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An integrated market solution to enable active distribution network to provide reactive power ancillary service using transmission–distribution coordination

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Abstract
The active distribution network (ADN) can provide the reactive power ancillary service (RPAS) to improve the operations of the transmission network operations (such as voltage control and network loss reduction) as distribution generation grows. In this context, an RPAS market is required to motivate the ADN to provide the RPAS to the transmission network since the transmission system operator (TSO) and the distribution system operator (DSO) are different entities. Hence, to obtain the TSO–DSO coordination in the RPAS market, this study proposes a two-stage market framework on the basis of the successive clearing of the energy and RPAS markets. Additionally, a distributed market-clearing mechanism based on an alternating direction method of multipliers (ADMM) is adopted to guarantee TSO’s and DSO’s information privacy. Furthermore, a binary expansion (BE) method is used to linearise the non-convex bilinear terms in the market-clearing model. The effectiveness of the proposed RPAS market framework and distributed market-clearing mechanism is validated using two different test systems with different system scales.

KEYWORDS
active distribution network, distributed generation, reactive power ancillary service, transmission–distribution coordination

1 | INTRODUCTION

To continuously satisfy the increasing electricity demand, the transmission system operator (TSO) is required to operate the transmission system at higher loads and lower capacity [1, 2]. Concurrently, a shortage of the reactive power will occur in the overload nodes (particularly, the boundary nodes between the transmission system and the distribution system) because of the reduction of traditional generators. Consequently, the voltage problem becomes critical for the boundary nodes in the transmission network [3, 4], and the reactive power ancillary service (RPAS) is required to address the voltage issues.

Distributed generations (DGs) such as photovoltaic (PV) and wind turbines (WT) can enable the active distribution network (ADN) to satisfy its load by itself. When the ADN has a high DG penetration [5, 6], there is an increased power interaction between the transmission network and the ADN. Additionally, the DGs can inject the reactive power into the power grid via the grid-connected inverter [7–9]. Hence, the ADN can provide the RPAS to the transmission network and help the TSO to address its voltage issues [10, 11].

In this context, an effective coordination mechanism must achieve the coordination between the TSO and the distribution system operator (DSO) in the RPAS market. The coordination between the transmission network and distribution networks has been studied extensively. The authors in [12] proposed a coordinated transmission and distribution alternating current (AC) optimal power flow method to improve economic...
operations of the whole system. The authors in [13] proposed a new decentralised restoration scheme to achieve efficiently coordinated restoration of the coupled transmission and distribution system (TSO–DSO). A multilevel overload relief method for the transmission network using the capability from an ADN is proposed in [14] under the coordinated transmission and distribution management framework. The authors in [15–17] presented different coordination schemes between the TSO and the DSO in the ancillary service markets. The authors in [18] proposed a methodology that improves reactive power management by taking advantage of the full capabilities of the distributed energy resources (DER) and by reducing the injection of reactive power in the distribution network and reducing losses. Another methodology using a stochastic AC-OPF for the reactive power management by the DSO under renewable energy sources forecast uncertainty was proposed in [19], and the DSO is allowed to coordinate and supply reactive power services to the TSO. Similarly, the authors in [20] considered the provision of the voltage support as an ancillary service to the TN and proposed a centralised tractable OPF-based control scheme that optimises the real-time operation of ADNs. The authors in [21] demonstrated how small-scale photovoltaic systems in distribution systems can provide reactive power services to other networks. A dedicated voltage ancillary service strategy is achieved by the combined usage of DG and traditional resources to temporarily satisfy the system’s requirement for voltage stability temporarily [22]. The authors in [23] studied an RPAS market framework from the energy market perspective to encourage DGs to participate in reactive power optimisation. The authors in [24] proposed a reactive power optimisation strategy with the objective function of minimising network loss, voltage deviation, and reactive power cost. In [25], an alternative method was demonstrated how to provide an RPAS for the transmission network by using existing parallel transformers in the distribution network. However, the DSO privacy is not well protected in the aforementioned studies and the flexibilities of DGs are underused. An optimal way was presented to control a steady-state voltage in distribution feeders using the RPAS that is provided by synchronous DGs [26]. The studies of the reactive power pricing have also received widespread attention. The authors in [27, 28] used the locational marginal price (LMP) as the reactive power pricing. Nonetheless, they were not feasible because of the reactive power’s localised nature. Few studies adopt the distribution LMP for reactive power pricing in the distribution network because the network length of the distribution network is relatively shorter than the transmission network [29, 30]. Nevertheless, reactive power compensation is not considered in these studies.

Furthermore, information privacy is particularly important for both market participants (TSO and DSO). Moreover, the DSO system data are not fully accessible by the TSO. In summary, the traditional centralised solution of the TSO–DSO coupling operation problem does not apply to the aforementioned situation. Conversely, the distributed optimisation solution method that has the characteristics such as ensuring the independence of regional optimisation, information privacy, and reducing the amount of information transmission, is widely used in solving TSO–DSO and multiparty transactions. For example, the master–slave splitting method is used to solve the transmission and distribution network's distributed power flow problem [31]. In [32], the benders decomposition method is used to solve the transmission and distribution network's distributed dispatch and active power optimisation problems. The alternating direction method of multipliers (ADMM) method can control a few information interactions between adjacent regions using global variables. Additionally, ADMM has the characteristics of good convergence and significant robustness. Thus, ADMM has become a crucial method for solving the power systems distributed optimisation problems [36–38].

