}} \end{bmatrix} \leq 0 \quad ,\ \mathbf{B}_{\bar{s}} = \mathbf{0}^{\top}_{N(K+1)}.$$

The entire transmission network which the market covers is modeled here as a single price-region bid, thus keeping state variables $\mathbf{x}_{\bar{s}}$ independent from other bids. It however does not imply that the network is managed by a single operator. An individual transmission system operator may be responsible for only a subset of constraints (14a)-(14c).

Once the market is cleared, the sum of money $\boldsymbol{\lambda}^{\star\top}\mathbf{q}^{\star}_{\bar{s}}$ will be attributed to the price-region bid s. In the transmission case, this is referred to as a *congestion rent*. Some market operators may allocate this rent to the transmission network operators, while others will use it to remunerate owners of financial transmission rights.

3.2.4. Application to an energy storage unit as a system asset

Consider an energy storage unit indexed by s, located in a price area N_s, managed as a system asset. Let P_s denote its charging/discharging capacity, in MW, and η_s its charging/discharging efficiency. Let E_s denote its energy capacity, and E^0_s its initial energy content, both in MWh. Let x^+_{sk} and x^-_{sk} respectively denote the energy charged into and discharged from the storage at time k, in MWh, and $\mathbf{x}_s$ a vector which contains variables x^+_{sk}, x^-_{sk} for all k. Let V_s denote the economic value of stored energy at the end of the horizon, in €/MWh. For each s, the operational constraints of the energy storage unit are given by

$$q_{sN_sk} = \eta_s x^-_{sk} - \frac{x^+_{sk}}{\eta_s} \quad ,\ \forall k \in \{1, ..., K\}, \tag{15a}$$

$$0 \leq x^+_{sk} \leq P_s \quad ,\ \forall k \in \{1, ..., K\}, \tag{15b}$$

$$0 \leq x^-_{sk} \leq P_s \quad ,\ \forall k \in \{1, ..., K\}, \tag{15c}$$

$$0 \leq E^0_s + \sum_{t=1}^{k} \left(x^+_{sk} - x^-_{sk} \right) \leq E_s \quad ,\ \forall k \in \{1, ..., K\}. \tag{15d}$$

These constraints can be represented as a bid $(\mathcal{F}_s, \mathcal{C}_s)$, with

$$\mathcal{F}_s = \left\{ \mathbf{q}_s \;\middle|\; \exists\, \mathbf{x}_s \in \mathbb{R}^N \text{ s.t. } (\mathbf{q}_s, \mathbf{x}_s) \text{ feasible to (15a)-(15d)} \right\},$$

$$\mathcal{C}_s(\mathbf{q}_s) = V_s \sum_{k=1}^{K} q_{snk}.$$

More specifically, given that (15a)-(15d) is a set of linear constraints on $\mathbf{q}_s$ and $\mathbf{x}_s$, and that $\mathcal{C}_s$ is a linear function of $\mathbf{q}_s$, there exists a pair $(\mathbf{A}_s, \mathbf{B}_s)$ which rewrites $(\mathcal{F}_s, \mathcal{C}_s)$ as a price-region bid by satisfying

$$\mathbf{A}_s \begin{bmatrix} 1 \\ \mathbf{q}_s \\ \mathbf{x}_s \end{bmatrix} \leq 0 \;\Leftrightarrow\; \text{(15a)-(15d)} \quad ,\ \mathbf{B}_s = \Big[\, 0 \;...\; 0 \;\; \underbrace{V_s \;...\; V_s}_{\substack{\text{columns} \\ (N_s-1)K+1 \text{ to } N_sK}} \;\; 0 \;...\; 0 \,\Big].$$

3.3. Electricity market clearing with price-region bids

We model a forward electricity market, where a subset of market participants $\Omega^{\text{PR}} \subset \{1, ..., S\}$ submits price-region bids, while other actors participate using other bid formats, such as price-quantity bids or block bids. Given Definition 6, the market-clearing problem (1a)-(1c) can be rewritten as (16a)-(16d). Intermediate steps are provided in Appendix A.

$$\min_{\mathbf{q},\mathbf{x}} \sum_{s\in\Omega^{\text{PR}}} \mathbf{B}_s \begin{bmatrix} \mathbf{q}_s \\ \mathbf{x}_s \end{bmatrix} + \sum_{s\notin\Omega^{\text{PR}}} \mathcal{C}_s(\mathbf{q}_s) \tag{16a}$$

$$\text{s.t. } \mathbf{A}_s \begin{bmatrix} 1 \\ \mathbf{q}_s \\ \mathbf{x}_s \end{bmatrix} \leq 0 \quad , \forall s \in \Omega^{\text{PR}}, \tag{16b}$$

$$\mathbf{q}_s \in \mathcal{F}_s \quad , \forall s \notin \Omega^{\text{PR}}, \tag{16c}$$

$$\sum_{s=1}^{S} q_{snk} = 0 \quad : \lambda_{nk} \quad , \forall n \in \{1, ..., N\},\ k \in \{1, ..., K\}. \tag{16d}$$

Objective function (16a) is the total operational cost of the system, incurred by the feasible dispatch $\mathbf{q}$. Vector $\mathbf{x}$ includes variables x_{sl} for all s and l. Constraints (16b) and (16c) ensure that the injection profiles $\mathbf{q}_s$ are feasible for all actors according to their individual feasible regions, respectively for price-region bids and other bid types. Constraints (16d) ensure that the dispatch is feasible from the system perspective. The dual multipliers λ_{nk} are used to derive uniform market prices. If $\mathbf{q}^\star$ and $\boldsymbol{\lambda}^\star$ are optimal for (16a)-(16d), the market operator pays actor s the amount $\boldsymbol{\lambda}^{\star\top}\mathbf{q}_s^\star$. Price-region bids introduce linear costs and constraints, and thus do not challenge the computational tractability of the problem. In order to prove desirable market properties, we introduce the following condition:

Condition 1. *All bids $s \notin \Omega^{\text{PR}}$ are special cases of price-region bids.*

Some traditional bid formats such as price-quantity bids, as well as linear models of system assets, are special cases of price-region bids, meaning that a range of existing markets readily meet Condition 1. See Remark 1 for the case of price-quantity bids. Note that, when Condition 1 is satisfied, the market-clearing problem (16a)-(16d) is linear.

Propositions 1-3 below establish that the properties described in Section 2.3 hold under this condition, with common assumptions for markets based on uniform pricing: each bid admits $\mathbf{q}_s = [0, ..., 0]^\top$ as a feasible injection profile, and the market is perfectly competitive (Wilson 1977, Hobbs et al. 2004, Bose and Low 2018). The proofs of these propositions are provided in Appendix B. Property 3 (Incentive compatibility) is trivially implied by the assumption of perfect competition.

Proposition 1. *The pair $(\mathbf{q}^\star, \boldsymbol{\lambda}^\star)$ from the optimal primal-dual solution of the market-clearing problem (16a)-(16d) satisfies Property 1 (Efficiency) if the market is perfectly competitive and if Condition 1 is satisfied.*

Proposition 2. *All actors participating in the market cleared by (16a)-(16d) recover their costs, i.e., Property 2 (Cost recovery) is satisfied, if each bid admits $\mathbf{q}_s = [0, ..., 0]^\top$ as a feasible injection profile and if Condition 1 is satisfied.*

Proposition 3. *The pair* $(\mathbf{q}^\star, \boldsymbol{\lambda}^\star)$ *from the optimal primal-dual solution of the market-clearing problem (16a)-(16d) guarantees budget balance for the market operator and thus satisfies Property 4 (Revenue adequacy).*[2]

Propositions 1-3 imply that price-region bids are compatible with forward electricity markets based on special cases of price-region bids, in the sense that these markets can incorporate price-region bids without disrupting existing practices.

Non-linear or mixed-integer mechanisms such as discrete block bids do not meet Condition 1. While these cases are not studied in this paper, price-region bids can in principle be used in markets which feature non-linear or mixed-integer bid formats. However, in this case the above theoretical conditions would not hold.

In the following section, we investigate how price-region bids can also be used to generalize financial rights, and we prove revenue adequacy for actors participating in both auctions.

