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A decision making framework for incorporating fairness in allocating slots at capacity-constrained airports

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\begin{abstract}
Fairness constitutes an important criterion for allocating scarce resources among self-interested parties. The capacity of congested airports is a scarce resource which is allocated in the form of slots to airlines to operate their flights. Traditionally, the optimisation of slot allocation efficiency has been considered as the primary criterion in making slot allocation decisions at capacity-constrained airports. The efficiency of slot allocation is expressed as a function of displacement, which is defined as the difference between the time requested by the airlines and the time allocated to airlines to operate their flights. Recent research work has considered fairness as an additional slot allocation objective. However, the literature has not adequately addressed the following two key research questions: i) in the presence of alternative fairness measures, which fairness measure should be used to investigate the slot allocation efficiency-fairness trade-off, and how the choice of the efficient frontier describing this trade-off should be made? and ii) given that the choice of the efficient frontier has been made, which slot allocation solution should be selected for implementation, and how this choice should be made? To address these questions, we investigate three inter-airline fairness objectives, and we introduce a decision-making framework to assist decision-makers in identifying the most preferable acceptable slot allocation outcome. The proposed decision-making framework is based on a voting mechanism which incorporates the preferences of stakeholders. We use slot request and capacity data to determine the most preferable slot allocation outcome at a capacity-constrained airport.
\end{abstract}

1. Introduction

The rapid growth of demand for air transport services has amplified congestion at airports around the world. Since 2010, the number of level 3 coordinated airports has increased by 31.6%, from 155 to 204 (ICAO, 2013; IATA, 2019a). An airport is a level 3 coordinated airport when its demand significantly exceeds its capacity, an expansion of the airport infrastructure to meet demand is not possible in the short term, and a process of slot allocation is required (IATA, 2019b). A slot is defined as the time allocated to an airline to land and take-off from a given airport. The prevailing slot allocation mechanism outside US1 is based on the IATA’s World Scheduling Guidelines (IATA, 2019b).

The practical importance of slot allocation coupled with the complexity of the resulting slot scheduling problem has attracted the...
interest of the research community, and several models have been proposed for optimising slot allocation decisions. The slot allocation literature has evolved from single to multi-objective formulations. Single objective slot scheduling models aim to optimise schedule efficiency through the minimisation of the total schedule displacement, i.e. the sum of the absolute values of the difference between the requested and scheduled time slot (Zografos et al., 2012, Fairbrother and Zografos, 2020).

Fairness has been recognised as an important criterion in making decisions regarding the allocation of scarce resources in a broader context. The consideration of fairness is crucial because it could increase the acceptability of decisions. As established by Freeman and Evan (1990), stakeholders are willing to accept a fair contract that ensures that the interests of all stakeholders are taken into consideration. Bertsimas et al. (2011) suggested that when allocating scarce resources among multiple self-interested parties, a solution that optimises efficiency might not be implementable, because some of the parties might consider it “unfair”.

In the context of airport slot scheduling, although the IATA guidelines (IATA, 2019b) require that “fair and equal treatment is provided to all airlines serving the airport”, no explicit quantitative fairness measures have been defined and adopted by the industry. To address the slot allocation fairness issue, Zografos and Jiang (2016; 2019), Zografos and Jiang (2017), Jacqillat and Vaze (2018), Fairbrother et al. (2019) proposed models that incorporate inter-airline fairness measures, i.e. measures that seek to allocate fairly schedule displacement among airlines. Models considering inter-flight slot allocation fairness measures, i.e. measures that seek to allocate fairly schedule displacement among flights, have been also proposed in the literature (Jacquillat and Odoni, 2015; Zografos et al., 2018; Ribeiro et al., 2018; 2019). Although, alternative measures can be used in multi-objective formulations to express fairness the literature currently lacks a methodological framework that can assist decision makers to select the fairness measure that should be used to determine the most preferable efficient solution. The objective of this paper, which builds upon our work presented at the 96th Transportation Research Board Meeting (Zografos and Jiang, 2017), is to introduce a decision-making framework for selecting the most preferable airport slot allocation decision when considering explicitly efficiency-fairness trade-offs. Specifically, the proposed framework aims to provide support in answering the following two questions in the context of airport slot allocation: i) in the presence of alternative fairness measures, which fairness measure should be used to investigate the slot allocation efficiency-fairness trade-off, and how the choice of the efficient frontier describing this trade-off should be made? and ii) given that the choice of the efficient frontier has been made, which slot allocation solution should be selected for implementation, and how this choice should be made? The proposed decision-making framework is developed in the airport slot allocation context defined by the IATA World Scheduling Guidelines. In this context, the slot allocation decisions are made bi-annually, and their outcome determines the airport schedules for the Summer and Winter seasons. Therefore, the research questions identified above are called to provide answers to strategic slot allocation decisions. The slot allocation process involves the: i) airport operator which defines the supply side inputs for the problem through the establishment of the slot allocation coordination parameters for the scheduling period under consideration, e.g. airside and passenger terminal capacity, ii) the airlines, which through their slot requests, express the demand side of the slot allocation process, and iii) the slot coordinator, an independent authority, that allocates the slots according to the IATA slot allocation rules. The IATA slot allocation guidelines establish the criteria and priorities that should be applied in order to satisfy the slot requests of the various airlines as close as possible. A concise description of the slot allocation decision-making process in the context of the IATA World Scheduling Guidelines can be found in (Zografos et al., 2012).

The remainder of this paper is organised as follows. Section 2 discusses previous related work and establishes the contributions of this paper. Section 3 presents the proposed fairness metric, objectives, and bi-objective models, followed by the solution algorithm. Section 4 introduces a decision-making scheme for selecting the acceptable slot allocation solution. Numerical studies using real data are presented in Section 5. Finally, Section 6 summarises the conclusions derived from the application of the proposed models and provides directions for future research.

2. Literature review

In line with the objectives of this paper, we are focussing our literature review on the modelling of airport slot allocation decisions based on administrative slot allocation approaches. For a more comprehensive review of the airport slot allocation problem, the reader is referred to (Zografos et al., 2017). The airport slot allocation problem was first formulated as a single objective optimisation problem seeking to minimise total schedule displacement (Zografos et al., 2012). The minimisation of the maximum displacement, mini-max, has been used as an additional criterion in modelling airport slot allocation decisions (Jacquillat and Odoni, 2015; Zografos et al., 2018; Ribeiro et al., 2018; 2019). Slot displacement is defined as the absolute difference between the time requested and the time allocated to a slot request. The maximum displacement represents the displacement received by the worst-off flight, i.e. the flight with the maximum displacement, as is small as possible. In this sense, the mini-max criterion can be considered as a criterion expressing fairness among flights, i.e. inter-flight fairness. Jacquillat and Odoni (2015) proposed a bi-objective slot allocation model which considers the minimisation of the maximum (mini-max) displacement of the requested slots (inter-flight fairness) and the minimisation of the total (mini-sum) displacement (efficiency). The proposed model was solved lexicographically by optimising first the mini-max problem. At a subsequent step, the optimum value of the solution of the mini-max problem is used as a constraint to minimise the total schedule displacement. The model was used in an integrated framework to optimise strategic slot allocation decisions and to mitigate airport congestion at the tactical level. Zografos et al. (2018) presented two formulations for modelling the slot allocation problem in the IATA context. The first formulation minimises the maximum displacement of all slot requests (inter-flight fairness) and the total displacement over all slot requests in the IATA context, while the second formulation minimises the total displacement and the number of violated slots. Both models were solved using the ε-constraint method. In the first model, the inter-flight fairness objective (minimisation of maximum slot displacement) was introduced as a constraint, while the total displacement (efficiency) was used as the
optimisation objective. The trade-off between fairness and efficiency was examined by varying the (ε) value of the fairness constraint which defines the level of maximum acceptable displacement. Ribeiro et al. (2018, 2019) introduced a model that includes the following four objectives: i) minimisation of total displacement (efficiency objective), ii) minimisation of maximum displacement (fairness objective), iii) minimisation of unassigned slots, and iv) minimisation of displaced slots. A weighted method was used for minimising the sum of the four objectives by assigning a higher weight to the number of the unassigned slots, followed by the minimisation of the maximum displacement, the minimisation of total displacement overall slot requests, and the minimisation of the number of displaced slots. Jacquillat and Odoni (2015) studied the slot allocation problem in the US context, while Zografos et al. (2018) and Ribeiro et al. (2018) have modelled slot allocation decisions in the context of the IATA world scheduling guidelines. It is worth emphasising that the US context differs significantly from the IATA World Scheduling Guidelines leading to different slot allocation modelling requirements. Major differences, with significant modelling implications, include the IATA requirements to i) consider the allocation of the slots for the entire slot scheduling season, i.e. a period of about six months identified as Summer and Winter scheduling seasons, and ii) prioritise the allocation of slots according to a historical precedence rule regarding the use of the airport infrastructure. According to this rule, slot requests are satisfied with different priorities with the highest priority given to Historic requests, followed by New Entrant and Other requests.

