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Mapping Initial and General Recombination in Scanning Proton Pencil Beams

Jeppe Brage Christensen*1, Erik Almhagen2,3, Liliana Stolarczyk2,4, Małgorzata Liszka4, Guillermo Garrido Hernandez5, Niels Bassler6,7,8, Ole Nørrevang9, and Anne Vestergaard9

1Center for Nuclear Technologies, Technical University of Denmark, Roskilde, Denmark
2Skandionkliniken, Uppsala, Sweden
3Medical Radiation Sciences, Department of Immunology, Genetics and Pathology, Uppsala University, Uppsala, Sweden
4Institute of Nuclear Physics Polish Academy of Sciences in Krakow, The Bronowice Cyclotron Centre, Krakow, Poland
5Department of Physics and Astronomy, Aarhus University, Aarhus, Denmark
6Medical Radiation Physics, Department of Physics, Stockholm University, Stockholm, Sweden
7Department of Oncology and Pathology, Medical Radiation Physics, Karolinska Institutet, Stockholm, Sweden
8Department of Experimental Clinical Oncology, Aarhus University Hospital, Aarhus, Denmark
9Danish Center for Particle Therapy, Aarhus University Hospital, Aarhus, Denmark

Abstract

The ion recombination is examined in parallel-plate ionization chambers in scanning proton beams at the Danish Centre for Particle Therapy and the Skandion Clinic. The recombination correction factor $k_s$ is investigated for clinically relevant energies between 70 MeV and 244 MeV for dose rates below 400 Gy min$^{-1}$ in air. The Boutillon formalism is used to separate the initial and general recombination. The general recombination is compared to predictions from the numerical recombination code IonTracks and the initial recombination to the Jaffé theory. $k_s$ is furthermore calculated with the two-voltage method (TVM) and extrapolation approaches, in particular the recently proposed three-voltage (3VL) method. The TVM is in agreement with the Boutillon method and IonTracks for dose rates above 100 Gy min$^{-1}$. However, the TVM calculated $k_s$ is closer related to the Jaffé theory for initial recombination for lower dose rate, indicating a limited application in scanning light ion beams. The 3VL is in turn found to generally be in agreement with Boutillon’s method. The recombination is mapped as a function of the dose rate and proton energy at the two centres using the Boutillon formalism: the initial recombination parameter was found to be $A = (0.10 \pm 0.01) \ V$ at DCPT and $A = (0.22 \pm 0.13) \ V$ at Skandion, which is in better agreement with the Jaffé theory for initial recombination than previously reported values. The general recombination parameter was estimated to $m^2 = (4.7 \pm 0.1) \cdot 10^3 V^2 \text{nA}^{-1} \text{cm}^{-1}$ and $m^2 = (7.2 \pm 0.1) \cdot 10^3 V^2 \text{nA}^{-1} \text{cm}^{-1}$. Furthermore, the numerical algorithm IonTracks is demonstrated to correctly predict the initial recombination at low dose rates and the general recombination at high dose rates.

1 Introduction

Gas-filled ionization chambers retain the position as the recommended detector for photon, electron, and ion beams in protocols as the IAEA TRS-398 (Andreo et al., 2000) and AAPM TG-51 (Almond et al., 1999). Ionization chambers are attractive for both reference dosimetry and daily quality assurance (QA) but the charge collection efficiency

*E-mail: jepb@dtu.dk
remains an issue. The charge liberated between the electrodes drifts towards the electrode of opposite polarity with a possibility of recombining with other charge carriers: the probability for two charge carriers to recombine depends on several factors but in particular with the center-of-mass energy between the charge carriers. The uncertainty of the recombination cross sections data at low center-of-mass energies is currently too large for Monte Carlo transport methods to be applied for recombination corrections in ionization chambers.

The ion recombination is traditionally divided into two phenomena: the recombination between charge carriers liberated from the same particle (initial recombination) or the recombination between charge carriers from different particles (general or volume recombination). The former case depends strongly on the linear energy transfer (LET) and the ion track structure whereas the latter is related to the dose rate.

