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Published in:
IEEE Systems Journal

Link to article, DOI:
10.1109/JSYST.2022.3154811

Publication date:
2022

Document Version
Peer reviewed version

Link back to DTU Orbit

Citation (APA):
Fully Distributed Second-Order Cone Programming Model for Expansion in Transmission and Distribution Networks

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Abstract—As the penetration level of distributed energy resources (DERs) is increasing at distribution level, the stakeholders at the distribution level are no longer pure consumers and they play a more active role. DERs can be used for transmission needs, and the technical impacts of them may propagate to the transmission level. To unlock the full benefits of the transmission-distribution coordination, this article proposes a second-order cone programming (SOCP) based expansion model for coordinated transmission and distribution networks by decomposition and parallelization. First, we formulate an SOCP-based subnetwork expansion problem related to voltage phase angle variables which is valid for both transmission and distribution networks. Then, a fully distributed decision-making framework is presented to solve the coordinated transmission-distribution expansion problem iteratively. Taking the total power output of each subnetwork as the interacting variable, we deploy the operation optimization of substations as the master problem and the expansion optimization of subnetworks as the subproblems in the modified iteration algorithm. The feasibility and optimality of the iteration are proved and numerically verified. Simulation results indicate the advantages of the coordinated transmission-distribution expansion which can prevent line congestion, decrease boundary power mismatches and accommodate more DERs.

Index Terms—Distributed optimization, distribution, expansion, modified benders decomposition (BD), second-order cone programming (SOCP), transmission.

Sets and Indices

- \( I \): Set of buses
- \( I_q \): Set of buses in subnetwork \( q \)
- \( I_{ss} \): Set of buses in substation \( ss \)
- \( J \): Set of iterations
- \( L \): Set of lines
- \( L_q \): Set of lines in subnetwork \( q \)
- \( L_{ss} \): Set of lines in substation \( ss \)
- \( N \): Set of subnetworks
- \( S \): Set of scenarios
- \( o(ij), r(ij) \): Indices of the origin and receiving buses of line \( ij \)

Parameters

- \( C_{ij} \): Annual investment cost of line \( ij \)
- \( C^G \): Operation cost of generations at bus \( i \)
- \( C^{\text{loss}} \): Network loss cost of line \( ij \)
- \( C^P_+ \), \( C^P_- \): Active power demand variation penalty cost parameters ($/MWh$)
- \( C^Q_+ \), \( C^Q_- \): Reactive power demand variation penalty cost parameters ($$/MVarh$$)
- \( G_i, B_i \): Shunt conductance and susceptance at bus \( i \)
- \( P^L_{i,s}, Q^L_{i,s} \): Active (MW) and reactive (MVar) power demand decrements at bus \( i \) in scenario \( s \)
- \( R_{ij}, X_{ij} \): Upper and lower bounds of variable \( Y \)
- \( \sigma \): Initial turn ratio of each transformer
- \( k \): Turn ratio increment per step of each transformer

Variables

- \( a_{ij,1,d,s}, a_{ij,2,d,s} \): Continuous variables to express \( V_{j,s} \) in the \( d \)th equation

NOMENCLATURE

- Annual investment cost of line \( ij \)
- Operation cost of generations at bus \( i \)
- Network loss cost of line \( ij \)
- Active power demand variation penalty cost parameters ($/MWh$)
- Reactive power demand variation penalty cost parameters ($$/MVarh$$)
- Shunt conductance and susceptance at bus \( i \)
- Active (MW) and reactive (MVAR) power demands at bus \( i \) in Scenario \( s \)
- Active (MW) and reactive (MVar) power demand increments at bus \( i \) in scenario \( s \)
- Active (MW) and reactive (MVar) power demand decrements at bus \( i \) in scenario \( s \)
- Resistance and reactance of line \( ij \)
- Upper and lower bounds of variable \( Y \)
- Initial turn ratio of each transformer
- Turn ratio increment per step of each transformer
- Total active (MW) and reactive (MVar) power outputs of generations solved from master problem in subnetwork \( q \), scenario \( s \) and iteration \( k \)
- Sufficiently large positive number

Manuscript received August 2, 2021; revised October 24, 2021 and December 18, 2021; accepted February 23, 2022. This work was supported in part by the National Key R&D Program of China under Grant 2018YFE0208400 and in part by the Zhejiang Provincial Natural Science Foundation of China under Grant LQ21E070003 and Grant LQ22E070001. (Corresponding author: Zao Tang.)

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The roles of transmission system operator (TSO) and distribution system operator (DSO) have some similarities in implementing the planning, operation or control tasks of their systems [1], [2]. However, the traditional design of power systems indicates that the transmission network hosts the large-scale generations while the distribution network hosts the numerous consumers [3]. This outdated image is experiencing dramatic changes due to growing power network expansions and accelerating proliferations of distributed energy resources (DERs) which may impose new challenges, e.g., line congestion [4] and voltage violation [5]. If not dealt with reasonably, these impacts can propagate to the transmission network and potentially cause cascading technical problems. One approach to handle these challenges is to exploit the benefits of interactions between transmission and distribution networks in a parallel manner [6]. The advantage of facilitating TSO-DSO interactions in a distributed way is that the number of constraints and variables of the transmission-distribution coordination is reduced. It is then available for the solver to handle large-scale co-optimization problems [7]. Another advantage of the distributed optimization is that, by taking the substation between transmission and distribution networks as the boundaries, the data privacy of each subnetwork can be protected and all the subnetworks’ information required by the substations is communicated at the boundary buses [8].

To unlock both the transmission and distribution networks benefit from the coordination, early researches about the power flow calculation [9], optimal power flow (OPF) [10], risk evaluation [11], reactive power optimization [5], and network expansion [2] are studied. The models in these literatures unanimously use the distributed approaches to solve their problems, including the Lagrangian relaxation family and the optimality condition decomposition family. To be specific, the Lagrangian relaxation family primarily comprises auxiliary problem principle [2], alternating direction method of multipliers [12] and analytical target cascading [13] while the optimality condition decomposition family generally comprises Benders decomposition (BD) [5], [14] and heterogeneous decomposition [15]. The Lagrangian relaxation family requires to first relax the consistency constraints at the boundary buses, and then iterate until the convergence condition satisfies [2]. In general, the drawbacks of Lagrangian relaxation are the low convergence rate and an approximated solution guarantee [16]. Benders and heterogeneous decomposition formulate the master problem and subproblem in a parallel manner, and require no iterative parameter tuning procedure [17]. The Benders version generates the dual cuts from the subproblems to correspond with the master problem, whereas the heterogeneous version forms the extended Karush–Kuhn–Tucker-based Hessian matrixes for the master problem and subproblems [10]. The advantage of BD is that the optimality and convergence property of the master problem and subproblems can be guaranteed by the convexity of the formulated problems. Thus, a modified BD algorithm is proposed to coordinate the transmission and distribution networks in this article.

