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Including the power regulation strategy in aerodynamic optimization of wind turbines for increased design freedom

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Abstract

In wind turbine optimization, the standard power regulation strategy follows a constrained trajectory based on the maximum power coefficient. It can be updated automatically during the optimization process by solving a nested maximization problem at each iteration. We argue that this model does not take advantage of the load alleviation potential of the regulation strategy and additionally requires significant computational effort. An alternative approach is proposed, where the rotational speed and pitch angle control points for the entire operation range are set as design variables, changing the problem formulation from nested to one-level. The nested and one-level formulations are theoretically and numerically compared on different aerodynamic blade design optimization problems for AEP maximization. The aerodynamics are calculated with a steady-state blade element momentum method. The one-level approach increases the design freedom of the problem and allows introducing a secondary objective in the design of the regulation strategy. Numerical results indicate that a standard regulation strategy can still emerge from a one-level optimization. Second, we illustrate that novel optimal regulation strategies can emerge from the one-level optimization approach. This is demonstrated by adding a thrust penalty term and a constraint on the maximum thrust. A region of minimal thrust tracking and a peak-shaving strategy appear automatically in the optimal design.

KEYWORDS

control co-design, multi-disciplinary analysis and optimization, power regulation strategy, wind turbine blade design

1 | INTRODUCTION

Designing a modern wind turbine is a complex engineering task, due to the multi-disciplinary nature and the uncertain environment it must operate in. Many different objectives and requirements need to be considered during the design process: power extraction, reduction of costs, stability, noise, and so forth. In addition, the designer has to take into account the different disciplines involved with the goals of (i) accurately forecast the behavior of the final design and (ii) take advantage of the potential couplings between disciplines to improve the design. This makes numerical optimization an ideal tool to be used for wind turbine design. Numerous studies have focused on the development of numerical tools for optimization with the goal of handling the multi-disciplinary aspect of wind turbine design. For example, Bottasso et al[1] take in account the aero-elastic coupling in the blade, Ashuri et al[2] design simultaneously the rotor and the tower, and Bortolotti et al[3] allow the integrated design of the entire turbine.

With the ability to use couplings in the optimization, innovative wind turbine designs emerge that take advantage of new load alleviation techniques. For rotor design, the rotor size is limited by loads. Therefore, larger rotors with increased energy production can be designed when new load alleviation techniques are applied. Passive bend-twist coupling, a passive load alleviation technique, can be achieved by exploiting aerodynamic and structural interactions.[4,5] McWilliam et al[6] showed how further improvements could be achieved when bend-twist coupling is used in association with flaps with quasi-static control. This work suggests that the active load alleviation from the flaps and the passive load alleviation are coupled and give better performances when used together. However, there is a need to develop numerical tools to include quasi-static control design as a form of active load alleviation in wind turbine optimization in order to explore this topic further.

Including control design during the design process of dynamic systems in a so-called co-design optimization, has already been studied by Allison and Herber.[7] Control co-design (CCD) is a promising field for wind energy due to the importance of control in wind turbine dynamics and the complexity of the system.[8] In the study mentioned previously, flaps are used as a form of quasi-static control. A more established type of quasi-static control in wind energy is the power regulation strategy (also called control strategy or control set points). It describes the choice of rotational speed and pitch angle for a range of wind speeds to regulate power production. The most widely used regulation strategy is described by two operational regions: in region II, below rated wind speed, it follows the maximum power coefficient; in region III, above rated wind speed, the rated power is maintained while the rotational speed is constant.

In modern wind turbine optimization frameworks, the regulation strategy is often designed using this two-regions strategy, see, for example, Ashuri et al,[2,9] or using a transition region in the presence of a noise constraint as described by Bottasso et al,[10] and used in Bottasso et al,[1,11] and Bortolotti et al.[3] In these optimization frameworks, the regulation strategy is derived from the calculation of the maximum power coefficient in a nested optimization problem. It is also called a bilevel optimization, where the two levels of the problems can have different objectives. Typically, the optimum design of the lower level problem is decided independently from the upper level objective.[12] Therefore, when the regulation strategy is calculated through this type of nested optimization, there is the underlying assumption that the regulation strategy is designed for the only goal of power coefficient maximization.

