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# Self-supporting structure design with feature-driven optimization approach for additive manufacturing 

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#### Abstract

In this work, a topology optimization approach is developed for additive manufacturing (AM) of 2D and 3D self-supporting structures. Three important issues, i.e., overhang angle control, avoidance of the so-called V-shaped areas and minimum length scale control are addressed. 2D solid polygon and 3D polyhedron features are introduced as basic design primitives that are capable of translations, deformations and intersections to drive topological changes of the structure. The overhang angle control is realized in a straightforward way only by imposing upper bounds to related design variables without introducing any nonlinear constraint. The V-shaped area is avoided by simply limiting the positions of solid features. Minimum length scale control is controlled by a robust formulation. Numerical examples in 2D and 3D demonstrate the effectiveness of the proposed approach for various build directions, critical overhang angles and minimum length scales considered in AM.


Keywords: Self-supporting structures; Solid polygon/polyhedron features; Overhang angle control; V-shaped area; Minimum length scale; Additive manufacturing.

## 1. Introduction

Additive manufacturing (AM) greatly relieves conventional manufacturing constraints on design complexities in shape and topology of structures. The flexibility allowed by AM thus provides a great advantage for its integration with topology optimization (TO) [1-4]. To fully strengthen this integration between design and manufacture in practical applications, it is very important to incorporate AM constraints into TO [5-9]. To this end, much effort has been made at the design stage to eliminate enclosed voids [10-12], reduce thermal warping [13] and layer-induced anisotropy [14], etc. Among others, overhang angle control is considered as a typical AM constraint needed in TO for avoiding eventual deformation and warping of overhang portions. The structure is thus desired to be self-supporting, i.e., free of supports. Otherwise, high material/time costs and difficult post-processing operations will be inevitable.

The density-based method is the most studied for TO of self-supporting structures. Overhang constraints are enforced through filters [15-22] or by introducing constraints [23-31]. In the former, element densities are filtered layer by layer according to the density layout relations [15-20] or filtered as a whole to get a printable density field since violations can be measured directly by the time delay of front propagation [21, 22]. The latter focuses on formulating specific constraints limiting the designed overhang angles greater than the critical overhang angle (COA). In [31], the overhang constraint was established on a continuous logistic aggregate function and still followed the idea of [15-20] concerning element densities of different layers. Studies [23-30] utilized density gradient information to evaluate overhang angle and further point-wise or element-wise constraints were aggregated to perform angle restrictions. Specifically, Qian [24], Mezzadri et al. [23] and Luo et al. [29] formulated explicit overhang constraints according to the Heaviside projected density gradient. Wang [30] utilized the B-spline parameterization of the density field to do self-supporting design. Zhang et al. [27, 28] estimated the structural boundary normal by exploring local elemental density distribution. Garaigordobil et al. [25, 26] developed an overhang constraint based on an edge detection algorithm, Smallest Univalue Segment Assimilating Nucleus (SUSAN).

Level-set method is an alternative approach which uses a contour description of the structure boundary for the definition of overhang constraint. Overhang angle control is often achieved by imposing nonlinear constraints in the form of boundary or domain integrals [32, 33].

A feature-driven optimization method [34-37], also based on level-set function, has been developed recently. A structure is considered to be constructed by a set of solid or void geometric features, the movements and deformations of which will drive topological change of the structure. Guo et al. [38] imposed constraints on adopted solid super-ellipses and void closed B-spline features to do 2D self-supporting designs. In our previous work [39], void polygon features were introduced as basic primitives to perform 2D self-supporting designs with overhang angle control directly linked to the definitions of design variables. V-shapes, which are special unprintable cases that meet the angle requirements but still need supports from below, often appear if only angle restriction of boundaries is imposed. To hinder V-shapes we took measures of merging the intersecting void polygons and then re-optimizing. Fig. 1 gives an illustration with a cantilever example. However, the research still remains 2D as void replacements become very complicated when extended to 3D. An optimized result for a 3D cantilever beam is depicted in Fig. 2(a) where several inverted-cone shaped features appear due to the intersections of void polyhedra or the intersections of void polyhedra with the
boundary of the design domain. Obviously, it is hard to merge such intersecting void polyhedra highlighted in Fig. 2(b) into one new polyhedron.


Fig. 1. The two-step optimization process of 2D cantilever beam using void polygons [39].
There are several works related to V-shape suppressions. Qian [24] suggested a larger radius in the linear density filter to eliminate V-shapes. Mezzadri et al. [23] first detected tips of V -shapes and then formulated adaptive filters, constraints or penalizations. In [27, 28], the suppressions were realized by combining the overhang angle constraint and the minimum horizontal length scale control. Wang et al. [30] and Allaire et al. [32] constructed a triangle constraint and a mechanical constraint to solve the problem of V-shapes, respectively. The work of [38] eliminated V-shapes by not allowing any two features intersecting.

In addition, minimum length scale control as a basic manufacturability issue must be imposed to make designs more practical. Much effort has been focused on the density-based method [40-45] and level-set method [46-50]. In particular, the so-called robust approach [51-53] simultaneously considering multiple perturbations of the structures has achieved great results, in which the optimization is formulated as a worst case design problem and the minimum length scale is guaranteed by uniform erosions or dilations of the structures.


(a) Optimized result with inverted-cone shaped features using void polyhedron features and overhang angle control.

(b) Colored void polyhedron features.

Fig. 2. 3D cantilever beam using void polyhedron features.
In this work, solid polygon and polyhedron features are introduced as basic design primitives for topology optimization of 2D and 3D self-supporting structures instead of
void shapes. Similarly, design variables are properly defined and bounded to control the overhang angle directly without the imposition of nonlinear constraints. V-shaped, areas or volumes are eliminated in a geometric way by limiting positions of solid features. The robust formulation is applied to control the minimum length scale and the contour characteristics of the level set facilitate the transformations between different forms without first identifying the structural boundaries. Fig. 3 also gives the optimization process of cantilever beam using solid polygons. By contrast, it avoids reoptimization and the optimized result has better stiffness than the result driven by void polygons.


Solid polygon feature Design variables:

- Length design variables - Ratio design variables
- Center coordinates


The dilated result


Robust formulation ensures the minimum length scale

Feature position control realizes the elimination of V-shapes


Fig. 3. Illustration of proposed approach for self-supporting design of cantilever beam using solid polygon features.

This paper is organized as follows. Section 2 gives a presentation about the selfsupporting design method by means of solid features. In detail, focus is on design variables selected to implement overhang constraint, level-set function (LSF) used to define the boundaries of solid features or structures and feature positions limited to eliminate $V$-shaped areas. Section 3 concerns the fixed grid used for structural analysis. Sensitivity analyses of objective function and constraints with respect to design variables are also deduced. In addition, the erosion-dilation scheme to guarantee the minimum length scale is discussed. Section 4 concerns numerical examples solved to demonstrate the effectiveness and generality of the proposed method. Finally, conclusions are drawn in Section 5.

