An Asynchronous Online Negotiation Mechanism for Real-Time Peer-to-Peer Electricity Markets

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Published in: IEEE Transactions on Power Systems

Link to article, DOI: 10.1109/TPWRS.2021.3111869

Publication date: 2022

Document Version Peer reviewed version


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Abstract—Participants in electricity markets are becoming more proactive owing to the fast deployment of distributed energy resources (DERs) and the further development of demand-side management (DSM), which also boosts the emergence of Peer-to-Peer (P2P) market mechanisms. Moreover, the market is also required to operate in a real-time scheme in response to changes in generation and load to maintain power balance. It is highly desirable to propose and analyse novel approaches suitable for real-time P2P market mechanisms. These are challenging since most often involving a heavy computational burden, while the time available for negotiation in real-time is very short. Our core contribution is to design and analyse a novel asynchronous online optimization framework and related real-time P2P market negotiation mechanism, which can greatly reduce the computation and communication burden from two aspects. First, a novel online consensus alternating direction method of multipliers (ADMM) algorithm is proposed. It significantly reduces computation since only one iteration is performed for each agent at every time period. Second, the market operates in an asynchronous mode so that all agents can freely trade without waiting for idle or inactive neighboring agents. The sublinear regret upper bound is proved for our asynchronous online algorithm, which indicates that social welfare in the market can be maximized in the long run on average (over time). Simulations show that our algorithm enjoys good convergence performance, robustness, and fairness.

Index Terms—Real-time P2P markets, asynchronous online consensus ADMM, forgetting factor, non-stationary regret.

NOMENCLATURE

Manuscript received December 14, 2020; revised March 29, 2021, July 6, 2021 and August 30, 2021; accepted September 5, 2021. This work was supported in part by the National Key R&D Program of China (2020YFB1005601, 2019QY(Y)0602), in part by the Natural Science Foundation of China (62002011, 61972019, 61772538, 61532021, 61903179, 61873118, 91646203), in part by the Science, Technology and Innovation Commission of Shenzhen Municipality under Grant RCBS2020071414918317, in part by the Zhejiang Provincial Nature Science Foundation of China under Grant LZ21F030004, in part by the Key-Area Research and Development Program of Guangdong Province under Grant No. 2018A010107002, in part by the Stable Support Plan Program of Shenzhen Natural Science Fund under Grant No. 2020062000517470702, in part by the Zhejiang Soft Science Research Program 2021C35044. Paper no. TPWRS-02048-2020. (Corresponding author: Zaiyue Yang.)

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receivers. Recently, a novel idea of electricity markets has emerged: these so-called Peer-to-Peer (P2P) electricity markets rely on multi-bilateral trades among participants [2]–[9], who work in a spontaneous and collaborative manner without requiring a central operator to centrally schedule. Employing P2P market mechanisms can yield many advantages, e.g., empowerment of participants, product differentiation, resilience or reliability of power system, and protection of privacy [2], [5]. Existing works about P2P markets mainly focus on the following issues: reallocation of costs [3], product differences [4], [9], dispatch fairness [8], communication properties [6] and costs [7].

Meanwhile, in actual operation, due to changes in the weather, potential power system accidents, renewable power generation uncertainty, demand-side load variations, and other contingencies, the actual generation and load may have a large deviation from the schedule obtained at the day-ahead market (DAM) stage, power balance hence ought to be restored. Thus, it is required to rapidly reschedule the transactions among participants to maintain power balance in response to these changes, hence calling for real-time markets (RTMs) [10]–[13]. At present, electricity markets in many countries, such as Nord Pool in Northern Europe, PJM and ECROT in the United States, and NEM in Australia, mainly adopt a combination of DAMs and RTMs to achieve two-stage separate settlement. Since RTMs are an important part of the electricity market, it is necessary to also operate the P2P market mechanism in a real-time scheme, however, which is quite technically challenging. Compared with day-ahead and intraday markets, the negotiation time for RTMs before operation is usually set to 5 minutes or even shorter [14]. Thus, RTMs demand fast computation and communication. However, a P2P market mechanism requires a very large amount of information to be exchanged, much greater than that required for a centralized market mechanism [6], [7], [15]. In a real-time context, such exchanges run the risk of not having enough time to complete the negotiation before the deadline. Eventually, the main challenge for designing a real-time P2P electricity market negotiation mechanism is how to reduce the computation and communication complexity of P2P mechanisms so that it can be deployed in a real-time scheme. 

Note that some works already considered deploying P2P mechanisms in real-time operation [16]–[19]. For instance, in [16], bilateral contract networks were proposed as a new market design for P2P energy trading in real-time markets and authors designed a price-adjustment process to implement the negotiation mechanism; The authors in [17] presented a two-phase operation algorithm: Phase I focuses on the day-ahead scheduling of generation and controllable DERs, whereas Phase II is developed for hour-ahead or real-time operation of P2P energy trading between individual prosumers; A P2P electricity trading model for locally buying and selling electricity among plug-in hybrid electric vehicles was proposed in [18]. An iterative double auction mechanism was proposed to maximize social welfare in day-ahead and real-time stages; A real-time transaction distribution network market is established in [19] to enable surplus renewable electricity P2P trading to be fulfilled among different end users. However, these works ignored the computational efficiency trouble of the real-time P2P market, and did not consider how one could take advantage of previous negotiations to make current results better.

Table. I gives a comparison between our proposed asynchronous online consensus ADMM and the state-of-the-art algorithms. The standard alternating direction method of multipliers (ADMM) [8], consensus-based ADMM [3], [7], [20], relaxed consensus+innovation (RCI) [6], [9], and primal-dual gradient method [4] all require to operate in a synchronous mode to maximize social welfare. But they may need to perform multiple iterations to converge in each time period, and run the risk of exceeding the deadline of negotiation time period before reaching a convergence. The price-adjustment process [16], iterative double auction [18], and continuous double auction (CDA) [21], [22] match buyers and sellers who submit bid prices and amounts in current time period, but they cannot ensure that social welfare is maximized. For the online matching CDA method [23], although it can also implement asynchronous transactions, and each agent only takes one match in every time period, it suffers from sub-optimality and there is a upper-bounded gap between social welfare maximization.

As it may be too expensive to optimally solve a real-time P2P market at every short time period, one needs to think of appropriate and computationally cheaper approaches. In this work, we propose a novel asynchronous online consensus ADMM algorithm to enable the real-time P2P electricity market negotiation mechanism. Compared with the above existing methods, the computation and communication burden of P2P mechanisms are reduced via two aspects. First, we innovatively design an online optimization framework to enable the real-time P2P market negotiation mechanism, which can maximize social welfare in the long run on time average – this is computationally lighter and more tractable. Despite many large-scale applications of online optimization, such as network resource allocation [24], [25], demand response [26], [27], and energy management [28], our work is the first application for P2P electricity markets. The number of operations and communications among agents can be heavily reduced, and the complexity of our approach is less than other approaches since the online consensus ADMM algorithm only performs one iteration for every single agent in each time period. Second, the market operates in an asynchronous mode. We first propose the synchronous online consensus ADMM, which requires each agent to wait to receive all bid prices and quantities from neighboring agents to run the algorithm. Then, we improve it to an asynchronous mode, which means that agents can freely trade without waiting to receive bid prices and quantities from inactive or idle neighboring agents, so that the computational efficiency will not be restricted by low-reliability agents. Although the asynchronous online consensus ADMM cannot achieve the optimal solution in every time period, social welfare can be maximized in the long run on time average by proving the sublinear regret upper bound.

