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# CFD investigation of flow in scour protection around a mono-pile with the volume-averaged $k-\omega$ model

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## Abstract

The study numerically investigates the flow behaviour around a mono-pile with scour protection under steady and oscillatory flow conditions. A hydrodynamic model based on volume-averaged Reynolds-averaged Navier-Stokes (VARANS) equations with the volume-averaged  $k-\omega$  turbulence closure is developed and implemented in OpenFOAM. Three porosity transition types, i.e., constant, linear and parabolic, near the interface between stone cover and free flow are firstly evaluated in two-dimensional models. The simulated results, i.e., flow velocities, turbulence levels and bed shear stresses, are compared to previous experiments under steady and oscillatory flow conditions. The parabolic transition shows the best agreement with the measurements and is therefore used in the developed model. Under steady current, a three-dimensional model is validated against experimental measurements including flow features both inside and outside of the scour protection around a mono-pile and it exhibits relatively good performance. Further, the volume-averaged  $k-\omega$  model shows better agreement to experiments in porous medium compared to results from  $k-\omega$  SST and volume-averaged  $k-\epsilon$  models. The model is applied to investigate the flow patterns under oscillatory flow condition. The results show that a horseshoe vortex is formed and it penetrates the entire scour protection, which generates high flow velocities and bed shear stresses; erosion is most likely to occur at the area in the presence of vortex, which poses a threat to the pile stability. The simulations demonstrate the ability of the developed model to evaluate the flow behaviour in scour protection.

**Keywords:** monopile, scour protection, volume-averaged  $k-\omega$  turbulence model, porous flow, OpenFOAM

## 1. Introduction

Offshore wind energy is developing rapidly as a principal source of renewable energy. Over the years, large quantities of offshore wind farms have been installed. For the stability of the structure, its foundation is a crucial component that still needs to be optimized. In the marine environment, the seabed in the vicinity of the foundation is often composed of sand or silt, where severe erosion occurs easily. Subjected to waves and currents, the increased hydrodynamic loads on the fine sediments result in local erosion around a pile foundation [1–3]. Without protection, the generated erosion would threaten the stability of the foundation.

Over the past few decades, a large number of studies are carried out to investigate the scour around the unprotected pile foundations through experiments and field studies. Most of the results have been compiled in Breusers and Raudkivi [4], Hoffmans and Verheij [5], Whitehouse [6], Melville and Coleman [7] and Sumer and Fredsøe [3].

34 They reveal that the surrounding seabed without protection suffers from erosion, and scour holes are thereby formed  
35 in some cases. To predict the evolution of scour holes, extensive numerical models have been established [8–17]. It is  
36 indicated that, with the elapse of time, the scour depth has a tendency of gradual increase. As a result, the scour would  
37 affect the stability of the foundation if countermeasures are not properly taken.

38 To protect the seabed from scouring, a submarine porous medium system, i.e., scour protection, as one  
39 practical approach is commonly designed and installed. The scour protection, composed of stone layers, is placed  
40 around the piles. The layers usually consist of both a filter layer and an armour layer. The latter ensures the stability  
41 of the piles under storm conditions, while the former prevents fine sediment from winnowing out of the scour  
42 protection through the large armour stones. In the presence of scour protections, the flow patterns and the sediment  
43 transport around the pile are altered, whereby the scouring issue is somehow alleviated. However, significant local  
44 sinking is observed during surveys of submarine scour protections in service [18–22]. Previous studies primarily  
45 investigate the sinking problem [23–26], in which the failure mode of scour protections is mostly predicted in an  
46 empirical way. Hansen et al. [18] also concluded that the sinking is induced by the removal of the base sediment.  
47 Through extensive physical model measurements, Nielsen et al. [27, 28] indicated that the horseshoe vortex is a key  
48 flow feature governing the sinking process of the scour protection, and it is also the essential element to cause erosion  
49 around unprotected piles [16, 29]. If the horseshoe vortex is strong enough to cause sediment transport in the scour  
50 protection, and as a result, the sinking of the scour protection occurs. The development of sinking increases free length  
51 of the monopile, which brings about the structural fatigue and damage. To figure out these processes, a detailed  
52 understanding of the flow inside the scour protections is thus necessary.

53 The flow behaviour inside the scour protections such as flow velocity, turbulence level and bed shear stress  
54 have been experimentally investigated [1, 27, 30, 31, 32]. Limitations do exist with numerical models to accurately  
55 describe these flow features. That is due to the fact that a complete resolution of the porous structure in scour protection  
56 is not yet feasible. To simulate porous flow, several resistance-type porosity models without turbulence model have  
57 been developed and applied without resolving the actual pores [33–36]. Later, Jensen et al. [37] derived volume-  
58 averaged flow equations to model the porous flow without using a turbulence closure, and the resistance force was  
59 added to account for the effect of the porous medium. A shortcoming is that the turbulent kinetic energy inside the  
60 porous media is not studied. These models assume that turbulence inside the porous media is negligible, and the  
61 assumption is only applicable when the permeability of the medium is comparatively small. However, in some cases  
62 the turbulent levels are of great importance [38, 39], and therefore, the turbulence closure is supposed to be taken into  
63 account.

64 To simulate turbulence in scour protection, some researchers used the LES model to closure the volume-  
65 averaged Reynolds-averaged Navier-Stokes equations (VARANS) [40–45]. However, the LES is time consuming and  
66 expensive computationally. Due to the limited computing resources, the standard turbulence models, e.g.,  $k-\omega$ ,  $k-\omega$ -  
67 SST and  $k-\epsilon$  described in Wilcox [46], are typically applied. These closure models have to be modified in order to  
68 account for the porous media [37]. An example of the turbulent closure model for porous media is volume-averaged  
69  $k-\epsilon$  model [47]. However, the  $k-\epsilon$  model is not suitable to simulate the adverse-pressure gradient flows [48, 49], which  
70 are induced by the interaction between the local hydrodynamics and the complex geometries of scour protection.

71 In this regard, we applied the volume-averaged method to the  $k$ - $\omega$  turbulence model [50], and validated the  
 72 model against the experiments under steady current [51] and oscillatory flow [31, 51]. The computed results in the 2D  
 73 cases are reasonably in line with the existing experiments and the model can accurately estimate the porous flow.  
 74 However, in the presence of a monopile with scour protection, it is unclear whether the developed model is suitable  
 75 for 3D cases due to the complex flow features, e.g., horseshoe vortex and lee-wake vortices. Moreover, a constant  
 76 porosity of the porous medium is often assumed in existing numerical models for simplification, while it is not  
 77 representative of real scenarios. In engineering practice, the porosity varies significantly near the interface between  
 78 scour protection and free flow, but there are few studies analyzing the effect of porosity variations. These are the  
 79 motivations of the present work, which will evaluate the porosity transition types near the interface, demonstrate the  
 80 model performance and gives a detailed description of the flow pattern around a monopile with scour protection. For  
 81 illustration, both steady and oscillatory flow conditions are taken into account.

82 The present paper is organized as follows. In Section 2, the numerical model is described briefly. The  
 83 volume-averaged Reynolds-averaged Navier-Stokes equations (VARANS) developed by Jensen et al. [37] are  
 84 rewritten and implemented as the governing equations. The volume-averaged  $k$ - $\omega$  equations, proposed by Zhai and  
 85 Christensen [50], are employed as the closure. Section 3 analyses the effects of porosity transitions near the interface  
 86 in 2D cases. In Section 4, the 3D model under steady current is validated against the existing experiments [27], and  
 87 the results are also compared with those from the volume-averaged  $k$ - $\varepsilon$  and the  $k$ - $\omega$  SST models. Application of the  
 88 3D model under oscillatory flow is subsequently presented and discussed in Section 5. Some main conclusions and  
 89 remarks of the paper are finally given in Section 6.

## 90 2. Porous media model

### 91 2.1 Hydrodynamic equations

92 In the porous media model, the flow is governed by the volume-averaged continuity equation [52] and the  
 93 VARANS equations [37]:

$$\frac{\partial \langle \bar{u}_i \rangle}{\partial x_i} = 0 \quad (1)$$

$$(1 + C_m) \frac{\partial \rho \langle \bar{u}_i \rangle}{\partial t} + \frac{1}{n} \frac{\partial \rho \langle \bar{u}_i \rangle \langle \bar{u}_j \rangle}{\partial x_j} + \frac{1}{n} \frac{\partial \rho \overline{u'_i u'_j}}{\partial x_j} = - \frac{\partial \langle p \rangle^f}{\partial x_i} + g_j x_j \frac{\partial \rho}{\partial x_i} + \frac{1}{n} \frac{\partial}{\partial x_j} \mu \left( \frac{\partial \langle \bar{u}_i \rangle}{\partial x_j} + \frac{\partial \langle \bar{u}_j \rangle}{\partial x_i} \right) + F_i \quad (2)$$

94 For an oscillatory flow, the oscillating body force ( $B_i$ ) is introduced into the Eq. (2) expressed as:

$$(1 + C_m) \frac{\partial \langle \bar{u}_i \rangle}{\partial t} + \frac{1}{n} \frac{\partial \langle \bar{u}_i \rangle \langle \bar{u}_j \rangle}{\partial x_j} + \frac{1}{n} \frac{\partial \overline{u'_i u'_j}}{\partial x_j} = - \frac{1}{\rho} \frac{\partial \langle p \rangle^f}{\partial x_i} + g_j + \frac{1}{n} \frac{\partial}{\partial x_j} \nu \left( \frac{\partial \langle \bar{u}_i \rangle}{\partial x_j} + \frac{\partial \langle \bar{u}_j \rangle}{\partial x_i} \right) + \frac{F_i + B_i}{\rho} \quad (3)$$

95 where  $x_i$  = the Cartesian coordinates system, fluid density  $\rho = 1000 \text{ kg/m}^3$ ,  $F_i$  is introduced by volume-averaging  
 96 process and consists of convective term, drag term and friction drag term,  $C_m$  = the inertial resistance force considering  
 97 the transient effects of fluid accelerating around solid particles,  $t$  = time,  $n$  = the porosity,  $\nu$  = the kinematic molecular  
 98 viscosity,  $B_i$  = the body force,  $g_j$  = the acceleration of gravity,  $p$  = the pressure,  $\bar{u}_i$  = the mean component of the

99 velocity  $(\bar{u}, \bar{v}, \bar{w})$ . The fluctuating component of the velocity is  $u'_i$  and  $\langle \cdot \rangle$  is the superficial volume averaging operator  
100 defined by:

$$\langle C \rangle = \frac{1}{V} \int_{V_f} C dV \quad (4)$$

101 where  $C$  is a scalar, vector, or tensor,  $V$  represents the total volume of solid and fluid phase and  $V_f$  is the volume of  
102 fluid phase.

103 The intrinsic average denoted by  $\langle \cdot \rangle^f$  is defined as:

$$\langle C \rangle^f = \frac{1}{V_f} \int_{V_f} C dV \quad (5)$$

104 The relationship between these two averages is:

$$\langle C \rangle = n \langle C \rangle^f \quad (6)$$

105 where the  $n$  is given by:

$$n = \frac{V_f}{V} \quad (7)$$

106 According to the extended Darcy-Forchheimer relation, the  $C_m$  and the porous medium resistance force  $F_i$   
107 are obtained. For  $C_m$ , the formulation proposed by van Gent [35] is adopted:

$$C_m = \gamma_p \frac{1-n}{n} \quad (8)$$

108 where  $\gamma_p = 0.34$ .

109  $F_i$  is composed of the linear and non-linear resistance force, given by:

$$F_i = a\rho\langle\bar{u}_i\rangle + b\rho\sqrt{\langle\bar{u}_j\rangle\langle\bar{u}_j\rangle\langle\bar{u}_i\rangle} \quad (9)$$

110 where the  $a$  and  $b$  for oscillatory flow are expressed separately as:

$$a = \alpha \frac{(1-n)^2}{n^3} \frac{\nu}{D_{50}^2} \quad (10)$$

$$b = \beta \left(1 + \frac{7.5}{KC_s}\right) \frac{1-n}{n^3} \frac{1}{D_{50}} \quad (11)$$

111 where  $\alpha$  and  $\beta$  are empirical resistance parameters for resistance term, in this paper  $\alpha = 500$  and  $\beta = 2.0$  [50],  $D_{50}$  =  
112 the median diameter of the porous medium,  $KC_s = U_m T / (n D_{50})$  is the Keulegan-Carpenter number for stones,  
113 representing the ratio of the characteristic length scale of fluid particle motion to that of porous media [33, 37], in  
114 which,  $U_m$  = the maximum free stream velocity and  $T$  = the time period.