## 2. Modeling self-supporting structures with solid polygon and polyhedron features

### 2.1. Formulation of design variables for overhang angle control


(a) A structure requiring supports when $\beta_{0}=45^{\circ}$.

(b) A support-free structure when $\beta_{0}=45^{\circ}$.

Fig. 4. The inclination angle distributions in two structures.
Due to the layer-by-layer processing nature of AM, the existence of materials in one layer depends on whether materials exist in the lower layers. Generally, given a specific build direction $\mathbf{b}$, boundaries with inclined angles $\beta$ less than the critical overhang angle (COA) $\beta_{0}$ are considered to be insufficiently supported and need additional material supports below. For example, the red boundary shown in Fig. 4(a) may deform or collapse without support, while the structure shown in Fig. 4(b) fully satisfies the overhang constraint for angle $45^{\circ}$ everywhere and is free of supports.

(a) A 2D solid polygon.

(b) A 3D solid polyhedron.

Fig. 5. A solid polygon/polyhedron satisfying the overhang constraint.


Fig. 6. Definitions of design variables for a 12 -sided solid polygon.
Instead of void features in the previous work [39], solid polygons/polyhedra are constructed as design primitives and unlike void polygons, ratio design variables involved in solid polygons are used to limit the inclined angles of the lower parts. Fig. 5 provides a feasible shape of the solid polygon/polyhedron when $\beta_{0}=45^{\circ}$.

As an example, consider the 12 -sided (i.e., $n=12$ ) polygon in Fig. 6. First, $o v_{1}(o$ is the center point and $v_{i}$ is the $i$ th vertex of the polygon) is predefined in the direction opposite to the build direction $\mathbf{b}$. The angle $\alpha$ between any two adjacent control radii ( $o v_{i}$ and $o v_{i+1}$ ) is constant and remains unchanged during the optimization process to avoid self-intersections of deformed polygons.

Once the ratio design variable $\lambda_{i}$ representing the length ratio between $o v_{i}$ and $o v_{i+1}$, is bounded to be less than a critical ratio value $\overline{\lambda_{i}}$, the overhang constraint $\beta_{i} \geq$ $\beta_{0}$ will be fulfilled for the side $v_{i} v_{i+1}$. In the general case, for a solid $n$-sided polygon, $\lambda_{i}$ is defined as

$$
\lambda_{i}=\left\{\begin{array}{cl}
l_{i+1} / l_{i} & \text { if } i<n / 2  \tag{1}\\
l_{i} / l_{i+1} & \text { if } i>n / 2 \text { and } i \neq n \\
l_{i} / l_{1} & \text { if } i=n
\end{array}\right.
$$

where $l_{i}$ is the length of the $i$ th control radius $\left(o v_{i}\right) \cdot \overline{\lambda_{i}}$ is the exact ratio between $l_{i}$ and $l_{i+1}$ when the inclined angle $\beta_{i}$ of $v_{i} v_{i+1}$ reaches $\beta_{0}$.

Note that there is no need to impose ratio limits for control radii $o v_{5} \sim o v_{9}$ when $\beta_{0}=45^{\circ}$. This is because sides $v_{5} v_{6}, v_{6} v_{7}, v_{7} v_{8}$ and $v_{8} v_{9}$ always have inclined angles greater than $45^{\circ}$ no matter how the corresponding control radii change. Therefore, design variables involved in a solid 12 -sided polygon consist of 8 ratio design variables $\left(\lambda_{1}, \ldots, \lambda_{4}, \lambda_{9}, \ldots, \lambda_{12}\right), 4$ length design variables ( $l_{1}, l_{6}, l_{7}, l_{8}$ ) and center coordinates $\left(x_{0}, y_{0}\right)$. Note that ratio design variables and length design variables are all nonnegative in all cases.

With this formulation, design variables and the critical ratio values of a solid polygon not only depend upon the number of sides, but also the value of $\beta_{0}$. Fig. 7(a) shows the case of a 24 -sided polygon with $\beta_{0}=45^{\circ}$. The involved design variables then correspond to $\left\{\lambda_{1}, \ldots, \lambda_{8}, \lambda_{17}, \ldots, \lambda_{24}, l_{1}, l_{10}, \ldots, l_{16}, x_{0}, y_{0}\right\}$. However, when
$\beta_{0}=63.4^{\circ}$ illustrated in Fig. 7(b), design variables become $\left\{\lambda_{1}, \ldots, \lambda_{10}, \lambda_{15}, \ldots, \lambda_{24}, l_{1}\right.$, $\left.l_{12}, \ldots, l_{14}, x_{0}, y_{0}\right\}$ with four more ratio design variables and four less length design variables. Notice that any radius in red is independent of other radii and can change as free length design variable.


Fig. 7. Illustration of design variables.


Fig. 8. A solid polyhedron feature with polygonal cross sections.
As to 3D solid polyhedron features, design variables are selected in each longitudinally-cut polygon, as shown in Fig. 8. Similarly, the angle between any two adjacent control radii in one polygonal section is identical and represented by $\alpha_{1} . \alpha_{2}$ is also a constant value representing the angle between any two adjacent section planes. If $n_{1}$ longitudinally-cut polygons exist, design variables involved in the polyhedron are $\left\{\boldsymbol{d}_{p, 1}, \ldots, \boldsymbol{d}_{p, n_{1}}, x_{0}, y_{0}, z_{0}\right\}$ with $\boldsymbol{d}_{p, i}$ denoting ratio design variables and length design variables defined in the $i$ th polygonal cross section. When all ratio design variables are bounded by their critical ratios, any triangular patch on the polyhedron satisfies the angle control constraint.

### 2.2. LSF representations of solid polygon and polyhedron features



Fig. 9. The LSF representation of a solid domain $\Omega$.
According to Osher [54], a solid domain can be represented by the zero contour of a level-set function (LSF) $\Phi(\boldsymbol{x})$ defined in a higher dimensional space. As depicted in Fig. 9, the sign of the level-set values clearly distinguishes the inside, outside and boundary of the domain. Following relations thus hold for an arbitrary point $\boldsymbol{x}$.

$$
\begin{cases}\Phi(\boldsymbol{x})>0 & \forall x \in \Omega  \tag{2}\\ \Phi(\boldsymbol{x})<0 & \forall \boldsymbol{x} \notin \Omega \\ \Phi(\boldsymbol{x})=0 & \forall \boldsymbol{x} \in \partial \Omega\end{cases}
$$

where $\Omega$ denotes the solid domain, $\partial \Omega$ is the solid-void interface.
Suppose $m$ polygons or polyhedra defined as solid features by the LSFs $\phi_{P, 1}$, $\phi_{P, 2}, \ldots, \phi_{P, m}$ are involved in a design domain represented by the LSF $\phi_{D}(\boldsymbol{x})$. The LSF of the whole structure, $\Phi$, can then be constructed through Boolean operations of these solid features and the design domain. Mathematically, Boolean operations related to union $U$ and intersection $\cap$ are equivalent to the $\max$ and $\min$ so that

$$
\begin{equation*}
\Phi=\min \left(\phi_{D}, \max \left(\phi_{P, 1}, \phi_{P, 2}, \ldots, \phi_{P, m}\right)\right) \tag{3}
\end{equation*}
$$

The max and min operations are realized and approximately calculated by the KS (Kreisselmeier-Steinhauser) function [55] to ensure differentiability.

$$
\begin{equation*}
K S=\frac{1}{w} \ln \left(e^{w \phi_{1}}+e^{w \phi_{2}}+\cdots+e^{w \phi_{q}}\right) \tag{4}
\end{equation*}
$$

in which $\phi_{1}, \phi_{2}, \ldots, \phi_{q}$ represent the LSFs of $q$ design primitives. Note that parameter $w>0$ and $w<0$ denote the union and intersection, respectively.


