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Vibration damping of an offshore wind turbine by optimally calibrated pendulum absorber with shunted electromagnetic transducer

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Abstract

This paper investigates the use of an augmented pendulum absorber for damping of tower vibrations in monopile-supported wind turbines. A shunted electromagnetic transducer is proposed to replace the classic viscous dashpot for pendulum absorbers, in order to improve performance and durability of the absorber. A series RCL (resistive-capacitative-inductive) network is designed as the supplemental shunt for the electromagnetic transducer, which together with the intrinsic resistive-inductive properties of the transducer coil results in an additional resonance. The system equations of motion are established, from which closed-form expressions for optimal calibration are derived using pole-placement. A fully-coupled 14-degree-of-freedom aero-hydro-servo-elastic wind turbine model is established to evaluate the pendulum absorber performance under realistic conditions by non-linear time-domain simulations. Both frequency- and time-domain results show that the calibration procedure results in an optimal absorber that outperforms the classic dashpot-based pendulum absorber with respect to vibration mitigation.

Keywords: wind turbine; pendulum absorber; shunted electromagnetic transducer; tower vibration; closed-form calibration; damping

1. Introduction

As offshore wind turbines (WTs) grow in size, the blades and towers become more flexible and hence more vulnerable to dynamic excitations. For monopile-supported WTs, the fatigue loads caused by the combination of wind and wave loading normally dictate the dimensioning of monopiles, especially at sea crest level [1]. Damping plays a very important role in dynamics of offshore WTs [2]. A significant increase in the fatigue life is found with an increased damping ratio in monopile-supported offshore WTs [3]. The tower vibrations can be decomposed into two directions: the fore-aft (FA) direction along the mean wind direction and the cross-wind side-side (SS) direction. Although the amplitude of the tower SS vibrations is normally smaller than that of FA vibrations, the aerodynamic damping in the SS direction is much lower during
operational conditions [4]. As a result, the FA vibrations are mainly quasi-static, due to very high aerodynamic damping, while the SS vibrations are dominated by dynamic response, resulting in comparable fatigue damage in the tower.

One way to damp the WT tower vibrations is the use of vibration absorbers. Many different types of vibration absorbers for WTs have been investigated including the classic tuned mass damper (TMD) [5, 6, 7], the roller damper [8, 9], the tuned liquid damper [10, 11], and the pendulum absorber [12]. Unlike TMDs or liquid dampers that need to be installed inside the nacelle where space limitation might be an issue, the pendulum absorber is installed inside the hollow tower. Conventionally, linear viscous or oil dampers [12] have been employed as the damping mechanism for the pendulum absorber, and its maintenance due to overheating or leakage may be a potential problem. Therefore, it is of value to explore an alternative damping mechanism for pendulum absorbers.

The electromagnetic (EM) transducer is a specific type of electromechanical device [13] that may be installed in vibrating structures to suppress excessive vibrations [14, 15]. The electromechanical coupling effect in EM transducers constitutes the foundation for many types of industrial actuators, sensors and motors. Inspired by the concept of resonant piezoelectric shunt damping [16, 17], Behrens et al. [18, 19] proposed to use the EM transducers with supplemental passive shunts (electrical impedances) to realize a resonant vibration absorber for structural control. The transducer is shunted with a series resistive-capacitive (RC) circuit, and is applied to a single-degree-of-freedom (SDOF) system in parallel with the system spring and dashpot. The additional resonance introduced by the EM shunt damping must be precisely synchronized with the targeted resonance of the structure, in order to achieve the desired performance. Therefore, optimal calibration of the parameters becomes essential. Inoue et al. [20] demonstrated that the RC shunt components can be calibrated using the fixed-point theory, as used for the classic TMD [21]. A similar calibration approach has been used by Zhu et al. [22], with a slightly different tuning formula for the shunt resistance. Since the optimal calibration of a resonant absorber normally requires a small amount of energy dissipation, the inherent resistance of the EM transducer may be too large, and hence RC shunts with negative resistance have been investigated [23, 24].

More recently, an alternative shunt, with the inductance placed in parallel with the resistance, has also been proposed [25] for improving the bandwidth of the device. Besides installation directly inside the structure, shunted EM transducers have as well been proposed in combination with auxiliary vibration absorber (proof-mass absorber) [26, 27, 28] to improve the damping of structural vibrations. A feasibility study presented by Palomera-Arias et al. [15] demonstrates the physical and economical viability of the EM damper in large-scale civil engineering structures, although the state of currently available magnet technology implies an increased cost.

In the present paper, an augmented pendulum absorber using a shunted EM transducer, to replace the classic dashpot, is proposed for damping of tower vibrations in monopile-supported wind turbines. A series RCL (resistive-capacitative-inductive) network is designed as the supplemental shunt for the EM transducer. The shunt components are calibrated by initially establishing an equivalent mechanical model to the RCL-shunted electromagnetic transducer. Such analogy between mechanical absorbers and resonant electrical circuits has previously been widely investigated and specifically used for shunt calibration, as described for piezoelectric RL-shunts in [16, 29, 30, 17, 31] or for electromagnetic RC-shunts in [18, 19, 20, 22, 32]. For the present pendulum absorber, the specific RCL-shunt will, together with the intrinsic resistive-inductive properties of the EM transducer coil, result in an additional resonance because of its analogy to an equivalent mechanical spring-dashpot-inerter element. It is expected that this augmented double-resonance pendulum absorber will exhibit improved performance compared to the classic
single-resonance pendulum with only a dashpot as damping element. The equations of motion for a wind turbine tower structure with a pendulum absorber are initially established, taking into account that the transducer is attached to the tower at a lower location than the pendulum’s pivot point. By use of modal analysis the coupled structure-pendulum-transducer system is then reduced to a 3-DOF model, which is similar to that in [28] obtained for a shunted piezoelectric absorber. Based on the normalized sextic characteristic equation of this 3-DOF system, closed-form expressions for the optimal absorber calibration are derived by means of a root locus or pole-placement method [28, 33]. An equal modal damping criterion from [34] is first used for equivalent stiffness calibration, while the existence of a triple-root bifurcation point is subsequently used as the basis for the calibration of the equivalent mass and damping. Then a fully-coupled 14-DOF aero-hydro-servo-elastic wind turbine model is established, from which the performance of the optimized augmented pendulum absorber can be evaluated under realistic wind and wave loading conditions. From both the frequency-domain analysis (using the 3-DOF model) and time-domain simulations (using the 14-DOF wind turbine model) it is observed that the proposed EM shunted pendulum absorber is superior to the classic dashpot-based pendulum absorber in the mitigation of structural vibrations (for identical absorber mass), at the cost of a slightly amplified absorber stroke.

2. Electromagnetic pendulum absorber in wind turbine structure

2.1. Wind turbine structure with pendulum absorber

The pendulum absorber is attached to the top of the flexible wind turbine structure in Fig. 1(a), assuming a planar problem with unidirectional vibrations and linearized pendulum behavior. As indicated in Fig. 1(b), the pendulum mass $m_a$ is connected to the structure via an electromagnetic (EM) transducer. Fig. 1(c) illustrates the equivalent model of the coupled system, with a finite element model representing the wind turbine structure. The horizontal degree of freedom (DOF) at the tower top is denoted as DOF $k$, while the horizontal DOF $j$ represents the EM transducer location, only considering pure transverse forces by the pendulum and neglecting any static and non-linear contributions. At the top attachment point, the pendulum therefore only applies its horizontal force associated with DOF $k$, while below the entire transducer force $f_a$ acts horizontally between the pendulum mass $m_a$ and the associated transverse DOF $j$. In the equivalent numerical model, the linearized restoring force from the pendulum is represented by a spring stiffness $k_a = m_a g / l_p$, where $g$ is the constant of gravity and $l_p$ is the pendulum length.

