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## Research Papers

# Performance comparison of two water pit thermal energy storage (PTES) systems using energy, exergy, and stratification indicators

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## ABSTRACT

Water pit thermal energy storage systems have been demonstrated in Denmark and have proven effective in increasing the solar thermal fractions of district heating systems and in covering the mismatch between heat demand and production. This study analyzed five years of measurement data for two PTES systems in Denmark, namely Marstal and Dronninglund. Their efficiency was assessed using energy, exergy, and a seasonal efficiency indicator. The degree of stratification was investigated using the MIX number, the stratification coefficient, and a newly-introduced indicator, exergy destruction. Exergy destruction was shown to be a promising indicator for assessing stratification since it can be used to compare PTES systems with different heat losses, providing a quantitative evaluation of the amount of mixing. In addition, the seasonal efficiency was found to be suitable for estimating the long-term efficiency of combined seasonal and short-term storage systems. The storage in Dronninglund had 92% energy and 73% exergy efficiency, while Marstal had 63% energy and 48% exergy efficiency. All stratification and efficiency indicators showed that the storage in Dronninglund performed better overall than the one in Marstal.

## 1. Introduction

Approximately 64% of residential heat consumers in Denmark use district heating to cover their heat demand [1]. A benefit of district heating systems is that, in principle, they can use a variety of heat sources, both renewable and non-renewable. However, due to the intermittent nature of renewables, e.g., wind and solar, there is often a mismatch between heat demand and production. In order to accommodate a high share of renewables in district heating grids, a solution is needed to deal with this mismatch [2]. Research has demonstrated that an effective solution is the use of thermal energy storage (TES) in district heating systems. According to Sveinbjörnsson et al. [3], the mismatch between heat demand and production typically limits the solar thermal fraction of district heating systems to 20%. However, by using seasonal TES, solar thermal fractions higher than 40% can be achieved.

Obtaining a high storage efficiency is crucial in order to obtain a high system performance. For this reason, storage efficiency has been thoroughly studied in the literature, primarily in terms of energy and exergy. Historically, energy efficiency analysis has been favored as it is based on relatively simple concepts and is easy to implement. However, Rosen et al. [4] demonstrated that exergy analysis provides a more realistic and

accurate assessment of TES performance than energy analysis.

Rezaie et al. [5] investigated the performance of a TES in a district heating system in Germany and calculated an energy and exergy efficiency of 60% and 19%, respectively. Lake and Rezaie [6] presented similar results for a cold TES where the overall energy efficiency of the storage was 75%, while the exergy efficiency was only 20%. Exergy efficiency is lower than energy efficiency as it accounts not only for losses but also reversibility [7]. For example, exergy can identify temperature degradation in a storage system caused by ambient temperature but also due to mixing [8]. Also, exergy can account for temperature differences in storage systems with the same energy content, resulting in a performance indicator that is more accurate than one based only on energy.

The main factors impacting TES performance are heat losses and thermal stratification. Thermal stratification occurs when fluid is stored at different temperature levels in the same tank, creating a vertical temperature gradient. Rosen et al. [9] concluded that achieving a high degree of stratification is essential for high TES performance since it increases the amount of stored exergy. Additionally, Sifnaios et al. [10] showed that a stratified tank could increase a heat pump system's coefficient of performance (COP) by up to 32% compared to a fully mixed tank. The reason is that a stratified TES can have a higher exergy content

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## Nomenclature

### Latin letters

$C_p$	Specific heat [J/(kg·K)]
$d$	Diameter [m]
$E$	Energy [J]
$Ex$	Exergy [J]
$h$	Height [m]
$H$	Enthalpy [J]
$m$	Mass [kg]
$M_E$	Moment of energy [J·m]
$S$	Entropy [J/K]
$T$	Temperature [K]
$v$	Water speed [mm/s]
$V$	Volume [m <sup>3</sup> ]
$\dot{V}$	Volume flow rate [m <sup>3</sup> /h]
$z$	Distance [m]

### Greek letters

$\eta_E$	Energy efficiency [%]
$\eta_{E, s}$	Seasonal energy efficiency [%]
$\eta_x$	Exergy efficiency [%]
$\rho$	Density [kg/m <sup>3</sup> ]

than a fully mixed tank for the same heat input [11].

While stratification naturally occurs due to the temperature dependence of the buoyancy force (i.e., the density of water generally decreases with an increase of temperature), several phenomena decrease stratification. Stratification is decreased by mixing induced by the inlet flow, by heat diffusion caused by natural convection, and by downward thermal conduction [12]. As a consequence, thermal stratification is affected by the inlet/outlet design, storage geometry, and operation strategy of the TES. To decide if a TES performs adequately, it is necessary to evaluate the degree of stratification, usually by calculating stratification indicators.

Many stratification indicators have been investigated in the literature without determining which performs best. Castell et al. [13] compared many dimensionless numbers for assessing thermal stratification and concluded that the Richardson number is the most suitable for predicting the expected stratification. However, the Richardson number does not quantify the actual storage stratification and can only be used for comparing two or more similar storage systems. In addition, in a review performed by Han et al. [14], it was found that various studies report inconsistent interpretations of the relation between the Richardson number and mixing.

Haller et al. [15] reported a comprehensive investigation of most available stratification indicators. The study concluded that all of the assessed indicators had limitations in their application, e.g., some indicators could not be used for scenarios including charging and discharging, whereas others failed to separate the effects of heat losses from mixing. To overcome these limitations, Sifnaios et al. [16] suggested using the internal exergy destruction for elucidating the specific times at which mixing occurs. Compared to other indicators, a benefit of this method is that it does not rely on a reference storage simulation, which can be difficult to implement in real life. It can also be applied to storage systems regardless of their application (e.g., short term, long term, or a combination).

In the present paper, two large-scale water pit thermal energy storage (PTES) systems were assessed and compared in terms of their efficiency and degree of stratification. A water PTES is a large water reservoir used to store thermal energy for a period ranging from days to months, depending on the application. It is constructed by excavating a pit in the ground, which is subsequently lined with a watertight polymer

liner [17]. The storage is then filled with water and covered with an insulated floating lid [18]. Water PTES technology was developed in Denmark and demonstrated with solar collector fields as the heat source [19]. The main advantage of a water PTES is its low cost compared to other storage technologies. For example, the investment cost of a water PTES ranges from 20 to 40 €/m<sup>3</sup> while the investment cost of conventional storage tanks is from 150 to 320 €/m<sup>3</sup> [20].

