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Electrical Conductivity Measurement on Metallic Materials With a Cylindrical Resonator

Yunfeng Dong, Member, IEEE, Morten Stendahl Jellesen, Rune Juul Christiansen, Jesper Høvelskov, Jörgen Sundgren, and Kaj Bjarne Jakobsen

Abstract—This article presents a novel method for measuring the electrical conductivity of metallic materials using a cylindrical resonator. The theory and the process of measuring the electrical conductivity are explained. The cylindrical resonator is designed and proved using full-wave electromagnetic simulations. A prototype operating at 10.5 GHz is fabricated, and the scattering parameters are measured. The quality factor of the cylindrical resonator is calculated at the first step from the measured scattering parameters, and the electrical conductivity of the metallic material is derived using Maxwell’s equations. By replacing the top cover of the cylindrical resonator with different materials or metallic sheets, the electrical conductivities can be measured easily. For validating the method, three top covers and six metallic sheets are fabricated and tested. The measured electrical conductivities are compared with other published results and typical values. Besides, by adjusting the operating frequency of the cylindrical resonator, the proposed method can be applied to determine the electrical conductivity at specific frequencies.

Index Terms—Cylindrical resonator, electrical conductivity, Maxwell’s equations, quality factor, scattering parameters, surface resistivity.

I. INTRODUCTION

ELECTRICAL conductivity (σ) of a material, referring to the inverse of electrical resistivity (ρ), is a fundamental property that indicates the ability of conducting electric current. Though the typical values of the electrical conductivities for metallic materials such as copper, aluminum, brass, and steel have been widely used for decades, they are mainly characterized by direct current (DC) using Ohm’s law, whereas, for customized materials or applications operating at microwave frequencies, the electrical conductivity varies and individual characterization becomes necessary [1]–[3]. In addition, due to skin effect, metallic sheets with thin cladding layers, engineered composites, and alloys may exhibit different properties [4]–[8]. As is reported in [1], a TE_{011} mode cavity resonator is used for characterizing the electrical conductivities of metallic sheets. According to [9]–[12], electrical conductivities can be characterized by eddy current using Faraday’s law of induction. Other methods for measuring the electrical conductivity, permittivity (ε), and permeability (μ) are also published in the literature [13]–[17]. However, these methods either rely on complex structures or consist of dedicated components such as dielectric resonator, spacer, loop feed, magnet, and coil. Fig. 1 shows the overview of the proposed cylindrical resonator. In the closed cavity, electric waves reflect back and forth between the conductor walls forming a standing wave of TE_{111} mode. By replacing the top cover with different materials or metallic sheets with thin cladding layers, the electrical conductivities can be measured at the resonant frequency (f). While having a simple structure, the cylindrical resonator also provides a low-cost and easy way of measuring the electrical conductivity of metallic materials at microwave frequencies. Furthermore, by scaling the cavity and replacing SMA connectors with waveguides, the proposed method can be potentially used for characterizing electrical conductivities at millimeter-wave and terahertz frequencies.

In this article, a novel method for characterizing the electrical conductivity of metallic materials at microwave frequencies is addressed. The cylindrical resonator is designed and proved...
using full-wave electromagnetic simulations in ansys electronics desktop (EDT, formerly known as HFSS). A prototype is fabricated, and different metallic materials are tested. By measuring the scattering parameters of the proposed TE_{111} mode cylindrical resonator, the quality factor ($Q$) can be calculated from the measured half-power (3-dB) bandwidth. In the meanwhile, the quality factor of the cylindrical resonator is determined by the angular frequency ($\omega$), the average stored energy ($W$), and the power loss ($P$) due to using nonperfect metallic materials. As a result, the electrical conductivity of the conductor walls of the cylindrical resonator can be derived by the measured quality factor using Maxwell’s equations.

The rest of this article is organized as follows. In Section II, the theory of deriving the electrical conductivities of the conductor walls from the quality factor of a cylindrical resonator by using Maxwell’s equations is introduced in detail. In Section III, the fabrication of the proposed cylindrical resonator operating at 10.5 GHz is presented. The assembly structure and the way of measuring metallic materials are illustrated. In Section IV, the experimental results are presented and compared with the typical values as well as published results. In Section V, the accuracy and the uncertainties of the characterized electrical conductivities are discussed. Finally, Section VI concludes this article.