3.4. Implementation as financial rights

Considering these non-conventional flexibility resources as electricity network assets can help provide stable long-term revenues for them. In addition, it facilitates their day-to-day participation in forward electricity markets, and provides an optimal dispatch of their flexibility in the power system. In that context, the proposed price-region bids can be used to construct rights like financial transmission rights (Hogan 1992, Chao and Peck 1996, Chao et al. 2000) and financial storage rights (Taylor 2015, Muñoz-Álvarez and Bitar 2017), with application to general flexible resources. Other actors such as loads, generators, and traders can purchase these rights in forward auctions. The owner of a right is entitled to a payment in the spot market that may depend on nodal prices or congestion. In this section, we use price-region bids to define financial flexibility rights, which generalize both point-to-point transmission rights (Hogan 1992) and profile-based storage rights (Muñoz-Álvarez and Bitar 2017).

Definition 7. A **financial flexibility right** r is defined by a profile $\tilde{\mathbf{q}}_r \in \mathbb{R}^{NK}$ for a number of locations N and time periods K. Once the electricity prices $\boldsymbol{\lambda}^{\star\top}$ for these locations and time periods are revealed, the right entitles its owner to the payment $-\boldsymbol{\lambda}^{\star\top}\tilde{\mathbf{q}}_r \in \mathbb{R}$.

We propose to clear an auction for financial flexibility rights using a program analogous to (16a)-(16d). A right r can be bought or sold in this auction by placing a bid $(\tilde{\mathcal{C}}_r, \tilde{\mathcal{F}}_r)$, such as a price-region bid $(\tilde{\mathbf{A}}_r, \tilde{\mathbf{B}}_r)$.

A point-to-point transmission right (Hogan 1992) is a special case of a flexibility right $\tilde{\mathbf{q}}_r \in \mathbb{R}^{NK}$, with $N = 2$ and $\tilde{q}_{rn_1k} = -\tilde{q}_{rn_2k}$ for all k. For a transmission system operator, placing a bid $(\tilde{\mathbf{A}}_r, \mathbf{0})$ in an auction for financial flexibility rights, with $\tilde{\mathbf{A}}_r$ representing transmission constraints, is equivalent to the way transmission system operators offer point-to-point transmission rights in existing auctions (Hogan 1992). A profile-based storage right (Muñoz-Álvarez and Bitar 2017) can be written as a right $\tilde{\mathbf{q}}_r \in \mathbb{R}^{NK}$ with $N = 1$ and $\sum_{k=1}^{K} \tilde{q}_{rn_1k} = 0$. For a storage owner, placing a bid $(\tilde{\mathbf{A}}_r, \mathbf{0})$ in an auction for financial flexibility rights, with $\tilde{\mathbf{A}}_r$ representing storage constraints, is

[2] In some markets based on uniform pricing, the transmission system operators are not remunerated directly from market prices, which implies a budget surplus for the market operator. This surplus is non-negative (Wu et al. 1996), and may instead be used to remunerate owners of financial transmission rights (Hogan 1992, Chao and Peck 1996, Chao et al. 2000). We discuss this in the context of price-region bids in Section 3.4.

equivalent to the way storage owners are suggested to offer profile-based storage rights in (Muñoz-Álvarez and Bitar 2017). In their general form, financial rights traded with price-region bids can further accommodate a range of flexible resources which feature spatial and temporal coupling, and which have non-zero cost functions.

In Hogan (1992) and Muñoz-Álvarez and Bitar (2017), financial transmission and storage rights are sold in an auction by a set of transmission and storage assets, which are treated as system assets. In these frameworks, system assets do not have variable costs, and their owners only receive revenues from the auction for rights. A system operator then collects the revenues from congestion rents associated with these system assets on the forward electricity market, and is in charge of paying the sums entitled to right owners after electricity prices are revealed. The authors in Hogan (1992) and Muñoz-Álvarez and Bitar (2017) show that revenue adequacy holds for the system operator if the rights are traded under the cover of the transmission and storage constraints. This condition is referred to as *simultaneous feasibility test* (Hogan 1992, Muñoz-Álvarez and Bitar 2017, Philpott and Pritchard 2004). The price-region-based auction for rights generalizes the simultaneous feasibility test to any flexible resource treated as system asset. For system assets without variable operational costs, Proposition 4 below confirms that a system operator with a similar responsibility as in Hogan (1992) and Muñoz-Álvarez and Bitar (2017) is also revenue adequate. Extending this result to assets with variable costs of operation is not straightforward, as there may be multiple approaches to the sharing of costs and revenues between the asset owner and the system operator. Instead, we formulate Proposition 5, which states that an independent asset owner participating in both the auction for rights and the forward electricity market with a price-region bid is certain to recover its costs if its bids on both auctions are equivalent. Propositions 4 and 5 are proven in Appendix B.

Proposition 4. *Let set Ω^{sys} represent a collection of assets without variable operational costs, treated as system assets and participating both in an auction for rights and a forward electricity market with their feasible regions of operation. Each auction is cleared by a program in the form of (16a)-(16d) where Condition 1 is satisfied. The pairs $(\tilde{\mathbf{q}}^\star, \tilde{\boldsymbol{\lambda}}^\star)$ and $(\mathbf{q}^\star, \boldsymbol{\lambda}^\star)$ are issued from the optimal solutions of the respective auctions. Let a system operator collect the surplus associated with system assets on the forward electricity market $(\sum_{s\in\Omega^{sys}} \boldsymbol{\lambda}^{\star\top}\mathbf{q}_s^\star)$, and be responsible for paying right owners the sums they are entitled to $(-\boldsymbol{\lambda}^{\star\top}\tilde{\mathbf{q}}_r^\star$ for each $r \notin \Omega^{sys})$. Then, the system operator is revenue adequate, i.e., the following relation holds:*

$$\sum_{s\in\Omega^{sys}} \boldsymbol{\lambda}^{\star\top}\mathbf{q}_s^\star \geq -\sum_{r\notin\Omega^{sys}} \boldsymbol{\lambda}^{\star\top}\tilde{\mathbf{q}}_r^\star. \tag{17}$$

Proposition 5. *Let an actor place a bid $(\tilde{\mathcal{F}}_r, \tilde{\mathcal{C}}_r)$ in an auction for rights, and a bid $(\mathcal{F}_s, \mathcal{C}_s)$ in a forward electricity market. Each auction is cleared by a program in the form of (16a)-(16d), and the pairs $(\tilde{\mathbf{q}}^\star, \tilde{\boldsymbol{\lambda}}^\star)$ and $(\mathbf{q}^\star, \boldsymbol{\lambda}^\star)$ are issued from the optimal solutions of the respective auctions. If $\tilde{\mathcal{F}}_r = \mathcal{F}_s$, $\tilde{\mathcal{C}}_r = \mathcal{C}_s$, $\mathbf{0} \in \mathcal{F}_s$, and Condition 1 is satisfied in both auctions, then the total economic surplus this actor derives from both auctions is non-negative, i.e., the following relation holds:*

$$\boldsymbol{\lambda}^{\star\top}\mathbf{q}_s^\star - \mathcal{C}_s(\mathbf{q}_s^\star) + \tilde{\mathbf{q}}_r^\star(\tilde{\boldsymbol{\lambda}}^{\star\top} - \boldsymbol{\lambda}^{\star\top}) \geq 0. \tag{18}$$

We only consider rights based on locational prices for simplicity. In principle, we could also define general rights based on shadow prices in the style of flowgate transmission rights (Chao and Peck 1996) and capacity-based storage rights (Taylor 2015). This could, however, entail a large

number of shadow prices. For example, the large number of constraints that describe a district heating network could lead to an impractically large number of shadow prices, which could be too complicated for practical implementation.

4. Numerical examples

In this section we illustrate the application and benefits of price-region bids with a case study.

4.1. Case study

This case study simulates the participation of a district heating utility in a day-ahead market under different conditions. Section 4.1.1 describes the integrated power and heat system under study. The different conditions for market participation are then described in Section 4.1.2. Finally, Section 4.1.3 contains our simulation results.

4.1.1. System description

We simulate the clearing of a day-ahead electricity market for a two-node power system, where each node constitutes a price area. The market horizon is divided in 24 hourly time periods. The power system is composed of a set of actors which participate in the day-ahead market (see Figure 10). A transmission line allows power to be transferred between the two nodes.