However, studies have shown that the power regulation strategy can be designed for various objectives relevant to wind energy, including load alleviation. The peak-shaving or thrust-clipping method consists in pitching the blades before rated wind speed in order to respect a thrust constraint.[13,14] Markou and Larsen[15] have developed a storm-control or soft cut-out method to increase the operational range of the wind turbine for improving the robustness and stability of the energy production. Various de-rating studies focus on the different possible regulation strategies in region III. Lio et al[16] compare the performance of four different regulation strategies on blade fatigue and drive-train loads. Van Der Hoek et al[17] show that all the de-rating regulation strategies in the study reduce ultimate and fatigue load reduction to different degrees.

Most modern wind turbines are designed not only for power output maximization but also for limitation of fatigue damage, grid stability, reduction of cost of energy, and so forth. Therefore, when the regulation strategy is designed based only on power coefficient maximization, its load alleviation potential is not used for the design of the global system. In other words, the design freedom is reduced by ignoring the control coupling. In order to take full advantage of the coupling between power regulation strategy and loads on the turbine, the regulation strategy needs to be designed simultaneously with the system it controls. For example, Madsen et al[18] have set the pitch angle and rotational speed evolution in region II as design variables of the problem resulting in the optimizer automatically finding a peak-shaving region in the presence of a thrust constraint. However, there is, to the best of the authors' knowledge, no publicly available study where the entire regulation strategy is included as a design variable in wind turbine optimization. This knowledge is important to understand and quantify the benefits of using a control co-design approach through the design of the power regulation strategy, especially in the rated region.

This work focuses on the couplings between the design of the power regulation strategy and the wind turbine aerodynamics design. A method to include the regulation strategy in aerodynamic design optimization of wind turbine blades without using a nested optimization is presented. Instead of using a pre-defined regulation strategy, the constraints on the evolution of the rotational speed and the pitch angle are made explicit. The parametrization of the regulation strategy is chosen to allow full design freedom in the entire wind speed range of operation. This formulation of the optimization is called the one-level formulation, as opposed to the bilevel or nested formulation. The objective of this study is to investigate and quantify the potential benefits of this method compared with a nested formulation. In particular, it will look at the computational effort and improvements that can be achieved by exploiting increased design freedom. For this purpose, two sets of blade design and regulation optimization problems are presented. The first set of problems consists of two equivalent AEP maximization problems, one with the nested formulation and the other with the proposed one-level formulation. A second set of optimization problems is used to show how the aerodynamic optimum changes when the constraints on the regulation strategy are loosened.

The terminology used in this work is different than the terminology used to describe multi-disciplinary design optimization (MDO) architectures.[19] For MDO architectures, the focus is put on how the analysis is done for the different disciplines involved. In a simultaneous analysis and design (SAND) architecture, the state equations are set as constraint in the optimization, while a multi-disciplinary feasible (MDF) or nested analysis and design (NAND) architecture, the state equations are solved at each iteration of the optimization. In this work, we address how to determine the power regulation strategy in an optimization setting. The power regulation strategy is not associated to a state equation but it is usually determined by the maximum power coefficient, calculated with a nested optimization problem. We introduce a new method that is not based on a nested optimization. Therefore, the terminology used in this work describes the presence or not of a nested optimization, and does not refer to the analysis of the different disciplines involved.

The first part of the paper lists the assumptions and the models used for the aerodynamics and for the regulation strategy, and describes precisely the optimization problems of interest. The second part reports the comparison of the nested and one-level formulations, underlying benefits and drawbacks of each. The limitations and applications of this work are then discussed in a third part.

2 | MODELS AND PROBLEM FORMULATIONS

We focus on the minimum set of industrially relevant models to answer the research questions. Blade design optimization typically include unsteady analysis to have an accurate description of loads. In this study, our focus is to highlight the coupling between power regulation strategy and aerodynamic performances, and not to reproduce state-of-the-art blade design results. Since the regulation strategy is based on the steady-state of the system, a steady-state BEM model is used (Section 2.1). The choice of regulation strategy is linked to the desired power curve. We explain in Section 2.2 how different regulation strategies can give the same power output and how this degree of freedom can be used for load alleviation. In the optimization problems studied in this work, the regulation strategy can be designed through a nested optimization or added as design variable in order to take advantage of this degree of freedom. The two types of formulations are presented and compared in Section 2.3.

2.1 | Aerodynamic model

The aerodynamic model represents the impact of the wake of the wind turbine on its aerodynamic properties, here with an induced wind on the blades. It is calculated using a steady-state Blade Element Method (BEM) with a Prandtl tip loss correction.[20] We use a simplified version of the model presented by Madsen et al.[21] In this implementation, the axial induction factor is a third-order polynomial function of the thrust coefficient. The dependency on the polar grid is not taken in account, and the yaw error is assumed to be null. The wind speed V is assumed to be constant in time and uniform over the rotor area.