The linearized equation of motion for the wind turbine with the pendulum attached can be written in the frequency domain as

$$\left( -\omega^2 M + K + k_a b_k^2 b_k^T \right) q - k_a b_k u_a - b_j f_a = f$$

with $\omega$ being the angular frequency of the assumed harmonic motion and all dependent variables representing amplitudes. In the equation of motion (1), the components $M$ and $K$ are the mass and stiffness matrices of the wind turbine structure, respectively. Structural damping is neglected in the subsequent calibration procedure to facilitate the derivation of simpler closed-form formulas. For wind turbine towers made from steel, the structural damping ratio is quite low (about 1%). Furthermore, the main absorber target is the side-side vibrations due to their very low (sometimes negative) aerodynamic damping. Thus, neglecting damping in the main structure when calibrating the pendulum absorber is considered reasonable and common practice [34]. The vector $q$ contains all degrees of freedom of the structural model, while $u_a$ denotes
the (absolute) horizontal displacement of the absorber mass \( m_a \). The participation vectors \( b_j \) and \( b_k \) are zero vectors with a single unit entry at their respective horizontal DOFs \( j \) and \( k \). Thereby \( b_j \) represents the location of the equivalent spring \( k_a \), while \( b_k \) defines the lower location of the EM transducer. The external load is represented by the vector \( f \), while \( f_a \) is the force produced by the shunted EM transducer. The corresponding equation of motion for the pendulum absorber mass is given by

\[
(-\omega^2 m_a + k_a)u_a - k_a b_k^T q + f_a = 0
\]

governing horizontal force equilibrium with linearized pendulum motion for small angles relative to vertical.

2.2. Shunted electromagnetic transducer

An EM transducer operates by moving a magnetic core with a given velocity within windings of a conductive coil with two end terminals. Figure 2(a) shows a schematic drawing of the EM transducer between tower and pendulum mass, with the desired shunt governing the voltage \( V \) across the coil terminals. In the following, the elongation of the EM transducer is

\[
x = u_a - b_j^T q
\]
The oscillatory voltage amplitude induced by the vibratory structure motion across the EM transducer must balance the inherent coil impedance and the applied loading $V$ from the shunt. By Kirchhoff’s voltage law this balance can be written as

$$V_m = (R_m + i\omega L_m)I + V \quad (4)$$

A desired resonant EM absorber can be obtained by the series $RCL$-shunt in Fig. 2(a), with impedance relation

$$V = (i\omega L_s + R_s + \frac{1}{i\omega C_s})I \quad (5)$$

containing the shunt inductance $L_s$, resistance $R_s$ and capacitance $C_s$.

By elimination of voltage via $V_m = i\omega K_m$ and current by $I = f_a/K_m$, the electrical balance equation (5) is converted to the equivalent absorber force relation

$$x = \left(\frac{1}{k} + \frac{1}{i\omega c} + \frac{1}{-\omega^2 m}\right)f_a \quad (6)$$

introducing its resulting stiffness $k$, damping $c$ and inertance $m$ as

$$k = \frac{K_m^2}{L_m + L_s}, \quad c = \frac{K_m^2}{R_m + R_s}, \quad m = K_m^2 C_s \quad (7)$$

for the equivalent absorber model in Fig. 2(b) with a spring, dashpot and inerter in series. Details concerning the specific derivation of the equivalent mechanical absorber model can be found in [32]. By introducing $x = u_a - \mathbf{b}_j^T \mathbf{q}$ from (3), the transducer force $f_a$ can finally be expressed as

$$f_a = \frac{-\omega^2 mk}{-\omega^2 m + i\omega mk/c + k}(u_a - \mathbf{b}_j^T \mathbf{q}) \quad (8)$$

The dynamic characteristics of the wind turbine model are thus governed by the coupled equations in (1), (2) and (8).
2.3. Modal equations

The dynamic properties of the structure are commonly represented by its free vibration characteristics without the absorber. The vibration modes $u_n$ and the associated natural angular frequencies $\omega_n$ are therefore governed by the eigenvalue problem

$$ (K - \omega_n^2 M)u_n = 0 $$

(9)

In the following, the dynamic response is entirely represented by the single dominant structural model $n = s$, whereby

$$ q = \frac{u_s}{b_j^T u_s} $$

(10)

introduces the generalized coordinate $u$ as the structural vibration amplitude at transducer location because $b_j^T q = u$ when substituting $q$ from (10).

The structural modal equation of motion is obtained by substitution of (10) into (1), followed by pre-multiplication with $u_s^T / (b_j^T u_s)$. Hereby it can be written as

$$ \left( -\omega^2 m_s + k_s + k_a(1 + \Delta \nu)^2 \right) u - k_s(1 + \Delta \nu) u_a - f_a = f_s $$

(11)

introducing the modal mass, modal stiffness and modal load as

$$ m_s = \frac{u_s^T M u_s}{(b_j^T u_s)^2}, \quad k_s = \frac{u_s^T K u_s}{(b_j^T u_s)^2}, \quad f_s = \frac{u_s^T f}{b_j^T u_s} $$

(12)

while the change in modal vibration amplitude across the pendulum attachment points,

$$ \Delta \nu = \frac{b_j^T u_s - b_j^T u_a}{b_j^T u_s} = \frac{b_j^T u_s}{b_j^T u_s} - 1 $$

(13)

accounts for the influence of the pendulum length.

By substitution of the modal representation (10), the absorber equation (2) is given as

$$ \left( -\omega^2 m_a + k_a \right) u_a - k_s(1 + \Delta \nu) u + f_a = 0 $$

(14)

while the transducer force expression in (8) becomes

$$ f_a = \frac{\omega^2 m k}{\omega^2 m + \omega^2 m + i \omega k m/c + k} (u_a - u) $$

(15)

with $u$ appearing without a modal factor because of the specific normalization in (10).

For the subsequent absorber calibration, the three modal equations in (11), (14) and (15) constitute the governing equations for the reduced 3-DOF modal structure-pendulum-transducer system, with $u$, $u_a$ and $f_a$ as its corresponding DOFs.

3. Optimal calibration based on root locus method

In this section, closed-form expressions for optimal calibration of the pendulum absorber and the shunted EM transducer are derived based on a root locus or pole placement method described in [28] and similar to the approach outlined in [34, 39, 31]. The absorber mass ratio $\mu_a = m_a/m_s$
is chosen in advance, as for the classic TMD [21, 34]. Next, the absorber spring stiffness and the equivalent stiffness are calibrated so that two of the three free vibration modes have equal modal damping ratio, while the remaining third mode is highly damped. Then the equivalent mass and equivalent damping are calibrated based on a triple-root bifurcation point where the three system poles meet. The calibration procedure detailed in the following is therefore concerned with the proper placement of the complex system poles. It should be mentioned that in the present paper the complex system poles are equivalent to the complex frequencies of free vibration modes, which are also equivalent to the complex roots of the characteristic equation.

3.1. Normalized characteristic equation

The three governing modal equations (11), (14) and (15) are conveniently reformulated as

\[ (-p^2 + 1 + \kappa_0(1 + \Delta \nu)^2)u - \kappa_0(1 + \Delta \nu)u_a - \frac{f_s}{K_s} = \frac{f_a}{K_a} \]  
(16)

\[ (-p^2\mu_a + \kappa_a)u_a - \kappa_a(1 + \Delta \nu)u + \frac{f_s}{K_s} = 0 \]  
(17)

\[ \frac{f_a}{K_a} = \frac{-p^2\mu + i\mu\kappa/\beta + \kappa}{u_a} \]  
(18)

in which the dimensionless frequency ratio

\[ p = \frac{\omega}{\omega_s} \]  
(19)

has been introduced with \( \omega_s = \sqrt{k_a/m_a} \) being the angular eigenfrequency of the structure’s mode. Furthermore, the following normalized parameters of the pendulum absorber and the shunt have been introduced in the above equations,

\[ \mu_a = \frac{m_a}{m_s}, \quad \kappa_a = \frac{k_a}{k_s}, \quad \mu = \frac{m}{m_c}, \quad \kappa = \frac{k}{k_s}, \quad \beta = \frac{c}{\sqrt{m_c k_s}} \]  
(20)

where \( \mu_a \) and \( \kappa_a \) are respectively the mechanical mass ratio and mechanical stiffness ratio (related to the pendulum absorber), while \( \mu, \kappa \) and \( \beta \) are the equivalent mass ratio, stiffness ratio and damping ratio (related to the shunted EM transducer), respectively.