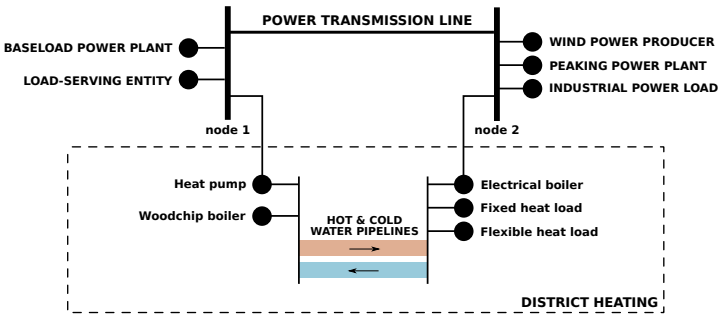


Figure 10: Overview of the integrated power and heat system.

A district heating system connected to both nodes is operated by a district heating utility which participates in the electricity market as a single entity. The two nodes of the district heating system are connected by a pair of water pipelines which allow the transfer of heat energy. The pipelines are operated with constant mass flows, which makes the resulting heat transport dynamics linear (Li et al. 2016). The physical and economic characteristics of the different actors in the market are all based on linear models. Mathematical formulations and numerical parameters are provided in Appendix C.

4.1.2. Conditions of market participation

Four cases are considered:

Case 1: As the least flexible case, the district heating utility bids an inflexible withdrawal profile in the electricity market. The profile is computed by the heating utility prior to market participation, and maximizes the expected economic surplus that can be derived from a set of electricity price forecast scenarios. Details on the generation of price scenarios are provided

in Appendix C. All other actors represent their characteristics using traditional bid formats. This case is used as a benchmark, and provides an upper bound for the total operational cost of the system.

Case 2: Following the current practice in electricity markets, the district heating utility submits price-quantity bids. Using the same set of price forecast scenarios as in Case 1, the heating utility finds an optimal withdrawal quantity for each price level, at each time period and node. For each time period and node, the price-quantity pairs are gathered into a piecewise linear demand curve. The bid quantities are constrained to ensure that the demand curve is non-increasing, which is a necessary condition for representing it with price-quantity bids. As in Case 1, all other actors represent their characteristics using traditional bid formats.

Case 3: As proposed in this paper, the district heating utility places a price-region bid in the electricity market, directly derived from its linear model of operation. This strategy does not rely on electricity price forecasts. In this case, all actors are able to represent exactly their characteristics in the market.

Case 4: As an idealized case, the electricity and heat sectors are co-optimized, based on linear models of the different assets. This case is used as an ideal benchmark, providing a lower bound for the total operational cost of the system. The information available to the central coordinator in this case is equivalent to the information available to the electricity market operator in Case 3.

4.1.3. Results

Price-region bids can more precisely represent flexible resources like district heating networks and cascaded hydro power plants in electricity markets. Our simulations show that this leads to improved utilization of flexible resources. Table 1 displays the total operational cost of the system in different cases, which includes: variable costs from electricity suppliers, variable costs from heat-only production in the district heating system, and costs of load curtailment. Load curtailment only occurs in the power system in Case 1, where the bid from the district heating utility is most inflexible. In Case 2, the district heating system provides some flexibility to the power system, reducing the operational cost of the system by 14%. In Case 3, the flexibility from the district heating system is used more efficiently, and the operational cost of the system is reduced by 35% as compared to Case 1. This suggests that allowing price-region bids in an electricity market with intermittent sources of renewable energy leads to increased social welfare, due to a better utilization of flexible resources. Case 3 performs as well as the ideal benchmark Case 4 as the information available to the electricity market operator in Case 3 is equivalent to the information available to the central coordinator in Case 4. This confirms that the flexibility from the heat sector is truly valued in the electricity market when the district heating utility participates with price-region bids.

Table 1: Comparison of the total operational cost of the system in four cases

Case	Cost (k€)	Cost reduction relative to Case 1 (%)
1	5.2	-
2	4.5	-14
3	3.4	-35
4	3.4	-35

Figures 11(a)-11(c) illustrate how the feasible region of the district heating utility described by price-quantity bids (areas delimited by red dashes in the figures) shrinks the full feasible region described by a price-region bid (areas delimited by a solid black line). Note that the feasible regions are defined in 48 dimensions. Figures 11(b) and 11(c) represent projections of the feasible regions onto pairs of hourly withdrawal quantities, while Figure 11(a) represents feasible regions for the aggregate withdrawal at each node. The region described by price-quantity bids is an inner approximation of the full feasible region. Figure 11(a) highlights how the full feasible region cannot be accurrately represented by a box region. It also shows that the solution from Case 3 (black star) is not feasible to the approximate region, and thus differs from the conservative solution in Case 2 (red circle). Figures 11(b) and 11(c) show how these solutions deviate substantially from each other when looking at the dispatched quantities in individual hours.

5. Conclusion and future perspectives

Existing bid formats in current forward electricity markets cannot accurately value certain types of flexible resources. To enable complicated flexible resources such as interconnected infrastructures to precisely represent their capabilities in electricity markets, we propose a new price-region bid format which allows the expression of any linearly-constrained feasible region of operation and convex piecewise linear cost function. As shown with two motivating examples based on district heating utility and cascaded hydropower plants, a price-region bid can be straightforwardly derived from a linear operations model including continuous state variables.

The price-region bid format generalizes existing bid formats such as price-quantity bids as well as linear models of system assets, and is shown to be compatible with markets that rely on these mechanisms. We show that market clearing with price-region bids can be carried out as a linear program, and that important desirable market properties hold under common assumptions.

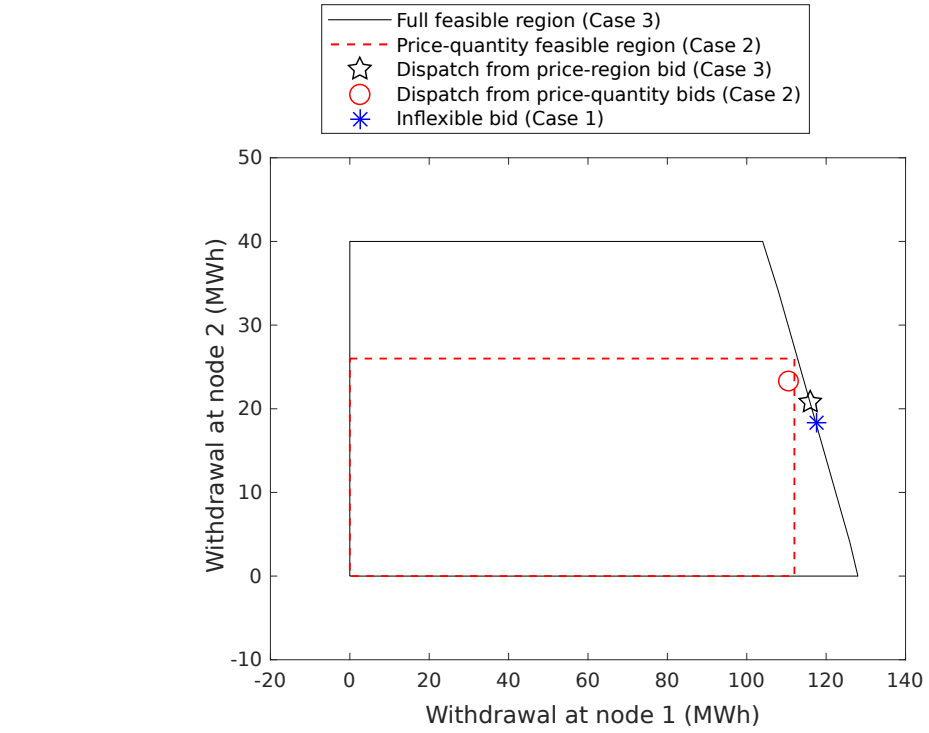
Price-region bids derived directly from operational models may feature a large number of state variables and constraints. Despite linearity, a market-clearing problem including bids with high dimensionality may be challenging to solve. Therefore, in practice, price-region bids may need to meet requirements fixed by the market operator, such as limits on the number of constraints and/or the number of state variables used in the bid. In those conditions, methods for price-region bid dimension reduction would be of interest, so that market participants with complex characteristics can derive optimal bids under dimension restrictions. Some state variables and constraints may be superfluous and could be omitted without losing information. But in cases where only few variables and constraints are allowed, the flexibility characteristics may need to be approximated, e.g., using inner approximation of the feasible region.