To the authors' knowledge, there has never been a study assessing the performance and stratification of a PTES using actual measurement data. In addition, although simple energy efficiencies have been calculated for individual storage systems, e.g., by Dahash et al. [21], a comparison of the performance of two PTES systems in terms of energy, exergy, and stratification indicators has never been performed.

For this reason, the efficiency and thermal stratification of two water PTES systems were determined experimentally in this study. The two systems were located in Denmark, in Marstal and Dronninglund. Long-term measurements at each of the two PTES systems were carried out. The data obtained were analyzed using a number of energy and exergy efficiency expressions. Their suitability for evaluating PTES performance was assessed. Lastly, an exergy destruction indicator was for the first time applied to actual storage systems to determine its suitability for assessing stratification.

## 2. Methods

The construction and operation of the water PTES systems in Marstal and Dronninglund are briefly described below, followed by an account of the measurements that were made and the definitions of the stratification and efficiency indicators.

### 2.1. The storage systems investigated

#### 2.1.1. Marstal

The water PTES system in Marstal was constructed in 2012, with a capacity of approximately 6000 MWh. The heat storage system was integrated with the local district heating system, which supplies heat to approximately 1600 consumers. The pit was excavated in the shape of an inverse truncated pyramid with a volume of 75,000 m<sup>3</sup>. This PTES system is used for seasonal storage of heat generated by a solar collector field consisting of 15,000 m<sup>2</sup> of flat-plate collectors.

The core of the lid construction consists of three layers of close cell structure polyethylene (PE) foam insulation (Nomalén 28N manufactured by the company NMC Termonova [23,24]) with a total thickness of 24 cm. The layers of Nomalén were enclosed between two high-density polyethylene (HDPE) liners. Weight pipes were used to ensure a slope towards the center of the lid, where a pump well was located to remove rainwater from the surface of the lid. Fig. 1 shows an aerial photo of the PTES and the solar thermal collector field in Marstal. The heat storage system was primarily charged during the spring and summer periods and discharged during autumn and winter.

The district heating supply temperature in Marstal was approx. 73 °C [25], while the average return temperature was 40 °C (lower in winter and higher in summer). When the temperature at the top of the storage was higher than the supply temperature, heat was supplied directly to the district heating grid. However, when the temperature at the top of the storage was between 70 °C and 73 °C, water from the storage system was mixed with higher temperature water from a biomass boiler in order to reach the desired supply temperature. Whenever the storage temperature dropped below 70 °C, the PTES acted as the heat source for a heat pump that supplied heat to the district heating network. The heat pump cooled down the storage to approximately 15–20 °C.

The storage system was charged and discharged using three double-plated diffusers located at the top, middle, and bottom of the storage (see Fig. 2). In the same figure, it may be seen that the pipes leading to the diffusers were uninsulated and passed through the storage side wall. The diameter of the diffuser plates was 3 m, and the vertical spacing between



Fig. 1. Aerial photo of the PTES in Marstal in 2013 [22].



Fig. 2. Diffuser installation in Marstal [22].

the diffuser plates was 0.85 m. The diffusers were constructed of steel, and the average exit velocity for the water was 2.2 mm/s, corresponding to a Reynolds number of 5731 (for water at 90 °C). Eq. (1) was used to calculate the average diffuser velocity for an average volume flow rate of 63 m<sup>3</sup>/h.

$$v_{diffuser} = \frac{\dot{V}}{d_{diffuser} \cdot h_{diffuser} \cdot \pi} \quad (1)$$

where  $\dot{V}$  is the volumetric flow rate,  $d_{diffuser}$  is the diameter of the diffuser plate, and  $h_{diffuser}$  is the vertical spacing between the diffuser plates.

The lid insulation in the PTES in Marstal started to degrade in the course of 2015 due to contact with water at high temperatures. In 2017, multiple piercings in the HDPE liner led to water entering the lid (rainwater from above and storage water from below), causing a large increase in the rate of heat loss. In 2018 the heat storage operation was interrupted during the replacement of the lid. The data used in this study are thus from 2014 to 2017.

### 2.1.2. Dronninglund

The water PTES system in Dronninglund was constructed in 2014, based on the same design as the storage in Marstal, but including some improvements (Fig. 3). It had a volume of 60,000 m<sup>3</sup> and an

approximate storage capacity of 5500 MWh. The storage system was charged from a 35,573 m<sup>2</sup> flat-plate solar collector field and supplied heat to 1350 consumers. As in the storage system in Marstal, the lid was made of three layers of Nomalén enclosed in two HDPE liners. However, the HDPE liner used in Dronninglund had a longer lifetime at high temperatures.

The main difference between the two storage systems was in their operation. The storage in Dronninglund was used both as seasonal and short-term heat storage. This means that not only was heat stored from summer to winter, but the storage was also used to even out the diurnal variation, i.e., heat charged during the day might be discharged in the evening and at night. This operation strategy increased the yearly charged and discharged energy compared to the PTES system in Marstal.

The supply temperature of the district heating network in Dronninglund was 75 °C [26], while the average return temperature was 42 °C. When the temperature at the top of the storage was higher than the supply temperature, heat was supplied directly to the district heating grid, but when the storage temperature was lower than 75 °C, the system was used as a heat source for a heat pump. The heat pump was only operated in winter and cooled down the storage to approximately 10 °C.

The Dronninglund PTES system was also charged using three diffusers, placed at the top, middle, and bottom of the pit, as shown in Fig. 4. However, the diffuser construction in Dronninglund differed



Fig. 3. Aerial photo of the PTES in Dronninglund in 2019.



Fig. 4. Diffuser installation in Dronninglund [27].

somewhat from the one in Marstal. For example, Fig. 4 shows that the pipes leading to the diffusers were insulated and entered the storage from below, thus minimizing any temperature exchange with the stored water. The diffusers were made of stainless steel and were slightly smaller than those in Marstal, with a plate diameter of 2.5 m and vertical spacing of 0.58 m. Consequently, the average flow rate was higher, with an average diffuser exit velocity of 5 mm/s at 80 m<sup>3</sup>/h volume flow rate. The corresponding Reynolds number for this exit velocity was 8887 (for water at 90 °C).