115  $B_i$  is implemented to drive the flow and determined by differentiating the horizontal velocity. For oscillating  
116 flow, the periodic velocity is given by:

$$u(t) = U_m \sin(\omega_0 t) \quad (12)$$

117 where  $\omega_0 = \frac{2\pi}{T}$ .  $B_i$  is determined by:

$$B_i = \frac{du}{dt} = \omega_0 U_m \cos(\omega_0 t) \quad (13)$$

118 *2.2 Volume-averaged k- $\omega$  equations*

119 The VARANS equations are solved by utilizing the volume-averaged  $k$ - $\omega$  turbulence model as the closure.  
 120 The turbulence closure model involves transport equations for the turbulent kinetic energy  $\langle k \rangle$  and the specific  
 121 dissipation rate  $\langle \omega \rangle$  [50]:

$$\frac{\partial \langle k \rangle}{\partial t} + \frac{1}{n} \frac{\partial \langle k \rangle \langle \bar{u}_j \rangle}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ (v + \langle v_T \rangle \sigma^*) \frac{\partial \langle k \rangle}{\partial x_j} \right] - \frac{\langle \bar{u}_i' \bar{u}_j' \rangle}{n} \frac{\partial \langle \bar{u}_i \rangle}{\partial x_j} - \frac{1}{n} \beta^* \langle \omega \rangle \langle k \rangle + n \beta_\infty^* \omega_\infty k_\infty \quad (14)$$

$$\frac{\partial \langle \omega \rangle}{\partial t} + \frac{1}{n} \frac{\partial \langle \omega \rangle \langle \bar{u}_j \rangle}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ (v + \langle v_T \rangle \sigma) \frac{\partial \langle \omega \rangle}{\partial x_j} \right] - \frac{1}{n} \frac{\gamma \langle \omega \rangle}{\langle k \rangle} \langle \bar{u}_i' \bar{u}_j' \rangle \frac{\partial \langle \bar{u}_i \rangle}{\partial x_j} - \frac{1}{n} \beta_1 \langle \omega \rangle^2 + n \beta_1 \omega_\infty^2 \quad (15)$$

122 where the superficial averaged eddy viscosity read:

$$\langle v_T \rangle = \gamma^* \frac{\langle k \rangle}{n \langle \omega \rangle} \quad (16)$$

123 where  $\gamma^* = 1$ . The model constants,  $k_\infty$  and  $\omega_\infty$ , are defined separately by:

$$k_\infty = 0.85 \frac{(1-n)}{\sqrt{n}} (\langle \bar{u}_j \rangle)^2 \quad (17)$$

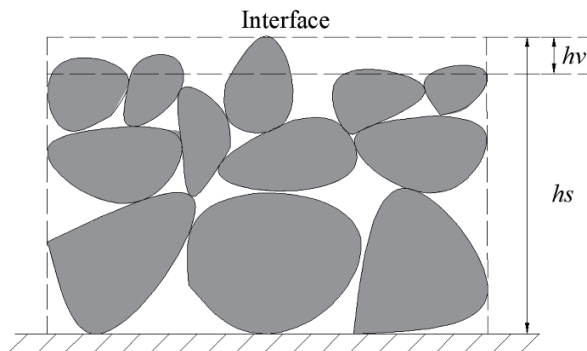
$$\omega_\infty = 117 \frac{(1-n)^{1.5}}{D_{50} \sqrt{n}} (\langle \bar{u}_j \rangle)^2)^{1/2} \quad (18)$$

124 and the closure coefficients are:  $\gamma = 0.52$ ,  $\sigma^* = 0.5$ ,  $\beta^* = 0.09$ ,  $\sigma = 0.5$  and  $\beta_1 = 0.075$ . For a detailed description of the  
 125 process, see our recent paper [50].

126 **3. Analysis of porosity transition near the interface**

127 The scour protection is usually modelled as a homogeneous medium with constant properties in the entire  
 128 porous zone, which does not in a realistic way accommodate variations near the interface between free flow and porous  
 129 medium. To illustrate the variations, Fig. 1 shows a sketch of a realistic interface with multi-layer stones. Near the  
 130 interface, the porosity,  $n$ , is not a constant and we can assume a transition from the value in porous zones to 1 in free  
 131 flow. Let  $h_s$  = the height of the stone cover. Larger ratio of  $D_{50}/h_s$  increases the transition effect. For a single layer  
 132 with large stones,  $D_{50}/h_s = 1$  the concept of porosity loses its meaning as it is intended for a medium with individual  
 133 solids smaller than the scale of the entire medium.

134 To illustrate the effect of the porosity variation near the interface, we analyse three transition types, i.e.,  
 135 constant, linear variation and parabolic variation. The transition height  $h_v$  is associated with the geometry of stones  
 136 and is described as a linear relation with  $D_{50}$ ,  $h_v = \lambda D_{50}$ . For stone cover with one, two and three layers, the  $\lambda$  was  
 137 determined empirically by Stevanato et al. [31] according to the measured velocity profiles inside stone cover.  
 138 Assuming that the porosity variation follows the velocity variation, then  $\lambda = 0.45$  for a single layer of stones and  $\lambda =$   
 139 1.05 for two and three layers.



140  
141 Fig. 1 Sketch of a realistic interface between porous medium and free flow.

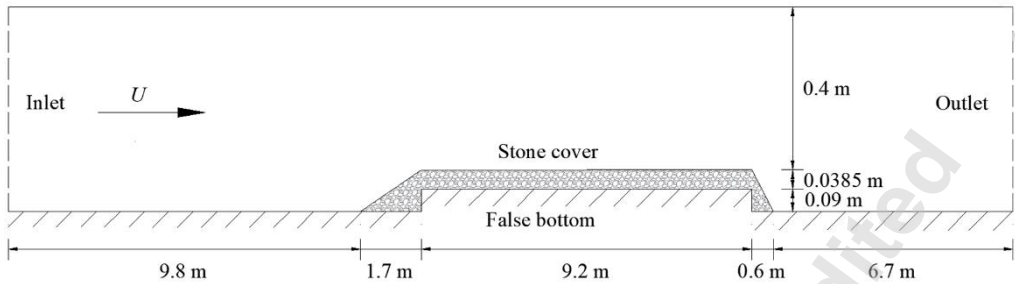
142 Data from previous experimental studies are used to determine the variation of porosity,  $n$ , near the interface.  
143 Sumer et al. [1] and Stevanato et al. [31] conducted flow experiments over stone covers under steady and oscillatory  
144 flow conditions. The measured results in terms of velocity profile, bed shear stress and turbulent kinetic energy are  
145 utilized for the analysis of the porosity transition type.

### 146 3.1 Steady current

147 Under steady state, the experiments of Sumer et al. [1] were carried out in a flume with cyclical water supply.  
148 Fig. 2 sketches the experimental layout in terms of geometry and dimensions. Its length, width and height were 28.0,  
149 0.6 and 0.8 m. A 9.2 m long and 0.09 m high false bottom was placed on the flume bottom as the working section, to  
150 get a complete side view for flow visualization. The false bottom was covered by stones ( $D_{50} = 38.5$  mm and  $n = 0.4$ ).  
151 In the working section, the water depth was 0.4 m. The Laser Doppler Anemometer (LDA) was used to measure  
152 velocity at five cross-sections, and the mean flow velocity  $U$  (averaged over water depth) = 38.7 cm/s. The parameter  
153 of friction velocity  $U_f = 5.22$  cm/s based on the logarithmic profile of the flow above the stones. More detailed  
154 information is given by Sumer et al. [1] and Fredsøe et al. [53].

155 Our numerical model in 2D is set up to match the experiment. The computational domain is resolved with  
156 65,709 non-uniform computational cells to ensure that the distance  $y_c^+$  from the wall-adjacent cell centers to the wall  
157 is about 30. The computational mesh is automatically constructed by the snappyHexMesh tool. Grid convergence is  
158 made through three sets of mesh (coarse, medium and fine) with reference to the velocity, and the medium mesh  
159 (65,709 cells) is shown to be the optimum. The boundary conditions are defined at the inlet, outlet and walls. For  
160 velocity, a uniform  $U = 33$  cm/s is specified at the inlet resulting in a mean velocity of  $U = 38.7$  cm/s at the working  
161 section. A zero normal gradient condition ( $\frac{\partial u}{\partial x} = 0$ ) is used at the outlet; a slip boundary condition is imposed at the  
162 top wall to model the flow with free surface and a non-slip condition is specified at the bottom wall. In regards to  
163 pressure, zero pressure is specified at both inlet and outlet; a zero pressure gradient is given at the top and bottom  
164 walls. At the inlet,  $k = 1.36 \times 10^{-4} \text{ m}^2/\text{s}^2$  and  $\omega = 0.16 \text{ s}^{-1}$ , which are based on the turbulent intensity of 5% of the inlet  
165 velocity, with a length scale of 20% of the channel width; at the outlet,  $k$  and  $\omega$  are both specified with a zero gradient  
166 condition. The timestep is set as 0.0001 s and the total simulation time is 1000 s, which is found to provide a stationary  
167 solution. The selected timestep is determined by calculating the Courant number (preferably  $\leq 0.4$ ), so that the

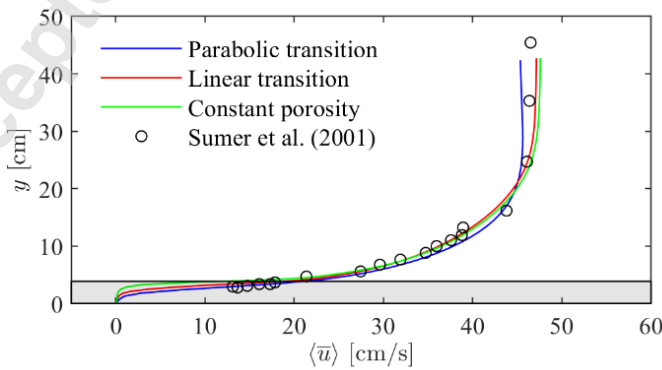
168 information propagation of the physical flow is captured. About 15.2 m of the inlet, the simulated results are extracted  
169 for comparison with the measured data.



171 Fig. 2 Sketch of the experimental layout for steady current through a porous medium.

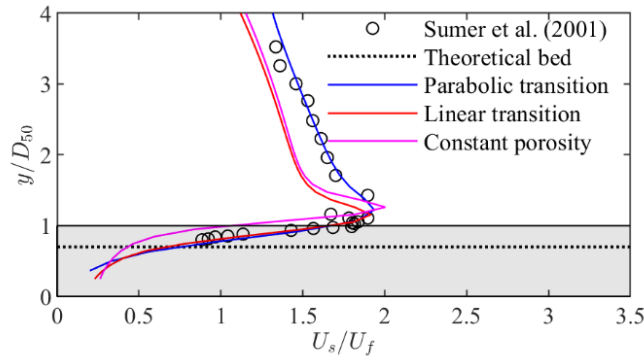
172 To illustrate the effect of the porosity transition on the velocity distribution, the simulated velocity profiles  
173 with three different transitions and the measured velocities are depicted in Fig.3. All computed velocity profiles follow  
174 a similar pattern and are in line with the measured results outside the porous media. With a constant porosity, the  
175 velocities inside the porous medium tend to be underestimated, while in the free flow they are slightly overestimated.  
176 Compared to measured values, the velocity profile from the linear porosity exhibits some slight deviations, but still  
177 better than the results with the constant porosity.

178 Turbulence is essential for sediment transport in the porous medium, and turbulent kinetic energy  $k$  is the  
179 crucial parameter to estimate the turbulence level. In the porous medium, the  $k$  is related to turbulence fluctuation  $U_s$   
180 by  $U_s = \sqrt{0.6k}$  according to Zhai and Christensen [50]. The ratio between the turbulent fluctuation and the friction  
181 velocity,  $U_s/U_f$ , is compared to experimental results in Fig. 4. In the vertical direction, the profiles in simulated  $U_s/U_f$   
182 for all porosity transitions are similar to the measured one.  $U_s/U_f$  increases from the free flow towards the porous  
183 medium and peaks closely to the interface. From the interface it decreases further down into the media. In the porous  
184 medium, the  $U_s/U_f$  profiles from parabolic and linear porosity transitions show good agreement with measurements,  
185 while the profile for constant porosity presents obvious deviations from the measurements. In the outer flow, the  
186 computed  $U_s/U_f$  results with the parabolic transition match well with the measured data, while the results for the linear  
187 transition and constant porosity have appreciable deviations. Therefore, for steady current, the numerical model  
188 provides better velocity and turbulence calculations when the parabolic transition in the porosity zone is implemented.