The mini-max criterion incorporated in the models discussed above addresses does not consider how fairly the airlines operating at an airport have been treated, i.e. how fairly the displacement associated with a given slot allocation outcome is apportioned among the airlines (inter-airline fairness). To address this issue, inter-airline fairness has been considered in modelling airport slot allocation decisions (Zografos and Jiang, 2016, 2019; Jacquillat and Vaze, 2018; Fairbrother et al., 2019). Zografos and Jiang (2016; 2019) modelled inter-airline fairness decisions in the IATA context. They used the principle of proportionality to allocate total displacement among all airlines operating at an airport. Manley and Sherry (2010), analysed performance and fairness within the context of the Ground Delay Programme and used an equity rule that allocates delay among airlines proportionally to the number of their flights. The proportionality metric used in Zografos and Jiang (2016; 2019) postulates that the total displacement allocated to an airline should be proportional to the slot requests made by each airline. This metric was subsequently used to define a fairness objective seeking to minimise the maximum absolute distance between the value of the fairness of each airline and the average fairness metric of all airlines. Fairbrother et al. (2019) adopted the fairness metric introduced by Zografos and Jiang (2017), and used the concept of demand-based fairness to define a proportionality metric that considers only the peak-period requests for allocating the total displacement among all airline. Jacquillat and Vaze (2018) proposed a model to maximise inter-airline equity via lexicographically minimising airlines’ disutility, which was computed through the weighted average of per-flight displacements. They showed that ignoring inter-airline equity (i.e. considering efficiency-based objectives exclusively, or, in some cases, requiring maximum efficiency) may lead to highly inequitable outcomes, highlighting the importance of incorporating equity objective in the scheduling problem. Pellegrini et al. (2017) modelled slot-scheduling fairness at the airport-network level through the minimisation of the maximum cost of missed allocations over all airlines, and the minimisation of the maximum total cost across all airlines.

The literature review suggests that alternative measures can be used to express fairness and that fairness can be incorporated either at inter-flight or inter-airline level in multi-objective formulations in order to study trade-offs between fairness and other competing objectives, e.g. efficiency. However, the airport slot allocation literature lacks a methodological framework that will help decision makers to answer the following two questions: i) how the fairness measure that should be used to investigate the efficiency-fairness trade-off in airport slot allocation should be selected? and ii) how the slot allocation solution that should be implemented should be selected? To address these questions, we introduce a decision-making framework which incorporates the preferences of the airlines and of the slot coordinator in order to select the most preferable slot allocation solution. In the proposed framework, the airlines express their preference regarding the inter-airline fairness measure that should be used. Our approach uses a utility function to identify which airlines are favoured or disfavoured when using alternative fairness measures. A majority rule based voting mechanism is used to aggregate individual preferences (views of the airlines) about the fairness measure that should be used by the slot coordinator. At a subsequent step, the slot coordinator selects the solution that will be implemented by selecting the solution that minimises the weighted normalised distance from the selected efficient frontier. The proposed framework can be used in conjunction with different inter-airline fairness measures. In this paper, we are applying the proposed framework using, in addition to the fairness measure that minimises the maximum distance from the average fairness (Zografos and Jiang, 2019), two other inter-airline fairness objectives, namely: the minimisation of the maximum distance from the absolute fairness, and the minimisation of the Gini index. The three alternative fairness objectives are introduced (one at a time) into a bi-objective efficiency-fairness model to generate the corresponding efficient frontiers. We are using slot requests and airport capacity data from a congested airport to demonstrate the use of the proposed framework. In addition, we perform sensitivity analysis to examine the effect of the importance assigned by the airlines and the coordinator on the efficiency and fairness objectives.

3. The proposed modelling framework

This section first provides the notation used throughout this paper, followed by the definition of the fairness metric and fairness objectives incorporated in the proposed framework.
3.1. Notation

Sets

- $T$: set of coordination time intervals
- $M_a$: set of movements requested by airline $a$
- $M$: set of movements requested by all airlines
- $P$: set of movement pairs $(m_{arr}^p, m_{dep}^p) \in P$, where $m_{arr}^p$ is the arrival movement and $m_{dep}^p$ is the departure movement
- $C$: set of airport capacity constraints
- $T_c$: set of slots associated with capacity constraint $c$
- $A$: set of airlines

Parameters

- $t_m$: originally requested time interval for movement $m$
- $\omega_p$: minimum turnaround time for movement pair $p$
- $u_c$: declared capacity associated with capacity constraint $c$
- $\epsilon_i$: parameters for the $\epsilon$-constraint model, $i \in \{\text{MMA, MMR, MGI}\}$

Decision variables

- $x_{tm}^a$: $x_{tm}^a = 1$ if movement $m$ is allocated to slot $t$; otherwise, $x_{tm}^a = 0$

Functions

- $\rho_a$: fairness metric for airline $a$ evaluated with all requests
- $d_a$: displacement allocated to airline $a$
- $D$: total displacement of all airlines
- $\bar{d}_a$: average displacement of airline $a$

3.2. Fairness metric

The fairness metric used in this paper is based on the principle of proportionality. In a distribution process, the principle of proportionality requires that the outputs must be distributed among different parties proportionally to their inputs (Lootsma, 1999). The fairness metric used herein, Eq. (1), was introduced by Zografos and Jiang (2016; 2019) to model airport slot allocation decisions. According to Eq. (1), the total displacement of a slot allocation outcome should be distributed among airlines proportionally to their slot requests. This metric expresses inter-airline fairness and is used to define the fairness objectives (see Sections 3.2.1 - 3.2.3) that will be introduced in the bi-objective models (see Section 3.4) and the proposed decision-making framework (see Section 4). As discussed in the introductory section of this paper, the proposed decision-making framework relates to strategic slot allocation decisions. Therefore the proposed metric is static and measures inter-airline fairness associated with the slot allocation decisions for an entire slot scheduling season.

$$\rho_a = \frac{d_a}{M_a}, \forall a \in A$$ (1)

The denominator in Eq. (1) represents the proportion of slots requested by airline $a$, while the numerator expresses the proportion of displacement allocated to airline $a$. $d_a$ and $D$ are defined by Eqs. (2) and (3),

$$d_a = \sum_{m \in M_a} \sum_{t \in T} |t - t_m^a| x_{tm}^a, \forall a \in A$$ (2)

and

$$D = \sum_{a \in A} d_a$$ (3)

Eq. (2) computes the schedule displacement for a given airline $a$. The schedule displacement is defined as the time difference between the allocated and requested slot, and it has been used as a proxy of slot scheduling efficiency (Koesters, 2007; Zografos et al., 2012). Eq. (3) calculates the total schedule displacement, i.e. the displacement encountered by all airlines.