Hence, this study proposes a two-stage market framework to realise the coordination between the TSO and the DSO in the RPAS market to address the aforementioned challenges. In the proposed market framework (the energy market and RPAS market), TSO, DSO, and independent market operator (IMO) are the market participants. The DSO is responsible for the balancing of the distribution grid and has priority to use resources (DGs) from the local grid. The DSO selects the necessary bids for local use and aggregates and transfers the remaining resources to the market. Similar to the DSO, the TSO needs to balance the transmission grid and integrates bids from power generation companies. The IMO is introduced to manage a third-party market to protect TSO’s and DSO’s information privacy. The TSO and the DSO submit their market information to IMO and IMO will handle exchanged information from the TSO and the DSO. The IMO has the benefit that the market will be operated by an entity that might already have experience in this field. The presence of the IMO must guarantee neutrality, which is essential for each market participant. Only in this way can IMO be qualified to share market information with market participants and handle exchanged information during the market-clearing process.

Furthermore, a distributed market-clearing mechanism based on ADMM is adopted to protect TSO’s and DSO’s privacy. Moreover, the non-convex bilinear terms in the model are linearised using the binary expansion (BE) method. The main contributions of this study are summarised as follows:

1. An RPAS market-clearing framework, which enables the ADN to provide the RPAS to the transmission network is proposed. The proposed market framework can not only make full use of the DGs’ capabilities in the ADN but can also promote the interaction between the transmission network and the ADN.
2. A distributed market-clearing mechanism is proposed to protect TSO’s and DSO’s information privacy under the framework.
3. The transmission system operator and the DSO are in the same position in the market mechanism when compared with other traditional TSO–DSO coordination schemes. The
clearing price and purchased quantity are jointly decided by the TSO and the DSO during the transaction process.

This study is structured as follows. Section 2 introduces the issues and frameworks of the energy and RPAS markets. Section 3 presents the clearing model of the energy and RPAS markets. This section also mentions the solution of the non-linear term of the product of two variables. Section 4 introduces the scheme-solving algorithm. Section 5 discusses the clearing results of the energy and RPAS markets. Furthermore, this section demonstrates the effectiveness of the RPAS market and the proposed market mechanism by comparing whether the DSO participates in the RPAS market. Finally, Section 6 presents the conclusions.

2 | PROBLEM DESCRIPTION

The operation state of the distribution network becomes more flexible as the new energy power generation grows. The power flow between the transmission network and the ADN coupling progressively changes from one-way to two-way, which significantly enhances the coupling relationship between them. The ADN with strong coordination ability can provide the RPAS to the transmission network [13]. In this study, the RPAS market is different from several conventional TSO–DSO decentralised RPAS market models [15].

Assuming that DSOs and DGs are integrated, the TSO cannot access the DGs of the ADN. The DGs do not only take responsibility for local network constraints but also provide the RPAS for the transmission network as the main body [39]. To address the different clearing periods between the energy market and RPAS markets, the energy market-clearing results are taken as the active power generation plan and the input data of the RPAS market. Additionally, the power capacity in the RPAS market is determined using the active power output of the DG supply in the energy market.

In this study, an IMO is introduced to protect the information privacy of the participants (TSO and DSO). The IMO is a third-party entity that handles exchanged information from the TSO and the DSO. The TSO and the DSO only provide the boundary information of their systems to the IMO to ensure information transmission privacy. The aforementioned market framework is applied to the energy market and RPAS markets.

Figure 1 shows the market framework. The RPAS market is cleared based on the basis of the energy market's clearing result. The active power outputs of the transmission generation units, the active power outputs of the DGs, and the purchased quantity in the energy market will be the initial RPAS market data. Moreover, the reactive power interaction between the transmission network and the ADN is determined using a constant power factor after clearing the energy market. Additionally, the DSO calculates the capacity of the conventional DGs in the RPAS market on the basis of the determined power factor combined with the energy market output data, whereas the capacity of PVs is based on the inverter capacity and the energy market output data.

2.1 | Framework of the day-ahead energy market

The power generation companies' bid on the TSO is the first step in the energy market framework (shown in Figure 2), and the TSO and the DSO perform optimisation calculations based on their system constraints. The TSO calculates its power demand on the basis of the lowest price combined with the bid from power generation companies and subsequently submits to IMO with the lowest expected price. The DSO should adjust DG's power outputs and purchase power on the basis of the expected TSO price. Afterwards, the DSO reports its power demand and bidding price to IMO. The aforementioned information is exchanged only by IMO to protect the TSO's and DSO's privacy. This action is repeated until both the TSO and the DSO agree on the price and the purchased quantity. Moreover, the market-clearing process is iterative. The TSO and the DSO interact and modify the clearing price and the purchased quantity until the difference between two adjacent iterations is less than a threshold. Finally, the market is cleared successfully (i.e. both the TSO and the DSO agree with the market's price and the purchased quantity).

2.2 | Framework of the RPAS market

The framework of the RPAS market shown in Figure 3 is similar to that of the energy market. This study assumes that
the boundary reactive power is balanced after the energy market clears and the reactive power capacity of DGs in the RPAS market is determined.

First, the TSO calculates the RPAS reactive power based on the system data and submits a bidding price to IMO. Concurrently, the DSO submits the reactive power capacity of DGs and the expected price to the IMO. After the IMO exchanges the information with the TSO and the DSO, they update their trading information based on the return information and submit it again to the IMO. The two parties alternate repeatedly to form an iterative process. Finally, when they agree on a clearing price and purchased quantity of the RPAS, the iteration stops and the market clears (the clearing process is the same as in the energy market).

