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Bunker Purchasing in Liner Shipping

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Abstract

The cost for bunker fuel represents a major part of the daily running costs of liner shipping vessels. The vessels, sailing on a fixed roundtrip of ports, can lift bunker at these ports, but prices in each port may be differing and fluctuating. The stock of bunker on a vessel is subject to a number of operational constraints such as capacity limits, reserve requirements and sulphur content. Contracts are often used for bunker purchasing, ensuring supply and often giving a discounted price. A contract can supply any vessel in a period and port, and is thus a shared resource between vessels, which must be distributed optimally to reduce overall costs. An overview of formulations and solution methods is given, and computational results are reported for some representative models.

Keywords: Bunker purchasing, Liner shipping, Mathematical programming, Maritime optimization, Decomposition methods

1. Introduction

Liner shipping companies are at the core of the major supply chains in the world, providing relatively cheap and reliable transport to and from any corner of the world. This industry has grown massively in the last decades, often with two digit percentage growth rates. Lately the supply of vessels have exceeded the demand for container transport, resulting in many liner carriers being loss giving. The profit margins in liner shipping are very slim, with marginal changes resulting in a company loss instead of profit.

This has shifted the shipping industry from a revenue optimizing focus, to use more resources on controlling and minimizing their costs. An example is the spend on bunker fuel, as this constitutes a very large part of the variable operating cost for the vessels. Also, the inventory holding costs of the bunker on board may constitute a significant expense to the liner shipping company.

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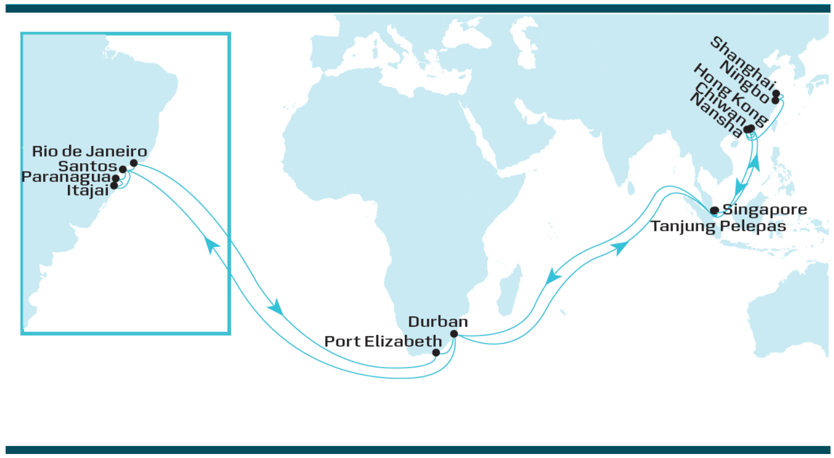

Figure 1: The ASAS 2 Service, transporting containers between East Coast South America, South Africa and the Far East.

For liner shipping companies in particular, the purchasing of bunkers can be planned some months ahead, as the vessels are sailing on a fixed schedule allowing for planning, as opposed to other types of shipping. An example of a liner shipping service can be seen in Figure 1, where the vessels are sailing between East Coast South America, South Africa and the Far East. This service allows for bunkering in three distinct markets, making it attractive to plan with a long time horizon.

Bunker prices are fluctuating and generally correlated with the crude oil price, but there are significant price differences between ports of up to 100 \$/mt (of a $\approx$ 600 \$/mt price). The price differences between ports are not stable, and the cheapest port on a roundtrip today may not be the cheapest tomorrow. In Figure 2 the prices for five important ports have been plotted for a time period of 18 months, illustrating how much the prices fluctuate. This creates the need for frequent (daily) reoptimization of the bunker plan for a vessel, to ensure the lowest bunker costs.

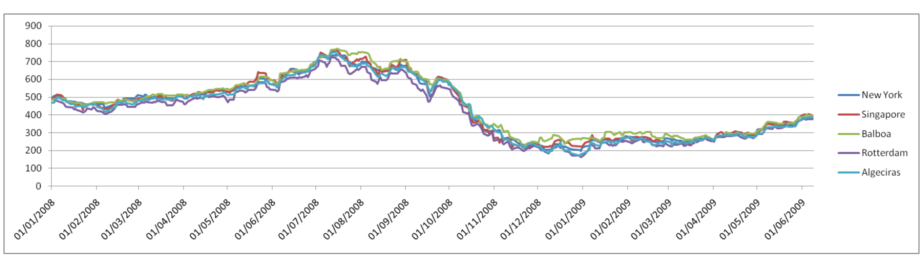

Figure 2: Bunker prices in New York, Singapore, Balboa, Rotterdam and Algeciras plotted from January 2008 to June 2009. On January 1st 2009, Balboa was almost 30% more expensive than Rotterdam and New York.

Port Id	Departure stock (mt)		Consumption (mt)		Purchase (mt)		Spot Price ($/mt)	
	LSFO	HSFO	LSFO	HSFO	LSFO	HSFO	LSFO	HSFO
NLROT	462	648	5	0	0	0	481	455
DEBRV	457	648	100	0	0	0	500	490
GBFXS	357	648	97	648	0	0	1000	675
USNWK	260	1004	0	134	0	186+**818**	491	465
USCHS	260	870	0	425	0	0	490	477
USSAV	260	445	0	74	0	0	493	457
USMIA	260	371	0	211	0	0	1000	1000
USHOU	260	1456	0	122	0	1296	1000	442
USMOB	260	1334	183	201	0	0	1000	1000
USNFK	77	1133	24	555	0	0	484	469
GBFXS	53	578	8	0	0	0	1000	641
NLROT	1053	4314	0	2340	1008	3737	442	421
DEBRV	1053	1974	0	447	0	0	457	447
USNWK	1053	1527	2	110	0	0	490	466
USCHS	1051	1417	0	25	0	0	495	482
USSAV	1051	1392	0	82	0	0	502	471
USMIA	1051	1310	0	211	0	0	1000	1000
USHOU	1051	1099	0	128	0	0	1000	451
USMOB	1051	971	0	365	0	0	1000	1000
USNFK	1051	606	221	606	0	0	510	495
NLROT	830	4021	21	0	0	4021	436	415
GBFXS	809	4021	65	0	0	0	1000	652
DEBRV	744	4021	98	511	0	0	467	456
USNWK	646	3510	0	161	0	0	485	464
USCHS	646	3349	0	19	0	0	496	483
USSAV	646	3330	0	98	0	0	499	468
USMIA	646	3232	0	183	0	0	1000	1000
USHOU	646	3049	0	135	0	0	1000	465
USMOB	646	2914	0	388	0	0	1000	1000

Table 1: An example of a bunker plan for a vessel operating a schedule with 29 ports.

The bunker purchasing problem is to satisfy the vessels consumption by purchasing bunkers at the minimum overall cost, while considering reserve requirements, and other operational constraints. Bunker can be purchased on the spot market when arriving to a port, but normally it is purchased some weeks ahead of arrival. Long-term contracts between a liner shipping company and a port can result in reduced bunkering costs by committing the company to purchase a given amount of bunker. Bunkering contracts may cover several vessels sailing on different services, making the planning quite complex.

An example of a bunkering plan can be seen in Table 1. *Departure stock* is the stock of bunker at departure of the port, as calculated by the model. *Consumption* is, the estimated consumption of bunker from this port to the next. *Purchase* is the quantity of bunker purchased at the port and $SpotPrice$ is the market price of bunker at the spot market. LSFO denotes low sulphur bunker, while HSFO denotes high sulphur bunker. Quantities are given in metric tonnes (mt). Possible bunker contracts are not shown. At the fourth port call 186 mt HSFO is bought at the spot market and 818 mt HSFO through a contract.