The price-region bid format proposed in this paper is a generalized linear bid. Extensions to mixed-integer linear or convex non-linear bid formats are also of interest. The formulation of mixed-integer linear price-region bids may not need to differ greatly from this paper's formulation. Allowing the use of discrete state variables would readily enable price-region bids to generalize mixed-integer bid formats, such as the discrete block bids in Nord Pool (2020) and O'Connell et al. (2016). However, non-convexity of a mixed-integer market-clearing problem poses challenges as uniform pricing cannot straightforwardly be applied (Scarf 1994, Gribik et al. 2007, Liberopoulos and Andrianesis 2016, Fuller and Celebi 2017).

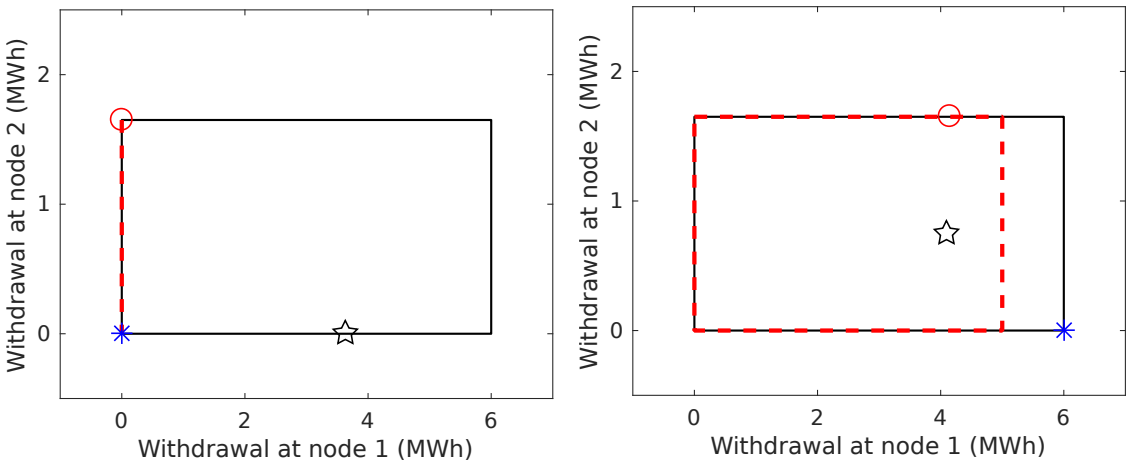
As our price-region bid format proposal is made to fit within the current electricity markets that clear the market in a deterministic manner, we have solely focused on highlighting its benefits in deterministic markets. There are currently a very few real-world electricity markets that use

a stochastic program for market clearing under uncertainty, e.g., the Swiss operational reserve market (Abbaspourtorbati and Zima 2016). It is worth noting that the proposed price-region bid format would still be compatible with stochastic market frameworks, as long as the convexity and linearity of the market-clearing problem are preserved. This could be the case if the potential uncertainty is modeled using, for example, scenario-based stochastic programming. An another example, chance-constrained programming can also model the uncertainty and be analytically reformulated in a linear way, but it is necessary to make some assumptions, e.g., related to properties of the probability distribution of the uncertainty (Nemirovski and Shapiro 2007). Additionally, future work should investigate the application of the proposed price-region bids to model different ancillary services, such as operating and contingency reserve products, which are currently cleared in European electricity markets in sequential and independent market trading floors (Dominguez et al. 2019). The proposed price-region bid format can provide a more accurate representation of the flexibility region of unconventional flexibility providers, increase their availability in ancillary services and energy markets, and reduce the need for contingency reserves. Following the hierarchical approach by Dvorkin et al. (2018), future work should also investigate how to set ancillary service requirements in the current sequential and deterministic market framework to approximate the robust dispatch proposed by Zugno and Conejo (2015).

Finally, it is of interest to explore whether the proposed price-region bid format affects the potential market power of sufficiently large participants and their strategic behavior. On the one hand, while the increased complexity of this bid format may result in transparency issues, it is also expected to increase the privacy of the market participants by making inference of their costs or parameters more complex, and therefore, reduce their ability to anticipate each other's behavior. Future work should investigate the impact of the proposed bid format on transparency and privacy of market participants by comparing the performances of different approaches for estimating supply and demand curves in electricity markets (Vázquez et al. 2014, Mitridati and Pinson 2017, Ruiz et al. 2013). On the other hand, these transparency issues may challenge market monitoring and allow the flexibility providers to exercise more market power. In this case, an additional study from regulatory perspective is required, aiming to investigate the potential market solutions to monitor strategic behavior and possibly mitigate market power.



(a) Total electricity withdrawal over 24 hours



(b) Electricity withdrawal in hour 20

(c) Electricity withdrawal in hour 21

Figure 11: District heating feasible region and optimal dispatch in Cases 1-3.

Appendix A. Reformulations

Appendix A.1. Intermediate steps for equation (5c)

Equation (4) can be written as

$$\begin{aligned} c^\star(\mathbf{q}_s) = &\min_{x_s}\Big\{x_s(C^w - \text{VOLL}) + (q_{sn_1k_1} + q_{sn_2k_2})\text{VOLL},\ \text{s.t. (3a)-(3c)}\Big\} \\ = &\min_{x_s}\Big\{-200x_s + 300(q_{sn_1k_1} + q_{sn_2k_2}),\ \text{s.t. (3a)-(3c)}\Big\} \\ = &-\max_{x_s}\Big\{300(-q_{sn_1k_1} - q_{sn_2k_2}) + 200x_s - 600,\ \text{s.t. (3a)-(3c)}\Big\}. \end{aligned}$$

Equation (4) also implies that

$$\begin{aligned} c^\star(\mathbf{0}_4) = &\min_{x_s}\Big\{x_s(C^w - \text{VOLL}),\ \text{s.t. } 0 \le x_s \le 3 \text{ and } x_s \le 5\Big\} \\ = &-600. \end{aligned}$$

It follows that

$$\begin{aligned} \text{WTP}(\mathbf{q}_s) = &\ c^\star(\mathbf{0}_4) - c^\star(\mathbf{q}_s) \\ = &\max_{x_s}\Big\{300(-q_{sn_1k_1} - q_{sn_2k_2}) + 200x_s - 600,\ \text{s.t. (3a)-(3c)}\Big\}. \end{aligned}$$

Appendix A.2. Price-region-based market-clearing reformulation

For a given pair $(\mathbf{A}_s, \mathbf{B}_s)$, let V_s^0 be a scalar defined as

$$V_s^0 = \min_{\mathbf{x}_s}\left\{\mathbf{B}_s\begin{bmatrix}\mathbf{0}\\ \mathbf{x}_s\end{bmatrix},\ \text{s.t. }\mathbf{A}_s\begin{bmatrix}1\\ \mathbf{0}\\ \mathbf{x}_s\end{bmatrix} \le 0\right\}.$$

It follows that the cost function expressed in (10b) may be rewritten as

$$\mathcal{C}_s(\mathbf{q}_s) = \min_{\mathbf{x}_s}\left\{\mathbf{B}_s\begin{bmatrix}\mathbf{q}_s\\ \mathbf{x}_s\end{bmatrix},\ \text{s.t. }\mathbf{A}_s\begin{bmatrix}1\\ \mathbf{q}_s\\ \mathbf{x}_s\end{bmatrix} \le 0\right\} - V_s^0. \tag{A.1}$$

Let a market receive a set of price-region bids from actors $\Omega^{\text{PR}} \subset \{1, ..., S\}$. Note that

$$\begin{aligned} \sum_{s\in\Omega^{\text{PR}}}\mathcal{C}_s(\mathbf{q}_s) \overset{\text{(A.1)}}{=} &\sum_{s\in\Omega^{\text{PR}}}\min_{\mathbf{x}_s}\left\{\mathbf{B}_s\begin{bmatrix}\mathbf{q}_s\\ \mathbf{x}_s\end{bmatrix},\ \text{s.t. }\mathbf{A}_s\begin{bmatrix}1\\ \mathbf{q}_s\\ \mathbf{x}_s\end{bmatrix} \le 0\right\} - \sum_{s\in\Omega^{\text{PR}}}V_s^0 \\ = &\min_{\mathbf{x}}\left\{\sum_{s\in\Omega^{\text{PR}}}\mathbf{B}_s\begin{bmatrix}\mathbf{q}_s\\ \mathbf{x}_s\end{bmatrix},\ \text{s.t. (16b)}\right\} - \sum_{s\in\Omega^{\text{PR}}}V_s^0. \end{aligned}$$