Our contributions mainly lie as follows.

- We propose an online optimization framework for P2P real-time electricity markets, meaning that, instead of
fully solving a complete P2P market at each time period, we use a recursive approach with a single iteration/optimization performed at each time step based on new information at the current time. The computation complexity of the P2P mechanism is highly reduced since only one iteration is performed for each agent at every time period (while offering some performance guarantees).

- We first propose the online consensus ADMM algorithm to enable the market negotiation mechanism. However, the synchronous mechanism requires each agent to wait to receive all bid prices and quantities from neighboring agents to update, and the computational efficiency may be highly limited by low-reliability agents. To overcome this drawback, we further improve it to an asynchronous mechanism, where agents can freely trade with each other without waiting for idle or slow neighboring agents.

- The sublinear non-stationary regret upper bound for our asynchronous online consensus ADMM algorithm is proved, which implies that social welfare will be maximized in the long run on time average.

The rest of the paper is organized as follows: Section II presents the real-time P2P electricity market model. Section III proposes the synchronous and asynchronous market negotiation mechanisms, followed by the regret and market properties analysis in Section IV. Numerical results and comparisons are presented in Section V. Finally, conclusions and limitations are drawn in Section VI.

### II. REAL-TIME P2P ELECTRICITY MARKET MODEL

We consider a real-time P2P electricity market with a set $\Omega$ of $N$ agents, who can be producers, consumers or prosumers over a period of time $T$. The time period for negotiation before operation and delivery is set to 5 minutes. Compared with a centralized market, a P2P electricity market is much more decentralized, which relies on multi-bilateral direct trades among a community of agents with flexible consumption or production.

The network is divided into physical layer and virtual layer. The physical layer includes electrical connection of all agents, whereas the virtual layer is the communication network which is used for information exchange among agents. We assume that the communication network is predetermined, and the communication subsets $\omega_n$ for each agent $n$ are forever preserved as fixed communication constraints.

As it is classically done, agents are supposed rational as in [29], i.e., always objectively taking the most beneficial decisions, and non-strategic, i.e., not anticipating actions and reactions of other agents.

#### A. Peer-to-Peer Trading

![Image](https://example.com/image.png)

To model the trading process, the total sold or purchased power $E_{n,t}$ of agent $n \in \Omega$ at time period $t$ is split into a summation of bilaterally transaction quantities with a set of neighboring agents $m \in \omega_n$ as

$$E_{n,t} = \sum_{m \in \omega_n} E_{nm,t}, \quad \forall n \in \Omega, \quad t = 1, ..., T \quad (1)$$

The neighboring agents of agent $n$ are the agents who are connected to agent $n$ in the communication network. A positive value of $E_{nm,t}$ corresponds to a sale/production and a negative value to a purchase/consumption. To tighten notations, $E_{n,t} = \{E_{n,1,t}, ..., E_{n,T,t}\}$ is used to denote the whole transactions of agent $n$ at time period $t$. The total sold or purchased power of agent $n$ at time period $t$ is constrained as

$$E_{n,t} \leq E_{n,t} \leq E_{n,t}, \quad \forall n \in \Omega, \quad t = 1, ..., T \quad (2)$$

Here, for renewable generators, the upper bound $E_{n,t}$ is set to the smaller value of actual power generation and maximal capacity. Each agent can be a producer ($0 \leq E_{n,t} \leq E_{n,t}$), a consumer ($E_{n,t} \leq E_{n,t} \leq 0$) or a prosumer ($E_{n,t} \leq E_{n,t} \leq 0$). Hence, $E_{nm,t}$ are constrained as

$$\begin{cases} E_{n,t} \geq E_{nm,t} \geq 0, & \forall (n,m) \in (\Omega_p,\omega_n), \quad t = 1, ..., T, \\
E_{n,t} \leq E_{nm,t} \leq 0, & \forall (n,m) \in (\Omega_c,\omega_n), \quad t = 1, ..., T, \\
E_{n,t} \leq E_{nm,t} \leq E_{n,t}, & \forall (n,m) \in (\Omega_{ps},\omega_n), \quad t = 1, ..., T, \end{cases} \quad (3)$$

where $\Omega_p$, $\Omega_c$ and $\Omega_{ps}$ are the set of producers, consumers and prosumers, respectively. Finally, the market equilibrium between production and consumption is represented by a set of reciprocity balance constraints as

$$E_{nm,t} + E_{mn,t} = 0, \quad \forall (n,m) \in (\Omega,\omega_n), \quad t = 1, ..., T. \quad (4)$$

We denote the production cost and consumer utility of agent $n$ at time period $t$ by convex function $NC_{n,t}$, which is supposed to denote cost net of benefit (i.e., cost minus benefit). For conventional or renewable generators, the function $NC_{n,t}$ reflects the costs by producing the power $E_{n,t} > 0$; while for consumers, the function represents the level of

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Category</th>
<th>Computational effort per iteration</th>
<th>Number of iterations per time period</th>
<th>Convergence performance for social welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard ADMM [8]</td>
<td>distributed/synchronous</td>
<td>high</td>
<td>high</td>
<td>maximize in each time period but maybe time out</td>
</tr>
<tr>
<td>Consensus-based ADMM [3], [7], [20]</td>
<td>decentralized/synchronous</td>
<td>high</td>
<td>high</td>
<td>maximize in each time period but maybe time out</td>
</tr>
<tr>
<td>Primal-dual gradient [4]</td>
<td>decentralized/synchronous</td>
<td>low</td>
<td>high</td>
<td>maximize in each time period but maybe time out</td>
</tr>
<tr>
<td>Price-adjustment process [16]</td>
<td>decentralized/asynchronous</td>
<td>high</td>
<td>medium</td>
<td>cannot guarantee social welfare maximization</td>
</tr>
<tr>
<td>Iterative double auction [18]</td>
<td>distributed/asynchronous</td>
<td>high</td>
<td>medium</td>
<td>cannot guarantee social welfare maximization</td>
</tr>
<tr>
<td>Online matching CDA [21], [22]</td>
<td>distributed/asynchronous</td>
<td>low</td>
<td>medium</td>
<td>cannot guarantee social welfare maximization</td>
</tr>
<tr>
<td>Asynchronous online consensus ADMM</td>
<td>distributed/asynchronous</td>
<td>high</td>
<td>low</td>
<td>maximize in the long run on time average</td>
</tr>
</tbody>
</table>
comfort/satisfaction obtained by consumers by consuming the power $E_{n,t} < 0$ [30]. A positive value of $NC_{n,t}$ corresponds to a cost, and a negative value to benefit. Fig. 1 gives the illustrative functions $NC_{n,t}$ for generators and consumers.