Literature. For a broad introduction to shipping and the importance of bunker spend refer to Stopford [27] and for an introduction to operations research within the maritime industry Christiansen et al. [8], Christiansen et al. [9] and Christiansen et al. [10] provide excellent overviews. A detailed description of Liner

Shipping Network Design and the impact of bunker usage and other relevant factors appears in Brouer et al. [5]. The authors also introduce **LINER-LIB 2012**, a benchmark data suite, consisting of liner shipping relevant data and benchmarks specifically for liner shipping network design problems. The work of Plum et al. [24] designs a liner shipping network taking bunker consumption into account. Details on the bunkering industry in relation to shipping can be found in Boutsikas [4].

The effect of the bunker price on Liner Shipping Network Design has been studied in a number of recent papers, as Wang and Meng [32] and Meng et al. [19]. The effect of bunker usage by the maritime industry in relation to the bunker price is investigated by Corbett et al. [11] with the aim of reducing CO_2 emissions by imposing tax on bunkers. The work of Acosta et al. [2] considers factors impacting the choice of bunker port. Fagerholt et al. [12] considers the optimal speed and route for a ship with respect to bunker costs. Other work on bunker costs and its impact on maritime transportation includes Notteboom and Vernimmen [20], who consider how slow steaming and the cost structure of liner shipping networks are affected by changes in bunker costs, and Ronen [25], who considers the bunker price's effect on speed and fleet size. The recent work of Wang et al. [33] provides an overview of available bunker optimization methods in shipping.

Fuel Purchasing. The problem of reducing fuel costs by optimizing fuel purchase has been investigated in a number of papers, studying different transport modes. Vilhelmsen et al. [31] investigated how tramp ships can be routed, while considering the impact on bunker costs. Oh and Karimi [21] plan bunker purchases for multi parcel tankers considering a fixed route under uncertain prices. This problems resembles bunker purchasing for liner vessels except that the vessel must make route deviations for bunker purchasing, thus making the problem partly a route selection problem, giving a different problem structure.

Research investigating how to refuel a transportation fleet has also been done for other transportation areas as the airline industry (Stroup and Wollmer [28], Abdelghany et al. [1]), trucking industry (Suzuki [29], Suzuki [30]) and in more general (Lin [17]). These papers take offset in the specific operational reality of the transport mode and possibly generates routes for the transport vehicle at the same time. This gives somewhat different optimization problems, not directly applicable to liner shipping bunker purchasing problems.

Bunker Purchasing in liner shipping. For a vessel sailing on a given port to port voyage at a given speed, the bunker consumption can be fairly accurately predicted. This gives an advantage in bunker purchasing, when a vessel has a stable schedule known for some months ahead. The regularity in the vessel schedules in liner shipping allows for detailed planning of a specific vessel, as considered in the works of Plum and Jensen [22], Besbes and Savin [3], Kim et al. [16], Kim [15], Sheng et al. [26] and Yao et al. [34]. These papers consider variants of a bunker optimization problem considering a single vessel.

Besbes and Savin [3] consider different refueling policies for liner vessels and presents some interesting considerations on the modeling of stochastic bunker prices using Markov processes. This is used to show that the bunkering problem in liner shipping can be seen as a stochastic capacitated inventory management problem. Capacity is the only considered operational constraint.

The work of Plum and Jensen [22] considers multiple tanks in the vessel and stochasticity of both prices and consumption, as well as a range of operational constraints. Yao et al. [34] does not consider stochastic elements nor tanks, but has vessel speed as an variable of the model. The work of Kim et al. [16] minimizes bunker costs as well as startup costs and inventory costs for a single liner shipping vessel. This is done by choosing bunker ports and bunker volumes but also having vessel roundtrip speed (and thus the number of vessels on the service) as an variable of the model; Kim [15] presents a different algorithm for a similar problem scoping.

In Sheng et al. [26] a model is developed which considers the uncertainty of bunker prices and bunker consumption, modelling their uncertainty by markov processes in a scenario tree. The work can be seen as an extension of Yao et al. [34], as it considers vessel speed as a variable within the same time window bounds. Capacity and fixed bunkering costs is considered, as is the holding / tied capital cost of the bunkers. Improved solutions are found, as compared to Yao et al. [34], credited to the uncertainty modelling. It only plans bunker purchases for a single roundtrip.

The studies described above do not consider bunker contracts, and all model the bunker purchasing for a single vessel. The work of Farina [13] is an extension of Plum and Jensen [22] with the additional consideration of bunker contracts, where a MIP model is presented capable of solving a 50 vessel instance for a 6 month period, falling short of solving real world instances of hundreds of vessels. Plum et al. [23] presents a decomposition algorithm for bunker purchasing with contracts, and showed that the model is able to solve even very large real-life instances involving more than 500 vessels, 40,000 port calls, and 750 contracts.

In the following section we will define the basics of bunker purchasing in liner shipping, and discuss all relevant constraints. The next section presents a basic model for bunker purchasing of a single vessel, using spot prices, and discuss extensions and variations presented in the literature. Finally, the model is extended to handle several vessels having shared contracts for buying bunker at fixed prices. This problem is more complex, and a decomposition model is needed to solve the problem to optimality for large instances. Finally, the chapter is concluded with a summary of the most important results, and directions for future research.

2. The bunkering problem

In this section we define the bunkering problem more formally, and introduce all relevant terms and constraints: We first introduce different bunker *grades*, and then describe how the *prices* for bunker at given dates and ports are obtained. Next, we describe the rules dictating how bunker is requested, *ordered*

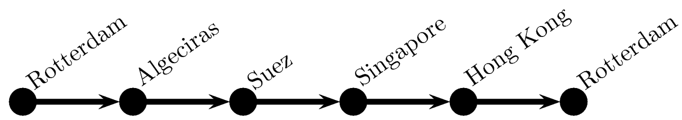

Figure 3: A simple example of a *service* starting and ending in Rotterdam.

and *delivered*. We then describe how the vessels' bunker *consumption* is calculated, the *tanks* used for storage, and bunker *reserve*. Finally we describe the testing of bunker and the *quarantine* periods this invokes, and various constraints for *mixing* different bunkers.

A vessel sails along a *path* of ports during a period of time. A *path* of length n can be divided into $n - 1$ *legs*. A *service* is a *path* starting and ending in the same port, a *round trip*. An example can be seen in Figure 3. For each port visit the vessel arrives and departs at specific dates and times, these are estimated time of arrival (ETA) and estimated time of departure (ETD).

All paths and the corresponding ETA and ETD are assumed to be fixed and can thus not be changed by a bunker purchasing model. The services of the vessels are constructed by shipping company considering other factors such as market demand for container transportation, etc.

It follows from this that interaction between vessels is impossible. Each vessel follows its own path and visits the ports at the specified times. Since no interaction is possible between the vessels we do only need to consider exactly *one* vessel in the model.

We have a given set of bunkers available (described in Section 2.1) in each port, and since we know exactly when the vessel visited each port, we can apply a fixed price for each bunker, for each port. This implies that we do not need to include time in the model, since we for each port visited have a fixed mapping from a bunker to a price.

2.1. Bunker Grades

Crude oil is refined into a variety of different products, ranging from jet fuel to gasoline, to bunker, to asphalt, see Figure 4. Bunker fuel oil is one of the heavier distillates from the distillation process.

Bunker fuel is sold in different grades, mainly distinguished by their viscosity, but also characteristics such as density, sulphur content and others are relevant. Some characteristics of the main bunker grades can be seen in Table 2. We mention RMF 180, RMG 380 etc. simply as 180, 380 and 700. For the case of RMG 380 15 we say 380 low sulphur.

Two of the characteristics are relevant for the optimization problem: the viscosity of the bunker and its sulphur content. The rest of the factors do not affect the modelling or the solution.