II. THEORY

A. Electrical Conductivity

As shown in Fig. 2, the proposed cylindrical resonator is a hollow closed conductor, where the electromagnetic waves form a standing wave of TE_{111} mode inside the cavity at the resonant frequency. When being filled with air, the relative permeability ($\mu_r$) and the relative permittivity ($\varepsilon_r$) of the cylindrical resonator are assumed to be 1, and the variation within frequency is neglected in the calculation.

In this case, the radius and the height of the air-filled cylindrical resonator are represented by $a$ and $d$, respectively. Based on Maxwell’s equations, the electric field ($E$) and magnetic field ($H$) components of the TE_{nm} mode in the $\rho\varphi z$-coordinate system can be written as

$$E_\rho = \frac{jk\mu a^2 n H_0}{P'_{nm\rho}} J_n \left( \frac{P'_{nm\rho}}{a} \right) \sin(n\varphi)\sin \left( \frac{l\pi z}{d} \right)$$

$$E_\varphi = \frac{jk\mu a H_0}{P'_{nm\rho}} J_n \left( \frac{P'_{nm\rho}}{a} \right) \cos(n\varphi)\sin \left( \frac{l\pi z}{d} \right)$$

$$E_z = 0$$

by using Maxwell’s equations. In (1c), $J_n$ is the first kind of Bessel function, $J'_n$ represents the derivative of $J_n$, and $P'_{nm}$ refers to the $n$th root of $J'_n$. Other parameters, including the propagation constant ($\beta$), the wave number ($k$), the intrinsic impedance ($\eta$), and the arbitrary amplitude ($H_0$), can be calculated as

$$\beta = \sqrt{k^2 - \left( \frac{P'_{nm\rho}}{a} \right)^2}$$

$$k = 2\pi f \sqrt{\varepsilon_r} = 2\pi f \sqrt{\mu_r \varepsilon_0 \varepsilon_r}$$

$$\eta = \sqrt{\frac{\mu_r}{\varepsilon_r}} = \sqrt{\frac{\mu_0 H_0}{\varepsilon_0 \varepsilon_r}}$$

$$H_0 = -2j A^+$$

where $\mu_0$ and $\varepsilon_0$ represent the permeability and permittivity of free space, respectively, and $A^+$ is the arbitrary amplitudes of the forward traveling wave.

For the proposed TE_{111} mode cylindrical resonator, the resonant frequency ($f_{111}$) can be expressed as

$$f_{111} = \frac{c}{2\pi \sqrt{\varepsilon_0 \varepsilon_r}} \sqrt{\left( \frac{P'_{111}}{a} \right)^2 + \left( \frac{\pi}{d} \right)^2}$$

By using the electric field components introduced in (1) as

$$W = W_e + W_h = 2W_e$$

$$W = \frac{\varepsilon}{2} \int_0^d \int_{\varphi=0}^{2\pi} \int_{\rho=0}^{a} \left( |E_\rho|^2 + |E_\varphi|^2 \right) \rho d\rho d\varphi dz$$
If the conductor wall of a resonator is made by a perfect electric conductor (PEC), there will not be any conductor loss due to infinite electrical conductivity. That means the corresponding power loss also becomes zero and results in an infinite quality factor. As shown in Fig. 4, nonperfect metallic materials are used for the cylindrical resonator in reality. In this case, due to different metallic materials, the side and bottom surfaces have a surface resistivity of $R_{S1}$, while it is $R_{S2}$ for the top surface. Since the surface resistivity is the inverse of electrical conductivity, it can be written as

$$R_{S1,2} = \sqrt{\frac{2\pi \eta}{2\sigma_{1,2}}}$$

(5)

where $\sigma_{1,2}$ represents the electrical conductivities of the corresponding metallic materials. In other words, once the surface resistivity is known, the electrical conductivity can be found.

