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Evolution of dynamic properties of cross anisotropic sand subjected to stress anisotropy

Mostafa Zamanian¹, Meghdad Payan², Fardin Jafarzadeh³, Navid Ranjbar⁴, Kostas Senetakis^{5*}

ABSTRACT

Understanding the evolution of dynamic properties of sand subjected to induced anisotropy is indispensable for an accurate seismic analysis of geo-structures. In this study, several bender element and hollow cylinder experiments along various anisotropic stress paths are performed on two types of sands to evaluate the soil dynamic properties over a wide range of small to large strain levels. Accordingly, the influences of both the major principal stress direction (α -direction) and the intermediate principal stress, $b=(\sigma_2-\sigma_3)/(\sigma_1-\sigma_3)$, are examined. The data suggested that α -direction affects both stiffness and damping ratio, especially at higher confining pressures. However, b -parameter has a little influence on stiffness and fairly negligible impact on damping ratio. Based on the experimental results, an empirical model published in the literature is extended by introducing correction factors so as to incorporate the significant contribution of induced anisotropy into the predictions of shear stiffness and damping ratio of sands.

Keywords: Stiffness; Damping ratio; Stress anisotropy; Major principal stress direction; Intermediate principal stress

1. INTRODUCTION

Inherently anisotropic structure of granular materials is commonly formed during the deposition process where particles with variable shapes would be arranged to be in their most stable position. This anisotropic structure continuously varies when the soil is subjected to

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induced shear strains. Thus, the evolution of soil structure, which is of great significance to predict soil behavior, could be quite distinct under different anisotropic stress states or along various stress paths. Several constitutive models have been developed to predict the response of an anisotropic soil fabric subjected to specific loading scenarios, but only a few models have considered the evolution of soil fabric during shearing (Dafalias et al. 2004; Jafarzadeh et al. 2008; Storesletten 1998). A number of attempts have been made to overcome this deficiency by incorporating rotational or kinematic hardening concepts into the constitutive models (Gajo and Wood 2001; Li 2002; Rahimi et al. 2016). However, it has been shown that these models could only reasonably consider the loading history, but are not capable of incorporating the evolution of fabric anisotropy. Recently, there have been a few constitutive models (e.g., Tager sand) developed to consider the evolution of soil fabric during anisotropic stress state adopting a progressive calibration procedure (Gao et al. 2014; Tasiopoulou and Gerolymos 2016). These models have a high potential to predict the sand behavior by incorporating Rowe's dilatancy theory, the critical state concept, and combining the bounding surface plasticity concept with Bouc–Wen type hysteresis. The evolution of different properties associated with the physical structure of sand was an essential aspect of the corresponding models. However, the coupling effect of inherent and induced anisotropy on the evolution of soil dynamic parameters during shearing has not been well investigated yet. Discrete element modeling (DEM) has also been appropriately implemented to simulate the fabric evolution of anisotropic structure through strain–displacement–fabric relationships which, in turn, have established analytical macro–micro relationships in granular mechanics (Kruty and Rothenburg 2019; Wang *et al.* 2017, 2020).

Several experimental studies have been performed to investigate the relevant parameters affecting the inherent anisotropic structure of soils by measuring their small strain parameters along different directions (Wang and Mok, 2008; Gu *et al.*, 2013; Li and Zeng, 2014; Payan *et*

al., 2016a; Senetakis *et al.*, 2017; Payan and Chenari, 2019; Payan and Senetakis, 2019). The effect of stress anisotropy on the inherent anisotropic structure of the soil has also been thoroughly examined through measuring small-to-medium strain parameters along various stress paths (Payan *et al.*, 2016a, 2017; Senetakis and Li, 2017; Prashant *et al.*, 2019; Jafarian and Javdanian, 2020; Le *et al.*, 2020). Moreover, several DEM studies have been performed to examine the behavior of granular materials subjected to stress anisotropy (Yuan *et al.*, 2019; Liu *et al.*, 2020; Shi *et al.*, 2020). However, determination of continuous evolution of soil structure and large-strain soil parameters, such as friction angle, secant shear modulus and damping ratio, has evidently proven to be challenging due to the coupling effect of inherent and induced anisotropy (Guoxing *et al.*, 2016; Cai *et al.*, 2018).

The soil behavior in medium-to-large strain levels has been well determined to be heavily influenced by the coupling effect of inherent and induced anisotropy (Kuhn *et al.*, 2015; Pietruszczak and Oboudi, 2017; Jafarian *et al.*, 2018; Jafarzadeh and Zamanian, 2018; Zamanian *et al.*, 2020). One of the most important aspects of stress anisotropy is the variation in both magnitude and direction of principal stresses. Rotation of the major principal stress direction is a typical loading condition in most geo-structures during seismic excitation and is *a priori* function of (1) the amplitude of seismic loading and (2) the magnitude of the initial static stresses (Al-rkaby et al. 2017; Georgiannou et al. 2018; Prasanna et al. 2020). Thus, numerous anisotropic stress states can be encountered where the stress path is a function of the initial static stresses and the imposed seismic loading, making it difficult to evaluate the evolution of soil parameters under anisotropic stress conditions for all loading combinations in a constitutive model.