We will assume that all bunker types show approximately the same fuel efficiency. If this is not the case, it is easy to modify the proposed models to

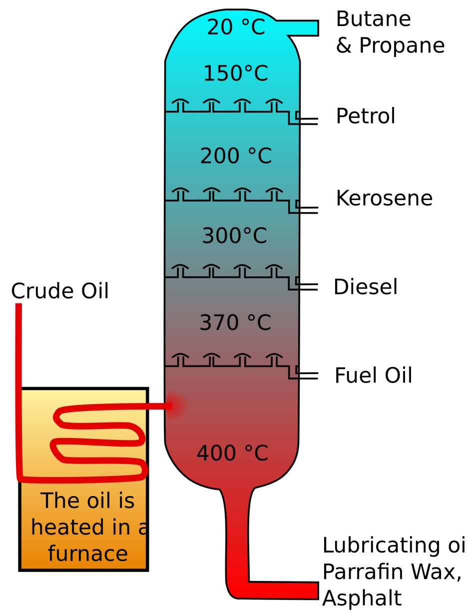

Figure 4: Illustration of the distillates from crude oil (source: Wikimedia).

take into account the efficiency. Since vessels generally only use one viscosity of bunker, the bunker consumption per nautical mile is easily adjusted to the designed bunker type.

2.1.1. Viscosity

Because of the high viscosity of bunker fuel oil, it is heated to 60-80°C prior to injection into the engine. The vessel's engine is designed to handle bunker of a certain viscosity, thus an engine can always burn more viscous bunker than the minimum grade it can take, but not less viscous bunker. E.g. a vessel engine which can burn a 380 bunker as the lowest quality grade, can also burn any 180

Parameter	Unit	Limit	RMF 180	RMG 380	RMG 380 15	RMK 700
Density at 15 °C	kg/m^3	Max	991.0	991.0	1010.0	1010.0
Viscosity at 50 °C	mm^2/s	Max	180.0	380.0	380.0	700.0
Water	% V/V	Max	0.5	0.5	0.5	0.5
Sulphur	% (m/m)	Max	4.5	4.5	1.5	4.5
Alum. + Silicon	mg/kg	Max	80	80	80	80
Flash point	°C	Min	60	60	60	60
Pour point, Summer	°C	Max	30	30	30	30
Pour point, Winter	°C	Max	30	30	30	30

Table 2: Characteristics of typical bunker grades. The RMG 380 15 is a low sulphur bunker and the rest are high sulphur bunkers.

bunker, but not a 700 bunker.

The prices of the bunkers are directly proportional with their viscosity, so a shipping company will always purchase the bunker with the lowest quality grade / highest viscosity available, which can be burned in the vessel's engine. Moreover, since bunker above the engine's viscosity limit cannot be burned, these bunkering options can be removed in a preprocessing phase.

2.1.2. Sulphur contents

Apart from categorizing a bunker by its density, viscosity, etc. it can be categorized by its sulphur content. Bunkers containing sulphur below a given limit are classified as *low sulphur* bunker. In certain parts of the world, known as SECA-areas (SO_x Emission Control Area), vessels are only allowed to burn low sulphur bunkers to limit air pollution. The Baltic Sea has been a SECA-area since May 19th 2006 by the MARPOL Annex VI protocol The North Sea became a SECA-area by November 22nd 2007. Also an area of 24 miles of California coast is a SECA area. The sulphur limits for fuel in SECA are 1.0% until January 2015, and 0.1% after January 2015. The general sulphur limits in other sea areas are 3.5% until January 2020, and 0.5% after January 2020. The future dates may be postponed if political agreement cannot be reached.

2.2. Bunker Prices

In practice there are two types of orders done by bunker traders: spot and contract orders. Spot orders are handled by a trader requesting bunker prices from one or more suppliers of the day and then places an order based on the price quotes. Contract orders are done on the basis of a contract, where the shipping company is obliged to purchase a certain volume of bunker within a certain period of time.

The market for bunker trading is commoditized and liquid, the use of contracts for a specified amount, port and price (or discount to some price-index) is widespread. This is done to reduce both delivery and price risk and to leverage the strength of being a large player on this market.

Liner shipping companies engage in contracts for the purchase of bunkers at ports where they have a large and regular demand. This is done both to gain a discount compared to the spot market, by leveraging on the large volumes involved, and to increase supply certainty. Bunker contracts will usually concern total lifted volumes within a calendar month, with specified minimum and maximal quantities.

The price can be agreed on in different manners, usually by using a fixed discount below the monthly average of a bunker index (Bunkerwire [6]) of the port in question. A contract is for one or more bunker grades and one or more ports, which usually will be located geographically close and considered as the same market. Many contracts can be available in a port for a bunkering vessel, and it must then be chosen which, if any, to purchase bunker from. *Spot* bunker is assumed freely available at all ports with a given *price quotes*. For an example of the bunker price development see Figure 5.

Port	**Rotterdam**		**Algeciras**		**Suez**		**Singapore**		**Hong Kong**	
	price	price	price	price	price	price	price	price	price	price
Grade	low	high	low	high	low	high	low	high	low	high
380	327	330	339	349	345	347	355	356	368	370
180	350	352	364	365	357	359	365	366	381	382
Marine Diesel	590	595	672	677	647	650	645	650	645	650

Table 3: Price quotes per metric tonne for 5 ports on April 24th 2006.

2.2.1. Ordering Time Window

An order for bunker must be placed before the vessel arrives at port. In order to prepare the suppliers and ensure that the bunker is available when the vessel enters the port, the order must be placed at some time interval before arrival (generally 72 hours). The order can be placed up to two weeks before the vessel arrives at the port, but normally happens 3-7 days before.

2.2.2. Price Structure

The objective of all bunker purchasing models is to minimize the total cost of bunker purchases. When calculating the bunker purchase costs, a number of factors should be taken into account:

Barge, Startup Costs

Generally the bunker pricing scheme differs between ports in America and ports in the rest of the world. European and other ports will price a fixed cost times the amount of bunker purchased. US ports will add a *barging* cost, for each of the barging vessels used to load bunker onto the vessel. (usually a vessel gets the bunker from barges holding around 2000 tons of bunker, which sails up to the vessel). This could be modelled using a piecewise linear objective function.

Different Price Over/Under 500 Tons

Some bunker retailers operate with different per ton price for purchases over and under 500 tons of bunker. This is a different version of the non-linear pricing

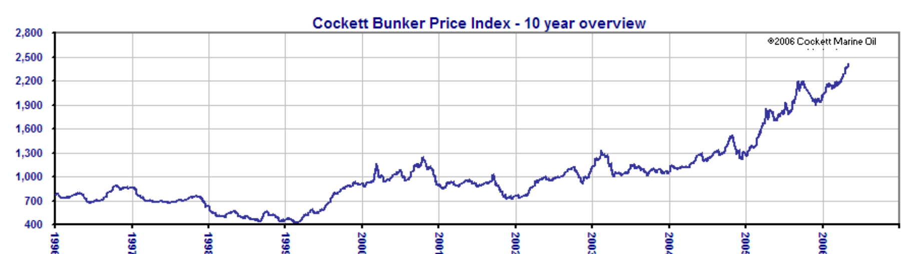

Figure 5: Price development for bunker oil over the last 10 years. The index was started at 1,000 on January 1st 1986, and represents the global price movement of bunkers.

structure mentioned above, but less used. This structure can also be modeled with a piecewise linear objective function.

Cost Of Capital Tie Up
The amount of bunker in a fully loaded vessel can represent a value of several million dollars. This value is tied-up in the bunker and cannot be used to generate interest nor be invested elsewhere. This represents a loss of profit which can be modelled by including the capital tie up term in the objective function.

Cost Of Bunker Carriage
The heavier a vessel is, the more bunker it will consume.

Bunker Tests
The price of a bunker is related to its specifications as described in Section 2.1.1. This gives the vendors of bunker an incentive to dilute their products with heavier components until it is just barely fulfilling the requirements of the bunker's ISO specifications. Hence it sometimes happens that the vendors sell products that does not satisfy the specifications they claim. These bunkers are *off-spec*. The consequences of burning off-spec bunker can be grave if the bunker already was the lowest possible burnable in the engine. The engine can malfunction and leave the vessel immobilized.

In many major ports it is possible to purchase bunker already tested by independent laboratories, so called *pre-tested* bunker. When this is available it can be trusted that the bunker is within the specification limits as claimed by the vendor, and the vessel can hence use the bunker immediately after purchasing.

For any purchased bunker a *post-test* is also carried out at an appointed analysis institute. This is also the case even for bunkers for which pre-test is available. Usually this process will take at most 5 days. The purchased bunker cannot be used in this time period.

