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A New Virtual Inductance Control Method for Frequency Stabilization of Grid-Forming Virtual Synchronous Generators

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Abstract—Frequency stabilization is the premise of guaranteeing grid-friendly integration of virtual synchronous generator (VSG). Based on that premise, this study, focused on frequency stability, establishes the small signal model of grid-forming VSG system. At first, the mechanism of frequency oscillation occurring in system of active- and reactive-power coupling is analyzed by the defined feedback effect factor (FEF) in this study. Besides, a new dynamic model is established for identifying dynamic interaction of voltage magnitudes and frequency by means of feedback effect. The analytical results of established models agree with that of eigenvalue analysis. Furthermore, a new virtual inductance control strategy is proposed to mitigate the unstable oscillation of frequency and powers, enhance damping performance, and improve stability margins. Unlike the conventional virtual inductance control, which is reliable on dual-loop control framework, the proposed virtual inductance control in this study is based on principle of energy conservation and can be applicable for the grid-forming inverter without inner dual-loop control structure. Finally, the proposed modeling as well as virtual inductance control method is experimentally verified.

Index Terms—virtual synchronous generator (VSG), feedback effect, frequency stabilization, rate of change of frequency (RoCoF), frequency offset (FO), virtual inductance control

I. INTRODUCTION

ENVIRONMENTAL pollution as well as energy crisis is getting more and more attentions. To cope with those issues, more and more distributed renewable energy generations are penetrating into modern power systems [1]. Low-inertia and poor-damping are remarkable characteristics of modern power systems due to the grid-integration of renewable energy. That characteristic directly leads to fast rate of change of frequency (RoCoF) and inevitable frequency nadir (FN) [2], which threatens secure operation of the AC power systems.

Virtual synchronous generator (VSG) control, which emulates the inertia and damping characteristics of conventional synchronous generator (SG), has been an emerging technique to improve inertia and damping of the system [3]. At present, the study of VSG are mainly focused on two aspects, i.e., control [4]-[7], and stabilization [8]-[16].

For example, a virtual inertia control strategy was proposed to provide inertia for AC power grid and improve FN [4]. However, this type of inertial control links DC voltage coupled with frequency, which is at the cost of drop of DC voltage. To rectify that drawback, a new compensator was proposed to alleviate the offset of DC voltage [5]. To overcome the shortcomings originated from fixed control parameters, an adaptive inertial control strategy was proposed to improve the stability of virtual synchronous machine (VSM) under various grid strengths [6]-[7]. Besides of this, an improved virtual inertia control was proposed to suppress frequency fluctuation of three-phase voltage source inverters [8]. No matter how the improved VSG control is implemented, the core idea is focused on the emulation of rotor swing equation of conventional SG.

However, the premise that VSG provide inertia and damping support for the system is that VSG itself or grid-integration VSG system can maintain stable operation.

For example, a new modeling and parameters design methodology have been developed for VSG to guarantee secure operation of the system [9]. However, it is not involved in voltage stability or frequency stability. To fill in this gap, voltage and frequency stability analysis were discussed with a grid-forming VSG [10]. However, the proposed magnitude-phase motion equation (MPME) simplifies the model and does not consider the reactive power control coupling dynamics, which plays a predominant role in the system stability.

It was pointed out that grid-forming VSG can maintain good stability even when attaching to weak grid [13]-[14]. However, the stability mechanism is clarified from impedance model. The study in this paper shows that there is a risk of divergent oscillation appearing, when VSG is attached to power grid of resistance-inductance impedance, due to negative damping and positive feedback formed in the system. To fill in the gaps, this study builds a more complete MPME model to clarify the cause of frequency instability. Besides, the physical significance is
discussed in various perspectives.

Although there have been several virtual impedance control strategies put forward to mitigate the oscillation of inverter or grid-forming VSG [15]-[17], all of them are realized by formulating the control links between current signal and voltage signal. However, dual-loop control structure is necessary for the implementation of virtual impedance. Therefore, to the best knowledge of the author, virtual impedance control is seldom proposed for grid-forming VSG without dual-loop control structure. Therefore, this study fills in the gaps.