The DistFlow equations have been used to exactly capture ac power flows in radial distribution networks [18], and replaced with second-order cone programming (SOCP) based relaxed model for distribution network expansion [19] and multi-area distribution network operation [20]–[22]. Zheng et al. [20] solves the fully distributed SOCP-based OPF by alternating direction method of multipliers for multi-area distribution networks. The results indicate that the distributed SOCP-based OPF preserves convexity to guarantee the optimality and convergence for radial distribution networks. Xu et al. [21] present a parallel primal-dual interior-point algorithm with second-order convergence to handle the distributed OPF for radial distribution networks. This peer-to-peer communication-based method can achieve a global optimum as it employs a linearized ac DistFlow model. Xu et al. [22] further investigates the same problem by a distributed quasi-Newton algorithm which can be extended to the SOCP-based ac OPF problems. However, the SOCP framework [23] and [24] is only valid for radial distribution networks and cannot directly solve the ac OPF problem for mesh transmission networks. This is because the model does not comprise constraints associated with voltage phase angle variables. To address this problem, [25] proposes a distributed SOCP-based ac OPF model for multiarea transmission networks. Thus, both the transmission and distribution ac OPF can be unanimously relaxed to the SOCP framework.

Some studies show that the transmission-distribution coordination can improve utilization of the generation resources in both networks and reduce their operation costs [26]–[28]. A hierarchy for reactive power optimization with TSO-DSO interactions is proposed in [26], in which curve fitting is employed to provide a cost function of the subproblem for the master problem. Hassan and Dvorkin [27] exploit the coordination between transmission and distribution networks that optimizes investments in energy storage for providing grid support in both networks. To address the drawbacks of the isolated transmission contingency analysis, a global power flow-based transmission contingency analysis method is presented [28], which also coordinates the interactions of TSO-DSO. Considering the line congestion regulation of the transmission-distribution coordination, the line expansion in
both the transmission and distribution networks should combine with it to unlock the potential technical and economic benefits.

However, most of studies have just focused on the isolated expansion in either transmission [29]–[31] or distribution network [32], [33]. Unlike [29], [30], [31] solves a distributed expansion model for multi-area transmission networks using auxiliary problem principle. As an improvement, this approach formulates specific power flows in existing and alternative tie-lines. The expansion models in [29]–[31] use a linearized dc power flow approximation to model the transmission network and neglect distribution network constraints. The expansion models in [32] and [33] unanimously assume that the upper transmission network is simplified as a voltage source at the boundary and ignore the transmission network constraints. Thus, the models in [29]–[33] could miscalculate the solutions of both the transmission and distribution expansion since additional flexibility cannot be harvested through the coordinated TSO-DSO interactions. In our previous work [2], a distributed stochastic expansion method is developed for integrated transmission and distribution networks, which is in fact a convex relaxation problem, guaranteeing the convergence of analytical target cascading. This decision-making framework is very flexible but suffers from computation complexity. Thus, coordinated transmission and distribution expansion should be further studied to exploit its full advantages.

This article presents a fully distributed SOCP-based decision-making framework to optimize line investments in both the transmission and distribution networks, while modeling the coordinated TSO-DSO interactions. The use of the SOCP relaxation is motivated by two factors. First, it is well-suited to preserve convexity to make the iteration process converge theoretically. Second, the SOCP framework is valid for both the transmission and distribution networks since it includes constraints related to voltage phase angle variables. Taking the total power output of each subnetwork as the interacting variable, we decompose the expansion problem into a master problem (substation operation problem) and several subproblems (subnetwork expansion problems). The modified BD algorithm is proposed to solve the distributed SOCP-based expansion problem in parallel. The main contributions of this article are summarized as follows.

1) A fully distributed SOCP-based optimization model is developed in order to make investment decisions in both the transmission and distribution lines. The formulated master problem and subproblems are decomposed by a modified BD algorithm, which provides a parallel computing accelerated framework to tackle the complexity of coordinated transmission and distribution expansion.

2) The feasibility and optimality of the modified BD algorithm are proved and numerically verified. The power increments and decrements for all the buses in the subnetworks are allowed to guarantee the feasibility of all the subproblems in the modified BD algorithm. The computation efficiency (reducing computation time and computer RAM requirement) can be improved by the parallel-based BD iteration.

A schematic diagram of the proposed methodology is depicted in Fig. 1, in order to better illustrate the model. The rest of this article is organized as follows. Section II describes the SOCP-based expansion model. Section III achieves the fully distributed decision-making solution based on a modified BD algorithm. The formulations of the distributed problems are detailed. The feasibility and optimality of the formulated master problem and subproblems in the modified iteration algorithm are analytically proved. The parallel computing framework is also proposed to accelerate the convergence in this section. Section IV discusses numerical results on three test cases, and compares with the centralized and isolated approaches to demonstrate the performance of the distributed method. Finally, Section V concludes this article.

II. SOCP-BASED EXPANSION MODEL

The existing nonconvex expansion model is relaxed to the following SOCP-based model in (1)–(24), which integrates constraints associated with voltage phase angle. The convexity, accuracy and applicability of this model can be guaranteed using the big-M method and the transformation [25]. The expansion objective of the TSO or DSO is to minimize the total cost of investment (i.e., line construction) and operation (i.e., generation operation and network loss), which can be any convex function of the decision variables

$$\min_{\Omega} \sum_{s \in S} f_{inv} + \sum_{s \in S} f_{oppe}$$

subject to

$$f_{inv} = \sum_{ij \in L} C_{ij} u_{ij}, f_{oppe} = \sum_{i \in I} C_{i} P_{i,s}^{G} + \sum_{ij \in L} c_{ij} P_{ij,s}^{loss}$$

(2)

$$P_{i,s}^{G} - P_{i,s}^{L} = \sum_{ij \in \partial (i)} P_{ij,s} - \sum_{ij \in \partial (i)} (P_{ij,s} - P_{ij,s}^{loss})$$

$$+ G_{i} V_{i,s}, \forall i \in I, s \in S$$

(3)

$$Q_{i,s}^{G} - Q_{i,s}^{L} = \sum_{ij \in \partial (i)} Q_{ij,s} - \sum_{ij \in \partial (i)} (Q_{ij,s} - Q_{ij,s}^{loss})$$

$$- B_{i} V_{i,s}, \forall i \in I, s \in S$$

(4)

$$u_{ij} P_{ij,s}^{loss, max} \geq P_{ij,s}^{loss} \geq \frac{P_{ij,s}^{2} + Q_{ij,s}^{2}}{V_{i,s}^{2}} R_{ij} \forall i \in L, s \in S$$