The general recombination was studied for several decades mainly in photon and electron beams and theories were developed accordingly for uniform charge carrier distributions and low-LET beams without initial recombination (Thomson, 1899; Mie, 1904; Boag, 1950; Greening, 1964). Boag and Currant (1980) developed the foundation for the widely applied two-voltage method (TVM) which is recommended by IAEA TRS-398 and AAPM TG-51 among other protocols to correct the recombination in clinically relevant beams. The model was later extended to include a free-electron component (Boag et al., 1996), which reduces the ion recombination, as the free electrons are almost immediately collected. Rossmome et al. (2017) observed that the TVM is not applicable to hydrogen and heavier ion beams where the initial recombination cannot be neglected: the foundation of the TVM relies on uniform charge carrier distributions in photon and electron beams and the does not include the initial recombination in high-LET ion beams.

Initial recombination in parallel-plate ionization chambers irradiated with ion beams was treated theoretically by Jaffé (1913, 1929) and extensively experimentally confirmed in light and heavy ion beams (Kanai et al., 1998; Rossmome et al., 2016). The Jaffé theory for initial recombination in an ion track was generalized numerically in the open-source code IonTracks (Christensen et al., 2016). IonTracks applies amorphous track structure theory to model the initial charge carrier distributions and a general advection equation to govern the ion movements and recombination. Thus, IonTracks accounts for the interaction between multiple ion tracks and numerically calculates both initial and general recombination in ion beams. In a similar manner, Han et al. (2020) derived an analytical expression for the integral collection efficiency for scanning proton beams based on the work by Boag.

Recently, Mirandola et al. (2019) investigated the LET dependence of recombination in the Bragg peak region of proton and carbon pencil beams with Monte Carlo methods. Shortly after, Rossmome et al. (2020) proposed a three-voltage method (3VL-method) relying on the linear part of the charge collection region. Whilst the 3VL-method relies on data points in the linear region and has the potential to provide more accurate and stable corrections than traditional extrapolation methods, the use of three data points in a narrow range could make it more prone to measurement fluctuations.

The different recombination correction methods for initial and general recombination, applicable at different energy and dose rate regimes, motivates an investigation of the recombination in scanning proton pencil beams. The recombination in a gas-filled parallel-plate ionization chamber is in the present work mapped at the Skandion Clinic (Uppsala, Sweden) and the Danish Centre for Particle Therapy (DCPT, Aarhus, Denmark) with Roos-type chambers. The initial and general recombination is separated using the approach developed by Boutillon (1998) and extended by Palmans et al. (2006).
Monte Carlo models of each beam line is used to extract the LET along the central beam axis for all experiments, which is used to model the initial recombination with the Jaffé theory and IonTracks. Furthermore, the Monte Carlo models are used to estimate an effective dose rate of the spot-scanning proton beams as suggested by Liszka et al. (2018). The ionization chamber is irradiated with protons at different energies and dose rates in order to map the recombination as a function of dose rate and perform an inter-comparison between the two proton therapy centers. Finally, the IonTracks numerical calculations of the recombination is compared to recombination results calculated with the Boutillon and TVM methods.

The mapping of the ion recombination as a function of proton energy and dose rate enables an estimate of a lower dose rate threshold, where initial recombination dominates, and the TVM is no longer applicable for daily reference dosimetry. Furthermore, the mapping enables the prediction and correction of the recombination in spread out Bragg peaks.

2 Materials and Methods

2.1 Experimental setup and Monte Carlo

The recombination was measured at Skandion and at DCPT with the same Roos-type ionization chamber irradiated in 10 cm $\times$ 10 cm mono-energetic layer consisting of 41 $\times$ 41 spots at 2 cm reference depth in water. The ionization chamber was irradiated three times for each dose rate at each of the polarization voltages 50 V, 67 V, 83 V, 125 V, and 200 V to avoid charge multiplication at higher voltages (Berg and Noerrevang, 2004; Palmans et al., 2010). The recombination is generally mapped at 200 V which relates the correction values to the daily QA. The ion collection time in the ionization chamber varied between 0.1 ms (at 200 V) and 0.5 ms (at 50 V). The Monte Carlo scoring of the fluence-averaged LET ($\Phi$-LET) follows the recommended implementation given by Cortés-Giraldo and Carabe (2015).