3 | MATHEMATICAL MODEL

3.1 | Energy market-clearing models

(1) Market-Clearing Model of Distribution Network

The distribution network's market clearing is modelled using a second-order cone programming [40] (SOCP)-based AC optimal power flow (ACOPF) model, and the objective function is to minimise each distribution network's operating cost.

\[
\min \sum_{e \in A} (c^D_{\text{pc}, e} P^D_{\text{pc}, e} + c^P_{\text{DC}} P_{\text{DC}, e}) \tag{1a}
\]

s.t.
\[
\sum_{i \in j(\ell)} (P^D_{\ell,i} - P^D_{\ell,i,x_{ij}}) + P^D_{\text{DC}, e} = \sum_{i \in j(\ell)} P^D_{\ell,i} \tag{1b}
\]
\[
\sum_{i \in j(\ell)} (Q^D_{\ell,i} - Q^D_{\ell,i,x_{ij}}) + Q^D_{\text{DC}, e} = \sum_{i \in j(\ell)} Q^D_{\ell,i} \tag{1c}
\]
\[
u^D_{\ell,i} = u^P_{\ell,i} - 2\left(r_{ij} P^D_{\ell,i,j} x_{ij} + \left( \left( r_{ij} \right)^2 + (x_{ij})^2 \right) i^D_{\ell,i} \right) \tag{1d}
\]
\[
V^2_{\ell,i,\text{min}} \leq u^D_{\ell,i} \leq V^2_{\ell,i,\text{max}} \tag{1e}
\]
\[
P^D_{\text{DC}, \text{min}} \leq P^D_{\text{DC}, e} \leq P^D_{\text{DC}, \text{max}} \tag{1f}
\]
\[
Q^D_{\text{DC}, \text{min}} \leq Q^D_{\text{DC}, e} \leq Q^D_{\text{DC}, \text{max}} \tag{1g}
\]

Equations (1b) and (1c) impose the active power and reactive power balance in the transmission system, respectively. Equations (1d) and (1f) are the voltage drop equation of branch \(ij\) and upper and lower limits of the voltage amplitude of node \(i\). The active and reactive power output constraints of DGs are described in Equations (1f) and (1g). Equation (1h) is the line flow constraint after SOC relaxation. Since the branch current, \(i^2\), and node voltage amplitude \(v^2\) are quadratic
variables of strongly non-convex forms, they are replaced with the primary variables $I_{ij}^{DM}$ and $u_{ij}^{DM}$.

(2) Market-Clearing Model of Transmission Network

The transmission network part in the energy market-clearing model adopts the direct current optimal power flow (DCOPF) model. The objective function is to maximise the social welfare of the transmission network, which is expressed as Equation (2a).

$$\max \sum_{b \in G} \left( c^T_{pe,ct} P_{pe,ct}^T - c^T_{p,sh,ct} P_{p,sh,ct}^T \right)$$  \hspace{1cm} (2a)

s.t.

$$\sum_{b \in G} P_{sh,ct}^T - \sum_{w \in PCC} p_{w,ct}^T - \sum_{d \in D} P_{d,ct}^T = 0$$  \hspace{1cm} (2b)

$$P_{sh,ct}^T \leq P_{sh,ct}^{t,\min} \leq P_{sh,ct}^{t,\max}$$  \hspace{1cm} (2c)

$$\text{Rate}A_{i,min} \leq \sum_{b \in G} GSF_{l,b} \cdot P_{sh,ct}^T - \sum_{d \in D} GSF_{l,d} \cdot P_{d,ct}^T$$

$$- \sum_{w \in PCC} GSF_{w} \cdot p_{w,ct}^T \leq \text{Rate}A_{i,max}$$  \hspace{1cm} (2d)

Equation (2b) imposes the power balance in the transmission system. Equation (2c) shows the output limit of each generator. Equation (2d) represents the transmission capacity limits of each line.

3.2 | RPAS market-clearing models

(1) Market-Clearing Model of Distribution network

In this model, the distribution network's market clearing is modelled using an SOCP-based ACOPF model, and the objective function is to maximise each DSO’s RPAS profit.

$$\max \sum_{v \in A} \left( b_{v,ct}^D \Delta Q_{v,ct}^D - b_{v}^Q Q_{v,ct}^{V,ar} \right)$$  \hspace{1cm} (3a)

s.t.

$$\sum_{i \in y(j)} \left( P_{ij,ct}^{V,ar} - Q_{ij,ct}^{V,ar} \right) + P_{DA,ct}^{v,ar} + P_{p,ct}^{v,ar} - P_{d,ct}^{V,ar} = \sum_{v \in y(j)} p_{ij,ct}^{V,ar}$$  \hspace{1cm} (3b)

$$\sum_{i \in y(j)} \left( Q_{ij,ct}^{V,ar} - Q_{ij,ct}^{V,ar} \right) + Q_{e,ct}^{V,ar} + Q_{z,ct}^{V,ar} - Q_{d,ct}^{V,ar}$$

$$+ Q_{p,ct}^D - \Delta Q_{p,ct}^D = \sum_{v \in y(j)} Q_{ij,ct}^{V,ar}$$  \hspace{1cm} (3c)

$$V_{ij,ct}^{V,ar} = u_{ij,ct}^{V,ar} - 2 \left( r_{ij} P_{ij,ct}^{V,ar} + x_{ij} Q_{ij,ct}^{V,ar} \right) + \left( r_{ij}^2 + x_{ij}^2 \right) I_{ij,ct}^{V,ar}$$  \hspace{1cm} (3d)

In the RPAS market, the active power of DGs is considered to be fixed values based on the energy market clearing results. Equations (3b) and (3c) impose the active power and reactive power balance in the distribution system, respectively. Equation (3d) describes the voltage drop equation and Equation (3e) represents the nodal voltage limit. The DGs could independently adjust active and reactive power using power electronic devices or conventional rotary motor interfaces. In this context, the reactive residual capacity of DG participating in the RPAS market can be calculated by the fixed power factor and its active power output in the energy market as shown in Equations (3g) and (3h). Similarly, Equation (3i) represents PV’s reactive power capacity in the RPAS market calculated from its active power output in the energy market.