It then follows that (1a)-(1c) can be written as

$$
\begin{aligned}
& \min_{\mathbf{q}} \left\{ \sum_{s=1}^{S} \mathcal{C}_s(\mathbf{q}_s),\ \text{s.t. (1b)-(1c)} \right\} \\
\equiv\ & \min_{\mathbf{q}} \left\{ \sum_{s\in\Omega^{\mathrm{PR}}} \mathcal{C}_s(\mathbf{q}_s) + \sum_{s\notin\Omega^{\mathrm{PR}}} \mathcal{C}_s(\mathbf{q}_s),\ \text{s.t.} \left\{ \mathbf{q}_s \in \mathcal{F}_s \quad ,\ \forall s \in \Omega^{\mathrm{PR}}, \text{(16c)} - \text{(16d)} \right\} \right\} \\
\equiv\ & \min_{\mathbf{q}} \left\{ \min_{\mathbf{x}} \left\{ \sum_{s\in\Omega^{\mathrm{PR}}} \mathbf{B}_s \begin{bmatrix} \mathbf{q}_s \\ \mathbf{x}_s \end{bmatrix},\ \text{s.t. (16b)} \right\} + \sum_{s\notin\Omega^{\mathrm{PR}}} \mathcal{C}_s(\mathbf{q}_s), \right. \\
& \qquad \left. \text{s.t.} \left\{ \exists\ \mathbf{x} \text{ feasible to (16b)} \right\}, \text{(16c)-(16d)} \right\} \\
\equiv\ & \min_{\mathbf{q},\mathbf{x}} \left\{ \sum_{s\in\Omega^{\mathrm{PR}}} \mathbf{B}_s \begin{bmatrix} \mathbf{q}_s \\ \mathbf{x}_s \end{bmatrix} + \sum_{s\notin\Omega^{\mathrm{PR}}} \mathcal{C}_s(\mathbf{q}_s),\ \text{s.t. (16b)-(16d)} \right\} \\
\equiv\ & \text{(16a)-(16d)}.
\end{aligned}
$$

Appendix B. Proofs

Appendix B.1. Proof of Proposition 1 – part I

A profit-maximizing actor participates in the market so as to maximize its *economic surplus*. For a given dispatch $\mathbf{q}$ and prices $\boldsymbol{\lambda}$, let $\mathrm{ES}_s(\mathbf{q}_s, \boldsymbol{\lambda})$ denote the economic surplus of the actor placing bid s, i.e., the difference between its income from market prices $\boldsymbol{\lambda}$ and the cost associated with the profile $\mathbf{q}_s$. It is written as

$$\mathrm{ES}_s(\mathbf{q}_s, \boldsymbol{\lambda}) = \boldsymbol{\lambda}^\top \mathbf{q}_s - \mathcal{C}_s(\mathbf{q}_s). \tag{B.1}$$

Let $\mathrm{SW}(\mathbf{q}, \boldsymbol{\lambda})$ denote the sum of individual economic surpluses as defined in (B.1), i.e.,

$$\mathrm{SW}(\mathbf{q}, \boldsymbol{\lambda}) = \sum_{s=1}^{S} \left(\boldsymbol{\lambda}^\top \mathbf{q}_s - \mathcal{C}_s(\mathbf{q}_s) \right).$$

If $(\mathbf{q}, \boldsymbol{\lambda})$ is a feasible solution to (16a)-(16d), maximizing $\mathrm{SW}(\mathbf{q}, \boldsymbol{\lambda})$ is equivalent to minimizing the objective function of (16a)-(16d), owing to the following relation:

$$\mathrm{SW}(\mathbf{q}, \boldsymbol{\lambda}) = \sum_{s=1}^{S} \sum_{n=1}^{N} \sum_{k=1}^{K} \left(\lambda_{nk} \sum_{s=1}^{S} q_{snk} \right) - \sum_{s=1}^{S} \mathcal{C}_s(\mathbf{q}_s) \overset{\text{(16d)}}{=} - \sum_{s=1}^{S} \mathcal{C}_s(\mathbf{q}_s).$$

This proves the first item of Property 1, i.e., the optimal solution to (16a)-(16d) maximizes the social welfare. ■

Appendix B.2. Proof of Proposition 1 – part II

If all bids $s \notin \Omega^{\mathrm{PR}}$ can be written as special cases of price-region bids, i.e., for each $s \notin \Omega^{\mathrm{PR}}$ there exists a price-region bid $(\mathbf{A}_s, \mathbf{B}_s)$ which perfectly represents $(\mathcal{F}_s, \mathcal{C}_s)$, then the problem (16a)-(16d)

can be written as

$$\min_{\mathbf{q},\mathbf{x}} \sum_{s=1}^{S} \mathbf{B}_s \begin{bmatrix} \mathbf{q}_s \\ \mathbf{x}_s \end{bmatrix} \tag{B.2a}$$

$$\text{s.t. } \mathbf{A}_s \begin{bmatrix} 1 \\ \mathbf{q}_s \\ \mathbf{x}_s \end{bmatrix} \leq 0 \quad , \forall s = 1, ..., S, \tag{B.2b}$$

$$\sum_{s=1}^{S} (q_{snk}) = 0 \quad : \lambda_{nk} \quad , \forall n = 1, ..., N, k = 1, ..., K. \tag{B.2c}$$

In the following, we show that the Karush-Kuhn-Tucker (KKT) conditions for optimality of the market-clearing problem (B.2a)-(B.2c) match the KKT conditions for the individual price-taker profit-maximizing problems.

For all s, let $\boldsymbol{\mu}_s$ be the Lagrange multipliers of constraints (B.2b). Let $A_{s,j,i}$ denote the element in the j^{th} row and i^{th} column of the matrix $\mathbf{A}_s$, and $B_{s,p}$ the p^{th} element of $\mathbf{B}_s$. The optimality condition of (B.2a)-(B.2c) are

$$B_{s,(K(n-1)+k)} + \sum_{j=1}^{J_s} (\mu_{s,j} A_{s,j,(K(n-1)+k+1))}) - \lambda_{nk}$$
$$= 0 \quad , \forall\ s = 1, ..., S, n = 1, ..., N, k = 1, ..., K, \tag{B.3a}$$

$$B_{s,(KN+l)} + \sum_{j=1}^{J_s} (\mu_{s,j} A_{s,j,(KN+l+1)}) = 0 \quad , \forall s = 1, ..., S,\ l = 1, ..., L, \tag{B.3b}$$

$$\boldsymbol{\mu}_s \geq 0 \quad , \forall s = 1, ..., S, \tag{B.3c}$$

$$\boldsymbol{\mu}_s \circ \mathbf{A}_s \begin{bmatrix} 1 \\ \mathbf{q}_s \\ \mathbf{x}_s \end{bmatrix} = 0 \quad , \forall s = 1, ..., S. \tag{B.3d}$$

Now let $\boldsymbol{\lambda}^\star$ denote the optimal value of $\boldsymbol{\lambda}$ in (B.2a)-(B.2c), i.e., the vector of market-clearing prices. The below optimization maximizes the economic surplus of actor s given prices $\boldsymbol{\lambda}^\star$:

$$\max_{\mathbf{q}_s} \left\{ \text{ES}_s(\mathbf{q}_s, \boldsymbol{\lambda}^\star),\ \text{s.t. } \mathbf{q}_s \in \mathcal{F}_s \right\}. \tag{B.4}$$

The problem may be rewritten as

$$(\text{B.4}) \overset{(\text{B.1})}{\equiv} \max_{\mathbf{q}_s} \left\{ \boldsymbol{\lambda}^{\star\top} \mathbf{q}_s - \mathcal{C}_s(\mathbf{q}_s), \text{ s.t. } \mathbf{q}_s \in \mathcal{F}_s \right\} \tag{B.5a}$$

$$\equiv \min_{\mathbf{q}_s} \left\{ \mathcal{C}_s(\mathbf{q}_s) - \boldsymbol{\lambda}^{\star\top} \mathbf{q}_s, \text{ s.t. } \mathbf{q}_s \in \mathcal{F}_s \right\} \tag{B.5b}$$

$$\overset{(10b)}{\equiv} \min_{\mathbf{q}_s} \left\{ \min_{\mathbf{x}_s} \left\{ \mathbf{B}_s \begin{bmatrix} \mathbf{q}_s \\ \mathbf{x}_s \end{bmatrix}, \text{ s.t. } \mathbf{A}_s \begin{bmatrix} 1 \\ \mathbf{q}_s \\ \mathbf{x}_s \end{bmatrix} \leq 0 \right\} - \boldsymbol{\lambda}^{\star\top} \mathbf{q}_s \text{ s.t. } \mathbf{q}_s \in \mathcal{F}_s^{\text{bid}} \right\} \tag{B.5c}$$

$$\overset{(10a)}{\equiv} \min_{\mathbf{q}_s, \mathbf{x}_s} \left\{ \mathbf{B}_s \begin{bmatrix} \mathbf{q}_s \\ \mathbf{x}_s \end{bmatrix} - \boldsymbol{\lambda}^{\star\top} \mathbf{q}_s \text{ s.t. } \mathbf{A}_s \begin{bmatrix} 1 \\ \mathbf{q}_s \\ \mathbf{x}_s \end{bmatrix} \leq 0 \right\}. \tag{B.5d}$$