2.2.3. Purchasing Limits

Other constraints apply to the bunkering in a port. A vessel will usually stay in a port between 12 and 24 hours from arrival to departure. The bunker can be loaded onto the vessel in 2 different ways. Larger ports have piping facilities in the actual docks, which can be connected to the vessel directly, and through these pipes large volumes of bunker can quite quickly be loaded onto the vessel. But this only exists in larger ports and only at certain anchorage places. When the vessel is lying at other ports or anchorage places, the bunkering is done with barges loading bunker from large land-side tanks, sailing to the vessel and then loading onto the vessel. The barges will usually take up to 2000 mt. of bunker each.

At busy times the maneuverability of the barges can be hindered by traffic from other vessels and port congestion can prevent the barges from sailing to the vessels, and thus prevent bunker from being loaded onto the vessel. Other minor ports might not have enough barges available. Together, these factors may impose a limit on how much bunker can be loaded onto the vessel. Therefore, a model should be able to handle an upper limit on the bunkering capacity, although most ports in reality do not have an upper limit.

A proper model should also enforce that some ports have a minimum amount of bunker that can be purchased, since vendors will refuse selling too small quantities of bunker. Typically this lower limit is 200 mt.

2.3. Bunker Consumption

The consumption of the vessel can (roughly) be regarded as a function of bunker type, weight of bunker, weather conditions, ocean currents, container load and speed of the vessel. Typically, the bunker type, speed of vessel and approximate container load is known some time in advance, while weather conditions and currents can change the bunker consumption significantly. A deterministic model for bunker purchasing must use average historic values to estimate bunker purchasing. The bunker consumption can also be considered as a stochastic variable in a model, leading to a more correct formulation that, however, is more difficult to solve.

2.4. Bunker Tanks

The vessels have multiple tanks. Some of these, typically four to 16 tanks, are larger tanks, holding up to some thousands of metric tonnes. A number of smaller special tanks in close connection to the vessel's engine exist as well, among these the day-tank which is directly connected to the engine, and holds enough bunker for 24 hours usage.

Some tanks are placed symmetrically at each side of the vessel. In order to keep the vessel's balance stable during travel, consumption must take place more or less symmetrically.

Certain tanks can be dedicated for some types of bunkers. Specifically since we are purchasing more expensive low sulphur bunkers such as RMK 380 15 to use when the vessel is sailing in the SECA area, we need to ensure that this bunker is not contaminated with bunker with a higher sulphur content. Therefore vessels sailing in the SECA area will have at least one tank dedicated to low sulphur bunkers. We can still use low sulphur bunker outside the SECA area.

2.4.1. Tank Limits

The volume of bunker grows with increasing temperature, so in order to avoid overflowing tanks, the tanks may only be filled to around 98% of their maximal capacity (at 25°C). This can easily be handled in a preprocessing phase, where the volume of the tanks is decreased to compensate for the fill limit.

The suction systems of the tanks are relying on the weight of the bunker in the tanks to extract the bunker to the engine. This means that under normal conditions the tanks cannot be emptied completely.

It is possible to move bunker between tanks, and this could be an advantage for an optimization model in some cases in order to avoid quarantine or mixing situations.

2.4.2. Bunker Reserves

When the vessel is sailing on the ocean it is paramount for any solution that the vessel can overcome any eventualities on its own. The cost of recovering a stranded vessel far exceeds any cost to the actual bunker. Therefore a solution needs to ensure that the vessel carries adequate reserves of tested bunker at all times.

Shipping companies have a number of requirements to bunker reserves which a valid fueling strategy must satisfy. Generally a vessel must hold 8 days of bunker in reserve at all times. This requirement can be lowered to 4 days reserve, when the next port on the vessel's service offers pre-tested bunker, see Section 2.2.2.

The actual amounts needed to be kept in reserve should be calculated for each port in the preprocessing face, based on the estimated consumptions per day of the vessel.

2.5. Bunker Quarantine Time

It is needed to ensure, that the bunker is not off-spec before it is used as fuel. As described in Section 2.2.2 this is done by possibly pre-testing and post-testing, which can take up to 5 days.

Meanwhile we cannot use the bunker, and it is therefore quarantined for 5 days. If we have mixed the bunker it is also quarantined, see the next section.

We only detail our model to how much bunker we use at which leg of the vessel's service, hence the 5 days quarantine will be translated to how many legs on the service the bunker must be in quarantine before we can use it. This can be calculated effectively for each port in preprocessing.

2.6. Mixing Of Bunkers

As mentioned in Section 2.4 each vessel is equipped with a number of tanks in different sizes. It is desirable to store the different bunker types separately, i.e. in different tanks in order to avoid the mixing of different bunker types. Two bunkers are considered as different types if at least one of the following properties holds:

- Differing fuel grades (density, viscosity, sulphur content, etc.)
- Same fuel grade bought at different ports
- Same fuel grade bought in the same port at different times
- Same fuel grade bought in the same port at different bunker vendors.

If two incompatible bunker types are mixed it will cause the creation of sediments, in this context asphalt, which when injected into the engine of the vessel can cause its malfunction. In this case the warranty from the manufacturer of the engine does not cover and shipping company will have to pay for a new engine plus they will loose money from lost earnings.

The risk of producing a bad mix is proportional to the density and viscosity of the bunkers and also depends on the ratio of the mixed bunkers. E.g. there is low risk of producing a bad mix when blending bunkers with low density, low viscosity and one bunker being added in a higher proportion than the other.

When a tank with a bunker volume less than 10% of the tank capacity subsequently is filled with at least 9 times as much bunker of a different type it is not considered a mixing of bunkers. The tank is then considered as containing the bunker of the highest volume. We call this a 9:1 mix.

It is only permissible to mix two bunkers at a time, so mixes with three or more constituents should not be considered. This also implies that two mixed bunkers, cannot be mixed again.

The amount of tanks where mixing takes place should be minimized. This takes precedence over minimizing the ratio of mixture. A scenario could be considered where the ratio of mixture is lowered by spreading the original bunker over a number of tanks, and mixing in each of these. This strategy should not be encouraged by an optimization model.

3. A model for single-vessel bunker purchasing

We will now introduce a basic model for bunker purchasing considering only one vessel. This means that bunker is bought on the spot market, or through contracts only covering the studied vessel.

We use a deterministic model, based on the formulations from Besbes and Savin [3] and Plum and Jensen [22], hence assuming that the bunker consumption for every leg is known in advance. We will also assume that a sufficient amount of tanks are present for each vessel, so that the mixing constraints discussed in Section 2.6 does not need to be taken into account. This is the case for most large vessels. The basic model is given without special cost structures such as piece-wise linear bunker costs and capital start up cost. Also some operational constraints are omitted, such as quarantine requirements, several bunker tanks and possibility of mixing bunkers. These cost structures and operational constraints are described in Section 4 as extensions to this basic bunker purchasing model.

We introduce the mathematical notation used throughout this chapter. Let $v \in V$ be the set of vessels. Let $i \in I$ be an ordered set of *port calls*, the vessel's schedule. A port call i will be uniquely defined by a port, a vessel, $v(i)$ and the date of arrival. Let $init(v)$ and $term(v)$ be the first and last considered port call of vessel v. Let $b \in B = \{L, H\}$ be the two considered bunker types. The startup cost for bunkering at a port call i is $startcost_i$. Each vessel, v has a capacity $D_{v,b}$ for each bunker type, b. For each leg i of the schedule, the vessel consumes $F_{i,b}$ bunker, between port call i and $i + 1$.

Variables used in the model are as follows: $l_{i,b}$ is the purchase of bunker for each port i, bunker type b. The binary variable $\delta_{i,b}$ is set to one iff a purchase of a bunker type b is made at a port call i. The volume $h_{i,b}$ of bunker after a vessel leaves port is a continuous variable, as is the consumption $f_{i,b}$ of each bunker type on vessel between port i and $i+1$.