Fig. 10. Polygon represented by line segments.
For a polygon depicted in Fig. 10, there are $n$ angle intervals divided by control radii. Suppose the LSF of the $i$ th side $v_{i} v_{i+1}$ is a signed distance function (SDF) $\phi_{s, i}=$ $a_{i} x+b_{i} y+c_{i}$ with three normalized coefficients $a_{i}, b_{i}$ and $c_{i}$ determined by the coordinates of two vertices $v_{i}$ and $v_{i+1}$. How to represent the polygon in the LSF form $\phi_{P}$ is the basic issue. If the expression is defined as a piecewise function, the level-set value of an arbitrary point $(x, y)$ will then be calculated by

$$
\phi_{P}=\left\{\begin{array}{lc}
\phi_{s, 1} & -\frac{\pi}{2} \leq \theta<-\frac{\pi}{2}+\alpha  \tag{5}\\
\cdots & -\frac{\pi}{2}+(i-1) \alpha \leq \theta<-\frac{\pi}{2}+i \alpha \\
\phi_{s, i} & -\frac{3}{2} \\
\cdots & -\frac{\pi}{2}+(n-1) \alpha \leq \theta<\frac{3 \pi}{2}
\end{array}\right.
$$

with the inclined angle $\theta$ defined below.

$$
\theta=\left\{\begin{array}{cl}
\arctan \left(\frac{y-y_{0}}{x-x_{0}}\right) & x \geq x_{0}  \tag{6}\\
\arctan \left(\frac{y-y_{0}}{x-x_{0}}\right)+\pi & x<x_{0}
\end{array} \theta \in\left[-\frac{\pi}{2}, \frac{3 \pi}{2}\right)\right.
$$



Fig. 11. The LSF of an arbitrary polygon described by Eq. (5) with 0 -contour, $\gamma$ contour and $-\gamma$-contour highlighted ( $\gamma$ is a positive value).

$\phi_{P}=\min \left(\phi_{s, i-1}, \phi_{s, i}, \phi_{s, i+1}\right)$


$$
\phi_{P}=\max \left(\min \left(\phi_{s, i}, \phi_{s, i+1}\right), \phi_{s, i-1}\right)
$$


$\phi_{P}=\max \left(\phi_{s, i-1}, \phi_{s, i}, \phi_{s, i+1}\right)$

$\phi_{P}=\max \left(\min \left(\phi_{s, i-1}, \phi_{s, i}\right), \phi_{s, i+1}\right)$

Fig. 12. Different LSF expressions for the $i$ th angle interval in case of vertices $v_{i}$ and $v_{i+1}$ located concavely or convexly. (Red dots mean vertices located concavely and blue dots indicate vertices located convexly)


Fig. 13. The modified LSF.
Fig. 11 plots the LSF of an arbitrary polygon described by Eq. (5). Unfortunately, jumping exists at non-zero contours. This seriously influences the sensitivity analysis and the minimum length scale control to be presented in section 3.2.

To remedy this, the LSF is modified by allowing sides of a polygon to cross the intervals. For the $i$ th angle interval, the LSFs of the ( $i-1$ )th side, the $i$ th side and the $(i+1)$ th side are used together, instead of one single side, for the definition of LSF. Fig. 12 describes four cases for vertices $v_{i}$ and $v_{i+1}$ with different concave and convex locations leading to different LSF expressions for the $i$ th angle interval. In this way, the continuity of contours and equal widths between iso-contours can be guaranteed to a certain extent, as shown in Fig. 13. The uniformity of LSF can be further improved considering more sides but it has to be weighed against the increased computational cost.


Fig. 14. The definition of $\theta_{1}$ and $\theta_{2}$.
-*Similarly, a polyhedron in 3D can be regarded as an enclosed geometry surrounded by triangular facets. Two different angles ( $\theta_{1}$ and $\theta_{2}$ ) illustrated in Fig. 14 are used for the division of angle intervals and facet selections when calculating the level-set value at any point.

$$
\begin{align*}
& \theta_{1}=\arctan \frac{z-z_{0}}{\sqrt{\left(x-x_{0}\right)^{2}+\left(y-y_{0}\right)^{2}}}
\end{align*} \quad \theta_{1} \in\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] ~\left\{\begin{array}{cl}
\arctan \left(\frac{y-y_{0}}{x-x_{0}}\right) & x \geq x_{0}  \tag{7}\\
\theta_{2}= & \theta_{2} \in\left[-\frac{\pi}{2}, \frac{3 \pi}{2}\right) \tag{8}
\end{array}\right.
$$

The LSF of the polyhedron in the $i$ th angle interval is not only closely related to the LSF of the $i$ th facet, but also includes the construction of LSFs of the three surrounding facets to eliminate the jumping at the transitions of angle intervals. Here, the LSF of the $i$ th facet is expressed as $a_{i} x+b_{i} y+c_{i} z+d_{i}$ with $a_{i}, b_{i}, c_{i}$ and $d_{i}$ calculated in terms of the positions of three vertices of the facet. Moreover, the shared edges in red which is shown in Fig. 15 determine the operations of LSFs of four facets. In other words, convexly and concavely located edges correspond to the intersections and unions of LSFs, respectively. In this way, the LSF of the polyhedron in each angle interval can be obtained, and the LSF is continuous and differentiable at all angle interval transitions.


Fig. 15. Central facet and three surrounding facets.

### 2.3. Elimination of $V$-shaped areas

Fig. 16(a) depicts the situation where all boundaries of a structure meet the overhang constraint but supports are still needed at the V-shaped areas marked by black dashed circles.

(a) V-shaped areas marked by dashed circles.

(b) Solid polygon distributions.

Fig. 16. Optimized result of cantilever beam with only overhang angle control.
For the void polygons model [39], V-shaped areas only occur when two void polygons intersect or void polygons intersect with the outer boundaries of the design domain. In this work, the appearance of V-shaped areas is due to the exposure of the lower parts of solid polygons or polyhedra, as indicated in Fig. 16(b). Therefore, constraints on the positions of solid polygons/polyhedra are introduced.


Fig. 17. Solid polygons arranged into different layers in the initial design domain.

(a) A solid polygon in the third layer.

(b) A solid polygon in the first layer.

Fig. 18. The allowable areas of the first vertex $v_{1}$ of an arbitrary polygon.