![Illustrative functions NC for (a) generators, (b) consumers.](image)

Fig. 1: Illustrative functions $NC$ for (a) generators, (b) consumers.

### B. Social Welfare Maximization Problem

Since agents do not know the information from the future, or in other words, the information is incomplete, they can only make decisions based on current updated and past information. Thus the market has to be running in an online manner instead of optimizing the market for all $T$. To be more specific, at the beginning of the time period $t$, each agent will be aware of the current updated information, including cost or utility function coefficients, power boundaries, demand requirements, and renewable generation. After obtaining the information, agents will decide the trading prices and amounts between neighboring agents for later time until the next decision is made. All agents negotiate with each other to reach an agreement on their final transactions while maximizing the social welfare. Since the cost or utility functions of all agents are convex, the equilibrium point will exist and be unique [31]. Therefore, mathematically speaking, the social welfare maximization problem can be regarded as an equivalent minimization problem formulated as

$$\min_{\{E_{n,t}\}} \sum_{t=1}^{T} \left( \sum_{n \in \Omega} NC_{n,t}(E_{n,t}) \right)$$

subject to

$$1 - \epsilon \leq V_i \leq 1 + \epsilon, \quad \forall i \in \mathcal{N},$$

$$1 - \epsilon \leq V_i \leq 1 + \epsilon, \quad \forall i \in \mathcal{N},$$

If we want to exactly and optimally solve above problem, a double loop algorithm is needed. In the outer loop, the function $NC_{n,t}$ will change over time, while in the inner loop, a decentralized algorithm will run iteratively for agents until the balance constraint (4) is satisfied. However, in a P2P market, the number of communications will increase with the square of the number of agents [7]. Thus, for the problem (5), if we want to obtain the optimal solution in each time period, all agents have to complete many iterations, which results in a very heavy communication and computation burden. Thus, one needs to think of appropriate and computationally cheaper approaches. To this end, we propose an asynchronous online optimization framework for real-time P2P market negotiation mechanisms, which is more practical and applicable.

### C. Extension to Include Physical Network Constraints

In this section, we introduce how to include the physical network constraints. To this end, we propose to add a system operator (SO) to the market, who helps complete the calculation of power flows and bus voltage magnitudes by solving an optimization problem converted from the linearized LinDistFlow model [32], [33]. The SO only behaves as a single agent in the market, and would not intervene the P2P trading among agents. Thus, the P2P trading structure is not broken.

To be specific, in a distribution network, the power flow model can be formulated as below [32]

$$p_i = P_i - \sum_{k \in \Delta(i)} P_k + r_i l_i, \quad \forall i \in \mathcal{N}$$

$$q_i = Q_i - \sum_{k \in \Delta(i)} Q_k + x_i l_i, \quad \forall i \in \mathcal{N}$$

$$v_i = v_{\pi(i)} + 2(r_i P_i + x_i Q_i) - (r_i^2 + x_i^2) l_i, \quad \forall i \in \mathcal{N}$$

$$l_i = \frac{P_i^2 + Q_i^2}{v_i}, \quad \forall i \in \mathcal{N}.$$

Here, $p_i/q_i$ is the active/inactive power injection in bus $i$, $P_i/Q_i$ is the active/inactive power flow from $i$ to $\pi(i)$, $r_i/x_i$ is the line resistance/reactance between $i$ and $\pi(i)$. Let $I_i$ be the complex current flowing from $i$ and $\pi(i)$, $V_i$ be the complex voltage of bus $i$. We assume the complex voltage $V_0$ at the substation node is given and fixed, and we define $I_i := |I_i|^2$, $v_i := |V_i|^2$. The active power injection $p_i$ equals to the cumulative active power of all agents in bus $i$, i.e., $\sum_{n \in \mathcal{N}} E_{n,t}$.

Constraint (6d) is nonconvex since it has quadratic terms. To eliminate this non-convex term, we can linearize the power flow model constraints using LinDistFlow model as below

$$p_i = P_i - \sum_{k \in \Delta(i)} P_k, \quad \forall i \in \mathcal{N}$$

$$q_i = Q_i - \sum_{k \in \Delta(i)} Q_k, \quad \forall i \in \mathcal{N}$$

$$V_i = V_{\pi(i)} - \frac{r_i P_i + x_i Q_i}{V_0}, \quad \forall i \in \mathcal{N}$$

where, $\epsilon$ is the maximum allowed voltage deviation and usually set to 0.05. Above LinDistFlow model constraints can be equivalently converted into an optimization problem for SO as below

$$\{P, Q, V\} = \arg \min_{\{P, Q, V\}} \sum_{i \in \mathcal{N}} \left[ p_i - P_i + \sum_{k \in \Delta(i)} P_k \right]^2$$

$$+ \left[ q_i - Q_i + \sum_{k \in \Delta(i)} Q_k \right]^2$$

subject to

$$1 - \epsilon \leq V_i \leq 1 + \epsilon, \quad \forall i \in \mathcal{N}$$

Above optimization problem (8) for SO is convex and easy to solve, thus, the computational efficiency of the market will not be highly reduced.
Compared with current market-based operation of electric energy systems, where system operators (and/or market operators) are required to centrally solve the economic dispatch problem or optimal power flow (OPF) problem, the SO does not directly intervene in the trading process. It behaves like an agent in the P2P market, being involved in each step of the solution approach and algorithm, by coupling the P2P market to a network-constrained feasible operation outcome. Unlike the other agents, the SO is non-profit, and just helps the market to a network-constrained feasible operation outcome.

This is in line with the original vision of Wu and Varaiya [34], for which the SO has to be in the loop, in the case of a multi-bilateral trading based approach to electricity markets (so, a P2P setup). Also, for convex class of problems (so, DC linearization of the OPF problem, LinDistFlow, etc.) having (so, a P2P setup). Also, for convex class of problems (so, DC linearization of the OPF problem, LinDistFlow, etc.) having this decentralized approach with the SO as an agent focused on networked constrained operation can be shown to be equivalent to a centralized setup.

III. ASYNCHRONOUS REAL-TIME P2P ELECTRICITY MARKET NEGOTIATION MECHANISM

In this section, we first propose a synchronous real-time P2P market mechanism, but it is highly restricted by low-reliability agents. To overcome this drawback, it is further improved into an asynchronous market mechanism.

