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Approaching optimum phase measurement in the presence of amplifier noise

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In fundamental papers from 1962, Heffener and Haus showed that it is not possible to construct a linear noiseless amplifier [Proc. IRE 50, 1604 (1962); Phys. Rev. Lett. 128, 2407 (1962)]. This implies that amplifier intrinsic noise sources induce random perturbations on the phase of the incoming optical signal, which translates into spectral broadening. Achieving the minimum induced phase fluctuation requires a phase measurement method that introduces minimum uncertainty, i.e., optimum phase measurement. We demonstrate that a measurement method based on heterodyne detection and extended Kalman filtering approaches optimum phase measurement in the presence of amplifier noise. A penalty of 5 dB (numerical) and 15 dB (experimental) compared to quantum limited spectral broadening is achieved. Spectral broadening reduction of 44 dB is achieved, compared to when using the widely employed phase measurement method, based purely on the argument of the signal field. Our results reveal new scientific insights by demonstrating a phase measurement method that enables to approach minimum phase fluctuation, induced by amplifier noise. An impact is envisioned for phase-based optical sensing systems, as optical amplification could increase sensing distance with minimum impact on the phase. © 2021 Optical Society of America under the terms of the OSA Open Access Publishing Agreement

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1. INTRODUCTION

High-power narrow-linewidth lasers are essential for optical fiber sensing, gravitational wave detection, optical space communication, and high-capacity optical fiber communication [1–7]. Typically, high-power performance is achieved by amplifying a low-noise seed laser with a single or several stages of fiber-optic amplification.

Apart from increasing the amplitude, an optical amplifier will induce random fluctuations on the phase of the incoming optical signal. The minimum induced phase fluctuation, (quantum limit) due to amplifier noise, denoted by $\Delta\phi_{ql}$, is a consequence of Heisenberg's uncertainty principle [8,9]. Moreover, to reach the minimum value, the detector must measure the photon number and phase with the same relative uncertainties as those introduced by the amplifier, i.e., optimum detector [8,9]. As the optical amplifier reduces the signal-to-noise-ratio of the incoming signal, the magnitude of $\Delta\phi_{ql}$ is inversely proportional to it. The random fluctuation of the optical phase, $\Delta\phi_{ql}$, will translate into spectral broadening, denoted by $\Delta\nu_{ql}$. For applications of high-power narrow-linewidth lasers in optical sensing and communication, increasing the laser's spectral width translates into system performance degradation, and it should therefore be kept at its minimum.

To quantify the impact of the amplifier noise, a measurement of the optical phase and corresponding spectral width, with and without optical amplification, and a comparison to quantum limits $\Delta\phi_{ql}$ and $\Delta\nu_{ql}$ are needed. A practical optimum phase measurement method is needed to approach $\Delta\phi_{ql}$ and $\Delta\nu_{ql}$ and thereby minimize the impact of amplifier noise.

So far, the approach for quantifying the impact of amplifier noise has been based on measurement of the phase power spectrum density (PSD) before and after amplification [7,10–15]. However, the reported experimental results do not fully agree, and are even contradictory in some cases. What makes measurements challenging is that the impact of amplifier noise is mostly visible at high frequencies (> 10 MHz) of the phase PSD and for low input signal power levels. State-of-the-art optical phase measurement methods, including spectrum analyzers, suffer from limited sensitivity as well as limited dynamic and frequency ranges. This makes it challenging to measure optical phase below the thermal measurement noise floor, which typically lies around -150 dBrad²/Hz, and at frequencies exceeding a few MHz. Thus, observing and quantifying the impact of amplifier noise requires a highly sensitive optical phase measurement method. Finally, to fully quantify the impact of amplifier noise, a comparison to the quantum limit $\Delta\phi_{ql}$ is necessary.

In this paper, we investigate the impact of amplifier noise on the phase fluctuation and spectral broadening of the incoming optical

signal, as a function of input signal power into the amplifier, and compare it to quantum limits. For measuring the optical phase, a heterodyne receiver, in combination with an extended Kalman filter (EKF), is proposed. The phase measurement method is a practical implementation of theoretically optimal phase measurement in the presence of amplifier noise. We demonstrate highly accurate phase measurement resulting in a penalty of 5 dB (numerically) and 15 dB (experimentally) compared to the quantum limit. Finally, we show that the commonly employed phase measurement method results in a large penalty for the increasing system bandwidth and decreasing signal power.