Fig. 17 depicts the 2D case. First, solid polygons will be arranged in the initial design domain and classified into different layers along the build direction $\mathbf{b}$. Note that layers will not be identified again even if polygons are moved to different places during the optimization process. To avoid V-shaped areas, only two cases are allowed for each polygon during the optimization process. One is that a polygon can completely move outside the design domain. That is, the intersection volume between the polygon and the design domain is $V_{\mathrm{n}}=0$. The other is that a polygon exists within the design domain partly or completely but its first vertex $v_{1}$ should be located in the interior of any polygon in the lower layer/layers or in the area below the design domain. The two specific areas are shaded in green and blue in Fig. 18(a). Specially, for the polygons in the bottom layer, their first vertices should be located below the design domain, as shown in Fig. 18(b). Hence, the $\operatorname{LSF} \phi_{A, i}(\boldsymbol{x})$ of the allowable area for $v_{1}$ of the $i$ th polygon can be expressed as

$$
\begin{equation*}
\phi_{A, i}(\boldsymbol{x})=\max \left(\phi_{L f, i}(\boldsymbol{x}), \phi_{B D}(\boldsymbol{x})\right) \tag{9}
\end{equation*}
$$

in which $\phi_{L f, i}(\boldsymbol{x})$ and $\phi_{B D}(\boldsymbol{x})$ denote the LSF of the overall lower-layer features within the design domain and the LSF of the area below the design domain, respectively. If there are $k$ polygons in the lower layers, $\phi_{L f, i}(\boldsymbol{x})$ is

$$
\begin{equation*}
\phi_{L f, i}(\boldsymbol{x})=\min \left(\phi_{D}(\boldsymbol{x}), \max \left(\phi_{P, 1}(\boldsymbol{x}), \ldots, \phi_{P, k}(\boldsymbol{x})\right)\right. \tag{10}
\end{equation*}
$$

Therefore, two feasible cases allowed for the $i$ th feature can be written as

$$
\begin{equation*}
V_{\cap, i}=0 \text { or } \phi_{A, i}\left(\boldsymbol{x}_{v 1, i}\right) \geq 0 \tag{11}
\end{equation*}
$$

in which $\boldsymbol{x}_{v 1, i}$ represents the coordinates of $v_{1}$ of the $i$ th feature. Considering the intersection volume $V_{n, i}$ cannot be less than 0 , Eq. (11) can be replaced with one constraint $\xi_{P, i}$ using max operation.

$$
\begin{equation*}
\xi_{P, i}=\max \left(-V_{n, i}, \phi_{A, i}\left(\boldsymbol{x}_{v 1, i}\right)\right) \geq 0 \tag{12}
\end{equation*}
$$

During the optimization process, all polygons/polyhedra can move freely while satisfying the constraint. But there are only two possibilities for the $i$ th feature: move out of the design domain completely or its first vertex $v_{1}$ satisfies the position requirement. Then, all $m$ constraints are aggregated into one constraint.

$$
\begin{equation*}
\xi=\min \left(\xi_{P, 1}, \ldots, \xi_{P, m}\right) \geq 0 \tag{13}
\end{equation*}
$$

Note that the max and min operations are again realized through KS functions defined in Eq. (4).

## 3. Topology optimization of self-supporting structures

### 3.1. Structural analysis with fixed mesh



Fig. 19. The analysis procedure with fixed mesh.
A fixed mesh technique is adopted for the structural analysis. Fig. 19 illustrates an arbitrary solid entity within an irregular design domain $\Omega$. The first step is to discretize the design domain with a sufficient number of finite elements. Then, the level-set value at the center point of each element is calculated and further projected by the modified Heaviside function $\bar{H}(\cdot)$ for the assignment of material property.

$$
\begin{equation*}
E_{e}=\bar{H}\left(\Phi\left(\boldsymbol{x}_{c, e}\right)\right) E_{0} \tag{14}
\end{equation*}
$$

where $\boldsymbol{x}_{c, e}$ represents the coordinates of the center point of the $e$ th element, $E_{0}$ and $E_{e}$ are Young's modulus of solid material and the eth element, respectively. $\bar{H}(\cdot)$ is in the form of piecewise polynomial applied to smooth the material distribution.

$$
\bar{H}(\Phi)=\left\{\begin{array}{cc}
1 & \Phi \geq \Delta  \tag{15}\\
\frac{3(1-\varepsilon)}{4}\left(\frac{\Phi}{\Delta}-\frac{\Phi^{3}}{3 \Delta^{3}}\right)+\frac{1+\varepsilon}{2} & -\Delta \leq \Phi<\Delta \\
\varepsilon & \Phi<-\Delta
\end{array}\right.
$$

This means that the $e$ th element will be attributed with solid material if $\Phi\left(\boldsymbol{x}_{c, e}\right) \geq$ $\Delta$ and weak material is attributed to void elements if $\Phi\left(\boldsymbol{x}_{c, e}\right)<-\Delta$. Here, $\varepsilon=10^{-6}$ is used to prevent the singularity of the element stiffness matrix in structural analysis. Gray elements within the narrow-band $-\Delta \leq \Phi\left(\boldsymbol{x}_{c, e}\right)<\Delta$ are attributed by intermediate material. Note that $\Delta$ determines the transition width and here takes half the element size [12].

With a total of $N E$ elements, the overall stiffness matrix $\boldsymbol{K}$ and total volume $V$ accumulated over the elements can be stated as

$$
\begin{gather*}
\boldsymbol{K}=\sum_{e=1}^{N E} \boldsymbol{k}_{e}=\sum_{e=1}^{N E} \bar{H}\left(\Phi\left(\boldsymbol{x}_{c, e}\right)\right)\left(\int_{\Omega_{e}} \boldsymbol{B}_{e}{ }^{T} \boldsymbol{D}_{e} \boldsymbol{B}_{e} d \Omega\right)=\sum_{e=1}^{N E} \bar{H}\left(\Phi\left(\boldsymbol{x}_{c, e}\right)\right) \boldsymbol{k}_{0, e}  \tag{16}\\
V=\sum_{e=1}^{N E} V_{e}=\sum_{e=1}^{N E} \bar{H}\left(\Phi\left(\boldsymbol{x}_{c, e}\right)\right) V_{0, e} \tag{17}
\end{gather*}
$$

where $\boldsymbol{k}_{e}$ and $V_{e}$ denote the stiffness matrix and the volume of the $e$ th element and they hold the same form with respect to their solid element terms $\boldsymbol{k}_{0, e}$ and $V_{0, e}$.

### 3.2. Length scale control

Length scale control limits the minimum size of each strut in the structure. To visualize this, Fig. 20 gives the optimization result of the MBB beam. Every polygon satisfies the overhang angle constraint due to its bounded ratio design variables and inexistence of V-shaped area with the lower parts of polygons forced to connect to other polygons by the constraint $\xi$ defined in Eq. (13). However, issues might occur at weak connections with only one point connected to satisfy the requirement of vertex position. Fig. 20(b) illustrates two such cases circled with dotted lines. Fig. 20(d) indicates the existence of only one gray element at these connections after the Heaviside projection.