To overcome this difficulty, the evolution of any soil characteristic at different stress paths can be determined while keeping other contributing factors constant during a controlled set of experiments. To this end, in this study, a comprehensive experimental program is carried out

on different specimens subjected to stress anisotropy in order to characterize the evolution of shear modulus and damping ratio of sand in the range of medium-to-large shear strains. Accordingly, the effects of the major principal stress direction and intermediate principal stress parameter on the dynamic properties of sands have been scrutinized in a systematic testing procedure. Two types of sands with different gradations and shape characteristics are examined using bender element tests and a hollow cylinder apparatus so as to evaluate the coupling effect of inherent and induced anisotropy on the response of granular soils. The modified hyperbolic relationships, proposed by Oztoprak and Bolton (2013) and Wichtmann and Triantafyllidis (2013), are used as the basic correlations into which the significant influence of stress anisotropy could be incorporated. The influences of major principal stress direction, confining pressure, relative density and soil type as the main parameters and the intermediate principal stress as the less important one, on the modulus reduction and damping ratio curves are subsequently examined. The interactions of these controlling parameters with stress anisotropy are also discussed in this study.

2. MATERIALS & EXPERIMENTAL APPROACH

2.1. Material characterization

Two types of sands, i.e. Babolsar and Toyoura, were used as test materials in this study. Babolsar sand is a natural soil collected from Babolsar beach in Mazandaran Province in the north of Iran. Representative optical and scanning electron microscopy (SEM - using a Hitachi SU3500) images of these sands are presented in S-Fig. 1.

In order to classify the sand grains in terms of their shape, three shape descriptors, including roundness (R), sphericity (S) and regularity (ρ), were evaluated through SEM image analysis of a minimum of 30 randomly chosen particles from each sand. The tested materials in this study were uniformly distributed sands with relatively similar particle size and shape. Therefore, selecting 30 random grains was visually acceptable to accurately quantify the

particle shape descriptors (Payan *et al.*, 2016a, b). The sand particle shape could be evaluated either by the shape characterization chart illustrated in S-Fig. 2a (Krumbein and Sloss, 1963; Cho *et al.*, 2006; Payan *et al.*, 2016b) or by using mathematical relationships and the calculation method of the shape descriptors indicated in S-Fig. 2b. Considering both techniques in the analysis, the resultant mean values and standard deviations of R , S and ρ are presented in Table 1. It is worth noting that the effect of particle position on the evaluation of shape descriptors was not considered in this study. The sphericity indices of Babolsar and Toyoura sands are around 0.65 and 0.5, respectively. Toyoura sand particles are more angular ($R = 0.30$) compared with Babolsar sand which has more rounded grains ($R = 0.45$). S-Fig. 3 shows the particle size distribution curves of the two materials, which are classified as poorly graded sands, SP, based on the USCS (unified soil classification system). The index test results, including specific gravity of the sand particles (G_s) (ASTM D854 2003) and minimum and maximum void ratios (e_{max} and e_{min}) of the soils (ASTM D4253 2013; ASTM D4254 2006) are summarized in Table 1. It should be noted that the relatively high specific gravity of Babolsar sand is due to the concentration of heavy oxides of TiO_2 ($\sim 0.8\%$), MgO ($\sim 2.5\%$) and Fe_2O_3 ($\sim 5\%$).

2.2. Testing procedure

In this study, the damping ratio and shear modulus reduction curves over medium-to-large strain levels were measured using hollow cylinder tests. The shear modulus was then normalized with respect to the small-strain shear modulus obtained from companion bender element tests on triaxial specimens which were prepared at different angles of bedding plane. In order to prepare uniform specimens for the hollow cylinder tests, the moist tamping method was implemented in this study taking into account under-compaction effects (Selig and Ladd 2008). Each specimen was prepared in five layers and the surface of each layer was gently scratched before placing the next layer so as to ensure maximum continuity and integrity of the specimens. A wide range of dense and loose specimens, with relative densities of about 75%

(71%-79%) and 25% (22%-28%) respectively, were prepared for medium-to-large strain experiments.

For the bender element tests, in order to measure the small-strain shear modulus at different angles of bedding plane, undisturbed specimens were obtained from the frozen sand according to Zamanian *et al.* (2020). In this method, the water-pluviated sand specimens were constructed in a cubic chamber placed in a refrigerator on an adjustable pedestal. Undisturbed samples were extracted by continuous coring of the frozen specimen. Afterwards, their top and bottom ends were carefully cut in order to acquire specimens with 10 cm of height and 5 cm of diameter ready for bender element tests. The bender element inserts were then fixed on top and bottom of the specimen before placing the triaxial cell.

In both sets of tests, full saturation of the soil samples was achieved by applying constant increments of cell pressure and back pressure under an effective stress of 15 kPa. Isotropic consolidation commenced when a minimum Skempton's B parameter of 0.96 was reached during the saturation process. The specimens were consolidated under isotropic effective confining stresses (p') of 50 and 150 kPa for hollow cylinder tests and 50, 100, 150, 200, 300 and 350 kPa for bender element tests.