The proposed fairness metric has a reference point which represents the perfect fairness condition for an airline. Perfect fairness represents the situation in which the proportion of displacement assigned to an airline is equal to the proportion of its requests. Therefore, each airline could easily identify how fairly it has been treated.

3.3. Fairness objectives

The fairness metric ($\rho_a$) discussed in Section 3.2, expresses the fairness condition of each airline ($a$) associated with a given slot allocation outcome. A slot allocation outcome results in a total displacement ($D$) for airlines and in a displacement ($d_a$) for each airline. Based on this metric we are introducing three fairness objectives: i) minimise the maximum deviation from the absolute fairness (MMA), ii) minimise the maximum deviation from the average fairness (MMR), and iii) minimise the Gini Index (MGI). In what follows, we are providing the rational and the formal mathematical definition of these objectives.
3.3.1. Minimise the maximum deviation from the absolute fairness (MMA)

As discussed in Section 3.2, an airline is treated fairly when its fairness metric is equal to one ($\rho_a = 1.0$). Nevertheless, due to airport scheduling constraints (see Eqs. (16) and (17) in Section 3.4), it is not possible to treat all airlines with absolute fairness to achieve ($\rho_a = 1.0$). Therefore, the slot allocation outcome at a congested airport will include favoured airlines, i.e. airlines that the proportion of displacement allocated to them will be less than the proportion of their requested slots ($\rho_a < 1$); and airlines that will be disfavoured, i.e. airlines that the proportion of displacement that will be allocated to their slot requests will be more than the proportion of the requested slots ($\rho_a > 1$) (Zografos and Jiang, 2019). The value of the absolute difference between the fairness value of a given airline and the absolute fairness value indicates how favourably or dis-favourably an airline has been treated in a given slot allocation decision. The maximum value of this difference is associated with the airline that has been most or least favourably treated. Therefore, the minimisation of the maximum value of this difference (see Eq. (4)) leads to a slot allocation outcome that avoids extreme unfairness conditions. Therefore it can be used to express inter-airline fairness. This measure is appropriate and impartial. From a managerial point of view, a measure is appropriate when its equity outcome is understandable by the decision makers/stakeholders, while an equity measure is considered impartial when the resulting equity outcome does not depend on the characteristics of the recipients (Marsh and Schilling, 1994). The managerial appropriateness of the MMA measure derives from its definition, which provides to airlines and the slot coordinator to compare the fairness of each airline with the absolute fairness. Furthermore, it is impartial, since the resulting equity does not depend on the characteristics of the airlines. We call this objective "absolute fairness objective" (Zografos and Jiang, 2017; Fairbrother et al., 2019) because it expresses fairness in relation to the value representing the absolute fairness condition. The mathematical expression of this objective is provided by Eq. (4).

$$\min r_{MMA} = \min \max_{a \in A} |\rho_a - 1|$$

(4)

3.3.2. Minimise the maximum deviation from the average fairness (MMR)

In making slot allocation decisions, airlines may be interested in assessing how fairly have been treated in relation to all other airlines participating in the slot allocation process at a given airport. Therefore, the average fairness can be used as the reference point for comparing the fairness of a given airline with the fairness condition of all other airlines collectively. The maximum value of the absolute difference between the fairness of an airline and the average fairness (see Eq. (5)) indicates the airline that has received the most unequal treatment in comparison to all other airlines. The minimisation of the maximum difference leads to slot allocation decisions that avoid large deviations from the average fairness and, therefore, it can be used to express inter-airline equity. This objective provides useful information to airlines regarding their equity conditions since it allows the airlines and the slot coordinator to compare the fairness of each airline with the average fairness. Furthermore, it is impartial since the resulting equity does not depend on the characteristics of the airlines. We call this "relative fairness objective" (Zografos and Jiang, 2019) because it expresses fairness in relative terms. The mathematical expression of this objective is provided by Eq. (5).

$$\min r_{MMR} = \min \max_{a \in A} \left| \rho_a - \frac{\sum_{a' \in A} \rho_{a'}}{|A|} \right|$$

(5)

If the value of Eq. (5) becomes 0, then $\rho_a = \sum_{a' \in A} \rho_{a'}/|A|, \forall a \in A$, implying that the fairness metric values of all airlines are identical. In contrast to the absolute fairness objective, the relative fairness objective seeks to ensure that each airline has a fairness value of each airline is as close as possible to the average fairness treatment of all airlines. This fairness measure has also been used in Zografos and Jiang (2019) to investigate the efficiency-fairness trade-off in airport slot allocation.

3.3.3. Minimise the Gini index (MGI)

In addition to developing fairness objectives based on the proposed fairness metric, i.e. Eq. (1), we also adopt a well-acknowledged equity measure in socio-economic studies, the Gini index (Sen, 1973). The traditional Gini index is computed using population size and income level. We adapt the Gini index to reflect inter-airline fairness in the slot allocation context. In this context, the population under consideration is the airlines operating at a given airport, while the characteristic of the population examined is the average displacement allocated to each airline. The resulting formulation of the Gini index for the slot allocation problem is given by

$$\min r_{MGI} = \min \frac{\sum_{a \in A} \sum_{a' \in A} |d_a - d_{a'}|}{2|A| \sum_{a \in A} d_a}$$

From Eq. (6), we can derive that if the Gini index equals 0, then each airline receives the same average displacement, implying that the Gini index leads to the same fairness condition as the absolute fairness objective defined by Eq. (4). The minimisation of the Gini index leads to a slot allocation outcome that distributes the total schedule displacement as equitably as possible and, therefore, it can be used to express inter-airline equity. The Gini index based objective is normalised and takes a value between 0 and 1, and it is impartial since the resulting equity does not depend on the characteristics of the airlines.

3.3.4. Relationship among the three fairness metrics

In what follows, we examine the relationship between the fairness metrics discussed in the previous section. First, we observe that the theoretical lower bound of all three metrics is equal to zero. Furthermore, we investigate the boundary conditions between absolute and relative fairness.

Proposition: Given $r_{MMA}$, the upper and lower bounds of $r_{MMR}$ can be written as
max \left\{ r_{\text{MMA}} - \frac{1}{|A|w_{\text{min}}} - 1, 0 \right\} \leq r_{\text{MMR}} \leq 2r_{\text{MMA}} \tag{7}

where \( w_{\text{min}} = \min_{a \in A} \{|M_a|/|M|\} \) is the minimum proportion of requests of all airlines.