Fig. 20. Self-supporting design of the MBB beam.
These places are quite vulnerable and hard to be realized by AM. Inspired by the robust approach [51-53], three-structure forms, i.e., eroded, intermediate and dilated designs are considered in the topology optimization where the structure directly constructed by solid features is regarded as the eroded design so that these one-point connections caused by the constraint $\xi$ imposed on feature positions later will be thickened to ensure the minimum length scale of the intermediate blue print design.

The LSFs of the eroded, intermediate and dilated designs can be expressed as

$$
\begin{gather*}
\Phi^{\text {ero }}=\Phi  \tag{18}\\
\Phi^{\text {int }}=\Phi^{\text {ero }}+\gamma_{1}  \tag{19}\\
\Phi^{\text {dil }}=\Phi^{i n t}+\gamma_{2} \tag{20}
\end{gather*}
$$

in which $\gamma_{1}$ and $\gamma_{2}$ are positive values representing the erosion width and the dilation width, respectively. Fig. 21 gives a more intuitive illustration of these two widths. The
structure boundaries are offset outward/inward by translating the level-set field up/down the robustness widths.

(a) The boundaries of eroded, intermediate and

(b) The corresponding LSFs dilated designs.
Fig. 21. A three-structure scheme in robust approach.
The introduction of the robust approach largely circumvents problems with small details. However, as illustrated in Fig. 22, the same robust formulation can nevertheless result in undesired spike-like artefacts in the dilated structure due to strong elongation of thin features from the eroded design. This, however, only happens for polygons with large ratios of adjacent control radii and can hence be avoided by introducing an additional constraint.

(a) Feature distribution in the eroded design.

(b) The intermediate result

(c) The dilated result.

Fig. 22. An undesirable case with two spike-like artefacts in the dilated structure.


Fig. 23. Constraints are imposed on the triangles involving independent control radii.

This constraint is only imposed on the independent control radii at the top region whose lengths act as design variables since the ratio control of other control radii in polygons can be directly realized by further limiting the ranges of ratio design variables.

Specifically, as depicted in Fig. 23, the constraint is evaluated on all triangles of polygons involving independent control radii. $\psi_{t}$ is calculated for the triangle $o v_{i} v_{i+1}$.

$$
\begin{equation*}
\psi_{t}=\frac{P_{t, i}}{\sqrt{A_{t, i}}} \tag{21}
\end{equation*}
$$

in which $P_{t, i}$ and $A_{t, i}$ denote the perimeter and the area of the triangle $o v_{i} v_{i+1}$. Since inclined angle $\alpha$ between $o v_{i}$ and $o v_{i+1}$ remains unchanged, the value of $\psi_{t}$ is directly determined by the relative length ratio between $o v_{i}$ and $o v_{i+1}$ which has the smallest value if $o v_{i}$ and $o v_{i+1}$ share the same length, and the value increases along with the enlargement of the length difference. Fig. 24 gives the curve of $\psi_{t}$ with respect to the length ratio of $o v_{i+1}$ to $o v_{i}$ when $\alpha=30^{\circ}$.


Fig. 24. The curve of $\psi_{t}$ with respect to the length ratio of $o v_{i+1}$ to $o v_{i}$ when $\alpha=$ $30^{\circ}$.
Now we introduce another max operation that aggregates the values of $\psi_{t}$ of all $\bar{n}$ triangles involved in all polygons. $\bar{\psi}$ decides the maximum allowable gap between adjacent control radii.

$$
\begin{equation*}
\Psi=\max \left(\psi_{t, 1}, \ldots, \psi_{t, \bar{n}}\right)-\bar{\psi} \leq 0 \tag{22}
\end{equation*}
$$

Tests have been done and show that the situation in Fig. 22 can be avoided if the length ratios of adjacent control radii are controlled within 5 times, and the specific value of $\bar{\psi}$ can be calculated according to the critical length ratio.

In 3D polyhedrons, the triangles involved in the calculation of $\Psi$ are selected from the longitudinally-cut polygons.

### 3.3. Optimization formulation and sensitivity analysis

This work is focused on standard minimum compliance optimization. Following the worst-case rule in the robust approach, the eroded structures are used for the calculation of objective due to the monotonous dependence of structural stiffness on quantity of materials. In addition, the volume constraint is imposed on the dilated structures for stable convergence. The intermediate structures are the final outputs of
optimization, which ensures a minimum length scale of $2 \gamma_{1}$. Based on above considerations, the topology optimization formulation finally holds the following form.

$$
\begin{align*}
& \operatorname{Min} J=\boldsymbol{F}^{T} \boldsymbol{U}\left(\Phi^{\text {ero }}\right) \\
& \text { s.t. }\left\{\begin{array}{c}
\boldsymbol{K}\left(\Phi^{\text {ero }}\right) \boldsymbol{U}\left(\Phi^{\text {ero }}\right)=\boldsymbol{F} \\
V\left(\Phi^{\text {dil }}\right) / V^{*} \leq f^{\text {dil }} \\
\xi \geq 0 \\
\Psi \leq 0 \\
\underline{\boldsymbol{d}}_{i} \leq \boldsymbol{d}_{i} \leq \overline{\boldsymbol{d}_{i}} \quad i=1,2, \ldots, m
\end{array}\right. \tag{23}
\end{align*}
$$

Here, $J$ denotes the structural compliance and $\boldsymbol{U}$ is the vector of nodal displacement. $\boldsymbol{d}_{i}$ refers to the vector of design variables involved in the $i$ th feature with the lower bound $\underline{\boldsymbol{d}}_{i}$ and the upper bound $\overline{\boldsymbol{d}}_{i}$. The composition of design variables is closely related to the adopted solid features and the COA. It consists of ratio design variables, the lengths of some control radii and the coordinates of centers.
$V^{*}$ represents the total volume of the design domain. $V\left(\Phi^{\text {dil }}\right)$ denotes the volume of the dilated structure, which is limited by the volume fraction $f^{\text {dil }}$. In practice, the value of $f^{\text {dil }}$ will be updated every 20 iterations during the optimization process to make the volume of the intermediate result, i.e., $V\left(\Phi^{i n t}\right)$ less than the prescribed volume fraction $f^{*}$.

$$
\begin{equation*}
f^{d i l}=\frac{V\left(\Phi^{d i l}\right)}{V\left(\Phi^{i n t}\right)} \cdot f^{*} \tag{24}
\end{equation*}
$$

The sensitivity of the compliance with respect to $\boldsymbol{d}_{i}$ is as usual given by

$$
\begin{equation*}
\frac{\partial J}{\partial \boldsymbol{d}_{i}}=2 \boldsymbol{U}^{T} \boldsymbol{K} \frac{\partial \boldsymbol{U}}{\partial \boldsymbol{d}_{i}}+\boldsymbol{U}^{T} \frac{\partial \boldsymbol{K}}{\partial \boldsymbol{d}_{i}} \boldsymbol{U}=-2 \boldsymbol{U}^{T} \frac{\partial \boldsymbol{K}}{\partial \boldsymbol{d}_{i}} \boldsymbol{U}+\boldsymbol{U}^{T} \frac{\partial \boldsymbol{K}}{\partial \boldsymbol{d}_{i}} \boldsymbol{U}=-\boldsymbol{U}^{T} \frac{\partial \boldsymbol{K}}{\partial \boldsymbol{d}_{i}} \boldsymbol{U} \tag{25}
\end{equation*}
$$

