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Møller, Randi N.; Krenk, Steen; Svendsen, Martin N.

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Time simulation of turbulent wind response and flutter of long-span bridges

Randi N. Møller^{a,b,*}, Steen Krenk^a, Martin N. Svendsen^b

^a*Department of Mechanical Engineering
Technical University of Denmark, DK-2800 Lyngby, Denmark*

^b*Rambøll, Hannemanns Alle, DK-2300 Copenhagen, Denmark*

Abstract

An efficient procedure for simulation of time domain response analysis of long-span bridges to turbulent wind load is presented. The wind load is simulated by a convected three-dimensional orthotropic wind field with separate turbulence length-scales and intensities in the along wind and the two transverse directions. Efficiency of the simulation is obtained by using eight memory steps with exponentially increasing distance from current time, leading to a very accurate representation of the statistical properties of the wind field. The self-induced aerodynamic forces are implemented in the analysis via a discrete form of a local filter with one or two steps, and an efficient procedure is developed for determination of the filter parameters. The local form of the filter enables reduction of the total equation system to the size of the bridge model in still air. The detailed representation of the wind field automatically includes the feature that points with large horizontal separation exhibit reduced coherence of the low-frequency components, an important feature, verified in several recent measurement programs, that reduces the turbulent loading on long-span bridges and thereby increases the flutter velocity. Furthermore, the representation of the wind field contains a separate transverse length scale of importance for the resulting load. The simulation method is sufficiently general to permit response analysis of the full bridge structure with two layers of cables, but examples demonstrate rather small effect on the lower modes from the wind load on the cables.

Keywords: Long span bridge, Time domain response, Sequential wind field simulation, Quasi-static condensation, Momentum-based time integration.

1. Introduction

In the design of long span bridges the response due to wind loading plays an important role. In recent years, significant research attention has been given to this field which includes representation of turbulent wind, analysis of wind-structure interaction and simulation of both the wind load and the structural response in time domain response calculations. The time domain response calculations is of special interest when considering wind load on vehicles, fatigue and non-linear dynamics.

*Corresponding author

Email addresses: rnm@ramboll.dk (Randi N. Møller), sk@mek.dtu.dk (Steen Krenk), mnns@ramboll.dk (Martin N. Svendsen)

A correct representation of turbulent wind is crucial for representing the dynamic loading on wind-sensitive structures. The transverse coherence is of particular interest for long-span bridges where the integral length scale of the turbulence is smaller than the size of the considered structure. The turbulent wind field is most often represented by the auto-spectral density of the wind components of interest to give the stochastic single-point loading, supplemented by a coherence function representation to provide the transverse field properties. The covariance function is generally chosen on the empirical form $\exp(-C\omega\Delta y/U)$ suggested by Davenport [1]. This representation suggests a positive correlation at any separation Δy , which is in conflict with a full representation of an incompressible field, where it is evident that the turbulence components must be negatively correlated at certain distances to provide a zero mean flow. To address this issue an alternative covariance representation was suggested by Krenk [2] based on homogenous isotropic turbulence and a von Kàrmàn representation of the along wind spectral density [3, 4]. Recent data studies have demonstrated the improved representation of measured wind data [5, 6, 7], although only an exponential simplification was used.

The simulation of turbulent wind can either be done by an inverse Fourier transform of a spectral representation of the wind field or by sequential models as e.g. the auto-regressive moving-average (ARMA) models. The former has been widely used for time domain simulation of bridge loading [8, 9]. The basic format of a multi-component Fourier simulation was introduced by Shinozuka and Jan [10] and since then a number of contributions have been made in order to increase computational efficiency, applicability to large systems etc. [11, 12, 13]. For large systems the Fourier simulated wind fields are challenged by large storage requirements. Various formulations are discussed in [14]. Alternatively, application of sequential simulation methods allow for step-wise updates of the wind load based on a relatively short data memory. Early contributions concerning sequential multivariate random process simulations were made by Mignolet and Spanos [15, 16] and the multivariate ARMA representation of wind fields in relation to dynamic response of structures was discussed by Li and Kareem [17]. The idea of utilizing a full-field description of the turbulent wind to apply a low order auto-regressive model was introduced by Krenk [18], and recently a sequential conditional mean-field simulation procedure was proposed, in which the coefficient matrices are determined explicitly in terms of conditional means and conditional covariances of the wind field, using a compact non-uniformly spaced set of previous field values, [19].

The aerodynamic wind-bridge interaction is another main field of interest for bridge response as well as bridge stability calculations. The basic principles are carried over from Theodorsen's classic airfoil theory [20] that describes the actions on an oscillating flat plate in a fluid flow, and subsequent application to bridge deck flutter was introduced by Scanlan and Tomko [21]. The fluid-structure interaction is typically described by the frequency dependent aerodynamic derivatives, and thus it is necessary to transform this representation into time domain. This can be done by indicial functions as suggested by Scanlan et al. [22], or by the computationally more efficient use of rational function representations introduced by Eversman and Tewari [23] and used for bridge deck flutter by Chen et al. [24] and Høgsberg et al. [25, 26]. More recently the aero-elastic forces have been introduced in a finite element framework via additional degrees of freedom by Øiseth et al. [27]. Methods based on quasi-steady aerodynamics have also been suggested as a computationally efficient, although less accurate alternative [28, 29].

This paper presents a state-space formulation for a long suspension bridge, including self-excited aerodynamic forces. The bridge response is investigated using an anisotropic

full-field sequential simulation procedure, recently developed in [19]. The combination of the recursive wind field simulation procedure using a memory with 6–10 non-equally spaced steps and a momentum-based time integration of the extended state-space equations, including the aerodynamic derivatives via a first-order filter representation, results in a very flexible model formulation in which the points used for wind velocity simulation and aerodynamic loads can be configured freely, e.g. including bridge deck, cables and hangers, or a smaller model including only the bridge deck. The results demonstrate highly accurate representation of the characteristics of the three-dimensional wind field and illustrate the rather strong influence of the transverse correlation length on the bridge response. Furthermore, the computations demonstrate the high computational efficiency and moderate storage requirements of the proposed procedure.

2. Aerodynamic system

The aerodynamic system in relation to long span bridges can be understood as the structural system determined by mass, stiffness and damping of the structure and an aerodynamic load representation. The aerodynamic loading consists of contributions from both the mean wind and from the wind turbulence, where the turbulent wind components of main interest are the along wind component and the vertical component. State-of-the-art calculations of aerodynamic response is neglecting the higher order turbulence contribution leaving a pure mean wind contribution and a mixed mean wind and turbulence contribution. The mixed contribution is referred to as the buffeting load and is treated as a direct load determined solely by the wind speeds, static aerodynamic coefficients, bridge deck geometry and admittance functions. The mean wind contribution is influencing the dynamic system due to the vibrations of the bridge deck and the loading of the structure is therefore referred to as self-excited forces. Figure 1 shows the bridge cross-section with cross-sectional forces $\mathbf{f} = [f_1, f_3, f_\theta]^T$, the mean wind speed U , and the turbulence components u and w .

Figure 1: Aerodynamic forces.