**Proof.** Given \( r_{\text{MMA}} \), the upper bound of \( r_{\text{MMR}} \) can be directly derived from Eq. (5).

\[
\begin{align*}
    r_{\text{MMR}} &= \max_{a \in A} \left| \rho_a - \frac{\sum_{\alpha \in A} \rho_{\alpha}}{|A|} \right| = \max_{a \in A} \left( \rho_a - 1 - \frac{\left( \sum_{\alpha \in A} \rho_{\alpha} \right)}{|A|} \right) = \max_{a \in A} \left( \rho_a - 1 + \frac{\sum_{\alpha \in A} (1 - \rho_{\alpha}) - |A|}{|A|} \right) \\
    &\leq \max_{a \in A} \left( \rho_a - 1 + \frac{\sum_{\alpha \in A} (1 - \rho_{\alpha}) - |A|}{|A|} \right) = \max_{a \in A} \left( \frac{\sum_{\alpha \in A} (1 - \rho_{\alpha}) - |A|}{|A|} \right) \tag{8}
\end{align*}
\]

\[
\begin{align*}
    &= r_{\text{MMA}} + \frac{\sum_{\alpha \in A} (1 - \rho_{\alpha}) - |A|}{|A|} \\
    &\leq r_{\text{MMA}} + \frac{\sum_{\alpha \in A} (\rho_{\alpha} - 1)}{|A|} \\
    &= r_{\text{MMA}} + \frac{\sum_{\alpha \in A} (\rho_{\alpha} - 1)}{|A|} \tag{9}
\end{align*}
\]

To derive the lower bound, we first reformulate Eq. (4) as

\[
\begin{align*}
    r_{\text{MMA}} &= \max_{a \in A} |\rho_a - 1| = \max_{a \in A} \left| \rho_a - \frac{\sum_{\alpha \in A} \rho_{\alpha}}{|A|} - \frac{\sum_{\alpha \in A} \rho_{\alpha}}{|A|} + \frac{\sum_{\alpha \in A} \rho_{\alpha}}{|A|} - 1 \right| \\
    &\leq \max_{a \in A} \left( \rho_a - \frac{\sum_{\alpha \in A} \rho_{\alpha}}{|A|} + \frac{\sum_{\alpha \in A} \rho_{\alpha}}{|A|} - 1 \right) \\
    &= \max_{a \in A} \left( \rho_a - \frac{\sum_{\alpha \in A} \rho_{\alpha}}{|A|} \right) + \max_{a \in A} \left( \frac{\sum_{\alpha \in A} \rho_{\alpha}}{|A|} - 1 \right) = r_{\text{MMR}} + \frac{\sum_{\alpha \in A} \rho_{\alpha}}{|A|} - 1 \tag{9}
\end{align*}
\]

The upper bound of the second term on the right-hand side can be derived via the following steps. To begin with, we simplify Eq. (1) as

\[
\rho_a = \frac{d_a}{w_a D} \forall a \in A \tag{10}
\]

where \( w_a = |M_a|/|M| \) is introduced to represent the proportion of requests made by airline \( a \). By substituting \( w_a \) into \( \sum_{\alpha \in A} \rho_{\alpha} \), we have

\[
\sum_a \rho_a = \sum_{a \in A} \frac{d_a}{w_a D} = \frac{1}{D} \sum_{a \in A} \frac{d_a}{w_a} \tag{11}
\]

Denote \( w_{\text{min}} = \min_{a \in A} w_a \) as the minimum proportion of requests among all the airlines, Eq. (11) can be simplified as

\[
\sum_a \rho_a = \frac{1}{D} \sum_{a} \frac{d_a}{w_a} \leq \frac{1}{D} \sum_{a} \frac{d_a}{w_{\text{min}}} = \frac{1}{D} w_{\text{min}} \sum_{a} d_a = \frac{1}{w_{\text{min}}} \sum_{a} d_a \tag{12}
\]

Therefore, the upper bound of the second term on the right hand of Eq. (9) is

\[
\left| \frac{\sum_{\alpha \in A} \rho_{\alpha}}{|A|} - 1 \right| \leq \frac{1}{|A| w_{\text{min}}} - 1 \tag{13}
\]

Substituting Eq. (13) into Eq. (9), we have
\[ r_{\text{MMA}} \leq r_{\text{MMR}} + \frac{1}{|A|w_{\text{min}}} - 1 \]
\[ \Rightarrow r_{\text{MMR}} \geq r_{\text{MMA}} - \frac{1}{|A|w_{\text{min}}} - 1 \] (14)

From the definition of the absolute value, we have \( r_{\text{MMR}} \geq 0 \). Therefore, it is concluded that the lower bound is given by
\[ \max\left\{ r_{\text{MMA}} - \frac{1}{|A|w_{\text{min}}} - 1 , 0 \right\} . \]

This completes the proof. \( \square \)

Furthermore, based on Eq. (7), it is straightforward to rearrange the equation, resulting in a proposition corresponding to the expression of the bound of \( r_{\text{MMA}} \) as a function of \( r_{\text{MMR}} \) as
\[ \frac{1}{2}r_{\text{MMR}} \leq r_{\text{MMA}} \leq \frac{1}{|A|w_{\text{min}}} - 1 + r_{\text{MMR}} \] (15)

In a nutshell, the proposition indicates that if the absolute fairness objective (MMA) is used, then it is always possible to guarantee the worst performance of the relative fairness (MMR) objective.

3.4. Bi-objective modelling framework

Using the fairness objectives described above, i.e. Eqs. (4)-(6), we develop the following mathematical framework to model the airport scheduling problem which incorporates both equity and efficiency objectives.

\[ \min_{x \in \mathcal{X}_1} Z(x) = \{ D, r_i \} \] (16)
\[ \sum_{m \in M} \sum_{c \in C} x_{mc}^{c} \leq u_c, \forall c \in C \] (17)
\[ \sum_{c \in \{0,1,2\}} x_{m_{arr}}^{c} + \sum_{c \in \{0,1,2\}} x_{m_{dep}}^{c} \leq 1, \forall \left( m_{arr}^{p,n}, m_{dep}^{p,n} \right) \in P, k \in [a^p, n) \] (18)
\[ \sum_{t \in T} x_{m}^{t} = 1, \forall m \in M \] (19)
\[ x_{m}^{t} \in [0,1], \forall t \in T, m \in M \] (20)

Objective function (16) minimises the total displacement (efficiency) defined in Eq. (3), and a fairness objective, where \( r_i \) represents one of the fairness objectives introduced in Section 3.3. Please note that the consideration of the different fairness objectives, i.e. MMA, MMR, and MGI, leads to the development of three alternative bi-objective models. Eq. (17) is the capacity constraint requiring that the declared capacity of the airport should not be violated, i.e. the number of slots allocated per time interval should be less than or equal to the maximum number of slots available during this time interval. The airport declared capacity is an input to the slot allocation process and expresses the number of aircraft movements (landings, and/or take-offs) that can be accommodated per unit time. This is a rolling constraint when considering different time intervals to express the airport’s runway capacity. For example, if there are 5 time intervals (i.e. intervals 0,1,2,3,4) and the airport runway capacity for arrivals is 15 for a period that involves 3 intervals, constraint ensures that the requests for arrivals that will be allocated to intervals \( 0,1,2 \), \( 1,2,3 \), \( 2,3,4 \) will be no more than 15. Denoting the three sets as \( C = \{ c_1, c_2, c_3 \} \), correspondingly, in constraint we set \( u_{c_1} = u_{c_2} = u_{c_3} = 15 \), \( T_1 = [0,1,2] \), \( T_2 = [1,2,3] \) and \( T_3 = [2,3,4] \). Eq. (18) expresses the aircraft turnaround time constraint, and it requires that the time interval between a slot allocated to an arrival and the slot allocated to the corresponding departure should be greater than or equal to the aircraft turnaround time. \( \left( m_{arr}^{p,n}, m_{dep}^{p,n} \right) \) represents a movement pair such that \( m_{arr}^{p,n} \) is the arrival that corresponds to departure \( m_{dep}^{p,n} \) and \( \omega^p \) is the turnaround time for movement pair \( p \), i.e. the time required for the aircraft to be prepared for its subsequent flight (which may depend on the aircraft type of the request since larger aircraft require more time to service). Eq. (19) stipulates that every request should be allocated to one and only one time-interval. Constraints - are similar to those introduced in Zografos et al. (2012) and Zografos and Jiang (2016; 2019).