$$Q_{e,ct}^{V,ar} = P_{e,ct}^{V,ar} \tan \varphi$$  \hspace{1cm} (3g)

$$Q_{e,ct}^{V,ar} \leq Q_{e,ct}^{V,ar} + Q_{e,ct}^{V,ar} \leq Q_{e,ct}^{V,ar}$$  \hspace{1cm} (3h)

$$P_{DA,ct}^{V,ar} + Q_{z,ct}^{V,ar} \leq S_{z,ct}^{V,ar}$$  \hspace{1cm} (3i)

(2) Market-Clearing Model of Transmission Network

In this section, the SOCP-transformed ACOPF model [41] is used in the transmission network clearing model. This study discusses establishing the penalty factor of total network loss and forming the penalty term of total network loss to achieve the economic benefits of the RPAS market to the TSO. The multi-objective function in Equation (4a) is to minimise the operating cost of the transmission network comprising the network loss penalty term, reactive power generation cost, and the RPAS. Equation (4b) is the loop phase angle constraint of the transmission network.

$$\min \sum_{b \in G, d \in D} \left( b_{b,ct}^Q Q_{b,ct}^{T} + b_{loss} P_{loss,ct} + b_{a,ct}^T \Delta Q_{p,ct}^T \right)$$  \hspace{1cm} (4a)

s.t.

$$\sum_{m \in y(\tau)} \left( P_{m,ct}^{T} - i_{m,ct}^{T} \right) + P_{b,ct}^{T}$$

$$- \sum_{w \in PCC} P_{w,ct}^{T} + \Delta P_{t} = \sum_{k \in \nu(n)} P_{nk}^{T}$$  \hspace{1cm} (4b)
\[ \sum_{m \in V_{i}} \left( \frac{Q_{mn,t}^{\ast} - i_{mn}x_{mn}}{1} \right) + Q_{b_{mn},t}^{\ast} - Q_{d_{mn},t} + \Delta_{pc_{mn},t}^{\ast} - Q_{pc_{mn},t}^{\ast} \]

(4c)

\[
\begin{align*}
&\begin{pmatrix}
2P_{mn,t}^{\ast} \\
2Q_{mn,t}^{\ast} \\
I_{mn,t} - u_{mn,t}^{\ast}
\end{pmatrix} \\
&\begin{pmatrix}
\leq
\leq
\leq
\end{pmatrix} \\
&\begin{pmatrix}
\leq
\leq
\leq
\end{pmatrix}
\end{align*}
\]

(4g)

\[
C_{x_{mn}}P_{mn,t}^{\ast} - C_{y_{mn}}Q_{mn,t}^{\ast} = 0
\]

(4h)

where \( \Delta b = \left( \bar{b} - b \right) / M_1 \) and \( x_k \) is a binary variable. Log2(M1) variables are also required here.

By multiplying both sides of Equation (5a) with another decision variable, \( \Delta Q_{pc_{mn}}^{\ast} \), we can replace \( x_k \Delta Q_{pc_{mn}}^{\ast} \) with a new variable \( z_k = x_k \Delta Q_{pc_{mn}}^{\ast} \) as shown in Equation (5b).

\[
b_{at}^{Q} \Delta Q_{pc_{mn}}^{\ast} = b \Delta Q_{pc_{mn}}^{\ast} + \Delta b \sum_{k=0}^{K_1} 2^{k} z_k
\]

(5b)

Furthermore, two inequality constraints Equations (5c) and (5d) must reflect the relationship \( z_k = x_k \Delta Q_{pc_{mn}}^{\ast} \)

\[
0 \leq \Delta Q_{pc_{mn}}^{\ast} - z_k \leq G(1 - x_k)
\]

(5c)

\[
0 \leq z_k \leq Gx_k
\]

(5d)

where the constant, \( G \), must be large enough for the constraints in (5c) and (5d) to be relaxed when \( x_k = 0 \) and \( x_k = 1 \), respectively.

### 4.3 Solution of the non-linear term of the product of two variables

Since it is difficult to solve the product form of clearing price and purchased quantity in the objective function of the entire model, so the BE method [42] is used to solve this problem in this study. We can use \( b_{at}^{Q} \Delta Q_{pc_{mn}}^{\ast} \) in Equation (4a) as an example. First, the continuous decision variable \( b_{at}^{Q} \) will be approximated by a set of discrete values \{\( b_{mn} \), \( m = 0, 1, 2, \ldots, M_1 \)\}, where \( M_1 = 2^{K_1} \) and \( K_1 \) are non-negative integer variables. Then, assuming the clearing price \( b_{at}^{Q} \) is in the range of \( [b \bar{b}] \), \( b_{at}^{Q} \) can be replaced by Equation (5a).

\[
b_{at}^{Q} = b + \Delta b \sum_{k=0}^{K_1} 2^{k} x_k
\]

(5a)

### 4.1 System decoupling

In this study, the general ADMM consistency optimisation method [43] is used to solve the market-clearing model to further achieve the privacy protection of both the TSO and the DSO. It is necessary to decouple the constraints of coupling lines between the transmission and distribution systems since they are connected by substations. Using the global variable in ADMM to ensure consistency before and after the boundary interactive power replication, the transmission lines between adjacent subregions can be copied to adjacent subregions, respectively, and then, the constraints of transmission lines between regions can be decoupled. Figure 4 shows that the interactive active and reactive powers are considered coupling variables in the energy and RPAS markets, respectively.