According to Eq. (16) and Eq. (17), the sensitivity of the global stiffness matrix, i.e., $\partial \boldsymbol{K}\left(\Phi^{e r o}\right) / \partial \boldsymbol{d}_{i}$ is written as

$$
\begin{equation*}
\frac{\partial \boldsymbol{K}\left(\Phi^{e r o}\right)}{\partial \boldsymbol{d}_{i}}=\sum_{e=1}^{N E} \frac{\partial \boldsymbol{k}_{e}\left(\Phi^{e r o}\left(\boldsymbol{x}_{c, e}\right)\right)}{\partial \boldsymbol{d}_{i}}=\sum_{e=1}^{N E} \bar{\delta}\left(\Phi^{e r o}\left(\boldsymbol{x}_{c, e}\right)\right) \cdot \frac{\partial \Phi\left(\boldsymbol{x}_{c, e}\right)}{\partial \boldsymbol{d}_{i}} \cdot \boldsymbol{k}_{0, e} \tag{26}
\end{equation*}
$$

and $\partial V\left(\Phi^{d i l}\right) / \partial \boldsymbol{d}_{i}$ corresponds to

$$
\begin{equation*}
\frac{\partial V\left(\Phi^{d i l}\right)}{\partial \boldsymbol{d}_{i}}=\sum_{e=1}^{N E} \frac{\partial V_{e}\left(\Phi^{d i l}\left(\boldsymbol{x}_{c, e}\right)\right)}{\partial \boldsymbol{d}_{i}}=\sum_{e=1}^{N E} \bar{\delta}\left(\Phi^{d i l}\left(\boldsymbol{x}_{c, e}\right)\right) \cdot \frac{\partial \Phi\left(\boldsymbol{x}_{c, e}\right)}{\partial \boldsymbol{d}_{i}} \cdot V_{0, e} \tag{27}
\end{equation*}
$$

in which the delta function $\bar{\delta}(\cdot)$ denotes the derivative of the modified Heaviside function.

$$
\bar{\delta}(\Phi)=\frac{\partial \bar{H}(\Phi)}{\partial \Phi}=\left\{\begin{array}{cc}
\frac{3(1-\varepsilon)}{4}\left(\frac{1}{\Delta}-\frac{\Phi^{2}}{\Delta^{3}}\right) & -\Delta \leq \Phi<\Delta  \tag{28}\\
0 & \text { else }
\end{array}\right.
$$

Obviously, $\bar{\delta}(\Phi)$ is non-zero only when values of the corresponding LSF fall into the transition interval $[-\Delta, \Delta]$, as depicted in Fig. 25. This means that only gray elements will participate in the sensitivity calculations of compliance and volume.


Fig. 25. The graph of delta function $\bar{\delta}(\Phi)$, Eq. (28).
$\Phi, \xi$ and $\Psi$ are all assembled by KS function described earlier and the sensitivity calculations follow the chain rule. Specifically, according to Eq. (3), the sensitivity of the LSF of the whole structure, i.e., $\partial \Phi / \partial \boldsymbol{d}_{i}$ is closely related to the sensitivities of the LSFs of $m$ solid features $\partial \phi_{P, 1} / \partial \boldsymbol{d}_{i}, \ldots, \partial \phi_{P, m} / \partial \boldsymbol{d}_{i}$ which still follow piecewise forms. Similarly, the constraint $\xi$ including the level-set values at the first vertices of solid features and intersection volumes between solid features and design domain ultimately concerns the sensitivities of the LSFs of solid features. $\Psi$ involves perimeters and volumes of all triangles in polygons/polyhedra and sensitivity calculation is easily realized since each triangle is only related to a few design variables.

## 4. Numerical examples

In this section, both 2D and 3D numerical examples are dealt with to illustrate the effectiveness of the proposed method. The 2D example mainly focuses on the effects of feature numbers and robustness widths, while 3D examples focus on build direction $\mathbf{b}$ and COA $\beta_{0}$ considered as two important parameters in self-supporting designs.

In addition, an unconstrained topology optimization problem (only volume constraint imposed) is studied where the lengths of all control radii are considered as free design variables without ratio limits. The corresponding design results are used as references. In all examples, the Young's modulus of solid material and Poisson's ratio are set to $E_{0}=1$ and $v=0.3$ with dimensionless geometric data and loads. The Globally Convergent Method of Moving Asymptotes (GCMMA) [56] is adopted as the optimizer.

### 4.1. The MBB beam

The MBB beam is illustrated in Fig. 26. The design domain has a dimension of $480 \times 80$ with a vertical unit load applied on the middle point of the top side. Only half of the model is considered due to the symmetry. Plane stress state (with unit thickness) is assumed and finite element analysis is based on four-node square elements of size 1 $\times 1$. The prescribed upper bound of the volume fraction constraint is $50 \%$. In this
example, a specific value of $\mathrm{COA}=45^{\circ}$ is used and the length ratios between adjacent control radii are set to be less than 5 times.


Fig. 26. MBB model.
Table 1. Five cases for free-form and self-supporting optimization.

| Case | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Numbers of polygons | 7 | 17 | 38 | 63 | 93 |
| Numbers of design <br> variables | 182 | 442 | 998 | 1638 | 2418 |

Five cases are tested with 2 to 6 layers of solid polygons, as listed in Table 1. Each solid polygon has 24 control radii and will be constantly deformed and translated throughout the optimization process.

Table 2 lists the polygon distributions before and after optimization without overhang angle control. Clearly, the design freedom is gradually enlarged with the increasing number of polygons so that more elaborate topologies are obtained. A comparison of case 1 and case 5 is a good illustration of this. Two solutions hold completely different topologies and more holes obviously appear in the latter. In addition, the compliance reduces at a slow pace and is almost unchanged when the number of polygons exceeds 38 . However, there exist many violations of the overhang angle constraint. The red boundaries indicated in the last column of Table 2 are at high risk of deformations and collapse during fabrications.

In comparison, self-supporting structures are also optimized in five cases with a bottom-to-up build direction. Table 3 shows polygons in the initial and final states, respectively. Note that polygons with different border colors denote different layers and black dots indicate the positions of the first vertices. It can be observed that the same effect holds in the self-supporting design with the increasing number of polygons. The stiffness degradations in five cases are within $12 \%$ when compared with solutions of the free-form topology optimization in Table 2. The last column of Table 3 shows the iteration curves of objective function and constraints in each case. The compliance curves are considerably influenced by the values of volume fraction, and continue to rise at the beginning but gradually become stable when the volume constraint is satisfied. The other two constraints $\xi$ and $\Psi$ are actually aggregations of a set of constraints. Due to the aggregation, the optimization is not directly affected by the
number of constraints. However, we do find that more polygons require more iterations and sometimes slight stability problems during intermediate iteration steps.