The outline of the paper is as follows: in Section 2, a theoretically achievable minimum (quantum limited) variance for phase fluctuations, due amplifier noise, is presented for the conditions under consideration. We also present a practical method based on extended Kalman filtering for approaching the optimum phase estimation and achieving minimum phase fluctuation. In Section 3, numerical results are presented and compared to the quantum limits. We also illustrate the impact of amplifier noise by computing the phase PSDs. In Section 4, the impact of amplifier noise on the phase PSD is experimentally investigated, and the corresponding spectral broadening is computed and compared to the quantum limit. Finally, in Section 5, the main findings are summarized, and the impact of results is discussed. In Section 6, suggestions for future works are presented.

2. THEORETICAL FRAMEWORK

A. Minimum Phase Fluctuation of Amplified Time-Varying Phase Signal

In the analysis to follow, only intrinsic (fundamental) noise sources are included. This assumption is valid, as we are interested in the minimum phase fluctuation induced by amplifier noise.

We assume that the input to the linear amplifier is provided by a laser source with angular frequency $\omega = 2\pi\nu$ and randomly time-varying phase $\phi(t)$:

$$E_{\text{out}}(t) = \sqrt{P_{\text{in}}} \sin[\omega t + \phi(t)], \quad (1)$$

where P_{in} is the input signal power. The minimum fluctuation of $\phi(t)$ is governed by the amplified spontaneous emission within the laser cavity (quantum noise) and can be approximated by a Wiener process [16]:

$$\frac{d\phi}{dt} = \sqrt{\kappa}\Gamma(t), \quad (2)$$

where κ is the rate of phase diffusion and is related to laser coherence time τ_c . $\Gamma(t)$ is a Langevin noise source with Gaussian distribution. It has a zero mean and delta correlation function. The PSD of a laser signal with time-varying phase described by a Wiener process has a Lorentzian shape and is centered around ω . Its spectral width, $\Delta\nu_c$, is inversely proportional to coherence time τ_c and can be computed using

$$\begin{aligned} \Delta\nu_c &= \frac{(\int_0^\infty S(\nu)d\nu)^2}{(\int_0^\infty S^2(\nu)d\nu)} \\ &= \left[\int_{-\infty}^\infty \left| \frac{\langle U^*(t)(t)U(t)(t+\tau) \rangle}{\langle U^*(t)U(t) \rangle} \right|^2 d\tau \right]^{-1} = \frac{1}{\tau_c}, \end{aligned} \quad (3)$$

where $S(\nu)$ is the PSD of the laser signal $E_{\text{in}}(t)$, and $U(t) = \sqrt{P_{\text{in}}}e^{j(\omega t + \phi(t))}$.

The signal $E_{\text{in}}(t)$ is now applied to the linear amplifier with gain, G . The output of the amplifier can be expressed as a coherent superposition of the amplified signal and amplifier noise [8,9]:

$$E_{\text{out}}(t) = \sqrt{P_s} \sin[\omega t + \phi(t)] + n_a(t), \quad (4)$$

where $P_s = GP_{\text{in}}$. $n_a(t)$ is a noise term, added by the amplifier. It has a zero mean Gaussian distribution and minimum noise power $P_N = h\nu(G-1)B$ [8,9]. B is the bandwidth, and h is Planck's constant.

To quantify the impact of amplifier noise on the phase of the incoming signal, $\phi(t)$, a receiver with bandwidth B is used. The receiver must employ a phase estimation method that introduces minimum uncertainty in the presence of amplifier noise. This is accomplished by performing a statistically optimum (and thereby theoretically most accurate) phase estimation. It is achieved by solving the following maximum *a posteriori* (MAP) phase estimation problem [17]:

$$\begin{aligned} \phi^{\text{MAP}}(t) &= \arg \max_{\phi'} p[\phi'(t)|E_{\text{out}}(t)] \\ &= \arg \max_{\phi'} \frac{p[E_{\text{out}}(t)|\phi'(t)]p[\phi'(t)]}{p[E_{\text{out}}(t)]}, \end{aligned} \quad (5)$$

where $\phi'(t)$ represent various phase evolutions (different trail phases). $p(\cdot)$ and $p(\cdot|\cdot)$ denote marginal and conditional probability densities, respectively. An analytical solution to Eq. (5) does not exist due to the nonlinear relationship between $E_{\text{out}}(t)$ and $\phi(t)$ (nonlinear estimation problem). To find the analytical solution, linearization needs to be performed.