From equations (16) to (18) the normalized amplitude of the structural response is obtained as the frequency response function

\[ \frac{u}{u_a} = \frac{\sqrt{\mu^2 - \mu^2(\mu + \kappa_0\mu + \mu_0\mu + \mu_0\kappa_0(1 + \kappa_0 + (1 + \Delta \nu)^2)\kappa_0)}}{\sqrt{\mu^4 + \mu^2(\mu + \kappa_0\mu + \mu_0\mu + \mu_0\kappa_0(1 + \kappa_0 + (1 + \Delta \nu)^2)\kappa_0)^2}} \]  
(21)

from which the sextic characteristic equation in \( p \) is obtained by setting the denominator equal to zero,

\[ -p^3\mu_0\mu + p^4\left[\kappa(\mu_0 + \mu) + \kappa_0\mu + \mu_0\mu_0(1 + \kappa + (1 + \Delta \nu)^2)\kappa_0\right] \]

\[ -p^2\left[\kappa(\mu_0 + \mu) + \kappa_0\mu + \mu_0\kappa_0(1 + \mu\Delta \nu^2 + (1 + \Delta \nu)^2)\mu_0\right] + \kappa_0k \]  
(22)

In [34] it is shown that optimal calibration of the classic TMD is guaranteed by equal modal damping of the two free vibration modes. This is further secured by the two complex poles (or
complex frequencies) being inverse points with respect to a reference frequency $\omega_r$, representing
the natural frequency of the system when the absorber mass is rigidly attached to the structure
[34]. To facilitate the optimal calibration, a new dimensionless frequency ratio $\xi$ is thus defined
using this reference frequency $\omega_r$

$$\xi = \frac{\omega}{\omega_r} = \frac{p}{\Omega_r}, \quad \Omega_r = \frac{\omega_r}{\omega_s} \quad (23)$$

with $\Omega_r$ being the reference frequency ratio, whose optimal calibration formula is also derived in
the following. By substitution of (23) into (22), the normalized characteristic equation can then
be written in terms of $\xi$ as

$$-\xi^6 + \xi^4 \left[ a \left( \frac{1}{\mu} + \frac{1}{\mu} \right) + \frac{s}{\mu} + 1 + \kappa + (1 + \Delta \nu)^2 \kappa \right]$$

$$-\xi^4 \left[ a \left( \frac{1}{\mu} + \frac{1}{\mu} \right) + \frac{s}{\mu} + \frac{s}{\mu}(1 + \mu \Delta \nu^2 + (1 + \Delta \nu)^2 \mu s) \right] + \frac{\kappa}{\mu} = 0 \quad (24)$$

By introducing $\xi$, the condition that two complex roots are inverse points with respect to $\omega_r$
is changed into the condition that they are inverse points with respect to 1.

In the following, the root loci in the complex $\xi-$ plane are estimated by the placement of the
undamped poles along the real-valued $\xi-$ axis. In (24) these undamped limits are obtained either
by $\beta \to \infty$ with (24) reducing to a sextic polynomial equation in $\xi$ but with only even power
terms, or $\beta \to 0$ whereby the real-valued poles are governed by the fourth order polynomial in $\xi$
inside the bottom square brackets in (24). They are equivalent to a cubic polynomial in $\xi^2$ and
a quadratic polynomial in $\xi^4$, respectively. The latter limit implies that the equivalent dashpot in
Fig. 2(b) transfers no force, and the two roots from this quadratic equation therefore represent
the complex frequencies of the 2-DOF structure-pendulum system without influence from the
transducer. Conversely, the former limit $\beta \to \infty$, associated with the even power terms $\xi^6$ to
$\xi^4$, represents the system in which the equivalent dashpot fully locks and thus instantaneously
transfers the force, whereby the equivalent inerter $m$ in Figure 2(b) is activated as the third DOF,
resulting in the undamped 3-DOF structure-pendulum-shunt system (without the dashpot).

3.2. Stiffness ratio calibration

To satisfy the requirement of equal modal damping in the complex $\xi-$ plane, two of the three
roots must be inverse points with respect to a unit reference (quarter) circle between $\xi=1$ on the
real axis and $\xi=1$ on the imaginary axis. The remaining third complex root must therefore lie on
this unit quarter circle. By varying the equivalent damping ratio $\beta$ from 0 to infinity, the so-called
root locus diagram can be obtained. Fig. 3 shows the conceptual illustration of three possible root
loci diagrams in the complex $\xi-$ plane, depending on the calibration of specifically the equivalent
mass ratio, as shown in the following. The two circle markers on the horizontal axis indicate
the two real-value roots $\xi_{0\omega}$ and $\xi_{0\omega}$, obtained in the limit $\beta \to 0$, while the three square markers
indicate the three real-value roots $\xi_{\omega\omega}$, 1 and $\xi_{\omega\omega}$, obtained in the limit $\beta \to \infty$. It is worth
noting that the root loci can also be plotted in $\xi^2-$ plane [28] instead of $\xi-$ plane, whereby the
usual symmetry about the imaginary axis is replaced by symmetry about the real axis. However,
the shape, the arrow direction and the inherent property (no bifurcation point, two double-root
bifurcation points, or single triple-root bifurcation point) of the root loci are unchanged when displaced in the first quadrant.

On the real-valued (horizontal) \(\xi\) axis, the inverse point condition with respect to \(\xi = 1\) is fulfilled by

\[ \xi_{\infty} - \xi_{\infty+} = 1, \quad \xi_{0} - \xi_{0+} = 1 \]  \hspace{1cm} (25)

for the two \(\beta\) limits. In the limit \(\beta \to \infty\), the cubic polynomial in \(\xi^2\) thus becomes

\[ (\xi^2 - 1)(\xi^2 - \xi^2_{0+})(\xi^2 - \frac{1}{\xi^2_{0+}}) = 0 \quad \Rightarrow \quad \xi^6 - \xi^4(1 + \xi^2_{0+} + \frac{1}{\xi^2_{0+}}) + \xi^2(1 + \xi^2_{0+} + \frac{1}{\xi^2_{0+}}) - 1 = 0 \]  \hspace{1cm} (26)

which shows that the ratio between the coefficients to the 6th order term and the constant term should be \(-1\). In (24) this condition leads to

\[ \frac{k_0 \kappa}{\mu_0 \mu} \frac{1}{\Omega^4_r} = 1 \]  \hspace{1cm} (27)

The similar inverse point condition \(\xi_{0} - \xi_{0+} = 1\) in (25) must be fulfilled, whereby the quadratic polynomial in \(\xi^2\) becomes

\[ (\xi^2 - \xi^2_{0+})(\xi^2 - \frac{1}{\xi^2_{0+}}) = 0 \quad \Rightarrow \quad \xi^4 - \xi^2(\xi^2_{0+} + \frac{1}{\xi^2_{0+}}) + 1 = 0 \]  \hspace{1cm} (28)

demonstrating that the ratio between the coefficients to the 4th order term and the constant term must be 1. For the characteristic equation (24) this provides the relation

\[ \frac{k_0}{\mu_0} \Omega^2_r = 1 \]  \hspace{1cm} (29)

for \(\beta \to 0\). By combining (27) and (29) the reference frequency ratio \(\Omega_r\) is obtained as

\[ \Omega^2_r = \frac{k}{\mu} \]  \hspace{1cm} (30)
while the relation between the mechanical mass-stiffness ratio and the equivalent mass-stiffness ratio

$$\frac{\kappa_a}{\mu_a} = \left(\frac{\kappa}{\mu}\right)^2 = \Omega_r^4$$  \hspace{1cm} (31)

is subsequently found directly from (29).