Figure 5 shows that the coupling variables \( c_{pc_{mn}}^{T}, P_{pc_{mn}}^{T}, b_{at}^{T}, \) and \( \Delta Q_{pc_{mn}}^{\ast} \) of the TSO, and \( c_{pc_{mn}}^{D}, P_{pc_{mn}}^{D}, b_{at}^{D}, \) and \( \Delta Q_{pc_{mn}}^{\ast} \) of the RPAS Market.

![Diagram of Transmission and Distribution Network Coupling](image-url)

**Figure 4** Transmission and distribution network coupling diagram
DSO is obtained from the clearing results in the energy market and RPAS markets, respectively. Coupling variable values are obtained after optimisation in the subregion, and the updated global variables $Z_i - Z_a^k$ are the mean values of the coupled variables connected to them. Each subregion receives the modified global variables, which are used in the subregion’s next optimisation. Cycling the information interaction between the optimisation within the region and the subregions until the variables meet the ADMM convergence conditions.

4.2 Applying ADMM to the market-clearing problems

The ADMM algorithm for market clearing is described in Algorithm 1. The interaction process of the energy and RPAS market are the same. Therefore, this subsection only uses the RPAS market as an example. The overall objective function for the RPAS market is to minimise the entire system’s operating cost. The augmented Lagrangian form of the objective function is shown in Equation (6a).

Algorithm 1 ADMM for Market Clearing

```plaintext
procedure ADMM Loop
1: Local optimisation for TSOs/DSOs, 
   Obtaining the initial value of coupling variables; 
2: Coupling variables update, 
   for the TSO: 
   \[
   c_i^{k+1} = \arg \min_{c_i^{k+1}} \left( \frac{c_i^{k+1}}{p_{i,t}^{k+1}} + \frac{c_i^{k+1}}{c_{pcc,t}^{k+1}} / \Delta Q_{pcc,t}^{k+1} \right)
   \]
   for all DSOs: 
   \[
   c_i^{k+1} = \arg \min_{c_i^{k+1}} \left( \frac{c_i^{k+1}}{p_{i,t}^{k+1}} + \frac{c_i^{k+1}}{c_{pcc,t}^{k+1}} / \Delta Q_{pcc,t}^{k+1} \right)
   \]
3: TSO broadcasts $cT^{k+1} pcc, i, t/b T^{k+1} a, i, t$ & $PT^{k+1} pcc, a, t$ to all DSOs; 
   DSOs broadcast $cD^{k+1} pcc, i, t/b D^{k+1} a, i, t$ & $PD^{k+1} pcc, a, t$ to the TSO; 
   TSOs/DSOs obtain new bidding prices and quantity transmitted; 
4: Global variable update: 
   for the energy market: 
   \[
   \frac{c_i^{k+1}}{c_{pcc,t}^{k+1}} + \frac{c_i^{k+1}}{c_{pcc,t}^{k+1}} \frac{c_i^{k+1}}{c_{pcc,t}^{k+1}} = \frac{c_i^{k+1}}{c_{pcc,t}^{k+1}} + \frac{c_i^{k+1}}{c_{pcc,t}^{k+1}} \frac{c_i^{k+1}}{c_{pcc,t}^{k+1}}
   \]
   for the RPAS market: 
   \[
   \frac{c_i^{k+1}}{c_{pcc,t}^{k+1}} + \frac{c_i^{k+1}}{c_{pcc,t}^{k+1}} \frac{c_i^{k+1}}{c_{pcc,t}^{k+1}} = \frac{c_i^{k+1}}{c_{pcc,t}^{k+1}} + \frac{c_i^{k+1}}{c_{pcc,t}^{k+1}} \frac{c_i^{k+1}}{c_{pcc,t}^{k+1}}
   \]
5: Lagrangian multiplier update: 
   for the energy market: 
   \[
   \lambda_i^{k+1} = \lambda_i^{k+1} + \rho (c_i^{k+1} - Z_i^{k+1})
   \]
   for the RPAS market: 
   \[
   \lambda_i^{k+1} = \lambda_i^{k+1} + \rho (c_i^{k+1} - Z_i^{k+1})
   \]
6: Convergence conditions: 
   for the energy market: 
   \[
   \| S_1^{k+1} \|_2^2 \leq \xi_1
   \]
   for the RPAS market: 
   \[
   \| S_2^{k+1} \|_2^2 \leq \xi_1
   \]
end procedure
```
Each variable is updated, Equation (6h) is used to ascertain if ADMM convergence conditions are satisfied. The two-norm square of the dual residuals $s_1 - s_4$ and the original residuals $r_1 - r_2$ must be less than the relative stop thresholds $\xi_1$ and $\xi_2$. $\xi_1$ and $\xi_2$ are usually set to extremely small numbers, such that they are approximately equal to zero. When the convergence condition is satisfied, the two parties can be considered to agree on the agreement on the clearing price and the purchased quantity.

TSO and DSO subproblems are independent of each other except for the boundary constraints Equations (6b)--(6h). The total optimisation problem must satisfy the distribution network’s internal constraints.