2.1.1 Measurements at the Skandion Clinic

The Roos-type ionization chamber was irradiated with 70 MeV, 150 MeV, and 226 MeV protons corresponding to the available energy range of the Proteus C235 cyclotron (IBA, Belgium). The cyclotron delivers the pulses with 106 MHz and the spot duration varied between 3 ms at 226 MeV and 20 ms a 70 MeV. The delivery of a single spot is treated as a continuous beam as the spot duration is much longer than the charge collection time in the ionization chamber.

The Proteus C235 cyclotron system optimizes the beam delivery time, and the dose rate is varied by increasing the number of monitor units (MUs) per spot. The dose rate or beam current does not increase linearly with the number of MUs per spot and is limited by the maximum available beam current. The recombination was investigated for each of the three energies for 0.08 MU/spot, 0.2 MU/spot, 0.4 MU/spot, and 1 MU/spot.

The energy deposition and proton LET in the ionization chamber and water at Skandion is calculated with a Geant4 (Agostinelli, S., Allison, J., and Amako, K. et al., 2003) Monte Carlo model of the gantry and water phantom as detailed in Almhagen et al. (2018).
2.1.2 Measurements at the Danish Centre for Particle Therapy

The ProBeam Proton Therapy System (Varian Medical Systems, USA) at DCPT was used to irradiate the ionization chamber at 70 MeV, 150 MeV, 226 MeV, and 244 MeV. The dose rate in the 72.8 MHz ProBeam system is adjusted with the dose rate factor (DRF), where e.g. \( \text{DRF} = 0.2 \) for a given energy approximately corresponds to \( 1/5 \) of the dose rate relative to \( \text{DRF} = 1.0 \). The dose rate is varied for \( \text{DRF} = 0.2, 0.5, \) and \( 1.0 \) in the present work. The spot duration times extracted from logfiles last from 2.6 ms (for 244 MeV and \( \text{DRF} = 1.0 \)) to 22 ms (for 70 MeV and \( \text{DRF} = 1.0 \)).

The dose deposition and LET at DCPT during the experiments are calculated using an in-house developed beam model based on the Geant4 toolkit TOPAS (Perl et al., 2012).

2.1.3 Effective dose rate calculations

The calculation of the dose rate in the spot scanning beam depends on several parameters, in particular the dose and deposition time. The dose deposited for a spot map is measured with the ionization chamber and related to the energy deposition per proton through a Monte Carlo model of each of the two proton beams. The time it takes to deposit each spot is extracted from the beam log files. The dose deposition and spot deposition time is then for each beam configuration converted into an effective dose rate following the procedure detailed in Liszka et al. (2018).

2.2 Theory

The general recombination of low-LET beams is often corrected by means of extrapolation, where the collection efficiency \( f \) is related to the polarization voltage \( V \). The recombination in pulsed beams exhibit a linear behaviour when \( f^{-1} \) is plotted versus \( V^{-1} \), whereas a linearity is observed for continuous beams for \( f^{-1} \) as a function of \( V^{-2} \). The TVM (TRS-398) is applicable for a continuous beam and corrects the recombination as

\[
k_s = \frac{\left(\frac{V_2}{V_1}\right)^2 - 1}{\left(\frac{V_1}{V_2}\right)^2 - \frac{Q_1}{Q_2}},
\]

where \( Q_i \) is the charge collected at polarization voltage \( V_i \) for \( i = 1, 2 \). The TVM, however, is developed for photon and electron beams with approximately uniform charge distributions and its application to recombination in light ion beams with distinct track structures has been demonstrated to be questionable (Palmans et al., 2006; Rossomme et al., 2017). The 3VL method proposed in Rossomme et al. (2020) aims at remedying the known issues with the TVM and extrapolation methods in light ion beams by extrapolating the recombination correction factor from three data points in the linear region.

2.2.1 The Jaffé theory for initial recombination

The Jaffé theory (Jaffé, 1913, 1929) models the initial recombination in ion tracks based on a Gaussian track structure. The theory is outlined in appendix A.1 along with the relevant proton track parameters.

The Jaffé theory is in the present work used to compute the initial recombination correction factor \( k_s^{\text{Jaffé}} \) in a Roos-type ionization chamber in figure 1(a). The \( \Phi\text{-LET} \) at 2 cm water depth, corresponding to the effective point of measurement in the ionization
chamber, is calculated with Geant4. The analytical and numerical solutions to initial recombination in figure 1(b) is treated in section 2.2.3.