(5)
\[ P_{ij,s} X_{ij} = Q_{ij,s}^\text{loss} + R_{ij} \forall i,j \in L, s \in S \]  
\[ V_{i,s} - V_{j,s} \leq 2R_{ij} P_{ij,s} + 2X_{ij} Q_{ij,s} - R_{ij} P_{ij,s}^\text{loss} - X_{ij} Q_{ij,s}^\text{loss} \]  
\[ + (1 - u_{ij}) \forall i,j \in L, s \in S \]  
\[ V_{i,s} - V_{j,s} \geq 2R_{ij} P_{ij,s} + 2X_{ij} Q_{ij,s} - R_{ij} P_{ij,s}^\text{loss} - X_{ij} Q_{ij,s}^\text{loss} \]  
\[ - (1 - u_{ij}) \forall i,j \in L, s \in S \]  
\[ Z_{ij,s}^H = X_{ij} P_{ij,s} - R_{ij} Q_{ij,s} \forall i,j \in L, s \in S \]  
\[ Z_{ij,s}^H \geq v_{ij,s} \sin \theta_{ij,s} + Z_{ij}^\text{pa,min} v_{m,s} - v_{ij,s} \sin \theta_{ij,s} \forall i,j \in L, s \in S \]  
\[ Z_{ij,s}^H \geq v_{ij,s} \cos \theta_{ij,s} + Z_{ij}^\text{pa,max} v_{m,s} - v_{ij,s} \cos \theta_{ij,s} \forall i,j \in L, s \in S \]  
\[ \forall i,j \in L, s \in S \]  
\[ Z_{ij,s}^\text{pa,min} \geq \cos \left( \frac{\theta_{ij,s}}{2} \right) \left( \theta_{ij,s} + \frac{\beta_{ij,s}}{2} \right) - \sin \left( \frac{\beta_{ij,s}}{2} \right) \]  
\[ \forall i,j \in L, s \in S \]  
\[ Z_{ij,s}^\text{pa,max} \leq \cos \left( \frac{\theta_{ij,s}}{2} \right) \left( \theta_{ij,s} - \frac{\beta_{ij,s}}{2} \right) + \sin \left( \frac{\beta_{ij,s}}{2} \right) \]  
\[ \forall i,j \in L, s \in S \]  
\[ V_{i,s} \geq v_{i,s} \forall i \in I, s \in S \]  
\[ V_{i,s} \leq (v_{i,s} + v_{i,s}^\text{min}) u_{i,s} - v_{i,s}^\text{min} \forall i \in I, s \in S \]  
\[ V_{i,s}^\text{min} \leq V_{i,s} \leq V_{i,s}^\text{max} \forall i \in I, s \in S \]  
\[ P_{i,s}^G \geq P_{i,s}^\text{max} \forall i \in I, s \in S \]  
\[ Q_{i,s}^G \geq -Q_{i,s}^\text{max} \forall i \in I, s \in S \]  
\[ \text{Cost}_{SP} = \min \sum_{i \in L_q, \forall j \in L_q} \left( f^{\text{inv}} + \sum_{s \in S} f^{\text{ope}} \right) \]  
\[ \text{subject to (2)–(24)} \forall i \in L_q, ij \in L_q, s \in S \]  
\[ \sum_{i \in L_q} P_{i,s} = P_{\text{sum}}(\mu_{q,s,k}) \forall q \in N, s, k \in J \]
\[
\sum_{i \in I_q} Q_{i,s}^G = \sum_{q,s,k} Q_{q,s,k}^\text{sum} : (\mu^Q_{q,s,k}) \forall q \in N, s \in S, k \in J
\] (28)

where (26) represents the power flow constraints for all buses and lines in subnetwork \( q \). Constraints (27) and (28) further enforce the active and reactive power outputs of generations in subnetwork \( q \).

To guarantee the feasibility of all the subproblems in the iterative algorithm, uncertainty related to load variation is considered, and load increments or decrements at all the buses in each subnetwork are formulated in this article. Thus, the power balance constraints (3) and (4) in (26) are rearranged

\[
P_{i,s}^G - P_{i,s}^L = \sum_{ij \in L_q, \sigma(ij) = i} P_{j,s} - \sum_{ij \in L_q, \rho(ij) = i} (P_{ij,s} - P_{ij,s}^{\text{loss}})
+ G_i V_{i,s} + \Delta P_{i,s}^L - \Delta P_{i,s}^L, \forall i \in I_q,
\]

\[
Q_{i,s}^G - Q_{i,s}^L = \sum_{ij \in L_q, \sigma(ij) = i} Q_{j,s} - \sum_{ij \in L_q, \rho(ij) = i} (Q_{ij,s} - Q_{ij,s}^{\text{loss}})
- B_i V_{i,s} + \Delta Q_{i,s}^L - \Delta Q_{i,s}^L, \forall i \in I_q, i,j \in L_q, s \in S.
\] (29)

The load increments and decrements in each scenario are penalized in the objective using the penalty cost parameters \( C_i^P, C_i^Q, C_i^{\text{MP}}, \) and \( C_i^{\text{DMP}} \). To guarantee the voltage variable consistencies between subnetworks and substations, a quadratic penalty term with \( C_i^v \) and \( C_i^g \) for the voltage variables is also regarded in the objective of the subproblem. Note that the substation voltage variables are solved from the formulated master problem (32)–(34) of the iterative algorithm. Then, the objective of subproblem \( q \) in (31) is to minimize the quadratic total cost over a convex feasible region, which is a convex optimization problem

\[
\text{Cost}^{SP}_{q,k} = \min_{V_{i,s} \in V_{i,s}^\text{min} \leq V_{i,s} \leq V_{i,s}^\text{max}} \left( f_{\text{inv}}^{\text{sum}} + \sum_{s \in S} f_{\text{ope}}^{\text{sum}} \right) + \sum_{i \in I_q, s \in S} \left( C_i^P \Delta P_{i,s}^L + C_i^Q \Delta Q_{i,s}^L + C_i^{\text{MP}} \left( V_{i,s} - V_{i,s}^{\text{MP}} \right)^2 + C_i^{\text{DMP}} \left( \theta_{i,s}^\text{ref} - \theta_{i,s}^{\text{DMP}} \right)^2 \right).
\] (31)

The master problem of the modified iteration algorithm is formulated as

\[
\min \text{Cost}^{\text{MP}} = \sum_{q \in N} \text{Cost}^{SP}_q
\] (32)

subject to
\[
(2) - (24) \forall i,j \in L_{a}, s \in S
\] (33)

\[
\text{Cost}_{q,k} \geq \text{Cost}_{q,k-1} + \sum_{s \in S} \left( \mu^P_{q,s,k} \left( P_{q,s,k}^\text{sum} - P_{q,s,k-1}^\text{sum} \right) + \mu^Q_{q,s,k} \left( Q_{q,s,k}^\text{sum} - Q_{q,s,k-1}^\text{sum} \right) \right)
\forall q \in N, s \in S, k \in J
\] (34)

where (33) represents the power flow constraints for all buses and lines in substation \( s \). Equation (34) relates the expanding Benders cuts generated from the subproblems in (25)–(28). Here, \( \mu^P_{q,s,k-1} \) and \( \mu^Q_{q,s,k-1} \) are the dual variables of (27) and (28) in subnetwork \( q \), scenario \( s \), and iteration \( k - 1 \). \( P_{q,s,k-1}^\text{sum} \) and \( Q_{q,s,k-1}^\text{sum} \) are the decisions of the previous iteration \( k - 1 \) which are treated as parameters in the current iteration. The decisions of \( P_{q,s,k}^\text{sum} \) and \( Q_{q,s,k}^\text{sum} \) are made in the master problem (32)–(34) by incorporating the expanding Benders cuts (34) and substation constraints (33). Thus, the SOC\( \text{P}\)-based operation of each substation is taken as the master problem in the modified iteration algorithm.

