Intrusion Detection, Measurement Correction, and Attack Localization of PMU Networks

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Intrusion Detection, Measurement Correction, and Attack Localization of PMU Networks

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Abstract—Accurate state estimation is essential for correct supervision of power grids. With the existence of cyber-attacks, state estimation may become inaccurate which can eventually lead to wrong supervisory decision making. To detect cyber-attacks in power grids equipped with PMUs, a new intrusion detection system based on clustering approach (PMUIDS) is proposed in this paper. After solving the optimal PMU placement in N-1 contingency, several static state estimations are obtained by removing the measurements of one PMU in each time. The resulting state vectors are clustered in two steps: 1- Subtractive clustering is employed to obtain the number of clusters which determines the number of integrity attacks, 2- Fuzzy C-means clustering assigns the state vectors to the corresponding clusters which determines the attacked PMUs. In addition, two theorems are proved which indicate that the attacker cannot coordinate successful stealth attacks in cases that by removing attacked PMUs from state estimation, the power system still remains fully observable. Furthermore, in the case of possible stealth attacks, the attacker cannot falsify the estimation of any arbitrary state variable. The hardware-in-the-loop (HiL) results on a sample power system show that the proposed approach can detect integrity attacks, determine the number of attacks, obtain the correct state vector, and localize the attacks, even in case of multiple simultaneous attacks.

Index Terms—Attack localization, intrusion detection, measurement correction, PMU network, static state estimation

I. INTRODUCTION

Secure state estimation is a challenging issue in future smart grids equipped with PMUs [1]–[5]. The PMU network is based on Information Technology (IT) networks and protocols which are vulnerable to a variety of cyber-attacks [6]. These attacks may impose false information to the control center, which could lead to false control decisions [7]–[9].

Among the cyber-attacks, those attacks that make power system state estimation inaccurate, are especially dangerous since they can lead to wrong supervisory decision makings. Attacks in this class are usually referred to as false (bad) data injection (or integrity) attacks [10]–[11]. Such attacks are usually detected by a number of bad data detection methods [12]–[14] such as sum detector [15], $\chi^2$ detector [16], and learning-based methods [17], [18]. However, these approaches only detect the attack and they fail to localize the attack and present correct measurements. On the other hand, invaders can orchestrate coordinated false data injection attacks (such as those in [19]–[21]) to make them unobservable by conventional detection methods. Such attacks are called stealth (or interacting) attacks.

In order to detect stealth attacks, a number of methods have been proposed in the literature [1], [22]–[25]. It should be noted that in addition to these researches, several methods are available to mitigate the stealth attacks using authentication, multi-path routing or encryption devices. However, they increase the implementation cost and complexity, and also impose additional delay [26], [27].

In this paper, a new intrusion detection system based on clustering techniques, called PMUIDS, is proposed which has some advantages over the previous researches, which are described next. In PMUIDS, first the optimal PMU placement is employed such that the network still remains full observable even if a single contingency happens (observability in N-1 contingency). Then, in each time step, measurements of one PMU are removed and the state vector is calculated. In the next step, the resulting state vectors are clustered. For this purpose, subtractive clustering obtains the number of clusters and, fuzzy C-means clustering assigns the state vectors to the correct cluster. The number of one-member clusters indicates the number of integrity attacks. These one-member clusters are also used to determine the attacked PMUs and obtain the correct state vector.

In this paper, the most related works on stealth attack detection are also reviewed and quantitatively compared with the proposed method. For this purpose, the following properties are checked for each of the algorithms: (1) Obtaining Correct Measurement (OCM): The capability of the attack detection methods in obtaining the correct value of state vector is checked through this property. (2) Attack Localization (AL): This feature checks whether the algorithm is able to determine the location of attacks or not. (3) Concurrent Random Attacks Detection (CRAD): This property shows the capability of the algorithm in detecting several random (non-coordinated)
attacks. (4) Stealth Attack Detection (SAD): Obviously, this feature investigates the ability of detection methods in detecting stealth attacks. (5) Obtaining the Number of Attacks (ONA): The ability of attack detection methods in obtaining the number of random (or coordinated) attacks is shown through this feature. (6) Requiring Non-Electrical Information (RNEI): This feature deals with the property that whether an attack detection method requires non-electrical (e.g. cyber security) information to detect stealth attacks or not. For example, in addition to power grid information, [28] needs security information such as data collected from intrusion detection sensors or a database of firewall rules. However, this approach causes the design process to be more complicated and imposes additional delays.

By specifying the above properties in Table I, comparison between the previous researches and PMUIDS is straightforward. Results in Table I demonstrate that methods proposed in [25] and [28] are more similar to PMUIDS. However, as discussed earlier, method of [28] requires non-electrical information and the method proposed in [25] requires secure PMUs for detecting stealth attacks. Secure PMUs need security controls such as encryption and authentication which impose additional delays [25].