3.1. Model

A basic model for Bunker Purchasing for Liner Shipping can be formulated as the following Mixed Integer Program:

$$\min \sum_{i\in I}\sum_{b\in B}(\delta_{i,b} \cdot startcost_i) + \sum_{i\in I}\sum_{b\in B}(p_{i,b} \cdot l_{i,b})$$

Subject to

$$h_{i,b} = h_{i-1,b} + l_{i,b} - f_{i-1,b} \quad \forall i, b \tag{1}$$

$$f_{i,b} \le h_{i,b} \quad \forall i, b \tag{2}$$

$$\sum_{b\in B} f_{i,b} = F_{i,H} \quad \forall i \tag{3}$$

$$f_{i,L} \ge F_{i,L} \quad \forall i \tag{4}$$

$$h_{i,b} \le D_{v(i),b} \quad \forall i, b \tag{5}$$

$$l_{i,b} \le \delta_{i,b} \cdot D_{v(i),b} \quad \forall i, b \tag{6}$$

The objective minimizes startup costs and bunker cost. The constraint (1) ensures flow conservation at each port, vessel and bunker type. Constraint (2) ensures that no more bunker than available is used between port i and $i+1$. Constraints (3) and (4) maintains the consumption of bunker, allowing LSFO to substitute HSFO, but not the other way around. The bunker capacity of the vessels are enforced by constraint (5). The decision variables $\delta_{i,b}$, indicating if any bunker is purchased by vessel b at port i, are set by constraint (6).

Finally, initialization and termination criteria for start and end bunker volumes must also be set. Let $S_{v,b}$ and $T_{v,b}$ be the start and terminal volume of bunker b on vessel v. This leads to the following constraints:

$$h_{init(v),b} = S_{v,b} \quad \forall v, b \tag{7}$$

$$\sum_{b\in B} h_{term(v),b} \ge \sum_{b\in B} T_{v,b} \quad \forall v \tag{8}$$

$$h_{term(v),L} \ge T_{v,L} \quad \forall v \tag{9}$$

Constraint (7) sets the start volume for both bunker types. Constraint (9) is the terminal volume for low sulphur bunker, while constraint (8) is the terminal volume for high sulphur bunker, allowing substitution.

The domains of the variables are:

$$h_{i,b}, l_{i,b}, f_{i,b} \in \mathbb{R}^+ \quad \forall i, b \tag{10}$$

$$\delta_{i,b} \in \{0, 1\} \quad \forall i, b \tag{11}$$

A usual time horizon for a single vessel model is 3 to 6 months, where a vessel may call up to a 100 ports. The MIP resulting from such problem instances can be solved by state of the art commercial MIP solvers, usually in a matter of seconds.

4. Operational Constraints

In practice bunker purchasing in liner shipping is influenced by a wide range of operational, commercial and financial factors, which dictates the properties of a good bunker plan. Some of these factors are modeled as MIP constraints in the following. Refer to the earlier mentioned literature for an elaborate discussion of other factors.

Bunker Reserves and Startup costs
As the consumption of bunker on a leg is an uncertain parameter due to factors as changed schedule (and thus speed), wind, current, waves and hull roughness, a good bunker plan will allow for variation in the bunker consumption. A way to handle this is to enforce a minimum reserve requirement of bunker at port arrival. This can be modeled as in (12), where $F_{i,b}$ is the minimal reserve requirement at port arrival.

Besides the startup cost for bunkering, $startcost_i$, bunker suppliers will usually require a minimum quantity to be purchased at each bunkering, this can be handled with constraints (13), where $\underline{L}_{i,b}$ is the minimal quantity.

$$F_{i,b} \leq \sum_{b \in B} (h_{i,b} - l_{i,b}) \quad \forall i, b \tag{12}$$

$$\delta_{i,b} \cdot \underline{L}_{i,b} \leq l_{i,b} \quad \forall i, b \tag{13}$$

Capital and carriage cost
The capital costs of bunker is extensive, due to the large volumes and high prices. A model could consider this by adding this cost (or lacking interest) to the objective, proportional to the average load of bunker on the vessels. Assuming an interest rate of α and a bunker cost of C \$/mt, we can estimate a daily capital cost per mt of bunker as:

$$(\sqrt[365]{1+\alpha} - 1) \cdot C \ \$/mt \tag{14}$$

A typical value for the interest rate is $\alpha = 10\%$ and a typical value of bunker cost is 600 \$/mt, leading to the daily capital cost per mt of bunker:

$$(\sqrt[365]{1.1} - 1) \cdot 600 \ \$/mt = 0.157 \ \$/(day \cdot mt) \tag{15}$$

Let d_i denote the number of days a vessel uses to travel from port i to port $i+1$. Then the cost of capital tie up can be included in the objective function as:

$$0.157 \cdot d_i(\sum_{b \in B} h_{i,b} - \frac{F_{i,b}}{2}) \tag{16}$$

Similarly a vessel carrying a large volume of bunker will, all things equal, have a larger draft. This will in general (but not always, due to specifics in the vessels design as the bulb) imply an increased bunker consumption proportional to an increased load. This term could be considered in the objective in the same manner as the capital costs.

Alternatively this can be modeled directly as an increased consumption proportional to increased bunker load. If we assume an increased consumption of $\gamma \cdot mt_{extra}/(day \cdot mt_{carried})$, the corrected bunker consumption per leg, $F_{i,b}^{corr}$ becomes:

$$F_{i,b}^{corr} = F_{i,b} + \gamma \ d_i(\sum_{b \in B} h_{i,b} - \frac{F_{i,b}}{2}) \tag{17}$$

We can then replace the term $F_{i,b}$ with $F_{i,b}^{corr}$ to model the increased cost of carrying bunker.

California sales tax
The California bunker sales tax, as described by California Legislative Analyst's Office [7], imposes a tax on bunker bought in California, which necessarily must be used en-journey to the first out of state port. I.e. if a vessel arrives with 1000 mt at an Californian port and requires 2000 mt to reach the first non-Californian port on its schedule, it must pay a tax for the first 1000 mt purchased. With additional decision variables this can be modelled and included in the objective.

Quarantine
A sample is usually taken from purchased bunker, to be analyzed for its specific content of carbohydrates, sulphur, water, ashes, etc. The sample must be within the ISO specifications of the purchased bunker grade. Until the result of the laboratory test are received, the bunker may not be used. This test can take three to five days. This can be handled by increasing the reserve requirements at port calls with bunker purchased within the last five days, $i' \in Quar(i)$:

$$F_{i,b} \leq \sum_{b \in B} \left(h_{i,b} - l_{i,b} - \sum_{i' \in Quar(i)} l_{i',b} \right) \quad \forall i, b \tag{18}$$

Other Constraints
Other more detailed operational constraints can be handled in preprocessing of data for the model or by adding new constraints. This includes:

- Vessels that cannot bunker at a port due to wielding works imposing fire hazards. In such ports the maxlift limit can be set to: $\overline{L}_{i,b} = 0$.

- Ports with limited quantity k_1 of bunkers available can be handled by setting the maxlift limit to: $\overline{L}_{i,b} = k_1$.
- Vessels with stability or air draft requirements requiring high drafts. This can be imposed by forcing k_2 mt of bunkers at arrival by increasing the reserve requirements $F_i = k_2$.
- Tank fill limits can be handled by lowering the tank limits as follows: $D^*_{v,b} = 0.98 \cdot D_{v,b}$.
- A maximal number N_{Bunker} of bunkerings can be enforced by the constraint: $\sum_{i \in I} \sum_{b \in B} \delta_{i,b} \leq N_{Bunker}$.

The above constraints can be added to the model without increasing its complexity significantly. All of them can be formulated linearly and only relate to a single vessel at a time, allowing them to be considered in a vessel specific subproblem.