The equation of motion of the full aerodynamic system is written with the structural terms on the left hand side and with the buffeting load and the self-excited forces on the right hand side of the equation

$$\mathbf{M}_s \ddot{\mathbf{q}}(t) + \mathbf{C}_s \dot{\mathbf{q}}(t) + \mathbf{K}_s \mathbf{q}(t) = \mathbf{f}_a(t) + \mathbf{f}_e(t). \quad (1)$$

Here $\mathbf{M}_s$, $\mathbf{C}_s$ and $\mathbf{K}_s$ are the structural mass, damping and stiffness matrices, respectively, and $\mathbf{q} = [q_1, q_3, q_\theta]^T$ is the displacement vector, $\mathbf{f}_e$ is the buffeting load, and $\mathbf{f}_a$ represents the self-excited aerodynamic forces. The self-excited forces depend on the displacement history

of the structure as expressed by the convolution integral

$$\mathbf{f}_a = \int_0^\infty \mathbf{Q}(\tau) \mathbf{q}(t - \tau) d\tau, \quad (2)$$

where $\mathbf{Q}(\tau)$ is the kernel function by which the self-excited forces are included as a sum of effects from time $t - \tau$, where τ is the time lag. Assuming a harmonic force and response

$$\mathbf{f}_a = \bar{\mathbf{f}}_a e^{i\omega t}, \quad \mathbf{q} = \bar{\mathbf{q}} e^{i\omega t}, \quad (3)$$

where the bar denotes the complex amplitude, the self-excited forces can be expressed in the frequency domain by the relation

$$\bar{\mathbf{f}}_a = \bar{\mathbf{Q}}(\omega) \bar{\mathbf{q}}, \quad (4)$$

where $\bar{\mathbf{Q}}(\omega)$ is the Fourier transform of the kernel function. The aerodynamic, self-excited forces are most commonly defined by the geometry-specific, non-dimensional aerodynamic derivatives P_n^* , H_n^* , A_n^* , $n = 1, 2, \dots, 6$, whereby the Fourier transform of the kernel function takes the form, [21],

$$\frac{\bar{\mathbf{Q}}}{\frac{1}{2}\rho U^2 B} = \frac{\omega_*^2}{B} \begin{bmatrix} P_4^* + iP_1^* & P_6^* + iP_5^* & B(P_3^* + iP_2^*) \\ H_6^* + iH_5^* & H_4^* + iH_1^* & B(H_3^* + iH_2^*) \\ B(A_6^* + iA_5^*) & B(A_4^* + iA_1^*) & B^2(A_3^* + iA_2^*) \end{bmatrix}, \quad (5)$$

with the reduced frequency $\omega_* = \omega B/U$ and with B and ρ as the reference width of the bridge deck and the air density.

The buffeting load is defined from geometrical considerations of how the turbulent wind affects the structure. For the geometrical considerations the deck is considered motionless wherefore the buffeting contribution is reduced by frequency dependent admittance functions to include effects of the dynamic structure. The frequency representation of the wind loading of the bridge is

$$\bar{\mathbf{f}}_e = \mathbf{B}_f(\omega) \bar{\mathbf{u}}(\omega) \quad (6)$$

where $\bar{\mathbf{u}} = [\bar{u}, \bar{w}]^T$ is the frequency representation of the the wind velocities found as the Fourier transform of the velocity vector $\mathbf{u}(t) = [u(t), w(t)]^T$. The matrix $\mathbf{B}_f(\omega)$ gives the relation between the relevant wind components and the bridge deck loads. From buffeting theory – see e.g. [30] – the relation is given as

$$\mathbf{B}_f(\omega) = \frac{\rho U B}{2} \begin{bmatrix} 2(D/B)\bar{C}_D A_{yu}(\omega) & ((D/B)C'_D - \bar{C}_L)A_{yw}(\omega) \\ 2\bar{C}_L A_{zu}(\omega) & C'_L + (D/B)\bar{C}_D A_{zw}(\omega) \\ 2B\bar{C}_M A_{\theta u}(\omega) & BD'_M A_{\theta w}(\omega) \end{bmatrix} \quad (7)$$

where $A_{yu}(\omega), \dots, A_{\theta w}(\omega)$ are the frequency dependent aerodynamic admittance functions, $\bar{C}_D$, $\bar{C}_L$ and $\bar{C}_M$ are the static coefficients related to drag, lift and moment and $\bar{C}'_D$, $\bar{C}'_L$ and $\bar{C}'_M$ are the coefficient derivatives with respect to the angle α of the bridge deck as shown in Fig. 1.

2.1. Extended state space representation

The convolution integral in (2) is computationally expensive and a rational function approximation with increased efficiency has been suggested first for airfoil theory [23] and later

introduced in bridge flutter assessment [24, 25, 26]. The idea is to represent the aerodynamic force by instantaneous terms proportional to the displacement, velocity and acceleration as well as one or more memory terms that can be included in the equation of motion as additional state variables. This suggests writing the kernel function in the form

$$\mathbf{Q}(\tau) = -\mathbf{M}_a \ddot{\delta}(\tau) - \mathbf{C}_a \dot{\delta}(\tau) - \mathbf{K}_a \delta(\tau) + \sum_{j=1}^J \mathbf{Q}_j(\tau), \quad (8)$$

where $\delta(\cdot)$ is the generalized delta function, $\mathbf{M}_a$, $\mathbf{C}_a$ and $\mathbf{K}_a$ are referred to as the aerodynamic mass, damping and stiffness and the functions $\mathbf{Q}_j$ represent the memory part of the kernel function. Substitution of this into the convolution integral in equation (2) yields the aerodynamic forces

$$\mathbf{f}_a = -\mathbf{M}_a \ddot{\mathbf{q}}(t) - \mathbf{C}_a \dot{\mathbf{q}}(t) - \mathbf{K}_a \mathbf{q}(t) + \sum_{j=1}^J \mathbf{f}_j, \quad (9)$$

where

$$\mathbf{f}_j = \int_0^\infty \mathbf{Q}_j(\tau) \mathbf{q}(t - \tau) d\tau. \quad (10)$$

This representation of the self-excited forces suggests that the contribution from the mean wind can partly be considered as a modification of the overall system mass, damping and stiffness dependent on the mean wind speed. When introducing the expression of the aerodynamic self-excited forces (9) into the equation of motion (1), it takes the form

$$\mathbf{M} \ddot{\mathbf{q}}(t) + \mathbf{C} \dot{\mathbf{q}}(t) + \mathbf{K} \mathbf{q}(t) = \sum_{j=1}^J \mathbf{f}_j + \mathbf{f}_e(t), \quad (11)$$

with system mass, damping and stiffness matrices $\mathbf{M} = \mathbf{M}_s + \mathbf{M}_a$, $\mathbf{C} = \mathbf{C}_s + \mathbf{C}_a$ and $\mathbf{K} = \mathbf{K}_s + \mathbf{K}_a$, respectively. The memory part of the aerodynamic forces is causal, and this implies that $\mathbf{Q}_j = 0$ for $\tau < 0$. In the present paper the aerodynamic properties of each cross-section are identical. When the kernel functions are assumed of exponential form this corresponds to the format

$$\mathbf{Q}_j = \mathbf{D}_j e^{-\gamma_j \tau}, \quad \tau \geq 0, \quad (12)$$

where the global matrix $\mathbf{D}_j$ contains identical diagonal block matrices, each representing the properties of the individual cross-section. If the aerodynamic properties of the cross-sections vary along the bridge deck, each diagonal block in the global matrix $\mathbf{D}_j$ may be different and associated with individual values of the decay parameter γ_j for different cross-sections.

When inserting the exponential representation of the kernel function into the convolution integral in (10), the memory part of the self-excited forces are found in the form

$$\mathbf{f}_j(t) = \int_0^\infty \mathbf{D}_j e^{-\gamma_j \tau} \mathbf{q}(t - \tau) d\tau. \quad (13)$$

In order to represent the memory forces as additional state variables in the equation of motion the objective is to find a first order differential equation giving the relation between the memory forces and the displacements. The first time derivative of the memory force is found by differentiating the displacement vector, whereby integration by parts gives

$$\dot{\mathbf{f}}_j(t) = -\mathbf{D}_j \left[e^{-\gamma_j \tau} \mathbf{q}(t - \tau) \right]_{\tau=0}^\infty - \gamma_j \int_0^\infty \mathbf{D}_j e^{-\gamma_j \tau} \mathbf{q}(t - \tau) d\tau. \quad (14)$$

It follows from (13) that the integral defines the force $\mathbf{f}_j$, leading to the first-order differential equation

$$\dot{\mathbf{f}}_j + \gamma_j \mathbf{f}_j = \mathbf{D}_j \mathbf{q}. \quad (15)$$

Each memory term of the aerodynamic forces can then be introduced in the equation of motion as an additional state-space variable.