In summary, the PMUIDS features are as follows: (1) Detecting integrity attacks on power grids using a novel clustering approach. (2) Localizing the attacks. (3) Obtaining the correct state vector when removing the measurements of attacked PMUs, the power system still remains full observable. (4) Requiring only electrical information (by placing additional PMUs) which does not cause additional delay. (5) Decreasing the probability of successful stealth attacks. The main contributions of this paper are the novel PMUIDS based on clustering technique with the above features, and two theorems that one mathematically proves by an appropriate PMU placement, the successful interacting attack cannot happen, and the other one proves if PMUs are not placed appropriately, then the successful interacting attack can happen. However, even in this situation, the measurements of some parts of the power system can be corrected by the PMUIDS.

The rest of the paper is organized as follows: Section II reviews the overall structure of PMU networks and static state estimation algorithm. In addition, two theorems about coordination of interacting attacks are presented in this section. Section III presents the suggested PMUIDS and the clustering-based approach. The HiL results on a sample power system is given in Section IV. Finally, conclusions of this research are given in Section V.

II. INTEGRITY ATTACK DETECTION IN PMU NETWORKS EXPLOITING STATIC STATE ESTIMATION

PMU networks are set up on the IT-based protocols. They are constituted from PMUs, Phasor Data Concentrators (PDCs) and communication networks for data transmission in a hierarchical structure which is expanded in a wide area measurement system (WAMS) and at the top level, transfers data to the control center [33], [34]. The network has two different parts; regional and backbone networks (Fig. 1). While the regional networks are responsible for transferring data from each PMU to the related PDC [3], the backbone network has the main role of communication in a higher level among PDCs and the control center. The received data in the control center can be exploited for different studies, i.e., state estimation, stability assessment, etc. [35]. The main difference between the PMU and SCADA systems is that in the PMU system, the real and imaginary parts of the voltages and currents are available and the measurement model is inherently linear. Moreover, the PMU systems are more resilient against attacks. Though, they are still vulnerable to some kinds of attacks [36].

![Fig. 1. Hierarchical structure of PMU networks [34].](image)

In [36], false data injection attack (FDIA) on power systems

<table>
<thead>
<tr>
<th>Method</th>
<th>OCM</th>
<th>AL</th>
<th>CRAD</th>
<th>SAD</th>
<th>Main idea for detecting stealth attack</th>
<th>ONA</th>
<th>RNEI</th>
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<tbody>
<tr>
<td>[1]</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Partially</td>
<td>Using Redundant PMUs</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>[16]</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Kalman Filter and ( x_2 ) detector</td>
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<td>No</td>
</tr>
<tr>
<td>[18]</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Partially</td>
<td>Perturbing grid topology</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>[22]</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Partially</td>
<td>Employing distributed state estimation using power system decomposition</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>[25]</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Partially</td>
<td>Optimal placement of secure PMUs (i.e. PMUs that cannot be attacked)</td>
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<tr>
<td>[26]</td>
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<td>Yes</td>
<td>Yes</td>
<td>Using a fully distributed algorithm for power system state estimation</td>
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<tr>
<td>[28]</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Combining cyber data with power grid information</td>
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<td>[29]</td>
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<td>Yes</td>
<td>Distance between probability distribution function of measurement variation</td>
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<td>[30]</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Adding protected meters to be sure about some measurements</td>
<td>No</td>
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<tr>
<td>[31]</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Needs some meters to be absolutely protected</td>
<td>No</td>
<td>Yes</td>
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<td>[32]</td>
<td>Yes</td>
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<td>Yes</td>
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<td>Increases the number of PMUs so that FDI cannot impact estimation</td>
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<td>No</td>
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<tr>
<td>PMUIDS</td>
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<td>Yes</td>
<td>Yes</td>
<td>Partially</td>
<td>Optimal placement of PMUs in N-1 contingency</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>
equipped with PMUs is studied and by exploiting static state estimation, the voltages in the power system are estimated. Then, some solutions (such as redundant PMU) are suggested to avoid load shedding in the power system. Similarly, in [32], FDIA on WAMSs is considered and an algorithmic solution is proposed to address the issue of additional PMU installation and placement with cyber security consideration. However, those approaches do not offer all properties of the suggested PMUIDs listed in Table I. For detecting integrity attacks in such networks, this paper combines the theories of static state estimation and data clustering approaches. In the static state estimation and static attack detection method, the system operating point changes slowly that the fast transient response is negligible [37]. Then, the values of currents and voltages for that operating point are used.

In the following, the theory is briefly reviewed and its capability in integrity attack detection is discussed. Furthermore, two theorems about the possibility of interacting attacks in PMU networks are presented.

A. Static State Estimation

In general, the measurement equation for an n-bus system is as follows [38]:
\[ Z_n = HV + e = (H_v + jH_m) (E_v + jF_v) + e \]  
where \((H_v + jH_m) \in \mathbb{C}^{m \times n}\) is the Jacobian matrix of measurements, \(e = (e_1, ..., e_m)^T\) is the measurement noise vector and \(V = (E_v + jF_v) \in \mathbb{C}^{n \times 1}\) and \(Z_n = (A + B) \in \mathbb{C}^{m \times 1}\) are the state vector and the measurements vector, respectively and \(A = H_v E_v - H_m F_v, B = H_m E_v + H_r F_v\).