Figure 1: (a) The recombination correction for initial recombination as predicted by the Jaffé theory in a parallel-plate chamber with 2 mm electrode spacing for the relevant proton energies and chamber voltages. (b) The correction factor for initial recombination in the same air-filled ionization chamber along the central axis of a 100 MeV proton beam in water calculated with the Jaffé theory (lines) and IonTracks (markers) for 2 polarization voltages. The dose is shown in (b) for reference.

2.2.2 Separation of initial and general recombination

An approach to separate initial and general recombination in a continuous beam was suggested by de Almeida and Niatel (1986), generalized by Boutillon (1998), and later extended by Palmans et al. (2006) to include temporal variations in the beam structure. With $I_V$ denoting the ionization current at polarization voltage $V$, the initial and general recombination can be estimated by measuring the current at lower voltages $I_{V/n}$, where $n > 1$, as

\[ \frac{I_V}{I_{V/n}} \approx 1 + (n - 1) \frac{A}{V} + (n^2 - 1) \frac{m^2 g}{V^2} I_V, \quad (2) \]

where the second term on the right-hand side is the contribution from initial recombination and the third term represents the general recombination. $m^2$ is related to the general recombination while $A$ is a geometrical parameter related to initial recombination. $g = d^4/(6v)$ for a plane-parallel chamber with electrode gap $d$ and nominal ionization chamber volume $v$ (Palmans et al., 2006). After the parameters $A$ and $m^2$ have been experimentally determined, the recombination correction factor in the Boutillon approach is estimated as

\[ k_s^B \simeq 1 + \frac{A}{V} + \frac{m^2 g}{V^2} I_{V, \text{eff}}, \quad (3) \]

where $I_{V, \text{eff}}$ is the effective current in the ionization chamber at voltage $V$. Eq. (3) is henceforth referred to as the Boutillon model.
The recombination contribution from initial recombination $A/V$ for small currents, i.e. when the track overlap is insignificant, is traditionally regarded as a constant related to the ionization chamber (Boutillon, 1998; Palmans et al., 2006). However, the initial recombination varies with the proton energy and other factors as defined in appendix A.1, and the term $A/V$ is in the present work assessed directly against the Jaffé theory through $A/V \approx k_{\text{Jaffé}} - 1$.

2.2.3 Numerical recombination calculations

The software IonTracks is able to simulate the recombination in parallel-plate ionization chambers by randomly sampling ion tracks in time and space corresponding to a given dose rate and beam energy. The track structure model and recombination-diffusion equation is outlined in appendix A.2.

IonTracks is available for download along with an implementation of the Jaffé theory eq. (4) in a parallel-plate ionization chamber\(^1\). The $\Phi$-LET along the central axis of a 100 MeV proton beam is used to calculate the initial recombination in figure 1(b) with IonTracks (plotted with markers) and the Jaffé theory (lines) with geometrical parameters of a Roos-type ionization chamber. These calculated recombination correction factors at the Bragg peak region are in agreement with the results presented in Mirandola et al. (2019).

The recombination calculations with IonTracks are based on a 3 dimensional array where the upper and lower parts of the array represent the parallel electrode plates and the height of the array corresponds to the electrode gap $d$. Proton tracks are sampled over an area as shown in figure 2(a)–(b) according to a dose rate and proton energy. An example of the charge carrier densities in the virtual ionization chamber is shown in figure 2(a)–(b) for 5 Gy/min and 100 Gy/min dose rates with a polarization voltage of 200 V and $d = 0.2$ cm electrode gap. The ion tracks in figure 2(a) hardly overlap and initial recombination dominates. The situation differs in figure 2(b) where tracks coinciding in time and space increase the general recombination greatly.

Figure 2(c) shows the number of collected and recombined ions for 100 Gy/min and 400 Gy/min dose rates. The densities and, in particular the recombination, fluctuates which reflects the fact that sometimes tracks are sampled randomly to overlap or sufficiently far apart to be treated as independent. The numbers of recombined $n_{\text{recombined}}$ and collected $n_{\text{collected}}$ charge carriers at any time is directly related to the correction factor via $k_s = 1 + n_{\text{recombined}}/n_{\text{collected}}$. Hence, IonTracks accounts for both initial and general recombination in ion beams. A more detailed description of IonTracks can be found in Christensen et al. (2016).