In order to ensure reproducibility of the results, Appendix A reports in details the equations used to calculate the aerodynamic power P, the thrust T, the power coefficient C_P and the thrust coefficient C_T from the value of the wind speed, the rotational speed Ω and the pitch angle θ.

2.2 | Model for the regulation strategy

The power regulation strategy is described in this work by the choice of the rotational speed Ω_i and the pitch angle θ_i for each wind speed V_i in the operational range. The reference regulation strategy is characterized by two regions. In region II, the pitch angle is constant, the rotational speed increases in order to follow the maximum power coefficient. In region III, the rotational speed is constant and the pitch angle is increased to maintain the power to the rated power (see Figure 1). Throughout this paper, this strategy is named *Const-Ω*.

The regulation strategy can be represented on the power coefficient surface as a function of the tip speed ratio $\lambda = \Omega R/V$, where R is the rotor radius. Figure 2 illustrates the trajectory of the Const-Ω strategy on this surface. Region II is represented by the point of maximum power coefficient, noted C_P^{II}. Region III is represented by the dashed line. In this region, maintaining rating power means that the power coefficient $C_P^{\mathrm{III}}(V)$ decreases.

The Const-Ω regulation strategy is calculated from the pitch angle θ^{II} and tip speed ratio λ^{II} giving the maximum power coefficient C_P^{II}, and from the rated wind speed $V_r = \sqrt[3]{\frac{2P_r}{\rho A C_p^{II}}}$, with ρ the air density and A the rotor area. The pitch angle and rotational speed $(\boldsymbol{\theta}, \boldsymbol{\Omega})$ at different wind speeds $V_1, \ldots, V_m$ can be expressed mathematically as follow:

$$\mathcal{F} : (\theta^{II}, \lambda^{II}, V_r) \mapsto \begin{cases} \theta_i = \theta^{\mathrm{II}} & \text{and} \quad \Omega_i = \lambda^{\mathrm{II}} \dfrac{V_i}{R} \text{if} V_i \leq V_r \\ P(\Omega_i, \theta_i, V_i) = P_r, \theta_i > \theta^{\mathrm{II}} & \text{and} \quad \Omega_i = \lambda^{\mathrm{II}} \dfrac{V_r}{R} \text{if} V_i > V_r \end{cases} \tag{1}$$

When $V_i \leq V_r$, the pitch angle and rotational speed are set to follow C_P^{II}: the pitch is constant and the rotational speed increases linearly. When $V_i > V_r$, the rotational speed is constant and as a consequence the tip speed ratio follows $\lambda(V_i) = \lambda^{II} V_r / V_i$. For this choice of tip speed ratio, two choices of pitch angle give the same power coefficient $C_P^{\mathrm{III}}(V_i)$ due to the properties of C_P: one below θ^{II} (pitch-to-stall) and one above (pitch-to-feather). Choosing the pitch-to-feather strategy means that for each wind speed, there is a unique choice of tip speed ratio and pitch angle giving

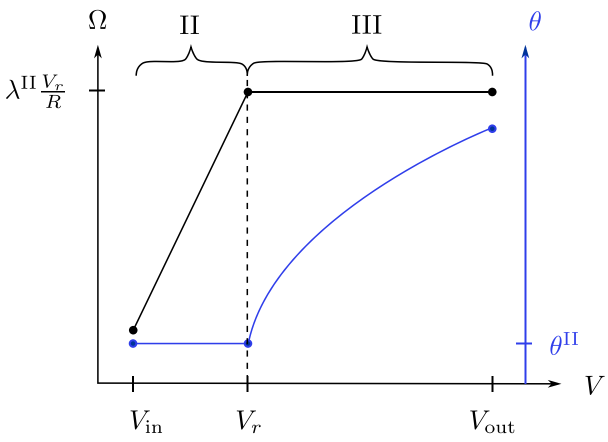

FIGURE 1 Power regulation strategy for Const-Ω: rotational speed and pitch angle evolution. There are two regions: region II below rated wind speed V_r and region III above