The RPAS market-clearing problem is divided into subproblems of the TSO and the DSO:

(1) **TSO Subproblem:** Considering the Lagrangian function Equation (6a), the TSO subproblem can be represented as follows:

$$
\min \text{ } L_{TSO} = b_{n,t}^{Q DG} Q_{n,t} + b_{m,t}^{Q DQ} Q_{m,t} + b_{los,t}^{P loss,t} + \lambda_{t}^{T} (b_{t, \lambda}^{T} - Z_{t}) \\
+ \lambda_{t}^{T} \left( \Delta Q_{PC, DQ}^{T} - Z_{t} \right) + \left( \rho / 2 \right) \left( \| b_{t, \lambda}^{T} - Z_{t} \|^2 \right) \\
+ \left( \rho / 2 \right) \left( \| \Delta Q_{PC, DQ}^{T} - Z_{t} \|^2 \right)
$$

(6i) s.t. Equations (4b)--(4h)

(2) **DSO Subproblem:** Considering the Lagrangian function Equation (6a), the subproblem for the DSO can be represented as follows:

$$
\min \text{ } L_{DSO} = b_{n,t}^{Q DG} Q_{n,t} - b_{m,t}^{Q DQ} Q_{m,t} + b_{los,t}^{P loss,t} + \lambda_{t}^{T} (b_{t, \lambda}^{D} - Z_{t}) \\
+ \lambda_{t}^{T} \left( \Delta Q_{PC, DQ}^{D} - Z_{t} \right) + \left( \rho / 2 \right) \left( \| b_{t, \lambda}^{D} - Z_{t} \|^2 \right) \\
+ \left( \rho / 2 \right) \left( \| \Delta Q_{PC, DQ}^{D} - Z_{t} \|^2 \right)
$$

(6j) s.t. Equations (3b)--(3i)

5 | CASE STUDIES

The effectiveness of the proposed market-clearing framework and mechanism is verified using the two test systems. The first test system, T5-D33, comprises a PJM-5 system and three modified IEEE 33-bus ADNs at bus 2. The second one, T30-D33, comprises IEEE 30-bus test system and three modified IEEE 33-bus ADNs at bus 26. The capacity of the DGs in each ADN is different, and the specific data [44] are shown in Table 1.

The test applications run on a 64-bit version of Windows 10. The CPU is an Intel Core i5-7200 K with 2.50-GHz master frequency and 8-GB memory. In this section, the clearing results of the energy and RPAS markets will be reflected through the test system. The following two cases are conducted for comparison.
Case 1: Energy market clearing with DSO's participation.
RPAS market-clearing without DSO's participation.
Case 2: Energy market clearing with DSO's participation.
RPAS market-clearing with DSO's participation.

5.1 Test on T5-D33

Figure 6 shows that all three ADNs set DGs at nodes 3, 6, 10, and 18. ADN_1 and ADN_3 connect two DGs, particularly, at node 13. The reactive power demand is assumed to be 30% of the active power demand on each node [45]. The voltage amplitude of each node in the transmission network is set to 0.95–1.05 p.u. and the ADMM convergence conditions, $\xi_1$ and $\xi_2$ are set to $10^{-3}$. The solving time of the energy market and the RPAS market-clearing model is 4649.699624 s and 442 s, respectively.

| TABLE 1 Capacity parameters of photovoltaic (PVs) and fossil fuel distributed generations (DGs) in active distribution networks (ADNs) |
|---|---|---|---|---|
| ADN_1 | # | PV1 | PV2 | PV3 | DG6 |
| Maximum output (MW) | 0.35 | 0.5 | 0.25 | 5 |
| ADN_2 | # | PV1 | PV2 | DG5 |
| Maximum output (MW) | 0.4 | 0.6 | 3 |
| ADN_3 | # | PV1 | PV2 | PV3 | DG6 |
| Maximum output (MW) | 0.8 | 0.35 | 0.25 | 6 |

(1) Energy Market-Clearing Result

Figure 7 shows the transmission network's load prediction curve. The maximum loads for three ADNs are 6.88, 4.99, and 7.35 MW. The power transmission system is in a high load state during 18:00–21:00. The transmission network requires the RPAS, and the RPAS market is activated during this period. The RPAS should address the high network loss and the low voltage on a transmission grid at the high-load period. In this context, it is necessary to set up the corresponding high-capacity generation units to satisfy the system's high-load requirements in this study, and Table 2 shows the data of generation units.

The clearing price and purchased quantity in the energy market are set as variables. The clearing price in this study is not calculated using the LMP, and a reasonable clearing price range is set based on previous historical data and references [46, 47]. Moreover, the load and capacity of DGs in ADN_2 are lower than other ADNs. To boost market competitiveness, DSO_2's clearing price is also cheaper than DSO_1 and DSO_3. Table 3 shows the bidding price range.

Figure 8 shows that the clearing prices change mainly at 10:00 and 18:00 because of the time characteristics of PVs. Compared with the other two ADNs, the purchased quantity of ADN_2 is also at a lower level due to its lower load level.

(2) RPAS Market-Clearing Result

This study assumes that PVs only supply active power in the energy market because of their characteristics.

---

**FIGURE 6** Topology diagram of the T5–D33 test system
Furthermore, the reactive power capacity is determined based on the inverter constraint and the active power output. The DCOPF model is the transmission network model in the energy market. Hence, the problem of an active power flow mismatch exists. This study assumes that the ADNs generate active power to balance the power flow to address the aforementioned problem. The power factor of the other conventional DGs is 0.95. The reactive capacity of all DGs is determined using the energy market-clearing results and their characteristics. Table 4 shows the transmission network’s generation unit data and the generation unit’s reactive power generation cost. The bidding price range in the RPAS market is set to 0–2 $/MVARh [48].

Figure 9 shows the clearing result. DSO2’s clearing price decreases after period 4 and the other DSOs’ are stable. This is because the ADN2 has the lowest load and more capacity of DGs to participate in market transactions of the RPAS market. The purchased quantity of three ADN changes is the same. Figure 10 shows the comparison of the total output of DGs in three ADNs between cases 1 and two; the total output of DGs in case 2 is much higher than that in case 1.

Table 5 also shows the costs and profits of TSOs and DSOs in the RPAS market in case 1 and case 2. The DSO can reduce operating costs and even make a profit by participating in the RPAS market. This implies that the participation of the DSO in the RPAS market can effectively reduce the system’s operating cost.