When introducing the velocity as a state-space variable $\mathbf{v}$ via the momentum relation $\mathbf{M}\mathbf{v} = \mathbf{M}\dot{\mathbf{q}}$ a convenient first order format of the equation of motion can be written as

$$\begin{bmatrix} \mathbf{C} & \mathbf{M} & \mathbf{0} \\ \mathbf{M} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{J \times J} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}} \\ \dot{\mathbf{v}} \\ \dot{\mathbf{f}} \end{bmatrix} + \begin{bmatrix} \mathbf{K} & \mathbf{0} & -\mathbf{I}_{1 \times J} \\ \mathbf{0} & -\mathbf{M} & \mathbf{0} \\ -\mathbf{D} & \mathbf{0} & \mathbf{\Gamma} \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ \mathbf{v} \\ \mathbf{f} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_e \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad (16)$$

where the matrices $\mathbf{D}$ and $\mathbf{\Gamma}$, and the vector $\mathbf{f}$ from the J convolution integrals are defined as

$$\mathbf{D} = \begin{bmatrix} \mathbf{D}_1 \\ \vdots \\ \mathbf{D}_J \end{bmatrix}, \quad \mathbf{\Gamma} = \begin{bmatrix} \gamma_1 \mathbf{I} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \gamma_J \mathbf{I} \end{bmatrix}, \quad \mathbf{f} = \begin{bmatrix} \mathbf{f}_1 \\ \vdots \\ \mathbf{f}_J \end{bmatrix}, \quad (17)$$

and the block diagonal unit matrix $\mathbf{I}_{J \times J}$ and the row of unit matrices $\mathbf{I}_{1 \times J}$ complete the equations in a consistent way. If the aerodynamic properties vary along the deck, each diagonal block matrix $\mathbf{D}_j$ will consist of individual block matrices corresponding to each cross-section, and similarly the global matrix $\gamma_j \mathbf{I}$ is replaced by individual diagonal sub-matrices representing each cross-section. The implementation of the general format is quite straight-forward.

Stability of the system is closely related to the energy balance, and in the numerical integration of the system equations it is of interest to reproduce the energy balance equation as closely as possible. An energy balance equation for the wind-loaded structure is obtained by pre-multiplication of the equations of motion (16) with the block vector $[\dot{\mathbf{q}}^T, -\dot{\mathbf{v}}^T, \mathbf{0}^T]$, leading to

$$\frac{d}{dt} \left(\frac{1}{2} \mathbf{v}^T \mathbf{M} \mathbf{v} + \frac{1}{2} \mathbf{q}^T \mathbf{K} \mathbf{q} \right) = \mathbf{v}^T \left(\mathbf{f}_e + \sum_j \mathbf{f}_j \right) - \mathbf{v}^T \mathbf{C} \mathbf{v}, \quad (18)$$

where the velocity $\mathbf{v} = \dot{\mathbf{q}}$ has been introduced. The left hand side of the equation is the rate of change of the energy of the structure, consisting of the sum of the kinetic energy and the elastic energy. The right hand side contains the rate of work of the sum of the external force $\mathbf{f}_e$ and the contributions $\mathbf{f}_j$ to the aerodynamic forces, and the rate of dissipation induced by the damping matrix $\mathbf{C}$. It is noted that the system matrices $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{C}$ contain aerodynamic contributions according to (11). In principle, a more detailed energy balance can be obtained by including self-induced aerodynamic forces $\mathbf{f}$ in the last position of the pre-multiplication vector. However, this has not been included here, as it is difficult to use directly due to its non-conservative character.

2.2. Aerodynamic matrices and parameter identification

The relation between the aerodynamic derivatives and the aerodynamic system parameter matrices, $\mathbf{M}_a$, $\mathbf{C}_a$, $\mathbf{K}_a$, $\mathbf{D}$ and $\mathbf{\Gamma}$ is now established, followed by the introduction of a simple algorithmic procedure for identifying the aerodynamic system parameters numerically. In this section the matrices represent the variables associated with an individual cross-section

without introducing a special notation for this. The parameter relations are found via the Fourier transform of the kernel function

$$\bar{\mathbf{Q}}(\omega) = \int_0^\infty \mathbf{Q}(\tau) e^{-i\omega\tau} d\tau. \quad (19)$$

The exponential representation of the memory part of the kernel function is substituted from equation (12) whereby the Fourier transform of each memory part is found on the simple form

$$\bar{\mathbf{Q}}_j(\omega) = \mathbf{D}_j \int_0^\infty e^{-(\gamma_j + i\omega)\tau} d\tau = \frac{\mathbf{D}_j}{\gamma_j + i\omega}. \quad (20)$$

The Fourier transform of the entire kernel function presented in equation (8) is then found using the same normalization as in (5)

$$\frac{\bar{\mathbf{Q}}}{\frac{1}{2}\rho U^2 B} = \omega_*^2 \mathbf{M}_a^* - i\omega_* \mathbf{C}_a^* - \mathbf{K}_a^* + \sum_j \frac{\mathbf{D}_j^*}{\gamma_j^* + i\omega_*}, \quad (21)$$

where the normalized aerodynamic system matrices and memory parameters are given as

$$\mathbf{M}_a^* = \mathbf{M}_a / (\frac{1}{2}\rho B^3), \quad \mathbf{C}_a^* = \mathbf{C}_a / (\frac{1}{2}\rho U B^2), \quad \mathbf{K}_a^* = \mathbf{K}_a / (\frac{1}{2}\rho U^2 B) \quad (22)$$

and

$$\mathbf{D}_j^* = \mathbf{D}_j / (\frac{1}{2}\rho U^3), \quad \gamma_j^* = \gamma_j B / U. \quad (23)$$

The relations between the aerodynamic derivatives and the aerodynamic system matrices can be established by comparing the Fourier transform of the kernel function in (5) with the expression in (21). For clarity the relation is split into the real part

$$\mathbf{M}_a^* - \frac{1}{\omega_*^2} \mathbf{K}_a^* + \frac{1}{\omega_*^2} \sum_j \frac{\gamma_j^*}{\gamma_j^{*2} + i\omega_*^2} \mathbf{D}_j^* = \frac{1}{B} \begin{bmatrix} P_4^* & P_6^* & BP_3^* \\ H_6^* & H_4^* & BH_3^* \\ BA_6^* & BA_4^* & B^2 A_3^* \end{bmatrix} \quad (24)$$

and the imaginary part

$$-\frac{1}{\omega_*} \mathbf{C}_a^* - \frac{1}{\omega_*} \sum_j \frac{1}{\gamma_j^{*2} + i\omega_*^2} \mathbf{D}_j^* = \frac{1}{B} \begin{bmatrix} P_1^* & P_5^* & BP_2^* \\ H_5^* & H_1^* & BH_2^* \\ BA_5^* & BA_1^* & B^2 A_2^* \end{bmatrix} \quad (25)$$

A method for parameter identification was presented by Scanlan et al. [22] using non-linear least squares fitting. This method has been adopted by Caracoglia and Jones [31] with a discussion of lack of robustness, and an improved iterative algorithmic approach for parameter fitting was later presented by Zhang et al. [32] providing a numerical scheme with multiple nested loops to search for the optimal exponentials. Here, a simple identification procedure is presented based on the least squares solution of an over-determined equation system. The equally weighted minimization process is written as

$$\min_{\gamma_j \in \mathbb{R}_+} \{(\mathbf{A}\mathbf{x} - \mathbf{b})^T (\mathbf{A}\mathbf{x} - \mathbf{b})\} \quad (26)$$

where $\mathbf{A}$ is the system matrix containing the relations between the aerodynamic derivatives and the aerodynamic system matrices as they appear of (24) and (25),

$$\mathbf{A} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & -(1/\omega_*^2)\mathbf{I} & \gamma_1/(\gamma_1^{*2} + \omega_*^2)\mathbf{I} & \cdots & \gamma_n/(\gamma_n^{*2} + \omega_*^2)\mathbf{I} \\ \mathbf{0} & -(1/\omega_*)\mathbf{I} & \mathbf{0} & \gamma_1/(\gamma_1^{*2} + \omega_*^2)\mathbf{I} & \cdots & \gamma_n/(\gamma_n^{*2} + \omega_*^2)\mathbf{I} \end{bmatrix} \quad (27)$$

The number of rows in $\mathbf{A}$ is $18m$, where m is the number of data points. The vector $\mathbf{b}$ contains the frequency dependent aerodynamic derivatives via the Fourier transform of the kernel function

$$\mathbf{b}^T = \left[\text{Re}[\tilde{\mathbf{Q}}^*]^T, \text{Im}[\tilde{\mathbf{Q}}^*]^T \right]. \quad (28)$$