A. Synchronous Market Mechanism

Since the market operates in an online framework, the problem (5) is decomposed into each single time period. The problem of maximizing social welfare in the time period $t$ is formulated as below

$$
\min \sum_{n \in \Omega(t)} (NC_{n,t}(E_n)) + \sum_{m \in \Omega_n} \frac{\eta}{2} (E_{nm,t-1} - E_{nm})^2
$$

s.t. \hspace{0.5cm} (1) - (4), \hspace{0.5cm} (9)

where $\eta$ is the penalty factor and the Bregman divergence term $\frac{\eta}{2} (E_{nm,t-1} - E_{nm})^2$ is appended to make the results close to previous value $E_{nm,t-1}$ in order to speed up the convergence process [35], [36]. The rationale behind the Bregman divergence term is taking advantage of previous negotiation results to make current results better so that the gap between optimal solutions can be bounded. A novel online consensus ADMM algorithm is proposed to implement the real-time P2P market negotiation mechanism, which produces the following updates:

- **Power Updates**: Each agent $n$ updates their transaction power with neighboring agents by solving the following individual optimization problem with constraints (1)-(3):

  $$
  E_{n,t} = \arg\min_{E_n} NC_{n,t}(E_n) + \sum_{m \in \Omega_n} \lambda_{nm,t-1}(F_{nm,t-1} - E_{nm}) + \frac{\rho}{2} (F_{nm,t-1} - E_{nm})^2 + \frac{\eta}{2} (E_{nm} - E_{nm,t-1})^2,
  $$

  $\lambda_{nm,t-1} = \{\lambda_{n1,t-1}, ..., \lambda_{nm,t-1}\}$ is used to represent the whole transaction prices of agent $n$ to neighboring agents $m \in \Omega_n$ for time period $t - 1$. $F_{nm,t-1}$ is the consensus variable defined as $\frac{E_{nm,t-1} - E_{nm,t-1}}{2}$ and $F_{n,t} = \sum_{m \in \Omega_n} F_{nm,t-1}$. Then, each agent broadcasts $E_{n,t}$ to neighboring agents.

- **Price Updates**: All agents update their prices to neighboring agents $m \in \Omega_n$ as:

  $$
  \lambda_{nm,t} = \lambda_{nm,t-1} - \rho (E_{nm,t} + E_{nm,t})/2. \hspace{0.5cm} (11)
  $$

The online consensus ADMM operates in a synchronous manner, which means each agent has to wait to receive all bid prices and quantities from neighboring agents for updates. Under the synchronous mechanism, the market efficiency will be highly restricted by slow and low-reliability agents.

B. Asynchronous Market Mechanism

In order to overcome the drawback of the synchronous mechanism, we propose a novel asynchronous negotiation algorithm for the real-time P2P market. Certain methods can also speed up the market negotiation process, e.g., adaptive penalty factor [37] and alternative stopping criterion [7]. The adaptive penalty factor method focuses on reducing the number of iterations, while the alternative stopping criterion concentrates on reducing the communication times. However, neither of them can deal with the trouble causing by low-reliability and poor-quality agents.

![Fig. 2: Description diagrams of synchronous and asynchronous mechanisms.](image-url)
agent, the market efficiency will be heavily reduced. Thus, the synchronous mechanism is not suitable for the real-time P2P market. To overcome this issue, we propose a novel asynchronous negotiation mechanism for the real-time P2P market, where agents can freely trade with each other without waiting for inactive or idle neighboring agents.

A simple example is given to show the asynchronous market mechanism. In Fig. 2(b), in the time period \( t = 1 \), agents 1-3 are active and they will negotiate with each other to determine the transaction prices and quantities without waiting for agent 4, whose transaction prices and quantities between agents 1-3 will remain unchanged. Then, in the time period \( t = 2 \), all four agents are active, and they can trade with each other. In the asynchronous mechanism, no agent has to be synchronized with all neighboring agents, nor does it need to wait for the slow lengths. Compared with the synchronous mechanism, the length of the time period can be shorter, and the market can run more frequently.

Let \( A_{n,t} \subseteq \omega_n \) denote the set of active neighboring agents of agent \( n \) at time period \( t \). For example, in Fig. 2, \( A_{1,1} = \{2, 3\} \) and \( A_{2,2} = \{1, 3, 4\} \). We use \( A_{n,t}^c \) to denote the complementary set of \( A_{n,t} \), i.e., \( A_{n,t} \cap A_{n,t}^c = \emptyset \) and \( A_{n,t} \cup A_{n,t}^c = \omega_n \). We define \( E_{n,t}^A \) to be the whole transaction quantities of agent \( n \) for active neighboring agents \( m \in A_{n,t} \) at time period \( t \).

Based on the synchronous online consensus ADMM, we further propose the asynchronous online consensus ADMM. Let \( t_n \) be the last time period before \( t \) where agent \( n \) was active. \( v \in (0, 1) \) is the forgetting factor, which has been proposed and developed in [38]–[41]. The forgetting factor is given to control the amount of older information that is used for current computation. A value of 1 results in no forgetting while decreasing values increase the amount of forgetting. Values slightly less than 1 are generally preferred. The asynchronous online consensus ADMM algorithm produces the following updates.

- **Price Updates**: All agents update their prices to neighboring agents \( m \in \omega_n \) as:

\[
\lambda_{nm,t} = \begin{cases} 
\lambda_{nm,t-1} - \rho \frac{E_{nm,t} + E_{nm,t-1}}{2}, & \forall m \in A_{n,t} \\
\lambda_{nm,t-1}, & \forall m \in A_{n,t}^c
\end{cases}
\]

(14)

However, since each pair of two agents only have one negotiation in each time period, the power balance between two agents may not be balanced, i.e., \( E_{nm,t} + E_{nm,t-1} \neq 0 \). To address this problem, we design a projection-based power update process to ensure the power balance.

- **C. Projection-based Power Update Process**

In the electricity market, it is necessary to ensure the balance for trade among agents. Thus, a projection-based power update process is designed to determine the final transaction quantities. This process is executed after the power and price updates (12)-(14). In this process, iterations are indexed with \( k \).

First, \( \hat{E}_{nm,t}^{k+1} \) is set to be the intermediate value of the transaction quantities between active agents \( n \) and \( m \) from last iteration as

\[
\hat{E}_{nm,t}^{k+1} = \left( \hat{E}_{nm,t}^{k} - \hat{E}_{nm,t}^{k-1} \right) / 2, \quad \forall m \in A_{n,t}.
\]

(15)

Here, the initial value \( \hat{E}_{nm,t}^{1} \) is obtained after the power update (12), i.e., \( \hat{E}_{nm,t+1}^{1} = E_{nm,t+1} \). Then the total power is obtained as

\[
\hat{E}_{n,t}^{k+1} = \sum_{m \in A_{n,t}} \hat{E}_{nm,t}^{k+1} + \sum_{m \in A_{n,t}^c} E_{nm,t}.
\]

(16)

After projecting into the feasible region (2), the projected power \( E_{n,t}^{proj,k+1} \) is updated as

\[
E_{n,t}^{proj,k+1} = \max \left\{ \min \left\{ \hat{E}_{n,t}^{k+1}, E_{n} \right\}, E_{n,t} \right\}.
\]

(17)

Then the trade \( \hat{E}_{nm,t}^{k+1} \) to active neighboring agent \( m \) is updated as

\[
\hat{E}_{nm,t}^{k+1} = \frac{\hat{E}_{nm,t}^{k+1} - E_{n,t}^{proj,k+1}}{E_{n,t}}, \quad \forall m \in A_{n,t}.
\]

(18)

Finally, all active agents send the updated quantities \( \hat{E}_{nm,t}^{k+1} \) to its active neighboring agents, and check if the balance equation \( \hat{E}_{nm,t}^{k+1} + \hat{E}_{nm,t}^{k-1} < \varepsilon \) is satisfied, where \( \varepsilon \) is the allowed maximal violation; if not, repeat the processes (15)-(18) until balance.

