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Current Distribution on Parallel Conductors for Planar Transformers at High Frequency

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Abstract—In high frequency and high current applications, use of parallel winding connection is becoming common practice in planar transformers. However, the current distribution on each parallel conductor is hard to predict. This paper investigates the effect of the insulation thickness and frequency on the current distribution of parallel conductors. When the copper thickness is much larger than the skin depth, the current on each parallel conductor becomes constant. Different layers have varying insulation thicknesses, which is often observed when printed circuit boards (PCB) are fabricated. This also result in unbalanced currents. Based on a 1D model, this paper proposes a methodology for determining the parallel current distribution, specifically used to the $N:1$ transformer when the copper thickness is much larger than the skin depth. No complex calculation algorithm or simulation software is required. Both finite element analysis (FEA) and theoretical model support the effectiveness of the proposed methodology, which can be used for many parallel structures. The experiment further verifies the theoretical analyses: the current on each parallel conductor is always constant at high frequency; the insulation thickness affects the current distribution.

Index Terms—High frequency, planar transformers, parallel current distribution.

I. INTRODUCTION

Planar transformers have been widely adopted in many switched-mode power supplies to achieve both high efficiency and high power density, because they are recognized for their low profile, excellent repeatability of construction, ease of manufacture and predictable parasitic components [1]-[6].

In high current high frequency (up to MHz) applications, a low output voltage tends to be required for many DC/DC power conversions, such as data centers and telecoms [7]-[10]. The secondary winding usually has fewer number of turns and sometimes only a single turn. Connecting several conductors in parallel can handle the large secondary current. This is recognized as common practice in order to reduce the DC current density. Furthermore, the parallel connections also allow the implementation of interleaved winding layouts to reduce the winding loss at high frequency [11]-[13].

However, the current on each parallel conductor is usually unbalanced, as discussed in [14]. Fig.1 shows a 2:1 transformer with three different winding layouts. 2 layers are connected in parallel to form one turn secondary winding. The ANSYS 2D simulation illustrated different current distributions on parallel conductors. Even the interleaved winding arrangements yield different current distributions.

Much excellent research has been carried out to investigate the current distribution in parallel conductors. FEA simulation tool is applicable, but extremely time-consuming for optimal

design. A number of studies [15]-[18] compare several parallel winding layouts via FEA simulation, but no mathematical model or analytical solution is given. Previous modeling efforts offer several analytical approaches to determine current sharing among parallel conductors [19]-[24]. Paper [23] proposes a systematic approach to predict AC resistance and inductance based on a lumped circuit model. Literature [24] presents a mathematical model to calculate the current on each parallel conductor. However, both of these methods require complex calculations for multiple turns in parallel. The current distribution at high frequency has not been investigated.

This paper provides insight into the current distribution on parallel conductors at high frequency. The analysis of the effect of insulation thickness and frequency in this paper reveals that the current on every parallel conductor does not change with frequency when the copper thickness h is much larger than the skin depth δ_w , and the interleaved winding layouts with different insulation thicknesses may result in a loss of the advantage of good current sharing.

Based on a 1D model, this paper proposes a straight forward methodology consisting of three rules for determining the current on each parallel conductor, specifically used for the $N:1$ transformer when $h \gg \delta_w$. This method is not limited to fully interleaved winding layout, but can be applied to many parallel connections. No complex calculation algorithm or simulation tool is required. This methodology is applied to a fully interleaved winding arrangements to determine the parallel current distribution. FEA simulations support the validity of the proposed approach. The calculated AC resistance demonstrate excellent agreement with the FEA simulated AC resistance.

The experiment results further prove the validity of theoretical analyses: when $h \gg \delta_w$, the current on each parallel conductor does not change with frequency; the insulation thickness affects the current distribution.