For calculating the power loss of the cylindrical resonator, the magnetic field components introduced in (1) also need to be rewritten for each surface as

**Side surface:**

$$H_{\varphi,(z=d)} = \frac{\beta a^2 n H_0}{P_{nm}^2} J_n(P_{nm}' a) \sin(n\varphi) \cos(l\pi)$$

(6a)

$$H_{z,(z=d)} = H_0 J_n'(P_{nm}' a) \sin(l\pi)$$

(6b)

**Bottom surface:**

$$H_{\rho,(z=0)} = \frac{\beta a H_0}{P_{nm}^2} J_n'(P_{nm}' a) \cos(n\varphi)$$

(6c)

$$H_{\varphi,(z=0)} = -\frac{\beta a^2 n H_0}{P_{nm}^2} J_n(P_{nm}' a) \sin(n\varphi) \cos(l\pi)$$

(6d)

**Top surface:**

$$H_{\rho,(z=a)} = \frac{\beta a H_0}{P_{nm}^2} J_n'(P_{nm}' a) \cos(n\varphi) \cos(l\pi)$$

(6e)

As a result, by taking into account the surface resistivities of the corresponding metallic materials, the power loss ($P_1$) of a TE$_{nm}$ mode cylindrical resonator can be expressed in terms of three surface integrals using the magnetic field components introduced in (6) as

$$P_1 = \frac{R_{S1}}{2} \int_{z=0}^{d} \int_{\varphi=0}^{2\pi} \left( |H_{\varphi,(\rho=a)}|^2 + |H_{z,(\rho=a)}|^2 \right) \rho d\rho d\varphi$$

$$+ \frac{R_{S2}}{2} \int_{\rho=0}^{\rho=a} \int_{\varphi=0}^{2\pi} \left( |H_{\rho,(z=0)}|^2 + |H_{\varphi,(z=0)}|^2 \right) \rho d\rho d\varphi$$

$$+ \frac{R_{S2}}{2} \int_{\rho=0}^{\rho=a} \int_{\varphi=0}^{2\pi} \left( |H_{\rho,(z=d)}|^2 + |H_{\varphi,(z=d)}|^2 \right) \rho d\rho d\varphi$$

$$= \frac{R_{S1}}{2} \pi H_0^2 J_n^2(P_{nm}') \left( \frac{na}{P_{nm}'} \right)^2 \rho d\rho \left( 1 + \frac{\beta a^2 n^2}{P_{nm}^4} \right)$$

(7a)

$$+ \frac{R_{S1}}{2} \pi H_0^2 J_n^2(P_{nm}') \left( \frac{\beta a^4}{2P_{nm}^2} \right) \left( 1 - \frac{n^2}{P_{nm}^2} \right)$$

$$+ \frac{R_{S2}}{2} \pi H_0^2 J_n^2(P_{nm}') \left( \frac{\beta a^4}{2P_{nm}^2} \right) \left( 1 - \frac{n^2}{P_{nm}^2} \right)$$

(7b)

So far, the total stored energy and the power loss of a TE$_{nm}$ mode cylindrical resonator are derived in (4c) and (7b), respectively. The quality factor ($Q$) can then be written as

$$Q = \frac{\omega W}{P_1} = \frac{k}{\eta} \frac{W}{P_1}$$

(8a)

$$Q = k \frac{\varepsilon k^2 a^2 H_0^2 \rho d}{8P_{nm}^2} \left[ 1 - \left( \frac{n}{P_{nm}'} \right)^2 \right] J_n^2(P_{nm}') \rho d$$

(8b)

where $R_{S1}$ and $R_{S2}$, representing the corresponding surface resistivities, are the only unknowns in the equation once the resonant mode and the dimensions of the cylindrical resonator are specified. By simplifying the equation and isolating $R_{S2}$, (8b) can be rewritten in terms of $Q$ and $R_{S1}$ as

$$R_{S2} = \left[ \frac{k^3 \eta a^4 d \left( 1 - \frac{n^2}{P_{nm}^2} \right)}{2P_{nm}^2 Q} - R_{S1} d \left( 1 + \frac{\beta a^2 n^2}{P_{nm}^4} \right) \right]$$