189  
190 Fig. 3 Comparisons of simulated and measured velocity profiles.





191  
 192 Fig. 4 Comparisons of simulated and measured turbulent kinetic energy.

193 *3.2 Oscillatory flow*

194 Under oscillatory flow condition, Stevanato et al. [31] carried out the flow experiments over stone covers.  
 195 The measured results that included velocity profile, bed shear stress and turbulent kinetic energy are utilized for the  
 196 analysis of the porosity transition type.

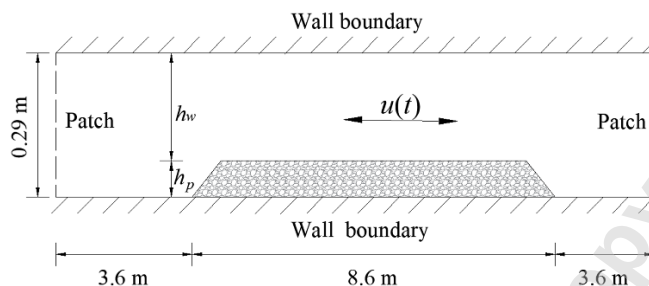
197 The experiments were performed in an oscillatory U-tube device with risers installed at the tube’s both ends  
 198 to generate the oscillatory flow. The horizontal rectangular working section was 15.8 m long, 0.39 m wide and 0.29  
 199 m high. Stones were placed on the bottom to form a 8.6 m long and 0.108 m high stone cover. The median stone  
 200 diameter was,  $D_{50} = 36$  mm, and the porosity  $n = 0.4$ . The maximum free stream velocity was,  $U_m = 0.96$  m/s with a  
 201 period of  $T = 9.73$  s. Based on the expression given in Section 2.1, the  $KC_s = 648.67$ .

202 LDA was used to measure the flow velocities. The velocities above the stones were measured at the centreline  
 203 of the tube to reduce effects of secondary flows. To avoid the laser beams being blocked by stones, the velocities in  
 204 the pores were measured nearby the sidewall. A Dantec hot-film probe measured the bed shear stress beneath the stone  
 205 layer. Further details are given in Stevanato et al. [31] and Sumer et al. [54].

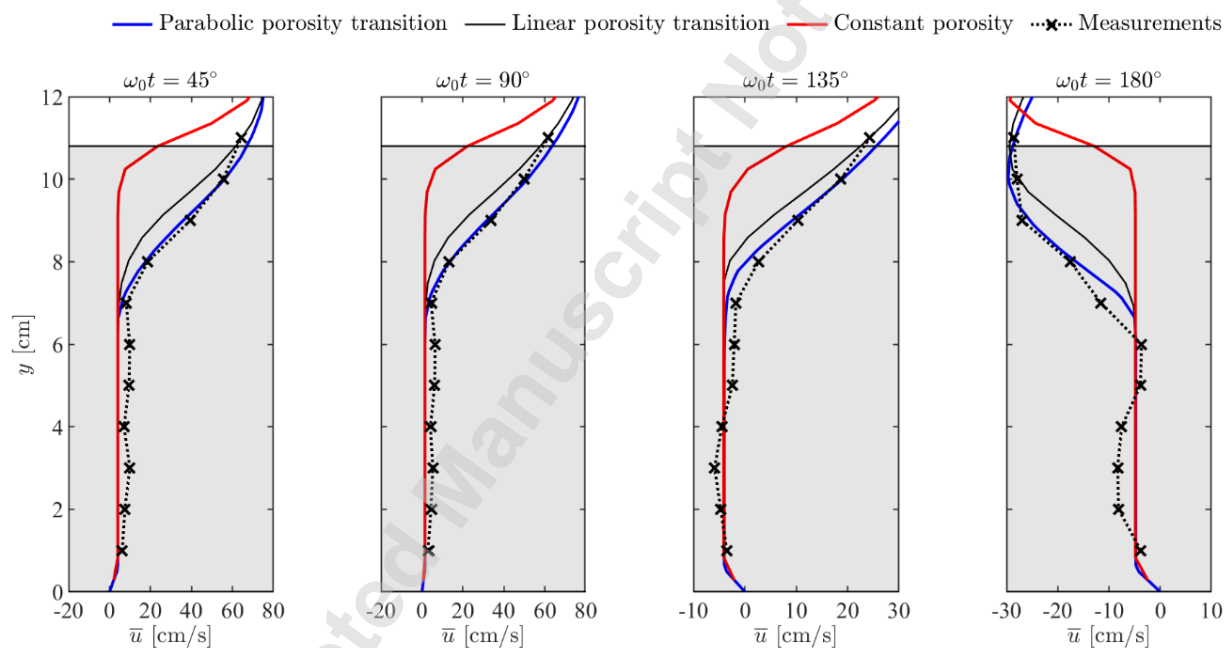
206 The numerical model reproduces the experiment with the same geometry of the horizontal working section.  
 207 Fig. 5 shows the sketch of the numerical setup. The model in 2D domain is resolved by 56,588 non-uniform  
 208 computational cells. For the velocity, the applied oscillatory body force is imposed over the domain after an iterative  
 209 process to reach the experimental  $U_m$ ; a no-slip boundary is specified at the walls and a zero gradient at the inlet and  
 210 outlet. For the turbulent quantities, zero gradients are specified at the inlet and outlet; the wall functions are used at  
 211 the walls. For pressure, we use zero gradients at walls; the cyclic boundary conditions at the inlet and outlet based on  
 212 the analysis of pressure boundary conditions in Schippers et al. [55]. To achieve stable results, the time step is 0.0001  
 213 s and the total simulation time is set to 350 s. The results in the period  $291.9 \text{ s} < t < 301.6 \text{ s}$  are selected for analysis  
 214 according to Zhai and Christensen [50].

215 Under the oscillatory flow conditions, the influence of porosity transition on the velocity distribution is  
 216 analysed. Fig. 6 shows the velocity profiles at four different phases ( $\omega_0 t = 45^\circ, 90^\circ, 135^\circ$  and  $180^\circ$ ). In the lower  
 217 layer ( $y \leq 7$  cm), the profiles from the three porosity transitions are almost straight lines and coincide with each other  
 218 at each phase, which implies that the velocities are almost constant. The computed profiles, irrespective of any  
 219 transition type, fit well with the measured ones, especially at the phase of  $45^\circ, 90^\circ$  and  $135^\circ$ . In the upper layer ( $y > 7$

220 cm), the velocity profile with the parabolic transition shows a better agreement with measured results for all four  
 221 phases. The model with the linear transition apparently under-predicts the velocity although it gives a similar profile  
 222 to the measurements. At all four phases, the velocity profiles with the constant porosity deviate from the experimental  
 223 results. In the case of the constant porosity, the velocity decreases quickly from the free flow to the top of the porous  
 224 medium, which is mainly attributed to the sharp porosity transition. In addition, the velocity at the interface with  
 225 constant porosity is about 70% smaller than that with the parabolic transition.

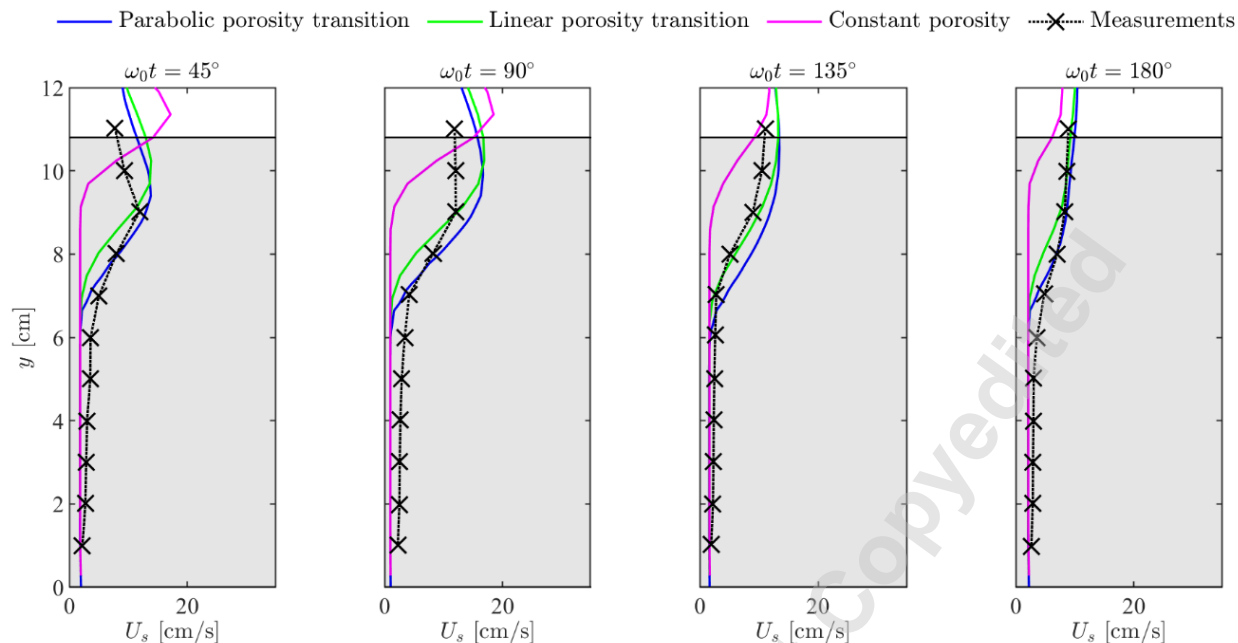


226  
 227 Fig. 5 Sketch of the numerical model setup for oscillatory flow through a porous medium.



228  
 229 Fig. 6 Comparisons of velocity profiles with different porosity transitions near the interface.

230 At the interface, the turbulent kinetic energy is mainly induced by the velocity gradients. To illustrate the  
 231 effect of porosity transition on turbulent level, the numerical and experimental results of  $U_s$  over the depth at the same  
 232 four phases are depicted for comparison in Fig. 7. The experimental results show the  $U_s$  varies noticeably from the  
 233 interface to  $y = 7$  cm, implying that the turbulence dissipates in a swift manner, which is also confirmed by the  
 234 simulations.



235  
 236 Fig. 7 Comparisons of turbulent fluctuations with different porosity transitions near the interface.

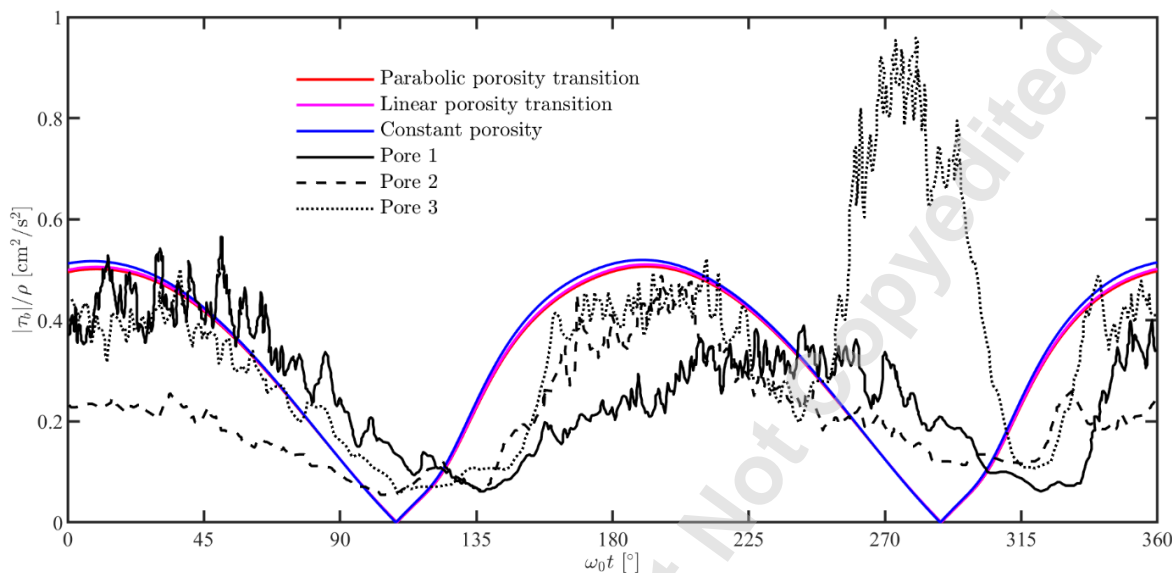
237 For  $y \leq 7$  cm, the simulated  $U_s$  are roughly in a line with the experimental results, which illustrates the  
 238 porosity transition type has probably no effect on the turbulence variation in the deeper part. For  $y > 7$  cm, the  
 239 simulations with parabolic and linear transitions show similar  $U_s$  profiles with measurements, although the magnitude  
 240 differs by and large. The  $U_s$  is overestimated under the two transition types, and the former seems to be better  
 241 especially at the interface in terms of  $45^\circ$  and  $90^\circ$ . However, with the constant porosity, the  $U_s$  is underestimated for 7  
 242 cm  $< y \leq 10$  cm and is overestimated for  $y > 10$  cm, presenting a significant deviation from the measured profile. It  
 243 is also noted that in the deeper part, it has relatively small velocities and thereby producing less turbulence. Near the  
 244 interface, the porosity transitions affect the flow resistance and result in the larger velocity gradients, which generate  
 245 large turbulence dissipation.