Assuming that the phase diffusion constant κ is small enough such that $\sin[\phi(t)] \approx \phi(t)$, and setting $\omega = 0$ for convenience, estimating the phase becomes a linear estimation problem since $E_{\text{out}}(t) \propto \phi(t) + n_a(t)$, which has an analytical solution [18]. Under the aforementioned approximation, the solution to Eq. (5) is obtained by finding a phase $\phi'(t)$ that minimizes the mean square error (MSE) defined as $E[(\phi(t) - \phi'(t))^2]$. The phase that is the solution to the MSE is denoted as $\phi'(t) = \phi^{\text{MAP}}(t)$. The minimum phase fluctuation due to amplifier noise is then expressed as

$$\Delta\phi_a^{\text{MAP}} = \sqrt{E[(\phi(t) - \phi^{\text{MAP}}(t))^2]}. \quad (6)$$

The results from the quantum phase estimation state that the expression for the minimum phase fluctuation, $\Delta\phi_a^{\text{MAP}}$, then becomes [19,20]

$$\Delta\phi_a^{\text{MAP}} = \frac{1}{\sqrt{\sqrt{2N_p}}}, \quad (7)$$

where N_p describes the mean number of photons per laser coherence time and is defined as

$$N_p = \frac{P_s \tau_c}{h\nu(G-1)} = \frac{P_s}{h\nu(G-1)\pi\Delta\nu_{\text{FWHM}}}, \quad (8)$$

where $\tau_c = 1/\pi\Delta\nu_{\text{FWHM}}$ is coherence time of a laser with phase modeled as a Wiener process, and $\Delta\nu_{\text{FWHM}}$ is the full width at half maximum (FWHM) of the Lorentzian laser spectrum denoted by $S(\nu)$. It should be stressed that the minimum phase fluctuation

expressed by Eq. (7) assumes that the receiver bandwidth $2B$ is matched to the laser spectral width $\Delta\nu_c = \Delta\pi\nu_{\text{FWHM}}$ [19,20].

We now need to find a practical receiver architecture that can perform statistically optimal phase estimation and approach $\Delta\phi_a^{\text{MAP}}$. In the following, we consider two different cases.

B. Minimum Phase Fluctuation of Amplified Single Frequency Constant Phase Signal

If we assume a constant phase signal ($\phi(t) = \phi$), the PSD becomes a delta function centered around ω . Applying a small detector bandwidth, B , such that the PSD of the amplified signal is confined to a relatively narrow band in the neighborhood of signal frequency (high SNR condition), the optimum phase estimation becomes [21,22]

$$\begin{aligned}\phi_{\tan^{-1}} &= \arg[(E_{\text{out}}(t) + j\mathcal{H}\{E_{\text{out}}(t)\})e^{-j\omega t}] \\ &= \arg[(I + jQ)e^{-j\omega t}] = \tan^{-1}(Q/I),\end{aligned}\quad (9)$$

where $\mathcal{H}\{\cdot\}$ denotes Hilbert transform. Using Eq. (9), the resulting minimum phase fluctuation (standard deviation) due to amplifier noise can then be computed as [8,21]

$$\Delta\phi_a = \sqrt{\frac{P_N}{2P_s}} = \sqrt{\frac{bv(G-1)B}{2P_s}} = \sqrt{\frac{1}{2\text{SNR}}} = \frac{1}{\sqrt{2N_p}},\quad (10)$$

where $N_p = \frac{P_s}{bv(G-1)B}$. The phase estimation method in Eq. (9) and the subsequent limit in Eq. (10) are widely employed in the literature for investigating the impact of amplifier noise [7,10–15]. However, for many practical applications within optical metrology and transmission, a relatively large receiver bandwidth is needed (up to a few GHz), and the input signal power to the amplifier may be low. Finally, the constant phase condition is hard to satisfy in practice, as the input to the amplifier is always provided by a laser source that has phase noise (randomly time-varying phase). Under the aforementioned practical conditions, the phase estimation provided in Eq. (9) is no longer optimal, and the corresponding minimum phase fluctuation governed by Eq. (10) no longer presents a lower bound for minimum phase fluctuation.