4.1. Complexity

The problem (1) to (6) is NP-hard to solve, which can be seen by reduction from the knapsack problem. The knapsack problem in minimization form as described in Kellerer et al. [14] can be formulated as follows: Given a set N of items having profit p_i and weight w_i and a knapsack of capacity c, the problem is to fill the knapsack at minimum overall profit, such that the overall weight is at least c. Given an instance of the knapsack problem, we construct an instance of the bunker purchasing problem by having one vessel, visiting N ports. The fuel consumption between each pair of ports is 0, except the leg after the last port visit, where the consumption is c. In each port, we have a contract of maximum w_i, and the minimum limit for lifting bunker is also w_i. The cost of buying the quantity w_i is p_i. It is easily seen that solving the bunker purchasing problem also solves the knapsack problem.

5. Bunker Purchasing with Contracts

The single-vessel model presented in the previous section does not take into account volume refueling discounts, which can only be fully exploited using more than one vessel. Thus we will now present a multi-vessel optimization model, taking into account various contracts covering multiple vessels.

The model considered in this chapter uses a *crystal ball* approach, i.e. using data not known at decision time, to benchmark the quality of already executed decisions. As the actual price of the contract is not known before a month has passed, the model will use after-the-fact prices for calculations.

Contract bunker

Contract bunker must be purchased according to details given by a number of contracts $c \in C$, minimal and maximal quantities are given by $\underline{q}_c$ and $\overline{q}_c$. The specified quantities are soft constraints, which can be violated by paying a high cost, $\underline{w}$, for violating the minimum volume and a lower cost for breaking the maximal constraint, $\overline{w}$. Contract c may cover several ports and multiple vessels can call at these ports in the duration of the contract. Each contract will give rise to a number of purchase options, $m \in M$, i.e. discrete events where a specific port call i, and thus vessel v, calls within a time period, allowing it to purchase bunker from a contract c. Purchases on a purchase option m will be done at a price p_m, specified by the contract c. To simplify modelling and to increase the density of the derived model, the sets of port calls, $i \in I$ and purchase options, $m \in M$ will be used instead of their underlying sets: ports, vessels and contracts, which could give an equivalent but much larger model.

The possibility of purchasing on the spot market, is considered as a special type of contract. The minimal and maximal volumes are relaxed as $\underline{q}_c = 0$ and $\overline{q}_c = \infty$. All port calls i have two spot purchase options m for LSFO and HSFO, with prices set at the corresponding spot price of the day and port. For ports where bunker prices are not published, we assume a high cost.

The model makes use of the following variables: l_m is the purchase of bunker for each purchase option m. The binary variable $\delta_{i,b}$ is set iff a purchase of a bunker type b is made at a port call i. The volume $h_{i,b}$ of bunker b after a vessel leaves port i is a continuous variable, as is the consumption $f_{i,b}$ of each bunker type b between port i and $i+1$. The contract violation or slack variables are $\underline{s}_c$ and $\overline{s}_c$. We let $M(c)$ denote the set of purchase options specified by contract c, and $M(i,b)$ the set of purchase options for bunker b in port i.

5.1. Model

The Bunker Purchasing with Contracts Problem (BPCP) can be formulated as the following Mixed Integer Program:

$$\min \sum_{i \in I} \sum_{b \in B} (\delta_{i,b} \cdot startcost_i) + \sum_{m \in M} (p_m \cdot l_m) + \sum_{c \in C} (\underline{s}_c \cdot \underline{w} + \overline{s}_c \cdot \overline{w})$$

subject to

$$h_{i,b} = h_{i-1,b} + \sum_{m \in M(i,b)} l_m - f_{i-1,b} \quad \forall i, b \tag{19}$$

$$f_{i,b} \leq h_{i,b} \quad \forall i, b \tag{20}$$

$$\sum_{b \in B} f_{i,b} = F_{i,H} \quad \forall i \tag{21}$$

$$f_{i,L} \geq F_{i,L} \quad \forall i \tag{22}$$

$$h_{i,b} \leq D_{v(i),b} \quad \forall i, b \tag{23}$$

$$\underline{q}_c - \underline{s}_c \leq \sum_{m \in M(c)} l_m \leq \overline{q}_c + \overline{s}_c \quad \forall c \tag{24}$$

$$\sum_{m \in M(i,b)} l_m \leq \delta_{i,b} \cdot D_{v(i),b} \quad \forall i, b \tag{25}$$

The objective minimizes startup costs, bunker cost and contract violation penalties. Constraint (19) ensures flow conservation at each port, for the given vessel and bunker type. Constraint (20) ensures that between port i and $i+1$ bunker can only be used if available. Constraints (21) and (22) maintains the consumption of bunker, allowing LSFO to substitute HSFO, but not opposite. The bunker capacity of the vessels are enforced by constraint (23). The minimal and maximal quantity required by the contracts are ensured by the double sided constraints (24), allowing for violations. The decision variables $\delta_{i,b}$ are set by constraint (25).

Initialization and termination criteria for start and end bunker volumes must also be set:

$$h_{init(v),b} = S_{v,b} \quad \forall v, b \tag{26}$$

$$\sum_{b \in B} h_{term(v),b} \geq \sum_{b \in B} T_{v,b} \quad \forall v \tag{27}$$

$$h_{term(v),L} \geq T_{v,L} \quad \forall v \tag{28}$$

The first constraint defines the start volume, while the last two constraints define the terminal volume, allowing low sulphur bunker to substitute high sulphur bunker.

Finally the domain of the variables is given as follows:

$$h_{i,b}, l_m, f_{i,b}, \underline{s}_c, \overline{s}_c \in \mathbb{R}^+ \qquad \forall i, b, m, c \tag{29}$$

$$\delta_{i,b} \in \{0,1\} \qquad \forall i, b \tag{30}$$

5.2. Bunker Contracts - Operational Constraints

Contracts may have minimum and maximal volumes that must be lifted per purchase, $\underline{N}_{i,b,c}$ and $\overline{N}_{i,b,c}$. This can be modelled similarly to the minimum lift constraints. As can purchases at port calls have maximal lift restrictions, $\overline{L}_{i,b}$, due to short port stays or limited supply:

$$\delta_{i,b} \cdot \underline{N}_{i,b,c} \leq \sum_{m \in M(i,b,c)} l_m \quad \forall i, b, c \tag{31}$$

$$\sum_{m \in M(i,b,c)} l_m \leq \delta_{i,b} \cdot \overline{N}_{i,b,c} \quad \forall i, b, c \tag{32}$$

$$\sum_{m \in M(i,b)} l_m \leq \delta_{i,b} \cdot \overline{L}_{i,b} \quad \forall i, b \tag{33}$$

6. Advanced Model Extensions

In order to get a closer correspondence with reality, the model can be extended to handle uncertainty in bunker consumption and prices. Moreover modelling of multiple bunker tanks and the properties of mixing different batches of bunker can add precision to the model. However, not all shipping companies have sufficient data quality to justify these extensions. Vessel speed can also be considered a variable which can be adjusted by the model.

6.1. Uncertainty

The previously described models assumes deterministic problem instances. Like in all models of real world problems this is an approximation of the full problem. In particular the uncertainty applies for the consumption of bunker and the price of bunkers, neither of which are deterministic.

Consumption Uncertainty
The bunker consumption for a given vessel, distance, speed and displacement can be fairly accurately predicted. Still the bunker consumption is affected by uncertainty due to factors as: unforseen changes in distance (changed schedule), speed (earlier / later arrival), displacement (more or less cargo), uneven speed (lowest consumption is attained at an even speed throughout), weather and many other factors.

The model can be translated into a multi stage stochastic program, working on a generated scenario tree taking offset in the consumption estimates. For more details on this please refer to Plum and Jensen [22] or Sheng et al. [26].

Price Uncertainty
Bunker prices which are correlated with crude oil prices are also stochastic and much harder to predict than bunker consumption, though some interesting atempts are done in Sheng et al. [26]. A wealth of literature and experts devote their time in how to predict such commodity prices and this text will not add to this. Instead we use the current bunker price at a port as the basic estimator for the future price. One enhancement could be to use the direction of the bunker forward price for the region of the port to predict the direction of the price.

6.2. Bunker Tanks

The vessels have multiple tanks as described in Section 2.4. Handling the tanks in an optimization model imposes a number of extra constraints.