**Table 2** Generation unit data of the transmission network

<table>
<thead>
<tr>
<th>Number</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum output (MW)</td>
<td>50</td>
<td>200</td>
<td>320</td>
<td>200</td>
<td>160</td>
</tr>
<tr>
<td>Bidding price ($/MWh)</td>
<td>14</td>
<td>15</td>
<td>25</td>
<td>22</td>
<td>20</td>
</tr>
</tbody>
</table>

**Table 3** TSO-DSO bidding price range in the energy market

<table>
<thead>
<tr>
<th>TSO–DSO1</th>
<th>TSO–DSO2</th>
<th>TSO–DSO3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bidding price range ($/MWh)</td>
<td>22–30</td>
<td>20–28</td>
</tr>
</tbody>
</table>
5.2 | Test on T30-D33

Figure 12 shows the T30-D33 test system, which comprises an IEEE 30-bus transmission network and three ADNs. The transmission network has six generation units, and the data are shown in Table 6. The reactive power demand is also assumed

<table>
<thead>
<tr>
<th>Number</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum output (MVAR)</td>
<td>80</td>
<td>150</td>
<td>100</td>
<td>160</td>
<td>120</td>
</tr>
<tr>
<td>Bidding price ($/MVARh)</td>
<td>1.4</td>
<td>1.5</td>
<td>2</td>
<td>3</td>
<td>1.6</td>
</tr>
</tbody>
</table>

**TABLE 4** Generation units reactive power data of transmission network

![Clearing price for the RPAS market](image1)

**FIGURE 9** Clearing result of the reactive power ancillary service (RPAS) market in each period

![Comparison of total output of DGs in ADN1](image2)

**FIGURE 10** Comparison of the total output of distributed generations (DGs) in three active distribution networks (ADNs)
to be 30% of the active power demand on each node. The voltage amplitude of each node in the transmission network is set to 0.95–1.05 p.u. The same system data used in the T5-D33 test system is applied for the three ADNs. According to the transmission network’s load curve (Figure 13), we set the clearing period of the RPAS market as 16:00–20:00. The energy market and RPAS market-clearing models are solved in 5254 s and 1032 s, respectively.

(1) Energy Market-Clearing Result

Figure 14 shows that the clearing prices exhibit a slightly increasing trend at 8:00 and 18:00–22:00. This is because the time characteristics of PVs cause insufficient power supply in the ADN during this period, resulting in higher clearing prices.
Because of the sufficient power supply of PVs around 13:00–16:00, ADN2 does not require purchasing energy from the transmission network in this period. Additionally, clearing prices remain stable because of the lower load level and power supply capacity of ADN₂ when compared with other ADNs.

(2) RPAS Market-Clearing Result

The TSO purchases from the three DSOs are roughly the same trend since the clearing prices of the three DSOs in the RPAS market remain unchanged (Figure 15). Table 7 shows that generation units 2, 3, and 4 have relatively low generation cost and their capacity can satisfy the system’s load. The reactive power output of other units is almost ignored. It is

<table>
<thead>
<tr>
<th>Table 7</th>
<th>Generation unit reactive power data of the transmission network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>#1</td>
</tr>
<tr>
<td>Maximum output of reactive power (MVAr)</td>
<td>150</td>
</tr>
<tr>
<td>Bidding price ($/MVArh)</td>
<td>2.5</td>
</tr>
</tbody>
</table>
only necessary to compare the output of generation units 2, 3, and 4 in cases 1 and 2. Figure 16 shows that the output of each generation unit in case 2 is significantly lower than that in case 1 after 12 periods. Concurrently, the output of DGs in the three ADNs of case 2 (Figure 17) is significantly higher than that in case 1. This demonstrates that DSO's participation in the RPAS market can reduce the output of generation units in transmission networks with higher generation costs by activating DG output. Figure 18 shows that comparing system operating costs in each period indicates that DSO’s participation in the RPAS market can lower the system operating cost by up to 15.63%. Table 8 illustrates

![Figure 16](image1.png)

**FIGURE 16** Reactive power output of transmission network generation units in each period

![Figure 17](image2.png)

(a) Comparison of total output of DGs in ADN1

(b) Comparison of total output of DGs in ADN2

(c) Comparison of total output of DGs in ADN3

**FIGURE 17** Comparison of the total output of distributed generations (DGs) in three active distribution networks (ADNs)
that the DSO obtains profits by participating in the RPAS market, which reduces the total system’s operating cost by 9.09%. It is worth mentioning that the T30-D33 test system's total operating cost is much lower than that of the T5-D33 test system due to the low-load level of the T30-D33 system.

6 | CONCLUSIONS

In this study, an RPAS market-clearing framework and a distributed market-clearing mechanism are proposed to achieve the coordination between the TSO and the DSO in the RPAS. The RPAS market's role is to promote the ADN to provide the RPAS to solve the coupling node's voltage over-limit problem in the transmission network via market mechanisms. The following conclusions were drawn based on numerical studies:

(1). The RPAS market with DSO's participation can improve the usage rate of DGs and activate DGs to generate a reactive power in the ADN to provide ancillary service for the transmission network.

(2). The energy and RPAS markets can be cleared using a few information interactions under the proposed market framework and distributed clearing mechanism.

(3). The RPAS market with DSO's participation can reduce the system's operating cost compared with that without DSO's participation.

In our future works, the clearing method of the RPAS market considering the overvoltage problem of boundary nodes between the transmission network and the distribution network will be studied. Also, to optimise the solution efficiency of the model, an optimal iteration step selection method will be integrated into the model.

