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# Hyperchaos in coupled Colpitts oscillators

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## Abstract

The paper suggests a simple solution of building a hyperchaotic oscillator. Two chaotic Colpitts oscillators, either identical or non-identical ones are coupled by means of two linear resistors  $R_k$ . The hyperchaotic output signal  $v(t)$  is a linear combination, specifically the mean of the individual chaotic signals,  $v(t) = (v_1 + v_2)/2$ . The corresponding differential equations have been derived. The results of both, numerical simulations and hardware experiments are presented. The coupling coefficient  $k \propto 1/R_k$  should be small to avoid mutual synchronisation of the individual oscillators. The spectrum of the Lyapunov exponents (LE) have been calculated versus the coefficient  $k$ . For weakly coupled oscillators there are two positive LE indicating hyperchaotic behaviour of the overall system.

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## 1. Introduction

The classical Colpitts oscillator with special parameter settings exhibits chaotic behaviour [1,2], thus can be used to generate noise-like broadband signals. The oscillator is very flexible—the fundamental frequency can be tuned from several kilohertz to several gigahertz, i.e. to the microwave range. The complexity of chaotic oscillations is characterized by the number of positive Lyapunov exponents (LE), i.e. the number of directions in the phase space along which the phase trajectories are unstable. Chaotic Colpitts oscillator has only one positive LE. Meanwhile for chaos based communications more complicated oscillators, characterized by multiple positive LE are needed [3,4]. Dynamical systems having more than one positive LE are called *hyperchaotic* ones.

The first hyperchaotic electronic circuit has been suggested by Matsumoto et al. [5]. During the past five years a large number of various hyperchaotic oscillators have been proposed by several authors, e.g. [6–25]. Among them are the fourth and higher order circuits composed of non-linearly coupled second order linear oscillators [6,7,10,12,13,15], high-order high-pass filter based oscillators [11,14], the sixth and higher order arrays of coupled third order non-linear chaotic oscillators [16–19], also the so-called “infinite” dimensional, delay line based oscillators [22–25], operating at kilohertz frequencies [22,23], as well as in the microwave range [24,25]. Some of the early hyperchaotic oscillators are described in the review papers [8,9]. Possible applications of hyperchaotic circuits to communications, specifically to secure communications are discussed and illustrated in [17,18,20,21].

In the present paper we suggest a simple solution of building a hyperchaotic Colpitts circuit. Two chaotic Colpitts oscillators, either identical or non-identical ones are coupled by means of linear resistors. The output hyperchaotic signal is a linear combination, specifically the mean of the individual chaotic signals.

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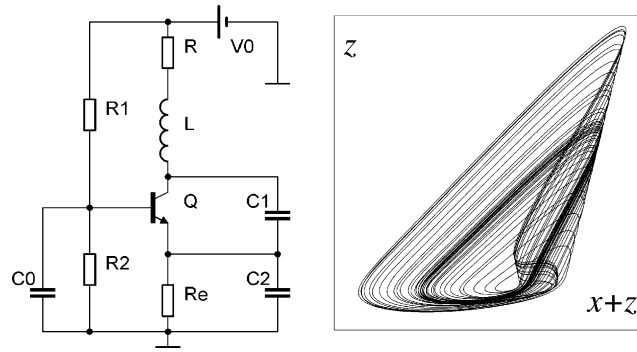


Fig. 1. The Colpitts oscillator: circuit diagram (left) and phase portrait (right).  $\varepsilon = 1$ ,  $a = 30$ ,  $b = 0.8$ ,  $c = 20$ ,  $d = 0.08$ ,  $e = 10$ .

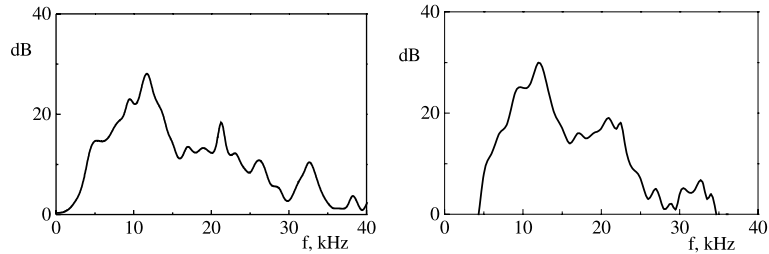


Fig. 2. Power spectra from the Colpitts oscillator: numerical (left) and experimental (right).  $\varepsilon = 1$ ,  $a = 30$ ,  $b = 0.8$ ,  $c = 20$ ,  $d = 0.08$ ,  $e = 10$ , Q – 2N3904,  $L = 850 \mu\text{H}$ ,  $C_1 = C_2 = 470 \text{ nF}$ ,  $C_0 = 47 \mu\text{F}$ ,  $R = 36 \Omega$ ,  $R_e = 510 \Omega$ ,  $R_1 = R_2 = 3 \text{ k}\Omega$ ,  $V_0 = 15 \text{ V}$ .