2.3. Test specification & interpretation

2.3.1. Small strain measurements

Small-strain stiffness measurements were performed on specimens of solid cross-section, 50 mm in diameter and 100 mm in height, inside a standard triaxial apparatus equipped with a pair of bender elements attached at the top cap and bottom pedestal. The bender element inserts were made of piezoelectric ceramic bi-morphs with a width, thickness and penetration depth of 11 mm, 1.2 mm and 2 mm, respectively (S-Fig. 4a). A sinusoidal-type mode of shear excitation with 15mV amplitude was transmitted from the bender element at the top and received at the bottom of the specimen. In this study, the so-called time-domain analysis was

performed to evaluate the optimized peak-to-peak arrival time of the shear waves propagating through the soil specimen (Jovičić et al. 1996; Lee and Santamarina 2005; Payan et al. 2016b). An input wave frequency of 7.5 kHz was adopted in the study as it was observed to render the most distinguishable received signals. The tip-to-tip distance between the bender elements was considered as the wave travel distance (L) and the peak-to-peak time interval between transmitted and received signals was denoted as travel time (t). Accordingly, shear wave velocity is defined as:

$$V_s = \frac{L}{t} \quad (1)$$

Consequently, the small strain shear modulus (G_{max}) of the soil samples could be quantified using the following correlation:

$$G_{max} = \rho V_s^2 \quad (2)$$

where ρ is the soil density.

2.3.2. Medium-to-large strain measurements

The hollow cylinder apparatus (HCA-Wykeham Farrance Company) used in this study had a 150 mm triaxial cell enclosed by hollow specimens with 100 mm of outside diameter, 60 mm of inner diameter and 200 mm of height. The apparatus is capable of applying ± 10 kN axial load and ± 200 N.m. torsional force. The stroke of the axial actuator was ± 25 mm and the torsional actuator had 90° rotational freedom. The transducer resolutions for axial load and axial displacement encoder were smaller than 1 N and 0.01 mm, respectively. The torque and rotational encoders had a resolution of about 0.01 N.m. and 1.5 μ m, respectively. The hollow cylinder apparatus utilized in this study along with typical stress-strain cyclic loops imposed on an element of the hollow cylinder specimen is depicted in S-Fig. 4b.

Fig. 1a shows typical variations of the four loading axes, including vertical load, torque, and inner and outer cell pressures in loose specimens. A representative hysteretic loop to calculate

secant shear stiffness and damping ratio is presented in Fig. 1b. The secant shear stiffness at each cycle of loading was determined as the slope of the line passing from the origin of the coordinates and the maximum and minimum shear stress values of the respective cycle. The damping ratio or the amount of energy loss in a given cycle was also calculated using the inscribed area of its corresponding stress-strain loop divided by the elastic stored energy.

2.4. Experimental program

In order to investigate the effect of stress anisotropy on the stiffness over medium to large strain levels, a comprehensive set of torsional shear tests on saturated specimens were performed for various combinations of the major principal stress direction with respect to the vertical direction, α (10° , 30° , 45° , 60° , and 80°) and intermediate principal stress parameter, $b = (\sigma_2 - \sigma_3) / (\sigma_1 - \sigma_3)$ (0.1, 0.5 and 0.9). For dense specimens, the inner and outer cell pressures were the same in magnitude during the tests and therefore, the major principal stress direction was related to b -parameter as $b = \sin^2(\alpha)$. Since the loading frequency has a relatively negligible influence on soil behavior in liquefaction testing (Boulanger *et al.* 1991; Polito *et al.* 2008), the cyclic tests were performed at a single loading frequency of 0.2 Hz. Several bender element tests were also performed to determine the small-strain stiffness of the sands of variable densities under different confining pressures. The detailed experimental program along with the specimen specifications are summarized in Table 2. It should be noted that stress non-uniformity may develop when the stress state is close to $b = 1.0$ and $\alpha = 90^\circ$ (Sayao and Vaid 1991; Naughton and O'Kelly 2007). Accordingly, the values of both α -angle and b -parameter were not selected in the vicinity of the border limits so as to avoid unacceptable levels of non-uniformity within the specimens. It is also worth noting that a more detailed experimental program is required for a comprehensive assessment of all influencing parameters, such as confining pressure, relative density and particle specifications, on the evolution of G/G_{max} .