However, as at Marstal, the HDPE liner enclosing the lid insulation experienced multiple piercings during 2020 and 2021, and eventually, the entire lid had to be replaced. The data from Dronninglund are thus from 2015 to 2019. Both storage systems had a different lid construction at the time of writing compared to those described, which are the ones that were in place when the measurements were made.

### 2.1.3. Measurement equipment

The temperature in each storage was measured using two vertical temperature sensor strings hanging from the middle of the lid. Each temperature string had 16 temperature sensors placed at 1 m intervals. The two strings were located next to each other and offset by 0.5 m; thus, the vertical temperature profile was measured every 0.5 m. The temperature sensors were Class A PT100, with an accuracy of  $\pm 0.15$  °C. The equipment used to measure the storage temperature was the same for Dronninglund and Marstal. The location of the water temperature sensors relative to the diffusers in the PTES is presented in Fig. 5.

The volume flow rate to and from the storage in Dronninglund was measured using electromagnetic flowmeters with an accuracy of 0.4%. In Marstal, the flow was measured using ultrasonic flowmeters with an estimated uncertainty of 2%. The temperatures in the inlet and outlet pipes were measured in both systems using immersed Class A PT100 sensors, with an accuracy of  $\pm 0.15$  °C.

### 2.2. Stratification indicators

The present study derived three indicators to assess thermal stratification: the MIX number, the stratification coefficient, and exergy destruction. In order to calculate these indicators, the storage systems were divided into discrete layers corresponding to the position of the installed temperature sensors. The temperature sensors in both storage systems were installed every 0.5 m from the bottom to the top of the water pit, and since both storage systems had a depth of 16 m, each storage was divided into 32 layers.

#### 2.2.1. MIX number

The MIX number is a dimensionless indicator assessing storage stratification by comparing the storage system temperature profile with a fully mixed and a fully stratified reference storage system [28]. The MIX number ranges between zero and one, corresponding to a perfectly stratified and a fully mixed storage, respectively. As presented in Eq. (2), the MIX number is calculated based on the moment of energy.

$$MIX = \frac{M_E^{stratified} - M_E^{actual}}{M_E^{stratified} - M_E^{fully\ mixed}} \quad (2)$$

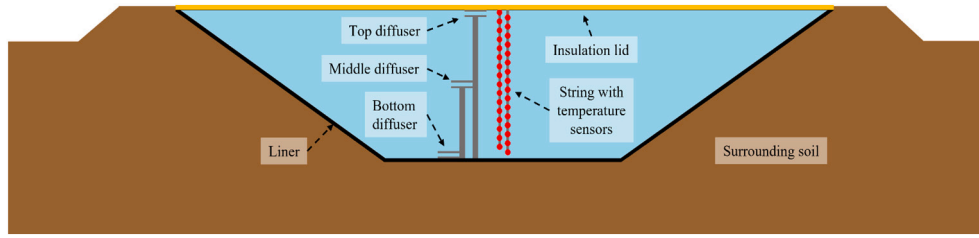


Fig. 5. Sketch of the location of the water temperature sensors (red dots) relative to the diffusers for the PTES in Dronninglund and Marstal. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The moment of energy for each scenario is calculated as the sum of the moment of energy of each layer. Each layer's moment of energy is calculated by multiplying the layer's energy content with the height from the bottom of the storage:

$$M_E = \sum_{i=1}^N \rho_i \cdot V_i \cdot C_{p,i} \cdot (T_i - T_{ref}) \cdot z_i \quad (3)$$

where  $N$  is the number of storage layers,  $\rho_i$  represents the water density of layer  $i$ ,  $V_i$  is the water volume of the layer,  $C_{p,i}$  is the specific heat, and  $T_i$  is the water temperature of the  $i$ th layer. The distance from the centroid of the layer to the bottom of the storage system is denoted as  $z_i$ , and  $T_{ref}$  is the reference temperature, i.e., the temperature at which the storage system is considered empty.

The moment of energy for the fully stratified and fully mixed storage systems is calculated such that they have the same energy content as the actual storage system.

### 2.2.2. Stratification coefficient

The stratification coefficient expresses the degree of thermal stratification based on the deviation of the storage temperature profile relative to the mean temperature. The expression developed by Wu and Bannerot [29] is presented below:

$$St = \sum_{i=1}^N \frac{m_i \cdot (T_i - T_{avg})^2}{m_{total}} \quad (4)$$

where  $T_i$  is the temperature and  $m_i$  is the mass of the  $i$ th layer,  $T_{avg}$  is the average storage temperature, and  $m_{total}$  is the total mass of the storage system.

### 2.2.3. Exergy destruction

The internal exergy destruction is calculated based on the exergy balance of the TES, using Eq. (5):

$$\Delta Ex_{destr} = \Delta Ex_{flow} - \Delta Ex_{store} - \Delta Ex_{loss} \quad (5)$$

Exergy destruction is the sum of the exergy destroyed due to mixing, heat conduction, and diffusion. This means that a storage system with high exergy destruction will experience a low level of stratification. It should be noted that the exergy destruction calculated in Eq. (5) includes the exergy lost directly due to heat losses. This is one of the main benefits of using exergy destruction compared to other methods, as it is possible to compare storage systems with different heat losses without having biased results. The exergies were calculated as:

$$\Delta Ex = \Delta H - T_0 \cdot \Delta S \quad (6)$$

where  $\Delta H$  is the change in enthalpy,  $\Delta S$  is the change in entropy and  $T_0$  is the dead state temperature. The dead state of a system is a state in which it is at temperature, pressure, elevation, velocity, and chemical equilibrium with its surroundings [30].

In order to be able to compare storage systems of different sizes fairly, the exergy destruction should be normalized with the storage volume. The normalized exergy destruction is therefore calculated as:

$$\Delta Ex_{destr, norm} = \frac{\Delta Ex_{destr}}{\sum_{i=1}^N V_i} \quad (7)$$

## 2.3. Efficiency indicators

Many different efficiency indicators have been proposed, although they can generally be expressed as the ratio between the useful output and input to the system:

$$\eta = \frac{\text{useful output}}{\text{input}} \quad (8)$$

In this study, two efficiency indicators were calculated, namely energy and exergy efficiency, as defined in the following sub-sections.