3 Results and Discussion

3.1 Calculations of the effective dose rates

The calculated effective dose rates are shown in figure 3 for the DCPT and Skandion measurements with typical standard uncertainties of 2 % as given in Liszka et al. (2018). The measurements with the Varian ProBeam at DCPT in figure 3(a) show a linearly increasing dose rate for an increasing DRF for the 150, 226, and 244 MeV energies. Thus, the recombination is expected to increase with the DRF.

\(^1\)https://github.com/jbrage/IonTracks
Figure 2: IonTracks simulations of recombination in parallel-plate chambers. Ions are initialized according to a dose rate of (a) 5 Gy/min and (b) 100 Gy/min inside the circle delineated with dashed lines. The ions in the 100 Gy/min beam exhibit track overlap and general recombination. (c) Examples of ion densities and recombination for 100 Gy/min and 400 Gy/min proton beams. \( \tau \) indicates the time it takes for an ion to drift across the chamber.

Figure 3: The calculated effective dose rates in the ionization chamber as a function of the dose rate variations at (a) DCPT and (b) Skandion. The markers in (b) are defined in the legend in (a).
The dose rates of the Proteus C235 cyclotron at Skandion in figure 3(b) exhibit a proportional increase with the number of MU/spot for the 150 MeV and 226 MeV energies until 0.4 MU/spot. However, the 70 MeV energy is relatively constant as a function of MUs/spot as the system increased the spot duration time rather than the beam current. Consequently, the recombination is expected to be constant for the 70 MeV measurements at Skandion.

3.2 Recombination correction factors

3.2.1 Linear and quadratic extrapolation

An example of the charge collected in the ionization chamber during irradiation with 226 MeV protons at DCPT is shown in figure 4. The collected charge is for each DRF normalized to the charge collected at 200 V where the $k_s$, obtained from both linear and quadratic extrapolation, is given in each legend. The recombination correction factor obtained with quadratic extrapolation is, however, overall associated with larger uncertainties as shown in figure 4(a) for all DRFs and energies and not applicable to light ion beams.

The non-linearity of the recombination in figure 4(a) as a function of the inverse voltage illustrates why the application of the TVM can be problematic: the Boag theory assumes a uniform charge carrier distribution with a linear relationship and the TVM may thus err in the present case. The inverse charge collection as a function of $1/V$ or $1/V^2$ in figure 4 is representative for all measurements at DCPT and Skandion, and only the linear extrapolation as a function of $1/V^2$ in figure 4(b) is used henceforth to extrapolate to $k_s$. The Jaffé theory in eq. (4) gives $k_{s_{\text{init}}} \approx 1.0002$ for a 226 MeV proton in air at 200 V with electrode spacing $d = 0.2$ cm and $k_{s_{\text{init}}} \approx 1.0004$ at 70 MeV with the same
conditions as given in figure 1(a). Thus, the recombination in figure 4 is dominated by general recombination.

### 3.2.2 Two- and three-voltage methods

The recombination correction factor is shown in figure 5 for the 3VL method and the TVM for each proton energy at a 200 V charge collection voltage. The Jaffé theory is used to estimate the initial recombination at 200 V which is plotted with horizontal dashed lines at each energy for reference. The results for the measurements at DCPT in figure 5(a) and Skandion in (b) show that several of the \( k_s \) factors obtained with the 3VL method are below the delineated initial recombination contribution. The discrepancy between \( k_s \) obtained with the TVM and the 3VL method is generally below 1%.

![Figure 5: The recombination correction factor \( k_s \) calculated with the TVM (triangles) and the 3VL-method (circles) at 200 V for all energies and dose rates. The Jaffé theory for initial recombination is plotted for reference for each energy with horizontal dashed lines. Dotted lines connect the \( k_s \) values obtained for the same energy and DRF to assist the eye. (a) The three DRF variations for each proton energy—except for 70 MeV—measured at DCPT. (b) The results from Skandion for the 4 dose rates for each proton energy with the same legend as in (a). The numbers denote the dose rate factor in (a) or MU/spot in (b).](image)

### 3.2.3 The Boutillon approach

The relative collected charge as a function of the effective currents \( I_{\text{eff}} \) is shown in figure 6 for the measurements at DPCT and Skandion. The Boutillon theory in eq. (2) is fitted to the data in each figure by minimizing the total least-square difference between the data and the fits. The initial and general recombination parameters obtained from each minimization is listed in table 1. The initial recombination parameter \( A \) in the Boutillon formalism is compared to the initial recombination estimated with the Jaffé theory, IonTracks, and estimates with two Roos-type chambers in a low-energy proton beam by Palmans et al. (2006).