246 As the porosity transition changes from constant to linear to parabolic, the peak position of  $U_s$  gradually  
 247 moves downward (Fig. 7) at the four phases. For instance, with  $\omega_0 t = 45^\circ$ , the maximum locates at  $y \approx 11$  cm for the  
 248 constant porosity and at  $y \approx 9$  cm for the parabolic transition. A similar change is also observed in the places where  
 249 the constant velocity occurs (Fig. 6), for example,  $y \approx 10$  cm for the constant porosity and  $y \approx 7$  cm for the parabolic  
 250 transition at  $\omega_0 t = 45^\circ$ . This is related to the increased porosity in the upper layer during this process. It indicates that  
 251 the greater the porosity, the deeper the turbulent flow penetrates into the porous medium.

252 To further illustrate the effect of porosity transitions on flow, the induced bed shear stress  $\tau_b$ , as a basis for  
 253 describing the sediment transport, is analyzed. Fig. 8 shows the comparison of the simulated and measured  $\tau_b$ . In the  
 254 experiment, the  $\tau_b$  was measured in three different layouts of the stones. Pore 1 and 2 were measured at the center of  
 255 a pore, and in the case of pore 3 there was a streamwise offset in the measured location. It presents that the  $\tau_b$   
 256 calculated with the three porosity transitions is in the right order of magnitude with measurements and is well  
 257 represented over all phases of a time period. The model produces similar  $\tau_b$  profiles irrespective of any transition type

258 considered, and the difference is minor. This is explained by the fact that the  $\tau_b$  is associated with the velocity gradients  
 259 and turbulence level, at the bed they are unaffected by the porosity transitions, as illustrated in Figs. 6 and 7.

260 Based on these analyses, the model with parabolic transition considered shows better agreement to the  
 261 measurements in terms of porous flow velocity, bed shear stress and turbulence levels. Therefore, the parabolic  
 262 transition near the interface is adopted in Section 4.

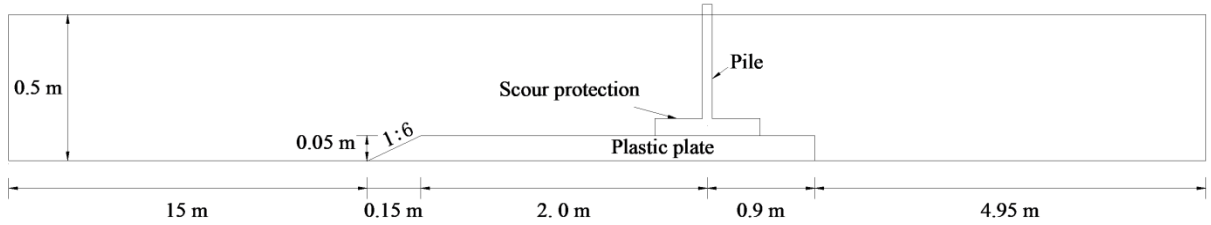


263  
 264 Fig. 8 Comparisons of bed shear stresses with different porosity transitions near the interface.

265 **4. Validation of the model under steady current**

266 To validate the proposed volume-averaged  $k-\omega$  turbulence model in the presence of a mono-pile, a 3D model  
 267 is set up under a steady current. The parabolic porosity variation is applied near the interface between the free flow  
 268 and the porous media. The calculated velocity profile and turbulence level are compared to experiments conducted by  
 269 Nielsen [27].

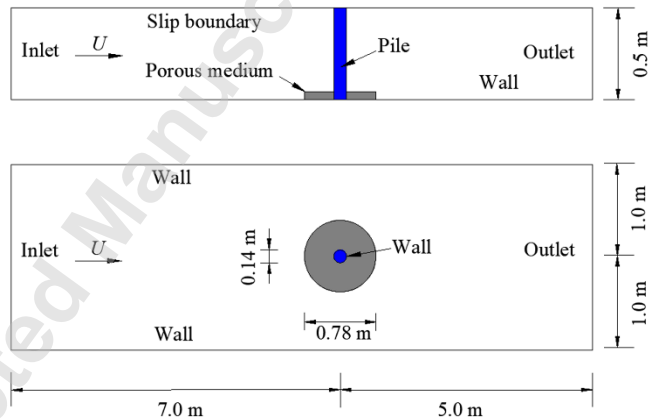
270 The experiments were conducted in a flume with recirculation pump to provide a steady flow. Fig. 9 shows  
 271 a sketch of the experimental layout. The flume was 23.0 m long, 2.0 m wide and 0.5 m high. About 15.0 m downstream  
 272 of the inlet, a false bottom (white plastic plate) was installed in the wave tank. Its length, width and height were 3.05  
 273 m, 2.0 m and 5.0 mm, respectively. The plate had a 15.0 cm long taper on the upstream end with a slope of 1:6. A pile  
 274 with a diameter of 14.0 cm was placed on the plate, whose centre point was approximately 0.9 m to the plate  
 275 downstream end and 1.0 m to the sidewall. The scour protection with a diameter of 78.0 cm was placed around the  
 276 pile and consisted of one layer of irregular crushed stones. The median diameter of the crushed stones was  $D_{50} = 4.3$   
 277 cm and the porosity  $n = 0.4$ . The water depth was 0.50 m and the incoming  $U = 0.40$  m/s. To measure the velocity,  
 278 the submerged pen-size LDA probe was adopted. More details are available in Nielsen et al. [28, 30].



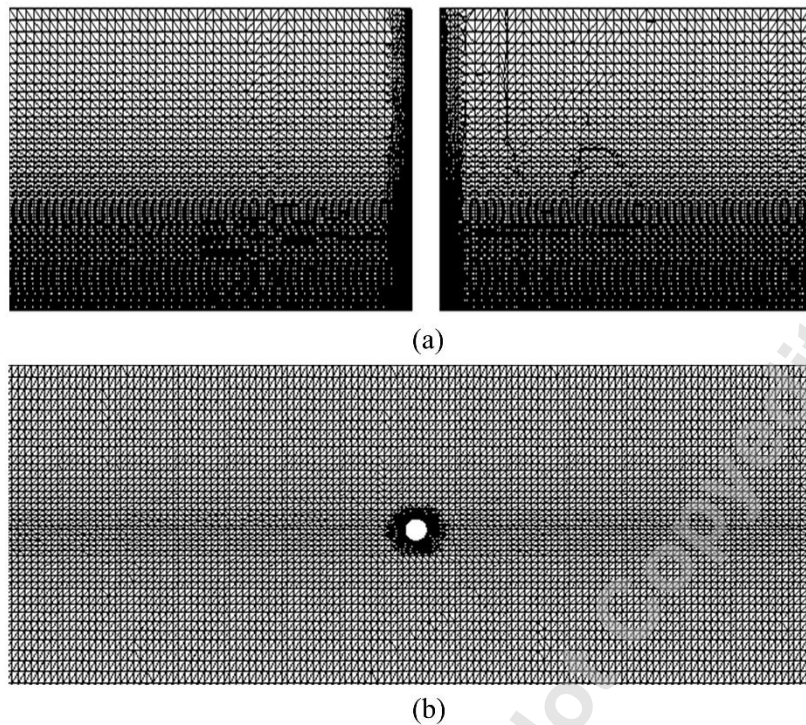
279  
 280 Fig. 9 A schematic layout of the experiment.

281 The numerical model has the same dimensions as the experiment with respect to the flume width and height,  
 282 while the length is shortened to 12.0 m to save computational resources. The shortening is made mainly on the seaside  
 283 of the structure. Fig. 10 shows a sketch of the model domain and boundary type. The pile and the scour protection  
 284 with the same dimensions as the experiment, are placed about 7.0 m downstream of the inlet. In view of the size of  
 285 the pile and scour protection, the up-and downstream distances are sufficient to remove any dependency of the  
 286 boundaries in the results.

287 In order to analyse the grid dependence on the numerical solutions, three different discretizations have been  
 288 adopted including a fine grid with 2,334,778 cells, a medium grid with 1,254,508 cells and a coarse grid with 702,576  
 289 cells. The computational mesh is constructed using snappyHexMesh generator. The results show that the force on the  
 290 pile obtained for the first two grids is almost identical, except for very small scale details. Therefore, in order to save  
 291 computational resources, the model domain is resolved with a total of 1,254,508 non-uniform cells, see Fig 11. These  
 292 cells are distributed over 16 computational cores on a high performance cluster. The grid refines toward the bottom  
 293 and the pile surface, where the wall-adjacent cell thickness is 0.0036 m and 0.0018 m.



294  
 295 Fig. 10 Sketch of the numerical model domain and boundary definitions.



296  
297 Fig. 11 Computational mesh with pile: (a) side view and (b) bird view.

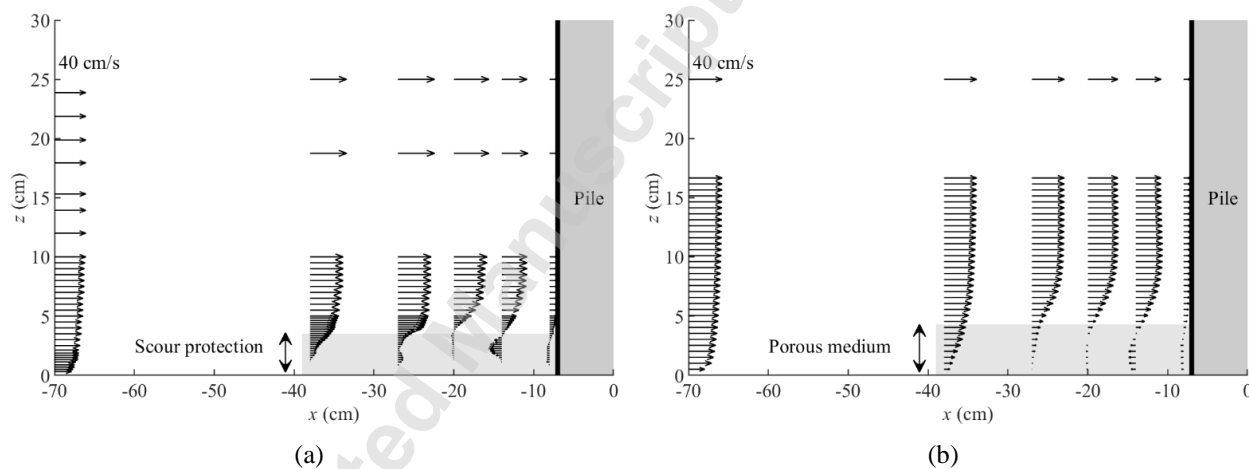
298 The boundary conditions are defined at the inlet, outlet and walls including the pile surface. A uniform  
299 velocity of  $U = 0.40$  m/s is specified at the inlet, and the zero normal gradient condition is used at the outlet; a slip  
300 boundary condition is imposed at the top wall to model the flow with free surface and a non-slip condition is specified  
301 at the bottom wall as well as the pile surface. In regards to pressure, a zero pressure gradient is given at both inlet and  
302 outlet, and the top, bottom and sides' walls. At the inlet,  $k = 2 \times 10^{-4}$  m<sup>2</sup>/s<sup>2</sup> and  $\omega = 0.258$  s<sup>-1</sup>; at the outlet,  $k$  and  $\omega$  are  
303 both specified with a zero gradient condition. At the walls, the wall functions for  $k$  and  $\omega$  are used. To obtain a  
304 stationary solution, the time step is 0.0001 s and the total simulation time is 1000 s.