(9)
where \( Q \) can be calculated from the measured half-power bandwidth, and \( R_{S1} \) needs to be provided or characterized. Then, the electrical conductivity of the top surface (\( \sigma_2 \)) can be calculated by rewriting (5) as

\[
\sigma_2 = \frac{2\pi f \mu}{2R_{S2}^2}. \tag{10}
\]

As shown in Fig. 4, \( R_{S1} \) in (9) represents the surface resistivity of the side and bottom surfaces of the cylindrical resonator. Instead of directly using the typical value or being achieved from the data sheet, it can also be characterized using the proposed method by replacing the metallic material of the top surface with the same metallic material as the side and bottom surfaces. In this case, the cylindrical resonator is covered by homogeneous conductor walls with a surface resistivity of \( R_{S1} \). By substituting \( R_{S2} \) with \( R_{S1} \) in (9b) and simplifying the equation, \( R_{S1} \) can be written as

\[
R_{S1} = \frac{k^3 \eta d (1 - \frac{n^2}{P_{nm}^2})}{4 P_{nm}^2 Q_0} \left[ \beta a^4 \left( 1 - \frac{n^2}{P_{nm}^2} \right) \right]^{-1} \tag{11}
\]

where \( Q_0 \) refers to the corresponding quality factor when the top cover of the proposed cylindrical resonator has a surface resistivity of \( R_{S1} \).

### B. Quality Factor

Though the quality factor of the cylindrical resonator can be calculated from the measured scattering parameters using half-power bandwidth, it is the loaded quality factor that cannot be put back into (9) and (11) directly. When two SMA connectors are inserted into the cavity, even if they are loosely coupled, the external quality factor must be taken into account. The relationship among loaded \( (Q_{\text{load}}) \), unloaded \( (Q_{\text{unload}}) \), and external \( (Q_{\text{ext}}) \) quality factors of the proposed cylindrical resonator can be expressed as

\[
\frac{1}{Q_{\text{load}}} = \frac{1}{Q_{\text{unload}}} + \frac{2}{Q_{\text{ext}}} \tag{12}
\]

where the last term refers to two identical SMA connectors. In practice, \( Q_{\text{unload}} \) is found using eigenmode simulations in EDT. Besides, the scattering parameters are measured at the beginning for the cylindrical resonator with homogeneous conductor walls, and the quality factor is calculated. By putting both \( Q_{\text{unload}} \) and \( Q_{\text{load}} \) into (12), \( Q_{\text{ext}} \) can be rewritten as

\[
Q_{\text{ext}} = \frac{2}{Q_{\text{load}}^{-1} - Q_{\text{unload}}^{-1}}. \tag{13}
\]

Once the external quality factor for the SMA connectors is determined, the value is kept constant and can be used for the following measurements. As a result, the quality factors involved in (9) and (11) can be calculated as

\[
Q = \frac{1}{Q_{\text{meas}}^{-1} - 2Q_{\text{ext}}^{-1}} \tag{14}
\]

where \( Q_{\text{meas}} \) represents the measured quality factor of the proposed cylindrical resonator with specific top covers.

### III. Prototype Fabrication

In order to validate the proposed cylindrical resonator as well as the method for characterizing the electrical conductivity, a prototype operating at 10.5 GHz was fabricated. Fig. 5 demonstrates the assembly structure of the proposed TE\(_{111} \) mode cylindrical resonator. The resonator is divided into two parts, where the top cover is fixed to the bottom cavity using four screws. Two SMA connectors are inserted and cut along the side surface of the cylindrical resonator for a loosely coupling. In addition, for measuring the metallic sheets, the sample can be clamped between the top cover and the bottom cavity forming a sandwich structure. The bulge on the top cover and the recess on the bottom cavity ensure the alignment and contact of the metallic sheets.