3.5. Solution algorithm

The proposed formulation (16)-(20) is a nonlinear integer bi-objective model. To solve this model, we adopt the well-established \( \varepsilon \)-constraint method (Ehrgott, 2006). We convert the fairness objective into a constraint, and we parametrically change the \( \varepsilon \)-value that sets the upper bound of fairness. The \( \varepsilon \)-constraints associated with models MMR, MMA, and MGI are, respectively, given by:
\[ |p_{a} - 1| \leq \varepsilon_{\text{MMA}}, \forall a \in A \] (21)
\[
\left| \rho_a - \frac{\sum_{a^i \in A} \rho_{a^i}}{|A|} \right| \leq \varepsilon_{\text{FMMR}}, \forall a \in A
\]  

\[
\sum_{a \in A} \left| \sum_{a^i \in A} d_a^i - d_a \right| \leq \varepsilon_{\text{FMI}}
\]  

where \( \varepsilon_{\text{FMMR}}, \varepsilon_{\text{FMMR}} \), and \( \varepsilon_{\text{FMI}} \) are different \( \varepsilon \) values associated with different models. Eqs. (21)-(23) are nonlinear, because their denominators contain the decision variable \( x_m^a \). Given the non-linear nature of Eqs. (20)-(22), we transform them into linear expressions by multiplying both sides of these inequalities with their respective denominator. This operation requires that the total displacement is positive. However, this condition always holds for the case of congested airports, where the total schedule displacement is always greater than zero. The resulting equations are expressed as follows:

\[
\sum_{m \in M} \sum_{t \in T} |t - t_m^a|x_m^a \leq \varepsilon_{\text{FMMR}} \cdot \sum_{m \in M} \sum_{t \in T} |t - t_m^a|x_m^a, \forall a \in A
\]  

\[
\sum_{m \in M} \sum_{t \in T} |t - t_m^a|x_m^a \leq \varepsilon_{\text{FMMR}} \cdot \sum_{m \in M} \sum_{t \in T} |t - t_m^a|x_m^a, \forall a \in A
\]  

\[
\sum_{a \in A} \left| \sum_{a^i \in A} d_a^i - d_a \right| \leq \varepsilon_{\text{FMI}} \cdot 2|A| \sum_{a \in A} d_a
\]  

Using the preceding \( \varepsilon \)-constraints (24)-(26), the bi-objective model (16)-(20) is transformed into the following single objective model,

\[
\min_{x} Z(x) = D
\]  

Subject to: Constraints (17)-(20) and \( \varepsilon \)-constraint

Please note that three alternative bi-objective models are generated depending on which of the constraints, or is used to express in the model the slot allocation fairness.

The preceding linear integer programming model can be solved using an extension of the solution algorithm developed in Zografos et al. (2012) by incorporating the \( \varepsilon \)-constraints. The algorithm optimises hierarchically the allocation of slots starting with the allocation of the Grand Father slot requests, followed by the allocation of New Entrant and Other slot requests. This algorithm iteratively reduces the value of \( \varepsilon \) by a predefined value (i.e. 0.05) and terminates when no feasible solution can be found given certain value of \( \varepsilon \). For a detailed description of the algorithm, we refer to our previous work in Zografos and Jiang (2019).

4. A decision-making framework for selecting an acceptable slot scheduling solution

The solutions obtained by the model described in Section 3.5 generate the Pareto (efficient) frontiers of the three bi-objective models under consideration. Please recall that these models correspond to the three alternative fairness objectives discussed in Sections 3.3.1 - 3.3.3. The generated efficient frontiers describe the efficiency-fairness trade-off corresponding to different \( \varepsilon \) values used for solving each of the three proposed models. Following the generation of efficient frontiers, the decision-maker(s)/stakeholders involved in and affected by the slot allocation decisions are confronted with the following questions: Which fairness measure should be selected (i.e. which of the three alternative models)? Which point of the alternative efficient frontiers, i.e. \( \varepsilon \) value, should be selected to allocate the airport slots? The choice of the most preferable solution among the generated efficient solutions requires the involvement of the relevant decision maker(s)/stakeholders. Hanowsky and Sussman (2009) and Hanowsky (2008) discuss how the stakeholders associated with the design of Ground Holding Programmes in Air Traffic Management should be selected and incorporated in the decision-making process. In the proposed framework, we incorporate the preferences of two key stakeholders, namely the airlines, and the slot coordinator. According to the IATA decision-making process, the choice of the slot allocation solution is made by the coordinator based on the inputs provided by the airlines. In the proposed framework the airlines express their preference regarding the choice of the efficient frontier, while the slot coordinator makes the final choice using the efficient frontier that has been proposed by the airlines to select the most preferable solution. Our approach uses a utility function to identify which airlines are favoured or
disfavoured by different slot allocation outcomes, and a voting mechanism, to aggregate the preferences of the airlines regarding the
efficient frontier that should be used to select the solution that will be implemented. At a subsequent step, the coordinator selects the
solution to be implemented by selecting the solution that minimises the weighted normalised distance from the selected efficient
frontier.

4.1. Voting mechanism and airlines’ utility function

The voting rule used in this paper assumes that all airlines have equal voting rights within each category of requested slots, i.e.
Historic, New Entrant, and Other. According to the IATA World Scheduling Guidelines, the airport coordinator\(^1\) should seek a uni-
versally accepted slot allocation solution. However, if such a solution does not exist, a solution that is supported by most of the airlines
could be adopted. We call this solution an acceptable solution, meaning that the majority of the airlines are satisfied with the cor-
responding slot allocation outcome. According to the proposed framework, each airline determines its vote for any given point on the
efficiency-fairness frontiers under consideration, according to the utility function expressed by Eq. (28).

\[
u_i(e_i) = \mu_{eff}^a \left( \frac{d_i^a - d_{i0}^a}{\max_{a \in A} \{|d_i^a - d_{i0}^a|\}} \right) + \mu_{fair}^a \left( \frac{\rho_i^a - \rho_{i0}^a}{\max_{a \in A} \{|\rho_i^a - \rho_{i0}^a|\}} \right), \quad \forall a, e_i, i \in \{\text{MMA, MMR, MGI}\}
\] (28)

where \(d_i^a\) and \(\rho_i^a\) represent an airline’s efficiency and fairness values in the benchmark solution. The benchmark solution is the solution
that corresponds to the optimisation of the slot allocation by considering only the efficiency objective. Therefore, the benchmark
solution is obtained by optimising the objective function expressed by Eq. (27), subject to constraints (17)-(20). In Eq. (28), \(d_i^a\) and \(\rho_i^a\) represent the efficiency and fairness values of airline (a) corresponding to a slot allocation having a fairness value \(r_i\), \(i \in \{\text{MMA, MMR, MGI}\}\) associated with \(e_i\). According to the utility function defined in Eq. (28), each airline may assign different weights to the efficiency
and fairness objectives. The weighting parameters associated with efficiency and fairness are given by \(\mu_{eff}^a\) and \(\mu_{fair}^a\), respectively, and satisfy \(\mu_{eff}^a + \mu_{fair}^a = 1\). The proposed utility function represents a linear combination of the normalised difference between the effi-
ciency and fairness values of a given point (solution) and those values of the benchmark solution. In practical terms, depending on the
weights assigned by each airline to the efficiency and fairness objectives, the utility function expresses how much an airline is willing to
sacrifice in terms of efficiency in order to improve the fairness of the slot allocation process.