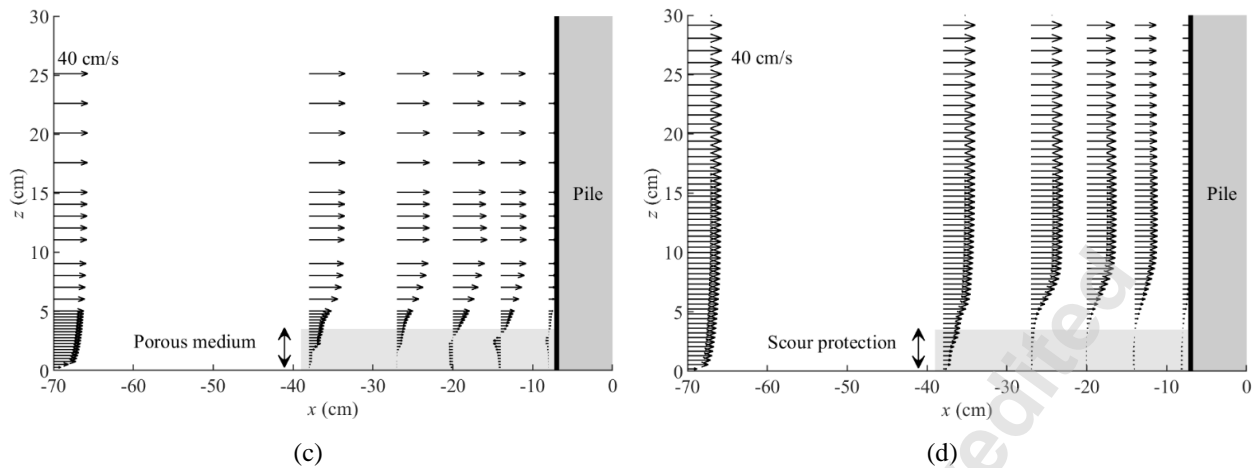
305 To compare the simulated velocity profiles with the measurements, Fig. 12 shows the profiles at six cross-  
306 sections upstream of the pile. The Figs. 12a, b correspond to the measured results and the simulated ones with the  
307 volume-averaged  $k$ - $\omega$  model, respectively. In general it shows a good agreement between model results and measured  
308 velocity profiles by Nielsen [27], although their magnitudes differ in a minor way inside the scour protection. An  
309 apparent return flow in the porous medium is observed up to about 10.0 cm from the edge of the pile, which is  
310 consistent with the flow measurements. In addition, the return flow exhibits a first increase and then decrease pattern,  
311 which is also visualized in the experiments. It is also noted that, in the range of  $-28.0$  cm  $< x < -7.0$  cm below  $z = 3.5$   
312 cm, the measured velocities (Fig. 12a) are slightly larger than the simulated ones (Fig. 12b). This implies that the  
313 present model does not prevent the horseshoe vortex in penetrating into the stone cover. The flow pattern upstream of  
314 the pile also coincides with that observed by, for example, Roulund et al. [16] and Graf and Yulistiyanto [56] for an  
315 unprotected pile.

316 The upstream directed flow near the bottom is generated by the horseshoe vortex that penetrates into the  
317 porous medium. The comparison in the Figs. 12a, b indicates that the vortex is predicted well by the numerical model.

318 In the present case, the horizontal size of the horseshoe vortex is approximately 13.0 cm, as shown in Fig. 12b, and it  
319 is a little smaller than the pile diameter (14.0 cm). That is because the size and strength of the vortex are determined  
320 by both pile diameter and flow velocity [27]. The horseshoe vortex is almost completely embedded in the porous  
321 medium due to the relatively thin protection layers. According to Roulund et al. [16], the vortex is believed to generate  
322 high bed shear stresses that cause sediment transport. This can also be illustrated by the high flow velocities near the  
323 bottom beneath the stone cover (Fig. 12). The vortex penetrating into the porous medium transports the sediment  
324 adjacent to the pile upstream. If it is strong enough, the scoured particles will be winnowed by the main flow and  
325 transported downstream, causing the sinking of scour protection. Otherwise, these particles will deposit in between  
326 the stones.

327 The results from the volume-averaged  $k-\epsilon$  model are also included for illustration (Fig. 12c). In comparison  
328 to the measurements (Fig. 12a), the numerical model also reproduces the similar flow patterns around the pile,  
329 especially outside the porous medium. However, inside the porous medium, the flow magnitudes differ appreciably  
330 although the horizontal flow directions are identical. For example, the simulated velocities at  $x = -20$  cm are larger  
331 than the measured ones, and the opposite is the case for the other cross-sections. It indicates that the simulated  
332 horseshoe vortex in size is comparatively larger than the measured one, while its intensity is relatively smaller. By  
333 comparing the results from the two volume-averaged models (Fig. 12b, c), they both reasonably capture the flow  
334 features, while regarding the return flow inside the porous medium, the volume-averaged  $k-\omega$  model gives a better  
335 picture with reference to the measurements.

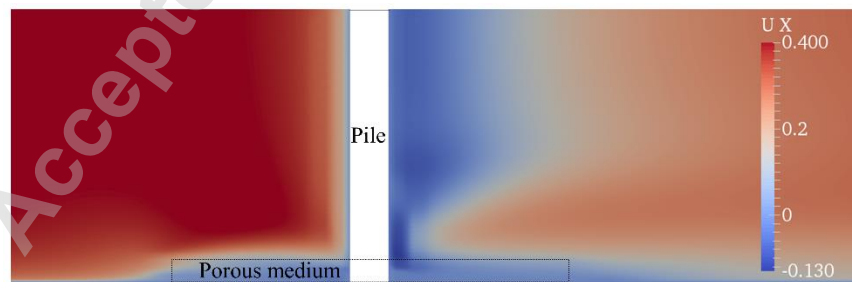




336 Fig. 12 Velocity profiles at six cross-sections upstream of the pile: (a) measurements, adapted from Nielsen et al. [28];  
 337 (b) simulations with the volume-averaged  $k-\omega$  model; (c) simulations with the volume-averaged  $k-\varepsilon$  model; (d)  
 338 simulations with the  $k-\omega$  SST model.

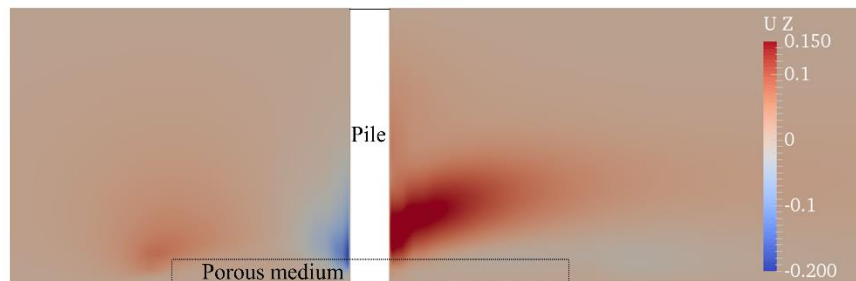
339 Moreover, one more simulation is carried out in which the  $k-\omega$  SST model is considered, and the velocity  
 340 profiles are shown in Fig.12d. The results show a much weaker return flow from the horseshoe vortex than the  
 341 measurements (Fig.12a) and the results with both volume-averaged models (Fig.12b, c). Without using the volume-  
 342 averaged procedures in the porous medium, too much resistance would be introduced for the horseshoe vortex, leading  
 343 to a smaller vortex in size and intensity. This further illustrates the improvement of the volume-averaged  $k-\omega$  model  
 344 in the prediction of the horseshoe vortex.

345 To further elaborate the flow features around the pile, Fig. 13 shows the horizontal and vertical velocity fields  
 346 corresponding to  $\bar{u}$  and  $\bar{w}$  from the volume-averaged  $k-\omega$  model. In Fig. 13a, positive values denote the streamwise  
 347 flow and negative ones return flow. In Fig. 13b, positive values denote the upflow and negative ones downflow. The  
 348 Fig. 13a presents that the return flow occurs both in the scour protection and immediately downstream the pile. The  
 349 former is a result of horseshoe vortex penetration from the free flow to the protection layer; the latter is attributed to  
 350 the lee-wake vortices. The simulated downstream  $\bar{u}$  is in general lower than the upstream one, and the  $\bar{u}$  in the scour  
 351 protection is also relatively small compared to that in the outer flow. The overall flow patterns are almost consistent  
 352 with the patterns observed by Nielsen et al. [28].



353  
 354 (a)

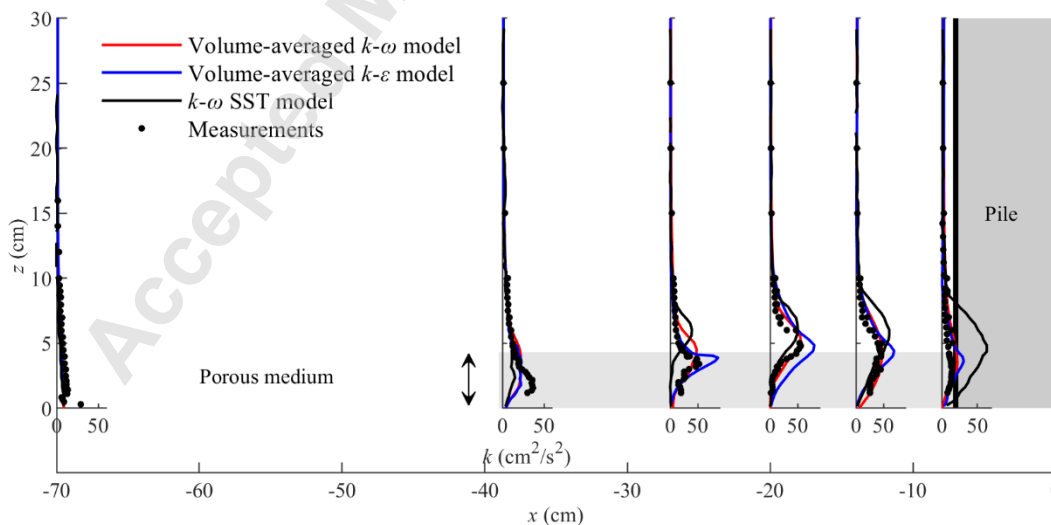




355  
 356 (b)  
 357 Fig. 13 Velocity fields around the mono-pile with scour protection: (a)  $\bar{u}$ -component in m/s, (b)  $\bar{w}$ -component in m/s.

358 In Fig. 13b, it appears that there is an obvious downflow in front of the pile, while it only exists on the top of  
 359 the scour protection. The reason is the porous resistance quickly dampens the downflow into the scour protection. The  
 360 downflow is driven by the deceleration of the flow, and it adds to the rotation of the horseshoe vortex. The upflow is  
 361 observed behind the pile and it becomes weak gradually along the streamwise direction, which is in relation to the  
 362 development of the lee-wake vortices.

363 In addition to the velocity profile, the  $k$  as the other essential quantity associated with sediment transport, is  
 364 also presented for analysis. Sumer et al. [57] pointed out that the sediment transport is augmented by a factor of 6  
 365 when the  $k$  near the bottom ascends by 20%. Together with the measured  $k$ , Fig. 14 plots the simulated  $k$  profiles at  
 366 the same six cross-sections upstream of the pile. In the figure, the black dots represent the measured values, the red  
 367 and blue lines refer to the simulated results with the volume-averaged  $k-\omega$  and  $k-\varepsilon$  models, respectively. The black  
 368 lines denote the simulated results with the  $k-\omega$  SST model. It illustrates that the numerical results with the two volume-  
 369 averaged turbulence models exhibit similar profiles and follow the distribution of the measured values. The  $k$  reaches  
 370 a peak roughly near the interface and it declines gradually towards the free flow and the bottom. However, the  $k-\omega$   
 371 SST model significantly overestimates the  $k$  values near the interface; the peak positions move upward and deviate  
 372 from the measurements, especially at  $x = -14.0$  and  $-8.0$  cm. This suggests that both volume-averaged models produce  
 373 reasonable turbulence levels in comparison with the measurements.

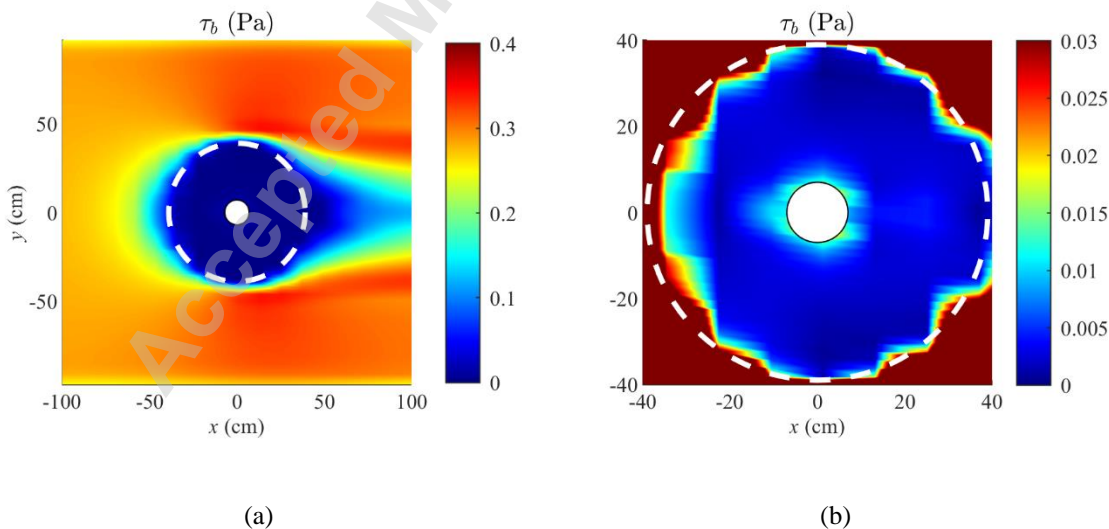


374

375 Fig. 14 Turbulent kinetic energy profiles at six cross-sections upstream of the pile. The black dots represent the  
376 measured values; the red and blue lines refer to the simulated results with the volume-averaged  $k-\omega$  and  $k-\varepsilon$  models,  
377 respectively; black lines denote the simulated results with the  $k-\omega$  SST model.