The detailed dimensions of the proposed TE\(_{111} \) mode cylindrical resonator and the metallic sheet are illustrated in Fig. 6. The radius (a) and the height (d) of the cylindrical resonator are 12 and 20 mm, respectively. The SMA connectors are located halfway between the top and bottom surfaces. The metallic sheets for characterizing the electrical conductivity must be large enough so that the top surface of the cylindrical resonator can be fully covered. Besides, the positions of the screws on the top cover need to be taken into account. As a consequence, the metallic sheets turn out to have a rectangular shape with corner cuts.
Once the dimensions are determined, the resonant frequency \( f_0 \) of the proposed TE\(_{111} \) mode cylindrical resonator can be estimated using (3) as

\[
f_0 = \frac{299792458}{2\pi \sqrt{1 \cdot 1}} \sqrt{(1.841 - 0.012)^2 + (0.02)^2} = 10.48 \text{ GHz.} \quad (15)
\]

When measuring the electrical conductivity of the metallic sheets, the thickness of the samples should be at least twice of the skin depth at the resonant frequency. Otherwise, the measured electrical conductivity might be affected by the top cover material. The skin depth \( \delta \) for nonmagnetic \( (\mu_r = 1) \) materials can be written as

\[
\delta = \sqrt{\frac{2}{\omega \mu \sigma}} = \sqrt{\frac{1}{\pi f_0 \mu_0 \mu_r \sigma}} = \sqrt{\frac{1}{\pi \cdot 10.48 \times 10^9 \cdot 4\pi \times 10^{-7} \cdot 1 \cdot \sigma}} \quad (16)
\]

where \( \sigma \) represents the electrical conductivity of the material. The relationship between the skin depth and the electrical conductivity is presented in Fig. 7. The markers on the solid line indicate the calculated skin depths of the corresponding metallic materials using their typical values of the electrical conductivity \([18],[19]\). For the material with a higher electrical conductivity, a smaller skin depth is achieved. In this case, for example, a thin cladding layer outside the original metallic material with a thickness of several micrometers (\( \mu m \)) should be enough to improve the electrical performance.

Fig. 8 demonstrates the fabricated cylindrical resonator. The prototype was fabricated using aluminum blocks by a milling process. Besides, extra brass and copper top covers were fabricated as references for comparing the characteristic electrical conductivity of the metallic sheets.

The fabricated metallic sheets shown in Fig. 9 include material samples made by steel (S235), stainless steel (304), brass, aluminum, and copper. These samples were cut for fitting the cylindrical resonator, and their measured thicknesses are much more than the corresponding skin depth at the resonant frequency. In order to have a lower and identical surface roughness, the raw metallic sheets were polished by using an abrasive paper with a grit designation of P2000. As a supplement, a sample made by the aluminum foil used in the kitchen with a thickness of 0.01 mm is also included.

**IV. EXPERIMENTAL RESULTS**

As shown in Fig. 10, the cylindrical resonator is connected to an Agilent E8361A vector network analyzer (VNA), which measures the scattering parameters of the device under test (DUT). In order to improve the accuracy, a two-port short-open-load-through calibration is carried out prior to the measurements using an Agilent 85052D calibration kit, so that the effects of the cables as well as the connectors are calibrated out from the measured scattering parameters.

At the beginning, the external quality factor of the SMA connector \( Q_{ext} \) needs to be found so that it can be used for the following measurements. The aluminum top cover is used, and the scattering parameters were measured. As shown in Fig. 11, the solid line represents the magnitude of the measured \( S_{21} \). In addition, \( f_0, f_1, \) and \( f_2 \) correspond to the resonant...
Meanwhile, the unloaded quality factor \( Q_{\text{unload}} \) is then calculated as

\[
Q_{\text{unload}} = \frac{f_0}{f_2 - f_1} = \frac{10474952000}{10476940000 - 10472964000} = 2634.5.
\]

(17)

Meanwhile, the unloaded quality factor \( Q_{\text{unload}} \) is estimated using an eigenmode simulation in EDT. By taking into account the electrical conductivity and the surface roughness of the fabricated cylindrical resonator, a value of 7800 is achieved for \( Q_{\text{unload}} \). Thus, \( Q_{\text{ext}} \) can be calculated by (13) as

\[
Q_{\text{ext}} = \frac{2}{\frac{1}{2634.5} - \frac{1}{7800}} = 7956.5.
\]

(18)

After that, the scattering parameters of the fabricated cylindrical resonator with different top covers and metallic sheets are measured so that the measured quality factors \( Q_{\text{meas}} \) are calculated from the corresponding half-power bandwidth. Since the external quality factor of the SMA connector is found by (18) and kept constant, the quality factor \( Q \) for each measurement can then be calculated by (14). Table I concludes the corresponding frequencies for the measured half-power bandwidth and the calculated quality factor.