Define a new measurement vector \(Z\) as follows [38]:
\[ Z = [A] \begin{bmatrix} H_v \\ H_m \\ H_r \\ F_v \end{bmatrix} + e = HX + e \]  
where \(X \in \mathbb{R}^{(2n-1) \times 1}\) is the new state vector and \(H \in \mathbb{R}^{2m \times (2n-1)}\) is the new linear measurement Jacobian matrix.

The estimation of state vector \(\hat{X}\) is procured as follows [39]:
\[ \hat{X} = \begin{bmatrix} \tilde{E}_v \\ \tilde{F}_v \end{bmatrix} = (H^T R)^{-1} H^T [A] \begin{bmatrix} E_v \\ F_v \end{bmatrix} = (H^T R H)^{-1} H^T R Z \]  
where \(R\) is the covariance matrix and determines the weight of each array in the measurement vector \(Z\). Without loss of generality, in the rest of this paper, it is assumed that \(R\) is equal to \(I\). It means that we used the known least square method to find the optimal static state estimation. Additionally, the matrix \(H\) should be non-singular, so that \(H^T R H\) is invertible. This issue is treated by the so-called numerical observability [38], in which not only the system is topological observable, but also as much PMUs are allocated as needed to make the system fully observable.

B. Interacting Attack Detection

Assume that an integrity attack happens in the PMU network and some measurements are altered. The vector of observed measurements after attack is \(Z_a = Z + a\) which consists of two parts; the real measurements \(Z = (z_1, ..., z_{2m})^T\) and the false data added to the real measurements \(a = (a_1, ..., a_{2m})^T\) that is called the attack vector. Assume that, \(\hat{X}_a\) refers to false estimation of state vector, \(\hat{X}_a = \hat{X} + W\), in which, \(W\) is the state estimation error due to the vector injected by the attacker.

In [12]–[14], for detection of bad data (such as attack) the constraint is to check \(\|Z - HX\|^2\) with a threshold and if this norm is less than the predefined threshold, it is concluded that there is no bad data in measurements. A category of attacks is interacting attacks, in which the attack vector has the specific form \(a = HW\) and do not result in a large deviation from threshold and cannot be detected by these methods. This kind of attack can be organized in two ways [20]:

- The attacker’s only intention is to deceive the result of state estimation. Therefore, \(W\) is assumed to be a random vector.
- The attacker aims to add specific errors to the estimation of particular state variables. Consequently, this kind of attack is called targeted false data injection attack. In other words, in targeted attacks, \(W\) is not a random vector.

Assume that the attacker wants to orchestrate random interacting attack. The matrices \(Q\) and \(K\) are defined as follows:
\[ Q = H (H^T H)^{-1} H^T, \quad K = Q - I \]  
Notice that \(Q\) is a \(Qa\) by \(Qa\) matrix. Multiplying this equation and \(a = HW\) by \(W\) and \(Q\) respectively, it can be concluded:
\[ a = HW \rightarrow Qa = QWH \]  
\[ QH = H \rightarrow QHW = HW = a \]  
\[ \rightarrow (Q - I)a = Ka = 0 \]  
(5)

Solving (5), the attack vector can be obtained.

Assume that the attacker’s access is limited and he can only forge few PMU’s data. Therefore, only the elements of vector \(\alpha\) that correspond to falsified PMUs, can be non-zero. Showing the nonzero elements of \(\alpha\) with vector \(\alpha’\) and the corresponding columns of matrix \(K\) with matrix \(K’\), (5) results in:
\[ K’ \alpha’ = 0 \]  
(6)

If \(K’\) is a full rank matrix, (6) has a unique solution \(\alpha’ = 0\). Therefore, it is highly unlikely for successful interacting attacks to happen. Otherwise, the equation has infinite number of non-zero solutions with the following general form [20]:
\[ \alpha’ = (I - K’ K)^{-1} \eta \]  
(7)

where \(\eta\) is an arbitrary non-zero vector with the same length as \(\alpha’\) and \(K’\) is pseudo inverse of matrix \(K’\). Consequently, when the attacker only has access to the measurements of few PMUs, the interacting attack may be detectable in some cases.