The major contributions of this study are organized as follows:
1) Based on the established small signal modeling of grid-forming VSG system, the motion dynamics of both RoCoF and frequency offset (FO) can be judged by feedback effect. It is found that frequency stability of grid-forming VSG system is closely related with the transients of RoCoF and FO. And the feedback effect brought by internal relationship between RoCoF and FO gives physical insights to the frequency stability.
2) A new coupling model between voltage magnitude and frequency is established to clarify the feedback interaction between voltage and frequency of grid-forming VSG system, and the results of this analytical model agree with that of eigenvalue analysis.
3) A new virtual inductance control strategy is put forward to mitigate divergent oscillation of VSG frequency and powers, and enhance damping performance of the system. It should be noted that this type of virtual inductance is different from conventional virtual inductance control, which is dependent on the dual-loop control architecture. The proposed virtual inductance control is based on principle of energy conservations. It can be applied for VSG control structure without inner dual-loop control in this study. Besides, the effectiveness of the proposed virtual inductance control strategy is demonstrated by vector analysis and eigenvalue analysis.
4) The proposed control algorithm and methodology can be analogously applicable to DC systems, which is our present focus.

The remainders of this study are organized as follows: The next Section discusses mechanism analysis of frequency oscillation in grid-forming VSG system by the established small signal model. The dynamic behaviors of RoCoF and FO are studied by identifying feedback effect. And dynamic coupling model is established to clarify dynamic interaction between voltage magnitudes and frequency of VSG. Section III introduces the proposed virtual inductance control strategy to mitigate oscillation and improve damping performance. Section IV gives the experiments validations. Section V draws the conclusion.

II. MECHANISM ANALYSIS OF FREQUENCY OSCILLATION OF GRID-FORMING VSG

In this Section, mechanisms of both frequency oscillation and power coupling are discussed by the small signal modeling.

\[ P_e = \frac{3}{2} \left( e_d i_d + e_q i_q \right) \frac{\omega_L}{s + \omega_L} \]
\[ Q_e = \frac{3}{2} \left( -e_d i_d + e_q i_q \right) \frac{\omega_L}{s + \omega_L} \]

where \( e_d, e_q, i_d, i_q \) are the voltage and current of \( d \)- and \( q \)-axis, respectively.

In \( dq \) frame, the voltage and current can be represented by:

\[ e_d = (R + L_s)i_d - \omega_L L_e \]
\[ e_q = \omega_L L_e i_d + (R + L_s)i_q \]
\[ i_d = \frac{(R + L_s)(E \cos \delta_e - U_e)}{(R + L_s)^2 + (\omega_L L_e)^2} \]
\[ i_q = -\omega_L L_e \cos \delta_e - U_e \]

where \( R = R_f + R_g, L = L_f + L_g \).

The VSG control dynamics, which emulates the inertia and damping of conventional SG, can be expressed as:

\[ P_{ref} - P_e = D_e (\omega_e - \omega_h) + J \omega_e (\omega_e - \omega_h) s \]
\[ Q_{ref} - Q_e = J_q (E - U_{ref}) s \]

where \( P_{ref}, Q_{ref} \) are the reference active- and reactive-power, \( P_e, Q_e \) are the output powers of the inverter. It should be noted that (4.a) and (4.c) mean frequency support for the system, and (4.b) represents voltage support for the system.
Combing the equation of (1)-(3), the relationships among active power, reactive power, magnitudes and phase angle of output voltage of inverter can be written as:

\[ P_e = f(\delta_e, E) \]
\[ Q_e = g(\delta_e, E) \]

Linearizing Eq. (5) around the equilibrium points, one can obtain that:

\[
\frac{\Delta P_e}{\Delta \omega_e} = G_{PE}(s)\Delta \delta_e + G_{PE}(s)\Delta E
\]
\[
\frac{\Delta Q_e}{\Delta \omega_e} = G_{QE}(s)\Delta \delta_e + G_{Qe}(s)\Delta E
\]

where \( G_{PE}(s) \), \( G_{PE}(s) \), \( G_{Qe}(s) \), \( G_{Qe}(s) \) are the coupling terms between voltage magnitude as well as phase angle and powers. According to the developed small signal model, closed-loop control system of active power-frequency is shown in Fig. 2.