II. PARALLEL CURRENT DISTRIBUTION PRINCIPLE

The method to predict the current distribution of the $N:1$ planar transformer is proposed in the subsequent part based on the following assumptions and constraints:

1. The transformer turn ratio is $N:1$ with primary windings connected in series and secondary windings connected in parallel;
2. Each layer has the same copper thickness h which is much larger than the skin depth δ_w ;
3. Conductors are closely spaced and the spacing (or the insulation thickness) is identical for every layer;
4. Edge and end effect of the conductors are not considered;

Fig.1 Three winding parallel schemes of a turn ratio 2:1 planar transformer with primary layers series connected and secondary layers parallel connected

Fig.2 Current distribution as a function of frequency sweeping from 10kHz to 5MHz for (a) Case I (b) Case II and (c) Case III

5. The magnetic core is ideal, i.e., $\mu_c = \infty$ and $\rho_c = \infty$, where μ_c and ρ_c are the relative permeability and the resistivity of the magnetic core, respectively;
6. A sinusoidal current flows through primary conductors:

$$i_p(t) = I_p \sin(\omega t) \quad (1)$$

The method consisting of three rules are given as follows:

- Rule 1: Primary and secondary currents attract each other. The current carried by the primary conductors is always equally divided to the top and bottom surfaces when the primary conductors are sandwiched by two secondary conductors. If primary conductors only have one secondary neighbor, all primary current tends to the surface adjacent to the secondary conductor.
- Rule 2: Currents carried by adjacent primary/secondary windings repel each other. If the parallel secondary conductors are placed in adjacent layers, the current on each conductor is drawn to the surface next to the primary windings, while no current is on the other surface contiguous with other secondary conductors. For primary winding, the current on each conductor tends to the surface adjacent to the secondary winding.
- Rule 3: The current on each secondary conductor is the sum of the primary current adjacent to its two surfaces.

TABLE I
WINDING DETAILS

Parameters	Values
Copper thickness (h)	70 μ m
Turn ratio ($n:I$)	2:1
Insulation thickness (t_h)	0.3mm
Window width (b)	9.275mm
Resistivity (ρ)	$1.71 \times 10^{-8} \Omega \cdot m$
Air permeability (μ_o)	$4\pi \times 10^{-7} H/m$
Magnetic core	E32/6/20
Mean turn length (MTL)	20.32mm

Three parallel winding layouts are shown in Fig.1 to support the proposed method. The primary conductors P_1 and P_2 are connected in series. A sinusoidal current with 1A peak is

injected to each primary conductor. The secondary conductors S_1 and S_2 are in parallel. As seen from the current density distribution made by ANSYS 2D simulation, the current on each parallel conductor is different.

TABLE I gives details about the winding arrangements. Fig.2 illustrates the current distributed on S_1 and S_2 as a function of the frequency sweeping from 10kHz to 5MHz. From above 884kHz where $h = \delta_w$, the two parallel conductors S_1 and S_2 always carry the constant current independent of frequency. In Case I, the S_2 conducts around 2A, while the S_1 has nearly zero current. In Case II, the S_2 conducts around 1A, which is almost identical to the S_1 . In Case III, the S_1 and the S_2 carry about 1.5A and 0.5A, respectively. At low frequency, the currents in parallel conductors are changing with frequency for both Case I and Case III. Constant current is observed in Case II at any frequency.

Fig.3. Modelling of two parallel layers

The current distribution is analyzed in the subsequent parts based on the same assumptions given initially in this Section. The model is illustrated in Fig.3. The current distribution in parallel conductors can be derived by Faraday's law and Kirchhoff's law. The magnetic flux Φ_{12} flowing through the space between conductors S_1 and S_2 can be written as functions of the current densities (J_1 and J_2) on the surfaces. Assume the primary windings perfectly couple with secondary windings. The sum of the secondary currents is thus $2i_p(t)$, such that

$$\underbrace{\rho l}_{\Omega \cdot m^2} \underbrace{(J_1 - J_2)}_{A/m^2} = \underbrace{j\omega \Phi_{12}}_V \quad (2)$$