Generally speaking, the broader the receiver bandwidth, the larger the number of photons associated with amplifier noise, with random phases that combine with the laser mode. This can lead to large spectral broadening unless we find an effective receiver able to distinguish between the photons associated with the laser phase $\phi(t)$ and amplifier noise.

C. Bayesian Filtering for Optimum Phase Estimation

For the practical case of large signal bandwidth, low signal powers, and time-varying phase, state-spaced-based Bayesian filtering is a highly accurate and practical approach for the MAP estimation problem in Eq. (5). State-space-based Bayesian filtering implements a recursive MSE approach for estimating the quantity of interest and can be easily implemented in analog or digital domain [17]. To perform the estimation, the state-space model needs to be defined. It consists of a measurement and a state equation. The measurement equation describes the relationship between the observable (measurable) quantity and the parameter we want to estimate, i.e., Eq. (4). The state equation describes the evolution of the quantity to be estimated, in our case the phase, i.e., Eq. (2).

Excellent treatment of Bayesian filtering techniques is provided in [17].

In this paper, we consider shot noise limited heterodyne coherent detection in combination with digital signal processing for phase estimation. This implies that the amplified optical signal is combined with an (ideal) local oscillator (LO) laser ($\kappa = 0$) in a 3 dB coupler, detected by the shot noise limited balanced receiver and sampled by an analog-to-digital (A/D) converter. The state-space model, including state and measurement equations, Eqs. (11) and (12) for phase estimation, is then expressed as

$$\phi[k] = \phi[k-1] + \Gamma[k-1],\quad (11)$$

$$y[k] = 2R\sqrt{P_s P_{\text{LO}}}\cos(\Delta\omega k T_s + \phi[k]) + n^{\text{sh}}[k] + n^b[k],\quad (12)$$

where $k = 1, \dots, K$ is an integer representing discrete time, and P_{LO} is the LO power. R is the responsivity of the photodetector, assumed to be one. $\Delta\omega$ is the difference angular frequency between the signal and the LO laser, T_s is the sampling time, and $\phi[k]$ is the laser source time-varying phase modeled as a Wiener process with a phase diffusion constant $\kappa^2 = 2\pi\Delta\nu_{\text{FWHM}}T_s$. The shot and beat noise terms, $n^{\text{sh}}[k]$ and $n^b[k]$, are modeled as Gaussian noise sources with zero mean and variances: $\sigma_{\text{shot}}^2 = 2qR(P_{\text{LO}})B$ and $\sigma_b^2 = 4P_{\text{LO}}bv(G-1)B$, respectively [23]. q is the elementary charge, and B is the 3 dB bandwidth of the balanced receiver. The state equation, (11), represents the discrete time Wiener process.

An exact solution to the phase estimation problem, given the state-space model described by Eqs. (11) and (12), is not achievable. The reason is the nonlinear relationship between y_k and ϕ_k . Therefore, approximation to Bayesian filtering in terms of an extended, cubature, or unscented Kalman filter needs to be employed [17]. In this paper, we choose to implement the EKF due to its faster processing time compared to the unscented and cubature Kalman filters. The state-space model described by Eqs. (11) and (12) can be directly plugged in to EKF filtering equation Eqs. (5.26) and (5.27) in [17]. The EKF will explore correlation properties of $\phi[k]$ specified by the state equation for tracking and estimation purposes, while ignoring the contribution from amplifier noise since there is no correlation between $\phi[k]$ and $n_a[k]$ and $n_b[k]$.

The estimated phase by the EKF will be denoted as $\phi_{\text{EKF}}[k]$. For comparison reasons, we also apply the phase estimation method expressed by Eq. (9), and denote the estimated phase by $\phi_{\tan^{-1}}[k]$. Using the aforementioned phase estimators, the phase fluctuation due to amplifier noise is then computed as a sample standard deviation:

$$\sigma_{\text{EKF}/\tan^{-1}} = \sqrt{\frac{1}{K} \sum_{k=1}^K (\phi[k] - \phi_{\text{EKF}/\tan^{-1}}[k])^2}.\quad (13)$$

D. Translating Phase Fluctuation into Spectral Broadening

Given an observation time $T = kT_s$ under which the time-varying phase is observed and a laser source with a Lorentzian spectrum, the phase fluctuation, $\Delta\phi$, results in the following spectral broadening, $\Delta\nu$ [14]:

$$\Delta\phi = \sqrt{\frac{2\pi\Delta\nu_c T}{\pi}} \Rightarrow \Delta\nu_c = \frac{\Delta\phi^2}{2T}.\quad (14)$$