Furthermore, equation (26) indicates that the ratio between the coefficients to the 4th and 2nd order terms should be $-1$ when $\beta \to \infty$. When applying this condition to the characteristic equation (24), the quadratic equation

$$\Delta \nu \mu_a \left(\kappa - (1 + \mu_a) \frac{\kappa}{\mu} + 1 \right) = 0$$  \hspace{1cm} (32)

in $\kappa/\mu$ is obtained. The accurate solution to (32) and its approximated expression (based on $\Delta \nu^2 \mu_a \ll 1$) are written as

$$\Omega_r^2 = \frac{1 + \mu_a}{2} \left(1 - \sqrt{1 - \frac{4 \Delta \nu^2 \mu_a}{(1 + \mu_a)^2}}\right) \approx \frac{1}{1 + \mu_a} \left(1 + \Delta \nu^2 \frac{\mu_a}{(1 + \mu_a)^2}\right) \approx \frac{1}{1 + \mu_a}$$  \hspace{1cm} (33)

whereby initially the square root is linearized by its Taylor-expansion, while the parenthesis in the final approximation is simply assumed close to unity. The accurate expression of $\Omega_r^2$ implies that an iterative procedure for obtaining $\Delta \nu$ is required when using (13), (27) and (33), while the approximated expression eliminates the need of iteration and is identical to that for a classic TMD [34]. It can be shown that the error from this approximation is negligible when the angular eigenfrequency of the targeted structural mode is larger than 1 rad/s and the mass ratio of the absorber $\mu_a$ is smaller than 0.1. Otherwise, the accurate expression of $\Omega_r^2$ should be used.

3.3. Equivalent mass ratio and damping ratio calibration

To derive the optimal equivalent mass ratio and damping ratio, the triple-root bifurcation point where the three system poles meet is initially investigated. As already indicated, in the limit $\beta \to 0$, the characteristic equation (24) is reduced to

$$\xi^4 - \Xi_0^2 \left(1 + (1 + \Delta \nu)^2 \kappa_a + \frac{\kappa_a}{\mu_a} + \frac{1}{\Omega_r^2}\right) = 0$$  \hspace{1cm} (34)

The two roots $\Xi_0^2$ and $\Xi_{0+}$ of (34) are then explicitly solved as

$$\Xi_{0-}^2 = C_0 \pm \sqrt{C_0^2 - 1}$$  \hspace{1cm} (35)

with

$$C_0 = \frac{1}{2} (\Xi_{0+}^2 + \Xi_{0-}^2) = \frac{1}{2} \left[\frac{1}{\Omega_r^2} + \Omega_r^2 (1 + \Delta \nu)^2 \mu_a + 1\right]$$  \hspace{1cm} (36)

representing the algebraic mean value of the two roots $\Xi_{0-}^2$ and $\Xi_{0+}^2$.

By substitution of the expressions (30), (31) and (36), the characteristic equation (24) can be rewritten in a more compact format as

$$-\xi^6 + \left(\xi^4 - \Xi_0^2\right) \left(1 + \frac{\mu}{\mu_a} + \mu + 2C_0\right) + 1 + i\xi \frac{\mu \Omega_r}{\beta} \left(\xi^4 - \Xi_0^2 2C_0 + 1\right) = 0$$  \hspace{1cm} (37)
The existence of the triple-root bifurcation point implies that the characteristic equation can also be expressed as

\[(\xi - \xi_*)^3(\xi - \bar{\xi}_*)^3 = 0\]  \hspace{1cm} (38)

where \(\xi_*\) is the triple-root bifurcation point with negative complex conjugate \(\bar{\xi}_*\) and the property \(|\xi_*| = 1\). When expanding (38) and equalizing coefficients with the corresponding equation (37), an expression of the equivalent mass ratio \(\mu_*\) at the triple-root bifurcation point is obtained as

\[\mu_* = 16(C_0 - 1)\frac{\mu_a}{1 + \mu_a}\] \hspace{1cm} (39)

together with an expression of the equivalent damper ratio \(\beta_*\)

\[\beta_* = \frac{\mu_\Omega_r}{\sqrt{54(C_0 - 1)}}\] \hspace{1cm} (40)

to adjust the optimal equivalent mass ratio \(\mu\) and damper ratio \(\beta\) relative to the bifurcation values. For the stiffness ratio calibration in section 3.2 and by using expressions (39) and (40) for equivalent mass ratio and damping ratio calibration, the triple-root bifurcation point, shown by the red star in Fig. 3, is achieved.

The triple-root bifurcation point is associated with the maximum modal damping ratio attainable when the equal modal damping condition is fulfilled [34]. However, it also implies that the three modes have the same angular frequency at this point, causing constructive interference between the modes. Therefore, calibration based on the triple-root bifurcation point is actually non-optimal. Instead, a slightly lower modal damping ratio will provide the optimal calibration, as already shown for example for a classic TMD in [34]. To obtain the optimal parameters, two scaling parameters \(\alpha\) and \(\eta\) are assigned to respectively the equivalent mass ratio \(\mu_*\) and equivalent damping ratio \(\beta_*\),

\[\mu = \alpha\mu_*\quad,\quad \beta = \frac{\beta_*}{\eta}\] \hspace{1cm} (41)

These two scaling parameters are to be obtained through a numerical optimization analysis. A linear relation between \(\alpha\) and the optimal value of \(\eta\) is observed, and the dependency on \(\Delta \nu\) also turns out to be insignificant. The slope of the linear relation ranges from 1.26 to 1.20 for \(\Delta \nu\) ranging from 0.05 to 0.6. When considering that the error introduced by assuming the slope being independent

...
on $\Delta \nu$ is negligible for small $\alpha$ values, a common linear relation $\eta_{opt} = 1.25\alpha + 0.03$ is chosen regardless of $\Delta \nu$. Therefore, the two scaling parameters are determined as $\alpha = 0.5$ and $\eta = 0.655$. Inserting these two values into the expressions in (39), (40) and (41) the optimal equivalent mass ratio and damping ratio are expressed as

$$\mu = 16\alpha(C_0 - 1)\frac{\mu_0}{1 + \mu_0} = 8(C_0 - 1)\frac{\mu_0}{1 + \mu_0}$$ (42)

and

$$\beta = \frac{\mu\Omega_r}{(1.25\alpha + 0.03)\sqrt{54(C_0 - 1)}} = \frac{\mu\Omega_r}{0.655\sqrt{54(C_0 - 1)}}$$ (43)

for $\alpha = 0.5$ introduced to obtain the final expressions. The complete calibration procedure for the augmented pendulum absorber is conveniently summarized by the flowchart in Fig. 6.

4. Results and discussion

In this section, performance of the augmented pendulum absorber is evaluated and compared with a classic pendulum absorber. First, the 3-DOF structure-absorber model, i.e. (11), (14) and
(15), are used to investigate the influence of absorber parameters on the dynamic amplification and the root loci. Next, the optimized absorber is incorporated into a 14-DOF aero-hydro-servo-elastic wind turbine model for non-linear time-domain simulations, evaluating its performance in a highly-coupled wind turbine system with realistic wind and wave loads.

4.1. 3-DOF structure-absorber model

The 3-DOF model (with SDOF representing the structure) assumes planar motion of the system. All results are presented in non-dimensional formats.

To calculate $\Delta \nu$ in (13), the mode shape of the considered mode is needed. Here, the fundamental SS tower mode shape of the 5 MW monopile-supported wind turbine (with the tower top height from mean water level to be 107.6 m) [40, 41] obtained from a finite element model is used, and the details can be found in [42]. Following the optimal mechanical stiffness ($k_a$) calibration procedure in Fig. 6, the pendulum length $l_p$ turns out to be 3.4 m, resulting in the value of $\Delta \nu$ being $\Delta \nu = 0.0674$.