To formulate the transformers in each substation as controllable devices in this article, the nonlinear model of transformer is precisely considered in the master problem and linearized using piecewise linear approximation. Thus, the branch flow (7) and (8) in (33) are replaced by

\[
V_{i,s} - V_{j,t,s} \leq 2R_{ij}P_{ij,s} + 2X_{ij}Q_{ij,s} - R_{ij}P_{ij,s}^{\text{loss}} - X_{ij}Q_{ij,s}^{\text{loss}} + M(1 - u_{ij,s}), \forall i,j \in L_{a}, s \in S
\] (35)

\[
V_{i,s} - V_{j,t,s} \geq 2R_{ij}P_{ij,s} + 2X_{ij}Q_{ij,s} - R_{ij}P_{ij,s}^{\text{loss}} - X_{ij}Q_{ij,s}^{\text{loss}} - M(1 - u_{ij,s}), \forall i,j \in L_{a}, s \in S
\] (36)

\[
V_{j,t,s} = \sigma^2 \sum_{d=1}^\infty \left( a_{ij,1,d,s}V_{j,min} + a_{ij,2,d,s}V_{j,max} \right) \left( k_{ij,d,s}^{\text{trans}} \right)^2
+ 2k_0^5 \sum_{d=1}^\infty \left( a_{ij,1,d,s}V_{j,min} + a_{ij,2,d,s}V_{j,max} \right) \left( k_{ij,d,s}^{\text{trans}} \right)^2
\times k_{ij,d,s}^{\text{trans}} + \left( k_0^{\text{trans}} \right)^2 V_{j,s} \forall i,j \in L_{a}, s \in S
\] (37)

\[
K_{ij,s,t} = \{ k_{ij,1,t}, k_{ij,2,t}, \ldots, k_{ij,T}^{\text{trans}} \} \forall i,j \in L_{a}, s \in S
\] (38)

where (35) and (36) are the branch flow equations for lines with transformers. Equations (37) and (38) represent the linearized transformer constraints using the piecewise linearization technique [35].

B. Modified Iteration Procedure

A fully distributed decision-making structure which consists of the master problem and subproblems in the modified iteration algorithm is illustrated in Fig. 2. The master problem is to
The TSO and each DSO individually formulate and solve their own expansion problem according to their own system. Thus, the model proposed is also able to independently perform valid functions in their own right and continue to work to fulfill those purposes when are separated from the overall system. The TSO and each DSO individually formulate and solve their own expansion problem according to their own available generation sources, network topologies, and forecasted loads.

Algorithm 1: Fully Distributed Optimization Algorithm

```
Initialize k=1;
do
    Formulate the SOCP based operation problem (32)-(38) for the substations:
    SOCP based Master Problem;
    Assign SOCP based Master Problem to Thread m;
    Execute Thread m;
    Send $\sum_{q,s,k} C^M_{q,s,k}$ to Thread q for $q < N$;
    $q=1$;
do
    Formulate the SOCP based expansion problem (25)-(31) for the subnetwork $q$: SOCP based Subproblem $q$;
    Assign SOCP based Subproblem $q$ to Thread q;
    while $q<qq^*$;
do
    If Thread $q$ is ready then
        Collect solutions from Thread $q$;
        Send $\sum_{q,s,k} C^M_{q,s,k}$ to Thread $m$;
        Release Thread $q$;
    while the SOCP based thread is nonempty;
    Release Thread $m$;
```

C. Feasibility and Optimality of the Iteration

Theorem 1: Since the isolated SOCP-based expansion model in Section II is feasible, the feasibility of the formulated master problem (32)-(34) in the modified iteration algorithm can be guaranteed.

Proof: Theorem 1 is analytically proved using the mathematical induction technique. We first prove that the solution for the formulated master problem is feasible in the first iteration. Then, we prove the formulated master problem is feasible for the next iteration with the feasibility assumption about the master problem in any iteration.

Step 1: For iteration $k=1$, assume that $\Omega_0 = \{u_{ij,0}, P^G_{i,s,0}, Q^G_{i,s,0}, P_{ij,s,0}, Q_{ij,s,0}, I_{ij,s,0}, P_{loss}^{ij}, Q_{loss}^{ij}, V_{i,s,0}, B_{ij,s,0}\}$ is a feasible solution for the isolated SOCP-based expansion model, and it is also feasible for constraints (33) in the formulated master problem. It is worthy to mention that $\Omega_0$ is not necessarily required to be the optimal solution for the isolated SOCP-based expansion model.

The feasible solution $P^\text{sum}_{q,s,k,0}$, $Q^\text{sum}_{q,s,k,0}$, $P^\text{sum}_{q,s,k-1}$, and $Q^\text{sum}_{q,s,k-1,0}$ for the master problem is constructed as

\[
F^\text{sum}_{q,s,k,0} = \sum_{i \in I_q} P^G_{i,s,0}
\]

\[
Q^\text{sum}_{q,s,k,0} = \sum_{i \in I_q} Q^G_{i,s,0}
\]

\[
P^\text{sum}_{q,s,k-1} = \sum_{i \in I_q} P^M_{i,s,0}
\]

\[
Q^\text{sum}_{q,s,k-1} = \sum_{i \in I_q} Q^M_{i,s,0}
\]

Afterwards, constraint (34) is rewritten as the following inequality constraint using $\Omega_0$, $P^\text{sum}_{q,s,k,0}$, $Q^\text{sum}_{q,s,k,0}$, $P^\text{sum}_{q,s,k-1}$, and $Q^\text{sum}_{q,s,k-1,0}$

\[
\text{Cost}_{q,k}^P \geq \text{Cost}_{q,k-1}^P + \sum_{s \in S} \left[ \mu^P_{q,s,k-1} (P^\text{sum}_{q,s,k,0} - P^\text{sum}_{q,s,k-1,0}) + \mu^Q_{q,s,k-1} (Q^\text{sum}_{q,s,k,0} - Q^\text{sum}_{q,s,k-1,0}) \right]
\]

\[
= \text{Cost}_{q,k-1}^P, \forall q \in N, s \in S.
\]

Constraint (43) holds regardless of the decisions of $\mu^P_{q,s,k-1}$ and $\mu^Q_{q,s,k-1}$, and then the feasible objective of the master problem becomes

\[
\text{Cost}_{\text{MP},0} \geq \min_{q \in N} \text{Cost}_{q,k}^P = \sum_{q \in N} \text{Cost}_{q,k}^P
\]

Thus, a feasible solution for the master problem as $\Omega_0$, $P^\text{sum}_{q,s,k,0}$, $Q^\text{sum}_{q,s,k,0}$ with objective Cost_{\text{MP},0} is achieved. This indicates that the feasibility of the master problem can be guaranteed in the first iteration.