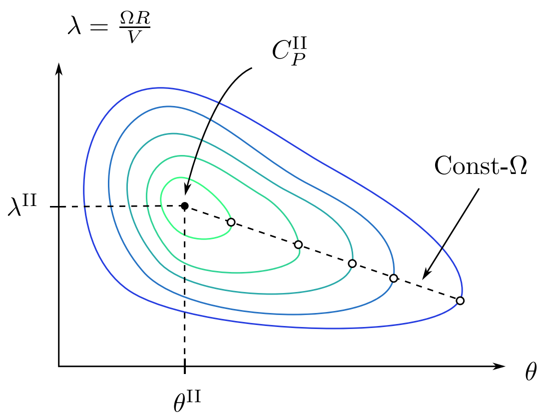

FIGURE 2 Trajectory of the regulation strategy Const-Ω on the power coefficient surface as a function of tip speed ratio (λ) and pitch angle (θ). The black point marks the maximum of the surface

the rated power. In addition, the pitch angle is defined by an inverse problem on the power. In other words, the pitch angle is defined implicitly with an equation on the power and its value can be found by solving this equation.

The Const-Ω strategy is not the only strategy giving the maximum power capture. In region III, there is an infinite number of pairs (λ, θ) giving the same power coefficient. The Const-Ω strategy makes this choice unique by setting the rotational constant and the pitch angle always above θ^{II}. Other possible strategies are: maximizing the rotational speed, maintaining the tip speed ratio constant or minimizing the thrust coefficient, see, for example, Mirzaei et al[22] and Lio et al.[16] Figure 3 represents the later: An isoline of C_P is drawn on top of contour lines of the thrust coefficient surface. Contrary to a Const-Ω strategy, the choice of the pair (λ, θ) is defined implicitly as the minimum of the thrust coefficient along the C_P isoline. This type of regulation strategy can be extended to any quantity that varies with the pitch and the tip speed ratio. It is a method to include a secondary objective in the design of the regulation strategy, for example aerodynamic torque. In order for the regulation strategy to be defined uniquely, it is necessary that the isolines of this secondary objective are not superposed to the isolines of C_P. From a design perspective, it means that the secondary objective and the power should be competing objectives.

2.3 | Problem formulations

Two sets of optimization problems are designed to compare the nested formulation to the one-level formulation. The first set consist of two equivalent optimization problems for maximization of the Annual Energy Production (AEP) (Problems (3) and (4)). It is used to compare the two formulations on their computational effort. The second set consists of two AEP maximization problems with thrust penalty (Problems (7) and (8)). It is used to demonstrate that the one-level optimization can incorporate additional problem information to generate totally new power regulation strategies.

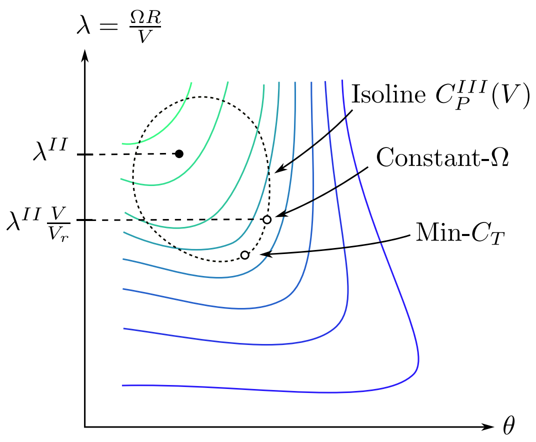

FIGURE 3 Contour lines of the C_T surface (colored lines) and isoline of C_P (dotted line)

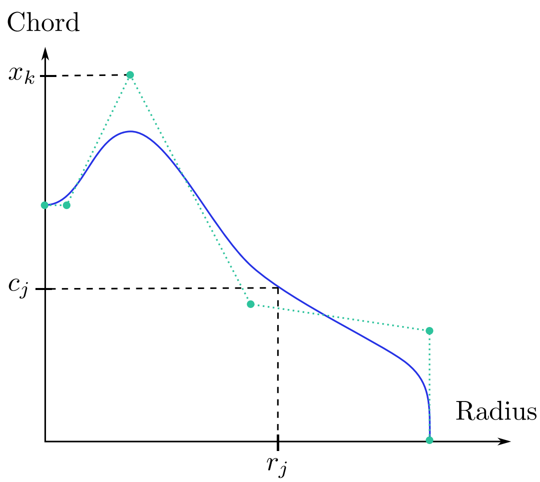

FIGURE 4 Representation of the B-spline parametrization of the chord distribution. The circles represent the control points