Let $\boldsymbol{\mu}_s$ denote the Lagrange multipliers of constraint (B.5d). The KKT conditions for optimality of (B.5d) write as

$$B_{(K(n-1)+k)} + \sum_{j=1}^{J} (\mu_{s,j} A_{s,j,(K(n-1)+k+1))}) - \lambda^{\star}_{nk} = 0 \quad , \forall\, n = 1, ..., N,\ k = 1, ..., K, \tag{B.5e}$$

$$B_{(KN+l)} + \sum_{j=1}^{J} (\mu_{s,j} A_{s,j,(KN+l+1)}) = 0 \quad , \forall\, l = 1, ..., L, \tag{B.5f}$$

$$\boldsymbol{\mu}_s \geq 0, \tag{B.5g}$$

$$\boldsymbol{\mu}_s \circ \mathbf{A}_s \begin{bmatrix} 1 \\ \mathbf{q}_s \\ \mathbf{x}_s \end{bmatrix} = 0. \tag{B.5h}$$

The KKT conditions {(B.2b), (B.3a)-(B.3d)} for the market-clearing problem are equivalent to the collection of KKT conditions (B.5d)-(B.5h) for the individual price-taker profit-maximizing problems. This implies that no price-taker market participant has incentives to deviate unilaterally from the optimal dispatch $\mathbf{q}^{\star}$, which proves the second item of Property 1. ■

Appendix B.3. Proof of Proposition 2

Let $(\mathbf{q}^{\star}, \boldsymbol{\lambda}^{\star})$ denote an optimal solution to the market-clearing problem (B.2a)-(B.2c). The equivalence of the KKT conditions {(B.2b), (B.3a)-(B.3d)} and (B.5d)-(B.5h) (see Appendix B.2) implies that, under Condition 1:

$$\text{ES}_s(\mathbf{q}^{\star}_s, \boldsymbol{\lambda}^{\star}) \geq \text{ES}_s(\mathbf{q}_s, \boldsymbol{\lambda}^{\star}) \quad , \forall \mathbf{q}_s \in \mathcal{F}_s,\ \forall s \in \{1, ..., S\}. \tag{B.6}$$

If all bidders allow an empty injection profile as a feasible solution, i.e., $\mathbf{0} \in \mathcal{F}_s\ \forall s$, then equation (B.6) implies, for all $s \in \{1, ..., S\}$,

$$\text{ES}_s(\mathbf{q}^{\star}_s, \boldsymbol{\lambda}^{\star}) \geq \text{ES}_s(\mathbf{0}, \boldsymbol{\lambda}^{\star})$$

$$\overset{(\text{B.1})}{\Leftrightarrow} \text{ES}_s(\mathbf{q}^{\star}_s, \boldsymbol{\lambda}^{\star}) \geq \boldsymbol{\lambda}^{\star\top} \mathbf{0} - \mathcal{C}_s(\mathbf{0}).$$

Following Condition 1, there exists, for all s, a vector $\mathbf{B}_s$ so that $\mathcal{C}_s(\mathbf{q}_s) = \mathbf{B}_s\mathbf{q}_s,\ \forall \mathbf{q}_s \in \mathcal{F}_s$. It follows that $\mathcal{C}_s(\mathbf{0}) = 0$ and

$$\mathrm{ES}_s(\mathbf{q}_s^\star, \boldsymbol{\lambda}^\star) \geq 0.$$

This verifies Property 2, i.e., the market participants recover the costs announced in their bids. ■

Appendix B.4. Proof of Proposition 3

Let $(\mathbf{q}^\star, \boldsymbol{\lambda}^\star)$ denote an optimal solution to the market-clearing problem (16a)-(16d). The net income of the market operator from the transactions prescribed by $(\mathbf{q}^\star, \boldsymbol{\lambda}^\star)$ is $\sum_{s=1}^{S} \boldsymbol{\lambda}^{\star\top}\mathbf{q}_s^\star$ and, given (16d), is always equal to zero. It follows that the market operator is budget-balanced and thus revenue adequate, i.e., Property 4 is satisfied. ■

Appendix B.5. Proof of Proposition 4

The system operator is responsible to pay the sum $\sum_{r\notin\Omega^{\mathrm{sys}}}(-\boldsymbol{\lambda}^{\star\top}\tilde{\mathbf{q}}_r^\star)$ to right owners. Given the balance constraints (16d) in the auction-clearing problem, this sum is equal to $\sum_{r\in\Omega^{\mathrm{sys}}} \boldsymbol{\lambda}^{\star\top}\tilde{\mathbf{q}}_r^\star$. The maximum value this amount can take is

$$\max_{\tilde{\mathbf{q}}_r,\ r\in\Omega^{\mathrm{sys}}} \left\{ \sum_{r\in\Omega^{\mathrm{sys}}} \boldsymbol{\lambda}^{\star\top}\tilde{\mathbf{q}}_r,\ \text{s.t. } \tilde{\mathbf{q}}_r \in \tilde{\mathcal{F}}_r,\ \forall r \in \Omega^{\mathrm{sys}} \right\}, \tag{B.7}$$

where $\tilde{\mathcal{F}}_r$ is the feasible region of system asset r declared in the auction for right. We assume the same feasible region is declared in the forward electricity market, i.e., $\mathcal{F}_s = \tilde{\mathcal{F}}_r$ for all $s = r \in \Omega^{\mathrm{sys}}$, and rewrite (B.7) as

$$\max_{\mathbf{q}_s,\ s\in\Omega^{\mathrm{sys}}} \left\{ \sum_{s\in\Omega^{\mathrm{sys}}} \boldsymbol{\lambda}^{\star\top}\mathbf{q}_s,\ \text{s.t. } \mathbf{q}_s \in \mathcal{F}_s,\ \forall s \in \Omega^{\mathrm{sys}} \right\} \tag{B.8}$$

$$= \sum_{s\in\Omega^{\mathrm{sys}}} \max_{\mathbf{q}_s} \left\{ \boldsymbol{\lambda}^{\star\top}\mathbf{q}_s,\ \text{s.t. } \mathbf{q}_s \in \mathcal{F}_s \right\}. \tag{B.9}$$

Given $\mathcal{C}_s = 0$ for all $s \in \Omega^{\mathrm{sys}}$, we rewrite (B.9) as

$$\sum_{s\in\Omega^{\mathrm{sys}}} \max_{\mathbf{q}_s} \left\{ \mathrm{ES}_s(\mathbf{q}_s, \boldsymbol{\lambda}^\star),\ \text{s.t. } \mathbf{q}_s \in \mathcal{F}_s \right\}. \tag{B.10}$$

We show, in Appendix B.2, that $\max_{\mathbf{q}_s} \{\mathrm{ES}_s(\mathbf{q}_s, \boldsymbol{\lambda}^\star),\ \text{s.t. } \mathbf{q}_s \in \mathcal{F}_s\} = \mathrm{ES}_s(\mathbf{q}_s^\star, \boldsymbol{\lambda}^\star)$. It follows that expression (B.10), and thus the sum the system operator is responsible to pay to right owners, is bounded by $\sum_{s\in\Omega^{\mathrm{sys}}} \mathbf{q}_s^\star$, i.e.,

$$\sum_{s\in\Omega^{\mathrm{sys}}} \boldsymbol{\lambda}^{\star\top}\mathbf{q}_s^\star \geq -\sum_{r\notin\Omega^{\mathrm{sys}}} \boldsymbol{\lambda}^{\star\top}\tilde{\mathbf{q}}_r^\star. \quad ■$$