If the utility of a given airline (a) for a value \((e_i)\) is zero, i.e. \(u_i(e_i) = 0\), this means that either both the displacement and fairness
values remain unchanged or the reduction in one of the two values is compensated by the improvement of the other value, i.e. any
reduction in efficiency is compensated by a fairness gain and vice versa. However, if \(u_i(e_i) > 0 (u_i(e_i) < 0)\), then either both
displacement and fairness improve (worsen) or the deterioration in the values of one of the two objectives, i.e. displacement or
fairness, is not compensated by the improvement of the value of the other objective.

Each airline votes for selecting a point \(e_i\) on the efficient frontier associated with each category of slot requests it has made, i.e.
Historic, New Entrant, and Other. Based on the value of the utility corresponding to a given point on the Pareto frontier of each fairness
objective, the airlines are categorised into one of the following two groups:

i. **Supportive airline:** Airline a supports a decision if \(u_a \geq 0\). The set of supportive airlines is denoted as \(S := \{a|u_a \geq 0, a \in A\}\);

ii. **Opposing airline:** Airline a opposes to a decision if \(u_a < 0\). The set of opposing airlines is denoted as \(O := \{a|u_a < 0, a \in A\}\).

Subsequently, the points on the efficient frontier are classified into two categories:

i. **Acceptable Points (decisions):** if the number of supportive airlines is larger than or equal to the number of opposing airlines, i.e. \(|S| \geq |O|\), then we call it as an acceptable point.

ii. **Unacceptable Points (decisions):** if the number of supporting airlines is smaller than the number of opposing airlines, i.e. \(|S| < |O|\), then we call it as an unacceptable point.

4.2. Most preferable solution

The most preferable decision \(e^*_1\) is the solution having the minimum normalised weighted distance to the Pareto frontier. The rationale
underlying the proposed definition of the most preferable acceptable decision is that the decision-maker ideally will seek to
achieve a Pareto optimum solution. However, as demonstrated in the case study (see Section 5), a Pareto solution may not be an
acceptable solution. In this case, the decision-maker will seek a solution that is as close as possible to the Pareto frontier.

Denote the set of \(e_i\) values on the Pareto frontier as \(\Psi^P\), then mathematically, the most preferable acceptable decision is given by,

\(^1\) The organization or individual responsible for slot allocation at a Level 3 airport. A coordinator is appointed to allocate slots to airlines and other
aircraft operators using or planning to use the airport as a means of managing the airport declared capacity (IATA, 2019b)
where $w^{\text{eff}}$ and $w^{\text{fair}}$ are the weighting parameters associated with the importance assigned by the decision-maker, i.e. the coordinator, to the efficiency and fairness objectives respectively. These weights satisfy $w^{\text{eff}} + w^{\text{fair}} = 1$. $D(\tau)$ and $r(\tau)$ are the efficiency and fairness objectives on the Pareto frontier. The denominator in Eq. (29) represents the difference between the maximum and minimum values for the objective considered. Therefore, the values in the bracket are normalised values between 0 and 1. This normalisation method is a widely used technique in multi-objective optimisation (Marler and Arora, 2004). Eq. (29) identifies the acceptable point that has the closest distance to the Pareto frontier. If $\epsilon_i^*$ is on the Pareto frontier, then the distance is equal to zero. Nevertheless, since a Pareto optimal solution may not always be an acceptable solution, $\epsilon_i^*$ may not necessarily lie on the Pareto frontier. Finally, in Step 4, once all $\epsilon_i^*$ are determined, the decision-maker selects the $\epsilon_i^*$ that is associated with the maximum number of supportive airlines. Following the above exposition, the process for determining the most acceptable slot allocation outcome is summarised as follows:

Step 1. Solve the bi-objective models MMA, MMR, and MGI using the ε-constraint method (see Sections 3.4 and 3.5).

Step 2. Determine the set of $\epsilon_i$ that results in acceptable solutions, i.e. solutions for which the number of supportive airlines is no less than that of opposing airlines. Denote the set as $\Omega_i$ for each fairness model $i \in \{\text{MMA, MMR, MGI}\}$.

Step 3. Identify $\epsilon_i^*$ from $\Omega_i$, where $\epsilon_i^*$ is the $\epsilon$ that results in the most preferable acceptable decision for fairness model $i \in \{\text{MMA, MMR, MGI}\}$. This is computed in Eq. (29).

Step 4. Find the fairness model and corresponding $\epsilon_i^*$, which leads to the maximum number of $|S| - |O|$.

5. Case study

The decision-making framework presented in Section 4 was used to determine the most preferable acceptable slot allocation outcome for allocating slots at a coordinated airport. The airport’s declared capacity data are provided in Table 1. For the summer scheduling season, we are using 439 slot requests.

For the ε-constraint method, the initial value of $\epsilon$ is set at 5.0 and the predefined step size is set at 0.5 for model MMA and MMR. For model MGI, the initial value of $\epsilon$ is set at 1.0, and the steps size is 0.01. The initial $\epsilon$ values are set to be sufficiently large to ensure that a feasible solution to the bi-objective optimisation problem exists. Furthermore, the initial $\epsilon$ values and step sizes are determined with the intention of generating the same number of solutions (i.e. 5/0.5 = 1.0/0.1 = 10) for all three models. The integer linear programming problem was solved by CPLEX 12.62. All the tests were run on a desktop with 32 GB RAM and Intel(R) Xeon(R) CPU E5264 V3@2.6 GHz.

5.1. Generating efficient frontiers for alternative fairness objectives

In this section, we are comparing the solution obtained from a single objective model that only minimises efficiency, i.e. the benchmark solution, without considering any fairness objectives with the solutions obtained from the three bi-objective models that consider fairness. The formulation of the single objective problem is obtained by considering only the displacement objective ($D$) in Eq. (16) of the model described by Eqs. (17)-(20). The solution of the single objective model is given in Table 2, the values of the three fairness objectives associated with the optimum value of the single objective model, are computed according to the expressions of the three fairness metrics included in Eqs. (4), (5), and (6) (see Section 3.3). Fig. 1 plots the Pareto frontiers of the three bi-objective models. The x-axis represents the efficiency objective, while the y-axis is the fairness objective corresponding to each model. For each model, three curves are plotted to represent the Pareto frontier for each slot request category, i.e. Historic, New Entrant, and Other. In each figure, the triangle mark stands for the solution obtained from the single objective model. The results depicted in Fig. 1 suggest that the Pareto frontier is neither continuous nor necessarily convex; this is exemplified by the frontier of Other slot requests. The observed discontinuity is the outcome of the integer nature of the formulation. The non-convexity for the Other slot requests is probably due to the hierarchical nature of the solution procedure. Please note that in the hierarchical solution procedure, the solution space for the category of Other requests depends on the solutions obtained for the Historic and New Entrant requests. It is also worth