4.1.1. Dynamic amplification

Fig. 7 shows the dynamic amplifications of the system response (with $\mu_a = 0.02$) for different values of the scaling parameter $\alpha$, while the mechanical stiffness has been optimally calibrated. $\Delta \nu = 0.0674$, $\eta = 1.25\alpha + 0.03$. (a) Structural response. (b) Relative pendulum absorber motion.
the classic TMD tuning principles [34, 21]. When $\alpha > 0.5$, the dynamic amplification becomes larger with two clearly visible peaks and an intermediate trough. On the other hand, it is observed that $\alpha = 0.3$ actually leads to an even lower dynamic amplification curve (with three local peaks and two troughs), in accordance with the results in Fig. 4 where $\alpha = 0.3$ gives the minimum response. However, a further reduction in $\alpha$ below 0.3 will lead to a significant increase of the dynamic amplification (not shown here), as also revealed by Fig. 4. When considering both the flat plateau and the sufficient margin away from the steep increase of structural amplification, $\alpha = 0.5$ can with good reason be considered as an optimal value determining $\mu$ by (41a). A similar performance has been obtained for an advanced shunted piezoelectric vibration absorber in [33].

Fig. 7(b) shows the corresponding dynamic amplification of the relative absorber motion (the damper stroke). Around the resonance frequency, the amplitudes of different curves are quite similar. Again, the optimal value $\alpha = 0.5$ leads to a fairly flat plateau. When $\alpha > 0.5$, there are two local peaks and one trough, while $\alpha = 0.3$ leads to three local peaks and two troughs. Furthermore, a smaller value of $\alpha$ leads to a broader plateau of the curve, implying that the relative absorber motion is maintained at a high level even when the frequency is shifted away from the resonance frequency. This further implies that the damper stroke increases as $\alpha$ decreases.

![Figure 8: Dynamic amplifications of the system response for different values of $\Delta \nu$. Mechanical stiffness optimally calibrated, $\alpha = 0.5, \eta = 1.2\% + 0.05$. (a) Structural response. (b) Relative pendulum absorber motion.](image)

By zooming in, it is observed that the curve corresponding to $\alpha = 0.5$ in Fig. 7(a) actually does not have a completely flat plateau. It is slightly inclined with larger amplitude in the low-frequency range. This is further investigated in Fig. 8 by varying the value of $\Delta \nu$. As explained in the flowchart of Fig. 6, a chosen value of the absorber mass ratio $\mu_k$ will lead to a fixed value of $\Delta \nu$ from the mechanical stiffness calibration. Nevertheless, here the value of $\Delta \nu$ is varied to show how the damper performance is changed with $\Delta \nu$. In practice, even with a given target angular frequency of the pendulum, it might still be possible to adjust the length of the pendulum (thus the value of $\Delta \nu$) by introducing a rotational spring at the attachment point of the pendulum at the tower top (the pivot). With this additional spring, a longer pendulum could effectively produce the stiffness of a shorter pendulum without affecting the optimal calibration. From Fig. 8(a) it is seen that as $\Delta \nu$ increases the inclination of the plateau increases, while the height of the plateau decreases (implying a larger response reduction by the pendulum absorber). When $\Delta \nu = 0$, the pendulum is reduced to a classic TMD with zero pendulum length, and the plateau becomes quite flat in accordance with the results for a classic TMD in [34, 39]. Fig. 8(b) shows that the increase of $\Delta \nu$ leads to smaller relative motion of the pendulum absorber (the damper stroke), which is very beneficial in practical applications. It should be noted that $\Delta \nu = 0.5$ corresponds to

14
a pendulum length of approximately 18.8 m which is not realistic, while $\Delta \nu = 0.1$ corresponds
to a pendulum length of approximately 5.0 m which is realizable by introducing the additional
rotational spring. To sum up, the increase of $\Delta \nu$ (with the pendulum eigenfrequency unchanged)
results in both the improved damping effect on structural response and the reduced damper stroke,
although the additional rotational spring is needed to maintain the same mechanical frequency
tuning, thus increasing device complexity.

4.1.2. Root loci diagram

![Root loci diagram](image)

Figure 9: Root loci diagram (by varying $\beta$ from zero to a large value) for $\mu_a = 0.02$, mechanical stiffness optimally
calibrated, $\Delta \nu = 0.0674$. The red crosses: the complex roots when the optimal value of $\beta$ is used (Eq. (43)). (a) $\alpha = 0.5$,
the optimal case. (b) $\alpha = 1.0$. (c) $\alpha = 2.0$.

Fig. 9 shows the root loci diagram (by varying the equivalent damping ratio $\beta$ from zero to a
large value) of the 3-DOF system for $\mu_a = 0.02$. Three different values of $\alpha$ have been used, i.e.
$\alpha = 0.5$, $\alpha = 1.0$ and $\alpha = 2.0$. When the optimal value $\alpha = 0.5$ is used as shown in Fig. 9(a), the
root loci behave as the conceptually illustrated possibility 1 (green and black curves) in Fig. 3,
and no bifurcation point exists. The red crosses indicate the three roots when (43) is used for
calculating $\beta$. Two roots are located at the two local closed loci, respectively, and they are inverse
points with respect to the unit circle (indicating equal modal damping ratio). The remaining third
root lies on the unit quarter circle. The modal damping ratios of the three modes are calculated
as 0.0673, 0.0673 and 0.1215. Fig. 9(b) shows the results with the scaling parameter $\alpha = 1.0$,
whereby the root loci behave as the conceptually illustrated possibility 2 (red and black curves)
in Fig. 3 with a triple-root bifurcation point. This makes sense as $\alpha = 1$ corresponds to $\mu = \mu_a$,
according to (41) with $\mu_a$ being calibrated by assuring the existence of the triple-root bifurcation
point. Again, the three red crosses indicate the three roots when the optimal value of $\beta$ is used.
Two roots located on the inner small half-circle are inverse points with respect to the unit circle,
while the third root is located above the bifurcation point on the unit circle. The modal damping
ratios for the three roots are determined as 0.0563, 0.0563 and 0.3886. For $\alpha = 2.0$, the root
loci in Fig. 9(c) exhibit two double-root bifurcation points, corresponding to the possibility 3
(blue and black curves) in Fig. 3. Similar to Fig. 9(b), two roots are located on the inner small
half-circle, while the third root on the unit circle. The modal damping ratios for $\alpha = 2.0$ turn out
to be 0.0506, 0.0506 and 0.8895. The very large damping ratio of the third mode implies that the
third root is located very high on the unit circle in Fig. 9(c).

4.1.3. Comparison with the classic pendulum absorber

Fig. 10 shows the performance comparison of the augmented pendulum absorber and the
classic pendulum absorber, both with a mass ratio $\mu_a = 0.02$ and both optimally calibrated fol-
Figure 10: Performance comparison of the augmented pendulum absorber (pendulum absorber with shunted EM transducer) and the classic pendulum absorber (with dashpot), \( \mu = 0.02 \), both optimally calibrated, \( \Delta \nu = 0.0674 \). (a) Dynamic amplification of the structural response. (b) Dynamic amplification of the relative absorber motion.

following the procedures in the flowcharts of Figs. 6 and 14, respectively. As indicated in Appendix A, the dashpot parameter of the classic pendulum absorber is determined by minimizing the peak frequency response of the structure, corresponding to \( H_\infty \) optimization. As a result, two peaks are visible in the curve corresponding to the classic pendulum absorber, similar to the Den Hartoog calibration result for TMDs. As seen in Fig. 10(a), the shunted EM transducer improves the performance of the pendulum absorber in vibration damping of the structural responses, in terms of the reduced dynamic amplification amplitude compared with the performance of the pure dashpot-based pendulum. The plateau becomes more flat and slightly more broad banded as well. Fig. 10(b) shows the corresponding comparison of the damper stroke. It is observed that the use of EM transducer also decreases the peak value of the dynamic amplification of the damper stroke, comparing with that by the dashpot. On the other hand, the plateau becomes broader when the shunted EM transducer is used, i.e. the damper stroke is maintained at a high level even when the frequency is shifted away from the resonance frequency. This actually indicates that the resulting damper stroke (consisting contributions from all frequencies) will turn out to be slightly larger when the EM transducer is used.

4.2. A fully-coupled offshore wind turbine model with the damper

In this subsection the performance of the augmented pendulum absorber is evaluated in a highly-coupled 14-DOF aero-hydro-servo-elastic wind turbine model [42] under realistic wind and wave loading conditions.