## 2. Chaotic Colpitts oscillator

Dynamics of the Colpitts oscillator (Fig. 1, left) can be described by the following differential equations:

$$\begin{aligned} \dot{x} &= y - aF(z), \\ \dot{y} &= c - x - by - z, \quad F(z) = \begin{cases} e - 1 - z, & z < e - 1, \\ 0, & z \geq e - 1. \end{cases} \\ \varepsilon \dot{z} &= y - dz, \end{aligned} \quad (1)$$

Here

$$\begin{aligned} x &= \frac{V_{C1}}{V^*}, \quad y = \frac{\rho I_L}{V^*}, \quad z = \frac{V_{C2}}{V^*}, \quad \vartheta = \frac{t}{\tau}, \quad \dot{u} \equiv \frac{du}{d\vartheta}, \quad \rho = \sqrt{\frac{L}{C_1}}, \quad \tau = \sqrt{LC_1}, \quad \varepsilon = \frac{C_2}{C_1}, \quad a = \frac{\rho}{r}, \\ b &= \frac{R}{\rho}, \quad c = \frac{V_0}{V^*}, \quad d = \frac{\rho}{R_e}, \quad e = \frac{R_2}{R_1 + R_2} c. \end{aligned} \quad (2)$$

Typical phase portrait is presented in Fig. 1 (right), while power spectra are illustrated in Fig. 2.

## 3. Coupled Colpitts oscillators

Two Colpitts oscillators, Colp1 and Colp2, specifically the collector nodes, are coupled to each other by two linear resistors  $R_k$  (Fig. 3). The output signal  $v(t)$  is simply the mean of the collector voltages ( $x_i + z_i$ ) of the individual oscillators:

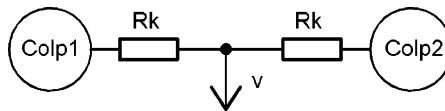


Fig. 3. Two coupled Colpitts oscillators.

$$v = \frac{(x_1 + z_1) + (x_2 + z_2)}{2}. \quad (3)$$

The coupled system is given by

$$\begin{aligned} \dot{x}_1 &= [y_1 - aF(z_1) + k(v - x_1 - z_1)]\omega_1, & \dot{x}_2 &= [y_2 - aF(z_2) + k(v - x_2 - z_2)]\omega_2, \\ \dot{y}_1 &= [c - x_1 - by_1 - z_1]\omega_1, & \dot{y}_2 &= [c - x_2 - by_2 - z_2]\omega_2, & \dot{z}_1 &= [y_1 - dz_1 + k(v - x_1 - z_1)]\omega_1, \\ \dot{z}_2 &= [y_2 - dz_2 + k(v - x_2 - z_2)]\omega_2. \end{aligned} \quad (4)$$

Here  $k = \rho/R_k$  is the coupling coefficient,  $\omega_1$  and  $\omega_2$  are the reduced cyclic frequencies of the oscillators.

### 3.1. Identical oscillators

The phase portraits of the system composed of identical oscillators ( $\omega_1 = \omega_2$ ) are shown in Fig. 4. Experimentally observed phase portraits correspond very well to the numerical ones.

The fine diagonal in Fig. 4d indicates full synchronisation of the individual oscillators at larger coupling coefficients. Obviously, the system of synchronized chaotic oscillators is not a hyperchaotic one. Visual inspection of the phase portraits at smaller coupling coefficients does not allow one to distinguish between simple chaotic and hyperchaotic states. This can be done by direct calculation of the LE (Fig. 5). At  $k = 0$  there are two positive LE, as expected. Meanwhile at  $k > 0$  the situation is rather complicated. There are certain parameter windows (Fig. 5, right) where the oscillations are either weakly chaotic ( $k \approx 0.02$ ; low value single positive LE) or even periodic ( $k \approx 0.03$ ; the largest LE is zero!).