**B. Dynamic Interactive Behavior and feedback effect between Frequency and Magnitude of VSG Output Voltage**

According to Fig. 2, the internal relationship from \( \Delta E \) to \( \Delta \omega_e \) (\( \Delta E \) divided by \( \Delta \omega_e \)) can be defined as \( M_{\omega E}(s) \). Likewise, the relationship from \( \Delta \omega_e \) to \( \Delta \omega_e \) can be defined as \( N_{\omega E}(s) \). By substituting (5.a) into (4.a) and combing (4.c), \( M_{\omega E}(s) \) is derived by linearization:

\[
M_{\omega E}(s) = \frac{\Delta \omega_e}{\Delta \omega_e} = \frac{-G_{PE}(s)/(J_\omega \omega_e s + D_p)}{[1 + G_{PE}(s)/(J_\omega \omega_e s^2 + D_p)]} \tag{7}
\]

Likewise, substituting (5.b) into (4.b), \( N_{\omega E}(s) \) can be obtained as follows:

\[
N_{\omega E}(s) = \frac{\Delta E}{\Delta \omega_e} = \frac{-G_{Qe}(s)/J_\omega s^2}{1 + G_{Qe}(s)/J_\omega s} \tag{8}
\]

It can be seen that Eq. (7) and Eq. (8) formulate the feedback loop to identify the dynamic interactions between frequency (\( \Delta \omega_e \)) and voltage magnitudes (\( \Delta E \)). To more intuitively identify the dynamic interactive behavior between \( \Delta \omega_e \) and \( \Delta E \), it is significant to define the \( i^{th} \) moment and the \( (i+1)^{th} \) moment of the state variables, e.g., \( \Delta \omega_e \) and \( \Delta E \). Provided that it is initial to define the \( i^{th} \) moment of \( \Delta \omega_e \) as \( \Delta \omega_{e,i} \), whereby state \( \Delta \omega_{e,j} \) of \( i^{th} \) moment of \( \Delta E \), is dependent on the effect of \( \Delta \omega_{e,j} \) with disturbance of \( \Delta \omega_{e,j-1} \), \( \Delta E_j \) will influence the state on \( \Delta \omega_{e,j} \) through \( M_{\omega E}(s) \). Therefore, the state of \( \Delta \omega_e \) will be accordingly updated, referred to as \( \Delta \omega_{e,i+1} \). Therefore, either positive feedback effect or negative feedback effect can be identified by comparing the updated state \( \Delta \omega_{e,i+1} \) with the earlier state \( \Delta \omega_{e,i} \). Moreover, the motion trajectory of VSG frequency can be judged by comparison between renewed state \( \Delta \omega_{e,i+1} \) and the original state \( \Delta \omega_{e,i} \), the coupling relationships between the renewed state and initial state can be described mathematically as follows:

\[
\begin{aligned}
N_{\omega E}(s) &= \frac{\Delta E_{i+1}}{\Delta \omega_{e,i+1}} \\
M_{\omega E}(s) &= \frac{\Delta \omega_{e,i+1}}{\Delta E_{i+1}}
\end{aligned}
\]

where \( G_{M\omega}(s) \) is the transfer function which can depict the feedback effect on state variable \( \Delta \omega_e \) and thus predict the motion trajectory of VSG frequency, i.e.,

\[
G_{M\omega}(s) = \frac{G_{PE}(s)G_{Qe}(s)/[J_\omega s^2(J_\omega \omega_e s + D_p)]}{[1 + G_{PE}(s)/(J_\omega \omega_e s^2 + D_p)](1 + G_{Qe}(s)/J_\omega s)}
\]

To better identify the motion state of VSG frequency, \( k \) is defined as feedback effect factor (FEF), i.e.,

\[
\Delta \omega_{e,i+1}\bigg|_{\text{due to } \omega_{e,i+1}} = k \Delta \omega_{e,i}\bigg|_{t}\text{ (CFP)}
\]

where the parameter \( k \) is the magnitude of \( G_{M\omega}(s) \) at the critical frequency point (CFP). Thus \( k \) can be recalculated as:

\[
k = 10^{A/10} > 1 \quad (A > 0)
\]
\[
k < 1 \quad (A < 0)
\]

where \( A \) represents the magnitude of transfer function \( G_{M\omega}(s) \) as the unit of Decibel (dB).

By rearranging Eq. (12), the RoCoF can be formulated as:

\[
\frac{\Delta \omega_{e,i+1}}{\Delta t} = k \frac{\Delta \omega_{e,i}}{\Delta t}
\]

By considering (13), a unified expression is derived to clarify the dynamic characteristics of RoCoF, i.e.,

\[
\frac{d \omega_{e,i}}{dt} = k^n \frac{d \omega_{e,0}}{dt}
\]

\[
\lim_{n \rightarrow 0} k^n = \begin{cases} +\infty & (k > 1) \\ 1 & (k = 1) \\ 0 & (k < 1) \end{cases}
\]

It can be seen in (16) that it forms positive feedback effect when \( k > 1 \). And the RoCoF gets greater and greater, indicating that divergent oscillation appears in the system. It is because that divergence oscillation of frequency indicates both RoCoF and FO get larger and larger periodically. Vice versa, it forms negative feedback effect when \( k < 1 \). And the oscillation will be dampened and frequency gets convergent to an equilibrium point.