Appendix B.6. Proof of Proposition 5

Let an auction for rights be cleared by a program analogous to (16a)-(16d) and satisfy Condition 1. The rights allocated in the auction write $\tilde{\mathbf{q}}^\star$ (analogous to $\mathbf{q}^\star$ in (16a)-(16d)), and the prices at which rights are traded write $\tilde{\boldsymbol{\lambda}}^\star$ (analogous to $\boldsymbol{\lambda}^\star$ in (16a)-(16d)). Let an actor place a bid $(\tilde{\mathcal{F}}_r, \tilde{\mathcal{C}}_r)$ in this auction. Now let a forward electricity market be cleared by (16a)-(16d) and satisfy

Condition 1. The optimal dispatch of this market is $\mathbf{q}^\star$ and the corresponding electricity prices $\boldsymbol{\lambda}^\star$. Let the same actor as above place a price-region bid $(\mathcal{F}_s, \mathcal{C}_s)$ in this market. As argued in Appendix B.3, the following relation holds:

$$\begin{aligned} & \mathrm{ES}_s(\mathbf{q}_s^\star, \boldsymbol{\lambda}^\star) \geq \mathrm{ES}_s(\mathbf{q}_s, \boldsymbol{\lambda}^\star) \quad , \forall \mathbf{q}_s \in \mathcal{F}_s, && \text{(B.11)} \\ \Leftrightarrow \quad & \boldsymbol{\lambda}^{\star\top}\mathbf{q}_s^\star - \mathcal{C}_s(\mathbf{q}_s^\star) \geq \boldsymbol{\lambda}^{\star\top}\mathbf{q}_s - \mathcal{C}_s(\mathbf{q}_s) \quad , \forall \mathbf{q}_s \in \mathcal{F}_s. && \text{(B.12)} \end{aligned}$$

If $\tilde{\mathcal{F}}_r = \mathcal{F}_s$, then $\tilde{\mathbf{q}}_r \in \mathcal{F}_s$ for all $\tilde{\mathbf{q}}_r \in \tilde{\mathcal{F}}_r$, including for $\tilde{\mathbf{q}}_r = \tilde{\mathbf{q}}_r^\star$. It follows that

$$\begin{aligned} & \boldsymbol{\lambda}^{\star\top}\mathbf{q}_s^\star - \mathcal{C}_s(\mathbf{q}_s^\star) \geq \boldsymbol{\lambda}^{\star\top}\tilde{\mathbf{q}}_r^\star - \mathcal{C}_s(\tilde{\mathbf{q}}_r^\star), && \text{(B.13)} \\ \Leftrightarrow \quad & \boldsymbol{\lambda}^{\star\top}\mathbf{q}_s^\star - \mathcal{C}_s(\mathbf{q}_s^\star) - \boldsymbol{\lambda}^{\star\top}\tilde{\mathbf{q}}_r^\star \geq -\mathcal{C}_s(\tilde{\mathbf{q}}_r^\star), && \text{(B.14)} \\ \Leftrightarrow \quad & \boldsymbol{\lambda}^{\star\top}\mathbf{q}_s^\star - \mathcal{C}_s(\mathbf{q}_s^\star) + \tilde{\mathbf{q}}_r^\star(\tilde{\boldsymbol{\lambda}}^{\star\top} - \boldsymbol{\lambda}^{\star\top}) \geq \tilde{\mathbf{q}}_r^\star\tilde{\boldsymbol{\lambda}}^{\star\top} - \mathcal{C}_s(\mathbf{q}_r^\star). && \text{(B.15)} \end{aligned}$$

Provided that $\mathbf{0} \in \tilde{\mathcal{F}}_r$, and according to Proposition 2 (see Appendix B.3), the auction for rights satisfies cost recovery for all participants, i.e.,

$$\tilde{\boldsymbol{\lambda}}^{\star\top}\tilde{\mathbf{q}}_r^\star - \tilde{\mathcal{C}}_r(\mathbf{q}_r^\star) \geq 0. \tag{B.16}$$

Using $\tilde{\mathcal{C}}_r = \mathcal{C}_s$, equations (B.15) and (B.16) imply

$$\boldsymbol{\lambda}^{\star\top}\mathbf{q}_s^\star - \mathcal{C}_s(\mathbf{q}_s^\star) + \tilde{\mathbf{q}}_r^\star(\tilde{\boldsymbol{\lambda}}^{\star\top} - \boldsymbol{\lambda}^{\star\top}) \geq 0. \quad \blacksquare \tag{B.17}$$

Appendix C. Case study models

Appendix C.1. District heating utility model

This section characterizes the physical characteristics of the district heating utility, shown in Figure 10, as a set of linear constraints. It then describes its bidding strategy in the day-ahead market.

Let n_1, n_2 denote the two price areas in the power system, and s the index of the bid from the district heating utility. Let K denote the number of market time periods in the bidding horizon, and $\tau = 1h$ the duration of each period. Let h_k^{wc}, h_k^{el} and h_k^{hp} denote the rate of heat production in time period k for the woodchip boiler, the electrical boiler and the heat pump, respectively. Their unit is GJ/h. Let $H^{\mathrm{wc}} = 37.5$ GJ/h, $H^{\mathrm{el}} = 5$ GJ/h and $H^{\mathrm{hp}} = 30$ GJ/h denote the maximum heat production level of these units. Let $\alpha^{\mathrm{el}} = 3$ GJ/MWh and $\alpha^{\mathrm{hp}} = 5$ GJ/MWh denote the power-to-heat conversion ratio of the electrical boiler and the heat pump, respectively. Equations (C.1a)-(C.1b) relate the withdrawal of electrical energy from n_1 and n_2 to the heat production by the heat pump and the electrical boiler. Equations (C.1c)-(C.1e) set bounds on the heat production of units, for all $k \in \{1...K\}$:

$$h_k^{\mathrm{el}} = -\alpha^{\mathrm{hp}} q_{sn_1k} \tag{C.1a}$$

$$h_k^{\mathrm{el}} = -\alpha^{\mathrm{el}} q_{sn_2k} \tag{C.1b}$$

$$0 \leq h_k^{\mathrm{hp}} \leq H^{\mathrm{hp}} \tag{C.1c}$$

$$0 \leq h_k^{\mathrm{el}} \leq H^{\mathrm{el}} \tag{C.1d}$$

$$0 \leq h_k^{\mathrm{wc}} \leq H^{\mathrm{wc}}. \tag{C.1e}$$

Let D_k^{fix} denote the heat demand rate of the inflexible load in time period k, in GJ/h. The inflexible load follows the demand profile displayed in Figure C.12. Let d_k^{flex} denote the rate at which heat is supplied to the flexible load in time period k, in GJ/h. Let $E^{\text{flex}} = 150$ GJ denote the total energy the flexible load requires over the horizon, and $R^{\text{flex}} = 10$ GJ/h the maximum rate at which it can be supplied. Equations (C.1f)-(C.1g) describe the flexible load constraints:

$$\sum_{k=1}^{K} \tau d_k^{\text{flex}} = E^{\text{flex}}, \tag{C.1f}$$

$$0 \leq d_k^{\text{flex}} \leq R^{\text{flex}} \quad , \ \forall\ k \in \{1...K\}. \tag{C.1g}$$

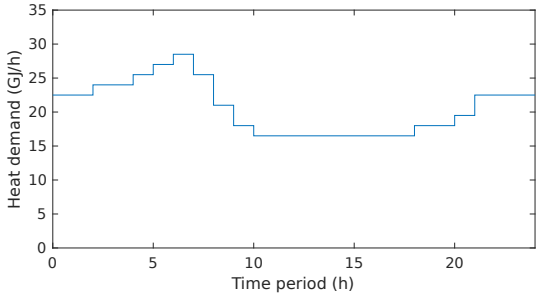


Figure C.12: Demand profile of inflexible heat load.