Finally, the vector $\mathbf{x}$ is found by solving the overdetermined linear system $\mathbf{A}\mathbf{x} = \mathbf{b}$ and contains the elements of the aerodynamic system matrices for a particular set of γ_j^* , $j = 1, 2, \dots, n$. This gives $\mathbf{x}$ in the form

$$\mathbf{x}^T = \left[\tilde{\mathbf{M}}^{*T}, \tilde{\mathbf{C}}^{*T}, \tilde{\mathbf{K}}^{*T}, \tilde{\mathbf{D}}_1^{*T}, \dots, \tilde{\mathbf{D}}_n^{*T} \right]. \quad (29)$$

The tilde denotes a rearrangement of the matrix so that the columns in the matrix are stacked in a vector, here exemplified by a matrix $\mathbf{V}$ containing vectors $\mathbf{v}_j$, $j = 1, 2, \dots, n$ as columns:

$$\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n] \Rightarrow \tilde{\mathbf{V}} = [\mathbf{v}_1^T, \mathbf{v}_2^T, \dots, \mathbf{v}_n^T]^T \quad (30)$$

The least square optimization requires an initial guess on the values of γ_j and from experience a good initial guess is $\gamma_j = 1$. Specific results are given in section 3

2.3. Time integration

Generalizing the momentum based time integration procedure, presented e.g. in [33], the discretized equations are obtained by integrating the extended first-order equations of motion (16) over a time interval h ,

$$\begin{bmatrix} \mathbf{C} & \mathbf{M} & \mathbf{0} \\ \mathbf{M} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{J \times J} \end{bmatrix} \begin{bmatrix} \int_h \dot{\mathbf{q}} dt \\ \int_h \dot{\mathbf{v}} dt \\ \int_h \dot{\mathbf{f}} dt \end{bmatrix} + \begin{bmatrix} \mathbf{K} & \mathbf{0} & -\mathbf{I}_{1 \times J} \\ \mathbf{0} & -\mathbf{M} & \mathbf{0} \\ -\mathbf{D} & \mathbf{0} & \mathbf{\Gamma} \end{bmatrix} \begin{bmatrix} \int_h \mathbf{q} dt \\ \int_h \mathbf{v} dt \\ \int_h \mathbf{f} dt \end{bmatrix} = \begin{bmatrix} \int_h \mathbf{f}_e dt \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad (31)$$

A discrete form is obtained by representing the integrals by the second order accurate trapezoidal rule in terms of the mean value, denoted by an overbar as e.g. $\bar{\mathbf{q}} = \frac{1}{2}(\mathbf{q}_n + \mathbf{q}_{n+1})$, where the subscripts $n+1$ and n refer to the end points of the time interval. When the increment is denoted by $\Delta\mathbf{q} = \mathbf{q}_{n+1} - \mathbf{q}_n$ the discretized form is

$$\begin{bmatrix} \mathbf{C} & \mathbf{M} & \mathbf{0} \\ \mathbf{M} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{J \times J} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{q} \\ \Delta\mathbf{v} \\ \Delta\mathbf{f} \end{bmatrix} + h \begin{bmatrix} \mathbf{K} & \mathbf{0} & -\mathbf{I}_{1 \times J} \\ \mathbf{0} & -\mathbf{M} & \mathbf{0} \\ -\mathbf{D} & \mathbf{0} & \mathbf{\Gamma} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{q}} \\ \bar{\mathbf{v}} \\ \bar{\mathbf{f}} \end{bmatrix} = h \begin{bmatrix} \bar{\mathbf{f}}_e \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}. \quad (32)$$

These equations serve as basis of the numerical time integration discussed below, but are also used for a linear stability analysis as discussed in Section 5.2.

An energy balance for the discretized equations is found by pre-multiplication of (32) by $[\Delta\mathbf{q}^T, -\Delta\mathbf{v}^T, \mathbf{0}^T]$. When the mean-value terms are expressed using the relation

$$\Delta\mathbf{q}^T \mathbf{K} \bar{\mathbf{q}} = \left[\frac{1}{2} \mathbf{q}^T \mathbf{K} \mathbf{q} \right]_n^{n+1}, \quad (33)$$

the energy balance equation of the discretized equations of motion takes the form

$$\left[\frac{1}{2} \mathbf{v}^T \mathbf{M} \mathbf{v} + \frac{1}{2} \mathbf{q}^T \mathbf{K} \mathbf{q} \right]_n^{n+1} = \Delta\mathbf{q}^T \left(\bar{\mathbf{f}}_e + \sum_j \bar{\mathbf{f}}_j \right) - \frac{1}{h} \Delta\mathbf{q}^T \mathbf{C} \Delta\mathbf{q}. \quad (34)$$

The discrete energy balance is a fairly direct reproduction of the differential form (18) in which the rate of change of the mechanical energy is replaced by its increment over the integration time interval h and the velocity is replaced by its integral mean $h^{-1}\Delta\mathbf{q}$ in the external work and dissipation terms on the right hand side of the equation.

When solving the full equations of motion with aerodynamic forces (32), the equations for the memory-based aerodynamic forces are expressed in terms of the individual contributions $\mathbf{f}_j$, and the corresponding equations are used to express the current unknown force $\mathbf{f}_{n+1}^j$ in terms of the value in the previous step $\mathbf{f}_n^j$ together with the displacement vectors $\mathbf{q}_{n+1}$ and $\mathbf{q}_n$. Similarly, the velocity $\mathbf{v}_{n+1}$ is expressed in terms of its previous value $\mathbf{v}_n$ and the displacement vectors $\mathbf{q}_{n+1}$ and $\mathbf{q}_n$. The first row in (32) can then be written as a set of equations for the displacement increment $\Delta\mathbf{q}$ in terms of the values of the state-space variables at step n . Thus, the size of the system of equations to be solved corresponds to the number of degrees-of-freedom in the structural model. The algorithm is presented in Table 1. It is noted that the inverse of the algorithmic stiffness matrix $\mathbf{K}_*$ should not be computed directly, but merely represents the solution of an equation system, conveniently solved in terms of the corresponding triangular factors, calculated outside the time-step loop.

Table 1: Numerical integration algorithm.

(1)	System matrices: $\mathbf{K}, \mathbf{C}, \mathbf{M}, \mathbf{D}, \mathbf{\Gamma}$
	$\mathbf{K}_* = \mathbf{K} + \frac{2}{h}\mathbf{C} + \left(\frac{2}{h}\right)^2 \mathbf{M} - \sum_j \frac{1}{2/h + \gamma_j} \mathbf{D}_j$
(2)	Initial conditions:
	$\mathbf{q}_0, \mathbf{v}_0, \mathbf{f}_0$
(3)	Displacement increment:
	$\Delta\mathbf{q} = 2\mathbf{K}_*^{-1} \left[\bar{\mathbf{f}}_e - \mathbf{K}\mathbf{q}_n + \frac{2}{h}\mathbf{M}\mathbf{v}_n + \sum_j \frac{1}{2/h + \gamma_j} \left(\frac{2}{h}\mathbf{f}_n^j + \mathbf{D}_j\mathbf{q}_n \right) \right]$
(4)	State vector update:
	$\mathbf{q}_{n+1} = \mathbf{q}_n + \Delta\mathbf{q}$
	$\mathbf{v}_{n+1} = \mathbf{v}_n + 2\left(\frac{1}{h}\Delta\mathbf{q} - \mathbf{v}_n\right)$
	$\mathbf{f}_{n+1}^j = \mathbf{f}_n^j + \frac{2}{2/h + \gamma_j} \left(\mathbf{D}_j\bar{\mathbf{q}} - \gamma_j\mathbf{f}_n^j \right)$
(5)	Return to (3) for a new time step or stop

The discrete dynamic equations (32) and the corresponding time integration algorithm in Table 1 represent a direct discretization of the system differential equations (16). For models with high-frequency modes there may be a need for so-called algorithmic damping, a mechanism for dissipating energy from high-frequency modes close to or beyond their aliasing limit. As demonstrated in [33], algorithmic damping that specifically addresses the high-frequency modes may be introduced in the present integration format on two levels – either as a generalized viscous damping, or by additional damping forces generated by a first-order filter. The generalized viscous damping consists in introducing two balanced terms proportional to the stiffness matrix $\mathbf{K}$ and the mass matrix $\mathbf{M}$, respectively, in the diagonal of the first matrix

Figure 2: The Sulafjord Bridge: (a) Rendering [37], (b) Finite element model.