During the characterization process, by using the calculated quality factors, the surface resistivity of aluminum is first derived by (11) and used as \( R_{\text{S1}} \) putting back into (9). In this case, the surface resistivities of other top covers as well as metallic sheets can be derived by (9) afterward. At the last step, the surface resistivities are converted into the electrical conductivities by (10). The characterized electrical conductivities are also summarized in Table I and compared with other published results in the literature.

Among all tested material samples, the metallic sheet made by steel (S235) achieves the lowest electrical conductivity of 0.52 MS/m. In addition, a similar performance can be observed for the metallic sheet made by stainless steel (304), which shows an electrical conductivity of 0.54 MS/m. For the material samples made by brass, the characterized electrical conductivities are 22.91 and 21.90 MS/m for the metallic sheet and top cover, respectively. By contrast, due to a better surface roughness, the foil exhibits a higher electrical conductivity of 29.25 MS/m. As for the metallic sheet and top cover made by copper, in comparison with other tested material samples, they achieve the highest electrical conductivities of 57.48 and 59.06 MS/m, respectively.

When compared with the characterized electrical conductivities in [1] and [2], due to different characterization methods, frequencies, and material properties, a reasonable agreement has been achieved. For the method introduced in [1], a TE\(_{011}\) mode cavity resonator is used for characterizing the electrical conductivity of metallic materials. The scattering parameters are measured using a VNA through two coupling loops on the top surface of the cavity. Besides, \( Q_{\text{unload}} \) of the cavity resonator is calculated from the measured insertion loss and \( Q_{\text{load}} \), while the conductor loss of the cavity resonator is not taken into account separately. For circular waveguides, TE\(_{01}\) mode not only has a low attenuation at microwave frequencies but also results in simplified field distributions. However, TE\(_{01}\) mode is not the dominant mode of circular waveguides so that the proposed TE\(_{011}\) mode cavity resonator has larger dimensions. When operating at 8.4 GHz, the diameter of the tested material samples is around 100 mm. By contrast, the proposed TE\(_{111}\) mode cylindrical resonator shown in Fig. 6 is more compact. When operating at 10.5 GHz, the length and the width of the tested material samples are 33 and 26 mm, respectively. Furthermore, the diameter of the tested material samples can be reduced to 25 mm when necessary.

In [2], a tunable cavity resonator with a dielectric resonator and a quartz spacer is proposed for characterizing the electrical conductivity of metallic materials. The input impedance is measured through a SMA connector with a loop, which is coupled to the magnetic field in the cavity. By using a tuning plunger controller, the measurement is performed over a small range of
TABLE I
MEASURED FREQUENCIES, CALCULATED QUALITY FACTOR, AND CHARACTERIZED ELECTRICAL CONDUCTIVITY AT 10.5 GHz