In addition, the FO (\( \delta \omega_j \)) is defined as the frequency difference between the VSG frequency and the grid frequency, and the \( i^{th} \) FO can be derived as shown:

\[
\delta \omega_{j,i} = \sum_{i=0}^{i-1} \frac{d \omega_{e,i}}{dt} \quad (i = 0, 1, 2, \ldots n \ldots \infty)
\]

Another index to identify frequency stability is Relative Frequency Offset (RFO). For any positive integers \( i, j \) and \( i > j \), if RFO (\( \rho_{ij} \)) is greater than zero, thereby the VSG frequency...
will be divergent and cannot return back to equilibrium point due to positive feedback effect, and vice versa.

\[ R_{0f} = \delta \eta_{P,i,j} - \delta \eta_{Q,i,j} \quad (i > 0, j > 0, i > j) \quad (18) \]

According to the above discussions, FEF (k) imposes a great effect on the motion trajectory of RoCoF and RFO. The FO gets larger and larger when FEF>1 that forms a positive feedback effect. However, FO gets smaller and smaller when FEF<1, and negative feedback effect is formed to stabilize VSG frequency.

To validate the proposed modeling, Fig. 3 obtains the results of eigenvalue analysis and bode diagram of \( G_{Mx}(s) \) with different \( R_{0f} \). The system parameters are shown in Table 1. In Fig. 3(a), the dominant eigenvalues are moving from left-half plane towards to the right-half plane as \( R_{0f} \) gets enlarged, which indicates that the greater size of \( R_{0f} \) is poorer for the stability of the system and has a larger risk of sub-synchronous oscillation. Fig. 3(b) shows bode diagram of \( G_{Mx}(s) \), from which it can be seen that a smaller \( R_{0f} \) means lower magnitude of FEF. The lower magnitude of FEF means lower risk of positive feedback effect, and this will improve stability margins of the system. It can be inferred that smaller \( R_{0f} \) can make the system more stable, and frequency stability is enhanced. According to the analysis of (14) and (16)-(18), it forms a negative feedback effect when FEF is below 1 (0 dB). And in this case, both RoCoF and FO are declined over time and gradually tend to be zero with the negative feedback effect. Therefore, it can be inferred that it forms a negative feedback when \( R_{0f} = 0.8 \Omega \).

However, the magnitude of \( G_{Mx}(s) \) is greater than zero at the CFP when \( R_{0f} = 1.2 \Omega \) and \( R_{0f} = 1.6 \Omega \), divergent oscillation (instability) of active power and frequency will appear. Therefore, the proposed modeling with feedback effect is well illustrated by bode analysis and eigenvalues.

### Table 1 System Specifications

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated active power</td>
<td>( P_e )</td>
<td>5 kW</td>
</tr>
<tr>
<td>Rated reactive power</td>
<td>( Q_e )</td>
<td>0 Var</td>
</tr>
<tr>
<td>DC voltage</td>
<td>( U_{dc} )</td>
<td>700 V</td>
</tr>
<tr>
<td>Filter inductance</td>
<td>( L_f )</td>
<td>4.4 mH</td>
</tr>
<tr>
<td>Resistance</td>
<td>( R_f )</td>
<td>0.1 \Omega</td>
</tr>
<tr>
<td>Grid voltage</td>
<td>( U_{G \text{RMS}} )</td>
<td>380 V</td>
</tr>
<tr>
<td>Sampling frequency</td>
<td>( f_s )</td>
<td>10 kHz</td>
</tr>
<tr>
<td>Rated frequency</td>
<td>( \omega_s )</td>
<td>314 rad/s</td>
</tr>
<tr>
<td>Inertia of Active Power</td>
<td>( J_p )</td>
<td>0.01</td>
</tr>
<tr>
<td>Damping coefficient</td>
<td>( D_p )</td>
<td>172.7</td>
</tr>
<tr>
<td>Inertia of Reactive Power</td>
<td>( J_q )</td>
<td>5</td>
</tr>
</tbody>
</table>