in (32),

$$\begin{bmatrix} \mathbf{C} + \frac{1}{2}\alpha h \mathbf{K} & \mathbf{M} & \mathbf{0} \\ \mathbf{M} & -\frac{1}{2}\alpha h \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{J \times J} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{q} \\ \Delta \mathbf{v} \\ \Delta \mathbf{f} \end{bmatrix} + h \begin{bmatrix} \mathbf{K} & \mathbf{0} & -\mathbf{I}_{1 \times J} \\ \mathbf{0} & -\mathbf{M} & \mathbf{0} \\ -\mathbf{D} & \mathbf{0} & \mathbf{\Gamma} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{q}} \\ \bar{\mathbf{v}} \\ \bar{\mathbf{f}} \end{bmatrix} = h \begin{bmatrix} \bar{\mathbf{f}}_e \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}. \quad (35)$$

The effect of the new terms can be seen from the energy balance equation, that now takes the form

$$\begin{aligned} \left[\frac{1}{2} \mathbf{v}^T \mathbf{M} \mathbf{v} + \frac{1}{2} \mathbf{q}^T \mathbf{K} \mathbf{q} \right]_n^{n+1} &= \Delta \mathbf{q}^T \left(\bar{\mathbf{f}}_e + \sum_j \bar{\mathbf{f}}_j \right) - \frac{1}{h} \Delta \mathbf{q}^T \mathbf{C} \Delta \mathbf{q} \\ &- \alpha \left(\frac{1}{2} \Delta \mathbf{v}^T \mathbf{M} \Delta \mathbf{v} + \frac{1}{2} \Delta \mathbf{q}^T \mathbf{K} \Delta \mathbf{q} \right). \end{aligned} \quad (36)$$

It is seen that in contrast to the original viscous dissipation term the additional dissipation terms are scaled by the time-step h and contain a quadratic form of the velocity increment $\Delta \mathbf{v}$. Hereby the algorithmic damping dissipates energy at high frequencies relative to the time-step, while the other terms are system specific and dissipate energy at certain frequencies independent of the time-step. The balanced algorithmic damping is a simple and computationally inexpensive implementation that provides damping with the same frequency dependence as the numerical damping of a traditional Newmark algorithm, [34], however without the distortion of the mechanical energy represented in the damped Newmark algorithm. Alternatively, first-order filters can be applied to provide improved algorithmic damping with frequency dependence corresponding to the generalized- α method, in which the damping is shifted towards higher frequencies [35]. The filter representation of the algorithmic damping force is similar to the present representation of the memory dependent aerodynamic forces, but does not contain any matrices and can therefore be implemented with negligible computational overhead as demonstrated in [33].

3. Bridge model

The structural model used for response calculation is a 3000 m suspension bridge proposed for crossing Sulafjorden in Norway in relation to the Coastal Highway E39 project [36, 37]. In Fig. 2 a rendering of the bridge structure is shown to the left and the finite element model used for response calculations is shown to the right. The suspension cables as well as the hangers are modelled with Green strain bar elements to ease the cable calibration procedure. However, the response calculations are performed on a linearised model. The towers are

Table 2: Still-air frequencies, S/A: symmetric/asymmetric, D/H/T: drag/heave/torsion.

Type	f [Hz]	Type	f [Hz]	Type	f [Hz]	Type	f [Hz]
SD1	0.037	SD2	0.117	SD3/ST1	0.134	AD3	0.182
AD1	0.074	AT1	0.118	AH2	0.137	ST3	0.193
AH1	0.076	SH2	0.126	ST2	0.145	AH3	0.209
SH1	0.091	AD2/AT2	0.129	SH3	0.175	AT3	0.221

modelled with elastic beam elements and the bridge deck is modelled using aero-elastic beam elements with self-excited forces represented via additional degrees of freedom in the system as presented in Section 2.1.

The structural properties are presented here providing information about the overall geometry, structural damping and still-air frequencies. A more detailed description of the structural properties is found in [36]. The distance between the pylons is 3000 m, the sag to span ratio of the suspension cables is 1:10 and the clearance height of the bridge deck is 74 m. Table 2 gives the lowest 16 still-air natural frequencies of the bridge.

The structural damping is implemented in the form of Rayleigh damping

$$\mathbf{C}_s = \alpha \mathbf{M}_s + \beta \mathbf{K}_s, \quad (37)$$

corresponding to the modal damping ratios

$$2\zeta_j = \frac{\alpha}{\omega_j} + \beta\omega_j, \quad j = 1, 2, \dots. \quad (38)$$

The parameters α and β are determined to provide a realistic level of damping with damping ratio $\zeta \simeq 0.32\%$ for the low modes, relevant for flutter, [38]. When setting the damping ratio to $\zeta_* = 0.32\%$ at two frequencies $\omega_a = \omega_1 = 0.232 \text{ rad/s}$ and $\omega_b = \omega_{10} = 0.861 \text{ rad/s}$, the parameters α and β are

$$\alpha = 2 \frac{\omega_a \omega_b}{\omega_a + \omega_b} \zeta_* = 1.17 \cdot 10^{-3} \text{ s}^{-1}, \quad \beta = \frac{2}{\omega_a + \omega_b} \zeta_* = 5.85 \cdot 10^{-3} \text{ s} \quad (39)$$

when the angular frequencies ω_1 and ω_{10} are determined from Table 2.

The bridge deck is a twin-box girder as shown in Fig. 1. However, as no detailed measurements of the aerodynamic coefficients are available for this cross-section at the present stage, the self-excited forces for heave and torsional motion are treated by the Theodorsen theory [20] for a flat plate. The self-excited forces related to drag are implemented based on quasi-steady theory corresponding to instantaneous action, see e.g [30]. The drag, lift and torsion aerodynamic coefficients are set to $\bar{C}_D = 1.20$, $\bar{C}_L = -0.15$ and $\bar{C}_M = 0.30$, and their first derivative with respect to the deck angle α are $C'_D = 0$, $C'_L = 6.3$ and $C'_M = 1.0$. The length of the bridge deck is $L_b = 3000 \text{ m}$, the height is $D = 2.5 \text{ m}$, and the reference width is $B = 13 \text{ m}$. Note, that this is the reference width for applying flat plate theory taking into account the expected effect of an aerodynamically shaped twin-box girder, [39]. The admittance functions are all set to one. The main suspension cables and the hangers have a diameter of $D_c = 1.19 \text{ m}$ and $D_h = 0.10 \text{ m}$, respectively, and the drag coefficient is set to 0.8 for all cables. The drag coefficient for the pylons is 2. Other aerodynamic coefficients are set to zero.

In order to determine the number of memory terms to be included in the representation of the self-excited forces the approximated aerodynamic derivatives for different number of

Figure 3: Aerodynamic derivatives, flat plate: (o) analytic plate, ($\cdot$ —, —, —) 0,1,2 memory terms.

memory terms are shown in Fig. 3. The black dashed-dotted line is without memory terms, the magenta dashed line represents a single memory term, and the blue solid line corresponds to two memory terms. The approximated aerodynamic derivatives are shown together with their theoretical counterparts for flat plate theory shown as black circles. Figure 3 shows that the representation without memory terms results in a rather poor representation of H_2^* , H_4^* , A_2^* and A_4^* . Also, it is seen that the representation of the remaining functions H_1^* , H_2^* , A_1^* and A_2^* is linear, introducing considerable arbitrariness in H_2^* and A_2^* . The single memory term representation is seen to be quite good for all eight derivatives, with only a modest improvement in the two-term representation, visible in the representation of H_4^* and A_4^* . This indicates that a single memory term representation provides sufficient accuracy in the case of flat plate theory. The single memory representation for the considered flat plate is given by the normalized aerodynamic system matrices and memory parameters

$$\mathbf{M}_a^* = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.105 & -0.019 \\ 0 & -0.053 & -0.060 \end{bmatrix}, \mathbf{C}_a^* = \begin{bmatrix} 0.036 & 0.012 & 0 \\ -0.023 & 0.281 & -2.051 \\ 0.600 & 0.913 & 3.545 \end{bmatrix}, \mathbf{K}_a^* = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.055 & -3.953 \\ 0 & 0.177 & -12.848 \end{bmatrix} \quad (40)$$

and

$$\mathbf{D}_1^* = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.017 & 0.593 \\ 0 & 0.056 & 1.927 \end{bmatrix}, \quad \gamma_1^* = 0.310, \quad (41)$$

where the normalization is defined in (22) and (23).