3. RESULTS & DISCUSSION

3.1. The effect of stress anisotropy on shear stiffness

The small strain shear modulus (G_{max}) of both Babolsar and Toyoura sands was evaluated by measuring the shear wave velocity at different confining pressures ($p' = 50$ kPa to 350 kPa), and plotting G_{max} against the pressure, as shown in Fig. 2. Regression analysis using a power-law type relationship for small-strain shear modulus against the confining pressure (Eq. 3) was applied on the test data so as to predict model parameters A and n (Cascante and Santamarina 1996; Santamarina *et al.* 2001; Menq 2003; Senetakis *et al.* 2012; Gu *et al.*, 2013; Enomoto, 2016; Payan *et al.* 2016b; Liu *et al.*, 2016; Jafarian *et al.*, 2018).

$$G_{max} = AF(e)\left(\frac{p'}{p'_a}\right)^n \quad (3)$$

In this equation, e is the void ratio and p'_a is the atmospheric pressure. The verified void ratio function in the form of $F(e) = e^d$, has been used herein to normalize G_{max} . This relationship was originally proposed by Jamiolkowski *et al.* (1991, 1999) for clays and was later used for sands. Payan *et al.* (2016a) observed that a mean constant value of $d = -1.29$ (Eq. 4) can be effectively utilized for the evaluation of the small strain shear modulus for a broad range of clean sands with different gradations and particle shapes.

$$F(e) = e^{-1.29} \quad (4)$$

An exponential curve fitting analysis showed that A and n constants are 52 and 0.50 for Babolsar sand and 43 and 0.55 for Toyoura sand, respectively (where A is expressed in MPa). The higher n value in Toyoura sand could be attributed to the more irregular shape of its grains compared with that of Babolsar sand, giving rise to greater particle interlocking at higher confining pressures (Cho *et al.*, 2006, Senetakis *et al.*, 2012, Payan *et al.*, 2016a, 2016b). Payan *et al.* (2016a) proposed the following correlations to evaluate model parameters A and n of clean sands based on their coefficient of uniformity (C_u) and particle shape descriptor of regularity (ρ):

$$A = 84 C_u^{-0.14} \rho^{0.68} \quad (5)$$

$$n = C_u^{-0.12} (-0.23 \rho + 0.59) \quad (6)$$

A and n values from these equations were obtained to be 52 and 0.49 for Babolsar sand and 43 and 0.52 for Toyoura sand, respectively, which are in good agreement with those estimated from the regression analysis in the present study. Small-strain shear modulus for different angles of bedding plane has been measured by Zamanian *et al.* (2020) for Babolsar sand. In that study the specimens were prepared by the freezing technique at different deposition inclinations (α) and a linear relationship for the prediction of G_{max} was proposed (Eq. 7). Similarly, the small-strain shear modulus of Toyoura sand was measured in this study at two extreme angles of bedding plane, i.e. $\alpha = 0^\circ$ and 90° , and an analogous linear correlation for the prediction of G_{max} could be developed, as shown in Eq. (8).

$$\text{Babolsar sand: } G_{max} = (53.6 - 0.3 \alpha) \times e^{-1.29} \times (p'/p_{atm})^{0.50} \quad (7)$$

$$\text{Toyourea sand: } G_{max} = (43 - 0.28 \alpha) \times e^{-1.29} \times (p'/p_{atm})^{0.55} \quad (8)$$

Results indicated that the growth rate (n) of the normalized small-strain shear modulus with the normalized pressure is nearly independent of the bedding plane direction and is solely a function of particle specifications (Wichtmann and Triantafyllidis 2009; Payan *et al.* 2016a; b; Payan and Jamshidi Chenari 2019; Zamanian *et al.* 2020). However, the amplitude factor (A) is linearly related to the bedding plane direction (Zamanian *et al.* 2020). This is primarily attributed to the role of particle interlocking which commonly occurs during the deposition process of sand grains. During deposition, the particles settle in their most stable position so as to tolerate the imposed surcharge stresses. Thus, particle interlocking significantly reduces by increasing the angle of bedding plane, which, in turn, causes considerably lower G_{max} values. The calculated G_{max} values from Eqs. (7) and (8) have been used in this study for normalization of the secant shear modulus of the specimens subjected to stress anisotropy at different α -directions. The strain-dependent G/G_{max} reduction curves of loose Babolsar sand specimens

tested at various loading combinations of α -angle and b -parameter under mean confining pressures of 50 kPa and 150 kPa are shown in Fig. 3. The upper and lower bounds in Fig. 3 are acquired from the following general modified hyperbolic relationship proposed by Oztoprak and Bolton (2013), extracted from the shear stiffness curves of 454 soils throughout the literature:

$$\left(\frac{G}{G_{max}}\right) = 1 / \left[1 + \left(\frac{\gamma - \gamma_e}{\gamma_{ref}} \right)^\xi \right] \quad (9)$$

where fitting parameter γ_{ref} refers to the shear strain corresponding to $G/G_{max}=0.5$; ξ is the coefficient of curvature; γ_e is the elastic threshold strain beyond which shear stiffness starts to decrease and its variation against strain becomes nonlinear. Accordingly, Oztoprak and Bolton (2013) proposed $\gamma_e = 0$; $\gamma_{ref} = 0.02\%$ and $\xi = 0.88$ for the lower bound and $\gamma_e = 0.003$; $\gamma_{ref} = 0.10\%$ and $\xi = 0.88$ for the upper bound of the stiffness reduction curves.