![Fig. 3. Eigenvalue loci and bode diagram with various \( R_{0f} \). (a) Dominant root locus, (b) \( G_{Mx}(s) \).](image)

To further discuss the frequency dynamics as well as system stability, by considering Eq. (6), (8) and Fig. 2, an active power closed-loop control diagram is shown in Fig. 4. The whole feedback paths include three parts, i.e., power-angle-related part (PARP) \( (h_f(s) \), voltage-angle-related part (VCRP) \( (h_E(s) \) and damping part \( (h_D(s) \). The outputs of the three feedback loops can be regarded as three equivalent active powers (i.e., \( \Delta P_h \), \( \Delta P_{E} \), and \( \Delta P_{D} \)) that interact with the virtual rotor, as depicted in Fig. 4. The dynamic behaviors of the virtual rotor are determined by the composite power \( (\Delta P = \Delta P_h + \Delta P_{E} + \Delta P_{D}) \), which can be used to evaluate the stability of the system.

Dynamic behaviors of system can be seen as superposition of dynamics of the feedback powers of three items, i.e.,

\[ \Delta P_h = h_f(s)\Delta \delta + h_E(s)\Delta \omega + h_D(s)\Delta \delta \]

where the transfer functions \( h_f(s) \), \( h_E(s) \) and \( h_D(s) \) are given by:

\[ h_f(s) = \frac{\Delta P_h}{\Delta \delta} \]

\[ h_E(s) = \frac{\Delta P_E}{\Delta \omega} \]

\[ h_D(s) = \frac{\Delta P_D}{\Delta \delta} \]

The amplitude and phase of three transfer functions at critical frequency point can be derived, and accordingly, the vector diagrams of the active powers are carried out to investigate the stability of the system. It should be emphasized that Fig. 5(a), (b) show the results of transfer functions of \( h_f(s) \), \( h_E(s) \), and \( h_D(s) \) with \( R_{0f} = 0.8 \Omega \) and \( R_{0f} = 1.2 \Omega \), respectively. Fig. 5(c), (d) presents the corresponding vectors diagram. \( \Delta P_h \), \( \Delta P_E \), and \( \Delta P_D \) represent PARP, VCRP, damping part, and total power [18].

Note that the phase of total active power at CFP is the focus of this paper. As for synchronous machine, the feedback active power can be divided into two items: damping power and synchronizing power, which are proportional to \( \Delta \omega \) and \( \Delta \delta \), separately. That vectors which are in the first quadrant means that the system is stable because of positive damping power and synchronizing power. From Fig. 5(c), the synthetic vector is located in the fourth quadrant with \( R_{0f} = 1.2 \Omega \). By decomposing the synthetic power \( \Delta P_h \) into damping and synchronizing ones, the negative damping power exists, which will trigger the oscillation instability. Different from Fig. 5(c), Fig. 5(d) shows the synthetic vector \( \Delta P_h \) which provides positive damping to suppress the frequency oscillation and compel the system...
frequency convergent to an equilibrium point. It can be inferred that the grid resistance plays an important role in the stability of the system. Besides, greater grid resistance indicates the stronger coupling effect between active- and reactive-power, which will aggravate stability of grid-forming VSG.

C. Influence of grid parameters on power stability and frequency oscillation

To reveal the influence of grid parameters on stability of grid-forming VSG system, eigenvalues with various \( R_e \) and \( L_g \) is developed in Fig. 6. From Fig. 6(a), the eigenvalues movement with \( R_e \) gradually increased from 0.3 to 1.6 Ω with \( L_g \) chosen as 8.8 mH. It can be seen that the coupling effect between active- and reactive-power becomes ever-increasing stronger. From partial enlarged view, a pair of dominant eigenvalues moves from the left half-plane to right half-plane, eventually arrives at unstable regions. Eigenvalue analysis results coincide with that of vector analysis. Fig. 6(b) shows the root locus with variations of \( L_g \) from 6.6 to 13.2 mH with \( R_e \)=1.2 Ω. The dominant eigenvalues move towards to the left half-plane as \( L_g \) gets greater, which indicates that the greater size of \( L_g \) enhance the stability of grid-forming VSG. In general, no matter increasing \( L_g \) or decreasing \( R_e \) aims to lessen coupling effect between active- and reactive-power, which can improve stability margins of the system.