3.1. Reduced model

The structural model is reduced by quasi-static condensation to improve the computational efficiency. The method of quasi-static condensation consists of selecting a number of ‘dynamic’ degrees of freedom, here denoted $\mathbf{q}_d$, and the remaining ‘static’ degrees of freedom $\mathbf{q}_s$. In the equations of motion corresponding to the ‘static’ degrees of freedom the inertia and damping terms are neglected, leading to a representation entirely in terms of the ‘dynamic’ degrees of freedom,

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}_d \\ \mathbf{q}_s \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{S} \end{bmatrix} \mathbf{q}_d, \quad \mathbf{S} = -\mathbf{K}_{ss}^{-1} \mathbf{K}_{sd} \quad (42)$$

The subscripts d and s on the stiffness matrix $\mathbf{K}$ refer to the blocks corresponding to dynamic or quasi-static degrees of freedom. Extending the method to include motion-induced forces suggests to treat the state proportional aero-elastic terms as the structural mass, damping and stiffness while the memory part of the motion-induced forces are reduced by considering the rate of work. The reduced mass matrix and force vector are found as

$$\bar{\mathbf{M}} = \mathbf{M}_{dd} + \mathbf{S}^T \mathbf{M}_{sd} + \mathbf{M}_{ds} \mathbf{S} + \mathbf{S}^T \mathbf{M}_{ss} \mathbf{S}, \quad \bar{\mathbf{f}} = \mathbf{f}_d + \mathbf{S}^T \mathbf{f}_s \quad (43)$$

and the reduction of the other system matrices and load vectors follows the same format.

The quality of the approximation depends on the difference between the quasi-static shape introduced between the master nodes, and the shape in the full dynamic model. Two different levels of reduction, 1:5 and 1:10, are considered to find a relevant level for the present structure. The methodology for the node selection is to choose every fifth or tenth node along the bridge deck and the corresponding nodes in the suspension cables as master nodes. Figure 4 shows

the principle of the master node selection for the case of the 1:5 reduction, with the master nodes indicated by black dots.

Figure 4: (a) Reduced system nodes, (b) second mode for reduction of 1:1, 1:5, 1:10.

Figure 4(b) shows the second symmetric heave mode determined with the full model at the top, with the 1:5 reduction in the middle, and the 1:10 reduction at the bottom. It is seen that the 1:5 reduction captures the mode shape well for this particular mode, whereas the 1:10 reduction is too coarse. Figure 5 shows the natural frequencies for the full model and the two reduced models. Figure 5(a) shows the natural frequencies for the still air system and Fig. 5(b) shows the natural frequencies for the aero-elastic system with wind speed $U = U_{cr}$, where $U_{cr} = 57$ m/s is the critical wind speed at which the system becomes unstable. Both plots indicate that the 1:5 reduction gives a good representation of modes up to around mode number 15, whereas the 1:10 reduction shows differences at mode 9 and up. The frequencies shown in Fig. 5 suggest that a 1:5 reduction maintains considerable accuracy while decreasing the computational effort, whereas the 1:10 reduction might be too coarse.

Figure 5: Modal frequencies: (a) Still-air, (b) at critical wind speed. Full model ($\circ$), 1:5 ($\star$), 1:10 ($\diamond$).

4. Wind field model

The wind field description used in this paper consists of a three-dimensional full-field representation of the turbulence described in 3D space and convected across the structure with the mean wind velocity U . The covariance structure of the turbulent wind velocity field is generated by a transformation of an equivalent isotropic field, thereby accounting for the different length scales in the along-wind and transverse directions. This representation is supplemented by an efficient sequential simulation model based on convected ‘frozen’ turbulence. These aspects are briefly outlined in the following subsections, and a detailed description of the underlying theory can be found in [19].

4.1. Stretched isotropic turbulence

The representation of the turbulent velocity field is based on a transformation of an equivalent isotropic velocity field, and therefore a compact resume of the properties of the isotropic velocity field is first given. In the equivalent homogeneous isotropic field the covariance matrix of the velocity vectors at two points is expressed in terms the vector $\mathbf{r}$ of length $r = |\mathbf{r}|$ connecting the two points. According to the classical theory of isotropic turbulence, [3], the covariance matrix of the velocity vector $\tilde{\mathbf{v}}$ at two points separated by the vector $\mathbf{r}$ is of the form

$$\mathbf{R}(\mathbf{r}) = \mathbf{E}[\tilde{\mathbf{v}}(\mathbf{r}_0 + \mathbf{r}) \tilde{\mathbf{v}}(\mathbf{r}_0)^T] = \sigma_{\text{iso}}^2 \left([f(r) - g(r)] \frac{\mathbf{r} \mathbf{r}^T}{r^2} + g(r) \mathbf{I} \right), \quad (44)$$

where σ_{iso}^2 is the variance of a single wind velocity component in the isotropic field. The functions $f(r)$ and $g(r)$ describe the lengthwise and transverse correlation, respectively.

The lengthwise correlation function $f(r)$ is introduced via its spectral wave-number representation

$$\sigma_{\text{iso}}^2 f(r) = \int_{-\infty}^{\infty} F(k) e^{ikr} dk, \quad (45)$$

where k is the wave-number corresponding to the spatial separation r . For convection in the x_1 -direction this defines a relation between the spatial x_1 -coordinate and the time t of the form $x_1 = -Ut$, and thus the spatial x_1 variation corresponds to a time history and the corresponding spectral densities are easily scaled to frequency form. In the present context it is convenient to represent the spectral density function $F(k)$ in terms of the normalized form of the von Kàrmàn spectral density, [4],

$$F(k) = \frac{1}{\sqrt{\pi}} \frac{\Gamma(5/6)}{\Gamma(1/3)} \frac{\sigma_v^2 \ell}{[1 + (k\ell)^2]^{5/6}}, \quad (46)$$

where $\Gamma(\cdot)$ is the gamma function, and ℓ is a length parameter, determined by the integral length-scale λ as $\ell = 1.339\lambda$. The sequential simulation procedure used here does not use the spectral density directly, but the covariance matrix $\mathbf{R}(\mathbf{r})$ expressed in terms of the corresponding correlation functions

$$f(r) = \frac{2}{\Gamma(1/3)} \left(\frac{r}{2\ell} \right)^{1/3} K_{1/3} \left(\frac{r}{\ell} \right), \quad g(r) = f(r) - \frac{2}{\Gamma(1/3)} \left(\frac{r}{2\ell} \right)^{4/3} K_{-2/3} \left(\frac{r}{\ell} \right), \quad (47)$$

where the transverse correlation function $g(r)$ follows from incompressibility of the velocity field, and $K_\nu(\cdot)$ denotes the modified Bessel function of the second kind of order ν . These functions are available in many computer software libraries.

In the isotropic turbulence model the length scale is the same in all directions and similarly the turbulence intensity σ_{iso}^2 identical for each of three velocity components. A simple generalization that conserves incompressibility can be obtained by a linear transformation of the spatial coordinates and performing a similar transformation of the corresponding wind velocity components,

$$\mathbf{x} = \mathbf{F} \mathbf{r}, \quad \mathbf{v} = \mathbf{F} \tilde{\mathbf{v}}. \quad (48)$$

In the present case orthotropic scaling, in which each of the axes are stretched, is used, corresponding to

$$\mathbf{F} = \frac{1}{\lambda} \begin{bmatrix} \lambda_x & & \\ & \lambda_y & \\ & & \lambda_z \end{bmatrix} \quad (49)$$

as illustrated in Fig. 6. The easiest way to implement this transformation is to transform the structure into the equivalent isotropic space, perform an isotropic wind field simulation, followed by the transformation (48) back to the physical coordinates and wind velocities.

Figure 6: Wind field stretching: (a) isotropic, (b) stretched.

In the current context of long-span bridge loading it is important to introduce a representative ratio between the standard deviation of the along wind and vertical wind velocity components σ_u and σ_w , and a representative ratio of the horizontal transverse length-scale λ_y to the axial length-scale λ_x . The stretch of the isotropic wind field is chosen based on the Norwegian standard, N400 [40]. The ratio of the vertical to along-wind turbulence intensities is set to $\sigma_w/\sigma_u = 0.5$, thereby defining the vertical spatial stretch of the field λ_z/λ_x . The horizontal transverse horizontal stretch is defined by the length scale ratio $\lambda_y/\lambda_x = 0.5$, with the implication that $\sigma_v/\sigma_u = 0.5$. The wind field parameters are given in Table 3 with U representing the mean wind speed and $I_u = \sigma_u/U$ being the turbulence intensity of the along-wind component. Due to the stretching procedure the parameters in Table 3 provide the complete field input for the representation of the full turbulence field, illustrating the basic simplicity of the stretched isotropic wind field description.