Step 2: For iteration $k = k' + 1$, suppose that $\Omega_{k'}$, $P^\text{sum}_{q,s,k',0}$, and $Q^\text{sum}_{q,s,k',0}$ is a feasible solution with master problem and subproblem objective solutions as Cost_{\text{MP},k'0}, Cost_{q,k'}. The feasible solution for iteration $k = k' + 1$ is constructed as

\[
P^\text{sum}_{q,s,k'+1,0} = P^\text{sum}_{q,s,k',0}
\]

\[
Q^\text{sum}_{q,s,k'+1,0} = Q^\text{sum}_{q,s,k',0}
\]
Then, the generated Benders cuts in iteration \( k = k' + 1 \) are
\[
\text{Cost}_{q,k}^{SP} \geq \text{Cost}_{q,k'}^{SP} \quad + \sum_{s \in S} \left[ \begin{array}{l}
\mu_{q,s,k'} \left( P_{q,s,k'}^{\text{sum}} - P_{q,s,k'}^{\text{sum},0} \right) \\
+ \mu_{q,s,k'}^{Q} \left( Q_{q,s,k'}^{\text{sum}} - Q_{q,s,k'}^{\text{sum},0} \right)
\end{array} \right] = \text{Cost}_{q,k'}^{SP} \quad \forall q \in N, s \in S. \quad (47)
\]

To guarantee the feasibility of the master problem (32)–(34), one feasible objective for iteration \( k = k' + 1 \) can be constructed as follows. If
\[
\text{Cost}_{q,k'}^{SP} \leq \text{Cost}_{q,k',0}^{SP} \quad (48)
\]
We replace these \( \text{Cost}_{q,k'}^{SP} \) by \( \text{Cost}_{q,k',0}^{SP} \). 

The feasible solution of Cost \( q \) is \( \{ \text{Cost}_{q,k'}^{SP} \forall q \in N' \in N \} \), and the feasible objective is
\[
\text{Cost}_{q,k',0}^{\text{MP}} = \sum_{q \in N' \in N} \text{Cost}_{q,k'}^{SP} \quad (49)
\]
Thus, the feasible solution for iteration \( k = k' + 1 \) is constructed by using the feasible solution for iteration \( k = k' \). In the presence of Steps 1 and 2, if the isolated SOCP-based expansion model is feasible, the feasibility of the master problem is proved.

**Theorem 2:** Since the isolated SOCP-based expansion model in Section II is feasible, the necessary and sufficient condition for the feasibility of the formulated subproblem (25)–(28) in the modified iteration algorithm is
\[
\sum_{i \in I_q} P_{q,i,s,k}^{G,\text{min}} \leq P_{q,i,s,k}^{\text{sum}} \quad \forall q \in N, s \in S, k \in J \quad (50)
\]
\[
\sum_{i \in I_q} Q_{q,i,s,k}^{G,\text{min}} \leq Q_{q,i,s,k}^{\text{sum}} \quad \forall q \in N, s \in S, k \in J. \quad (51)
\]

**Proof:** We first prove that (50)–(51) is necessary for the feasibility of subproblem (25)–(28). Assume that \( \Omega_k = \{ u_{i,j}, P_{i,s,k}^{G,0}, Q_{i,s,k}^{G,0}, P_{ij,s,k}^{\text{loss}}, Q_{ij,s,k}^{\text{loss}}, V_{i,s,k}, \theta_{ij,s,k} \} \) is a feasible solution of subproblem (25)–(28) in iteration \( k \) and thus
\[
\sum_{i \in I_q} P_{q,i,s,k}^{G,\text{min}} \leq P_{q,i,s,k}^{\text{sum}} \forall q \in N, s \in S, k \in J \quad (52)
\]
\[
\sum_{i \in I_q} Q_{q,i,s,k}^{G,\text{min}} \leq Q_{q,i,s,k}^{\text{sum}} \forall q \in N, s \in S, k \in J. \quad (53)
\]

Because \( \Omega_k \) is also the feasible solution for constraints (33), we have
\[
P_{q,i,s,k}^{G,\text{min}} \leq P_{q,i,s,k}^{G,\text{max}} \forall i \in I_q, s \in S \quad (54)
\]
\[
Q_{q,i,s,k}^{G,\text{min}} \leq Q_{q,i,s,k}^{G,\text{max}} \forall i \in I_q, s \in S. \quad (55)
\]

Obviously, (50), (51) hold combining (52), (53) and (54), (55).