### 3.2. Non-identical oscillators

The system of non-identical oscillators ( $\omega_1 \neq \omega_2$ ) has an advantage in the sense, that full synchronisation is impossible in this case. Moreover, no chaotic and periodic windows have been detected at small coupling coefficients (Fig. 6). The systems remains hyperchaotic with two positive LE up to  $k = 0.12$ .

In addition, it is worth to note, that weak coupling of non-identical oscillators, e.g.  $k = 0.05$  enables one to reduce the unevenness of the power spectrum (Fig. 7, middle).

The hardware experiments do confirm the numerical result of smoothing the power spectra by means of weak coupling of the oscillators (Fig. 8, middle).

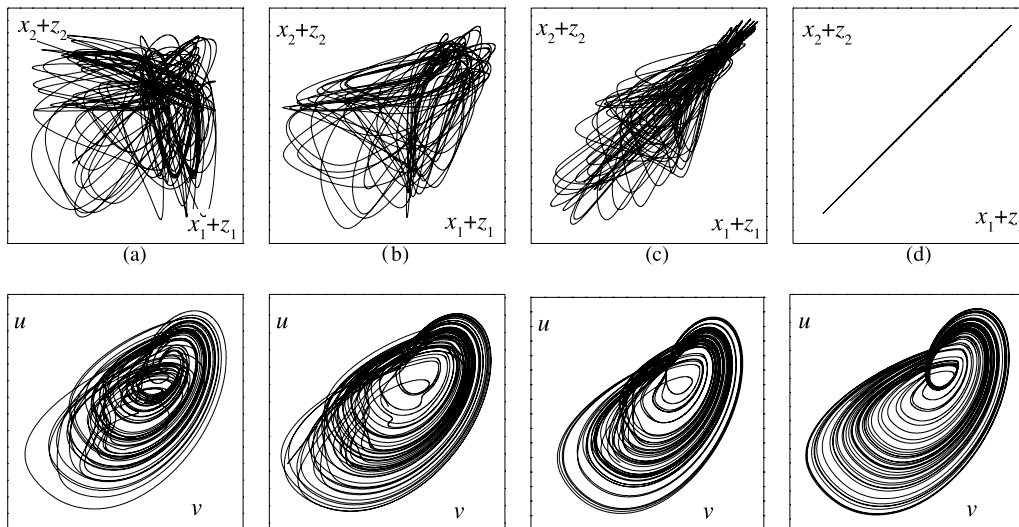


Fig. 4. Phase portraits of two coupled identical Colpitts oscillators at different coupling coefficients: (a)  $k = 0$ , (b)  $k = 0.1$ , (c)  $k = 0.3$ , (d)  $k = 1.0$ . In the top row the  $x_1 + z_1$  and  $x_2 + z_2$  is proportional to the collector voltage of the Colp1 and the Colp2 oscillator, respectively. In the bottom row the  $v$  is the mean collector voltage given by Eq. (3) and  $u$  is simulated from an auxiliary linear differential equation:  $\dot{u} = v - u$ .

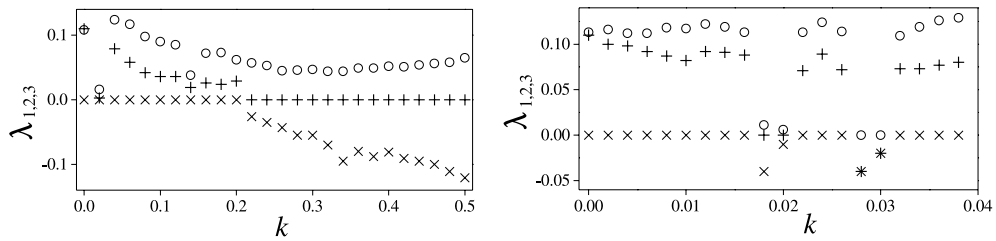


Fig. 5. Three largest LE,  $\lambda_1$ ,  $\lambda_2$ ,  $\lambda_3$  from Eqs. (3) and (4). Panorama view (left), detailed view (right). Identical oscillators,  $w_1 = w_2 = 1$ ,  $\varepsilon = 1$ ,  $a = 30$ ,  $b = 0.8$ ,  $c = 20$ ,  $d = 0.08$ ,  $e = 10$ .