<table>
<thead>
<tr>
<th>Material</th>
<th>Type</th>
<th>$f_0$ (GHz)</th>
<th>$f_1$ (GHz)</th>
<th>$f_2$ (GHz)</th>
<th>$Q_{\text{meas}}$</th>
<th>$Q^*$ (MS/m)</th>
<th>$\sigma$ (MS/m)</th>
<th>$[1]^*$ (MS/m)</th>
<th>$[2]^*$ (MS/m)</th>
<th>Typ.$^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>sheet, S235 grade</td>
<td>10.478 875 60</td>
<td>10.476 106 80</td>
<td>10.481 698 00</td>
<td>1892.3</td>
<td>3609.0</td>
<td>0.52</td>
<td>na</td>
<td>na</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>sheet, 304 stainless</td>
<td>10.475 113 40</td>
<td>10.472 367 50</td>
<td>10.477 903 10</td>
<td>1907.4</td>
<td>3664.3</td>
<td>0.54</td>
<td>na</td>
<td>na</td>
<td>1.1</td>
</tr>
<tr>
<td>Brass</td>
<td>sheet</td>
<td>10.476 176 00</td>
<td>10.474 140 25</td>
<td>10.478 204 25</td>
<td>2573.1</td>
<td>7284.6</td>
<td>12.93</td>
<td>13.2</td>
<td>15.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>top cover</td>
<td>10.475 004 80</td>
<td>10.472 975 30</td>
<td>10.477 036 70</td>
<td>2579.2</td>
<td>7333.7</td>
<td>13.65</td>
<td>na</td>
<td>na</td>
<td>15.0</td>
</tr>
<tr>
<td>Aluminium</td>
<td>sheet</td>
<td>10.476 173 10</td>
<td>10.474 182 90</td>
<td>10.478 165 70</td>
<td>2631.9</td>
<td>7777.2</td>
<td>22.91</td>
<td>23.1</td>
<td>21.3</td>
<td>38.2</td>
</tr>
<tr>
<td></td>
<td>top cover</td>
<td>10.475 111 00</td>
<td>10.473 122 30</td>
<td>10.477 108 70</td>
<td>2627.7</td>
<td>7740.4</td>
<td>21.90</td>
<td>23.1</td>
<td>21.3</td>
<td>38.2</td>
</tr>
<tr>
<td></td>
<td>foil</td>
<td>10.488 573 20</td>
<td>10.486 597 20</td>
<td>10.490 564 00</td>
<td>2654.0</td>
<td>7973.0</td>
<td>29.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Copper</td>
<td>sheet</td>
<td>10.476 475 00</td>
<td>10.474 535 50</td>
<td>10.478 409 00</td>
<td>2700.8</td>
<td>8411.1</td>
<td>57.48</td>
<td>46.8</td>
<td>55.4</td>
<td>58.1</td>
</tr>
<tr>
<td></td>
<td>top cover</td>
<td>10.475 054 30</td>
<td>10.473 120 20</td>
<td>10.476 996 50</td>
<td>2702.3</td>
<td>8425.8</td>
<td>59.06</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^*$ $Q_{\text{ext}} = 7956.5$ is used for calculations.

$^*$ In [1], the measurements were carried out at 8.4 GHz.

$^*$ In [2], the measurements were carried out at 3.5 GHz.

$^*$ The conductivities reported in [18] and [19] are used as the typical values.

The frequencies. However, it has a complex structure, and $Q_{\text{unload}}$ of the cavity resonator is obtained using a finite integration algorithm. The quartz spacer is also assumed to be lossless, and an estimated value is used as the electrical conductivity of the cavity resonator.

The electrical conductivities reported in [18] and [19] are referenced as the typical values in Table I. Since the measurements are carried out at dc using high-purity materials, some of the typical values are much higher than the characterized electrical conductivities. In particular, for aluminum, including the foil sample, the characterized electrical conductivities range from 21.3 to 29.25 MS/m, while the typical value is 38.2 MS/m. At microwave frequencies, the electrical conductivities are mainly affected by skin depth, impurities, oxidization, and surface roughness. As a result, for critical applications, instead of directly using the typical values, the electrical conductivities of the customized materials or samples are usually characterized at the operating frequency.

V. ACCURACY AND UNCERTAINTY

For the proposed method, as an essential parameter for extracting the external quality factor ($Q_{\text{ext}}$), the unloaded quality factor ($Q_{\text{unload}}$) of the cylindrical resonator needs to be determined at the beginning, and its accuracy also affects the following characterization process. However, the estimation of $Q_{\text{unload}}$ is solely based on an eigenmode simulation in EDT. Under this circumstance, the simulation accuracy and the uncertainty of the characterized electrical conductivities need to be taken into consideration. In order to improve the accuracy of the eigenmode simulation, aluminum is assigned as the material of the cylindrical resonator, while the roughness of the inner surfaces is set to be around 0.35 μm. Though the actual electrical conductivity and surface roughness of the fabricated cylindrical resonator might be slightly different from the values used for the eigenmode simulation, it represents an overall and comprehensive effect, which models the imperfections of the cylindrical resonator including fabrication tolerance, surface roughness, and structural discontinuity between the top cover and the bottom cavity.