The dotted and dashed lines in Fig. 3 are the predicted G/G_{max} curves by Oztoprak and Bolton (2013) and Wichtmann and Triantafyllidis (2013), respectively. Oztoprak and Bolton (2013) stated that ξ is slightly affected by soil uniformity coefficient (C_u); while γ_{ref} and γ_e are dependent on C_u , confining pressure (p'), void ratio (e) and relative density (D_r), as follow:

$$\gamma_{ref}(\%) = 0.01 C_u^{-0.3} \left(\frac{p'}{p_{atm}} \right) + 0.08 e \cdot D_r \quad (10)$$

$$\gamma_e = 0.002 + 0.012 \gamma_{ref} \quad (11)$$

$$\xi = C_u^{-0.075} \quad (12)$$

On the other hand, Wichtmann and Triantafyllidis (2013) proposed a more simplified formula to predict G/G_{max} (Eq. 13) based on γ_{ref} and a constant parameter, d . γ_{ref} in their study was observed to be simply predicted by the effective stress ratio (Eq. 14) and d was only dependent on C_u (Eq. 15).

$$\left(\frac{G}{G_{max}}\right) = \frac{1}{1 + d \left(\frac{\gamma}{\gamma_{ref}} \right)} \quad (13)$$

$$\gamma_{ref} = \sqrt{\frac{p'}{p_{atm}}} \quad (14)$$

$$d = 1093.7 + 1955.3 \ln(C_u) \quad (15)$$

As observed in Fig. 3, the relationship proposed by Wichtmann and Triantafyllidis (2013) can effectively predict the G/G_{max} curves of Babolsar loose specimens tested at $p' = 50$ kPa and 150 kPa. However, Oztoprak and Bolton (2013) predictions considerably underestimate the stiffness reduction curves (after, Akeju *et al.*, 2019). For the tests performed at $p' = 50$ kPa, the shear stiffness can be assumed to follow a decreasing trend with the increase in the α -direction. This is aligned with the observations made by several previous researchers (Sivathayalan and Vaid, 2002; Chaudhary *et al.*, 2002; Kumruzzaman and Yin, 2010). On the other hand, for the tests performed at $p' = 150$ kPa, the shear stiffness curves bear their minimum values when α approaches around 45° - 60° . This could be attributed to the cross anisotropic nature of the specimens resulted from horizontal bedding plane (Lade *et al.*, 2014; Sivathayalan *et al.*, 2015). All the above observations could be justified using a schematical illustration of particle packing and stress state. As demonstrated in Fig. 4, sample preparation method creates a cross anisotropic soil structure, consisting of a deposition direction and a bedding plane. In this structure, soil grains are commonly well packed to tolerate vertical loads. Therefore, the maximum resistance (in terms of peak strength and stiffness) is expected to occur along the vertical direction whereas the horizontal bedding direction is supposed to be the weakest plane (Oda, 1972; Sivathayalan and Vaid, 2002; Chaudhary *et al.*, 2002; Kumruzzaman and Yin, 2010; Zamanian *et al.*, 2020). However, the behavior of geomaterials is more pronouncedly influenced by the direction of the maximum shear stress with respect to the vertical alignment (α_r), which is deviated by 45° from α -direction. For specimens tested at α angles close to 45° - 60° , α_r - direction is inclined along the horizontal bedding plane deemed to be the weakest plane within the soil sample (Figs. 4b-4d). Similar observations have been made by Lade *et al.* (2014)

and Sivathayalan *et al.* (2015) by performing monotonic and cyclic hollow cylinder tests on clean sands, respectively. They observed that shear strength reduced to a minimum value when the major principal stress direction is aligned at 45° to 60° and then increased at higher α -directions. According to the test results in the current study and the abovementioned discussions, it can be concluded that the higher confining pressure increases the inter-particle forces and inherent anisotropy within the soil medium, thus posing the maximum shear stress to play a more significant role in the evolution of soil stiffness.

Another important observation revealed from Fig. 3 is that the confining pressure has amplified the influence of stress anisotropy on the stiffness, as more deviations between G/G_{max} reduction curves of the samples tested at various α -directions could be observed at higher confining pressures. This observation could be attributed to the role of inter-particle forces that affect particle movement during shearing. For a given relative density and sand type, by increasing confining pressure, the contribution of particle rearrangement (geometrical interface resistance) augments (Sadrekarimi and Olson 2011), which could be related to the higher influence of particle geometry and interlocking (inherent anisotropy). Accordingly, as stated earlier, by increasing the level of inherent anisotropy at higher confining pressures, further subsequent anisotropic behavior can be expected as a consequence of the coupling effect between induced and inherent anisotropy. Li and Senetakis (2020) also observed that the deviation between G/G_{max} curves of anisotropically loaded sands clearly increased at higher confining pressures, though their study was constrained on the rotation of the principal axes at 90° .

S-Fig. 5 shows G/G_{max} reduction curves of dense Babolsar and Toyoura sands tested at $p' = 150$ kPa. For these sands, the predicted G/G_{max} curves by Oztoprak and Bolton (2013) and Wichtmann and Triantafyllidis (2013) are converging. Similar to the loose specimens, there is a clear deviation between G/G_{max} of the specimens tested at various α -directions, so that G/G_{max}

reduction curves move downwards when α approaches 45° - 60° , which is more pronounced for Babolsar sand. The pink lines in S-Fig. 5a show the upper and lower bounds of the G/G_{max} reduction curves of loose Babolsar sand subjected to stress anisotropy. As can be observed, shear stiffness is influenced by the relative density as the reduction of the G/G_{max} values is relatively faster for the loose specimens than that of the dense specimens (Wichtmann and Triantafyllidis, 2013).