### 2.3.1. Energy efficiency

The energy balance of a thermal storage system can be expressed as:

$$E_{out} = E_{in} - E_{loss} - \Delta E_{int} \quad (9)$$

where  $E_{out}$  is the energy discharged from the storage system,  $E_{in}$  is the charged energy, and  $E_{loss}$  is the energy lost due to heat losses.  $\Delta E_{int}$  is the change in the internal energy of the storage system, i.e., the difference between the internal energy at the start and end of the period under consideration.

Two expressions have been used to calculate energy efficiency and are presented in Eqs. (10) and (11). It should be noted that these expressions are slightly different from Eq. (8) since they also account for the internal energy change of the storage system.

$$\eta_{E,1} = \frac{E_{out}}{E_{in} - \Delta E_{int}} = \frac{E_{out}}{E_{out} + E_{loss}} \quad (10)$$

$$\eta_{E,2} = \frac{E_{out} + \Delta E_{int}}{E_{in}} = \frac{E_{in} - E_{loss}}{E_{in}} \quad (11)$$

The main difference between the two expressions is the treatment of the internal energy change, i.e., either subtracting it from the charged energy or adding it to the discharged energy. Both expressions have been used in engineering reports and are included in this study to investigate whether they can be used interchangeably.

Nevertheless, energy efficiencies calculated in this way can be misleading when comparing seasonal storage systems (such as Marstal) with storage systems used for both seasonal and short-term storage (such as Dronninglund). The main reason is that the total energy charged and discharged in a seasonal and short-term storage system is much higher than a storage system that is only used for seasonal storage. However, the heat losses of a seasonal and short-term storage are not proportionally higher since they depend on the storage duration. For this reason, Jensen [31] proposed a seasonal efficiency, which attributes the heat losses only to the seasonally stored energy:

$$\eta_{E,S} = \frac{E_{seasonal}}{E_{seasonal} + E_{loss}} \quad (12)$$

where  $E_{seasonal}$  is the stored seasonal energy. The stored seasonal energy

can be calculated as the difference between the minimum and maximum energy content in the course of one year. Seasonal efficiency quantifies the efficiency of the storage system as if it was solely used for seasonal heat storage.

### 2.3.2. Exergy efficiency

Exergy is a measure of the quality of energy and indicates the work potential of a system relative to its environment [32]. Following the general format of efficiency indicators, Rosen et al. [4] calculated the exergy efficiency as:

$$\eta_x = \frac{Ex_{out}}{Ex_{in}} \quad (13)$$

where  $Ex_{in}$  is the exergy entering the system and  $Ex_{out}$  is the exergy exiting the system. Unlike energy efficiency, exergy efficiency also accounts for the level of stratification, as mixing reduces the exergy content.

## 3. Results

The first section of the results derives the stratification indicators for the PTES systems in Marstal and Dronninglund. The second section

calculates the energy and exergy efficiencies of the storages.

### 3.1. Stratification indicators

#### 3.1.1. Marstal PTES system

The storage temperatures, stratification indicators, diffuser energy supply, and normalized weekly flow rate for the PTES system in Marstal are shown in Fig. 6. The storage temperature, diffuser energy supply, and normalized volume flow rate are included in the plot to demonstrate the storage operation and assist in understanding the stratification indicators.

For the PTES temperature, the temperature of each storage layer is illustrated using a different color, namely green for the top of the storage and blue for the lowest level. Additionally, a thin black curve indicates the temperature for the top, middle, and lowest layers. It may be seen that the annual maximum temperature of the PTES system in Marstal decreased over the investigated period. As previously mentioned, this was because of the high heat losses due to the degradation of the lid insulation after 2014 (see also Table 2).

In Fig. 6, the MIX number and the stratification coefficient are presented for each day, while the exergy destruction and diffuser energy supply are tabulated for each month and the volume flow rate for each