<table>
<thead>
<tr>
<th>Movement type</th>
<th>Duration (min)</th>
<th>Capacity (No. of movements)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total movements</td>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>Total movements</td>
<td>60</td>
<td>10</td>
</tr>
<tr>
<td>Arrivals</td>
<td>60</td>
<td>4</td>
</tr>
<tr>
<td>Departures</td>
<td>60</td>
<td>6</td>
</tr>
</tbody>
</table>
noting that in certain instances, i.e. MMA and MGI Other requests, the Pareto frontiers contain points that provide better displacement values than the solution of the corresponding single objective problem (indicated with the green triangle). This is due to the fact that the corresponding bi-objective problems are optimised hierarchically, i.e. we first generate the Pareto Frontier for the Historic, followed by the Pareto frontiers of New Entrants and Others respectively. As a consequence of the hierarchical optimisation, the points that correspond to the optimum displacement (the leftmost point) on the MMA and MGI Other graphs, may correspond to points on the Pareto frontier of MMA and MGI New Entrants that have more displacement. Therefore, accepting more displacement for the New Entrants leads to an improved solution for the Other requests. To better illustrate the effect of the hierarchical solution of the bi-objective problem, we consider as an example, the MMA case graphs depicted in Fig. 1 (a). The optimum solution for the Historic (first level of the hierarchy) is represented by the left most point on the Parato frontier and is obtained for $\varepsilon = 1.1$. The solutions obtained using the same value of $\varepsilon$ for the New Entrant and Others are shown on the graphs of Fig. 1(a). Please note that the point that corresponds to $\varepsilon = 1.1$ in the New Entrants graph leads to a higher displacement value ($D = 148$) than the point indicated with the green triangle in the same graph ($D = 80$). The consideration of more displacement for the MMA New Entrants ($D = 148$ vs $D = 80$) leads to a smaller displacement for MMA Other. Another conclusion emerging from Fig. 1 is that the trade-off between the efficiency and fairness objectives always exists, regardless of the fairness objective used and the category of slot requests, meaning that any gain in fairness would always be accompanied by a loss in efficiency and vice versa.

Table 2
Single Objective (Minimisation of Total Displacement) Slot Allocation.

<table>
<thead>
<tr>
<th></th>
<th>Historic</th>
<th>New Entrant</th>
<th>Other</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>122</td>
<td>80</td>
<td>1057</td>
<td>1259</td>
</tr>
<tr>
<td>Computed MMA</td>
<td>1.55</td>
<td>2.74</td>
<td>4.96</td>
<td>–</td>
</tr>
<tr>
<td>Computed MMR</td>
<td>2.08</td>
<td>2.70</td>
<td>4.79</td>
<td>–</td>
</tr>
<tr>
<td>Computed Gini</td>
<td>0.74</td>
<td>0.71</td>
<td>0.59</td>
<td>–</td>
</tr>
</tbody>
</table>

Fig. 1. Pareto frontiers of the three models.
Fig. 1. (continued).
Fig. 2. Preferable acceptable decisions obtained from the three models.
5.2. Determining the most preferable acceptable slot allocation

This experiment is designed to illustrate how the most preferable acceptable decision is identified based on the decision-making framework proposed in Section 4. In this experiment, we set $\mu^\text{fair}_a = 0.3$ and $w^\text{fair} = 0.5$ to represent a scenario in which the fairness objective is moderately important to each airline (See Eq. (28)), but equally important to the coordinator (see Eq. (29)). The results are presented in Fig. 2 and Table 3. In Fig. 2, the x-axis represents the value of displacement, while the y-axis represents the value of the fairness objective. The blue line represents the Pareto frontier. The bar chart indicates the difference between the number of supportive airlines and that of opposing airlines corresponding to the Pareto frontier. If the value of the bar chart is positive, it means that the number of supportive airlines is higher than that of opposing airlines and the corresponding point on the Pareto frontier is acceptable.

In each graph, the most preferable acceptable decision, the acceptable solution with the minimum normalised weighted distance to the Pareto frontier (see Section 4), is marked by a triangle. The number in brackets next to the triangle represents the value of displacement, fairness objective, and the difference between the number of supportive airlines and that of opposing airlines, respectively.

For all three models, it is apparent that not all the points on the Pareto frontiers are acceptable. For example, in MMA Historic plot, the rightmost point on the Pareto frontier is associated with a negative blue bar, meaning that the number of opposing airlines is larger than that of supportive airlines. Furthermore, the number of acceptable points varies among the three models, represented by the

Table 3
Acceptable decisions obtained from the three models.

<table>
<thead>
<tr>
<th></th>
<th>Historic</th>
<th>New Entrant</th>
<th>Other</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model MMA</td>
<td>Displacement</td>
<td>122</td>
<td>140</td>
<td>1575</td>
</tr>
<tr>
<td></td>
<td>Value of MMA</td>
<td>1.07</td>
<td>1.14</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>S</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Model MMR</td>
<td>Displacement</td>
<td>123</td>
<td>92</td>
<td>1190</td>
</tr>
<tr>
<td></td>
<td>Value of MMR</td>
<td>1.88</td>
<td>1.98</td>
<td>2.05</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>S</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Model MGI</td>
<td>Displacement</td>
<td>125</td>
<td>97</td>
<td>1052</td>
</tr>
<tr>
<td></td>
<td>Value of Gini</td>
<td>0.49</td>
<td>0.45</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>S</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>
different number of positive blue bars in the figures. Following the framework proposed in Section 4, it is found that the most preferable acceptable decision points are on the Pareto frontier for the New Entrant and Other slot requests in models MMA and MMR. This means that, for the two models, the most preferable acceptable decisions are Pareto optimum solutions for New Entrant and Other slot requests, while this is not the case for the Historic slot requests.

Table 3 presents the most preferable acceptable decision obtained from the three models. We observe that, based on the decision-making framework described in Section 4, the most preferable acceptable solution corresponds to the MMR model. This is because the solution that attracts the maximum number of supporting airlines (i.e. 27) is generated by this model. It is worth noting that the number of supportive airlines varies among the different slot request categories. For example, in the MMR model, the difference between the number of supportive airlines and opposing airlines is 16 for Historic slot requests, while the number is only 4 and 7 for the New Entrant and Other slot requests, meaning that the fairness is perceived differently within each category.

As discussed in Section 4, the value of the utility function, expressed by Eq. (28) that determines the vote of the airlines in relation to a given slot allocation outcome, depends on the relative weight that each airline assigns to efficiency and fairness objectives. Furthermore, the final selection of the preferred solution among all the acceptable solutions, as determined by Eq. (29), depends on the relative importance assigned by the decision-maker (coordinator) to the two objectives. Therefore, it is crucial to study the sensitivity of the final slot allocation outcome, i.e. the selection of the slot allocation solution that will be agreed upon to be implemented, in relation to these parameters. In Sections 5.3 - 5.4, we are further experimenting with the values of these parameters in order to investigate their effect on the selection of the most preferable acceptable slot allocation outcome.

5.3. Effect of $\mu_{a}^{\text{fair}}$ on the number of acceptable solutions

This experiment is designed to illustrate the effect of airlines’ preferences regarding fairness. This is done by changing the value of the weighting parameter, $\mu_{a}^{\text{fair}}$. Please recall, that each acceptable solution corresponds to one $\epsilon$ value that results in more supportive airlines than opposing ones. Therefore, the value of $\mu_{a}^{\text{fair}}$ influences the number of acceptable solutions in the sense that it affects airlines’ utility function defined in Eq. (28). It is this utility function which determines whether an airline is supporting a given slot allocation outcome or not. If the number of supportive airlines is higher than that of opposing airlines, the solution is an acceptable solution. In this test, for simplicity, the value of $\mu_{a}^{\text{fair}}$ was set to be identical for all airlines and varied from 0 to 1. Fig. 3 illustrates the number of acceptable solutions corresponding to each $\mu_{a}^{\text{fair}}$ of all three models.
Fig. 3 shows that the number of acceptable decisions varies among the three models corresponding to the three different fairness objectives. In general, the number of acceptable solutions changes with respect to $\mu_{fair}^a$ in a stepwise fashion. Please observe, that as the value of $\mu_{fair}^a$ becomes larger, more solutions obtained from the bi-objective model become acceptable. Interestingly, the number of acceptable solutions drops when $\mu_{fair}^a$ approaches 1 in the MMA and MGI models. This is attributed to the trade-off between fairness and efficiency. The fairness improves at the cost of efficiency. For some airlines, with the increase in $\mu_{fair}^a$, their gain in fairness cannot compensate the loss in efficiency; therefore, they turn into opposing airlines, causing an acceptable solution to become an unacceptable solution.