The effect of b -parameter on G/G_{max} of the loose Babolsar specimens tested at the same α -directions is presented in S-Fig. 6. As observed, b -parameter has a slight influence on G/G_{max} of the specimens especially at higher confining pressure. For all α -directions, G/G_{max} of the specimens tested at $b = 0.5$ was slightly higher than $b = 0.1$ or 0.9 . As a justification for this observation, it should be noted that for a constant α -direction, the load combination at $b = 0.3$ - 0.55 is associated with the plane strain condition in which the soil exhibits greater amounts of shear strength (Symes *et al.*, 1984; Minh, 2006; Lade *et al.*, 2014).

As observed in Fig. 3 and S-Figs. 5-6, the relationships proposed by Wichtmann and Triantafyllidis (2013) can effectively predict the G/G_{max} reduction curves for both loose and dense sands, provided that the significant influence of stress anisotropy is accounted for in the constant parameter d . Accordingly, regression analyses have been performed on the test results using Eq. (13) as a basic relationship in order to incorporate the contribution of stress anisotropy into the constant parameter d in Eq. (15) (Akeju *et al.*, 2019). In this regard, a correction factor, denoted as λ , has been defined as the ratio of the constant parameter obtained from the regression analysis (d_{reg}) to the value predicted by Eq. (15) ($\lambda = d_{reg}/d_{Eq.13}$). Fig. 5 shows the variation of λ against the major principal stress direction (α) for loose and dense Babolsar and Toyoura sands. The data suggest that λ is *a priori* function of α -direction, b -parameter and confining pressure. Furthermore, it should be noted that for a given initial confining pressure and α -direction, the correction factor is different for Babolsar and Toyoura

sands (Fig. 5b). This observation is consistent with that of Oztoprak and Bolton (2013) who reported that the soil state parameter, defined as the product of void ratio and relative density ($e.D_r$), can influence the G/G_{max} reduction curve.

In order to incorporate the effects of all these parameters in the correction factor, a power-type relationship (Eq. 16) obtained from regression analysis (Fig. 5a) has been used as a basic equation. It is worth noting that these best-fit curves have been considered by disregarding the occasional increase or decrease in some trends discussed earlier for various α values. Therefore, the decreasing values of shear stiffness with increasing α is considered as the general trend (Sivathayalan and Vaid, 2002; Kumruzzaman and Yin, 2010; Zamanian *et al.*, 2020). The constant parameter X is independent of b -parameter (Fig. 5a) and relative density (Fig. 5b) and is only influenced by the confining pressure, where X is 0.56 for $p' = 50$ kPa and 0.31 for $p' = 150$ kPa. Thus, X can be simply expressed in terms of reference strain (Eqs. 14 and 17). The power Y , in turn, is influenced simultaneously by b -parameter, p' and the product ($e.D_r$) and can be expressed as a product of Y_1 and Y_2 , where Y_1 represents the effect of b -parameter and confining pressure and Y_2 depicts the contribution of soil state.

$$\lambda = X\alpha^Y = X\alpha^{Y_1 \times Y_2} \quad (16)$$

Fig. 6a shows a slight variation of Y_1 with b -parameter, which can be well predicted with a quadratic equation. The dotted lines are the predicted Y_1 by Eq. (18) with the consideration of simultaneous effects of b -parameter and confining pressure. The variation of Y_2/Y_{1ref} with $e.D_r$ is presented in Fig. 6b for specimens tested at a confining pressure of 150 kPa (Eq. 19). Thus, a comprehensive relationship for the prediction of G/G_{max} incorporating the effect of anisotropy is developed as shown in Eq. (20).

$$X = 0.4 \gamma_{ref} \quad (17)$$

$$Y_1 = 0.16 \gamma_{ref}^{0.45} (b^2 - b + 1.5 \gamma_{ref}^{0.25}) \quad (18)$$

$$\frac{Y_2}{Y_{1ref}} = 0.94 \exp(0.50 e.D_r) \quad (19)$$

$$\lambda = 0.4 \gamma_{ref} \alpha^{0.15} \gamma_{ref}^{0.45} (b^2 - b + 1.5 \gamma_{ref}^{0.25}) \exp(0.5 e.D_r) \quad (20)$$

The predicted G/G_{max} values by Eq. (20) have been compared with all measured data in Fig. 7 and S-Fig. 7. As can be observed, Eq. (20) can efficiently predict the measured G/G_{max} reduction curves of the loose and dense specimens subjected to stress anisotropy at different confining pressures with an acceptable accuracy. It is worth noting that the constants presented in the abovementioned equations could be dependent on sand index properties, including grain size characteristics and particle shape. In the study conducted by Oztoprak and Bolton (2013), however, parameter $e.D_r$ has been assumed to be a possible surrogate for particle morphologies.