III. PROPOSED VIRTUAL INDUCTANCE CONTROL STRATEGY FOR MITIGATION OF FREQUENCY OSCILLATION OF GRID-FORMING VSG

In this section, a virtual inductance feed-forward control is proposed to enhance the stability of grid-forming VSG system, thereby frequency oscillation of grid-forming VSG is mitigated by the virtual inductance control as well.

A. Operation principle of virtual inductance control

As discussed earlier, increase of inductance alleviates the coupling effect between active- and reactive-power. In order to achieve good dynamic behavior, a virtual inductance, which emulates the real inductance, is developed. It should be noted that the virtual inductance is implemented by the control loop, which does not add hardware costs. Besides, the principle as well as architecture of virtual inductance is different from that in [19], [20], since that no architecture of dual-loop control is contained in this type of VSG. The equivalent power with consideration of virtual inductance can be rewritten as:

\[
\begin{align*}
P_{\text{v}} &= \frac{3}{2}(e_{d_L} \delta_{d_L} + e_{q_L} \delta_{q_L}) \frac{\omega_r}{s + \omega_r} \\
Q_{\text{v}} &= \frac{3}{2}(-e_{d_L} \delta_{d_L} + e_{q_L} \delta_{q_L}) \frac{\omega_r}{s + \omega_r}
\end{align*}
\]

where \( e_{q_L} \), \( i_{q_L} \) is the \( d- \) and \( q- \) axis current with introduction of virtual inductance, i.e.,

\[
\begin{align*}
\delta_{d_L} &= \frac{[R + (L_L + L_s)s] \cos \delta_r - U_r + \omega_r (L_r + L_s) \sin \delta_r}{[R + (L_r + L_s)s]^2 + [\omega_r (L_r + L_s)]^2} \\
\delta_{q_L} &= \frac{-\omega_r (L_r + L_s) \cos \delta_r - [R + (L_r + L_s)s] \sin \delta_r}{[R + (L_r + L_s)s]^2 + [\omega_r (L_r + L_s)]^2}
\end{align*}
\]

Based on the energy conservation, the difference of powers between with virtual inductance and without virtual inductance can be derived as:

\[
\begin{align*}
P_{\text{v}} &= P_{\omega r} - P_r = m(\delta_r, E) \\
Q_{\text{v}} &= Q_{\omega r} - Q_r = n(\delta_r, E)
\end{align*}
\]
B. Discussion on virtual inductance to dampen frequency oscillation

This part explores the principle of virtual inductance for damping frequency oscillation of the system. Besides, it shows the positive damping effect brought by the proposed virtual inductance control method.

Fig. 8 shows the eigenvalue analysis result with virtual inductance varying from 0 to 17.6 mH. Critical oscillatory modes move toward the left half-plane as virtual inductance gets enlarged. It indicates that increase of virtual inductance can strengthen damping performance and enhance the system stability.

Analogous to (7) and (8), the relationship between $\Delta \omega_e$ and $\Delta E$, with introducing virtual inductance, can be reformulated as:

$$\Delta P_{\text{vir}} = K_{P_{\text{E-L}}} (s) \Delta \delta_e + K_{P_{E-L}} (s) \Delta E$$

$$\Delta Q_{\text{vir}} = K_{Q_{\text{E-L}}} (s) \Delta \delta_e + K_{Q_{E-L}} (s) \Delta E$$

where $\Delta P_{\text{vir}}$ and $\Delta Q_{\text{vir}}$ represent the control instructions feed into the active-power control loop and reactive-power control loop, respectively.

Therefore, effect of virtual inductance can be realized by the derived control blocks in (24), as shown in Fig. 7.

C. Physical significance of proposed virtual inductance control for mitigation of system oscillation

According to the above discussions, negative damping as well as positive feedback effect of the system is the predominant cause of divergence of system frequency. Hence, this part will introduce physical significance of the proposed virtual inductance for mitigation of system oscillation.