Then, we prove conditions (50) and (51) are sufficient for the feasibility of subproblem (25)–(28). Suppose that a feasible solution for the isolated SOCP-based expansion model is \( \Omega_0 = \{ u_{i,j}, P_{i,s,k}^{G,0}, Q_{i,s,k}^{G,0}, P_{ij,s,k}^{\text{loss}}, Q_{ij,s,k}^{\text{loss}}, V_{i,s,k}, \theta_{ij,s,k} \} \) where \( \{ u_{i,j}, P_{i,s,k}^{G,0}, Q_{i,s,k}^{G,0}, P_{ij,s,k}^{\text{loss}}, Q_{ij,s,k}^{\text{loss}}, V_{i,s,k}, \theta_{ij,s,k} \} \) is feasible for constraints (5)–(24) in constraint (26). The rest constraints of the formulated subproblem are (29), (30) and (27), (28). To guarantee the feasibility of the formulated subproblem, the feasible solution \( P_{q,i,s,k}^{G} \) and \( Q_{q,i,s,k}^{G} \) in iteration \( k \) is constructed as
\[
P_{q,i,s,k}^{G} = P_{i,s,k}^{G,0} + \Delta P_{i,s,k}^{L+,0} \forall i \in I_q, s \in S \quad (56)
\]
\[
Q_{q,i,s,k}^{G} = Q_{i,s,k}^{G,0} + \Delta Q_{i,s,k}^{L+,0} \forall i \in I_q, s \in S. \quad (57)
\]
By substituting (56) and (57) in (29) and (30), we have
\[
P_{q,i,s,k}^{G} - P_{i,s,k}^{L-} = \sum_{ij \in L_q \cap (i,j) = i} P_{ij,s,k}^{\text{loss}} + \sum_{ij \in L_q \cap (i,j) = i} (P_{ij,s,k}^{\text{loss},0} + \nabla_{ij,s,k}^{\text{loss}}) \forall i \in I_q, s \in S \quad (58)
\]
\[
Q_{q,i,s,k}^{G} - Q_{i,s,k}^{L-} = \sum_{ij \in L_q \cap (i,j) = i} Q_{ij,s,k}^{\text{loss}} \forall i \in I_q, s \in S \quad (59)
\]
which are feasible due to the feasibility of \( \Omega_0 \). To further construct the feasible solutions of load increments or decrements, the feasibility of constraints (27) and (28) is
\[
\sum_{i \in I_q} P_{q,i,s,k}^{G} = \sum_{i \in I_q} (P_{i,s,k}^{G,0} + \Delta P_{i,s,k}^{L+,0} - \Delta P_{i,s,k}^{L-,0}) = P_{q,i,s,k}^{\text{sum}} \forall q \in N, s \in S, k \in J \quad (60)
\]
\[
\sum_{i \in I_q} Q_{q,i,s,k}^{G} = \sum_{i \in I_q} (Q_{i,s,k}^{G,0} + \Delta Q_{i,s,k}^{L+,0} - \Delta Q_{i,s,k}^{L-,0}) = Q_{q,i,s,k}^{\text{sum}} \forall q \in N, s \in S, k \in J. \quad (61)
\]
Since (50), (51) hold, \( P_{q,i,s,k}^{G,0} \) and \( Q_{q,i,s,k}^{G,0} \) can be expressed as the following equations using \( \lambda_{i,s,k} \) where \( 0 \leq \lambda_{i,s,k} \leq 1 \)
\[
P_{q,i,s,k}^{G,0} = \lambda_{q,i,s,k} P_{i,s,k}^{G,\text{max}} + (1 - \lambda_{q,i,s,k}) P_{i,s,k}^{G,\text{min}} \quad (62)
\]
\[
Q_{q,i,s,k}^{G,0} = \lambda_{q,i,s,k} Q_{i,s,k}^{G,\text{max}} + (1 - \lambda_{q,i,s,k}) Q_{i,s,k}^{G,\text{min}}. \quad (63)
\]

Similarly, \( P_{q,i,s,k}^{G} \) and \( Q_{q,i,s,k}^{G} \) can be arranged as the following equations using \( \xi_{i,s,k} \) where \( 0 \leq \xi_{i,s,k} \leq 1 \)
\[
P_{q,i,s,k}^{G} = \lambda_{q,i,s,k} P_{i,s,k}^{G,\text{max}} + (1 - \lambda_{q,i,s,k}) P_{i,s,k}^{G,\text{min}} \quad (64)
\]
\[
Q_{q,i,s,k}^{G} = \lambda_{q,i,s,k} Q_{i,s,k}^{G,\text{max}} + (1 - \lambda_{q,i,s,k}) Q_{i,s,k}^{G,\text{min}}. \quad (65)
\]
Thus, the feasible solution of \( \Delta P_{i,s,k}^{L+,0} + \Delta P_{i,s,k}^{L-,0} + \Delta Q_{i,s,k}^{L+,0} + \Delta Q_{i,s,k}^{L-,0} \) is constructed as
\[
\Delta P_{i,s,k}^{L+,0} = \left| P_{q,i,s,k}^{G} - P_{i,s,k}^{G,0} \right| \quad \forall i \in I_q \quad (66)
\]
\[
\Delta P_{i,s,k}^{L-,0} = 0 \quad \forall i \in I_q \quad (67)
\]
\[
\Delta Q_{i,s,k}^{L+,0} = \left| Q_{q,i,s,k}^{G} - Q_{i,s,k}^{G,0} \right| \quad \forall i \in I_q \quad (68)
\]
\[
\Delta Q_{i,s,k}^{L-,0} = 0 \quad \forall i \in I_q. \quad (69)
\]
It is worthy to mention that (66)–(69) can guarantee the feasibility of (27) and (28) and the feasibility of \( P_{i,s,k}^{G} \) in (56) and (57).

**Theorem 3:** Since the isolated SOCP-based expansion model in Section II is feasible and conditions (50) and (51) hold, the converged solution is exactly optimal for the formulated distributed problems in the modified iteration algorithm.

**Proof:** Geoffrion [36] has proved the convergence property of the modified iteration algorithm is guaranteed for the convex
determined problems. Assume $\Omega_*$ is the optimal solution of the formulated distributed problems, which is

$$\Omega_* = \left\{ u_{i,j,s}, P_{i,s,s}, Q_{i,s,s}, P_{i,j,s,s}, Q_{i,j,s,s}, P_{ij,s,s}, Q_{ij,s,s}, V_{i,s,s}, \theta_{ij,s,s} \right\}. \quad (70)$$

Since $\Omega_*$ is also feasible for the distributed problems, $\Omega_0$ supposed in the proof of Theorem 1 can be set as $\Omega_*$. Similar to the proof procedure in Theorem 1, we prove $\Omega_*$ is always the feasible solution for the master problem (32)–(34).

In iteration $k = 1$, the feasible solution $P_{\text{sum}}^{\Omega_0, q,k,s,*}, Q_{\text{sum}}^{\Omega_0, q,k,s,*}$, and $P_{\text{sum}}^{\Omega_0, q,k-1,s,*}, Q_{\text{sum}}^{\Omega_0, q,k-1,s,*}$ is constructed as

$$P_{\text{sum}}^{\Omega_0, q,k,s,*} = \sum_{i \in I_q} P_{i,s,s} \quad (71)$$

$$Q_{\text{sum}}^{\Omega_0, q,k,s,*} = \sum_{i \in I_q} Q_{i,s,s} \quad (72)$$

$$P_{\text{sum}}^{\Omega_0, q,k-1,s,*} = \sum_{i \in I_q} P_{i,s,s} \quad (73)$$

$$Q_{\text{sum}}^{\Omega_0, q,k-1,s,*} = \sum_{i \in I_q} Q_{i,s,s}. \quad (74)$$

According to the step 1 in the proof of Theorem 1, $\Omega_*$ is the feasible solution for the master problem in iteration $k = 1$. Then, since $\Omega_*$ is feasible in iteration $k = k'$, we can construct the feasible solution in iteration $k = k' + 1$ as

$$\Omega_{k'} = \Omega_* \quad (75)$$

$$P_{\text{sum}}^{\Omega_0, q,k',0} = P_{\text{sum}}^{\Omega_0, q,k'+1,0} = \sum_{i \in I_q} P_{i,s,s} \quad (76)$$

$$Q_{\text{sum}}^{\Omega_0, q,k',0} = Q_{\text{sum}}^{\Omega_0, q,k'+1,0} = \sum_{i \in I_q} Q_{i,s,s}. \quad (77)$$

According to the step 2 in the proof of Theorem 1, $\Omega_*$ is feasible for the formulated master problem in iteration $k = k' + 1$ with the feasibility assumption about the master problem in iteration $k = k'$.