378 At the undisturbed flow area, e.g.,  $x = -70.0$  cm, the simulated results from both models agree with the  
379 measurements. As the incoming flow comes close to the medium, the turbulent kinetic energy,  $k$ , is slightly  
380 underestimated. This is understandable as the model mainly induces turbulence at the interface with the free flow  
381 owing to the flow shearing. The induced turbulence is transported downwards from the interface to the bottom, as the  
382 flow moves along the streamwise direction. At the upstream end of the porous medium, i.e.,  $x = -38.0$  cm, the flow is  
383 not very sheared and as a result, and less turbulence is generated compared to the measurements. When the flow  
384 approaches the pile, e.g., at  $x = -27.0$  cm,  $-20.0$  cm,  $-14.0$  cm and  $-8.0$  cm, the simulated  $k$  from the volume-averaged  
385  $k-\varepsilon$  model is significantly over predicted, especially near the interface. While, the result from the volume-averaged  $k-$   
386  $\omega$  model is generally in the right order of magnitude compared to the measurement, apart from minor local  
387 overestimation at  $x = -27.0$  cm and  $-8.0$  cm. This implies that the latter could evaluate the turbulence levels more  
388 accurately than the former. It is also observed that the  $k$  has relatively large values upstream of the pile, e.g., between  
389  $x = -20.0$  and  $-14.0$  cm, which is likely coupled with the fact that the flow at this locations is subjected to the horseshoe  
390 vortex. These are also confirmed by Zhai and Christensen [50] with 2D validation cases involving a scour protection.

391 It appears that no previous experimental results are available with respect to the  $\tau_b$  in the surrounding of the  
392 pile. However, the  $\tau_b$  is a key parameter that is closely associated with sediment transport. The distribution of  $\tau_b$  from  
393 the numerical model is presented in Fig. 15a. The enlargement view presenting the  $\tau_b$  in the scour protection is shown  
394 in Fig. 15b. In the figure, it is visualized by contours of  $\tau_b$  magnitude. The white circular area at the center is the  
395 mono-pile and the dashed circle is the porous medium. As expected, the  $\tau_b$  reduces considerably inside the porous  
396 medium, and also in the separation zones in front of and in the lee-wake side of the porous medium. Additionally, it  
397 shows that there is a concentration of bed shear stress between the front and the side edges of the pile. This is due to  
398 the strong presence of the horseshoe vortex.

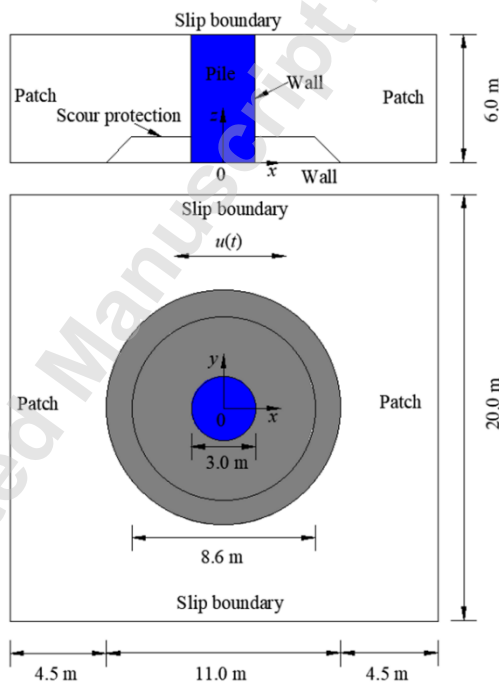


399  
400 (a) (b)  
401 Fig. 15 Simulated bed shear stress: (a) in the surrounding of the pile; (b) the enlargement view.

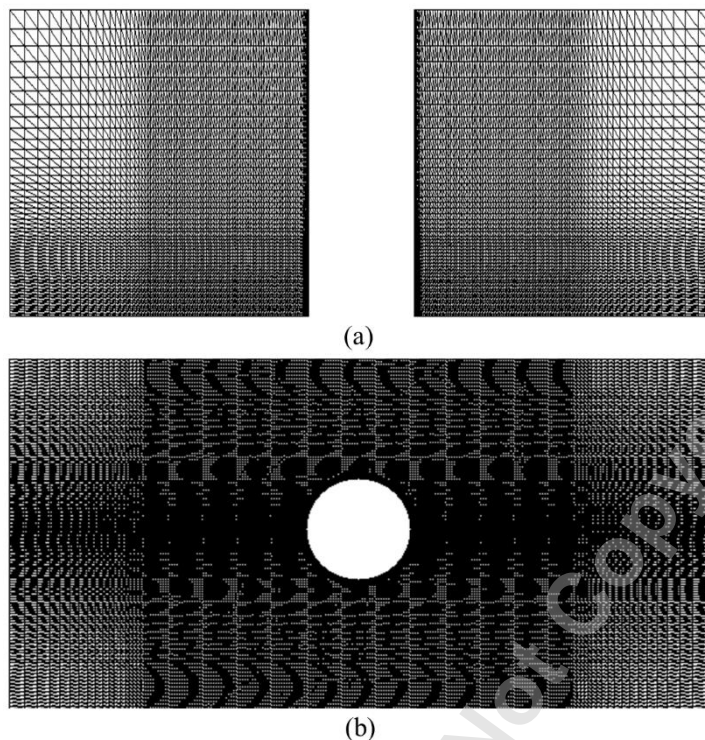
402 Taking the Figs. 12–15 into account, the velocities, turbulence levels and bed shear stresses upstream of the  
 403 pile are relatively high due to the penetration of horseshoe vortex into the protection layer. To conclude, the volume-  
 404 averaged  $k-\omega$  model has a relatively good performance for studying the flow around a pile both inside and outside of  
 405 the scour protection.

406 **5. 3D model application under oscillatory flow**

407 After validation of the model under steady state, it is used to investigate the flow around a full-scale pile with  
 408 scour protection under oscillatory flow. The schematic layout of the numerical setup is shown in Fig. 16. The model  
 409 domain is 20 m long, 20 m wide and 6 m high. The pile is placed centrally in the domain and its diameter,  $D_p$ , is 3 m.  
 410 The scour protection with three stone layers is modelled in the shape of a truncated conical section; its bottom diameter  
 411 is 11 m and top diameter 8.6 m. The median stone diameter is,  $D_{50} = 0.40$  m and the porosity is,  $n = 0.4$ . Similar to  
 412 Section 4, three discretizations, i.e., 2,000,000 cells, 2,259,600 cells and 3,390,400 cells, are considered to analyse the  
 413 grid convergence in terms of the force on the pile. The force obtained for the last two grids is almost identical, except  
 414 for very small scale details. Therefore, in order to save computational resources, the computational domain is resolved  
 415 with 2,259,600 non-uniform cells, see Fig. 17. These cells are distributed over 24 computational cores on a high  
 416 performance cluster. The grid refines toward the bottom and the pile surface where the wall-adjacent cell thickness is  
 417 0.0050 m and 0.0015 m.



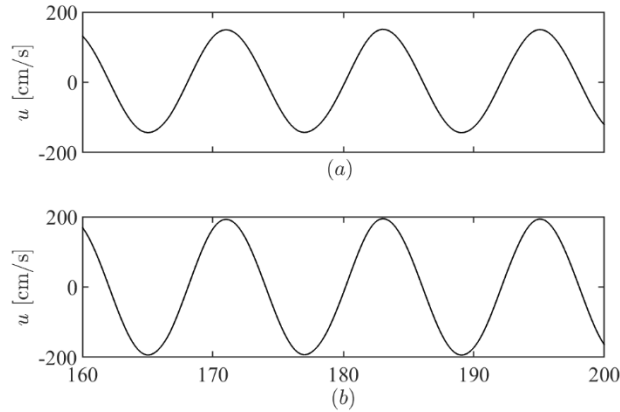
418  
 419 Fig. 16 Schematic layout of the numerical setup and boundary definitions.



420  
421 Fig. 17 Computational mesh with pile: (a) side view and (b) bird view.

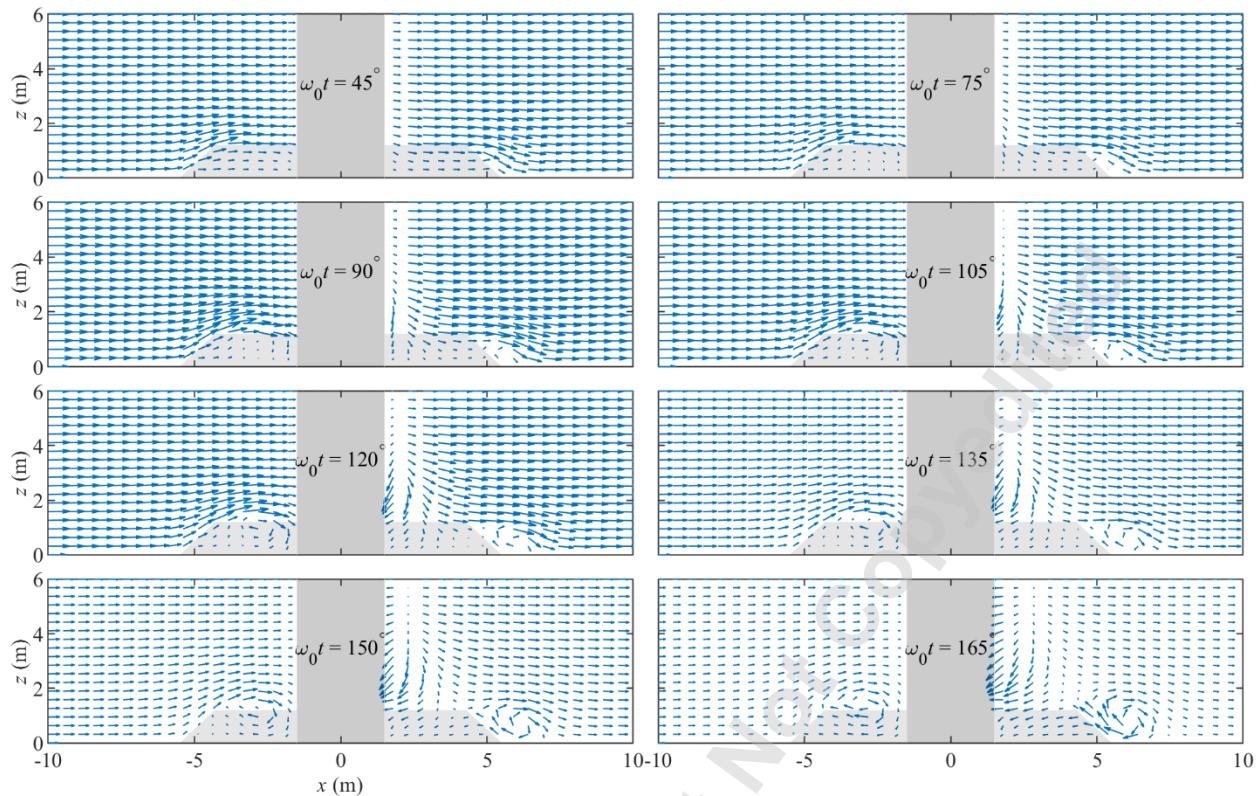
422 Same boundary conditions are defined at the inlet, outlet and walls as the ones in the 3D steady state model,  
423 while for the oscillatory flow, two kinds of flows with the same period  $T = 12$  s, are studied. One is the  $U_m = 1.50$  m/s  
424 and the other is  $U_m = 1.95$  m/s. The corresponding  $KC_s$  values are 112.50 and 146.25, respectively. The boundary  
425 definitions are also illustrated in Fig. 16. At the inlet,  $k = 1.36 \times 10^{-4}$  m<sup>2</sup>/s<sup>2</sup> and  $\omega = 0.16$  s<sup>-1</sup>; at the outlet,  $k$  and  $\omega$  are  
426 both specified with a zero gradient condition. When the pile is placed in oscillatory flow, the Keulegan-Carpenter  
427 number for pile,  $KC_p$ , has to be introduced. It defines the oscillatory motion related to the diameter of the pile,  $D_p$ ,  
428 which is calculated with  $KC_p = U_m T / D_p$ . Based on the given flow conditions and mono-pile diameter, the  $KC_p = 6.0$   
429 and 7.8, respectively, which meet the critical limit ( $KC_p = 6.0$ ) for the formation of a horseshoe vortex in the front of  
430 a pile without scour protection [58]. To achieve stable results, the time step is 0.0001 s and the total simulation time  
431 is 500 s.