Table 3: Wind field parameters

U [m/s]	I_u	λ [m]	λ_x/λ	λ_y/λ	λ_z/λ
44	0.13	300	1.0	0.5	0.5

4.2. Sequential wind field simulation

The wind-field is simulated with the sequential simulation procedure developed in [19], in which the wind velocity field at the current time is formed as the sum of the conditional mean for a given set of previous velocities and a stochastic contribution representing the random fluctuations. The wind velocity components at a set of selected points at the time step n are collected in the vector $\mathbf{u}_n$. The vector $\mathbf{u}_n$ is generated sequentially by the recurrence relation

$$\mathbf{u}_n = \mathbf{A}\mathbf{w}_n + \mathbf{B}\boldsymbol{\xi}_n, \quad n = 1, 2, \dots \quad (50)$$

in which the vector $\mathbf{w}_n$ contains the extended velocity vector $\mathbf{u}_i$ at j previous times, and the matrix $\mathbf{A}$ collects the corresponding regression matrices according to the format

$$\mathbf{A} = [\mathbf{A}_1, \dots, \mathbf{A}_j], \quad \mathbf{w}_n^T = [\mathbf{u}_{n-i_1}^T, \dots, \mathbf{u}_{n-i_j}^T]. \quad (51)$$

The vector $\boldsymbol{\xi}_n$ is formed by independent normalized random variables, and scaling and correlation are introduced via the coefficient matrix $\mathbf{B}$. The format described by (50) and (51) describes an auto-regressive stochastic process, but in the present context of a convected wind velocity field there are two important special features: the convection concept permits explicit determination of the matrices $\mathbf{A}$ and $\mathbf{B}$, and the spectral characteristics of the wind are efficiently represented by an exponential regression layout with indices $[i_1, i_2, \dots, i_j] = [2^0, 2^1, \dots, 2^{j-1}]$, illustrated in Fig. 7.

Figure 7: Wind simulation memory with 5 memory steps (distorted geometry).

Within the concept of a convected stationary wind field the turbulent wind velocity components are fully characterized by their covariance matrices

$$\mathbf{C}_{\mathbf{u}\mathbf{u}} = E[\mathbf{u}_n \mathbf{u}_n^T], \quad \mathbf{C}_{\mathbf{u}\mathbf{w}} = E[\mathbf{u}_n \mathbf{w}_n^T], \quad \mathbf{C}_{\mathbf{w}\mathbf{w}} = E[\mathbf{w}_n \mathbf{w}_n^T]. \quad (52)$$

These matrices are given directly in terms of the 3D spatial covariance function described in Section 4.1. The matrices $\mathbf{A}$ and $\mathbf{B}$ can then be determined by the following simple argument. It follows directly from the recurrence relation (50) that the first term on the right side is the conditional mean value of $\mathbf{u}_n$ for given $\mathbf{w}_n$. By moving this term to the left side of the equation and multiplying each side with its transpose it follows that the product $\mathbf{B}\mathbf{B}^T$ is

defined by the conditional covariance of $\mathbf{u}_n$ for given $\mathbf{w}_n$. These results can be expressed in the following compact form

$$\mathbf{A} = \mathbf{C}_{\mathbf{uw}}\mathbf{C}_{\mathbf{ww}}^{-1} \quad \text{and} \quad \mathbf{B}\mathbf{B}^T = \mathbf{C}_{\mathbf{uu}} - \mathbf{A}\mathbf{C}_{\mathbf{ww}}\mathbf{A}^T. \quad (53)$$

Full details of the wind field simulation procedure and its calibration have been given in [19].

4.3. Wind field simulation examples

Statistical properties of the wind field described by the data of Table 3 are here estimated from the simulated records and the performance and applicability of the sequential simulation method is discussed in relation to large scale bridge structures. Two different simulation fields are evaluated. Firstly, a full field simulation of all three wind components in 335 simulation points positioned according to the nodes of the three dimensional finite element bridge model including bridge deck, cables and towers. Secondly, a simulation record of a line of 101 simulation points located at the nodes along the bridge deck. For the second simulation record the along bridge wind component is considered redundant and due to symmetry the along wind and vertical wind component become statistically independent, thereby allowing separate simulations of the two remaining components.

For both simulation records the simulation frequency $f_s = 6 \text{ Hz}$, corresponding to an equivalent spatial separation of $\Delta x = U/f_s = 7.33 \text{ m}$, is used in order to obtain a suitable resolution in the time domain. The horizontal separation is $\Delta y = L_b/100 = 30 \text{ m}$. In order to permit predictions with low scatter the simulation time was 60 hours corresponding to $1.296 \cdot 10^6$ time steps. The regression layout is chosen with 8 memory steps, $[i_1, i_2, \dots, i_8] = [1, 2, 4, 8, 16, 32, 64, 128]$. Thus, the depth of the 8-step memory is $128/6 = 21.33 \text{ s}$.

Figure 8: Auto-spectral densities: (a) $S_{uu}(f)$, (b) $S_{wv}(f)$. Full-structure (—), horizontal line (---), analytic (—).

The frequency content in a simulation point is illustrated by the auto-spectral densities. In Fig. 8 the estimated auto-spectral density of the along wind and the vertical wind component are plotted together with the theoretical spectrum. It is seen that both the full-structure and the horizontal line simulation represent the energy content very well over the full frequency interval, indicating negligible influence of the difference from the different simulation point distributions.

Finally, the transverse coherence is estimated from the simulation records. In Fig. 9 the estimated (root) coherence between simulation records at points with a transverse separation of $\frac{1}{2}\lambda$, λ and $\frac{3}{2}\lambda$, respectively, are plotted together with the theoretical coherence given in [19]. The coherence functions are plotted as function of the wave number k_1 times the distance between simulation points y . It is seen that the single-component and the full-field simulation both give a good representation of the target coherence. It is interesting to note, that coherence functions of the general shape illustrated in Fig. 9 have been measured and reported in the recent publications [5, 6, 7].

Figure 9: Transverse coherence for separation $\frac{1}{2}\lambda$, λ and $\frac{3}{2}\lambda$: (a) $\psi_{uu}(k_1 y)$, (b) $\psi_{ww}(k_1 y)$. Full structure (—), horizontal line (—), analytic (—).

The simulated wind velocity records do not show any quality differences between the full-field simulation and the use of two independently simulated single-component records for u and w for points along a horizontal line.

Table 4: Computational time for 3 hours simulated wind velocity record.

	Sequential [19]	Fourier [8]
Full Structure	233 s	-
Deck, field simulation	10 s	134 s
Deck, two-component simulation	2×2 s	2×11 s

Table 4 shows the computation time for 3 hours simulated records obtained with the sequential simulation method. The scenarios covered are points distributed on the full bridge including cable and pylons, all velocity components in points along the bridge deck, and a case in which the vertical and the along-wind components are simulated independently at points along the bridge deck. The third case permits separate simulation of the vertical and the along-wind velocity components because they uncouple for points in a horizontal plane due to spatial symmetry. The last column in the table gives the time of a standard Fourier simulation based on [8]. The computations were carried out on a desktop computer with 32 GB memory and 2.70 GHz Intel Core i7-6820HQ CPU. The 3 hours time record represents the limit for full-field deck Fourier simulation without memory swapping, leaving direct Fourier simulation of the wind velocity field for the full bridge out of the present range. In this context

it should be mentioned that the computation time of the Fourier simulation can be decreased and the storage requirements reduced e.g. by interpolation of the decomposed matrices or by system truncation [9, 14]. Also, super fast Fourier simulation methods have been developed for line-like structures [11, 12] applicable for bridge deck loading.

5. Model performance and results

In this section the model performance is discussed in relation to the quality of the structural response. The wind field is defined by the parameters given in Table 3. Time integration is performed using the second order momentum based time integration algorithm shown in Table 1 with time step $h = 1/6 = 0.166$ s.