When the same material and surface roughness settings are used for simulating the cylindrical resonator with SMA connectors, the accuracy can be evaluated by comparing the loaded quality factors that are calculated from the measured and simulated scattering parameters. The dashed line in Fig. 11 represents the magnitude of the simulated $S_{21}$. The simulated loaded quality factor ($Q'_{\text{load}}$) is then calculated as

$$Q'_{\text{load}} = f'_0 \frac{f'_0}{f'_2 - f'_1} = 10474740000 \frac{10476702600 - 10472772300}{10472772300} = 2665.1$$

where $f'_0$, $f'_1$, and $f'_2$ refer to the simulated resonant frequency and lower and upper frequencies of the half-power bandwidth, respectively. In comparison with the measured loaded quality factor ($Q_{\text{load}}$) calculated by (17), a good agreement is achieved. Since the difference between $Q_{\text{load}}$ and $Q'_{\text{load}}$ is kept less than 1.2%, the material and surface roughness settings used in the simulations are validated. As a result, $Q_{\text{unload}}$ with a simulated value of 7800 is used for the proposed method.

The characterized electrical conductivities listed in Table I are calculated from the measured scattering parameters, while $Q_{\text{ext}}$ is also involved and extracted using $Q_{\text{unload}}$ of the cylindrical resonator. Thus, the uncertainty caused by the estimation of $Q_{\text{unload}}$ is transferred to the characterized electrical conductivities. However, the potential effect and the corresponding relationship are not straightforward. Fig. 12 illustrates the variations of the characterized electrical conductivity in terms of $Q_{\text{unload}}$ for different material samples. When $Q_{\text{unload}}$ changes within
the uncertainties for aluminum and brass are ±2.7% and ±3.9%, respectively. With the highest electrical conductivities at millimeter-wave and terahertz frequencies, the proposed cylindrical resonator operating at 10.5 GHz has been fabricated using aluminum blocks by a milling process. In order to validate the method, three top covers and six metallic sheets have been fabricated and tested. The characterized electrical conductivities are compared with other published results in the literature, and a reasonable agreement has been achieved. The uncertainties of the characterized electrical conductivities are also discussed. Furthermore, by adjusting the resonant frequency of the cylindrical resonator, the proposed method can be potentially used for characterizing electrical conductivities at millimeter-wave and terahertz frequencies.

VI. CONCLUSION

In this article, a novel method for characterizing the electrical conductivity of metallic materials using a cylindrical resonator has been presented. The average stored energy in the cylindrical resonator and the power loss due to nonperfect metallic materials are first derived using Maxwell’s equations. The quality factor is then expressed in terms of the surface resistivities of the top, bottom, and side surfaces. Under this circumstance, once the quality factor is measured and the surface resistivity of the bottom and side surfaces are known, the electrical conductivity of the top surface can be characterized at the resonant frequency. The proposed method provides an easy and realistic way of measuring the electrical conductivity of metallic materials at microwave frequencies. Due to skin effect, the method is suitable for testing not only conventional top covers and metallic sheets but also thin material samples such as foil and cladding layers. As a prototype, a TE_{111} mode cylindrical resonator operating at 10.5 GHz has been fabricated using aluminum blocks by a milling process. In order to validate the method, three top covers and six metallic sheets have been fabricated and tested. The characterized electrical conductivities are compared with other published results in the literature, and a reasonable agreement has been achieved. The uncertainties of the characterized electrical conductivities are also discussed. Furthermore, by adjusting the resonant frequency of the cylindrical resonator, the proposed method can be potentially used for characterizing electrical conductivities at millimeter-wave and terahertz frequencies.

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REFERENCES

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