From the view of attributes of the powers, the feedback active power with consideration of virtual inductance can be divided into two parts, $h_{\text{vir, } \delta}(s)$ and $h_{\text{vir, } E}(s)$, i.e.,

$$h_{\text{vir, } \delta}(s) = \frac{\Delta P_{\text{vir}}}{\Delta \delta_e} = K_{P_{\text{E-L}}},$$

$$h_{\text{vir, } E}(s) = \frac{\Delta P_{\text{vir}}}{\Delta E} = \left[ G_{\text{PE-L}}(N_{E_{\text{vir}}}) - N_{E_{\text{vir}}} \right] + K_{P_{E-L}} N_{E_{\text{vir}}}$$

Fig. 10 plots the frequency response results of $h_{\text{vir, } \delta}(s)$ and $h_{\text{vir, } E}(s)$ with various sizes of virtual inductance $L_v$, ($L_v=4.4$ mH, $L_v=8.8$ mH, $L_v=13.2$ mH, $L_v=17.6$ mH). It can be seen that the magnitude of $h_{\text{vir, } \delta}(s)$ is higher than that of $h_{\text{vir, } E}(s)$ in the
The small signal control system with virtual inductance is presented as shown in Fig. 11. Therefore, the synthetic powers can be derived as:

\[
\Delta P_f' = h_i(s)\Delta \delta + h_k(s)\Delta \delta + h_{\delta}(s)\Delta \delta + h_w(s)\Delta \delta
\]  

(29)

In order to illustrate how the virtual inductance impacts the dynamics of active power, the newly formed three feedbacks active powers are formulated as:

\[
\begin{align*}
\frac{h'_i(s)}{\Delta \delta} = & \frac{\Delta P_f'}{\Delta \delta} = G_{QE}(s) + K_{P_{\delta},L_{\delta}}(s) \\
\frac{h'_k(s)}{\Delta \delta} = & -\frac{1}{J_{QK}}\left(G_{QK}(s) + K_{Q_{\delta},L_{\delta}}(s)\right)\left(G_{PQ}(s) + K_{P_{\delta},L_{\delta}}(s)\right) \\
\frac{h'_{\delta}(s)}{\Delta \delta} = & D_{\delta} s
\end{align*}
\]  

(30)

Consequently, (30) can be rearranged as:

\[
\Delta P_f' = h'_i(s)\Delta \delta + h'_k(s)\Delta \delta + h'_{\delta}(s)\Delta \delta
\]  

(31)

Fig. 12 shows the frequency responses of\( h_i(s)\), \( h_k(s)\), \( h_{\delta}(s)\), \( h_w(s)\) and vector diagrams, which reflects the impact of virtual inductance on dynamic behavior of the grid-forming VSG system. Vectors are characterized by information of phase and magnitude at CFP. The synthetic power is located in the first quadrant, which illustrates the proposed virtual inductance control provides damping support for the system. Besides, negative damping power caused by strong power coupling effect can be offset by the effect of virtual inductance.

Fig. 13 shows the results of bodies as well as vector diagrams with varying size of virtual inductance (\( L_{v1}=4.4 \) mH, \( L_{v2}=8.8 \) mH, \( L_{v3}=13.2 \) mH, \( L_{v4}=17.6 \) mH). It shows that the greater virtual inductance results in stronger damping effect and negative feedback effect that can stabilize system. The effectiveness of the proposed virtual inductance control is reflected. Thereby the frequency stability is enhanced by the virtual inductance control.

IV. SIMULATION AND EXPERIMENT RESULTS

To verify the effectiveness of the proposed virtual inductance control method, Scenarios in different value of virtual inductance are built. Simulation results of frequency characteristic are displayed in Fig. 14. The attenuation speeds of the frequency with virtual inductance being 8.8 mH are much faster than that of virtual inductance being 4.4 mH. It inferred that the larger size of virtual inductance can make the
oscillation attenuated much faster and provide stronger damping for the system.

Specifically, the stability as well as dynamic performance of the grid-forming VSG (GFVSG) in this study is not only related with the short circuit ratio (SCR), but also concerned with the ratio of $R_g/X_g$. As can be seen in Fig. 15(a) and Fig. 15(c), higher level of $R_g/X_g$ leads to poorer stability of the system with weaker damping performance. The oscillation in Fig. 15(a) is divergent, but oscillation in Fig. 15(c) is convergent due to the different $R_g/X_g$, but the dynamic characteristics are all improved with the proposed virtual inductance control in Fig. 15(b) and (d). Generally, in all of the case studies, the proposed virtual inductance can take good effect in suppressing oscillation with larger inertia and stronger damping.

Fig. 15. Current response with and without proposed control. (a) Without proposed control ($R_g/X_g=0.11$), (b) With proposed control ($R_g/X_g=0.43$), (c) Without proposed control ($R_g/X_g=0.29$), (d) With proposed control ($R_g/X_g=0.29$).