In the following, two theorems are presented to determine whether or not a successful random interacting attack can be organized in PMU networks. In addition, if possible interacting attack happens, it is significant to know how the affected state variables are limited (successful targeted interacting attacks are restricted). Before mentioning the theorems, some definitions are given as follows:

- \(\varphi_1\) & \(\varphi_2\): The number of PMUs to guarantee full observability (topologically and numerically) in normal condition and \(N - 1\) contingency, respectively (\(\varphi_2 > \varphi_1\)).
- \(L = \{l_1, ..., l_{\varphi_2}\}\): The set of buses which define the locations of installed PMUs in \(N - 1\) contingency condition.
- \(a_l\): Each subset of \(L\), which makes power system full observable (observable set).
- \(O = \{o_l|l = 1, ..., N_o\}\): The set of observable sets.
- \(u_l\): Each subset of \(L\), which cannot make power system full observable (unobservable set).
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- \( U = \{ u_i | i = 1, \ldots, N_u \} \): The set of unobservable sets.
- \( S = \{ s_i | s_i = \{ L - \alpha_0 | i = 1, 2, \ldots, N_s \} \) where \( s_i \) is a set of PMUs which omitting their measurements’ still makes the power system full observable.
- \( \hat{S} = \{ \hat{s}_i | \hat{s}_i = L - u_i, i = 1, 2, \ldots, N_u \) where \( \hat{s}_i \) is a set of PMUs that omitting their measurements’ results in the power system not be full observable anymore.
- \( L_A \): The set of buses that the attacker gain access to the measurements of their PMUs.
- \( m \): The number of all measurements obtained from \( \varphi_2 \) PMUs.

**Theorem 1**: Assume that \( \varphi_2 \) PMUs are installed in the system. If the attacker only has access to a set of PMUs which belong to \( S \) (i.e. \( L - L_A \in O \)), the matrix \( K' \) is full rank and successful interacting attacks will not happen.

**Proof**: Select one observable set \( O_i \) from the set \( O \). The number of measurements related to eliminated PMUs \( (s_i) \) is shown by \( m_e \). These measurements are the ones that the attacker has access to them and can falsify them. The number of remaining PMU measurements which cannot be altered by the attacker is equal to \( m_o = (m - m_e) \geq n \). Since, interchanging rows or columns of a matrix doesn’t change the rank of matrix, without loss of generality, the matrix \( H \) can be considered as follows:

\[
H_{2m \times (2n-1)} = [H^T \ H^T]^{T}\tag{8}
\]

where \( H_e \) is a \( 2m_e \times (2n-1) \) matrix and consists of the rows of \( H \) that correspond to measurements of eliminated PMUs and \( H_r \) is a \( 2m_r \times (2n-1) \) matrix which is related to the remaining PMUs. The full observability (topologically and numerically) of power system before and after elimination of the specified PMUs \( (s_i) \), results in:

\[
\text{rowspace}(H) = \text{rowspace}(H_e) = \mathbb{R}^{2n-1}
\]

\[
\text{rank}(H) = \text{rank}(H_e) = 2n - 1
\]

In other words, any vector in \( \mathbb{R}^{2n-1} \) can be written as a linear combination of rows of \( H \) or \( H_e \). Define matrix \( Y \) as follows:

\[
Y = (H^T \ H)^{-1} \left( \begin{bmatrix} H^T \ H_e^T \end{bmatrix} \right)^{-1}\tag{10}
\]

The matrix \( Q \) will be obtained as the following:

\[
Q = H(H^T \ H)^{-1} H^T = \begin{bmatrix} H_e^T & H_r^T \end{bmatrix} \begin{bmatrix} H_e^T \ H_e^T & H_r^T \end{bmatrix}^{-1}\tag{11}
\]

Therefore, the matrix \( K \) will be represented as follows:

\[
K = Q - I = \begin{bmatrix} H_e^T & H_e^T \end{bmatrix} \begin{bmatrix} H_e^T & H_e^T \end{bmatrix}^{-1} = \begin{bmatrix} H_e^T & H_r^T \end{bmatrix} \begin{bmatrix} H_e^T & H_r^T \end{bmatrix}^{-1} l_{2m_e}
\]

Here, vector \( a' \) consists of the last \( 2m_e \) elements of vector \( a \) (those elements that the attacker has access to). Therefore, matrix \( K' \) is as follows:

\[
K' = \begin{bmatrix} H_e^T & H_r^T \end{bmatrix} \begin{bmatrix} H_e^T & H_r^T \end{bmatrix}^{-1} l_{2m_e} \tag{13}
\]

As it is mentioned before, if \( K' \) is a full rank matrix, the successful interacting attacks will not happen. If each vector in \( \mathbb{R}^{2m_e} \) is presented as a linear combination of rows of \( K' \), then the row space of \( K' \) is \( \mathbb{R}^{2m_e} \) and this matrix is full rank. Therefore, to prove the nonexistence of successful interacting attack, we need to prove that for each vector \( F \in \mathbb{R}^{2m_e} \) the equation \( G^T K' = F^T \) has an appropriate solution. Partitioning vector \( G^T \) as \( G^T = [g_1^T \ g_2^T] \), where \( g_1 \) and \( g_2 \) are \((2m_e \times 1)\) and \((2m_r \times 1)\) vectors respectively, concludes:

\[
G^T K' = g_1^T H_e Y_H^T + g_2^T H_r Y_H^T - g_2^T H_r^T \left( Z_r + a_r \right) \tag{14}
\]