Tank Limits
The volume of bunker grows with increasing temperature, so in order to avoid overflowing tanks, the tanks may only be filled to around 98 % of their maximal capacity (at 25 C). As mentioned, this should be considered in a data preprocessing phase, where the volume of the tanks should be decreased accordingly.

The suction systems of the tanks cannot be empty the tanks completely, leaving about 1 % of the capacity. This can be implemented as a reserve limit specific for each tank.

Commingling of bunkers
In general commingling of bunkers of different supplier, grade or batch is inadvisable as the combined properties are hard to predict and may form sediments or have unpredictable properties which the engine system can not handle. The

constant mL indicates the limit for when a mixing is hazardous, and $mL = 9$ indicates that if bunker is mixed at a ratio greater than 9 to 1, it is not considered a commingling.

Two approaches can be taken to ensure that bunker commingling does not take place:

- Tanks are not modelled explicitly, but it is assumed that the number of tanks will allow vessel crew to easily find a feasible solution of a concrete bunker plan.
- Bunker tanks can be modelled explicitly, adding constraints to ensure that commingling cannot take place.

Modelling bunker tanks
Bunker tanks can be modelled by adding an extra index $t \in T$ for each bunker tank t. This means that parameters and variables l_m, $\delta_{i,b}$, $h_{i,b}$, $f_{i,b}$, and $D_{v,b}$ are replaced by $l_{m,t}$, $\delta_{i,b,t}$, $h_{i,b,t}$, $f_{i,b,t}$, and $D_{v,b,t}$.

The original constraints will be replaced by a constraint for each $t \in T$ where applicable. The mixing constraint then becomes:

$$mL \cdot (h_{i-1,b,t} - f_{i-1,b,t}) - l_{m,t} \leq (1 - \delta_{i,b,t} + \gamma_{i,b,t}) \cdot mL \cdot D_{v,b,t} \tag{34}$$

The mixing constraint forces the volume $l_{m,t}$ of newly bought bunker in tank t, to be at least mL times as much as the current volume of bunker, $h_{i-1,b,t} - f_{i-1,b,t}$, unless a penalty indicated by $\gamma_{i,b,t}$ is paid in the objective. It is not practice to dilute 9 units of bunker with one unit of newly bought bunker, so this is not modelled. The term

$$\sum_{i \in I} \sum_{b \in B} \sum_{t \in T} mixpen \cdot \gamma_{i,b,t} \tag{35}$$

should be added to the objective, where $mixpen$ is the penalty for mixing. To avoid mixing the penalty should be set as $mixpen = \infty$. It is possible to move bunker between tanks, which can be used to avoid quarantine or mixing situations.

6.3. Speed Adjustment

The work of Yao et al. [34] extent the bunker purchasing problem by considering the speeds of the vessels on a fixed itinerary of ports, but with some flexibility on the departure and arrival times at the ports. The model of Kim et al. [16] control the full roundtrip speed and thus the roundtrip time and number of vessels assigned. This results in very slow service speeds, which arguably can be uncompetitive commercially. The variable speed is considered by letting the constant $F_{i,b}$ be a variable $F^{var}_{i,b}$ dependent on vessel speed following the equation:

$$F^{var}_{i,b} = k_b(k_1 V_i^3 + k_2) \tag{36}$$

Where V_i is the speed from port i to port $i+1$, k_1 and k_2 are constants and $0 \leq k_b \leq 1$ is the fraction of bunker of bunker type b used on leg b. Additional constraints impact V_i :

$$v_{min} \leq V_i \leq v_{max} \qquad \forall i \in I \tag{37}$$

$$A_i + t_i + \frac{d_i}{V_i} = A_{i+1} \qquad \forall i \in I \tag{38}$$

$$e_i \leq A_i \leq l_i \qquad \forall i \in I \tag{39}$$

Where constraint (37) ensures that the vessel does not exceed its min speed v_{min} and its max speed v_{max} on any legs. Constraint (38) sets the arrival times A_i and A_{i+1} in relation to the speed V_i, distance d_i and port time t_i. Constraint (39) ensures that the arrival time A_i are within the time windows of the port call e_i and l_i.

This model is cubic, but can be linearized as it is a convex function being minimized. For details please refer to Yao et al. [34]. For single vessel instances this problem can be solved by commercial solvers.

It should be noted that in practice the choice of port arrival time is impacted by many other factors than minimizing bunker costs. This could be the amount of cargo that needs to be loaded / unloaded at the port; When vessels that containers must be transshipped to arrive; When the port berth is available, etc. Due to the cubic nature of the bunker consumption curve, the best sailing speed for a vessel on a given rotation, will be an even speed throughout the rotation. This dictates that when buffer time is available at a port the leg, the leg with highest speed before / after the port, should use the buffer to lower the speed. This could easily be handled in preprocessing for the problem.

7. Solving the multi-vessel model

The fleet of a global liner shipping company may consist of hundreds of vessels, with many of these having overlapping schedules visiting the same hub ports. This means that the full problem can be of a very large size, making the MIP model impossible to solve for large instances as observed by Plum et al. [23]. This makes it interesting to consider a decomposition of the MIP model, to solve these large problem instances.

The arc flow model given by (19) - (30) is Dantzig-Wolfe decomposed on the variables l_m. Let R_v be the set of all feasible bunkering patterns for a vessel v, satisfying constraints (19) - (30), except (24). This set has an exponential number of elements. Each pattern $r \in R_v$ is denoted as a set of bunkerings. Let $u_r = \sum_{m \in M}(p_m \cdot l_m) + \sum_{i \in I} \sum_{v \in V} \sum_{b \in B}(\delta_{i,b} \cdot startcost_i)$ be the cost for pattern $r \in R_v$. Let λ_r be a binary variable, set to 1 iff the bunkering pattern r is used. Let $o_{r,c}$ be the quantity purchased of contract c by pattern r. The BPCP can then be formulated as:

$$\min \sum_{v \in V} \sum_{r \in R_v} \lambda_r \cdot u_r + \sum_{c \in C} (\underline{s}_c \cdot \underline{w} + \overline{s}_c \cdot \overline{w}) \tag{40}$$

Subject to

$$\underline{q}_c - \underline{s}_c \leq \sum_{v \in V} \sum_{r \in R_v} \lambda_r \cdot o_{r,c} \leq \overline{q}_c + \overline{s}_c \qquad \forall c \tag{41}$$

$$\sum_{r \in R_v} \lambda_r = 1 \qquad \forall v \tag{42}$$

$$\lambda_r \in \{0, 1\} \qquad \forall r \tag{43}$$

The objective minimizes the costs of purchased bunker, startup costs and slack costs. Constraints (41) ensures that all contracts are fulfilled. Convexity constraints (42) ensure that exactly one bunker pattern is chosen for each vessel.

7.1. Pricing Problem

Let $\underline{\pi_c} \leq 0$ and $\overline{\pi_c} \leq 0$ be the dual variables for the upper and lower contract constraints (41), due to the structure of these constraints at least one of these will be 0 for each contract c. Let $\theta_v \in \mathbb{R}$ be dual variables for the convexity constraints (42). Then the pricing problem becomes:

$$\text{Min: } u_r + \sum_{c \in C} (\underline{\pi_c} - \overline{\pi_c}) - \theta_v \tag{44}$$

Subject to constraints (19) - (30), except (24).

This pricing problem is a Mixed Integer Program, considering a single vessel. This size of problem can be solved in reasonable time by a standard MIP solver, as done in Plum and Jensen [22]. Columns λ_r with negative reduced cost will then be added to the master problem, also solved as a MIP.

7.2. Column Generation Algorithm

Due to the large number of columns in model (41) to (43) Plum et al. [23] proposed to solve the LP relaxed model by *Column Generation*. Using the generated columns from the LP-solution, the resulting problem is then solved to integer optimality using a MIP solver, leading to a heuristic solution for the original problem.