The hot water pipelines are operated at constant mass flows. Figure C.13 provides a schematic representation of the supply-and-return pipelines system. Energy is injected from the left-hand side by increasing the temperature of the water flowing from the return pipe outlet to the supply pipe inlet at a fixed flow rate. Energy is withdrawn from the right-hand side by cooling down the water flowing from the supply pipe outlet to the return pipe inlet at the same fixed flow rate. In this case study, it is assumed that it takes exactly one time period for the water mass to travel from one end to the other end of a pipe. This means that, in a given time period, the temperature of the water at the outlet of a pipe is equal to the temperature at the inlet of the pipe in the previous time period. Let t_k^{sup} denote the water temperature in the inlet of the supply pipe at time period k, and t_k^{ret} that of the return pipe, in °C. The initial temperatures are set to $t_0^{\text{sup}} = 70\,°\text{C}$, $t_0^{\text{ret}} = 50\,°\text{C}$, and temperatures at the end of the horizon are enforced to reach the same value (see equations (C.1h)-(C.1i)) so that operations in the next day are not compromised. Equations (C.1j) enforce lower and upper bounds for temperatures, respectively $T^{\min} = 30\,°\text{C}$ and $T^{\max} = 90\,°\text{C}$, for all $k \in \{1...K\}$. Equations (C.1k) enforce that the temperatures in the return pipe are never higher than in the supply pipe, for all $k \in \{1...K\}$:

$$t_K^{\text{sup}} = t_0^{\text{sup}}, \tag{C.1h}$$

$$t_K^{\text{ret}} = t_0^{\text{ret}}, \tag{C.1i}$$

$$T^{\min} \leq t_k^{\text{ret}}, \quad t_k^{\text{sup}} \leq T^{\max}, \tag{C.1j}$$

$$t_{k-1}^{\text{ret}} \leq t_k^{\text{sup}}, \quad t_k^{\text{ret}} \leq t_{k-1}^{\text{sup}}. \tag{C.1k}$$

Let $\alpha^{\text{pipe}} = 1.5$ GJ/°C denote the specific energy of the mass of water that travels the pipelines in one time period. The energy injected at area n_1 into the pipelines in time period k is $\alpha^{\text{pipe}}(t_k^{\text{sup}} -$

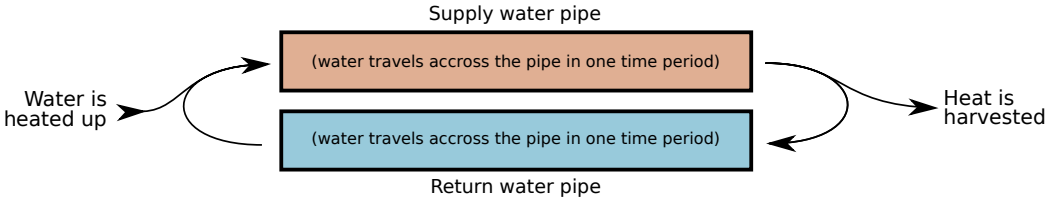


Figure C.13: Supply and return district heating pipelines.

t^{ret}_{k-1}). The energy withdrawn from the pipelines in area n_2 in a time period k is $\alpha^{\text{pipe}}(t^{\text{sup}}_{k-1} - t^{\text{ret}}_{k})$. Equations (C.1l)-(C.1m) respectively describe the conservation of energy in areas n_1 and n_2, for all $k \in \{1...K\}$:

$$\tau(h^{\text{wc}}_k + h^{hp}_k) = \alpha^{\text{pipe}}(t^{\text{sup}}_k - t^{\text{ret}}_{k-1}), \tag{C.1l}$$

$$\tau h^{\text{el}}_k + \alpha^{\text{pipe}}(t^{\text{sup}}_{k-1} - t^{\text{ret}}_k) = \tau(D^{\text{fix}}_k + d^{\text{flex}}_k). \tag{C.1m}$$

Let $\mathbf{q}_s$ denote the injection profile of the heating utility, and let a vector $\mathbf{x}_s$ include all variables $\{h^{\text{wc}}_k, h^{\text{el}}_k, h^{\text{hp}}_k, d^{\text{flex}}_k, t^{\text{sup}}_k, t^{\text{ret}}_k, k = 1...K\}$. The feasible region of electricity injection $\mathcal{F}_s$ is given by

$$\mathcal{F}_s = \left\{ \mathbf{q}_s \;\middle|\; \exists\, \mathbf{x}_s \in \mathbb{R}^{L_s} \text{ s.t. } (\mathbf{q}_s, \mathbf{x}_s) \text{ satisfies (C.1a)-(C.1m)} \right\}.$$

The district heating system in this case study has only one variable cost component: the fuel cost of the woodchip boiler, $C^{\text{wc}} = 40$ €/GJ. The cost function of the district heating utility is thus

$$\begin{aligned}\mathcal{C}_s(\mathbf{q}_s) = &\min_{\mathbf{x}_s} \left\{ \sum_{k=1}^{K} (C^{\text{wc}} h^{\text{wc}}_k), \text{ s.t. (C.1a)-(C.1m)} \right\} \\ &- \min_{\mathbf{x}_s} \left\{ \sum_{k=1}^{K} (C^{\text{wc}} h^{\text{wc}}_k), \text{ s.t. } (\mathbf{0}, \mathbf{x}_s) \text{ satisfies (C.1a)-(C.1m)} \right\}.\end{aligned}$$

Appendix C.2. Characteristics and bids of other market participants

This section describes the other actors in the case study and their participation on the day-ahead market. The transmission line is a system asset, modeled by the market operator based on physical characteristics communicated by the transmission system operator. It is modeled as a lossless transmission line, with a capacity of 5 MW.

The wind power producer has a variable electricity supply capacity in a range between 0 and 25 MWh/h. The available power is assumed to be well-known by the wind power producer a day ahead of delivery. It offers energy in the market by placing price-quantity bids with bid prices set to zero.

The load-serving entity has a variable electricity demand between 5 and 15 MWh/h. The demand profile is assumed to be well-known by the load-serving entity a day ahead of delivery. It bids for energy in the market by placing price-quantity bids with bid prices set to a value of lost load of 1000 €/MWh. This is a relatively high value so that the demand can be met even when electricity supply is scarce.

The peaking power plant has a flexible power output. Its variable cost of operation is positive and increases as the power output increases, following a piecewise linear function. This actor offers energy in the market by placing price-quantity bids.

The baseload power plant has a constant, predictable supply capacity of 10 MWh/h with no variable costs of operation. It offers this energy in the market by placing price-quantity bids with bid prices set to zero.

The industrial load has a predictable inflexible demand for electrical energy following the profile from Figure C.14. It bids for this energy in the market by placing price-quantity bids with bid prices set to a value of lost load of 1000 €/MWh.

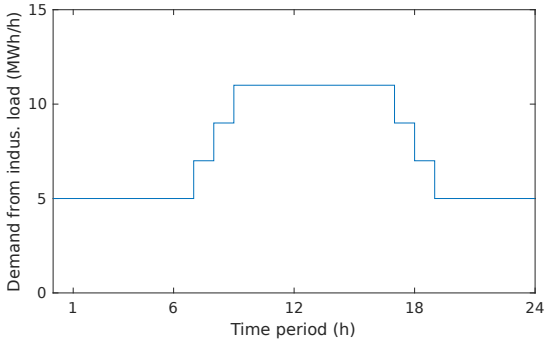


Figure C.14: Demand for electrical energy from the industrial load.

Appendix C.3. Generation of price scenarios

Cases 1 and 2 rely on a set of electricity price forecast scenarios. These are generated by simulating different outcomes of the market-clearing program, using slightly different wind power and load profiles. The intention is to obtain representative price series for the system under study, accounting for a certain degree of uncertainty in wind power and load. Figure C.15 below display the set of wind power and load profiles that are used to generate different scenarios. Combining the different profiles leads to 27 different scenarios. The highlighted profiles in Figure C.15 correspond to the realized profiles in the day-ahead market.

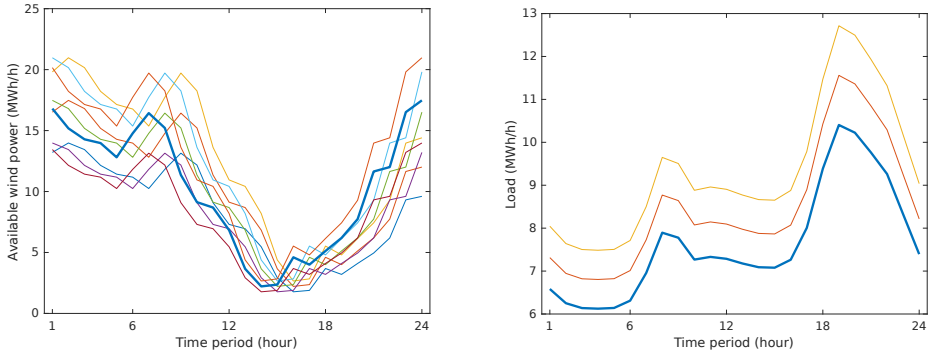


Figure C.15: Left: set of likely available wind power profiles. Right: set of likely load profiles (load serving entity).

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