Boutillon (1998) suggests a generic \( A = 0.25 \) value for photon beams, which is significantly larger than the initial recombination in proton beams as modelled in the Jaffé theory in eq. (4), which averaged over the 4 relevant proton energies is \( A = 0.045 \) in...
a Roos-type chamber with collecting voltage of 200 V. The $A$ values obtained at DCPT (for 150, 226, and 244 MeV) and Skandion (70, 150, 226 MeV) are both lower than the results presented by Palmans et al. (2006) but in better agreement with IonTracks and the Jaffé theory estimate as $A/V \approx k_{\text{Jaffé}} - 1$.

The smaller initial recombination value $A$ derived here is counterbalanced by a larger general recombination value $m^2$ than traditionally reported (Palmans et al., 2006) which however is within the limits of results caused by statistical fluctuations.

![Figure 6](image)

**Figure 6**: Relative charge collection as a function of the effective current measured at (a) DCPT and (b) at Skandion. The Boutillon theory in eq. (2) has been fitted simultaneously to all data in each figure.

**Table 1**: The initial $A$ and general $m^2$ recombination parameters (for $V = 200$ V) in the Boutillon formalism. The results obtained at Skandion and DCPT are compared to literature values and an estimate of the initial recombination from the Jaffé theory. The listed ($k = 2$) standard uncertainties are estimated from the fits.

<table>
<thead>
<tr>
<th>Data source</th>
<th>$V$</th>
<th>$\mu$m</th>
<th>$m^2$ [$V^2,\text{nA}^{-1},\text{cm}^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boutillon (1998)$^a$</td>
<td>0.25</td>
<td>—</td>
<td>$3.97 \cdot 10^3$</td>
</tr>
<tr>
<td>Palmans et al. (2006)$^b$</td>
<td>0.35 – 0.46</td>
<td>6.63 – 6.86</td>
<td>(2.71 – 2.93) \cdot 10^3</td>
</tr>
<tr>
<td>Jaffé theory$^c$</td>
<td>0.045</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>IonTracks fit</td>
<td>0.073</td>
<td>7.62</td>
<td>$7.4 \cdot 10^3$</td>
</tr>
<tr>
<td>DCPT data</td>
<td>0.10 ± 0.01</td>
<td>7.62</td>
<td>(4.7 ± 0.1) \cdot 10^3</td>
</tr>
<tr>
<td>Skandion data</td>
<td>0.22 ± 0.13</td>
<td>7.62</td>
<td>(7.2 ± 0.1) \cdot 10^3</td>
</tr>
<tr>
<td>DCPT &amp; Skandion</td>
<td>0.16 ± 0.13</td>
<td>7.62</td>
<td>(5.9 ± 0.1) \cdot 10^3</td>
</tr>
</tbody>
</table>

$^a$ Generic values estimated for initial recombination in beams.

$^b$ Listed ranges for two Roos-type chambers.

$^c$ Averaged over the 70, 150, 226, 244 MeV energies in the Roos-type chamber.
3.3 Recombination models

The recombination correction factors are plotted in figures 7(a)–(b) for the DCPT and Skandion measurements, respectively. The Boutillon formalism parameters in table 1, used for eq. (3), are compared directly to the initial recombination calculated with the Jaffé theory at two proton energies for a Roos-type chamber with a 200 V polarization voltage. The correction factors obtained with the TVM in eq. (1), calculated for the charge collected at 200 V relative to that at 67 V, is shown for comparison along with a linear function fitted to the TVM data. Furthermore, the generic $A = 0.25$ V initial recombination for photon beams is shown for reference.