Other existing results of self-supporting designs of the MBB beam are also displayed in Table 4. Here, included stiffness degradations are taken directly from the respective paper or calculated according to the data provided. The first two approaches exploit the ability of level set functions to handle the evolution and geometrical information of the structural boundary to meet the overhang angle condition, while the remaining density-based methods show flexibility to create finer structures in response to the angle restriction.
Table 2. Results of free-form topology optimization in five cases.

| Case | Feature distributions in initial structures | Feature distributions in final optimized results | Compliance values | Identifications of boundaries violating angle condition ( $\beta_{0}=$ $45^{\circ}$ ) |
| :---: | :---: | :---: | :---: | :---: |
| 1 |  |  | 102.66 |  |
| 2 |  |  | 95.37 |  |
| 3 |  |  | 95.05 |  |
| 4 |  |  | 94.77 |  |
| 5 |  |  | 94.63 |  |

Table 3. Topology optimization of self-supporting structures in five cases with a bottom-to-up build direction.

| Case | Initial structures | Initial feature distributions | Final self-supporting designs | Final feature distributions | Iteration curves of objective and constraints |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |
| 2 | $\begin{array}{ccccc} \Delta & \wedge & \wedge & \wedge & \Delta \\ \wedge & \wedge & \wedge \end{array}$ |  | $J=105.98$ |  |  |


| 3 |  |  | $J=102.60$ |  |  | ${ }_{-5}^{5}\left[\begin{array}{c} 15 \\ -50 \\ -5 \\ -5 \end{array}\right.$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 |  |  |  |  |  | $\left[\begin{array}{c} 15 \\ -5 \\ -0 \\ -5 \end{array}\right.$ |
| 5 |  |  |  |  |  | $\left[\begin{array}{l} 15 \\ -10 \\ -5 \\ -5 \end{array}\right.$ |

Table 4. Existing self-supporting designs of MBB example.

| Design results | Wang et al. [33] | Guo et al. [38] | Zhang et al. [28] | Langelaar [19] | Gaynor et al. [15] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Free-form <br> optimizations |  |  |  |  |  |
| Self-supporting <br> designs with $\beta_{0}=$ <br> $45^{\circ}$ | $17.59 \%$ | $14 \%$ | $3.84 \%$ | $6 \%$ | $19 \%$ |
| \% increase in <br> compliance |  |  |  |  |  |

Table 5. Self-supporting designs and corresponding element distributions.

| Robustness <br> widths | Eroded structures | Intermediate structures <br> (final outputs) | Dilated structures |
| :---: | :---: | :---: | :---: | :---: |

In addition, the influences of robustness widths are investigated based on the 38polygon case. Table 5 gives the self-supporting designs with 3 sets of widths and each of the designs shown in three forms. The eroded topologies in the second column are the direct feature-driven results constructed by the union of polygons. Since there are point-to-point connections and no disconnected regions, a minimum length scale of $2 \gamma_{1}$ is ensured in the intermediate results of the next column. The dilated structures in the last column are involved in the calculation of the volume constraint for convergence stability. The zooming views show that original one-element features are two or more elements distributed nearby according to the specific values of $\gamma_{1}$ and $\gamma_{2}$. A larger
robustness width, which means larger differences between the three forms, will result in a bigger compliance of intermediate solutions.

We observe that V-shaped areas are successfully suppressed in all results by the constraint $\xi$. Limited by ratio design variables and $\Psi$ constraint, no cusps like those in Fig. 22(c) appear.

### 4.2. 3D beam



Fig. 27. A 3D Beam.

(a) The initial structure.

(b) The distribution of polyhedra (reduced size) in the initial structure.

(c) The free-form optimization result $(J=$ 3343.04).

Fig. 28. Free-form topology optimization.
In Fig. 27, the 3D beam model studied in the work of Langelaar [18] is reconsidered to highlight the effect of build orientations on the self-supporting design. Here, one end of the domain is fully clamped and distributed forces are applied vertically along the lower edge of the other end. The model has dimensions of $150 \times$ $50 \times 50$. Only a half is meshed with 8 -node hexahedral elements of size $1 \times 1 \times 1$ and optimized according to the symmetry. The volume fraction is constrained to $30 \%$. The dilated widths are set to be the same with $\gamma_{1}=\gamma_{2}=0.5$.

Fig. 28(a) depicts the initial structure consisting of 28 polyhedra, each of which has 72 control radii. Thus, a total number of 2100 design variables are considered. Fig. 28(b) indicates the polyhedron distribution where polyhedra are reduced in size for better visualization. The result of the free-form topology optimization has an I-beam profile, as shown in Fig. 28(c), and satisfies the volume constraint.

However, as depicted in Fig. 29, the overhang surfaces with inclined angles exceeding the COA exist so that supports are required to prevent the collapse of the structure. It is seen that these infeasible areas are closely related to the build direction $\mathbf{b}$ and a good choice seems to be $\mathbf{b}=[-1,0,0]$ with the least violation of the overhang angle.


Fig. 29. Distributions of surface inclination angles for four different build directions.


Fig. 30. Initial structures and self-supporting designs for four different build directions.
Fig. 30 gives the self-supporting designs $\left(\beta_{0}=45^{\circ}\right)$ for four different build orientations. The results have similar topologies to the free-form design except for local variations of surface inclination angles. In this paper, the stiffness degradations due to the self-supporting constraint are $3.46 \%, 5.25 \%, 1.78 \%$ and $0.73 \%$ in four cases. $\mathbf{b}=[-1,0,0]$ is superior to other orientation selections just as predicted above. In particular, for the worst and best cases, the features of different layers distinguished by colors are depicted in Fig. 31.


Fig. 31. Polyhedron distributions in the initial and final structures for the worst and best build directions.


Fig. 32. Self-supporting designs from the work of Langelaar [18].
For comparison, Fig. 32 shows the results obtained by means of the layer-wise filtering procedure in the work of Langelaar [18]. The relative increased compliances compared to the reference design are $-1 \%,-2 \%, 1 \%$ and $0 \%$, which implies that the freely optimized result corresponds to a local optimum. Unlike the I-beams obtained in our work, most results are composed of two vertical webs connecting the top and the bottom. However, if we increase the number of polyhedron features to 126 , there will be two webs in the optimized results, as shown in Fig. 33.

Regarding the computation cost of optimization, finite element analysis and sensitivity analysis are two main aspects. The former is highly related to the number of elements, while the latter depends on the number of features and also element numbers. If the same finite element model is used but with an increasing number of features, the time spent on finite element analysis is unchanged, but the time cost of sensitivity analysis will gradually increase and possibly become the major part due to larger numbers of design variables. In the example, structural analyses are all based on $150 \times$ $25 \times 508$-node hexahedral elements. For the optimizations driven by 28 polyhedra, the
average time of one iteration is about 100 seconds (Intel(R) Core(TM) i7-10875H CPU @ 2.30 GHz 16G RAM) in which 50 seconds are spent on finite element analysis, 35 seconds are used to calculate the sensitivities of objective and constraints. When the number of polyhedra increases to 126 , it costs about 230 seconds to do one iteration with still 50 seconds for structural analysis but 160 seconds for sensitivity calculation.