To examine the effects of $\mu_{fair}^a$ in determining which fairness metric should be adopted, Fig. 4 is plotted, in which $\mu_{fair}^a$ was varied from 0.1 to 0.9, and the corresponding $|S| - |O|$ is plotted. The three curves fluctuate, and there does not exist any model that always dominates the others. This means that there does not exist a best fairness objective irrespective of the value of $\mu_{fair}^a$.

5.4. Effect of $w_{fair}^a$ on determining the most preferable acceptable slot allocation

In this section, we study the effect of weighting parameter, $w_{fair}^a$, on determining the most preferable acceptable point. The value of this weighting parameter represents the relative importance of fairness over efficiency. It applies when multiple slot-allocation solutions are indifferent with regards to the number of supporting/opposing airlines (i.e. solutions that have the same number of supporting airlines), and the decision-maker should select one of these indifferent solutions to be implemented. In the reported tests, the value of $w_{fair}^a$ was increased by 0.1 from 0.1 to 0.9. To be consistent with the tests in Fig. 2, $\mu_{fair}^a$ is fixed at 0.3. Table 4 reports the most preferable acceptable decisions for all models using different values of $w_{fair}^a$. In the brackets, the first number is the value of $w_{fair}^a$. 

<table>
<thead>
<tr>
<th>$w_{fair}^a$</th>
<th>Model MMA</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Historic</td>
<td>New Entrant</td>
<td>Other</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1–0.9</td>
<td>(122, 1.07)</td>
<td>(140, 1.14)</td>
<td>(1575, 1.15)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w_{fair}^a$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Model MMR</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Historic</td>
<td>New Entrant</td>
<td>Other</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1–0.8</td>
<td>(123, 1.88)</td>
<td>(92, 1.98)</td>
<td>(1190, 2.05)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>(122, 1.69)</td>
<td>(171, 1.66)</td>
<td>(1230, 1.40)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w_{fair}^a$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Model MGI</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Historic</td>
<td>New Entrant</td>
<td>Other</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.10–0.9</td>
<td>(125, 0.49)</td>
<td>(97, 0.45)</td>
<td>(1052, 0.49)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
displacement, while the second number is the value of the fairness objective under consideration. For models MMA and MGI, \( w_{\text{fair}} \) does not affect the most preferable acceptable decision at all. This is because all preferable acceptable points have identical displacement and fairness values. In MMA model, the most preferable acceptable decision only changes when \( w_{\text{fair}} \geq 0.9 \). These results suggest that \( w_{\text{fair}} \) has a minor effect in determining the most preferable acceptable decision. This is because in our case following the majority rule, a unique optimal slot allocation scheme that has the maximum number of supportive airlines usually can be obtained. Therefore, the slot allocation solution to be implemented is determined regardless of the value of \( w_{\text{fair}} \).

5.5. Efficient frontiers for the non-hierarchical

The results discussed so far refer to the hierarchical generation of efficiency-fairness frontiers. This approach incorporates the IATA guidelines, which suggest that there is a hierarchy for satisfying slot requests. According to this approach, a higher priority is assigned for the satisfaction of slot requests which have historical slot usage rights over New Entrant and Other slot requests. In this section, we investigate a non-hierarchical slot allocation approach in order to generate the efficient frontiers corresponding to the three fairness objectives under consideration, i.e. MMA, MMR, Gini. The non-hierarchical approach considers all slot requests simultaneously and therefore generates a single efficient-frontier (for all requests) for each fairness objective considered. For a detailed discussion regarding the generation of the non-hierarchical scenario, we refer to Zografos and Jiang (2019). We have used the non-hierarchical approach to generate the efficient frontiers when the MMA, MMR and Gini fairness objectives are used in the bi-objective efficiency-fairness model. The resulting efficient frontiers are presented in Fig. 5. These frontiers can be used, similarly to the hierarchical case presented in Sections 5.2 - 5.4, in the proposed framework, to select the most preferable solution.

6. Concluding remarks

In this paper, we investigated three fairness objectives, namely, absolute fairness, relative fairness, and the Gini index, for modelling inter-airline fairness in the airport slot scheduling problem. We also investigated the relationship between the absolute and relative fairness objectives, and we proved that given the value of the absolute fairness objective, the bound of relative fairness
objective can be derived. The three fairness objectives were incorporated into a bi-objective modelling framework, resulting in three alternative bi-objective efficiency-fairness formulations. The solution of the three bi-objective efficiency-fairness formulations led to the generation of three alternative Pareto frontiers. A voting mechanism, based on the majority rule, was used to identify the most preferable Pareto frontier. The proposed decision-making framework was applied to a setting resembling real world airport demand and supply conditions. For the case study under consideration (when the importance assigned by all airlines to fairness was 30%, i.e. $\mu_0^{fair} = 0.3$, and the importance assigned to fairness by the slot coordinator was 50%, i.e. $w_{air}^{fair} = 0.5$), it was found that the bi-objective formulation that involves the minimisation of the maximum deviation from the average fairness yields the most preferable acceptable solution. However, in general, the selection of the most preferable acceptable slot allocation solution depends on the relative importance that the airlines and the coordinator assigned to the conflicting efficiency and fairness objectives. Therefore, we performed extensive numerical experiments to investigate the sensitivity of the choice made in relation to the parameters expressing the relative importance of efficiency and fairness for both the airlines and the coordinator. The results of the sensitivity analysis for the case under consideration suggest that irrespectively of the fairness objective considered the number of acceptable slot allocation solutions increases as the importance assigned to fairness increases. It was also found, ceteris paribus, that the choice of the most preferable acceptable solution is not very sensitive to the importance assigned by the slot coordinator to the fairness objective.

In this paper, we have used a voting mechanism that allocates a single vote to each airline regardless of the airline characteristics. Furthermore, the proposed mechanism uses an additive majority rule to decide on the outcome of the voting. Future research may investigate the use of alternative vote allocation and vote decision rules (e.g. Yan et al., 2017). Furthermore, in this research, we have used the MMR, MMA, and MGI objectives to express inter-airline fairness. Given that the application of the proposed framework is not limited by the efficiency and fairness objectives investigated in this paper, it is useful to investigate the inclusion of fairness metrics that distinguish between peak and off-peak slot requests as in Fairbrother et al. (2019), or fairness measures that incorporate schedule displacement costs (Pellegrini et al., 2017). Another possible extension of our research relates to the potential enhancement of the utility function of the airlines. In the proposed framework, the airline utility function depends on the sum of the weighted normalised difference of the efficiency and fairness values of a given solution from the efficiency and fairness values of a benchmark solution, i.e. the solution that corresponds to optimum efficiency. However, airlines’ utility may depend on a host of factors that relate to their business model, commercially strategy, operational efficiency, and costs. Unveiling or better approximating the airline utility function could be a challenging research direction that has the potential to improve slot allocation decisions. Furthermore, the proposed framework currently incorporates the views of two stakeholders, i.e. airlines and slot coordinator, identifying and incorporating the preferences of other slot allocation stakeholders (Hanowsky, 2008; Hanowsky and Sussman, 2009) will contribute to the improvement of the decision-making framework.

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References