$$i_1(t) + i_2(t) = 2i_p(t) \quad (3)$$

where ρ is the resistivity of the conductor; l is the conductor mean turn length (MTL); J_1 is the current density along the top surface of the conductor S_1 ; J_2 is the current density along the bottom surface of the conductor S_2 ; ϕ_{12} is the magnetic flux going through the space between two parallel conductors; $i_1(t)$ and $i_2(t)$ are the currents conducted by S_1 and S_2 , respectively. J_1 , J_2 and ϕ_{12} can be expressed by the induced magnetic field H , which is expressed by $i_1(t)$ and $i_2(t)$. Combining (2) and (3), the current distributions on parallel conductors are solved.

Fig.4. Boundary conditions for i^{th} layer of a transformer

The magnetic field inside the conductor is described by the Helmholtz equation.

$$\frac{d^2 H_{s2}(x)}{dx^2} = \gamma^2 H_{s2}(x) \quad (4)$$

The complex propagation constant γ is

$$\gamma = \frac{1+j}{\delta_w} \quad (5)$$

The skin depth δ_w is given by

$$\delta_w = \sqrt{\frac{\rho}{\pi f \mu_o}} \quad (6)$$

where f is the frequency and μ_o is the air permeability.

The center of the coordinate system is located in the center of the core, as shown in Fig.4, where x_{in} and x_{im} represent the distance from y axis to the top and bottom surfaces of the i^{th} conductor, respectively. From the boundary conditions H_{im} and H_{in} , the magnetic field $H_i(x)$ inside the i^{th} conductor is given by

$$H_i(x) = \frac{H_{in} \sinh(\gamma(x - x_{im})) - H_{im} \sinh(\gamma(x - x_{in}))}{\sinh(\gamma h)} \quad (7)$$

The current density for the i^{th} conductor $J_i(x)$ is

$$J_i(x) = -\frac{dH_i(x)}{dx} \quad (8)$$

Based on the magnetic field in each insulation layer given in Fig.1, the magnetic flux ϕ_{12} for each case can be determined. In Case I, the magnetic flux ϕ_{12} is

$$\phi_{12} = u_o l t_h \cdot H_{s2}(x_{s2m}) \quad (9)$$

In Case II, the magnetic flux ϕ_{12} is

$$\phi_{12} = u_o l t_h \cdot H_{s2}(x_{s2m}) \quad (10)$$

In Case III, the magnetic flux ϕ_{12} is

$$\phi_{12} = u_o l (t_h \cdot H_{s2}(x_{s2m}) + t_h \cdot H_{s1}(x_{s1n})) \quad (11)$$

where t_h is the insulation thickness; $H_{s2}(x_{s2m})$ and $H_{s1}(x_{s1n})$ are magnetic fields in the insulation as given in Fig.1. $H_{s2}(x_{s2m})$ is the magnetic field along the bottom surface of the conductor S_2 and $H_{s1}(x_{s1n})$ is the magnetic field along the top surface of the

conductor S_1 . Combining (2), (3) and (7) to (11), $i_1(t)$ and $i_2(t)$ for each case are solved. For Case I, the expressions for $i_1(t)$ and $i_2(t)$ can be found in the appendix (A.1) and (A.2). For Case II, the expressions for $i_1(t)$ and $i_2(t)$ are

$$\begin{cases} i_1(t) = I_p \sin(\omega t) \\ i_2(t) = I_p \sin(\omega t) \end{cases} \quad (12)$$