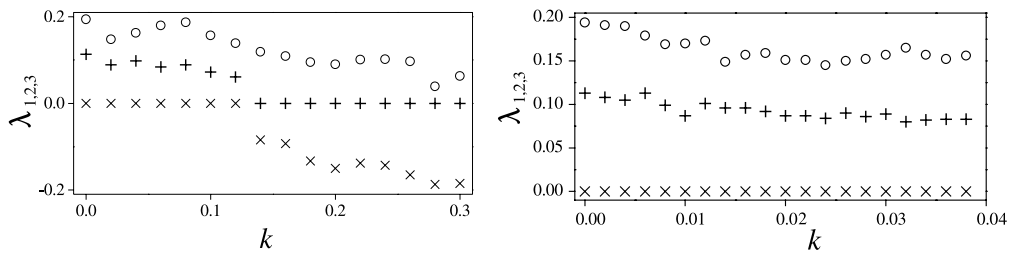


Fig. 6. Three largest LE,  $\lambda_1$ ,  $\lambda_2$ ,  $\lambda_3$  from Eqs. (3) and (4). Panorama view (left), detailed view (right). Non-identical oscillators,  $w_1 = 1$ ,  $w_2 = 1.6$ ,  $\varepsilon = 1$ ,  $a = 30$ ,  $b = 0.8$ ,  $c = 20$ ,  $d = 0.08$ ,  $e = 10$ .

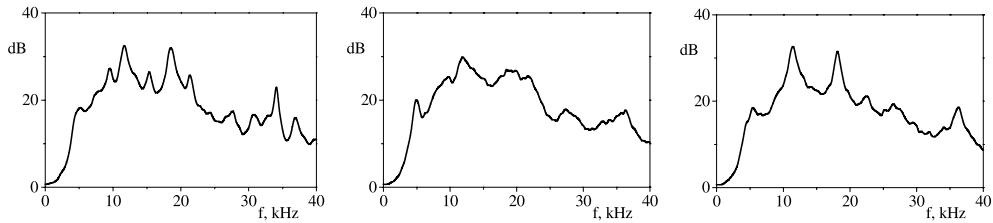


Fig. 7. Power spectra of non-identical oscillators ( $w_1 = 1$ ,  $w_2 = 1.6$ ) at different coupling coefficients:  $k = 0$  (left),  $k = 0.05$  (middle),  $k = 0.1$  (right).  $\varepsilon = 1$ ,  $a = 30$ ,  $b = 0.8$ ,  $c = 20$ ,  $d = 0.08$ ,  $e = 10$ .

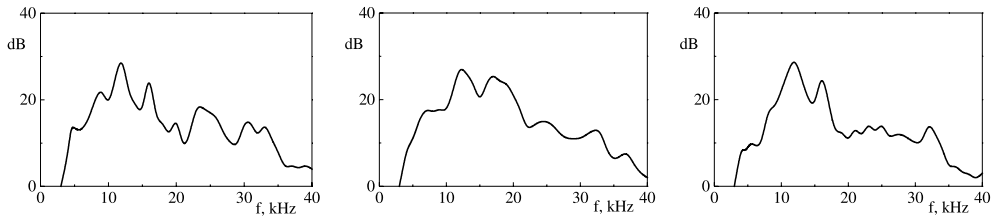


Fig. 8. Experimental power spectra of non-identical oscillators ( $f_1 \approx 11$  kHz,  $f_2 \approx 17$  kHz) at different coupling resistors:  $R_k = 5.1$  k $\Omega$  (left),  $R_k = 1.1$  k $\Omega$  (middle),  $R_k = 750$   $\Omega$  (right). Q – 2N3904,  $C_0 = 47$   $\mu$ F,  $R = 36$   $\Omega$ ,  $R_c = 510$   $\Omega$ ,  $R_1 = R_2 = 3$  k $\Omega$ ,  $V_0 = 15$  V. The LC tank parameters are as follows:  $L = 850$   $\mu$ H,  $C_1 = C_2 = 470$  nF (in Colp1) and  $L = 530$   $\mu$ H,  $C_1 = C_2 = 300$  nF (in Colp2).

#### 4. Conclusions

Two weakly coupled Colpitts oscillators exhibit hyperchaotic behaviour characterized by two positive LE. In the case of identical oscillators there are undesirable regions of coupling coefficient where weakly chaotic (small single LE) and even periodic oscillations are observed. Meanwhile, non-identical Colpitts oscillators with different fundamental frequencies have an advantage. The system oscillates hyperchaotically over sufficiently wide range of coupling without

any chaotic or periodic windows. In addition, certain carefully chosen coupling of non-identical oscillators provide relatively smooth power spectrum of the overall system. We expect that coupling of a larger number of chaotic Colpitts oscillators can produce hyperchaotic systems with multiple positive LE and rather smooth power spectra.

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