For the projection-based power update process, there is only one situation where agents cannot reach transaction balance, which is that the maximum power generation of all generators still cannot meet the minimum demand of all consumers. However this situation probably will not happen in actual operation unless there is a serious malfunction occurs in the power system. Therefore, as long as the market has feasible solutions, the power update process will ensure balances for trading among agents.

The asynchronous negotiation mechanism for the real-time P2P electricity market is detailed in Fig. 3.
This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TPWRS.2021.3111869, IEEE Transactions on Power Systems

Fig. 3: Flowchart of the asynchronous negotiation mechanism for real-time P2P electricity market

IV. MARKET ANALYSIS

In this section, we first analyze the convergence performance of the online algorithm. Under some standard assumptions, the sublinear regret upper bound for the asynchronous online consensus ADMM algorithm is proved, which indicates that social welfare will be maximized in the long run on time average. After that, four desirable properties of market mechanisms are analyzed.

A. Regret Analysis

The convergence performance of an online optimization algorithm is usually measured by the regret, which is the accumulated gap between the online solutions and the best solutions in hindsight [35], [42]–[44]. The regret for our problem is formulated as below.

$$R(T) = \sum_{t=1}^{T} \left( \sum_{n \in \Omega} NC_{n,t}(\hat{E}_{n,t}) - \sum_{n \in \Omega} NC_{n,t}(E_{n,t}^*) \right)$$  

where $\hat{E}_{n,t}$ is the online solution of agent $n$ in the time period $t$ using our asynchronous online consensus ADMM, and $E_{n,t}^*$ is the optimal solution. The goal of the online algorithm is to generate decisions making the regret $R(T)$ small. If the regret is bounded by a sublinear function of $T$, e.g., $R(T) = \mathcal{O}(\sqrt{T})$ [45], then the average regret converges to zero as $T \to \infty$, which indicates that the sequence of the decisions converges to the best fixed decisions in hindsight.

Before presenting the results, some needed rational assumptions are introduced to derive the sublinear regret upper bound to be presented later in Theorem 1.

Assumption 1.

(a) $G_{n,t}$ are convex with bounded subgradients for $m \in \omega_n$, i.e., $\frac{\partial G_{n,t}(E_{n,t})}{\partial E_{n,m,t}} \leq S$, $\forall(n,m) \in (\Omega, \omega_n)$, with $S$ being a positive constant.
(b) The initial values are set to zero, i.e., $\lambda_{n,1} = 0$ and $E_{n,0} = 0$, $\forall(n,m) \in (\Omega, \omega_n)$.
(c) The gap between optimal solutions and initial ones are bounded, i.e., $(E_{nm,t}^* - E_{nm,1})^2 \leq D_1$ and $(E_{nm,t}^* - F_{nm,t})^2 \leq D_2$, $\forall(n,m) \in (\Omega, \omega_n)$, with $D_1$ and $D_2$ being positive constants.
(d) The cost gap between online solutions and optimal ones are bounded, i.e., $NC_{n,t}(E_{n,t}) - NC_{n,t}(E_{n,t}^*) \leq J$, $\forall(n \in \Omega, J$ being a positive constant.
(e) Path variation is defined as the temporal change of the optimal solutions sequence, i.e., $V_{nm,T} = \sum_{t=1}^{T} |E_{nm,t}^* - E_{nm,t+1}^*|$. The optimal solutions do not change dramatically, or in mathematical sense, the path variation is bounded as $V_{nm,T} \leq P$, $\forall(n,m) \in (\Omega, \omega_n)$, with $P$ being a positive constant called variation budget.
(f) The electricity prices do not change dramatically, or in mathematical sense, the price variation between two time periods is bounded as $|\lambda_{n,m,t+1} - \lambda_{n,m,t}| \leq \Lambda$, $\forall(n,m) \in (\Omega, \omega_n)$, with $\Lambda$ being a positive constant.
(g) Let $\tau > 0$ be a maximal tolerable delay for all agents. For all $(n,m) \in (\Omega, \omega_n)$ and time periods $t \geq 0$, it must be that $m \in \mathcal{A}_{n,t} \cup \mathcal{A}_{n,t-1} \cdots \cup \mathcal{A}_{n,\max\{t-\tau+1,1\}}$.

In Assumption 1, (a) is an assumption that many may see as simplifying the reality of prosumers’ marginal utility functions. The issue of potential non-convexity in market clearing approaches is general in the electricity market literature, and not restricted to the case of the P2P market studied here. It should be noted the fact that function $G_{n,t}$, which acts as a smoother over the previous cost and utility functions, may actually help in having more well-behaved functions as input to the current optimization process. (b) is the settings of initial values. (c) and (d) are also reasonable since the online solutions and optimal solutions cannot be infinite. (a)–(d) are usually required in the online optimization settings. Generally, in the field of online optimization research, it is inevitable to make some assumptions about the real problems, and our assumptions as similar in essence to, e.g., [44], [46]–[49]. For the specific case of (e), it is reasonable to think that the optimal solutions do not change drastically or infinitely from one step to the next, otherwise, the power system would probably break down. Thus, we place mild restrictions on the possible variation of the optimums, as is also proposed in previous works [44], [47], [48]. (f) is reasonable in the context of electricity markets since electricity prices cannot be infinite. Finally for (g), any unbounded delay will jeopardize the market social welfare convergence. Therefore, throughout this paper, we assume that the asynchronous delay in this market is bounded. If it was not, this would mean an agent is not active anymore, and should be removed from the market. We then follow the popular asynchronous model [50] to make that assumption. It implies that every agent $n$ will trade with neighboring agents within the period $[t-\tau+1, t]$. In another word, the bid prices and quantities from neighboring agents received by each agent $n$ must be at most $\tau$ time periods delayed.

Bearing all the above in mind, the following theorem establishes the sublinear regret upper bound for the asynchronous online consensus ADMM algorithm.

Theorem 1. The asynchronous online consensus ADMM algorithm has the following sublinear regret upper bound by
setting \( \rho = \sqrt{T} \) and \( \eta = \sqrt{T} \).

\[
R(T) \leq \frac{N(N - 1)}{v^2} \left( \frac{D_1 + D_2 + \Lambda^2 + S^2}{2} + LP \right) \sqrt{T} + \frac{N J}{v^2(1 - v)} \tag{20}
\]

where \( L = \max_{n \in \Omega} 2 + (\tau_n - E_n) \).