For Case III, the expressions for $i_1(t)$ and $i_2(t)$ are

$$i_1(t) = I_p \left(\frac{jwu_o t_h \left(\sin(\omega t) - \frac{1}{2} \right) \cdot \sinh(\gamma h)}{jwu_o t_h \cdot \sinh(\gamma h) - \gamma \rho \cdot \cosh(\gamma h)} - \frac{\gamma \rho \left(\left(\sin(\omega t) - \frac{1}{2} \right) \cdot \cosh(\gamma h) + \frac{1}{2} \right)}{jwu_o t_h \cdot \sinh(\gamma h) - \gamma \rho \cdot \cosh(\gamma h)} \right) \quad (13)$$

$$i_2(t) = I_p \left(\frac{\left(\sin(\omega t) + \frac{1}{2} \right) \cdot jwu_o t_h \cdot \sinh(\gamma h)}{jwu_o t_h \cdot \sinh(\gamma h) - \gamma \rho \cdot \cosh(\gamma h)} - \frac{\gamma \rho \left(\left(\sin(\omega t) + \frac{1}{2} \right) \cdot \cosh(\gamma h) - \frac{1}{2} \right)}{jwu_o t_h \cdot \sinh(\gamma h) - \gamma \rho \cdot \cosh(\gamma h)} \right) \quad (14)$$

Fig.5. Comparison between the simulated and calculated current distribution for (a) Case I and (b) Case III

A. Frequency effect on parallel current

Fig.5 shows I_1 and I_2 as a function of the frequency for Case I and Case III. The calculated instants I_1 and I_2 are peak currents. There is a minor gap between the calculation and simulation, because we assume the primary winding perfectly couples with secondary winding. The theoretical results match the simulation: I_1 and I_2 are constant at high frequency. Case II is not given as the current on each parallel conductor is obviously constant from (12).

Fig.6 shows the simulated magneto motive force (MMF) for Case II from 100kHz to 5MHz. Following Ampère's circuital law which links the magnetic field to current, Fig.6 indicates that currents carried by adjacent primary and secondary conductors attract each other. This is primarily because the primary and secondary windings conduct currents in opposite directions. Considering the proximity effect and skin effect, it is reasonable to observe the current tend to surface next to the other side windings. Conversely, currents carried by two adjacent secondary conductors repel each other, as they are in the same direction.

Fig.6. MMF distribution of Case II

Fig.7. Coordinate system for P2 in Case III

Fig.8. Absolute H field distribution inside P_2 in Case III

It is also worthy to prove that at high frequency the primary current is always equally divided to the top and bottom surfaces when the primary conductor is sandwiched between the two secondary conductors, as shown in Case III. Fig.7 shows the boundary conditions for the conductor P_2 at $wt = \pi/2$. The center of the coordinate system is located in the middle of the P_2 left side. Based on the magnetic field general expression given in (7), the magnetic field H_{P2} in the conductor P_2 is

$$H_{P2} = \frac{(I_p - I_2) \cdot \sinh\left(\frac{\gamma(h-2x)}{2}\right) + I_1 \cdot \sinh\left(\frac{\gamma(2x+h)}{2}\right)}{b \sinh(\gamma h)} \quad (15)$$

Fig.8 illustrates the absolute $|H_{P2}|$ distribution in conductor P_2 at different frequencies. The H fields are almost identical at two boundaries, $x = -35 \mu\text{m}$ and $x = 35 \mu\text{m}$. Hence, following Ampère's circuital law, the primary current is always equally distributed to the two surfaces of the conductor when $h \gg \delta_w$, as long as the winding layouts satisfy the assumptions and constraints mentioned initially in this section. The sandwiched primary conductor set the two boundary conditions of their two neighboring secondary conductors. The other two boundary

conditions of the secondary conductors is determined by the other conductors which has no effect on the current distribution of the sandwiched primary conductor.

Fig.9. Case III with different insulation

Fig.10. I_1 and I_2 versus insulation ratio t_t/t_b with $t_t+t_b=0.6\text{mm}$ for Case III

B. Insulation effect on parallel current distribution

In the above analysis, the insulation thickness t_h is assumed to be 0.3mm for each layer. However, many PCBs' insulation thickness varies from layer to layer due to the fabrication technique. This affects the parallel current distribution dramatically.