432 For the two flow cases, the undisturbed flows are sampled after they reach stability. Fig. 18 depicts the time  
433 series of the outer flow velocity, sampled at  $x = -10.0$  m and  $z = 5.0$  m in the plane of symmetry  $y = 0$ . It shows that  
434 the signal follows a sinusoidal variation for the two cases. The symmetry in the oscillatory flows causes the symmetry  
435 of flow features between the crest and trough half-periods, thereby the results of the crest half-period are selected for  
436 analysis.



437  
 438 Fig. 18. Time series of the undisturbed free-stream velocity with  $T = 12$  s. (a)  $U_m = 1.50$  m/s; (b)  $U_m = 1.95$  m/s.

439 To examine the periodical flow patterns ( $U_m = 1.50$  m/s and  $T = 12$  s), eight phases, i.e.,  $\omega_0 t = 45^\circ, 75^\circ, 90^\circ,$   
 440  $105^\circ, 120^\circ, 135^\circ, 150^\circ$  and  $165^\circ$  in a crest half-period ( $0^\circ - 180^\circ$ ) are selected for illustration, as presented in Fig. 19.  
 441 The water flows from left to right. On the left side of the pile a horseshoe vortex emerges for phases at  $\omega_0 t = 105^\circ,$   
 442  $120^\circ, 135^\circ, 150^\circ$  and  $165^\circ$ . This is almost in line with the predictions that the vortex is expected to appear in the phase  
 443 interval  $110^\circ < \omega_0 t < 170^\circ$  without scour protection [3]. The size and strength of the vortex grows as the flow  
 444 develops ( $105^\circ < \omega_0 t < 165^\circ$ ). As the vortex strengthens inside the porous medium the return flow in terms of range  
 445 and size also increases. In front of the pile, we see that the near-bed flow velocity at the right hand side of the porous  
 446 medium edge first declines ( $45^\circ < \omega_0 t < 135^\circ$ ) and then increases ( $150^\circ < \omega_0 t < 165^\circ$ ) in the reverse direction. After  
 447  $\omega_0 t$  reaches about  $165^\circ$ , the vortex breaks down and washed out prior to the flow reversal ( $\omega_0 t = 180^\circ$ ), and  
 448 eventually disappear.



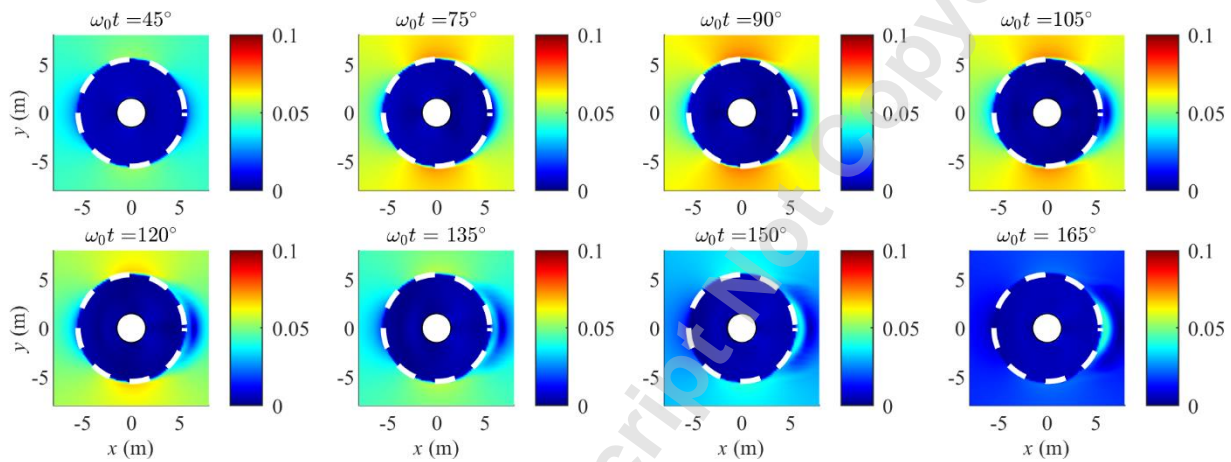
449  
 450 Fig. 19 Velocity fields around the pile with scour protection at different phases for the crest half-period ( $U_m = 1.50$   
 451 m/s,  $T = 12$  s and  $KC_p = 6.0$ ).

452 No wave and combined wave-current measurements are made around a pile within the scour protection.  
 453 Therefore, we draw an analogy to flow around a pile without scour protection [58], where we expect the horseshoe  
 454 vortex in the case of waves to be small. In such a case, the size and strength of the horseshoe vortex are determined  
 455 by both flow velocity and pile size. The vortex system is in a form of one dominating vortex, i.e., single horseshoe  
 456 vortex system. The reason is that the thickness of scour protection is smaller than the pile diameter, otherwise, two  
 457 clockwise rotating vortices might appear in the vicinity of the pile, which is revealed through experiments by Nielsen  
 458 et al. [28]. Similarly to the steady current in Section 4, the vortex penetrates into the entire scour protection. The down-  
 459 flow adjacent to the pile decreases from the interface to the bottom due to the porous resistance, and only a small  
 460 amount continues up and out from the porous medium.

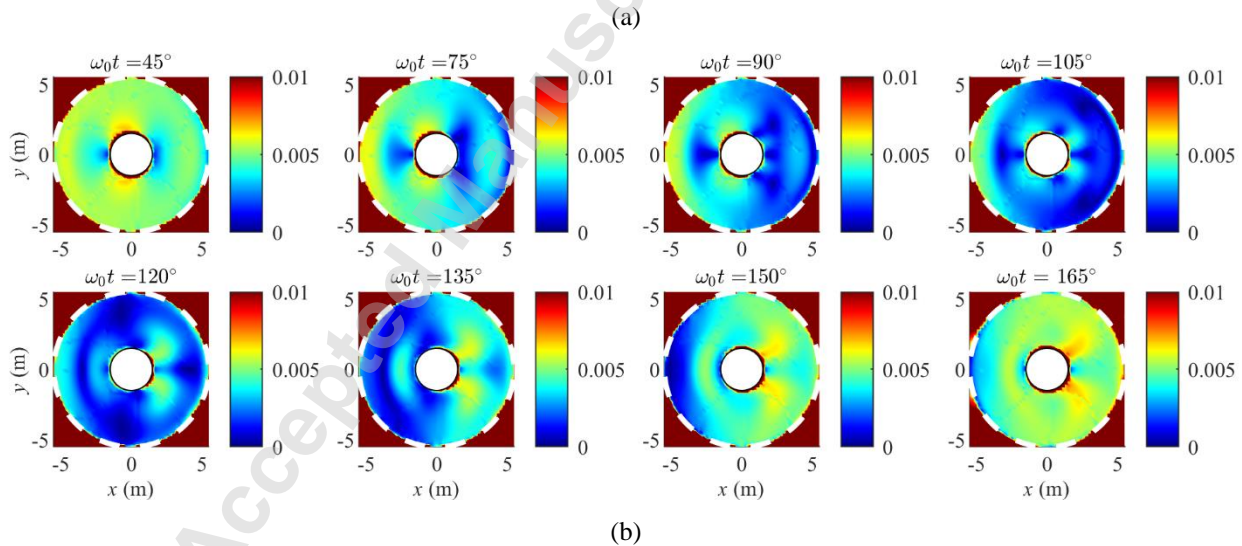
461 Immediately behind the pile, there occurs reverse flow with the increased phase, implying the occurrence of  
 462 lee-wake vortex. Compared to the horseshoe vortex, the attached lee-wake vortex near the porous medium first  
 463 emerges earlier at  $\omega_0 t = 90^\circ$ , while it in the free flow first appears later at  $\omega_0 t = 120^\circ$ . Similar to the horseshoe vortex,  
 464 the lee wake vortex grows in size and intensity gradually after its generation ( $\omega_0 t < 165^\circ$ ), which can be observed  
 465 from the increased reverse velocity behind the pile (Fig. 19). This results in that as  $\omega_0 t$  increases from  $75^\circ$  to  $165^\circ$ ,  
 466 the near-bed wake vortex penetrates deeper into the porous medium. Fig. 19 shows that the near-bed velocity first  
 467 decreases ( $45^\circ < \omega_0 t < 105^\circ$ ), and then increases inversely ( $105^\circ < \omega_0 t < 165^\circ$ ) behind the pile. At the downstream

468 edge of the scour protection, a distinct vortex rotating in clockwise direction occurs at  $\omega_0 t = 105^\circ$ ,  $135^\circ$ ,  
 469  $150^\circ$  and  $165^\circ$ .

470 Corresponding to the same phases, the bed shear stress field,  $\tau_b$ , around the pile and scour protection is  
 471 analyzed, as shown in Fig. 20a. The  $\tau_b$  distribution in the domain is symmetric with reference to  $y = 0$ . Outside the  
 472 porous medium, the bed shear stress,  $\tau_b$  is relatively large compared with bed shear stress in inside. The  $\tau_b$  shows an  
 473 increase from  $\omega_0 t = 45^\circ$  to  $90^\circ$ , followed with a decrease from  $\omega_0 t = 90^\circ$  to  $165^\circ$ . This is consistent with the variation  
 474 of flow velocity during the first half-period. The flow runs from left to right, and it is noted that behind the porous  
 475 medium, the location of  $\tau_b$  with minor values moves along the streamwise direction at a certain distance. Adjacent to  
 476 the scour protection on the right side, the area with enhanced  $\tau_b$  appears at  $\omega_0 t = 105^\circ$  and it amplifies obviously with  
 477 the increasing  $\omega_0 t$  from  $105^\circ$  to  $165^\circ$ , which coincides with development of vortices observed in Fig. 19.



478  
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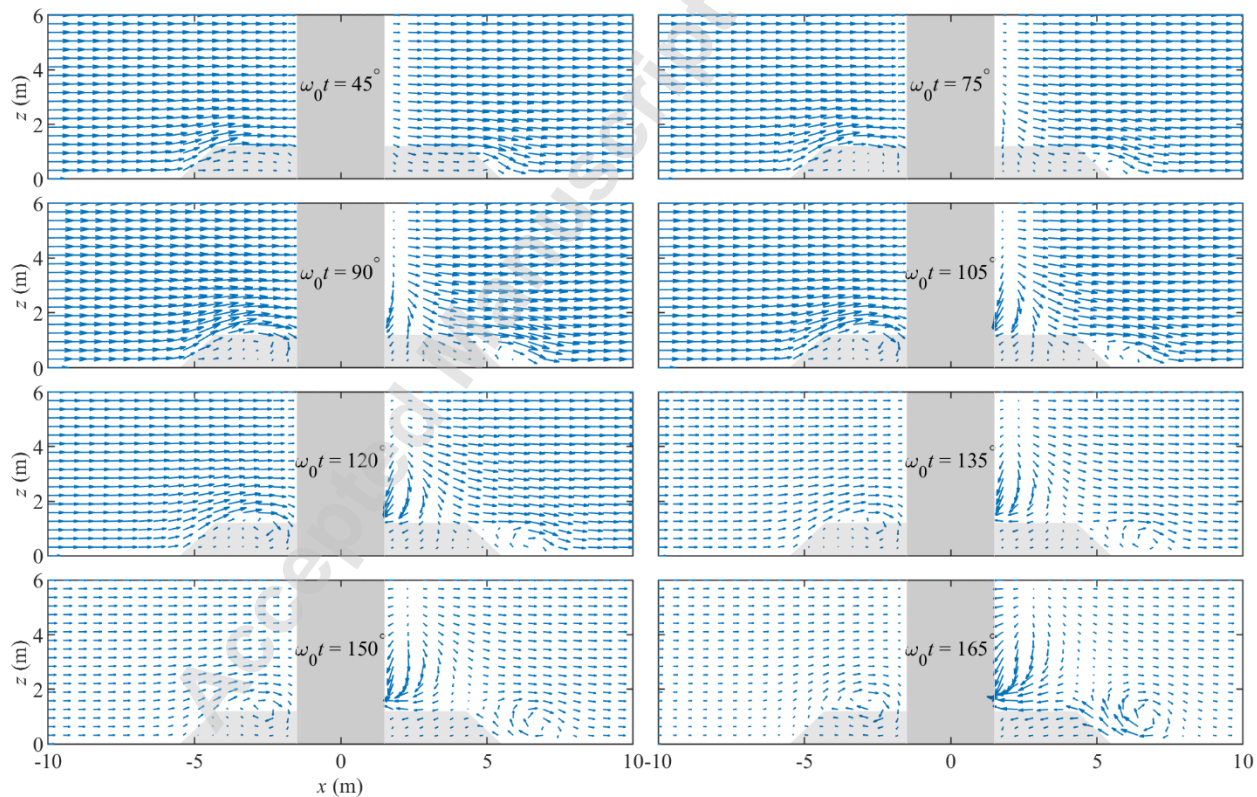
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482 Fig. 20 Bed shear stress around the pile with scour protection at different phases for the crest half-period ( $U_m = 1.50$   
 483 m/s,  $T = 12$  s and  $KC_p = 6.0$ ): (a) in the whole domain and (b) enlargement view inside the porous medium.