Proof. See Appendix A.

Since the regret has \( O(\sqrt{T}) \) upper bound, we have \( \lim_{T \to \infty} \frac{R(T)}{T} = 0 \), which implies the accumulated gap between our online solutions and optimal ones are approaching to zero in the long run on time average. In the market context, regret can be regarded as the cumulative total cost gap between online solutions and optimal ones. Thus, the sublinear regret upper bound also indicates that social welfare will be maximized in the long run on time average.

B. Desirable Properties of Market Mechanism

It is important and necessary to evaluate a market mechanism by checking the four desirable properties, which are market efficiency\(^1\), incentive compatibility\(^2\), cost recovery\(^3\), and revenue adequacy\(^4\). Based on the Hurwicz theorem [51], no mechanism is capable of achieving all those properties at the same time.

1) Market efficiency: According to Theorem 1, we have \( \lim_{T \to \infty} \frac{R(T)}{T} = 0 \), which implies that social welfare and market efficiency will be maximized in the long run and on average (over time).

2) Incentive compatibility: The P2P market mechanism we have proposed is based on Locational Marginal Pricing (LMP), and the biggest limitation of LMP markets is that incentive compatibility cannot be satisfied anyway. Hence, the fact we cannot assure incentive compatibility is due to the pricing mechanism itself and not to the fact we are working with a P2P market. However, the LMP mechanism is widely considered in actual electricity markets in practice as, for example, the PJM market in the USA [14]. Although our design cannot guarantee incentive compatibility, revenue adequacy is satisfied, which is usually more important in electricity markets.

3) Cost recovery: The individual profit for an agent \( n \) in time period \( t \) is

\[
\sum_{m \in \omega_n} \lambda_{nm,t} E_{nm,t} - NC_{nt}(E_{nt}) \tag{21}
\]

Since the quadratic cost function is convex, monotonically increasing, and passing through the origin, the agent can always set \( E_{nt} = E_{nm,t} = 0 \) to avoid a negative profit. In other words, the agent does not participate in the market during the time period \( t \). Thus, the cost recovery is satisfied.

\^1\ Market efficiency is maximized when outcomes maximize social welfare.
\^2\ A mechanism is said incentive-compatible if every participant can maximize its objective just by acting according to its true preferences.
\^3\ Cost recovery implies that individual profit is non-negative.
\^4\ Revenue adequacy implies that there is no financial deficit in the market.

4) Revenue adequacy: From (14), the prices between each pair of two agents are identical, i.e., \( \lambda_{nm,t} = \lambda_{mn,t} \), and after the projection-based power update process, the quantities between each pair of two agents are balanced, i.e., \( E_{nm,t} + E_{mn,t} = 0 \). Thus, the revenue adequacy is satisfied.

Summing up the above, our proposed real-time P2P market mechanism satisfies most of the desirable properties.

V. SIMULATION RESULTS

Our online algorithms are tested on a dataset of wind power generation of 20 wind farms in Australia [52] to show the convergence performance. In order to better display performance, uniformly distributed stochastic parameter settings are applied. We use Matlab R2017b on a PC with 1.6 GHz Intel Core 4 Duo CPU and 8 GB memory to perform simulation, and solve the convex optimization problem by CVX Sedumi solver. Since MATLAB cannot implement parallel computing, the agents run the procedure sequentially, i.e., one agent will start running after the previous one has completed. We record the computational time of each agent, and the time for all agents to complete an iteration is the maximal value of the computational time of all agents. The decentralized mechanism is achieved in this way. In the communication network setting, each agent is freely able to trade with any other agent, or in other words, the communication adjacent subset \( \omega_n \) is the set \( \Omega \) minus \( n \).

A. Convergence Performance

We establish a market consisting of 20 conventional generators, 20 users, and 20 wind generators with actual wind power generation data. The convergence performance of the synchronous and asynchronous mechanisms is tested. At any given moment, an agent has only two states, i.e., it can only be either active or inactive, and these two states are mutually exclusive. Thus we believe the Bernoulli distribution is suitable for modeling the active rates of agents. We set the active rate as \( p_n \) and the idle rate as \( 1 - p_n \). The active rates of agents can be collected according to the type of agents.

For conventional thermal generators, the active rates can be obtained according to the generation plans; for consumers, active rates can be obtained based on electric usage habits; for renewable generators, when to enter the market depends on the weather condition, which can be roughly estimated from historical data. The probability of the number of consecutive delayed time periods is shown in Fig. 4 under different active rates. It can be seen from the figure that agents are extremely unlikely to be infinitely delayed.

We operate the synchronous and asynchronous market mechanisms in two different active rate ranges, i.e., \( p_n \in (0.9, 1) \) and \( p_n \in (0.95, 1) \), lasting 200 time periods, and the convergence performance is shown in Fig. 5. For the asynchronous mechanism, it can be seen that agents can freely trade in each time period and the average regret \( R(T)/T \) drops rapidly in about the first 40 time periods. Since then, the average regret declines much slower, but it still keeps going down with small oscillations, mainly due to the large uncertainty in renewable power generation. The convergence performance is significantly better than the synchronous one.
Probability as there exists one agent who has a very low active rate, the number of agents is usually very large, which means as long as the active rate has a greater impact on the convergence performance, which shows the robustness of the asynchronous mechanism.

Fig. 4: The probability of the number of consecutive delayed time periods under different active rate.

As the active rate increases, the robustness of the asynchronous mechanism.

Fig. 5: Average regret $R(T)/T$ of synchronous and asynchronous markets with different active rate ranges.

B. Market Transaction Frequency

We operate the synchronous and asynchronous market mechanisms under five different active rate ranges to investigate the frequency of market transactions, which is measured by the average number of time periods between two market transactions. The markets run 50 times with different settings in 200 time periods. Fig. 6 shows the average number of time periods between two transactions. The figure shows that, as the lower bound of active rates decreases, the average number of time periods between two transactions for synchronous mechanism increases exponentially. However, for the asynchronous market, since agents can trade with each other in each time period, the value remains at 1. Therefore, the asynchronous market negotiation mechanism can operate more frequently and better catch up with the uncertainty of renewable power generation.

C. Fairness of Individual Profit

A agent is reliable if he can continuously trading with neighboring agents over a specific period of time without failure. In our work, a market mechanism is defined to be “fair” if a more reliable agent can get more profit than a less reliable agent. We believe that “fairness” defined in our work is an important metric that should be considered, since a fair market mechanism can encourage agents to voluntarily improve quality and reliability, and more reliable agents can help to increase the social welfare in return.