3. NUMERICAL RESULTS

In Fig. 1, the minimum theoretically achievable (quantum noise limited) phase fluctuation due to amplifier noise, $\Delta\phi_a^{\text{MAP}}$, and $\Delta\phi_a$, is plotted. We also plot the numerically computed phase fluctuation using Eq. (13), for phase estimation based on the EKF and Eq. (9), i.e., σ_{EKF} and $\sigma_{\tan^{-1}}$, respectively. We consider the receiver bandwidths B of 1 and 5 GHz. The laser linewidth $\Delta\nu_{\text{FWHM}}$ is chosen to be 10 Hz. The reason for choosing a relatively small linewidth is that we would like to minimize the penalty to $\Delta\phi_a^{\text{MAP}}$, which is derived for small diffusion factors (small linewidths).

It should be noticed in Fig. 1 that the minimum phase fluctuation $\Delta\phi_a^{\text{MAP}}$ is significantly lower compared to $\Delta\phi_a$ for $B = 5$ GHz. As the bandwidth is reduced to $B = 1$ GHz, $\Delta\phi_a^{\text{MAP}}$ still provides a lower phase fluctuation for input signal power levels up to 20 dBm.

The numerically computed phase fluctuation σ_{EKF} , exhibits only 5 dB of penalty compared to the theoretically achievable limit given by $\Delta\phi_a^{\text{MAP}}$. Most importantly, it does not increase as the bandwidth is increased from 1 to 5 GHz. Part of the penalty is due to the variance of the beat term, $n_b[k]$, being $4P_{\text{LO}}$ times larger than the variance of $n_a[k]$. In general, the results for σ_{EKF} illustrate the robustness of the EKF method for estimating the phase in the presence of amplifier noise and receiver bandwidths significantly larger than the spectral width. Finally, the numerically computed phase fluctuation $\sigma_{\tan^{-1}}$ closely follows the theoretical limit given by $\Delta\phi_a$. The penalty of approximately 1 dB, compared to $\Delta\phi_a$, is due to the variance of the beat term. Also, it should be noted that both $\Delta\phi_a$ and $\sigma_{\tan^{-1}}$ increase as the bandwidth is increased.

From a practical point of view, quantifying the impact of amplifier noise in terms of phase fluctuation is very challenging. A more practical approach would be to investigate the impact of amplifier noise on the phase PSD, and then compute the signal spectral broadening and compare it to the quantum limit.

In Fig. 2, the phase PSD obtained using the phase estimation method based on the EKF and Eq. (9) is plotted as a function of frequency. We also plot a reference PSD computed using $\phi[k]$. The input signal power, P_s , is varied from -20 to -5 dBm. Using phase estimation based on Eq. (9), the impact of amplifier noise is clearly observable. Amplifier noise results in a horizontal noise floor that increases as P_s is decreased.

The phase PSD computed using the EKF-based phase estimation method resembles, to a high degree, the reference spectra. The impact of amplifier noise is visible, as the deviation from the reference spectrum is observed. However, the deviation is not nearly as large as when Eq. (9) is employed for phase estimation. This illustrates the efficiency of the phase estimation method based on the EKF.

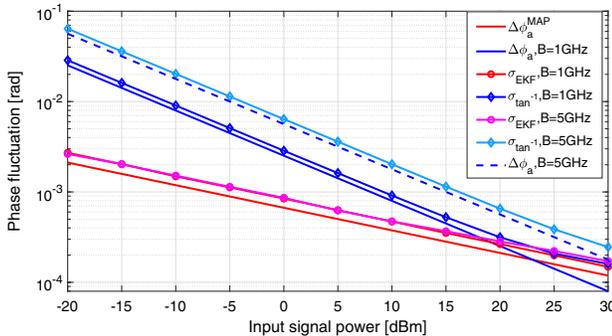


Fig. 1. Simulation: phase fluctuation as a function of input signal power P_s .