484 Inside the porous medium, the enlargement view of the bed shear stress,  $\tau_b$ , distribution is presented in Fig.  
 485 20b. From  $\omega_0 t = 45^\circ$  to  $90^\circ$ , the area directly in front of the pile features a small  $\tau_b$  and its range expands

486 symmetrically with reference to  $y = 0$ . A similar phenomenon is also observed at the lee-wake side of the pile. We  
 487 observe high bed shear stresses,  $\tau_b$ , between the front and the side edges of the pile, and its amplification at the lee-  
 488 wake side of the pile is small. This distribution is similar to the experimental enhanced  $\tau_b$  values with a symmetrical  
 489 distribution to the sides, which is attributed to the changes in size and intensity of the horseshoe vortex. At both sides  
 490 downstream of the pile, the  $\tau_b$  shows a symmetric distribution and its values increases significantly ( $90^\circ < \omega_0 t <$   
 491  $165^\circ$ ). This also illustrates that, behind the pile, a pair of symmetric attached vortices are formed.

492 Exposed to oscillatory flows, the flow features around a pile with scour protection is primarily related to the  
 493  $KC_p$ , and in influence on generation of the horseshoe vortex. To further understand the effect of  $KC_p$  on the flow  
 494 patterns, Fig. 21 presents a sequence of panels illustrating the development of the velocity fields during approximately  
 495 one half-period of the oscillatory flow for  $KC_p = 7.8$ . It shows that, in front of the pile, a horseshoe vortex exists at  
 496  $\omega_0 t = 90^\circ$  and it is not formed at  $\omega_0 t = 75^\circ$ , which is inferred that the vortex begins to emerge between  $75^\circ$  and  $90^\circ$ .  
 497 Once the vortex has emerged, it becomes enhanced with the increased phase (e.g., from  $90^\circ$  to  $165^\circ$ ) and eventually  
 498 disrupted and washed just prior to the reversed flow at  $\omega_0 t = 180^\circ$ . Without the scour protection, Sumer et al. [58]  
 499 predicted that the vortex appears in the phase interval  $90^\circ < \omega_0 t < 170^\circ$  for  $KC_p = 7.8$ . The vortex appears at a slightly  
 500 earlier phase in the case with scour protection, which obviously leads to larger lifespans. The reason is that, in front  
 501 of the pile, the flow velocity is increased due to the presence of the scour protection and large adverse pressure gradient  
 502 is generated at an earlier phase.

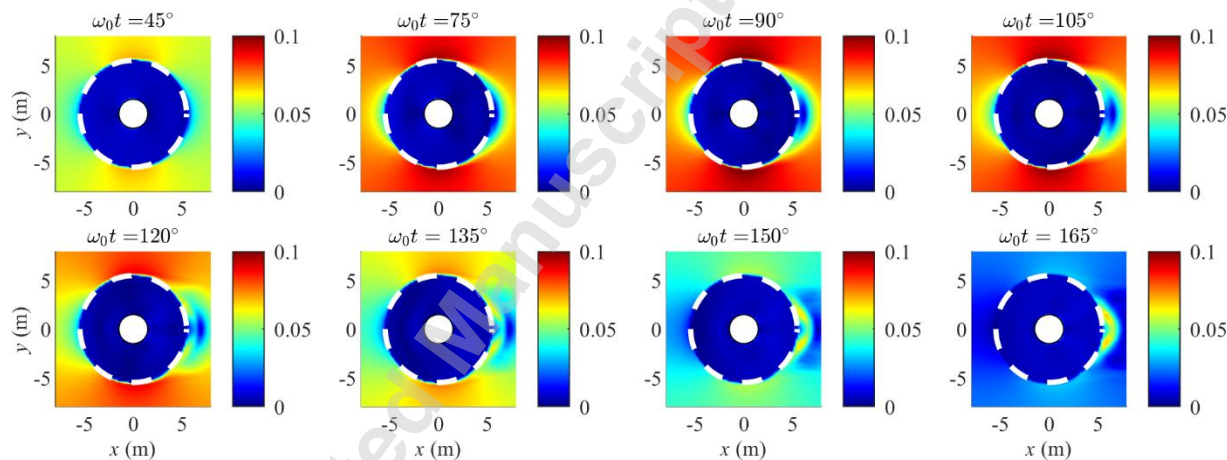


503  
 504 Fig. 21 Velocity fields around the pile with scour protection at different phases for the crest half-period ( $U_m = 1.95$   
 505  $\text{m/s}$ ,  $T = 12 \text{ s}$  and  $KC_p = 7.8$ ).



506 The lee-wake vortex near the porous medium occurs first at  $\omega_0 t = 75^\circ$  and it in the free flow emerges first at  
 507  $\omega_0 t = 105^\circ$ . Besides, at the downstream edge of the scour protection, a clockwise vortex emerges in the interval of  
 508  $\omega_0 t = 90^\circ - 165^\circ$ , and it grows in size with the phase. The vortices shift their locations in each half-period, and the  
 509 horseshoe vortex in front of the pile is supposed to be symmetric to that at the back of the pile, due to the symmetric  
 510 oscillatory flow. Compared to the case where  $KC_p = 6.0$ , the vortices inclusive of the horseshoe vortex, the lee-wake  
 511 vortices and the clockwise vortex all emerge earlier, and are maintained over a larger span of  $\omega_0 t$ . Similarly, their size  
 512 and intensity also increases with increasing  $\omega_0 t$ . This illustrates the variations of the vortices as the  $KC_p$  increases  
 513 from 6.0 to 7.8. These findings are in line with Sumer and Fredsøe [3], who made visualization of horseshoe vortex  
 514 in front of a pile without scour protection, and found the horseshoe vortex to increase in both size and lifespan as the  
 515  $KC_p$  increases.

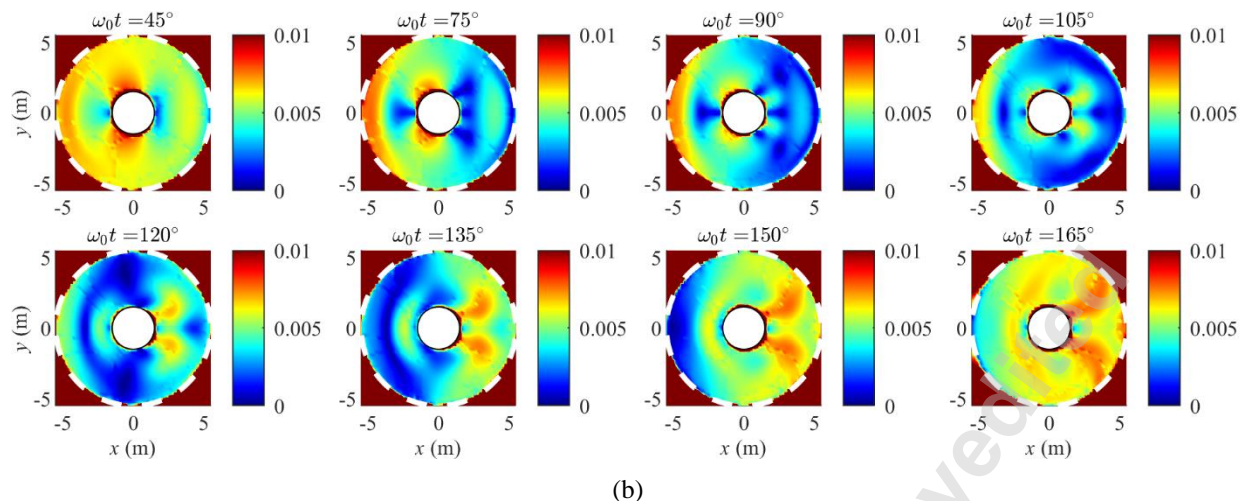
516 To further illustrate the effect of  $KC_p$  variations on the flow features, during the course of the same crest half-  
 517 period ( $0^\circ - 180^\circ$ ), the  $\tau_b$  for  $KC_p = 7.8$  around the pile and scour protection are analyzed, as shown in Fig. 22a. The  
 518 close-up of its distribution inside the porous medium is presented in Fig. 22b. Similar to the  $\tau_b$  distributions for  $KC_p$   
 519  $= 6.0$ , symmetry of the  $\tau_b$  distribution, with reference to  $y = 0$ , is also observed under all phases for  $KC_p = 7.8$ . In the  
 520 front of and in the lee-wake side of the pile, the  $\tau_b$  amplifies symmetrically with the development of the vortices. The  
 521  $\tau_b$  is significantly influenced by the  $KC_p$ , increasing the  $KC_p$  from 6.0 to 7.8, its value becomes large, which is  
 522 expected according to the flow fields.



(a)

523

524



525  
 526 (b)  
 527 Fig. 22 Bed shear stress around the pile with scour protection at different phases for the crest half-period ( $U_m = 1.95$   
 528 m/s,  $T = 12$  s and  $KC_p = 7.8$ ): (a) in the whole domain and (b) enlargement view inside the porous medium.

## 529 6. Conclusions

530 The present study investigates, using 2D and 3D numerical models, the flow around a mono-pile with scour protection.  
 531 Both steady and oscillatory flow conditions are examined. In the model, the volume-averaged Reynolds-  
 532 averaged Navier-Stokes (VARANS) equations are solved with the volume-averaged  $k-\omega$  turbulence closure. The main  
 533 conclusions are as follows:

- 534 (1) Near the interface between free flow and porous medium, the effect of porosity transition type on flow velocity,  
 535 turbulence kinetic energy and bed shear stress is analysed, which is based on the experiments performed by Sumer et  
 536 al. [1] and Stevanato et al. [31]. In comparison to the constant porosity and linear transition, the parabolic transition  
 537 shows better agreement with the measurements.
- 538 (2) In the presence of monopile with scour protection, the model with parabolic transition considered is validated  
 539 against the existing experiments [28]. The simulated flow patterns and turbulence levels are both consistent with the  
 540 measurements, illustrating a good performance of the model. It is shown that the horseshoe vortex in the upstream of  
 541 pile can penetrate into the scour protection, which induces high bed shear stress. As a result, the sediment particles  
 542 are supposed to be transported upstream, causing the sinking of the protection layer.
- 543 (3) The model subsequently studies the oscillatory porous flow features around a pile. It is found that the horseshoe  
 544 vortex and attached lee-wake vortices are formed at two sides of the pile, respectively. They penetrate into the entire  
 545 scour protection layer, generating high flow velocities and bed shear stresses.
- 546 (4) In oscillatory flow, the horseshoe vortex in front of a pile emerges later than the lee-wake vortices near the porous  
 547 medium. The lee-wake vortices first occur near the porous medium and gradually amplify to the free flow. At the right  
 548 toe of the scour protection, a distinct local vortex rotating in a clockwise direction is also observed. The vortices all  
 549 increase in both size and intensity with increasing phase in each half-period. The bed shear stress inside the scour  
 550 protection is smaller compared to outside. Under the protection, it reaches the maximum at the side edges of the pile.

551 (5) The flow features under oscillatory condition are strongly influenced by  $KC_p$ . As the  $KC_p$  increases, the vortices  
552 inclusive of the horseshoe vortex and the lee-wake vortices, both in size and lifespan augments; the bed shear stress  
553 both inside and outside the scour protection also amplifies.

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