Cases III involves two insulation layers to calculate the flux ϕ_{12} between two parallel secondary conductors. Assume the two insulation layers are now different, as shown in Fig.9, then the magnetic flux ϕ_{12} becomes

$$\phi_{12} = u_o l \left(t_i \cdot H_{S2}(x_{S2n}) + t_b \cdot H_{S1}(x_{S1n}) \right) \quad (16)$$

Expressions for $i_1(t)$ and $i_2(t)$ change to

$$i_1(t) = I_p \left(\frac{jwu_o t_i \cdot (2 \sin(wt) - 1) \cdot \sinh(\gamma h)}{jwu_o (t_b + t_i) \cdot \sinh(\gamma h) - 2\gamma p \cdot \cosh(\gamma h)} - \frac{\gamma p ((2 \sin(wt) - 1) \cdot \cosh(\gamma h) + 1))}{jwu_o (t_b + t_i) \cdot \sinh(\gamma h) - 2\gamma p \cdot \cosh(\gamma h)} \right) \quad (17)$$

$$i_2(t) = I_p \left(\frac{jwu_o (2t_b \sin(wt) + t_i) \cdot \sinh(\gamma h)}{jwu_o (t_b + t_i) \cdot \sinh(\gamma h) - 2\gamma p \cdot \cosh(\gamma h)} - \frac{\gamma p ((2 \sin(wt) + 1) \cdot \cosh(\gamma h) - 1)}{jwu_o (t_b + t_i) \cdot \sinh(\gamma h) - 2\gamma p \cdot \cosh(\gamma h)} \right) \quad (18)$$

As the parallel current is proved to be constant when $h \gg \delta_w$, the frequency is set at 1MHz to analyze the insulation effect. Fig.10 shows peak currents I_1 and I_2 changes with t_t/t_b under the constant $t_t+t_b=0.6\text{mm}$. Larger ratio t_t/t_b leads to an almost balanced current distribution. Lower ratio t_t/t_b shows more current on S_2 but much lower current on S_1 . The interleaved winding layout has lost its benefit on better current distribution.

III. INVESTIGATION OVER FULLY INTERLEAVED WINDING STRUCTURE

The proposed method is further verified by a fully interleaved parallel winding schemes for an 8-layer planar transformer with a 4:1 turn ratio, as shown in Fig.11. Each has 4 primary layers connected in series and 4 secondary layers in parallel. Other parameters keep the same with TABLE I. The insulation thickness is identical for each layer.

TABLE II
CASE IV: CURRENT DISTRIBUTION

P ₁		S ₁		P ₂		S ₂		S ₃		P ₃		S ₄		P ₄	
T	B	T	B	T	B	T	B	T	B	T	B	T	B	T	B
0	I	I	I/2	I/2	I/2	I/2	0	0	I/2	I/2	I/2	I/2	I	I	0
I		1.5I		I		0.5I		0.5I		I		1.5I		I	

T: Top B: Bottom

Fig.11. FEA simulated H field from 500kHz to 110MHz for Case IV

Three rules have to be applied to predict the current distribution for Case IV, namely P₁-S₁-P₂-S₂-S₃-P₃-S₄-P₄. Conductors P₁, P₂ and P₃ in Case V are placed in the same locations with conductors P₁, P₂, and P₃ in Case IV, respectively. The current distributions are also identical. Conductor P₄ is attracted by the S₄ and the current I tends to the top surface. Comes to secondary windings. Following Rule 1 and Rule 3, conductor S₁ carries $1.5I$ with I on the top surface and $0.5I$ on the bottom surface. Conductor S₄ conducts $1.5I$ with I on the bottom side and $0.5I$ on the top side. Rule 2 is applied to conductors S₂ and S₃, which are adjacent to each other. This means that currents on S₂ and S₃ repel each other, but are attracted by the current $0.5I$ from conductors P₂ and P₃, respectively. Hence, conductor S₂ carries $0.5I$ on the top surface, while conductor S₃ carries $0.5I$ on the bottom surface. TABLE II documents the current distribution for Case V.