In this section, the experiments are developed to demonstrate the proposed analysis framework as well as the proposed virtual inductance control strategy. In Fig. 16, StarSim experimental setups include: oscilloscope, host PC, Rapid control prototype (RCP), IO board, and Hardware-in-the-loop (HIL) model. The RCP upload the controller built in the matlab Simulink model. And the HIL is connected to the hardware of the Simulink model. The HIL-RCP based StarSim can validate the proposed control algorithm rapidly in real time. The parameters of grid impedance and virtual inductance in those cases are given in Table II.

**Table II Parameters of Grid Impedance and Virtual Inductance**

<table>
<thead>
<tr>
<th>Condition</th>
<th>Parameters $R_g/\Omega$</th>
<th>$L_v$/mH</th>
<th>$L_v$/mH</th>
<th>$R_g/\Omega$</th>
<th>$L_v$/mH</th>
<th>$L_v$/mH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before</td>
<td>0.3</td>
<td>8.8</td>
<td>0</td>
<td>1.1</td>
<td>8.8</td>
<td>4.4</td>
</tr>
<tr>
<td>After</td>
<td>0.3</td>
<td>8.8</td>
<td>0</td>
<td>1.1</td>
<td>8.8</td>
<td>8.8</td>
</tr>
<tr>
<td>Case I</td>
<td>0.3</td>
<td>8.8</td>
<td>0</td>
<td>1.1</td>
<td>8.8</td>
<td>4.4</td>
</tr>
<tr>
<td>Case II</td>
<td>0.3</td>
<td>8.8</td>
<td>0</td>
<td>1.1</td>
<td>8.8</td>
<td>8.8</td>
</tr>
<tr>
<td>Case III</td>
<td>0.3</td>
<td>8.8</td>
<td>0</td>
<td>1.2</td>
<td>8.8</td>
<td>4.4</td>
</tr>
</tbody>
</table>
control strategy was proposed to dampen oscillation of grid-forming VSG by increasing positive damping power, and the effectiveness is validated by the FEF analysis. In this framework, it does not rely on dual-loop control structure, which is different from the conventional virtual inductance control. What’s more, it is implemented based on energy conservation, and it can be adaptive to the power converter with another types of filters, like LC filters, and LCL filters. Besides, the LC filter is as similar as that of LCL filter since that the grid-side L filter of LCL filter can be integrated into the grid impedance as a whole. All of the analytical results of established models agree with the results of eigenvalue analysis. Finally, both proposed analytical model and virtual inductance control are verified experimentally.

**APPENDIX**

Combining (4.a) and (5.a) yields

\[ P_{\text{ref}} - f(\delta, E) = D_p (\omega_e - \omega_h) + J_p \omega_h (\omega_e - \omega_h) s \]  

(A1)

Linearizing (4.c), (A1) and combining (6), then (A2) can be obtained

\[ \Delta P_{\text{ref}} - (G_{pq} (s))^{-1} \Delta \omega_e + G_{pe} (s) \Delta E \]

\[ = D_p (\omega_e - \omega_h) + J_p \omega_h s (\Delta \omega_e - \Delta \omega_h) \]  

(A2)

According to (A2), the relationship from \( \Delta E \) to \( \Delta \omega_e \) can be derived as

\[ M_{\text{ide}} (s) = \frac{\Delta \omega_e}{\Delta E} = \frac{-G_{pe} (s) / (J_p \omega_h s + D_p)}{1 + G_{pq} (s) / (J_p \omega_h s^2 + D_p s)} \]  

(A3)

Similarly, combining (4.b) and (5.b) yields

\[ Q_{\text{ref}} - g(\delta, E) = J_q (E - U_{\text{ref}}) s \]  

(A4)

Linearizing (4.c), (A4) and combining (6), then (A5) can be obtained

\[ \Delta Q_{\text{ref}} - (G_{qq} (s))^{-1} \Delta \omega_e + G_{qe} (s) \Delta E = J_q (\Delta E - \Delta U_{\text{ref}}) s \]  

(A5)

According to (A5), the relationship from \( \Delta \omega_e \) to \( \Delta E \) can be derived as

\[ N_{\text{ide}} (s) = \frac{\Delta E}{\Delta \omega_e} = \frac{-G_{qe} (s) / J_q s}{1 + G_{qg} (s) / J_q s} \]  

(A6)

**REFERENCES**


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