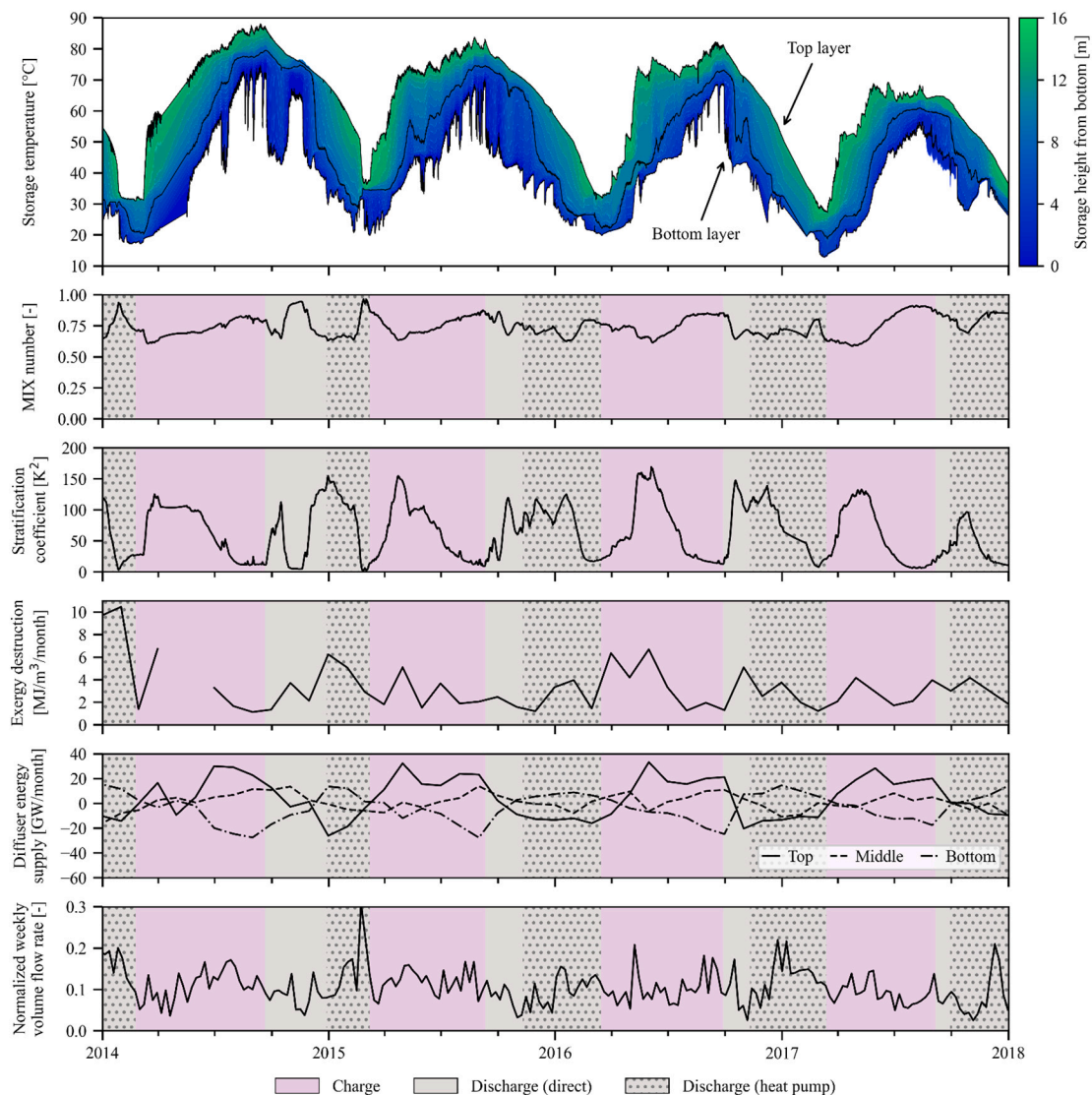


Fig. 6. Storage temperature and stratification indicators for PTES in Marstal. The gap for exergy destruction in spring 2014 is due to missing flow rates from the dataset, probably due to data logging errors.

week. Ideally, all of the indicators should be calculated for each day. However, the exergy destruction and heat losses can only be calculated with reasonable accuracy on a monthly basis due to the low spatial resolution of the temperature in the storage. The volume flow rate was plotted weekly since the daily flow was very variable. The charging periods are illustrated in Fig. 6 using a pink background, while the discharging periods are shown in grey. The heat pump discharge periods are illustrated using a dotted pattern so as to be distinguishable from the periods of direct discharge of the PTES. As mentioned before, the duration of the direct discharge period varies from year to year, depending on the storage temperature. It may be seen that for years with low storage temperatures, e.g., in 2017, the storage system was almost exclusively used as a source for the HP.

The maximum value of the MIX number in Marstal was 0.97, and the minimum was 0.56. The average MIX number value for the entire period under investigation was 0.74, indicating a high degree of mixing. It may be seen that the MIX number generally approaches one towards the end of each charging and discharging period when there is an almost uniform temperature in the PTES system.

Comparing the MIX number with the stratification coefficient, it can be stated that they have a negative correlation, i.e., when one increases, the other decreases. For example, at the end of each charging and discharging period, the stratification coefficient approaches its lower value, i.e., zero, indicating a uniform temperature in the PTES. However, the variation of the values of the stratification coefficient is more pronounced, providing more information about the mixing within the storage. The stratification coefficient reached its maximum value when there was the largest temperature difference between the top and bottom of the storage. In Marstal, this value was  $169 \text{ K}^2$  and occurred in the summer of 2016. Like the MIX number, the value of the stratification coefficient has no physical meaning, but it can be used to compare two different storage systems or different storage operations. The maximum value of the stratification coefficient was achieved approximately at the middle of each charge/discharge period. The small spikes during charge and discharge indicate mixing since a more uniform temperature decreases the value of the coefficient. Apart from in 2014, the stratification coefficient indicates better stratification during charge than discharge periods. However, these values are caused by higher temperatures at the top of the PTES system and not necessarily by better stratification.

Also, in Fig. 6, it is evident that, in general, there is high exergy destruction during the charging periods. The reason for this is that most of the heat losses take place from the top of the storage during charging, leading to the spiky temperature profile shown in Fig. 6. Heat losses can affect the stratification in the storage system by cooling down the top layer of water just under the lid, creating thermal inversion that induces mixing. Besides mixing caused by heat losses, heat conduction and diffusion in the storage system also generate mixing. These phenomena occur when there is a non-uniform temperature profile in the storage system. The exergy destruction indicator captures the effect of heat conduction and diffusion since it reaches its lowest values when the storage system approaches a uniform temperature (i.e., when fully charged or discharged). Last, some of the spikes in exergy destruction can be correlated with periods of high energy supply/extract from the diffusers, indicating mixing occurring during the diffuser operation. The mean annual exergy destruction in Marstal for the analyzed period was  $36 \text{ MJ/m}^3/\text{year}$ . This number indicates the amount of mixing in the storage and will be used for comparison with the Dronninglund storage.

Periods with high exergy destruction occurred when there were spikes in the stratification coefficient. As previously mentioned, a rapid decrease in the stratification coefficient indicates mixing, which is well captured by exergy destruction. However, it should be noted that the exergy destruction values were calculated monthly, so it is not easy to match them with the other stratification indicators.

From Fig. 6, it is apparent that there is no direct correlation between the normalized volume flow rate passing through the storage and the stratification indicators. This indicates that the diffusers were

functioning appropriately and were reducing the water velocity even when high flows were entering the PTES.

The mean annual value for the MIX number and stratification coefficient are presented in Table 1, along with the sum of exergy destruction per year for the two PTES systems under investigation.

### 3.1.2. Dronninglund PTES

Table 2 shows the energy balance for the PTES system in Dronninglund. It shows that more energy was being charged and discharged annually in Dronninglund than in Marstal due to their different modes of operation. In addition, the heat losses in the Dronninglund system were much lower than in Marstal, primarily because of lower heat losses from the lid, but also because of the mode of operation. For example, in Dronninglund, the mean low temperature of the PTES system was around  $10 \text{ }^\circ\text{C}$ , while in Marstal it was around  $20 \text{ }^\circ\text{C}$ , so the heat gains from the ground in wintertime were much greater in Dronninglund.

The effect of the lower heat losses can be seen in Fig. 7, where the temperature in the storage system is shown. Compared to Marstal, the storage temperature in Dronninglund remained at higher levels. It should be noted that the temperature spikes occurring in the top layers of the storage were mainly caused by the short-term storage cycles and not by heat losses, as they were in Marstal.

In Dronninglund, the MIX number for the period analyzed ranged from 0.20 to 0.67 with an average of 0.34, which is 46% lower than in Marstal. This value indicates a well-stratified storage as a value of zero represents a perfectly stratified storage. As in Marstal, there seems to be a negative correlation between the MIX number and the stratification coefficient.

However, the maximum value reached in Dronninglund for the stratification coefficient was  $538 \text{ K}^2$ , which is 3.2 times higher than in Marstal, indicating a higher degree of thermal stratification. Nonetheless, due to the short-term storage cycles, there were many more spikes in the stratification coefficient during the charge periods, indicating a higher degree of mixing during charge than in Marstal.