The Boutillon theory is generally larger than the TVM data in figure 7(a) for the measurements at DCPT but in better agreement with the TVM data in figure 7(b) for the measurements at Skandion. However, the Boutillon method converges towards the Jaffé theory for low dose rates as expected. That is in contrast to the generic Boutillon parameter $A = 0.25$ V which overestimates the recombination predicted with the Jaffé theory at any dose rate.

The literature Boutillon parameters for eq. (3) listed in table 1 differ from the values estimated in this work: the initial recombination parameter $A$ is reported larger in Palmans et al. (2006) which consequently gives a smaller general recombination parameter $m^2$ than calculated here. However, the initial recombination parameter $A$ values in table 1 obtained at DCPT and Skandion are slightly larger than predicted by the Jaffé theory.

The disagreements between the fits in this work and the literature values can be attributed to data fluctuations arising from the calculation of the effective dose rates and the charge collections.

The TVM is generally in agreement with the Boutillon theory above $100\text{ Gy min}^{-1}$. However, the TVM seems to underestimate the recombination for low dose rates where the general recombination is comparable to the initial recombination, also observed by Rossomme et al. (2017). Also, the 3VL exhibits more scatter than the TVM in figure 4, which is anticipated due to the extrapolation from a narrow range. The case is especially pronounced at the 70 MeV measurements at both DCPT and Skandion: the TVM and 3VL calculated $k_s$ values fall in both cases within 1–2 standard deviations of the initial recombination predicted by the Jaffé theory.

Hence, the TVM seems to be applicable at high-dose rates where the charge carrier distribution is approximately uniform but to err at low dose rates where the track overlap is scarce and the ion track structure is important.

3.3.1 Recombination model comparison

The recombination correction factors from the DCPT and Skandion irradiations are compared directly in figure 8. The predicted correction factors calculated with IonTracks are shown along with the Boutillon formalism obtained from fit to all measurements at DCPT and Skandion.

These results are in agreement with Rossomme et al. (2017) where the TVM was demonstrated to err at low dose rates with significant amounts of initial recombination. The TVM is, as shown in figure 5, generally in good agreement with the extrapolation method. The 3VL method, as the TVM, however seems to fail to predict the total recombination when the dose rate is low and the general recombination is comparable to the initial recombination as observed for the lowest dose rates for the 70, 150, and 226 MeV energies at DCPT and 70 MeV at Skandion.
Figure 7: The recombination correction factors $k_s$ calculated with the TVM (markers) as a function of dose rate in the ionization chamber for the (a) DCPT measurements and (b) Skandion measurements. The Boutillon theory (dashed line) is plotted with the parameters listed in table 1. The contributions from initial recombination predicted by the Jaffé theory are plotted for comparison along with the literature value $A = 0.25$ V of initial recombination in the Boutillon model.
Figure 8: The DCPT and Skandion TVM data compared to the IonTracks results, the Boutillon theory fitted to all data, and a linear fit to all TVM data. The TVM methods generally underestimate the recombination at low dose-rates. The lower figure shows the residuals relative to the Boutillon theory (dashed line) in the upper figure.

The IonTracks calculations of the initial and general recombination in figure 8 is within 0.3% of the Boutillon formalism. IonTracks is a generalized version of the Jaffé theory and thus gives the same correction factors as the Jaffé theory for initial recombination at low dose rates. The good agreement between the TVM and the Boutillon formalism for higher dose rates, where general recombination dominates, shows that IonTracks is applicable to correct the ion recombination in scanning proton beams.

Conclusion

The ion recombination in scanning proton beams at the Danish Centre for Particle Therapy and the Skandion Clinic is investigated with the Boutillon formalism, the two-voltage method, the three-voltage method, the open-source code IonTracks, as well as extrapolation methods and the Jaffé theory for initial recombination. The Boutillon theory enables a separation of initial and general recombination which is investigated for 70 MeV to 244 MeV and 20 Gy min$^{-1}$ to 400 Gy min$^{-1}$ dose rates.

The initial recombination parameter was found to be $A = (0.10 \pm 0.01)$ V at DCPT and $A = (0.22 \pm 0.13)$ V at Skandion, which is lower than previously reported values. The values are on the other hand in better agreement with the Jaffé theory for initial recombination and IonTracks. The general recombination parameter was found to $m^2 = (4.7 \pm 0.1) \cdot 10^3$ V$^2$ nA$^{-1}$ cm$^{-1}$ at DCPT data and $m^2 = (7.2 \pm 0.1) \cdot 10^3$ V$^2$ nA$^{-1}$ cm$^{-1}$ at Skandion. Both $m^2$ values are larger than typical literature values which reflects the fact that the initial recombination parameter $A$ is correspondingly lower.