Next, we prove $\Omega_*$ is the converged solution of the modified iteration algorithm using the contradiction approach. Assume that $\Omega$ is the converged solution, which is not equal to $\Omega_*$. As $\Omega_*$ is the optimal solution of the formulated distributed problems, we have

$$\sum_{q \in N} \text{Cost}^{SP}_q < \sum_{q \in N} \text{Cost}^{SP}_q = \text{Cost}^{MP}_q \quad (78)$$

where $\text{Cost}^{SP}_q$ and $\text{Cost}^{MP}_q$ are, respectively, the optimal and converged objective solutions for the subproblem (25)–(28). $\text{Cost}^{MP}_q$ is the converged objective solution for the master problem (32)–(34). $\text{Cost}^{MP}_q$ is set as the optimal objective solution for the master problem, and we have $\text{Cost}^{MP}_q < \text{Cost}^{MP}_q$. This expression contradicts with the convergence assumption about $\text{Cost}^{MP}_q$. Thus, the converged solution must be $\Omega_*$. It is worthy to mention that $\Omega_*$ is not required to be feasible for all the iterations of the subproblem (25)–(28). Since the master problem (32)–(34) can converge to $\Omega_*$, and send parameters $P_{q,k,s,*}$ and $Q_{q,k,s,*}$ to the subproblem, the final converged solution is $\Omega_*$. 

### IV. CASE STUDIES

To validate the effectiveness and high performance of the fully distributed SOCP-based expansion (hereafter denoted DIS) model, we implement it on the test cases (i.e., T24D9, T118D3, and T300D60) and solve them using GUROBI solver [37]. The planning horizon considered in all the cases is five years. As benchmarks of the investigations, we achieve the original centralized nonconvex expansion (hereafter denoted CEN) solutions by IPOPT solver [38]. Also, we compare our DIS model with the conventional isolated expansion (hereafter denoted ISO) model. All programs are implemented in MATLAB R2016a and run on a laptop with 3.60 GHz CPU and 8 GB RAM. The YALMIP toolbox [39] is deployed to implement the fully distributed optimization.

The remainder of this section is arranged as follows. T24D9 is employed to compare the DIS solutions with the ISO solutions for transmission (hereafter denoted ISO-T) and distribution (hereafter denoted ISO-D), respectively. With the model in Section II adopted, the power demands are regarded as constants in ISO-T while the transmission locational marginal prices are recognized as constants in ISO-D. T118D3 and T300D60, two relatively large cases, are further studied to unlock the benefits of the DIS method. Next, the computational performances of the fully distributed optimization algorithm in Section III-B are investigated for T24D9, T118D3, and T300D60. The results are then compared to those from other expansion models in terms of the optimality and convergence.

#### A. Case A-T24D9

The test case consists of an IEEE 24 bus transmission network [40] and nine IEEE 33 bus distribution network [41]. The schematic representation of the test case is displayed in Fig. 3, where the locations of DERs in distribution network 6 are shown in specifics. The peak load for the whole system is 2332 MW, and the obtained expansion solutions can guarantee system security under this particular condition. These nine distribution networks are connected to the transmission network at buses 1, 2, 3, 5, 6, 7, 10, 13, and 19, respectively. The boundary buses are all bus 1 on the distribution side. The total installed capacity of wind
turbine generation at transmission buses 3, 10, and 19 is 300 MW. The transmission generation data are the same as in [35]. The distribution networks also allocate DERs with the total installed capacity of 333 MW. The network assumes to be partitioned into five zones. The yearly profiles of power demands and renewable energy sources for each zone correspond to real measurements recorded into a region in Northwest China, and are reduced to 20 scenarios using the fuzzy C-means clustering approach from [42]. The representative scenarios to address the uncertainties associated with renewable energy and load variation are shown in Fig. 4.

1) Comparison of DIS, CEN, and ISO Solutions From the Transmission’s Perspective: This part investigates the optimal transmission line investments for different expansion methods and their effect on the cost performance of the transmission network. Table I gives the new lines, investment, and operation costs of the transmission network under different methods. It can be seen that the transmission expansion results for the DIS and CEN methods are almost the same, although the operation cost difference between the DIS and CEN methods is 0.16% which is acceptable. This indicates the effectiveness of the fully distributed SOCP-based expansion model presented in this article. The number of new transmission lines and their corresponding costs are consistently smaller in the DIS model than in the ISO model. These results indicate that the DIS model contributes to the reduction of the carbon emission and economic performance of the transmission operation.

For the DIS and ISO models, the optimized active powers of all transmission generations are compared in Fig. 5. The DIS model, with the coordination benefit, has lower transmission generations’ active power outputs. The maximum difference in the total active powers of transmission generations in the DIS and ISO models reaches 176.27 MW in scenario 14. This indicates that the DIS model can accommodate more renewable energy sources in the transmission network and adapt to the new environment of high share of renewable energy.

Fig. 6 shows the DIS and ISO results in terms of the load factors of new transmission lines 6–10 and 8–10. With DERs dispatched to supply some of the local loads of the distribution network 7 in the DIS model, the load factors of transmission lines 6–10 and 8–10 are always less than the ISO model in each scenario. This indicates that the DIS model contributes to mitigating and preventing the congestion that may occur in the ISO model. The DIS model has more significant load factor changes in the lines that are adjacent to the boundary buses. This result reveals that the DIS model can coordinate the distribution injections for transmission expansion needs. Also, the load factors of lines 6–10 and 8–10 range from 0.35 to 0.85, which validates the effectiveness of the transmission expansion solutions.

2) Comparison of DIS, CEN, and ISO Solutions From the Distribution’s Perspective: This part studies sensitivities of the distribution expansion results to different methods. Table II
TABLE II
DISTRIBUTION EXPANSION RESULTS FOR VARIOUS METHODS

<table>
<thead>
<tr>
<th>Distribution network no.</th>
<th>Investment cost (MS)</th>
<th>Operation cost (MS)</th>
<th>Total cost (MS)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DIS</td>
<td>CEN</td>
<td>ISO</td>
</tr>
<tr>
<td>D1</td>
<td>0.41</td>
<td>0.41</td>
<td>0.52</td>
</tr>
<tr>
<td>D2</td>
<td>0.36</td>
<td>0.36</td>
<td>0.44</td>
</tr>
<tr>
<td>D3</td>
<td>0.79</td>
<td>0.79</td>
<td>1.04</td>
</tr>
<tr>
<td>D4</td>
<td>0.22</td>
<td>0.22</td>
<td>0.29</td>
</tr>
<tr>
<td>D5</td>
<td>0.54</td>
<td>0.54</td>
<td>0.72</td>
</tr>
<tr>
<td>D6</td>
<td>0.51</td>
<td>0.51</td>
<td>0.67</td>
</tr>
<tr>
<td>D7</td>
<td>0.79</td>
<td>0.79</td>
<td>1.16</td>
</tr>
<tr>
<td>D8</td>
<td>1.14</td>
<td>1.14</td>
<td>1.26</td>
</tr>
<tr>
<td>D9</td>
<td>0.79</td>
<td>0.79</td>
<td>1.07</td>
</tr>
</tbody>
</table>

Fig. 6. Load factors of new transmission lines. (a) Line 6–10. (b) Line 8–10.