4.2.1. The 14-DOF wind turbine model

A 14-DOF model has been established using a combined multi-body and modal-based formulation, similar to the approach used in [43]. This model takes into account the most important characteristics of a monopile-supported offshore wind turbine, including coupled foundation-tower-blade-drivetrain vibrations, time-dependent system matrices, nonlinear aeroelasticity, as well as generator and pitch controllers.

The blade is modeled as a rotating Euler-Bernoulli beam in a local moving coordinate system. Each blade is associated with two DOFs \( q_j \) and \( q_{j+3} \), \( j = 1, 2, 3 \), indicating the flapwise vibration and edgewise vibration, respectively. The monopile-tower structure is modeled as a Euler-Bernoulli beam in a global fixed coordinate system. Since the damper is to be applied for damping of tower vibrations, the monopile-tower structure is described in more details, with three DOFs (vibrational modes) in each of the FA and SS directions. The DOF \( q_7 \) (the 1st modal
coordinate), \( q_8 \) (the 2nd modal coordinate) and \( q_9 \) (the 3rd modal coordinate) denote the three DOFs for tower FA vibrations, while \( q_{10} \), \( q_{11} \) and \( q_{12} \) are the three DOFs for tower SS vibrations. By normalizing all the related mode shapes to attain a unit value at the tower top, the tower top FA displacement \( q_{FA} \) and SS displacement \( q_{SS} \) can be written respectively as

\[
q_{FA}(t) = q_7(t) + q_8(t) + q_9(t), \quad q_{SS}(t) = q_{10}(t) + q_{11}(t) + q_{12}(t)
\]

Furthermore, the flexible drivetrain is modelled by DOFs \( q_{13} \) and \( q_{14} \) using St. Venant torsional theory. The DOFs \( q_{13} \) and \( q_{14} \) therefore indicate the deviations of the rotational angles at the hub and generator from the nominal rotational angles \( \Omega t \) and \( N \Omega t \), respectively, with \( \Omega \) being the nominal rotational speed of the rotor and \( N \) being the gear ratio.

A multi-body based formulation has been performed to obtain the total kinetic energy and potential energy of the system, from which the equations of motion for the 14-DOF model can be derived using the Lagrange equations [44], with details given in [42]. Moreover, a full-span collective pitch controller is modeled by a PI controller together with a first order filter representing the actuator dynamics [43]. A variable speed generator controller is also incorporated into the 14-DOF model, in order to adjust the generator torque based on the rotor speed [40]. Furthermore, the rotational sample turbulence is generated based on the recently developed multi-step auto-regressive (AR) model [45]. The random sea surface elevation is modelled as a zero-mean, stationary Gaussian process defined by the Joint North Sea Wave Project (JONSWAP) spectrum [46]. The blade element momentum (BEM) method [47] with Prandtl’s tip loss factor and Glauber’s correction is used to calculate the aerodynamic loads on the blades, while Morison’s equation [48] has been used to calculate the hydrodynamic loads on the monopile.

The developed 14-DOF model is verified by comparing the model outputs with those from the FAST (Fatigue, Aerodynamics, Structure and Turbulence) software [40], in terms of the transient responses due to a step input in the mean wind velocity as well as the steady-state responses [42]. Good agreement has been obtained for all responses, and the 14-DOF model can thus be used as a basis for evaluating the performance of the augmented pendulum absorber in a realistic condition.

4.2.2. Specification of the offshore wind turbine and load cases

The NREL (National Renewable Energy Laboratory) 5-MW reference wind turbine [40] together with the monopile-type support structure [41] have been used to calibrate the developed 14-DOF aero-hydro-servo-elastic model. Table 1 shows the system parameters of the monopile-supported offshore wind turbine investigated in the present paper.

Six different load cases (LCs) with turbulent wind (\( V_0 \) is the mean wind speed and \( f_c \) is the turbulence intensity) and irregular waves (\( H_s \) is the significant wave height and \( T_p \) is the peak wave period) have been defined for the monopile-supported wind turbine and used in the time-domain simulations, as given in Table 2. These LCs are under normal operational condition, and the parked condition is not considered. The mean wind speed has been chosen to range from the rated wind speed 11.4 m/s to the cut-out wind speed 25 m/s, with both the pitch and generator controllers being active. The corresponding significant wave height for each mean wind speed is chosen based on wind-wave correlations [49]. The peak wave period is determined by the relation \( T_p = \sqrt{\frac{\text{speed}}{\text{gravity}}} \) [50], with \( g \) being the constant of gravity. In the present study, the wave direction is assumed to be perpendicular to the mean wind velocity (90° wind-wave misalignment) to excite the tower SS vibration. This is because the aerodynamic damping is low in the SS direction, and thus the tower SS vibration is of greater concern.
Table 1: Properties of the NREL 5-MW turbine [40] mounted atop a monopile [41]. MWL: mean water level.

<table>
<thead>
<tr>
<th>Rating</th>
<th>5 MW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor orientation, configuration</td>
<td>Upwind, 3 blades</td>
</tr>
<tr>
<td>Pitch control</td>
<td>Variable speed, collective pitch</td>
</tr>
<tr>
<td>Generator control</td>
<td>Variable-speed controller</td>
</tr>
<tr>
<td>Drivetrain</td>
<td>High speed, multiple-stage gearbox</td>
</tr>
<tr>
<td>Rated rotor speed</td>
<td>12.1 rpm</td>
</tr>
<tr>
<td>Cut-in, rated, cut-out wind speed</td>
<td>3 m/s, 11.4 m/s, 25 m/s</td>
</tr>
<tr>
<td>Rotor diameter</td>
<td>126 m</td>
</tr>
<tr>
<td>Hub height above MWL</td>
<td>87.6 m</td>
</tr>
<tr>
<td>Elevation of tower base above MWL</td>
<td>10 m</td>
</tr>
<tr>
<td>Water depth</td>
<td>20 m</td>
</tr>
</tbody>
</table>

Table 2: Load cases: combined wind and waves

<table>
<thead>
<tr>
<th></th>
<th>$V_0$ m/s</th>
<th>$I_v$ [-]</th>
<th>$H_s$ [m]</th>
<th>$T_p$ [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC1</td>
<td>11.4</td>
<td>0.150</td>
<td>3.0</td>
<td>7.4</td>
</tr>
<tr>
<td>LC2</td>
<td>13.0</td>
<td>0.140</td>
<td>3.5</td>
<td>8.0</td>
</tr>
<tr>
<td>LC3</td>
<td>15.0</td>
<td>0.135</td>
<td>4.0</td>
<td>8.6</td>
</tr>
<tr>
<td>LC4</td>
<td>18.0</td>
<td>0.125</td>
<td>4.5</td>
<td>9.1</td>
</tr>
<tr>
<td>LC5</td>
<td>21.0</td>
<td>0.120</td>
<td>5.0</td>
<td>9.6</td>
</tr>
<tr>
<td>LC6</td>
<td>25.0</td>
<td>0.115</td>
<td>6.0</td>
<td>10.5</td>
</tr>
</tbody>
</table>

4.2.3. Performance of the damper in the 14-DOF model

As explained in section 2.1, a planar problem has been considered when deriving the optimal calibration formulas for the pendulum absorber. Furthermore, linearizations have been used by assuming small angles between the displaced pendulum and the tower central axis, which results in the equivalent mechanical stiffness $k_a = m_a g/l_p$. In reality, a wind turbine incorporated with a pendulum absorber exhibits three-dimensional motion, and the pendulum moves on a dome surface with the radius of the pendulum length. However, if a small angle condition is still fulfilled, the motion of the pendulum can be effectively decoupled into two independent translational motions in FA and SS directions, respectively. As a result, the influence of the pendulum absorber on the tower FA vibration and SS vibration can be considered independently. Two additional DOFs (representing the pendulum absorber with shunted EM transducer) is to be introduced and coupled to the tower FA vibrations, and similarly two DOFs will be introduced and coupled to the SS vibration. This leads to an 18-DOF wind turbine-absorber model.