Initially all dual variables are set to zero, a subproblem is constructed for each vessel and solved as a MIP problem. The first master problem is then constructed with one solution for each vessel as columns. This master is solved and the first dual values are found. The subproblems are resolved for all vessels (only the objective coefficients for the contracts needs updating) and new columns are generated for the master. This continues until no negative reduced cost columns can be generated, and the LP optimal solution is achieved.

In the next step, the problem is solved as a MIP, providing an integral solution. The subproblems only need to find *a* negative reduced costs column, to ensure progress of the algorithm. This means that initially they are allowed to return solutions with considerable subproblem gaps. As the algorithm progresses, the allowable subproblem gap is reduced, until it reaches the tolerance level.

7.3. Dual stabilization

A simple form of dual stabilization has been used in the implementation by Plum et al. [23] to speed up convergence. The Box-step method described in Marsten et al. [18] imposes a box around the dual variables, which are limited from changing more than π_{max} per iteration. This has been motivated by the dual variables only taking on values $\{-\underline{w}, \overline{w}, 0\}$ in the first iteration, these then stabilize at smaller numerical values in subsequent iterations.

7.4. Interpretation of dual values

The dual variables $\underline{\pi_c}$ and $\overline{\pi_c}$ for the upper and lower contract constraints (41) can be used to evaluate the gain of a given contract.

Using best estimates for bunker consumption and prices (current prices for instance) together with known or expected contracts, a baseline bunker purchasing plan could be run. A new scenario could then be constructed with the addition of the considered contract and by analyzing the output, it could be seen whether the overall costs of the scenario increased or decreased as compared with the baseline.

Another investigation could be to solely consider the baseline's final dual variables, $\underline{\pi_c}$ and $\overline{\pi_c}$, and depending on the magnitude of these evaluate the contracts effect. As these dual values are the same for all subproblems, they can be interpreted as balancing out the price of the contract, increasing the price if it is a popular contract or decreasing it otherwise, converging when they are in balance. The magnitude of this will be proportional to the contracts gain.

8. Computational Results

According to Plum and Jensen [22] the simple model proposed in Section 3.1 can be solved in a couple of seconds, since every vessel is considered independently.

The model with bunker contract presented in Section 5 is more difficult to solve as reported in Plum et al. [23]. To give an impression of the complexity, we will now present computational results for a number of real-life instances including up to 500 vessels, 40.000 ports, and 750 contracts. These instances are representative for what problems need to be solved in a major liner shipping company.

The **MIP** model described in Section 3.1 has been implemented in **CPLEX 12.2**, while the column generation algorithm outlined in Section 7.2 has been implemented in **ILOG OPL** as modelling language and **CPLEX 12.2** as LP/MIP solver. We will use **DW** to denote the column generation implementation.

Real life data for a large number of liner vessels describing their schedules, consumptions, tank capacities and other relevant data has been made available by **Maersk Oil Trading**, who have also supplied data on a large number of actual bunker contracts and spot prices available in a range of ports. Based on these data a number of instances have been constructed to test the scalability and performance of the implementations. Due to confidentiality reasons the price's have been distorted by $\pm 10\%$. This small amount of noise, however, does not affect the main structure of the problem. The penalty $\underline{w}$ for violating minimum volume is set at 200 \$/mt, and the penalty $\overline{w}$ for breaking the maximal constraint at 50 \$/mt. If a bunker price is not available at a port, the price is set at 1000 \$/mt. Details about the instances can be seen in Table 4.

Instance	Size	V	P	C
RULED	Small	6	1048	29
FRFSM	Small	8	2128	10
ZADUR	Small	49	5973	35
US_WC	Small	32	6022	68
USNWK	Medium	49	9048	69
USSAV	Medium	50	9194	23
PABLB	Medium	65	9817	27
AEJAL	Medium	80	15442	9
09_H2	Large	408	16214	307
11_H2	Large	572	18426	254
10_H1	Large	469	18704	332
10_H2	Large	534	21907	424
11_H1	Large	609	23453	376
HKHKG	Large	158	29177	20
10_FY	Large	535	40611	756

Table 4: Instances of varying sizes for the BPCP. **Instance** is the name of the instance, **Size** is a grouping of the instances. V the number of vessels, P the number of port calls, C the number of Contracts.

An overview of the performance and results can be found in Table 5. It can be seen that the **DW** model is able to solve the problem for all instances. For larger instances **MIP** runs out of memory and finds no solution, due to the size of the instances and their resulting MIPs. Both models find solutions with very small gaps, but still considerable absolute gap's to the optimal solution. **MIP** only finds optimal solutions for the smallest instances, for all medium and large instances the solver runs out of memory before it has closed the gap. **DW** is able to find solutions with relatively small gaps for even the largest problem instances covering all vessels and all contracts on a global level. In practice the resulting gaps of the algorithms, can be much smaller since we benchmark against a lower bound and not against the optimal solution.

Instance	Obj_{MIP}	LB_{MIP}	Gap_{MIP}	t_{MIP}	Obj_{DW}	LB_{DW}	Gap_{DW}	t_{DW}
RULED	5.404 e+7	5.404 e+7	0.00 %	1083	5.408 e+7	5.404 e+7	0.08 %	118
FRFSM	1.319 e+8	1.319 e+8	0.00 %	21	1.321 e+8	1.319 e+8	0.20 %	86
ZADUR	7.064 e+8	7.063 e+8	0.02 %	609	7.071 e+8	7.064 e+8	0.10 %	653
US_WC	6.628 e+8	6.626 e+8	0.03 %	481	6.654 e+8	6.627 e+8	0.41 %	1142
USNWK	9.067 e+8	9.063 e+8	0.03 %	834	9.077 e+8	9.066 e+8	0.11 %	1114
USSAV	9,830 e+8	9.826 e+8	0.04 %	775	9.830 e+8	9.829 e+8	0.00 %	399
PABLB	1.108 e+9	1.107 e+9	0.06 %	906	1.108 e+9	1.108 e+9	0.01 %	672
AEJAL	1.490 e+9	1.489 e+9	0.03 %	686	1.490 e+9	1.490 e+9	0.00 %	415
09_H2	2.115 e+9	2.113 e+9	0.10 %	1160	2.120 e+9	2.115 e+9	0.22 %	8642
11_H2	2.478 e+9	2.475 e+9	0.09 %	1107	2.479 e+9	2.477 e+9	0.07 %	9411
10_H1	2.255 e+9	2.253 e+9	0.09 %	1181	2.259 e+9	2.255 e+9	0.19 %	7267
10_H2	Out of Mem				2.529 e+9	2.526 e+9	0.12 %	10649
11_H1	Out of Mem				3.217 e+9	3.214 e+9	0.09 %	10075
HKHKG	Out of Mem				3.427 e+9	3.427 e+9	0.00 %	4344
10_FY	Out of Mem				4.835 e+9	4.807 e+9	0.59 %	28922

Table 5: Results and performance of **MIP** and **DW** implementation. **Instance** is the instance name. Obj_{MIP} is the best found solution for the **MIP** algorithm, and LB_{MIP} is the best found lower bound. Gap_{MIP} is the resulting gap between upper and lower bound and t_{MIP} is the time used in seconds. Obj_{DW} is the best found solution for the **DW** algorithm, and LB_{DW} is the best found lower bound. Gap_{DW} is the resulting gap between upper and lower bound and t_{DW} is the time used in seconds.

9. Conclusion and Further work

We have given an in-depth description of how to optimize bunker purchasing in liner shipping. First, a mathematical model for bunker bought on the spot market was presented, and various extensions from the literature were discussed. Next, a model for bunker purchasing with contracts was presented, and a novel solution approach based on decomposition was described.

Since bunker prices are stochastic of nature, future research should be focused on modeling the price fluctuation. However, the models tend to become quite complex and difficult to solve as observed by Plum and Jensen [22], while only adding small extra improvements to the results. So a trade-off must be done between model complexity and gain in bunker costs. The work of Sheng et al. [26] shows some promising developments in this important direction.

Also, instruments from finance could be used to control risk in bunker purchasing, and to increase the margins on oil trade. Bunker purchasing for liner ships constitutes such a big market that it deserves a professional trading approach.

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