On the other hand, any vector \( F^T H_e \in \mathbb{R}^{2m_e} \) can be written as a linear combination of rows of \( H_r \). Therefore, for each arbitrary vector \( F \), the equation \( g_1^T H_e = F^T H_e \) has a solution that is called \( g_1 \). As it is shown in the following, for each arbitrary vector \( F \), the vector \( \hat{G} = [g_1^T \ - F^T]^T \) is a solution for the equation \( G^T K' = F^T \):

\[
\begin{aligned}
\hat{G}^T K' &= g_1^T H_e Y_H^T - F^T H_r Y_H^T + F^T \Rightarrow \hat{G}^T K' &= F^T
\end{aligned}
\]

Therefore, it is shown that if the attacker has access to the measurements of a set of PMUs that belongs to \( S \), each vector \( F \in \mathbb{R}^{2m_e} \) can be presented as a linear combination of rows of \( K' \) which means the matrix \( K' \) is full rank. Therefore, the equation \( K' a' = 0 \) has a unique solution \( a = 0 \) and the successful interacting attacks will not happen.

**Theorem 2**: Assume that the attacker only gain access to a set of PMUs which belongs to \( \hat{S} \) (i.e. \( L - L_A \in U \)) and can organize an interacting attack in the form of \( a = HW \). In this case, the estimation error vector \( (W) \) will have a specified structure and all of its elements related to the buses that are still observable after removing the attacked PMUs \( (L_A) \) are zero. Therefore, the attacker can only falsify estimation of buses that become unobservable after omitting \( L_A \).

**Proof**: Consider the case that the attacker gain access to a set of PMUs that belongs to \( \hat{S} \) and can manage a successful interacting attack \( a = HW \). The estimation of state vector using all measurements after the attack is as follows:

\[
\tilde{X}_a = (H^T H)^{-1} H^T \tilde{Z}_a = (H^T H)^{-1} H^T (Z + HW) = \tilde{X} + W \tag{16}
\]

Since \( L - L_A \in U \), omitting measurements of attacked PMUs, makes some buses unobservable. Those elements of vectors \( W, \tilde{X} \) and \( \tilde{X}_a \) that are related to observable buses are shown by \( W_0, \tilde{X}_0 \) and \( \tilde{X}_a_0 \). Define vector \( a_r \) by removing the elements related to \( L_A \) from \( a \). In addition, define matrix \( H_r^{\perp} \) by removing the corresponding rows of \( H \). Therefore, the equation \( a_r = H_r W \) still holds.

Assume that matrix \( H_r^{\perp} \) is obtained by removing the zero columns related to unobservable state variables from matrix \( H_r \). Since these zero columns have no effect on the value of vector \( a_r \), after removing them, the equation \( a_r = H_r^{\perp} W_0 \) still holds. Although the power system is not full observable after omitting the measurements related to \( L_A \), the voltages of observable buses can be estimated as follows:

\[
\tilde{X}_a = (H_r^{\perp} H_r^{\perp})^{-1} H_r^{\perp} \tilde{Z}_a = (H_r^{\perp} H_r^{\perp})^{-1} H_r^{\perp} (Z_r + a_r) = (H_r^{\perp} H_r^{\perp})^{-1} H_r^{\perp} (Z_r + H_r^{\perp} W_0) = \tilde{X}_0 + W_0 \tag{17}
\]

On the other hand, the remaining measurements after omitting the attacked PMUs \( (L_A) \) are correct. Therefore, the estimation of observable buses will be correct, too.

\[
Z_{a-r} = Z_r \Rightarrow \tilde{X}_{a-r} = (H_r^{\perp} H_r^{\perp})^{-1} H_r^{\perp} Z_r = \tilde{X}_0 \tag{18}
\]

Comparing (17) to (18), it is concluded that \( W_0 = 0 \). Therefore, the attacker can’t target any arbitrary bus and only estimation of buses that become unobservable by removing measurements of attacked PMUs may be wrong.
In Theorems 1 and 2, the PMUs belong to the sets $S$ and $\bar{S}$, respectively. Depending the attacker access to the sets $S$ and $\bar{S}$, the attack success changes. From Theorem 1, one infers that gaining access to the measurement of those PMUs that belong to $S$, does not result in successful interacting attacks. Based on Theorem 2, gaining access to those PMUs that belong to $\bar{S}$ may falsify the estimation of buses that become unobservable after omitting the attacked PMUs. By using these two theorems, all set of PMUs that by gaining access to their measurements, organizing a successful interacting attack might be possible (i.e. $\bar{S}$) can be determined and the buses that falsifying their estimation is much easier than other buses (i.e. buses that become unobservable in most cases) can be identified.

### III. PMUIDS

In this section, the steps of PMUIDS for detecting attacks, localizing them, determining the number of attacks and obtaining the correct measurements are presented. In the suggested approach, different state estimation vectors are calculated and compared in the control center. The redundancy of PMUs is increased to make it possible to obtain several solutions for the state estimation problem. Then, clustering is employed to detect the number and location of attacks and obtain the correct estimation of state vector. Notice that in case of multiple simultaneous attacks, if by omitting measurements of falsified PMUs, the power system remains fully observable, the state vector can be estimated correctly. Otherwise, only the correct voltages of observable buses will be obtained.