The absorber has been optimally calibrated to the fundamental tower SS mode, since the lightly damped tower SS vibration is of more concern than the highly damped FA vibration. Hence, the absorber is suboptimal for the fundamental tower FA mode. With a chosen mass ratio $\mu_a = 0.01$ ($m_a = 4590$ kg), the optimal absorber parameters are calculated following the procedure in Fig. 6 and with specific results given in Table 3.

Fig. 11 shows the performance of the optimally-calibrated pendulum absorber (with shunted EM transducer) for damping of the tower SS vibrations, both in the time and frequency domains and with LC3 applied. The optimally-calibrated classic pendulum absorber (with dashpot) is also investigated for comparison. The dashpot parameter of the classic pendulum absorber is
Table 3: Optimal parameters of the augmented pendulum absorber, $\mu_a = 0.01$

<table>
<thead>
<tr>
<th>$m_a$ [kg]</th>
<th>$k_a$ [N/m]</th>
<th>$l_p$ [m]</th>
<th>$m$ [kg]</th>
<th>$c$ [Ns/m]</th>
<th>$k$ [N/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4.590 \times 10^3$</td>
<td>$1.474 \times 10^4$</td>
<td>3.06</td>
<td>204.0</td>
<td>$1.019 \times 10^3$</td>
<td>661.6</td>
</tr>
</tbody>
</table>

determined following Appendix A, same as for Figure 10. The optimal $m_a$, $k_a$ and $l_p$ of the classic pendulum absorber are exactly the same as in Table 3, while its optimal damping coefficient being $c_a = 1.047 \times 10^3$ Ns/m. Both absorbers effectively reduce the tower top SS displacement $q_{SS}(t)$ defined in (44). It is also observed that the absorber with shunted EM transducer slightly outperforms the pure dashpot-based pendulum absorber, which is in agreement with the findings in Fig. 10. The Fourier amplitude in Fig. 11(b) clearly indicates that the tower SS vibration is completely dominated by the fundamental mode due to the very low aerodynamic damping in this direction. The spectral peak that corresponds to the fundamental SS mode is significantly reduced when the absorbers are installed. The spectral peak corresponding to the case with EM transducer is slightly lower than that for pure dashpot absorber, again confirming the improved performance by the shunted EM transducer. The absorber has no effect on the other spectral peaks including the 2nd and 3rd tower SS eigenfrequencies (not observed in the figure because they are larger than 1.5 Hz).

Figure 11: Tower top side-side (SS) displacement, $\mu_a = 0.01$ ($m_a = 4.590 \times 10^3$ kg), LC3. (a) Time series. (b) Fourier amplitude.

Fig. 12 shows the corresponding results of the tower FA vibrations. As shown in Fig. 12(a), the tower top FA displacement $q_{FA}(t)$ is slightly reduced by the absorbers. Because of the high aerodynamic damping present in this direction, the vibration damping effect by both absorbers are much less pronounced compared to Fig. 11(a). The Fourier amplitude in Fig. 12(b) clearly indicates that the spectral peak corresponding to the fundamental tower FA mode is much lower when compared with Fig. 11(b). Both absorbers still effectively reduce this peak, while keeping all other peaks unaffected. By zooming in, it is observed that the peak corresponding to the EM transducer is actually slightly lower than that for dashpot absorber. Even though the main focus is on the tower SS vibration, and the absorbers have been optimally calibrated to the fundamental SS mode, it is encouraging to observe the positive effect on the FA vibrations as well.

Table 4 summarizes the performance of both pendulum absorbers with respect to damping of the dominant tower vibrations (in terms of the standard deviation), for all six LCs considered. For a given LC, the mean value of either FA vibration or SS vibration is unchanged when an absorber
is installed. Both absorbers significantly reduce the standard deviations of SS vibration for all LCs, with the reduction ratio (indicated in the parenthesis) ranging from 38.8% to 66.8%. The pendulum absorber with the shunted EM transducer always has a reduction in standard deviation compared to the pure dashpot-based counterpart, indicating the consistently superior damping effect of the proposed augmented pendulum absorber. The reductions by the absorbers on tower FA vibration are quite limited, ranging from 0.37% to 5.62%, due to the already existing high aerodynamic damping (as also shown in Figure 12). Still, the augmented pendulum absorber as well slightly outperforms the classic pendulum absorber in reducing FA vibrations for all LCs.

Table 4: Performance of the pendulum absorbers on reducing tower vibrations, $\mu_a = 0.01 \ (m_a = 4.590 \times 10^3 \text{ kg}), \text{six different LCs considered. STD: standard deviation, U: controlled, E: Electromagnetic, D: dashpot.}$

<table>
<thead>
<tr>
<th>LC</th>
<th>Side-side Mean [m]</th>
<th>STD, U [m]</th>
<th>STD, E [m]</th>
<th>STD, D [m]</th>
<th>Fore-aft Mean [m]</th>
<th>STD, U [m]</th>
<th>STD, E [m]</th>
<th>STD, D [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC1</td>
<td>-0.054</td>
<td>0.0478</td>
<td>0.0188</td>
<td>0.0195</td>
<td>0.440</td>
<td>0.1172</td>
<td>0.1161</td>
<td>0.1162</td>
</tr>
<tr>
<td>LC2</td>
<td>-0.058</td>
<td>0.0400</td>
<td>0.0185</td>
<td>0.0191</td>
<td>0.380</td>
<td>0.0992</td>
<td>0.0988</td>
<td>0.0988</td>
</tr>
<tr>
<td>LC3</td>
<td>-0.058</td>
<td>0.0632</td>
<td>0.0210</td>
<td>0.0218</td>
<td>0.315</td>
<td>0.0910</td>
<td>0.0901</td>
<td>0.0901</td>
</tr>
<tr>
<td>LC4</td>
<td>-0.058</td>
<td>0.0777</td>
<td>0.0265</td>
<td>0.0273</td>
<td>0.253</td>
<td>0.0814</td>
<td>0.0791</td>
<td>0.0792</td>
</tr>
<tr>
<td>LC5</td>
<td>-0.058</td>
<td>0.0767</td>
<td>0.0337</td>
<td>0.0352</td>
<td>0.222</td>
<td>0.0747</td>
<td>0.0715</td>
<td>0.0716</td>
</tr>
<tr>
<td>LC6</td>
<td>-0.056</td>
<td>0.0740</td>
<td>0.0438</td>
<td>0.0453</td>
<td>0.200</td>
<td>0.0891</td>
<td>0.0841</td>
<td>0.0844</td>
</tr>
</tbody>
</table>

The curves in Fig. 13 compare the relative absorber displacement (damper stroke) in the SS direction, $q_{\text{SS}}(t)$, of the two pendulum absorbers under LC3. As seen from Fig. 13(a), the damper stroke of the augmented pendulum absorber is slightly larger than that of the classic dashpot-pendulum absorber, which is in fine accordance with the previous results presented in Fig. 10(b). The Fourier amplitude in Fig. 13(b) actually resembles the dynamic amplification in Fig. 10(b), i.e. the frequency response of the damper stroke is more broad banded with the shunted EM transducer although the peak frequency response is higher with the dashpot. The
overall response is the integration of contributions from all frequencies, and thus the damper stroke of the augmented pendulum absorber is slightly larger. The same conclusion can be drawn from all other LCs.

Figure 13: Relative absorber displacement in the side-side (SS) direction, $\mu_a = 0.01$ ($m_a = 4.590 \times 10^3$ kg), LC3. (a) Time series. (b) Fourier amplitude.

5. Conclusions

This paper proposes an augmented pendulum absorber for damping of wind turbine tower vibrations, where the shunted electromagnetic (EM) transducer has been introduced to replace the classic dashpot-based absorber. Initially, the equations of motion for a wind turbine tower structure installed with a pendulum absorber are formulated. A series $RCL$ (resistive-capacitative-inductive) network has been designed as the supplemental shunt for the EM transducer. This leads to an equivalent mechanical system that is a resonance system with a spring, a dashpot and an inerter in series, adding an extra resonance to the pendulum absorber. Based on modal analysis, the coupled structure-pendulum-transducer system is reduced to a 3-DOF system, which acts as the basis for deriving the closed-form expressions for the optimal calibration.