This phenomenon is better illustrated in terms of exergy destruction, where the maximum values occur for the charge periods where the short-term storage cycles occur. Although these values are higher than those in Marstal in some cases, very little mixing took place in Dronninglund during discharge. Unlike the PTES in Marstal, in Dronninglund, there was no correlation between exergy destruction spikes and the diffuser operation, except from the summer of 2018. Overall, the mean annual exergy destruction for Dronninglund was  $34 \text{ MJ/m}^3/\text{year}$ , which is approximately 6% lower than for Marstal.

In both PTES systems, most charged/discharged energy was supplied using the top and bottom diffusers. However, in Dronninglund, the middle diffuser was used more than in Marstal, especially during discharge operations. The authors are not aware of whether the operation strategy differed between the two systems or if this was a result of the difference between their temperature profiles. In either case, increased usage of the middle diffuser would be expected to affect the storage stratification positively.

Higher flow rates were measured at Dronninglund than at Marstal on average, but the higher flow rates do not seem to have affected the stratification indicators.

Overall, all stratification indicators indicate that Marstal had a lower degree of stratification than Dronninglund. However, the MIX number and stratification coefficient are both affected by heat losses, favoring systems with low heat loss. Since exergy destruction is the amount of exergy lost only due to mixing, it is considered by the authors to be the best indicator for comparing storage systems with different heat losses. Additionally, its value has a physical meaning, quantifying the exergy lost through mixing. Consequently, it is the only indicator investigated that can present a quantitative evaluation of the amount of mixing that takes place in a storage system.

Calculating these indicators requires a knowledge of the vertical distribution of the storage system temperature. However, for calculating



**Table 1**

Mean annual values for MIX number and stratification coefficient and sum of the annual values for exergy destruction for the PTES in Marstal and Dronninglund.

	Marstal			Dronninglund		
	MIX number [-]	Stratification coefficient [K <sup>2</sup> ]	Exergy destruction [MJ/m <sup>3</sup> ]	MIX number [-]	Stratification coefficient [K <sup>2</sup> ]	Exergy destruction [MJ/m <sup>3</sup> ]
2014	0.75	60	38	–	–	–
2015	0.75	71	33	0.33	174	36
2016	0.73	75	42	0.34	187	39
2017	0.76	49	32	0.35	154	34
2018	–	–	–	0.32	115	31
2019	–	–	–	0.34	139	32
Overall	0.74	64	36	0.34	154	34

**Table 2**

Annual energy values for the storages in Marstal and Dronninglund. All quantities are in MWh.

	Marstal					Dronninglund				
	E <sub>in</sub>	E <sub>out</sub>	ΔE <sub>int</sub>	E <sub>loss</sub>	E <sub>seasonal</sub>	E <sub>in</sub>	E <sub>out</sub>	ΔE <sub>int</sub>	E <sub>loss</sub>	E <sub>seasonal</sub>
2014	7654	5124	1134	1396	4875	–	–	–	–	–
2015	7568	5571	–609	2606	3802	13,164	12,127	–559	1596	4551
2016	7452	5392	–695	2755	4823	12,107	10,842	13	1252	4317
2017	6975	3179	–822	4618	4021	11,442	11,555	–585	472	4225
2018	–	–	–	–	–	14,793	13,893	–159	1059	5101
2019	–	–	–	–	–	12,733	11,573	234	926	4803

the MIX number, a perfectly stratified and a fully mixed scenario have to be simulated as a reference for the actual storage system under investigation, increasing the complexity of the calculation. In addition, the MIX number can be affected by the choice of reference temperature for calculating the moment of energy. For this reason, it is essential to choose as reference temperature the one that indicates that the storage system is considered as empty. Choosing a lower reference temperature will cause the storage system to appear to be more mixed, while choosing a higher reference temperature will cause it to appear to be more stratified. Lastly, for calculating the MIX number, the moment of energy must be calculated for each of the layers in the PTES system. This is a more demanding task than for a tank due to the truncated pyramid geometry of the PTES systems under consideration.

### 3.2. Efficiency indicators

Table 2 sets out the annual energy values for the PTES systems in Marstal and Dronninglund. It may be seen that the energy input of each storage system,  $E_{in}$ , was similar each year. However, the discharged energy,  $E_{out}$ , varied, as it was affected by the heat losses for the specific year and the internal energy change of the storage,  $\Delta E_{int}$ . The internal energy change depends on the storage system's energy content at the start and the end of each year, and, as shown in Table 2, it can be positive or negative.

A visualization of the monthly charged and discharged energy and the energy content of the storage system is given in Figs. 8 and 9 for Marstal and Dronninglund, respectively. These figures illustrate the different operations of the two storages, e.g., the Dronninglund water pit is also discharged during the charging periods. In addition, the charged energy per month in Dronninglund is approximately twice that in Marstal since the size of the solar collector field is twice as large. Last, it may be seen that Dronninglund is discharged to a lower energy content than Marstal.

The calculated energy and exergy efficiencies for the storage systems in Marstal and Dronninglund are shown in Table 3. It may be seen that the efficiencies achieved in Dronninglund are very high. The main reasons for this are the lower heat losses and the utilization of low-temperature heat by the heat pump, which increases the amount of energy that can be discharged.

When considering energy efficiency, the results for Marstal differ whether  $\eta_{E, 1}$  or  $\eta_{E, 2}$  are used. The reason is that if the internal energy

change is negative (meaning the energy content of the storage is lower at the end of the year than at the start), then  $\eta_{E, 1}$  indicates higher efficiency than  $\eta_{E, 2}$ . The opposite occurs if the energy change is positive. For example, in Marstal, in 2014, the internal energy change was positive, leading to a higher  $\eta_{E, 2}$ , while in 2015, where  $\Delta E_{int}$  was negative,  $\eta_{E, 1}$  was higher.