The PMUIDS algorithm is as follows:

1) Perform PMU placement by taking $N-1$ contingency conditions into account.
2) Install PMUs on the buses determined in the previous step.
3) Omit the measurements received from one of the PMUs and calculate the estimation of the state vector $V_i$ by using the measurements of other PMUs.
4) In a row, consider each PMU data to be omitted once and exploit the other measurements for the required estimation data. After elimination of $i^{th}$ PMU data, go to Step 3 and calculate state vector $V_i$ and $\varphi_2$. Thus, different estimation results ($V_1$, $V_2$, ..., $V_{\varphi_2}$) will be acquired.
5) Use the following formula to calculate the mean of all state vectors:

$$V_{\text{Ave}} = \text{mean}(V_1, V_2, ..., V_{\varphi_2}) = \frac{1}{\varphi_2} \sum_{i=1}^{\varphi_2} V_i$$

(19)

6) Choose Euclidean norm as an appropriate measure to find the distance between each state and the mean.

$$\text{Dis}_i = ||V_i - V_{\text{Ave}}||_2$$

(20)

7) Find the state vector with the largest distance from the mean.

$$\text{Dis}^* = \max_{i} (\text{Dis}_i)$$

(21)

If this distance is smaller than a threshold, $\epsilon$, PMUIDS ignores it and concludes that no attack has occurred. Otherwise, if the largest distance from the mean is larger than a threshold, it can be concluded that an attack has occurred. The threshold $\epsilon$ is considered to remove the effect of measurement noises and it is the key to identify the integrity attack from the noisy measurements.

8) In this step, the number and locations of the attacks will be detected. First, by using subtractive clustering algorithm, the number of clusters will be obtained. Then, by using fuzzy c-mean clustering, each state vector will be assigned to the right cluster. The number of single-member clusters shows the number of attacks. In addition, those PMUs that their data was omitted in calculating the state vectors associated with single-member clusters are the attacked ones.

9) For single-attack cases, the state vector with maximum distance from the mean which was found in Step 7 shows the correct measurements. For multiple simultaneous attacks, the measurements of detected attacked PMUs will be omitted and state estimation will be done based on the remaining measurements. If the remaining measurements make the power system fully observable, the correct estimation of state vector will be obtained. Otherwise, only the correct voltages of observable buses will be estimated.

To prove the capability of PMUIDS, assume that only measurements of $PMU_i$ are forged. Implementing the above algorithm, it is obvious that when data of $PMU_i$ is omitted, the state estimation result is correct (Estimation$_1$). All the other cases contain the forged data and will lead to wrong estimations. Since the forged data affect all these estimated state vectors similarly, their results will be very close to each other, but different from Estimation$_1$. In other words, the state estimation results will form two clusters, where one of them is a single member cluster (Estimation$_1$). This scheme can be extended to the cases that measurements of more PMUs are altered by attackers.

It should be noted that the proposed algorithm does not have any limitation for $N-2$ or further assumption. Furthermore, the algorithm does not have any limitation for large-scale systems. It is just needed to have enough measurement to be sure that the matrix $H$ of (3) is nonsingular. Thereby, it is required to install as much PMUs as they guarantee the matrix non-singularity, which is the limitation of all static state estimation approaches.

### IV. HiL Results

The approach proposed in Section III is applied on the known IEEE 14-bus test system though the OPAL-RT HiL environment, shown in Fig. 2. The structure of the HiL setup consists of an OPAL-RT as a Real-Time Simulator (RTS) which simulates the IEEE 14-bus grid, seven virtual Phasor PMUs, and the static attack detection; a PC as the programming host; an amplifier that generates the current and voltage for the PMUs; two real PMU devices; and a satellite-synchronized clock. The optimal PMU locations are found by the method introduced in [40]–[41]. The optimal numbers of PMUs in normal and $N-1$ contingency conditions are equal to 4 and 9, respectively. The locations of PMUs in $N-1$ contingency condition is shown with $S_2 = \{2,4,5,6,7,8,9,11,13\}$. In Fig. 3, the modified version of the IEEE 14-bus system benchmark with the assumption of no connection between the buses 5 and 6 is shown. The PMU network is designed and 9 PMUs ($PMUs_i$,
$PMU_b$,...,$PMU_i$) are allocated. Additionally, the network for transferring data from PMU to PDC and the CC are selected. It is assumed that the network requires 3 PDCs, corresponding regional networks (R1, R2, & R3) and Ring Interface Units (RIU) and a back-bone network.

In the part of the static attack detection, initially an integrity attack is generated and added to data of the attacked real/virtual PMU devices to falsify the measurements. Then, the static attack detection method is performed.

These sets are utilized in Theorems 1 and 2 to analyze the possible attacks. The results are as follows:

- **(Single attack):** Since 9 PMUs are placed in the grid, if one random PMU is attacked (and detected by the PMUIDS), still 8 PMUs measurements are available. Based on Table II, there is no set of unobservable set (i.e. $U \in \emptyset$) and all possible sets are observable. Thereby, based on Theorem 1, the system is fully observable; and no successful attack can be performed.