3.2. The effect of stress anisotropy on damping ratio

Variations of damping ratio in loose and dense specimens tested at $p' = 50$ kPa and $p' = 150$ kPa subjected to induced anisotropy (different α -direction and b -parameter) are presented in Fig. 8 and S-Figs. 8-9, respectively. The solid lines show upper and lower bounds for damping ratio of clean sands proposed by Seed *et al.* (1972). The dotted line shows predicted damping ratio based on the relationship proposed by Wichtmann and Triantafyllidis (2013) for clean sands which is a function of shear strain and reference strain, γ_r .

$$\frac{D - D_{min}}{D_{max} - D_{min}} = \frac{(\gamma/\gamma_r)[1 + m \cdot \exp(-\gamma/\gamma_r)]}{1 + (\gamma/\gamma_r)[1 + m \cdot \exp(-\gamma/\gamma_r)]} \quad (21)$$

Wichtmann and Triantafyllidis (2013) suggested that $D_{min} = 0.006$, $D_{max} = 0.32$, $m = 843$, and $\gamma_r = \sqrt{p'/p_{atm}}$ could render satisfactorily appropriate predictions for a wide range of clean sands. As can also be observed, Eq. (21) can fairly be utilized for the prediction of damping ratios in this study; however, the constant parameter, m , in this equation needs to be modified in order to incorporate the effect of stress anisotropy. Independent of confining pressure and b -

parameter, lower damping ratio curves are observed for the specimens tested at $\alpha = 10^\circ$. Damping ratio increases when α approaches 45° - 60° beyond which it shows a decline. The higher damping ratios for α -directions of 45° - 60° could be primarily attributed to the lower shear strength along the bedding plane due to the direction of maximum shear stress. Similar trends can be observed for damping ratios of both dense Babolsar and Toyoura specimens tested at $p' = 150$ kPa (S-Fig. 9), where damping ratios increase when α -direction approaches 45° - 60° . The results also indicate that the b -parameter doesn't have any particular influence on the evolution of damping ratio. This may be due to the range of confining pressures tested in this study or particle specifications of the tested materials.

In order to incorporate the effect of stress anisotropy into the variation of damping ratio with shear strain, regression analyses were performed on the test data to determine the coefficient m for the specimens tested at different α -directions and b -parameters. To this end, a correction factor, β , has been defined as the ratio of m_{reg} obtained from the regression analysis to the proposed m value by Wichtmann and Triantafyllidis (2013). The variation of β with b -parameter for loose Babolsar specimens shows that confining pressure and b -parameter have negligible influence on damping ratio (Figs. 9c-d). Unlike G/G_{max} , the correction factor for damping ratio (β) for both loose and dense specimens is influenced, predominantly, by α -direction and can be well predicted by the following quadratic equation (Figs. 9a-b):

$$\beta = -0.0003 \alpha^2 + 0.03\alpha + 0.08 \quad (22)$$

Fig. 10 and S-Fig. 10 compare the measured and predicted damping ratios of loose and dense sands, respectively. As can be observed, Eq. (22) can be effectively incorporated into Eq. (21) to accurately reflect the significant influence of stress anisotropy on the prediction of the damping ratios of both loose and dense specimens.

4. CONCLUDING REMARKS

The dynamic properties (stiffness and damping ratio) of two different types of sands subjected

to anisotropic stress states were experimentally investigated through a comprehensive set of bender element and torsional shear tests covering small to large strain amplitudes. The effects of the major principal stress direction, intermediate principal stress, confining pressure and relative density on the dynamic parameters of the sands were studied. The following major conclusions, stemming from the study, are summarized.

1. For a lower confining pressure (equal to 50 kPa in the study), the shear stiffness was observed to follow a decreasing trend with the increase in the α -direction. On the other hand, for a higher confining pressure (equal to 150 kPa in the study), the stiffness reduction curves shift to lower G/G_{max} values (for a given shear strain amplitude) when α -direction approaches around 45° - 60° due to the direction of the maximum shear stress. Damping ratio increased as α increased to around 45° - 60° , beyond which point damping values decreased.
2. Greater deviation between G/G_{max} reduction curves was observed at higher confining pressures, which was attributed, on a qualitative standpoint, to the influence of the developed normal contact forces between the sand particles, amplifying in this way the influence of stress-induced anisotropy. However, neither confining pressure nor b -parameter had any significant influence on the damping ratio.
3. Unlike the negligible impact of the intermediate principal stress parameter on damping ratio, G/G_{max} reduction curves were observed to be slightly influenced by b -parameter. For all α -directions, shear stiffness of the specimens tested at $b = 0.5$ was slightly higher which was mostly attributed to its loading combination corresponding to plane strain condition.
4. Data analysis suggested that by introducing some correction factors, the previous well-established correlations can effectively be implemented for the prediction of stiffness and damping ratio of sands subjected to stress anisotropy.
5. The results indicated that α -angle and confining pressure (p') have significant influence on the evolution of G/G_{max} ; however, b -parameter and soil state (e, D_r) slightly influence the

evolution of G/G_{max} . Besides, the evolution of damping ratio was only influenced by α -angle and confining pressure, and the effects of b -parameter and soil state were negligible.