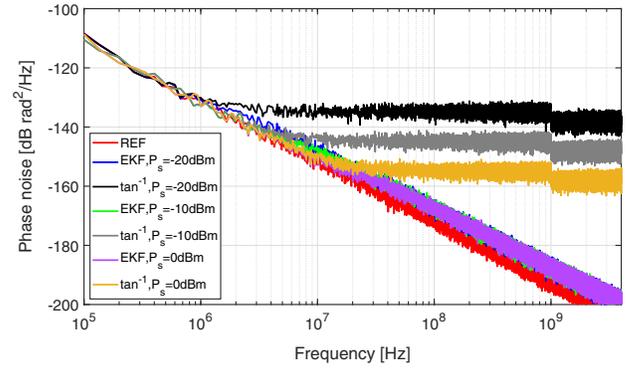


Fig. 2. Simulation: phase power spectral density as a function of frequency using different phase estimation methods.

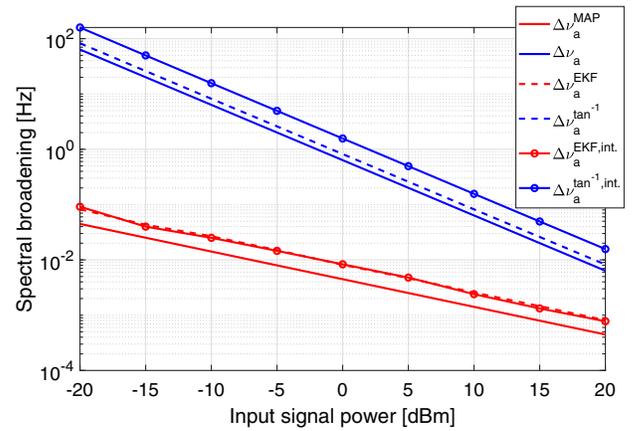


Fig. 3. Simulation: spectral broadening as a function of input signal power, P_s , for different phase estimation methods.

Next, in Fig. 3, we compute the corresponding spectral broadening due to amplifier noise as a function of input signal power P_s . This is achieved by performing the numerical integration of phase PSDs. The corresponding spectral widths are denoted by $\Delta\nu_a^{\text{EKF,int.}}$ and $\Delta\nu_a^{\tan^{-1},\text{int.}}$. The theoretical values for minimum spectral broadening (quantum limit) denoted by $\Delta\nu_a^{\text{MAP}}$ and $\Delta\nu_a$ are obtained by inserting Eqs. (7) and (10) into Eq. (14), respectively.

To investigate the accuracy of numerical integration, we convert the numerically computed phase fluctuations, σ_{EKF} and $\sigma_{\tan^{-1}}$, into spectral broadening using Eq. (14). The corresponding spectral broadening are denoted by $\Delta\nu_a^{\text{EKF}}$ and $\Delta\nu_a^{\tan^{-1}}$, respectively. The agreement between $\Delta\nu_a^{\text{EKF,int.}}$ and $\Delta\nu_a^{\text{EKF}}$ indicates that the error induced by numerical integration is small. A penalty of approximately 3 dB is observed between $\Delta\nu_a^{\tan^{-1},\text{int.}}$ and $\Delta\nu_a^{\tan^{-1}}$. A possible reason is that $\phi_{\tan^{-1}}[k]$ contains a higher degree of noise compared to $\Delta\nu_a^{\text{EKF}}$, which makes the numerical differentiation becomes less accurate.

It should be emphasized that the spectral broadening $\Delta\nu_a^{\text{EKF,int.}}$ is significantly below $\Delta\nu_a$ and $\Delta\nu_a^{\tan^{-1}}$. Finally, a penalty of only 5 dB is observed between $\Delta\nu_a^{\text{EKF,int.}}$ and a minimum spectral broadening $\Delta\nu_a^{\text{MAP}}$.

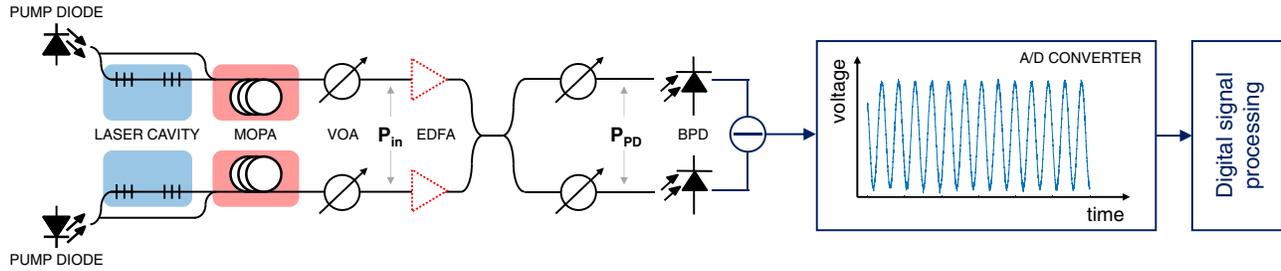


Fig. 4. Experimental setup for investigating the impact of amplifier noise on the phase fluctuation of the incoming signal.