We build a small market with 5 conventional generators, 5 users, and 5 wind generators. The parameters of the same type of agents are identical. The active rates of the five conventional generators are set to 0.6, 0.7, 0.8, 0.9, and 1, respectively, while the active rates of the other agents are all set to 1. By displaying the accumulated individual profits of conventional generators and users in 200 time periods, we compare the fairness of synchronous and asynchronous mechanisms. As shown in Fig. 7, the result indicates that synchronous mechanism could be very inefficient. In the real world, it is less likely that all agents are efficient and have high reliability. Thus, the asynchronous mechanism probably will not take more iterations (time periods) than the synchronous mechanism in practical applications.
the profits of the same type of agents are identical in the synchronous market since agents have to trade together. In the asynchronous market, agents with a higher active rate can trade more frequently and earn more profits. Therefore, the asynchronous market mechanism is more fair, and the deployment of it can encourage agents to improve the quality and reliability.

D. The Impact of Forgetting Factor

The impact of the value of forgetting factor on the convergence performance is tested. The forgetting factor is given to control the amount of older information that is used for current computation. By giving less weight to older data, the agent focuses more on the updated new information. Fig. 8 shows that the reduction of the forgetting factor value will increase the amount of forgetting and improve the convergence performance.

VI. CONCLUSION

P2P markets are considered as an evolution of the future electricity markets driven by the development of DERs and DSM. However, it is technically challenging to operate P2P market mechanisms in real-time, since they usually involve a heavy computation burden, while real-time trading demands fast calculations. How to reduce the computation complexity of P2P mechanisms to be within a real-time architecture remains a challenge. To this end, we propose a novel real-time P2P electricity market that integrates online optimization approaches and asynchronous mechanism, where each agent only performs one iteration and can freely trade with neighboring agents without waiting for the idle or inactive ones. Finally, we give proof of the sublinear regret upper bound for our asynchronous online consensus ADMM algorithm, which indicates that social welfare can be maximized in the long run on time average. Simulation results show that our asynchronous market mechanism has good convergence performance, robustness, and fairness compared with the synchronous mechanism.

The main drawback of the asynchronous mechanism are that if most agents are reliable, it may take a very long period of time to converge below an acceptable level. However, it is less likely that all agents are efficient and have high reliability in actual markets.

APPENDIX A

PROOF OF THE SUBLINEAR REGRET UPPER BOUND

Let \( \frac{\partial G_{n,t}(E_{n,t})}{\partial E_{nm,t}} \) be the gradient of \( G_{n,t}(E_{n}) \) at \( E_{nm,t} \). Since \( E_{n,t}^* \) minimizes (12), combining (14), we have

\[
\frac{\partial G_{n,t}(E_{n,t})}{\partial E_{nm,t}} = \lambda_{nm,t} + \rho(F_{nm,t-1} - F_{nm,t}) + \eta(E_{nm,t-1} - E_{nm,t}), \forall m \in A_{n,t}
\]

Since \( G_{n,t} \) is a convex function and its subgradient at \( E_{nm,t+1} \) is given in (22), for optimal solution \( E_{n,t}^* \) we have

\[
G_{n,t}(E_{n,t}) - G_{n,t}(E_{n,t}^*) \leq \sum_{m \in A_{n,t}} \frac{\partial G_{n,t}(E_{n,t})}{\partial E_{nm,t}} (E_{nm,t} - E_{nm,t}^*)
\]

\[
= \sum_{m \in A_{n,t}} \lambda_{nm,t}(E_{nm,t} - F_{nm,t}^*)
+ \sum_{m \in A_{n,t}} \rho(F_{nm,t-1} - F_{nm,t})(E_{nm,t} - F_{nm,t}^*)
+ \sum_{m \in A_{n,t}} \eta(E_{nm,t-1} - E_{nm,t})(E_{nm,t} - E_{nm,t}^*)
= \sum_{m \in A_{n,t}} \lambda_{nm,t}(E_{nm,t} - F_{nm,t}^*)
+ \sum_{m \in A_{n,t}} \frac{\rho}{2} [(F_{nm,t-1} - F_{nm,t}^*)^2 - (F_{nm,t} - F_{nm,t}^*)^2]
+ \frac{\eta}{2} [(E_{nm,t} - F_{nm,t})^2 - (E_{nm,t} - F_{nm,t-1})^2]
+ \frac{\eta}{2} [(E_{nm,t} - F_{nm,t})^2 - (E_{nm,t} - E_{nm,t-1})^2]
\]

According to the Fenchel-Young’s inequality [53], we have

\[
G_{n,t}(E_{n,t}) - G_{n,t}(E_{n,t}^*) \leq \sum_{m \in A_{n,t}} \frac{\partial G_{n,t}(E_{n,t})}{\partial E_{nm,t}} (E_{nm,t} - E_{nm,t})
\]

\[
\leq \sum_{m \in A_{n,t}} \frac{1}{2\rho} \left( \frac{\partial G_{n,t}(E_{n,t})}{\partial E_{nm,t}} \right)^2 + \frac{\rho}{2}(E_{nm,t} - E_{nm,t})^2
\]
\[
\begin{align*}
\frac{1}{2} \sum_{m \in A_{n,t}} \frac{\partial G_{n,t}(\hat{E}_{n,t})}{\partial \hat{E}_{n,t}} &+ \frac{\rho}{2} (F_{n,t} - E_{n,m,t})^2 \\
\frac{1}{2} \sum_{m \in A_{n,t}} \frac{\partial G_{n,t}(\hat{E}_{n,t})}{\partial \hat{E}_{n,t}} &+ \frac{\lambda_{n,m,t-1} - \lambda_{n,m,t}}{2 \rho}
\end{align*}
\]

(24)

For the penultimate inequality, because \( \hat{E}_{n,m,t} \) begins updating from \( F_{n,m,t} \), if the projection update finishes at the first time, which also means both agents do not touch the bound, the term \((E_{n,m,t} - E_{n,m,t})^2 \) reaches the maximal value \((F_{n,m,t} - F_{n,m,t})^2 \). The worst case is that the final trade reaches at the initial value \( E_{n,m,t} \) or \( E_{n,m,t} \), which means at the beginning, one of the agents \( n \) or \( m \) has reached the bound, then the term \((E_{n,m,t} - E_{n,m,t})^2 \) reaches the minimal value zero.

Combining (23)-(24), we have

\[
G_{n,t}(\hat{E}_{n,t}) - G_{n,t}(E_{n,t}^*) \\
\leq \sum_{m \in A_{n,t}} \lambda_{n,m,t} (E_{n,m,t} - F_{n,m,t})^2 + \frac{\rho}{2} (E_{n,m,t} - F_{n,m,t})^2 \\
+ \sum_{m \in A_{n,t}} \frac{\eta}{2} ((E_{n,m,t} - E_{n,m,t-1})^2 - (E_{n,m,t} - E_{n,m,t})^2) \\
+ \sum_{m \in A_{n,t}} \frac{1}{2 \rho} \left( \frac{\partial G_{n,t}(\hat{E}_{n,t})}{\partial \hat{E}_{n,t}} \right)^2 + \frac{\lambda_{n,m,t-1} - \lambda_{n,m,t}}{2 \rho}
\]

(25)