The recombination correction factor $k_s$ obtained with the two-voltage and three-voltage methods was found to be in agreement with the theoretical IonTracks software.
and the Boutillon theory \( \approx 100 \text{ Gy min}^{-1} \). However, the extrapolation methods and the TVM underestimate the recombination below that threshold, when the amount of initial recombination is comparable to the general recombination, which is in agreement with previous findings in light ion beams.

IonTracks was used independently to assess the recombination in the Roos-type chamber. The agreement with the Jaffé theory at low dose rates and both the TVM and the Boutillon theory at high dose rates validates the use of IonTracks to calculate the recombination in parallel-plate ionization chambers irradiated with proton beams. Further experiments are required to benchmark IonTracks in heavier ion beams and at higher dose rates.

The mapping of the recombination with the Boutillon formalism and IonTracks for the clinically relevant energies enables a general prediction of the correction factor \( k_s \) directly as a function of the proton energy and the dose rate.

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A Appendix

A.1 The Jaffé theory

Jaffé (1913, 1929) investigated the initial recombination in ionization chambers exposed to ions and deduced the collection efficiency as

\[
f_{\text{Jaffe}} = \frac{y_1}{y_2} \exp(-y_1) [\text{Ei}(y_1 + \ln(1 + y_2)) - \text{Ei}(y_2)] \tag{4}
\]

with

\[
y_1 = \frac{8\pi D}{\alpha N_0}, \quad y_2 = \frac{2dD}{\mu b^2 E}, \quad \text{and} \quad N_0 = \frac{\text{LET}}{W}, \tag{5}
\]

where Ei denotes the exponential integral, \( d \) is the distance between the collecting electrodes, \( E \) is the strength of the electric field parallel to the ion tracks, and \( N_0 \) is the linear ion density where \( W \) is the mean energy expended in air per formed ion pair taken from IAEA TRS-398.

The recombination coefficient \( \alpha \), ion mobility \( \mu \), and diffusion constant \( D \) are listed in Kanai et al. (1998). Kaiser et al. (2012) demonstrated that the Gaussian track structure applied in the Jaffé theory gave a better agreement with experiments than traditional amorphous track structure models. The Gaussian track radius \( b \) is depending on the LET and is implemented as suggested by Rossumme et al. (2017).

A.2 Numerical approach: IonTracks

IonTracks relies on amorphous track structure theory in the same manner as the Jaffé theory and models the density of charge carriers in a track as

\[
n(r) = \frac{N_0}{\pi b^2} \exp \left( -\frac{r^2}{b^2} \right), \tag{6}
\]
where $r$ is the radial distance from the center, $N_0$ is given in eq. (5). $b$ is, as for the Jaffé theory in section 2.2.1, derived from Rossomme et al. (2017) for a given proton LET. The movement of charge carriers in the ionization chamber in an electric field $\vec{E}$ is governed by

$$\frac{\partial n_\pm}{\partial t} = D \nabla^2 n_\pm \mp \mu \left( \vec{E} \cdot \nabla n_\pm + n_\pm \nabla \cdot \vec{E} \right) - a n_+ n_-$$

(7)

where $n_\pm$ denotes the densities of the positive and negative charge carriers respectively (Thomson, 1899). The first term in the brackets corresponds to the charge carrier drift in the externally applied electric field whereas the last term in the brackets represents the space-charge screening of the charged particles exerted by neighbouring charge carriers.

The space-charge screening effects in air are traditionally neglected for low-LET beams (Boag et al., 1996) corresponding to therapeutic relevant proton beams. The first term on the right-hand side is the charge carrier diffusion and the last a sink term corresponding to the recombination. IonTracks computes the recombination by solving eq. (7) subject to conditions given by the dose rate and the ion track structures as modelled by eq. (6). The movement of charge carriers inside the ionization chamber in the externally applied electric field induces a current which potentially affects the charge collection at the electrodes. An effect which currently is unaccounted for in IonTracks.

References