Fig. 33. The self-supporting results driven by 126 polyhedra for four different build directions.

### 4.3. 3D bracket

A 3D bracket example is shown in Fig. 34(a). It has an irregular design domain. Horizontal and vertical loads are applied at the centers of two rigid cylinders in red and four non-designable rings at the bottom are fixed. In view of the structural symmetry, only one quarter of the bracket domain is retained for topology optimization. In Fig. 34(b), the bracket is meshed with 140430 elements for the structural analysis. Hexahedral and pentahedral elements are used. Similarly, $\gamma_{1}=\gamma_{2}=0.5$ is used for the minimum size constraint of 1 . The prescribed volume fraction is $36 \%$ excluding nondesignable areas and rigid areas.


Fig. 34. A 3D bracket.


Fig. 35. Results of free-form topology optimization for the bracket example.
First, only the volume constraint is considered as a reference. 45 polyhedra are used. The result of the free-form optimization is shown in Fig. 35. In addition to the legs connecting the bolt holes, one can see two additional plates that produce force transmission. The printability of the upper plate seems to be very doubtful due to the lack of supports underneath. In details, Fig. 36 highlights surfaces with angles smaller than the specific three COAs. Clearly, an increasing value of the COA results in larger infeasible areas.


Fig. 36. Identification of infeasible areas requiring supports for three different COAs (only one quarter of the structure is shown here for clarity).

Concerning self-supporting designs, initial and optimized topologies are depicted in Fig. 37 and Fig. 38, respectively. Here, support-like structures are generated between two plates as polyhedron features. A greater COA tends to produce a more complex topology to mitigate the impact on structural performance at the cost of larger stiffness degradation. In this example, the three COAs result in stiffness degradations of $4.27 \%$, $5.58 \%$ and $9.86 \%$, respectively.

Fig. 39 illustrates the distributions of surface inclination angles in the three selfsupporting designs. Red colors indicate overhang angles violating the COA. In addition to the unavoidable inner surface of the non-designable ring at the top, infeasible areas also exist at some intersections of polyhedra especially when there are transitions from
$+\beta_{0}$ to $-\beta_{0}$. This is mainly because the sharp transition regions are represented by the approximation of smoothed KS function. In practice, small overhang sections are printable, so this is not expected to be a problem.


Fig. 37. Initial structure.

(a) Self-supporting design for $\beta_{0}=26.6^{\circ}(J=40261.40)$.

(b) Self-supporting design for $\beta_{0}=45^{\circ}(J=40766.56)$.

(c) Self-supporting design for $\beta_{0}=63.4^{\circ}(J=42417.90)$.

Fig. 38. Initial structure and self-supporting designs for different COAs.

(a) Self-supporting design for $\beta_{0}=$ $26.6^{\circ}$.

(b) Self-supporting design for $\beta_{0}=$ $45^{\circ}$.

(c) Self-supporting design for $\beta_{0}=$ $63.4^{\circ}$.

Fig. 39. Identification of areas requiring supports within the self-supporting solutions.

### 4.4. Torsion structure



Fig. 40. A torsion structure.


Fig. 41. Initial structure for free-form topology optimization.
Consider now the torsion structure with cylindrical design domain loaded by two opposite forces in Fig. 40. It is clamped by a bolt connection but we assume that the end plate is fully fixed. The overhang angle constraint $\beta_{0}=45^{\circ}$ is violated over one quarter of the cylindrical surface. Finite element analysis is carried out using 99050 irregular hexahedral elements. In this example, the solid volume of the cylinder is required to be reduced by $70 \%$.

In Fig. 41, 61 solid polyhedra are used and evenly distributed in the initial structure. As expected, the free-form design is a hollow cylinder providing the largest torsion moment of inertia, as shown in Fig. 42. In details, the cross-section is of almost equal thickness including about 3 or 4 layers of elements. However, the red inner cylinder surface at the top depicted in Fig. 42(c) is an infeasible region violating the overhang angle $45^{\circ}$. As a result, supports will be added inside the cylinder to allow the AM process.

(a) The optimized result in three views $(J=$ 16608.30).

(b) Element distribution.

(c) Inclination angle distribution with infeasible areas highlighted.
Fig. 42. Free-form optimization result.
Self-supporting design is now carried out. Fig. 43(a) shows the initial structure with the same number of polyhedra in the form of water drops. The distribution of polyhedra is the same as in Fig. 41(b). Fig. 43(b) shows the optimization result now with a non-uniform cross-section. Meanwhile, some holes appear on the top and some ribs are distributed on both sides of the upper surface inside the cylinder. From the element distribution in Fig. 43(c), it is seen that the lower part of the cylinder is thinned with two or three layers of elements, while more layers of elements are distributed on the upper part to meet the overhang angle control. Compared with the free-form optimization result, the need of supports is almost eliminated inside the cylinder, as indicated in Fig. 43(d).


Fig. 43. Self-supporting design without overhang angle control of the outer surface of the cylinder.
Final studies are made about the cylindrical design domain extended without the infeasible region, as illustrated in Fig. 44. Note that the material inside the extended part is not involved in the calculation of structural compliance but included in the volume constraint. By this means, the expanded part can be regarded as a specific support design area. In other words, the design is a simultaneous optimization of structure topology and AM supports.

Figs. 45(a)-(b) give a detailed depiction of the optimization result. Material distributed on the bottom of the cylinder plays a supporting role and two separate parts in Fig. 45(c) give a clear view. However, the final compliance is considerably higher than that in Fig. 43(b) since the actual material usage for structural stiffness is approximately reduced to $80 \%$.


Fig. 44. Extended design domain and the initial structure.

(a) Self-supporting result $(J=$ 23212.94).

(b) Inclination angle distribution with infeasible areas highlighted.

(c) Optimized structure separated by original design domain.

Fig. 45. The optimization result within the extended design domain.

## 5. Conclusion

Due to the special layer-wise manufacturing process of AM, additional supports need to be added in overhang places to prevent deformation and collapse. This paper suggests a design approach to eliminate or reduce the need for supports. Solid polygon/polyhedron features act as basic deformable primitives to perform optimization, and the ratio design variables defined provide the precise overhang angle control of features. Small overhangs occurring at the intersections of features and caused by the approximation of involved KS-functions are printable and acceptable.
V-shapes are special cases in structures which almost everywhere satisfy the angle constraint but still need supports. To avoid this, every feature located inside the design domain is required to connect to features in the lower layers or to the baseplate. Weak point-to-point connections are found in the optimized results and robust formulation together is adopted to ensure a reasonable minimum length scale.

Representative numerical examples verify the proposed method. It is found that refined topologies can be designed with increasing number of features. The elimination
of sacrificial support material always comes at the cost of reducing structural stiffness and the specific loss is closely related to build direction $\mathbf{b}$ and critical overhang angle $\beta_{0}$, which reminds us to introduce these two important factors as design variables in future works.

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