- **(Two attacks):** If two random PMUs are attacked, they are detected and the power system still has 7 available PMUs. In this case, 29 sets out of 36 sets are fully observable and based on Theorem 1, system states are fully observable; and, 7 sets out of 36 sets are unobservable and based on Theorem 2, their corresponding states are lost, meanwhile the other states are observable. This shows that only 7 out of 36 possible sets of two simultaneous attacks can be a successful interacting attack, which results into the loss of some buses’ states. Consequently, the probability of organizing targeted false data injection attack is highly unlikely. The possible interacting attacks on two PMUs and the buses that their estimation could be wrong are shown in Table III. Moreover:

  1. No one can deceive the estimation of buses (2, 4, 5, 6, 7, and 9) without being detected.
  2. To deceive the estimation of bus 10 without being detected, the attacker should have access to the measurements of buses 9 and 11, otherwise successful interacting attack is impossible.

- **(Three attacks):** In this case, 6 available PMUs’ measurements are available. Based on Table II, this means that only 45 out of 83 possible sets of three simultaneous attacks can be a successful interacting attack. Furthermore:

  1. No one can falsify the estimation of buses (4, 5, 6, and 7) without being detected.
  2. To falsify the estimation of bus 9 without being detected, the attacker should have access to the data of buses 4, 7 and 9, otherwise successful interacting attack is impossible.

### A. Analyzing the results of Theorems 1 and 2

In this part, it is considered that the proposed PMUIDS detects the attacked PMUs and the applicability of Theorems 1 and 2 is investigated. These theorems facilitate analyzing the feasibility of successful interacting attacks. To do this, the sets $S$ (or $O$) and $\tilde{S}$ (or $U$) are required and can be obtained based on the topology of IEEE 14-bus and the places of PMUs and PDC. Table II shows the number of observable and unobservable sets for different PMU numbers $4 \leq \gamma \leq 9$.

<table>
<thead>
<tr>
<th>PMU Numbers ($\gamma$)</th>
<th>Number of $U_i$</th>
<th>Number of $O_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>7</td>
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<td>29</td>
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<tr>
<td>6</td>
<td>45</td>
<td>39</td>
</tr>
<tr>
<td>5</td>
<td>106</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>125</td>
<td>1</td>
</tr>
</tbody>
</table>

### B. Evaluating the proposed PMUIDS

In this part, three scenarios are considered for evaluating PMUIDS on IEEE 14-bus system. In the first Scenario, the power system is working normal and no attack is carried out. This scenario indicates whether PMUIDS produces any false results or not. In the second scenario, one integrity attack is carried out. Results of this scenario show whether PMUIDS can successfully detect the attack, obtain the correct measurements, and localize the attack in case of one attack at a time or not.
Finally, in the third scenario, three attacks are simultaneously orchestrated on the IEEE 14-bus system. This test investigates whether PMUIDS can still detect intrusion, determine the number and location of attacks and correct estimation in case of simultaneous attacks or not. It should be noted that since PMUIDS employs mathematically proven techniques, in similar scenarios, similar results will be obtained.

B.1. Scenario 1: No Attack

First, consider the case that no attack happened in the PMU network. At each step, 8 out of 9 PMUs are selected and the state vector is estimated based on the method described in section III.A. Therefore, for calculating $V_A$, all measurements except information gathered from $PMU_A$ is used. Similarly, this procedure is repeated for all PMUs. Therefore, nine solutions $(V_b, V_c,..., V_I)$ are obtained for different state estimation problems which are shown in Fig. 4. It is obvious that all solutions are very close to each other. The small differences between the state vectors in this case is due to measurement noises. The Euclidean distance of each state vector to the $V_{ave}$ is presented in Table IV.

B.2. Scenario 2: One Attack

Assume that the measurements of $PMU_A$ (shown in Fig. 3) are falsified to simulate an integrity attack. This PMU transfers the information to PDC. Here, a series of actions are needed to check whether integrity attack (Man-in-the-Middle (MitM)) modified the PMU data. A similar procedure to Section IV.B.1 is repeated to find 9 state vectors while data from one of the PMUs is eliminated. The Euclidean distance of computed state vectors to $V_{ave}$ are presented in Table IV.

According to the Results, $V_A$ has the largest Euclidean distance to $V_{ave}$ which is greater than $\varepsilon = 0.05$. Therefore, PMUIDS can effectively detect the attack. The subtractive and fuzzy c-means clustering methods are exploited to determine the clustering information which is useful in attack determination.

The number of clusters that is calculated by subtractive clustering algorithm equals to 2. One of these clusters consists of 8 vectors $(V_b, V_c,..., V_I)$ and the second one, is a single-member cluster $(V_A)$ shown in Fig. 5. Thus, there is only one attack and $PMU_A$ that wasn’t used in calculation of $V_A$ contains the falsified data and its data is compromised. Since only one attack is happened in the network, state vector $V_A$ represents the accurate estimations.