Fig. 7. Active power outputs of DERs in scenario 17.

Fig. 8. Apparent power mismatches at the boundary buses in scenario 17.

compares the cost performance of the distribution networks for the DIS, CEN, and ISO models. It can be observed that the solutions of the DIS model are the same as the solutions of the CEN model in terms of the investment costs, and the results from the DIS model are very close to the results from the CEN model in terms of the operation costs. The maximal difference between the total costs solved by these two models is 0.78%. These results show the effectiveness of the DIS model with respect to distribution expansion. Furthermore, the investment and operation costs in the distribution networks are sensitive to the DIS and ISO models. Thus, compared with the ISO model, the DIS model can reduce investment costs further since more local loads of the distribution networks are supplied by DERs and the congestion is prevented by the DIS model. As expected, the DIS model yields smaller total costs for the distribution networks than the ISO model as the distribution expansion benefits from the transmission-distribution coordination.

The DER accommodation of distribution network 1 to distribution network 9 in scenario 17 is shown in Fig. 7. The ISO’s curtailment is now mitigated by the DIS, as the latter allows for the DSO’s regulation for transmission needs. Take distribution network 7 as an example, the advantages of the transmission-distribution coordination reach 5.36 MW. This indicates that the DIS model can contribute to accommodating more DERs in the distribution networks.

Compared with the DIS model, the ISO model may result in apparent power mismatches at the boundary buses. Take scenario 17 as an example, Fig. 8 shows these power mismatches between the transmission network and nine distribution networks. For the DIS model, there is no boundary power mismatches occurred. Conversely, the ISO model causes different power mismatches at the boundary buses. The maximum boundary power mismatch reaches 5.36 MVA at the distribution network 7. With the coordination between transmission and distribution networks, these significant boundary power mismatches are avoided by the DIS model, as the latter allows us to share the distribution’s available controls with the TSO. Large power mismatches can cause regulation difficulties and increase operation costs.
B. Case B-T118D3

In this case, a relatively large integral network with an IEEE 118-bus transmission network [43] and three practical distribution networks is further used to test the proposed SOCP-based expansion model. The root buses of the Dongxingyuan 43-bus distribution network (denoted D-DXY), Kangzhuang 57-bus distribution network (denoted D-KZ) and Yanqing 76-bus distribution network (denoted D-YQ) in North China are connected to the transmission network at buses 54, 62, and 80, respectively. In D-DXY, three distributed generators are installed at buses 8, 24, and 39. In D-KZ, distribution buses 15, 39 and 57 host distributed generators. In D-YQ, three distributed generators are also allocated at buses 22, 41, and 58. It is worthy to mention that the simulation parameters are set the same as Section IV-A.

Fig. 9 depicts the total cost of the DIS model which is 640.80 M$. To check the validity of the solutions, the CEN model is also investigated and the total cost is 645.15 M$. The difference between the total costs obtained by these two models is 0.67% which is acceptable. It is important to emphasize that, compared with the CEN model, the DIS model results in a slight decrease in total cost since the results of the proposed expansion model are relaxed solutions. Also, the total cost of the DIS model is smaller than that of the ISO model, indicating that the DIS model can improve the cost performance of the network reinforcement by transmission-distribution coordination.

C. Case C-T300D60

T300D60 is a modified IEEE 300-bus transmission network connected to 60 distribution networks. A total of 34 of the 60 distribution networks have a topology that is similar to the topology of active distribution grid 1 of T6D2 in [35], whereas the other 26 have a topology that is similar to the topology of active distribution grid 2 of T6D2. The generation cost functions of all the DERs are the same as in [35]. The whole network includes 61 subnetworks, 788 buses, and 237 generators overall. The fully distributed decision-making formulation takes 23.41 s to obtain an optimal solution, confirming its fast convergence features. This indicates that the performance of the proposed method is also robust against the problem’s scale. Moreover, the total cost of the DIS method is 496.35 M$. To check the optimality of the result, the CEN method is also investigated and the total cost is 497.28 M$. The difference between the total costs obtained by these two methods is 0.19% which is acceptable. This validates the accuracy of the proposed model when applied on large-scale systems.

D. Comparison of Different Expansion Models

To compare our DIS model with the SOCP-based expansion model in [19], we deploy both models in YALMIP and solve the models for Cases T24D9, T118D3, and T300D60 by GUROBI solver. As benchmarks of the comparisons, we achieve the solutions from MATPOWER which gives local optima of the nonconvex expansion model and the solutions from the CEN model which gives global optima of the nonconvex expansion model. MATPOWER uses MATLAB built-in Interior Point Solver to solve the nonconvex expansion problem.

The results are given in Table III. Compared with MATPOWER and CEN solutions, the total costs of all test cases from our DIS model are bit lower. This is because constraints related to voltage phase angle variables are included in our DIS model while these are not necessary for the nonconvex expansion model in MATPOWER and CEN model solved by IPOPT. Note that the results of transmission-distribution expansion from the model in [19] are much relaxed as this model does not integrate voltage phase angle constraints. As can be seen from Table III, lower total costs are achieved from the model in [19] compared with our DIS model. Thus, the solutions of our DIS model can guarantee more ac feasibility. In terms of computation time, the model in [19] requires the least computation time since this model encompasses the least number of variables and constraints. The CEN model requires the most computation time to find a global optimum. All the above testify the high performance of our DIS model compared with other existing models.

V. Conclusion

This article presents a fully distributed SOCP-based expansion model for exploiting the technical and economic benefits of interactions between transmission and distribution networks. Our SOCP-based expansion model can be used to optimize both transmission and distribution networks since it includes constraints related to voltage phase angle. Taking the total power output of each subnetwork as the interacting variable, the proposed expansion problem can be decomposed into a substation operation master problem and a series of subnetwork expansion subproblems in a distributed manner. A modified iteration algorithm is deployed to tackle the formulated subproblems in parallel. The feasibility and optimality, and convergence properties of the iteration are analytically and numerically proved.

Numerical results show that both transmission and distribution networks benefit from the coordination. Boundary power mismatches are reasonably decreased in the coordinated transmission-distribution expansion. Also, the TSO-DSO interactions contribute to accommodating more DERs in the distribution networks. The modified iteration algorithm is an efficient and robust algorithm with limited communication burdens to
solve the fully distributed expansion problem. Our SOCP-based expansion model outperforms others by achieving a preferential solution with limited computation time. The work investigated in this article can be directed on two future extensions. First, the coordinated transmission-distribution expansion can account for other forms of DERs like energy storage devices. Second, the optimal allocation of DERs could be integrated into the fully distributed decision-making framework.

REFERENCES