Based on the normalized sextic characteristic equation of the system, a root locus method has been employed to achieve the optimal parameter calibration. The absorber spring stiffness and the equivalent stiffness have been calibrated using the equal modal damping ratio criteria. The equivalent mass and equivalent damping have subsequently been calibrated relative to the triple-root bifurcation point, with two scaling parameters introduced to place the roots slightly lower than the triple-root bifurcation point, thus leading to apparent optimal response mitigation. The complete procedure for the optimal calibration has been developed and shown in Figure 6.

The developed closed-form calibration procedure has been evaluated by both a simple 3-DOF model in the frequency domain and a highly coupled 14-DOF aero-hydro-servo-elastic wind turbine model in the time domain. From the dynamic amplification analysis (using the 3-DOF model), it is observed that the closed-form calibration procedure indeed leads to optimality, in terms of the desired flat plateau in the response amplitude curve as well as a low sensitivity against parameter changes. The three possible root loci by different values of the parameter $\alpha$ agree with the theoretical predictions, verifying the theoretical rigor of the calibration procedure. Furthermore, it is seen from the dynamic amplification analysis that the augmented pendulum
absorber outperforms the classic pendulum in reducing structural response, at the cost of slightly larger damper stroke.

Nonlinear simulations have been performed using the 14-DOF model subject to turbulent wind and irregular wave excitation, with the pendulum absorbers equipped. Both absorbers effectively suppress the tower SS vibration, while their damping effects on the tower FA vibration are much less pronounced due to the high aerodynamic damping present in this direction. Both absorbers are good candidates for vibration damping of offshore WT towers. The augmented pendulum absorber has a consistently better vibration damping effect (although the improvement is slight) on both tower SS and FA vibrations than the classic pendulum absorber, for all load cases considered. In line with the frequency-domain results, the augmented pendulum absorber has slightly larger damper stroke (which should not be a concern for WT towers). The maintenance issues (for example oil leakage) can also be improved by the proposed pendulum absorber.

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Appendix A. Optimal calibration of a pendulum absorber with a classic viscous damper

The optimal calibration of a pendulum absorber with a classic dashpot (the classic pendulum absorber) is briefly covered here, using the same calibration method as in Section 4. This facilitates the performance comparison between the proposed augmented pendulum absorber and the classic pendulum absorber.

The classic dashpot only introduces a damping force, and thus does not introduce an extra resonance as in the case of the shunted EM transducer. Therefore, the resulting system is a 2-DOF system with equations of motion written as

\[
\begin{align*}
\left( -\omega^2 m_s + k_s + k_a(1 + \Delta \nu)^2 + i\omega c_a \right) u_s - (k_a(1 + \Delta \nu) + i\omega c_a) u_a &= f_s \\
\left( -\omega^2 m_a + k_a + i\omega c_a \right) u_a - (k_a(1 + \Delta \nu) + i\omega c_a) u_s &= 0
\end{align*}
\]  

in which \( c_a \) is the linear viscous damping coefficient of the dashpot. From equations (45) and (46) the frequency response function is obtained as

\[
\frac{u_s}{f_s/k_s} = \frac{-p^2 \mu_a + ip\beta_a + \kappa_a}{p^3 \mu_a - p^2 (\mu_a + \kappa_a + \mu_a \kappa_a(1 + \Delta \nu)^2) + \kappa_a - ip\beta_a (p^2 (1 + \mu_a) - 1 - \kappa_a \Delta \nu^2)}
\]  

where the following dimensionless parameters have been introduced,

\[
p = \frac{\omega}{\omega_s}, \quad \mu_a = \frac{m_a}{m_s}, \quad \kappa_a = \frac{k_a}{k_s}, \quad \beta_a = \frac{c_a}{\sqrt{m_s k_s}}
\]
The characteristic equation of this 2-DOF system is obtained from denominator of (47) as

\[ \xi^4 - \frac{\xi^2}{\Omega^2} \left( 1 + \frac{k_a}{\mu_a} + k_a(1 + \Delta \nu)^2 \right) + \frac{1}{\Omega^2} \frac{k_a}{\mu_a} \]

\[- \frac{i \xi}{\Omega} \beta_a \left( \frac{1}{\mu_a} \left( 1 + \frac{k_a}{\mu_a} \Delta \nu^2 \right) \right) = 0 \]  

(49)

where the new dimensionless frequency ratio \( \xi = p/\Omega \) has been introduced from (23).

The root loci diagram of the classic pendulum absorber is similar to that of a classic TMD [34], while the optimal calibration formulas will in principle be slightly different due to the presence of the parameter \( \Delta \nu \) indicating the change of modal vibration amplitude of the two ends of the pendulum. When the damping ratio \( \beta_a \to 0 \) there are two real roots \( \xi_0 - \) and \( \xi_0 + \) on the real axis of the complex plane, while there is only a single root \( \xi = 1 \) on the real axis for \( \beta_a \to \infty \).

By applying the inverse point condition to the characteristic equation (49) in the limit \( \beta_a \to 0 \), the ratio between the coefficients to the 4th order and the constant should be 1. This leads to exactly the same equation as in (29). Conversely for \( \beta_a \to \infty \), the condition of a single unit root leads to the exactly same equation as in (32). These results are expected since (29) and (32) are merely related to the frequency tuning of the pendulum absorber, which should be the same whether a dashpot or a shunted transducer is attached.

The optimal damping ratio is derived based on investigating the double-root bifurcation point by a similar procedure as for the triple-root bifurcation point in subsection 3.3. This initially results in the damping ratio

\[ \beta_a^* = \Omega_r \sqrt{8(C_0 - 1) \frac{\mu_a}{1 + \mu_a}} \]  

(50)

at the double-root bifurcation point, where \( \Omega_r \) is given in (33) and \( C_0 \) is defined by (36). As previously noted the maximum damping at the bifurcation point is associated with undesirable response amplification, whereby a numerical optimization is again applied by introducing a scaling parameter as for the shunted absorber in subsection 3.3. It should be noted that the criterion here is to minimize the peak frequency response of the structure, corresponding to \( H_{\infty} \) optimization and thus the Den Hartog damping calibration [51]. This is slightly different from the augmented absorber in subsection 3.3 where \( \alpha = 0.5 \) (which does not lead to the absolute minimum of the peak frequency response) is chosen, resulting in both the flat plateau and the sufficient margin away from the steep increase of structural amplitude. The optimal damping ratio for the viscous damper turns out to be

\[ \beta_a = \frac{3}{5} \left( 1 + \frac{\Delta \nu}{10} \right) \beta_{a^*} = \frac{3}{5} \left( 1 + \frac{\Delta \nu}{10} \right) \Omega_r \sqrt{8(C_0 - 1) \frac{\mu_a}{1 + \mu_a}} \]  

(51)

The complete calibration procedure of the classic pendulum absorber is summarized in the flow chart of Fig. 14.

References

Figure 14: Flowchart of the complete calibration procedure for a classic pendulum absorber.

2020; 210:209.
[8] Celik S, Basu B, Nielsen SRK. Realtime hybrid aeroelastic simulation of wind turbines with various types of 
Massachusetts Institute of Technology, 2005.
[16] Hageodd NW, von Flotow A. Damping of structural vibrations with piezoelectric materials and passive electrical 
and Acoustics, 2008; 130(4): 041003.
[22] Zhi S, Shen W, Qian X. Dynamic analysis between an electromagnetic shunt damper and a tuned mass damper. 
[24] Stabile A, Aghetti GS, Richardson G, Smet G. Design and verification of a negative resistance electromagnetic 
Smart Materials and Structures. 2015; 24(5): 055015.
[28] Högberg J. Vibration control by piezoelectric proof-mass absorber with resistive-inductive shunt. Mechanics of 
[29] Wu SY. Piezoelectric shunts with a parallel R-L circuit for structural damping and vibration control. Proceedings 


Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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**Jan Høgsberg**: Conceptualization, Investigation, Supervision, Writing- Original draft, Writing - review & editing