ACKNOWLEDGEMENT

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CONFLICT OF INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

NOMENCLATURE INDICES AND INDEX SETS

- $b$: Index of the transmission network's generator nodes
- $w$: Index of boundary buses between the transmission network and the ADN
- $d$: Index of the transmission network's load nodes
- $l$: Index of the transmission network lines
- $e$: Index of ADN's DG nodes
- $v_1(n)/v_2(n)$: Set of the initial/terminal node of line with terminal/initial node $n$ in the transmission network
- $ij$: Index of ADN branches
- $a$: Index of the ADN
- $z$: Index of ADN PV nodes
\( y_1(j)/y_2(j) \) Set of the initial/terminal node of line with terminal/initial node \( j \) in the ADN

\( G \) Set of generators at the transmission level

\( PCC \) Set of coupling nodes

\( D \) Set of loads at the transmission level

\( A \) Set of DGs in the ADN

**PARAMETERS**

- \( P_{bid}^G \) Genco's active power generation bid
- \( P_{bid}^{rG} \) Genco's reactive power generation bid
- \( P^G \) A DG's active power generation cost
- \( P^{rG} \) A DG's reactive power generation cost
- \( b_{loss} \) Penalty factor of transmission network's total network loss
- \( P_{d_{ij}}/Q_{d_{ij}} \) System active/reactive load of node \( d \) in the transmission network at time \( t \)
- \( P_{g{\min}}^D/P_{g{\max}}^D \) Minimum/maximum active power output of a generation unit
- \( RateA_{l{\min}}/RateA_{l{\max}} \) Minimum/maximum flow capacity of the transmission line \( l \)
- \( GSF \) Generation shift factor
- \( Q_{g{\min}}^e/Q_{g{\max}}^e \) Generation unit's minimum/maximum active power output
- \( P_{d_{ij}}/Q_{d_{ij}} \) Active/reactive load of node \( j \) in the ADN at time \( t \)
- \( r_{mn}/x_{mn} \) Resistance/reactance of line \( mn \) in the transmission network
- \( r_{ij}/x_{ij} \) Resistance/reactance of line \( ij \) in the ADN
- \( P_{DG_{\min}}^D/P_{DG_{\max}}^D \) DGs' minimum/maximum active power output at time \( t \)
- \( Q_{DG_{\min}}^e/Q_{DG_{\max}}^e \) Minimum/maximum reactive power output of DG
- \( V_{m{\min}}^2/V_{m{\max}}^2 \) Minimum/maximum voltage amplitude of node \( m \) in the transmission network
- \( V_{i{\min}}^2/V_{i{\max}}^2 \) Minimum/maximum voltage amplitude of node \( i \) in the ADN
- \( C \) Loop matrix in the transmission network
- \( \varphi \) DG's power-factor angle
- \( S_{pv{\max}} \) Maximum capacity of a PV inverter

**VARIABLES**

- \( c_{t{\max}} \) Expected price of the TSO in the energy market at time \( t \)
- \( c_{t{\min}} \) Expected price of the DSO in the energy market at time \( t \)
- \( b_{t{\max}} \) Bidding price of the TSO in the RPAS market at time \( t \)
- \( b_{t{\min}} \) Expected price of the DSO in the RPAS market at time \( t \)

\( P_{g,c_{ij}}^D/P_{s_{ij}}^D \) Active/reactive power delivered by the transmission network to the ADN at time \( t \)
\( \Delta Q_{g_{ij}}^\varphi/\Delta Q_{s_{ij}}^\varphi \) The purchased quantity in the RPAS market at time \( t \)
\( \Delta P_{ij} \) Active power compensated by the ADN at time \( t \)
\( P_{g_{ij}}^D \) Active power output of a generation unit at time \( t \)
\( P_{g_{ij}}^D \) Active power output of PV at time \( t \) in the energy market
\( P_{D_{ij}}^D \) Reactive power output of PV at time \( t \) in the RPAS market
\( P_{s_{ij}}^D \) Power purchased from the transmission network by the DSO at time \( t \)
\( Q_{s_{ij}}^\varphi \) Total network loss of the transmission network at time \( t \)
\( P_{g_{ij}}^D \) Generation unit's active power output at time \( t \)
\( P_{D_{ij}}^D \) Active/reactive power output of DG at time \( t \) in the energy market
\( Q_{e_{DG}}^\varphi \) Reactive power output of DG at time \( t \) in the RPAS market
\( Q_{e_{DG}}^\varphi \) The branch active/reactive flow of the ADN in the energy market
\( Q_{e_{DG}}^\varphi \) The branch active/reactive flow of the ADN in the RPAS market
\( I_{n_{mn}}^T \) Square of the voltage amplitude of node \( m \) of the transmission network in the RPAS market at time \( t \)
\( I_{n_{ij}}^T \) Square of the voltage amplitude of node \( i \) of the ADN in the energy/RPAS market at time \( t \)
\( I_{n_{mn}}^T \) Square of the current on line \( mn \) of the transmission network in the RPAS market at time \( t \)
\( I_{n_{ij}}^T \) Square of the current on line \( ij \) in the ADN in the energy/RPAS market at time \( t \)
\( P_{p_{ij}}^D \) The branch active/reactive flow of the transmission network in the RPAS market
\( P_{p_{ij}}^D \) Net active power injection of node \( j \) of the ADN in the energy/RPAS market at time \( t \)
\( Q_{p_{ij}}^\varphi \) Net reactive power injection of node \( j \) of the ADN in the energy market/RPAS market at time \( t \)
\( P_{n_{nk}}^T/Q_{n_{nk}}^T \) Net active and reactive power injection of node \( j \) of the transmission network in the RPAS market at time \( t \)

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