When comparing the results of using  $\eta_{E, 1}$  or  $\eta_{E, 2}$  for Dronninglund and Marstal, the two expressions give the same results for Dronninglund but are slightly different for Marstal. The reason is that in Dronninglund, due to the short-term storage cycles, the charged and discharged energy is much higher than the internal energy charge of the storage, so  $\eta_{E, 1}$  and  $\eta_{E, 2}$  give essentially the same results. However, for Marstal, since the charged and discharged energy are of the same order of magnitude as the internal energy charge, selecting one energy expression over the other leads to different PTES system efficiencies. The authors believe that the two expressions cannot be used interchangeably and that it is preferable to use  $\eta_{E, 1}$ . The reason is that subtracting the internal energy change from the charged energy is more appropriate since less energy has to be charged due to existing energy from the previous cycle. This results in the heat losses from each year being attributed to the heat discharged during the year. In contrast,  $\eta_{E, 2}$  attributes the heat losses to the energy charged during the year.

From the efficiencies in Table 3, it may be seen that the energy efficiencies  $\eta_{E, 1}$  and  $\eta_{E, 2}$  give higher efficiency values compared to the seasonal efficiency, primarily for Dronninglund. As explained in Section 2.3.1, the seasonal energy efficiency,  $\eta_{E, S}$ , estimates the efficiency of the storage system as if it had been used only for seasonal storage. This is why the energy efficiency and seasonal efficiency for Marstal are very close. In contrast, the seasonal efficiency is lower than the energy efficiency for Dronninglund, which is used for both seasonal and short-term storage. While the short-term storage operation increases the heat losses, the total amount is approximately the same regardless of the length of the storage cycle. The reason is that most heat losses occur from the lid, and the temperature at the top of the storage system is the same whether it is used for seasonal storage only or combined seasonal and short-term storage. Therefore, the seasonal efficiency is a good indicator for finding the equivalent efficiency of a PTES if it was only used for seasonal storage.

It was found that exergy efficiency is always lower than the energy efficiencies for all the investigated years. This is because it accounts for both heat losses and the amount of mixing in the storage, while the

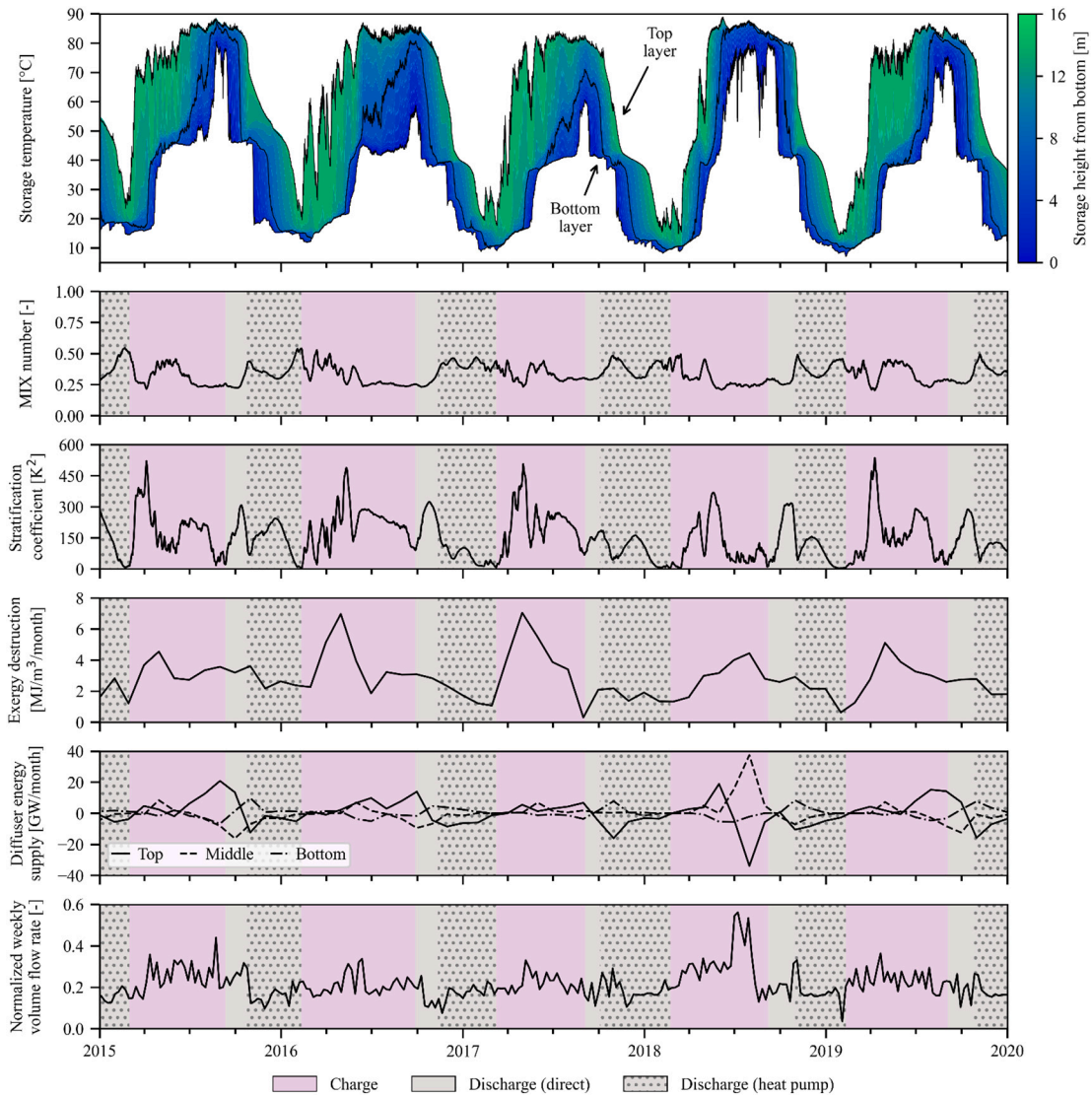


Fig. 7. Storage temperature and stratification indicators for PTES in Dronninglund.