For the first term, using \( E_{n,m,t} - F_{n,m,t} = \lambda_{n,m,t-1} - \lambda_{n,m,t} \), \( F_{n,m,t} + F_{n,m,t} = 0 \), \( \lambda_{n,m,t} = \lambda_{n,m,t} \) and summing up for all \( n \in \Omega \) yields

\[
\sum_{n \in A_{t}} \sum_{m \in A_{n,t}} \lambda_{n,m,t} (E_{n,m,t} - F_{n,m,t})^2 + \frac{\rho}{2} (E_{n,m,t} - F_{n,m,t})^2 \\
= \frac{1}{2} \sum_{(n,m) \in (A_{n,t})} \lambda_{n,m,t} (E_{n,m,t} + E_{n,m,t} - (F_{n,m,t} + F_{n,m,t})) \\
+ \frac{\rho}{2} (E_{n,m,t} - F_{n,m,t})^2 + \frac{\rho}{2} (E_{n,m,t} - F_{n,m,t})^2 \\
= \frac{1}{2} \sum_{(n,m) \in (A_{n,t})} 2 \lambda_{n,m,t} (E_{n,m,t} - F_{n,m,t}) \\
+ \frac{1}{2 \rho} (\lambda_{n,m,t-1} - \lambda_{n,m,t})^2 + \frac{1}{2 \rho} (\lambda_{n,m,t-1} - \lambda_{n,m,t})^2 \\
= \sum_{(n,m) \in (A_{n,t})} \frac{1}{2 \rho} (\lambda_{n,m,t-1}^2 - \lambda_{n,m,t}^2)
\]

(26)

For the second term

\[
(F_{n,m,t-1} - F_{n,m,t})^2 (F_{n,m,t} - F_{n,m,t})^2 \\
= (F_{n,m,t-1} - F_{n,m,t})^2 (F_{n,m,t} - F_{n,m,t})^2 \\
+ (F_{n,m,t} - F_{n,m,t})^2 (F_{n,m,t-1} - F_{n,m,t})^2 \\
\leq (F_{n,m,t} - F_{n,m,t})^2 + (F_{n,m,t-1} - F_{n,m,t})^2 \\
+ 2 (F_{n,m,t} - F_{n,m,t}) (F_{n,m,t} - F_{n,m,t}) (F_{n,m,t} - F_{n,m,t}) \\
\leq (F_{n,m,t} - F_{n,m,t})^2 + (F_{n,m,t} - F_{n,m,t})^2 \\
+ L |F_{n,m,t} - E_{n,m,t}|
\]

(27)

Similarly, for the third term

\[
(E_{n,m,t} - E_{n,m,t-1})^2 (E_{n,m,t} - E_{n,m,t})^2 \\
\leq (E_{n,m,t} - E_{n,m,t} - E_{n,m,t})^2 + (E_{n,m,t} - E_{n,m,t})^2 \\
+ L |E_{n,m,t} - E_{n,m,t}|
\]

(28)

Combining (25)-(26) and based on Assumption 1, we have

\[
\sum_{t=1}^{T} \sum_{n \in A_{t}} G_{n,t}(\hat{E}_{n,t}) - \sum_{t=1}^{T} \sum_{n \in A_{t}} G_{n,t}(E_{n,t}^*) \\
\leq \sum_{\forall (n,m) \in (A_{n,t})} \frac{1}{2 \rho} (\lambda_{n,m,t-1}^2 - \lambda_{n,m,t})^2 \\
+ \sum_{n \in A_{t}} \sum_{m \in A_{n,t}} \frac{\rho}{2} [(F_{n,m,t} - F_{n,m,t})^2 (F_{n,m,t} - F_{n,m,t})^2] \\
+ \sum_{n \in A_{t}} \sum_{m \in A_{n,t}} \frac{\eta}{2} [(E_{n,m,t} - E_{n,m,t})^2 (E_{n,m,t} - E_{n,m,t})^2] \\
+ \sum_{t=1}^{T} \sum_{n \in A_{t}} \sum_{m \in A_{n,t}} \frac{1}{2 \rho} \left( \frac{\partial G_{n,t}(\hat{E}_{n,t})}{\partial \hat{E}_{n,t}} \right)^2 \\
+ \sum_{t=1}^{T} \sum_{n \in A_{t}} \sum_{m \in A_{n,t}} \frac{\lambda_{n,m,t-1} - \lambda_{n,m,t}}{2 \rho} \\
\leq N(N-1) \left( \frac{\rho D_{2}}{2} + \frac{\eta D_{1}}{2} + \frac{(\rho + \eta) L P}{2} + \frac{T S^{2}}{2 \rho} + \frac{T A^{2}}{2 \rho} \right)
\]

(29)

Now, for each \( n \in A_{t} \), let \( \tilde{t}_{n} \) be the last iteration where agent \( n \) was active. Using Assumption 1(g), we have that

\[
\sum_{t=1}^{T} \sum_{n \in A_{t}} G_{n,t}(\hat{E}_{n,t}) - \sum_{t=1}^{T} \sum_{n \in A_{t}} G_{n,t}(E_{n,t}^*) \\
= \sum_{t=1}^{T} \sum_{n \in A_{t}} \sum_{l \in \tilde{t}_{n+1}} \left( NC_{n,l}(\hat{E}_{n,t}) - NC_{n,l}(E_{n,t}^*) \right) \\
\geq \sum_{t=1}^{T} \sum_{l \in \Omega} \sum_{n \in A_{t}} \left( NC_{n,l}(\hat{E}_{n,t}) - NC_{n,l}(E_{n,t}^*) \right) \\
- \sum_{n \in A_{t}} \sum_{l \in \tilde{t}_{n+1}} \left( NC_{n,l}(\hat{E}_{n,t}) - NC_{n,l}(E_{n,t}^*) \right)
\]

(30)

Noting that \( \hat{E}_{n,l} = \hat{E}_{n,t} \) for \( l = \tilde{t}_{n} + 1, \ldots, t \) and using Assumption 1(f), the last term in (30) can be bounded as follows:

\[
\sum_{n \in A_{t}} \sum_{l=\tilde{t}_{n+1}}^{T} \left( NC_{n,l}(\hat{E}_{n,t}) - NC_{n,l}(E_{n,t}^*) \right) \\
\leq NJ \left( \frac{1 - v^{-T}}{1 - v} \right) \leq \frac{NJ}{1 - v}
\]

(31)

where we have used \( A_{t} \in \Omega \). By using (31) and (30) in (29), we finally have

\[
R(T) = \sum_{t=1}^{T} \sum_{n \in \Omega} \left( NC_{n,l}(\hat{E}_{n,t}) - NC_{n,l}(E_{n,t}^*) \right)
\]
\[ \frac{N(N-1)}{v^{2}} + \frac{\eta D_{2}}{2} + \frac{\rho D_{1}}{2} + \frac{\rho + \eta}{2} \frac{L P}{2\rho} + \frac{T S^{2}}{2\rho} + \frac{T A^{2}}{2\rho} \] 

Setting \( \rho = \sqrt{T} \) and \( \eta = \sqrt{T} \) yields sublinear regret \( R(T) \).

REFERENCES


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