FEA simulation is performed to verify the feasibility of the proposed approach. The absolute magnetic field distributions with frequency sweeping from 500 kHz to 100MHz are given in Fig.12. The current assigned to every primary conductor is a sine wave with the amplitude 1A. Following Ampère's circuital law, the current amplitude can be easily calculated by $I = Hb/N$. 117A/m corresponds to 1A and 58.5 A/m corresponds to 0.5A. The simulation results match the previous theoretical analyses. It also proves that the current distribution on every secondary layer is always constant when $h \gg \delta_w$. S₁ and S₄ have the same current 1.5A in Case IV. S₂ and S₃ have the identical current 0.5A.

Acknowledging the current distribution on each conductor, the winding loss and AC resistance R_{ac} for each case can be calculated afterward. Previous efforts [25]-[29] have estimated the AC resistance of planar transformers. This paper utilizes the Dowell equation to calculate the AC resistance. Fig.13 shows comparisons between the calculated AC resistance and the FEA

simulated AC resistance as a function of the frequency. It illustrates excellent agreement.

Fig.12. Comparison between the calculated AC resistance and FEA simulated AC resistance for Case IV

IV. EXTENSIONS TO OTHER WINDING STRUCTURES

The proposed methodology used to predict the parallel current distribution is not limited to fully interleaved winding layout, but capable to be applied to many winding arrangements, such as partial-interleaved and non-interleaved winding structures.

Fig.13. (a) Sandwiching winding arrangement and (b) non-interleaved winding arrangement

TABLE III
CURRENT DISTRIBUTION FOR SANDWICHING WINDING ARRANGEMENT

P _T		S ₁		S _{other}		S _N		P _B	
T	B	T	B	T	B	T	B	T	B
0	NI	NI	0	0	0	0	NI	NI	0
NI		NI		0		NI		NI	

TABLE IV
CURRENT DISTRIBUTION FOR NON-INTERLEAVED WINDING ARRANGEMENT

P _T		S ₁		S _{other}		S _N	
T	B	T	B	T	B	T	B
0	NI	NI	0	0	0	0	0
NI		NI		0		0	

Fig.13(a) illustrates a sandwiched winding arrangement: $2N$ primary turns are connected in series, while $2N$ secondary conductors in parallel. Similar with previous analyses, Conductors P₁ to P_N and P_{N+1} to P_{2N} can be viewed as two separate conductors, namely P_T and P_B, separately. NI on conductor P_T inclines to the bottom surface. NI on conductor P_B tends to the top surface. In secondary winding, following Rule 2 and Rule 3, conductors S₁ and S_N each conducts NI , but no current flows through any other layers between S₁ and S_N. The current distribution is shown in TABLE III.

The non-interleaved winding layout is shown in Fig.13(b). Similar with the previous process, the current distribution is derived and documented in TABLE IV.

From the above analyses, it can be concluded that it is useless to reduce the current density simply through more parallel conductors. Interleaved winding layouts are recommended to have a better current distribution.

Fig.14. Current waveforms for (a) Case II P1-S2-S1-P2 and (b) Case III P1-S2-P1-S1

TABLE V
WINDING DETAILS

Parameters	Values
Copper thickness (h)	70 μ m
Turn ratio ($n:1$)	2:1
Window width (b)	5.9mm
Magnetic core	E22/6/16

V. EXPERIMENT

To verify the effectiveness of the theoretical analyses, two interleaved PCB winding layouts Case II P1-S2-S1-P2 and Case III P1-S2-P1-S1 were manufactured. TABLE V shows details about the transformer. The high-speed bipolar amplifier HSA4101 injected the sinusoidal current whose frequency changed from 100kHz to 1MHz to the primary winding. Secondary windings were short circuits. Current probes CWT Ultramini from PEM were used to measure the secondary currents. As shown in Fig.14, the parallel current waveforms for Case II and Case III were captured in different frequencies.