5.1. Buffeting response

Figure 10 shows the standard deviation of the drag, heave and torsional response σ_y , σ_z and σ_θ at quarter span as function of the mean wind speed for loading of the full structure and loading of the deck only. The load is evaluated based on the wind speeds at the nodal locations and is assumed linearly distributed over the elements. It is seen that the influence of including full structural loading is only significant for the along wind response, whereas the heave and torsional response for the two load cases are very similar.

Figure 10: Response at quarter-span: deck loading (—), full bridge loading (---).

Figure 11 shows the standard deviation of the displacement at quarter-span in terms of drag, heave and torsion as function of the normalized wind speed. Full structure turbulent loading is applied and the response is found using 0, 1 and 2 memory terms, respectively, in the representation of self-excited aerodynamic forces, corresponding to extending the state-space equation of motion in (16) with 0, 1 or 2 additional state-variable vectors $\mathbf{f}_j$. A quasi-steady representation was assumed for the wind-structure interaction related to drag motion, and thus the drag response is indifferent to the number of memory terms included in the representation of self-excited forces. Also, the heave response is seen to be only slightly influenced by including additional memory terms. However, the torsional response is seen to be overestimated using a zero memory representation of the self-excited forces. The difference in the response found with 1 and 2 memory terms, respectively, is seen to be small in the considered case of flat plate theory, suggesting the use of only one additional state-variable in the response calculations for this case. This result corresponds well with the representation of aerodynamic derivatives illustrated in Fig. 3.

Figure 11: Response standard deviations at quarter-span: 0,1,2 memory terms ($\cdot$ —, —, —) .

The time record of the full bridge model and the bridge model reduced by quasi-static condensation with a node selection ratio of 1:5 are shown in Fig. 12. The mean wind is set to $U = 0.6 U_{cr}$ and full structure turbulent wind load simulation is applied. The time records show the drag q_y , the heave q_z and the torsional q_θ steady-state response at quarter-span

Figure 12: Time response at quarter span: full model (—), 1:5 reduction (—).

in a five minute time interval. It is seen that the response obtained by the reduced model corresponds well with the response found with the full model. The calculation time for the five minute time history is reduced significantly from 59.3s to 0.7s by making use of the reduced model instead of the full model.

Figure 13: Response at quarter-span: frequency domain (—), time domain ($\circ$).

Figure 13 shows the response at quarter span as function of the mean wind speed for loading of the full structure. The solid line represents the response obtained with a frequency domain calculation based on the spectral representation of the isotropic turbulence field given in [2]. As for the time domain loading, stretching of the wind field is equivalent with inverse stretching of the structural model and the scaling of the turbulence component follows the scaling shown in (48). The response is evaluated for a number of frequencies in the interval $\omega = [0.001; 2]$ rad/s with a frequency interval of $\Delta\omega = 0.0014$ rad/s. This method is free of the approximations related to the auto-regressive simulation of the wind and the quality of the result is thereby only determined by the frequency interval and the discretization of the solution domain. For lightly damped systems a relatively fine resolution is needed around the system frequencies. The blue circles marks the time domain results evaluated based on 20 hours of steady-state response. Besides the approximations related to the wind field simulation using this method, the quality of the results also depend on the time-step size chosen for the time integration. The integration algorithm is second order accurate meaning the error introduced is proportional to $(\omega h)^2$, wherefore the number of time-steps per modal period of interest should be at least $2\pi \simeq 6$ to limit the time-step error. The results obtain by the two methods appear consistent as expected based on the statical properties estimated from the wind field, the simulation length, the time-step size and the frequency domain resolution.

5.2. Flutter

The critical wind speed, also referred to as the flutter wind speed, defining the stability limit of the aero-elastic system can be determined from the homogeneous form of the discrete equation of motion (32). The stability limit is found at the mean wind speed that will cause the magnitude of the complex valued amplification factor of the most critical mode of the combined aero-elastic system equations to exceed unity. This stability limit is independent of the external loading. However, in the design of long span bridges the flutter wind speed is sometimes defined as the wind speed at which the torsional and the vertical response exceed a certain limit [41]. In this particular reference the torsional and the vertical response limit is set to 0.035 rad and 1/500 times the span length, respectively.

Figure 14: Transverse correlation functions: (a) $C_{uu}(y)$, (b) $C_{ww}(y)$. Full-field (—), single component(---), analytic (—).

For long-span bridges the span exceeds the turbulence length-scale along the bridge deck. This leads to two effects: the non-trivial form of the coherence function that describes the covariance of turbulence components around a particular frequency, and the limitation of the horizontal covariance to distances of around two times the transverse length-scale. The coherence characteristics are illustrated in relation to the sequential wind field simulation in Fig. 9. The most characteristic feature is that even the low-frequency components have limited correlation over large distances, a feature that would be lost if transverse statistics were represented by the classic non-dimensional combined frequency-length parameter $\omega y/U$. In addition to this feature, demonstrated in several recent measurement campaigns [5, 6, 7], the transverse correlations depend on the length-scale of the turbulent field in the transverse direction. This effect is illustrated in Fig. 14 showing the normalized correlation of the along-wind turbulent velocity component u as well as the vertical velocity component w as a function of the transverse separation, normalized by the transverse length-scale λ_y . It is seen that both the u - and the w -component are effectively uncorrelated for distances exceeding twice the transverse length-scale, $y > 2\lambda_y$. For fully correlated components the response corresponds to that following from a harmonic analysis, whereas the magnitude of the response to uncorrelated components is accumulated via what amounts to a root-mean-square analysis. Thus, limited correlation results in reduced response, and as a result a reduced transverse length-scale will lead to a reduced response.

Figure 15: Response at quarter-span: $\lambda_y = 75$ m (—), $\lambda_y = 150$ m (— · —), $\lambda_y = 300$ m (---).

The influence of the across-wind turbulence length-scale on the response is illustrated in Fig. 15. The plots show the standard deviation of the response as function of the normalized mean wind speed. The response is calculated with simulated wind load on the full structure and is shown for three different transverse length scales, $\lambda_y = 75, 150, 300$ m, while the along wind length scale is held constant at $\lambda_x = 300$ m. As discussed, above a shorter transverse length-scale leads to a smaller part of the wind field being correlated and therefore results in a smaller response. The effect is quite noticeable, and it is seen that a doubling of the length scale results in around 50% increase of the response. This emphasizes that a representative value of the transverse turbulence length-scale is important in the design of long span-bridges.

6. Conclusions

A procedure has been presented for time-domain aerodynamic response analysis of long-span bridges subjected to turbulent wind. The wind load is generated by the mean-field simulation procedure recently developed in [19]. The procedure is based on convection of an isotropic homogeneous turbulent velocity field, which is generalized to orthotropic form by a simple stretching procedure. Based on data from the Norwegian standard [40] the transverse length-scale is set to half of the along-wind scale, and vertical turbulence component intensity is set to half of the along-wind component intensity. The structural model combines beam elements for bridge deck and pylons with bar elements for cables and hangers. It is demonstrated that a 1:5 reduction of the nodes along the deck and main cables leads to a quite accurate representation of the dynamics of the first 13 modes. The self-excited aerodynamic loads are represented via first-order filter terms in the equation of motion, and the full system is represented by a set of equations in which these filter variables are included as auxiliary degrees of freedom. The full equation system is discretized based on momentum considerations, and the discretized equations serve both as a tool for simulation via time integration and as basis for a linear eigenvalue analysis, determining the flutter velocity.

Three procedures were investigated for the simulated wind load: full 3D simulation at all nodes of the structure, full 2D simulation of the wind velocity components on the bridge deck, and a reduced form simulating only the vertical and along-wind components at the nodes of the bridge deck. It was found that the wind load on the cables had only minor influence on the response in relation to low-frequency deck response and flutter. This in turn permits to use the reduced two-component simulation method, in which the two components are statistically independent, hereby reducing the simulation effort further. The direct time-simulation procedure of both wind field and response is considerably faster than FFT-based simulation procedures, and permits simulation of wind fields with a larger number of simulation points, because the storage requirements are much smaller than for the FFT methods. The simulated wind field gives a very detailed representation of the transverse coherence, and examples demonstrate the quite large influence of this in combination with the use of a representative transverse turbulence length scale on the response magnitude.

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