4. EXPERIMENTAL RESULTS

The experimental setup for investigating the impact of amplifier noise on phase fluctuation and the corresponding spectral broadening is shown in Fig. 4.

The state-space model, employed for extended-Kalman-filtering-based phase estimation, is described by Eqs. (11) and (12). The state equation (11) assumes that the laser is dominated only by quantum noise, which is typically not the case for practical laser sources. A practical laser source will be dominated not only by intrinsic noise sources but also by technical/environmental noise. The impact of these is that the penalty may be expected.

We employ two similar fiber lasers, one as a signal and the other as the LO source. Both lasers have matching phase noise performances. The lasers use a fiber Bragg grating cavity to produce an output beam operating in a single mode. The maximum output power measured directly from the cavity is -2 dBm. The laser cavity is then followed by a master oscillator fiber amplifier (MOPA) to boost the signal power level up to 15 dBm. The signal and LO lasers are set at 1550.08 nm, with a frequency offset of approx. 200 MHz to minimize the impact of the DC response of the electronics at the receiver.

The laser modules can output the signal directly after the cavity and the MOPA; see Fig. 4. The output signals right after the cavity give us the opportunity to measure a reference phase PSD and its corresponding spectral width. Finally, after the signal and the LO laser modules (cavity plus MOPA), we add an additional erbium doped fiber amplifier (EDFA). The EDFAs are commercial dual-stage low-noise pre-amplifiers (noise figure approx. 5 dB) operated at maximum pump currents, corresponding to a target output power of approx. 15 dBm. Variable optical attenuators (VOAs) placed before the EDFAs allow to tune the input power P_{in} and thereby to vary the SNR to the receiver.

The two optical carriers are then combined in a 50:50 coupler and injected into the two inputs of a 43 GHz balanced photodetector used for coherent signal detection. A second set of VOAs at each photodetector inputs allows to keep the input power into each photodetector arm constant to $P_{PD} = 0.64$ mW.

The photodetector is followed by a digital storage oscilloscope (DSO) performing the A/D conversion. The DSO has a bandwidth of 13 GHz and operates at a sampling rate of $F_s = 40$ GHz. The sampled signal is then stored for offline signal processing. The memory of the sampling scope allows us to store $K = 256 \times 10^6$ samples. This results in the minimum phase PSD frequency of $f_{min} = F_s/K = 156$ Hz and maximum PSD frequency of $f_{max} = F_s/2 = 20$ GHz. Similar to the numerical results, we use the EKF- and Eq. (9) (\tan^{-1})-based phase estimation methods.

In Fig. 5, the phase PSD is shown as a function of frequency. To obtain the reference phase PSD, the laser outputs (signal

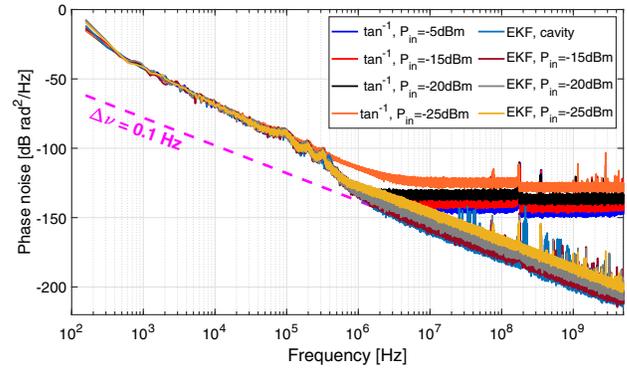


Fig. 5. Experiments: phase power spectral density as a function of frequency using different phase estimation methods and input power to the EDFA.

and LO) right after the cavities are measured. We use the EKF method for phase measurement due to its higher accuracy. Assuming equal contributions from the signal and the LO laser, the obtained phase PSD is divided by a factor of two. The reference phase PSD illustrates that the quantum limited laser linewidth (Schawlow–Townes linewidth) is around 0.1 Hz.