6. The current study could be a robust basis for future research in this area. Accordingly, it is recommended to perform similar tests on the samples with various gradations and particle shapes prepared at different relative densities, from very low to very high, under a wide range of low to high confining pressures (e.g. 50 kPa to 800 kPa); so that the modulus degradation and damping ratio of inherent anisotropic sands from very small to large strain levels could be thoroughly characterized. Discussed for the first time and a possible pioneer in this area, the major contributions made in the course of this study could shed light upon various aspects of induced anisotropy in granular materials, which, in turn, could be extended in future further research.

DATA AVAILABILITY STATEMENT: The experimental data that support the findings of this study are available from the corresponding author upon reasonable request.

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TABLES

Table 1. Physical properties of tested materials

Sand type	Specific gravity of sand grains (G_s)	Maximum void ratio (e_{max})	Minimum void ratio (e_{min})	Uniformity index (C_u)	Curvature index (C_c)	Roundness (R) (Mean $\pm$ stdev)	Sphericity (S) (Mean $\pm$ stdev)	Regularity (ρ) (Mean $\pm$ stdev)
Babolsar	2.753	0.790	0.531	1.79	1.32	0.45 ($\pm$ 0.10)	0.65 ($\pm$ 0.12)	0.55 ($\pm$ 0.11)
Toyoura	2.645	0.973	0.609	1.56	0.95	0.30 ($\pm$ 0.08)	0.50 ($\pm$ 0.12)	0.40 ($\pm$ 0.10)

Table 2. Testing programme

Test type	Sand Type	p' (kPa)	b	α (degree)	D_r (%)	e
HC ⁺	Babolsar	50	0.1, 0.5, 0.9	10, 30, 45, 60, 80	22-28	0.717-0.733
HC	Babolsar	150	0.1, 0.5, 0.9	10, 30, 45, 60, 80	22-28	0.717-0.733
HC	Babolsar	150	$\sin^2(\alpha)$	10, 30, 45, 60, 80	71-79	0.591-0.601
HC	Toyoura	150	$\sin^2(\alpha)$	10, 30, 45, 60, 80	72-79	0.689-0.715
BE ⁺⁺	Babolsar	50, 100, 150, 200, 300, 350	0	0	25, 52, 79	0.583-0.725
BE	Toyoura	50, 100, 150, 200, 300, 350	0	0 and 90	24, 50, 81	0.688-0.715

⁺HC: Hollow cylinder ⁺⁺BE: Bender element

Fig. 1. a) Representative curves showing the variation in inner and outer cell pressure, vertical load and torque; b) typical hysteresis loop and the way to calculation of shear modulus and damping ratio

Fig. 2. (a) Small strain shear modulus of (a) Babolsar sand and (b) Toyoura sand

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Fig. 3. Shear stiffness reduction curves of anisotropically loaded loose Babolsar sand at (a) $p' = 50$ kPa, $b=0.1$; (b) $p' = 50$ kPa, $b=0.5$; (c) $p' = 50$ kPa, $b=0.9$; (d) $p' = 150$ kPa, $b=0.1$; (e) $p' = 150$ kPa, $b=0.5$; (f) $p' = 150$ kPa, $b=0.9$ (solid lines are boundary values proposed by Oztoprak and Bolton (2013), dashed lines are predicted values by Wichtmann and Triantafyllidis (2013) and dotted lines are predicted values by Oztoprak and Bolton (2013))

Fig. 4. Schematic illustration of a cross anisotropic deposited sand loaded at various major principal stress directions

Fig. 5. Variation of correction factor, λ , with α -direction: (a) loose Babolsar sand $p' = 50$ kPa and 150 kPa; (b) dense Babolsar and Toyoura sands $p' = 150$ kPa

Fig. 6. (a) variation of Y_I with b -parameter; (b) variation of Y_2/Y_{Iref} with soil state ($e.Dr$)

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Fig. 7. Measured against predicted small strain shear modulus of anisotropic loaded loose Babolsar sand at (a) $p' = 50$ kPa (b) $p' = 150$ kPa

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Fig. 8. Damping ratio of anisotropic loaded loose Babolsar sand at (a) $p' = 50$ kPa, $b=0.1$; (b) $p' = 150$ kPa, $b=0.1$ (solid lines are boundary values proposed by Seed (1972), dashed lines are predicted values by Wichtmann and Triantafyllidis (2013))

Fig. 9. Variation of correction factor, β , with: (a) α - direction in loose Babolsar sand; (b) α - direction in dense Babolsar and Toyoura sand; (c) b - parameter in loose Babolsar sand at $p' = 50$ kPa; (d) b - parameter in loose Babolsar sand at $p' = 150$ kPa

Fig. 10. Measured against predicted damping ratio of anisotropically loaded loose Babolsar sand at (a) $p' = 50$ kPa, (b) $p' = 150$ kPa