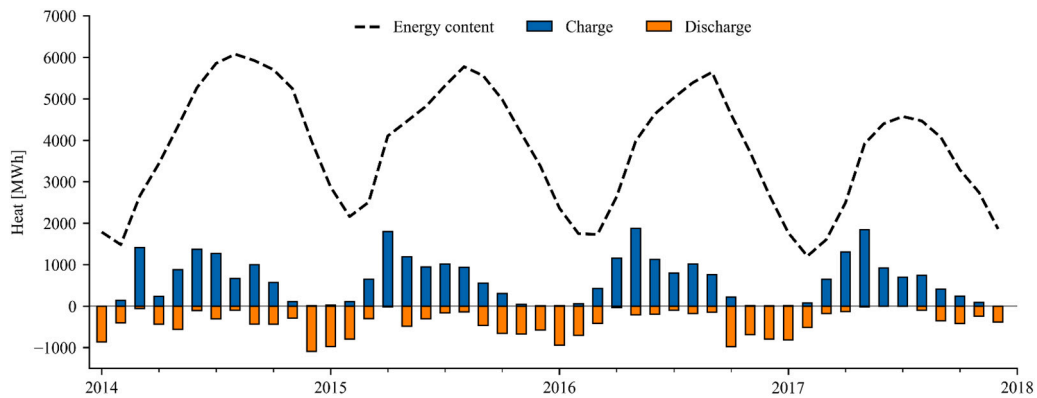


Fig. 8. Monthly charged/discharged energy and energy content for the PTES in Marstal.

typical energy efficiencies account only for heat losses. As a result, it can complement the energy efficiency expressions and provide a more detailed indicator for the comparison of different storage systems. However, it is apparent that it does not follow the same trend as the energy efficiencies, as it indicates different years as having the highest

exergy efficiency. For both PTES systems, the highest exergy efficiency values corresponded to years with the highest charged/discharged energy.

All the efficiency indicators considered show that the PTES system in Dronninglund had a higher efficiency than Marstal. The main reasons

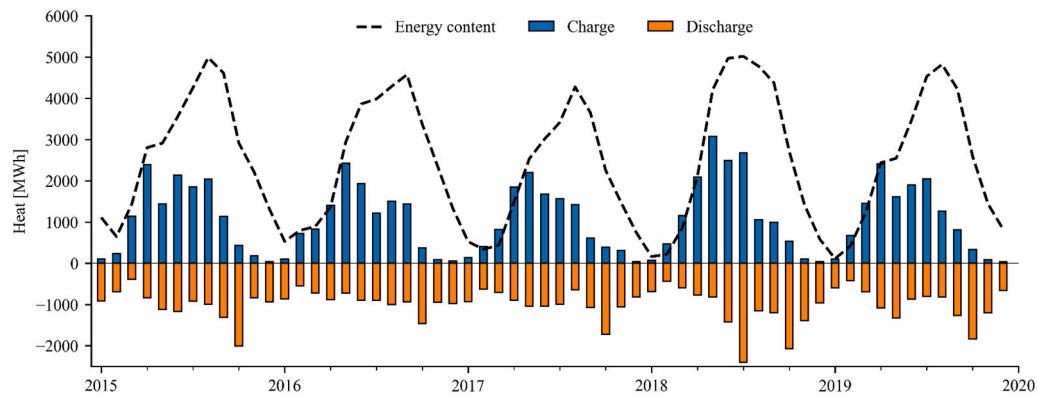


Fig. 9. Monthly charged/discharged energy and energy content for the PTES in Dronninglund.

**Table 3**  
PTES efficiencies for Marstal and Dronninglund.

	Marstal				Dronninglund			
	$\eta_{E, 1}$ (%)	$\eta_{E, 2}$ (%)	$\eta_{E, S}$ (%)	$\eta_X$ (%)	$\eta_{E, 1}$ (%)	$\eta_{E, 2}$ (%)	$\eta_{E, S}$ (%)	$\eta_X$ (%)
2014	79	82	78	53	–	–	–	–
2015	68	66	59	58	88	88	74	73
2016	66	63	64	52	90	90	78	69
2017	41	34	47	27	96	96	90	73
2018	–	–	–	–	93	93	83	79
2019	–	–	–	–	93	93	84	73
Total	63	62	61	48	92	92	81	73

seem to be the high heat losses and the lower degree of stratification in the Marstal PTES. The high heat losses in Marstal are most likely due to water entering the lid (either rainwater from above or storage water from below), which reduces the performance of the insulation. Thus, it is crucial that the lid insulation is maintained dry since the thermal barrier of the lid is key to limiting the heat loss of a PTES. Regarding stratification, research should be conducted on selecting the optimal diffuser construction and operation.

#### 4. Conclusions

This study investigated the efficiency and stratification of two existing large-scale water pit thermal energy storage (PTES) systems. Both systems were located in Denmark, namely in Marstal and Dronninglund. Long-term measurements at the two systems were analyzed.

The MIX number, stratification coefficient, and exergy destruction indicators were used for assessing the stratification of the PTES. Although all these indicators show that Marstal had a lower degree of stratification than Dronninglund, exergy destruction was considered the most promising indicator for assessing the amount of mixing in a storage. One of the benefits of exergy destruction is that it can be used to compare storage systems with different heat losses without necessarily favoring the one with the lowest heat losses, but instead providing a quantitative evaluation of the amount of mixing.

The storage efficiency was assessed using energy, exergy, and a seasonal efficiency indicator. It was found that the two energy expressions compared cannot be used interchangeably since they give different results for seasonal storage systems, in which the internal energy change is of the same order of magnitude as the charged and discharged energy.

The seasonal efficiency was found to be a suitable indicator for comparing storage systems with different operations. Calculating the equivalent efficiency of a PTES system as if it was used as seasonal storage enables a fair comparison between a seasonal and a combined seasonal and short-term storage.

Exergy efficiency is recommended as an indicator complementing

energy efficiency since it is able to account for the effects of heat losses and mixing in the storage. In addition, the exergy efficiency can more fairly compare storage systems with different operations.

In comparing the performance of the two PTES systems, all of the stratification and efficiency indicators considered in this study showed that the Dronninglund system performed better than the Marstal system. The reason is that the PTES system in Dronninglund had lower heat losses and better stratification.

Future work should investigate the impact of storage operations on efficiency and stratification by assessing the performance of PTES systems with different storage configurations. At the time of writing, there is one pit storage system that is under construction and another that is being planned in Denmark, both of which are designed to be used as short-term storage systems. They will both be connected directly to the district heating grid, and neither will use a heat pump. This means that measurements from short-term PTES systems operation will be available in the near future.

#### CRediT authorship contribution statement

**Ioannis Sifnaios:** Conceptualization, Methodology, Visualization, Investigation, Data analysis, Writing - Original draft. **Adam R. Jensen:** Methodology, Visualization, Data analysis, Writing - Review & editing. **Simon Furbo:** Writing - Review & editing, Supervision, Funding acquisition. **Jianhua Fan:** Writing - Review & editing, Supervision, Funding acquisition.

#### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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