Next, the input signal and the LO power to the EDFA amplifiers is decreased from -20 to -15 dBm while keeping the signal power to the balanced receiver at a constant level. A clear impact of amplifier noise on phase fluctuation and thereby the corresponding phase PSD is observed for the \tan^{-1} phase estimation method. Moreover, the impact of amplifier noise on the phase PSDs agrees well with the numerical simulations shown in Fig. 2.

For the phase PSD obtained using the EKF phase estimation method, the impact of amplifier noise is observable for input power levels ranging from -25 to -15 dBm. However, we stress that the EKF is able to filter out a significant amount of amplifier noise affecting the phase, thus resulting in PSDs that resemble the reference to a high degree. This is also in accordance with the numerical results. For input signal power of -10 dBm or higher, we did not observe any difference compared to the reference PSD. We have therefore not plotted it.

Finally, in Fig. 6, we compute the corresponding spectral broadening, induced by the EDFA, with respect to the signal after the MOPA, and plot it as a function of input signal power to the EDFA. Spectral broadening is computed by the numerical integration of the phase PSD and is denoted by $\Delta\nu_a^{\tan^{-1}, \text{int.}}$ and $\Delta\nu_a^{\text{EKF}, \text{int.}}$. For the reference, we also plot theoretically minimum achievable spectral broadening, i.e., $\Delta\nu_a$ and $\Delta\nu_a^{\text{MAP}}$.

It is worth noting that the computed spectral broadening $\Delta\nu_a^{\tan^{-1}, \text{int.}}$ closely follows the theoretical limit $\Delta\nu_a$. It is also

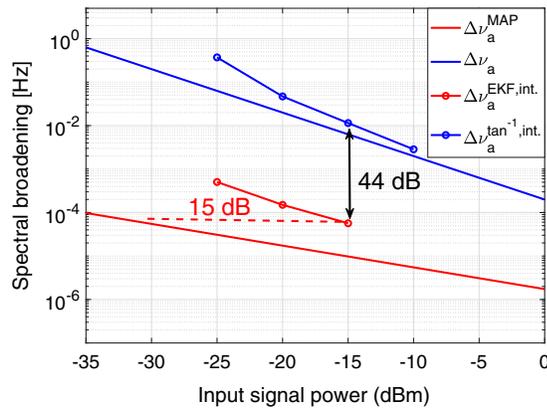


Fig. 6. Experiments: spectral broadening as a function of input signal power, P_{in} , for different phase estimation methods.

observed that the spectral broadening obtained using the EKF phase estimation method, $\Delta\nu_a^{\text{EKF,int.}}$, is significantly lower compared to $\Delta\nu_a^{\tan^{-1},\text{int.}}$. This is in accordance with the numerical results. By employing the EKF-based phase estimation method, a reduction of 44 dB in terms of spectral broadening is achieved, as illustrated in Fig. 6. Finally, $\Delta\nu_a^{\text{EKF,int.}}$ exhibits a penalty of approximately 15 dB compared to $\Delta\nu_a^{\text{MAP}}$. The penalty may be attributed to the fact that the state-space model employed for phase estimation assumes that the laser is dominated only by intrinsic noise sources.

5. CONCLUSION

The impact of amplifier noise on phase fluctuation and corresponding spectral broadening of the incoming signal has been investigated theoretically, numerically, and experimentally. We have shown that a heterodyne signal detection in combination with the EKF is a realization of a practical optimum phase measurement method in the presence of amplifier noise. The proposed phase measurement method has a relatively small penalty compared to the quantum limit. Most importantly, a significant reduction of the impact of amplifier noise has been demonstrated, both numerically and experimentally, compared to when using a widely deployed (sub-optimal) phase estimation method.

6. FUTURE WORK

The analysis presented in the paper is valid only for coherent signals, which implies that the input to the amplifier is provided by a coherent laser source (coherent-state light). For nonclassical states, the analysis needs to be revised, which indeed is a very interesting follow-up subject.

Moreover, for future work, it would be highly beneficial to consider the real-time implementation of the proposed EKF phase estimation method. Such an implementation should be feasible, as the state-space model for the EKF is relatively simple (single state and measurement equation). However, to implement it at high frequencies (> 10 GHz), a high degree of parallelization would be needed, which may lead to an increased penalty compared to the quantum limit.

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