Fig. 5. All solutions for the state estimation problem after one attack

B.3. Scenario 3: Three Attacks

Assume that the measurements of $PMU_A$, $PMU_D$ and $PMU_G$ are altered to simulate three integrity attacks. The same detection procedure is used here. The estimation results $(V_A, V_B, V_C,..., V_I)$ are presented in Fig. 6 and the corresponding Euclidean distance is presented in Table IV. The results show that the norm of difference between $V_B$ and $V_{ave}$ is the largest value, equal to 0.84103. Therefore, the attack is detected properly. The number of clusters in this case equals to 4. Three of these clusters, which are marked with dashed lines in Fig. 6, only consist of one state vector. Other state vectors $(V_B, V_C, V_E, V_F, V_G, V_D)$ form a cluster together. Therefore, the number of attacks is identified correctly. The PMUs, whose data were omitted in calculating the state vectors associated with single-member clusters, will be effectively determined as the location of attacks. Since after removing the detected forged PMUs, the power system remains full observable, the true values of bus voltages can be obtained.

By using the proposed PMUIDS, the state system vector is estimated and clustered via the approach presented in the Appendix. If one gets no specific cluster, it shows that all solutions are very close to each other and no attack does happen, which is evident in Scenario A. However, if more clusters are achieved, an attack is detected. Based on the number of clusters, it is possible to determine the number of simultaneous attacks. As can be seen in Scenarios B and C, one and three clusters are obtained, respectively which is in consistent with the number of

| TABLE IV. NORM OF THE DIFFERENCE BETWEEN ESTIMATED VECTORS AND THE MEAN VALUE |
|-------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
|                  | Dis1            | Dis2            | Dis3            | Dis4            | Dis5            | Dis6            | Dis7            | Dis8            | Dis9            |
| Before Attack    | 0.00349         | 0.00045         | 0.00048         | 0.00058         | 0.00049         | 0.00079         | 0.00108         | 0.00092         | 0.00084         |
| One Attack       | 0.58452         | 0.0599          | 0.10686         | 0.07179         | 0.06906         | 0.07292         | 0.07069         | 0.07576         | 0.07542         |
| Three Attacks    | 0.49932         | 0.14472         | 0.1602          | 0.84103         | 0.13346         | 0.1371          | 0.39936         | 0.15263         | 0.15551         |
In this paper, integrity attack on power systems was studied and a procedure based on the static state estimation and data clustering algorithms was presented to accurately detect the number and location of attacks and determine which PMU in the network contained malicious data. This detection can be done in the control center by comparing different state estimation results and data clustering approaches and it was called PMUIDS. The theory of detecting attacks by PMUIDS is proved by two theorems. The theorems show that in most cases, the successful interacting attacks cannot happen and PMUIDS can effectively detect any arbitrary false data that is added to the measurements. However, in some special cases that many PMUs are forged and the remaining ones cannot make the whole power system observable, the successful interacting attacks can happen. Even in such a case, the algorithm is still able to provide correct measurements for some parts of the system based on the number of unfalsified PMUs. The HiL results on a sample power system prove the PMUIDS capability in finding the forged PMU data. For future work, considering more complex large-scale systems is suggested. Further, considering the issue of PMU faulty measurements and distinguishing it from the attacked PMUs can be an interesting work.

**APPENDIX: DATA CLUSTERING ALGORITHM**

Each clustering analysis has two pre-requisites; determining the number of clusters and the clustering method. Here, for determining the number of clusters, subtractive clustering technique is utilized [42]. In each step of this iterative algorithm, the potential to be a cluster center is evaluated for all points in the data set and the point with the highest potential value, which has more data in its neighborhood, will be found. Then, by using some criteria, it is determined whether that point is a new cluster center or not. After obtaining a new cluster center, the potential values of the remaining data points will be modified and the process repeats for the remaining points till no new cluster center can be found. Based on subtractive clustering, the number of clusters and a good initial guess for cluster centers are obtained. However, data points of each cluster cannot be found in this method. This information can be derived by utilizing fuzzy c-mean (FCM) clustering. The FCM algorithm iteratively minimizes the following objective function [43]:

$$f = \sum_{i=1}^{n_d} \sum_{j=1}^{n_c} \mu_{ij}^d \|d_j - c_i\|^2$$  \hspace{1cm} (22)

where $n_d$ refers to the number of data samples, $n_c$ refers to number of clusters determined by subtracting clustering method, $d_j$ is the $j$-th data point, $c_i$ is the $i$-th cluster center, $\mu_{ij}$ refers to the membership degree of $j$-th data in the $i$-th cluster and is computed as follows:

$$\mu_{ij} = \frac{1}{\sum_{l=1}^{n_c} \left( \frac{\|d_j - c_l\|}{\|d_j - c_i\|} \right)^2}$$  \hspace{1cm} (23)

In the initial step of FCM algorithm, pre-fixed $n_c$ and initial values for cluster centers are determined. Then, the optimum value of the cluster centers and membership degrees are procured iteratively by minimizing the objective function.

**REFERENCES**


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