Analysis of Case II is given first. Fig.15 shows the measured RMS current ratio as a function of frequency. Each parallel conductor has an almost identical current. Recalling the theoretical analyses on Case II given in Section II, the same current is supposed to go through S2 and S1. Hence, the experiment results support the analytical solutions.

In Case III, Fig.16 shows the secondary RMS current on every parallel conductor over the average RMS current as a function of the frequency. The average RMS current is expressed by

$$I_{RMS_avg} = \sqrt{(I_{1_RMS}^2 + I_{2_RMS}^2) / 2}$$

The RMS current ratios shown in Fig.16 do not match Fig.5 (b), which are supposed to be 1.5 and 0.5 for conductors S2 and S1, respectively. To further analyze this phenomenon, the PCB winding is cut off to measure the insulation thickness. Fig.17 represents the cross-section view of the PCB winding captured by the Micromanipulator 2210-LS. The insulation $t_i=0.543$ mm is much larger than the $t_b=0.089$ mm. Taking the measured insulation thicknesses to (17) and (18), the calculated RMS current ratios as a function of frequency are plotted in Fig.16. It can be observed that calculations represent good agreement with experiment results at high frequency.

As a result, the experiment results support the theoretical analyses. The current distribution on parallel conductors is constant at high frequency. The insulation thickness significantly affects the current distribution.

Fig.15. RMS current ratio I2/I1 for Case II

Fig.16. Case III calculated and experimental RMS current ratio I_{RMS}/I_{RMS_avg}

Fig.17. Case III P1-S2-P1-S1 cross section view of the PCB winding

VI. CONCLUSION

This paper reveals that the current on every parallel conductor does not change with frequency when $h \gg \delta_w$, and the interleaved winding layouts with different insulation thicknesses may result in a loss of the advantage of good current sharing. This paper proposes a methodology consisting of three rules to predict the parallel current distribution specifically for $N:1$ transformers, which is supported by theoretical deductions and FEA simulations. No complex calculation algorithm or simulation tool is required, thereby greatly simplifying the winding loss calculation. The proposed methodology is not limited to the fully interleaved structures, but can be applied to many parallel schemes. The experiment further proves the effectiveness of the theoretical analyses on the effect of the

insulation thickness and current distribution on parallel conductors at high frequency.

APPENDIX

The current expressions $i_1(t)$ and $i_2(t)$ for Case I are represented in (A.1) and (A.2), respectively.

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$$i_1(t) = -2I_p \gamma \rho \frac{\sin(wt)(\cosh(2\gamma h) + \sinh(2\gamma h) + 1) - \cosh(2\gamma h) + 2\sinh(\gamma h) + 2\cosh(\gamma h) - \sinh(2\gamma h) - 1}{(j \cdot t_h u_o w - 2\rho\gamma)(\cosh(2\gamma\rho) + \sinh(2\gamma\rho)) - jt_h u_o w - 2\rho\gamma} \quad (\text{A.1})$$

$$i_2(t) = 2I_p \frac{\left[(j \cdot t_h u_o w - \gamma\rho) \sin(wt) - \gamma\rho \right] (\cosh(2\gamma h) + \sinh(2\gamma h)) - (j \cdot t_h u_o w - \gamma\rho) \sin(wt) + 2\gamma\rho \left(\cosh(\gamma h) + \sinh(\gamma h) - \frac{1}{2} \right)}{(j \cdot t_h u_o w - 2\gamma\rho)(\cosh(2\gamma\rho) + \sinh(2\gamma\rho)) - jt_h u_o w - 2\gamma\rho} \quad (\text{A.2})$$