2.3.1 | Design variables

The planform design of the blade is parametrized using cubic B-splines.[23] This type of parametrization allows to define a smooth surface with a small number of control points and to describe a wide variety of shapes. Spline interpolation is used in several works in the domain of wind turbine blade optimization, see, for example, previous works.[1,2,24,25] The control points for the chord and twist distribution are the planform design variables of the problem and gathered under the notation $\boldsymbol{x} = \{\boldsymbol{x}^c, \boldsymbol{x}^\beta\}$. The chord $\boldsymbol{c}$ and twist $\boldsymbol{\beta}$ at each radius r_j are calculated using the spline function, as illustrated in Figure 4. Considering the discretized nature of the problem, a matrix $\mathbf{N}_3$ is constructed representing the B-spline basis function evaluated at each radial position. The chord and twist are then calculated as

$$\boldsymbol{c}(\boldsymbol{x}^c) = \mathbf{N}_3 \cdot \boldsymbol{x}^c, \boldsymbol{\beta}(\boldsymbol{x}^\beta) = \mathbf{N}_3 \cdot \boldsymbol{x}^\beta$$

In the nested formulation of the optimization problem, only the control points for the chord and twist distribution are design variables. In the one-level formulation, the value of the rotational speed and the pitch angle at discretized wind speeds in the operational range $[V_1, V_m]$ are also design variables. They are noted $\boldsymbol{\Omega}$ and $\boldsymbol{\theta}$ and represent the regulation strategy. The rated wind speed is also a design variable in one problem studied in this work.

2.3.2 | AEP maximization

In the first set of optimization problems, an AEP maximization problem is formulated in two ways: one-level and nested formulations. The one-level problem is constructed to give the same solution as the nested one. The two formulations are different in the way the algorithm finds the optimal design. Therefore, this problem is used to outline the differences in computational effort. The objective function is the AEP, where P is the power calculated from the planform design $\mathbf{x}$, the rotational speed Ω_i, the pitch angle θ_i and the wind speed V_i.

$$AEP(\mathbf{x},\boldsymbol{\Omega},\boldsymbol{\theta}) = \sum_{i=1}^{m} w_i P(\Omega_i,\theta_i,V_i,\mathbf{x}) \tag{2}$$

The factors w_i represent the cumulative distribution function of a Weibull distribution for the wind speeds. The rated wind speed and the rated rotational speed are not fixed in the optimization problem, in order to keep the problem as general as possible. The sets $\mathcal{S}_x, \mathcal{S}_\Omega$ and $\mathcal{S}_\theta$ represent bounds on the control points, rotation speed and pitch design variables respectively.

The nested formulation problem is reported below.

$$\underset{\mathbf{x}\in\mathcal{S}_x}{\text{maximize}}\ AEP(\mathbf{x},\boldsymbol{\Omega}(\mathbf{x}),\boldsymbol{\theta}(\mathbf{x})) \tag{3a}$$

$$\text{subject to}\ \ \boldsymbol{\Omega}(\mathbf{x}) \in \mathcal{S}_\Omega, \boldsymbol{\theta}(\mathbf{x}) \in \mathcal{S}_\theta, V_r(\mathbf{x}) \in [V_1, V_m] \tag{3b}$$

$$\text{with}\qquad (\lambda^{II}(\mathbf{x}),\theta^{II}(\mathbf{x})) = \arg\max_{\lambda,\theta}\{C_p(\lambda,\theta,\mathbf{x})\} \tag{3c}$$

$$C_p^{II}(\mathbf{x}) = \max_{\lambda,\theta}\{C_p(\lambda,\theta,\mathbf{x})\} \tag{3d}$$

$$V_r(\mathbf{x}) = \sqrt[3]{\frac{2P_r}{\rho A C_p^{II}(\mathbf{x})}} \tag{3e}$$

$$(\boldsymbol{\Omega}(\mathbf{x}),\boldsymbol{\theta}(\mathbf{x})) = \mathcal{F}(\lambda^{II}(\mathbf{x}),\theta^{II}(\mathbf{x}),V_r(\mathbf{x}),\mathbf{x}) \tag{3f}$$

Equations (3c), (3d), and (3e) represent the lower optimization level to find the maximum power coefficient for a given design $\mathbf{x}$ and the corresponding tip speed ratio, pitch angle and rated wind speed. These three values are then used in Equation (3f) to calculate the value of the pitch angle and rotational speed at each wind speed V_i so that it respects the strategy Const-Ω (see Equation (1)).

In the one-level formulation, the rotational speed, the pitch angle and the rated wind speed are design variables instead of being calculated in a nested optimization.

$$\underset{\mathbf{x}\in\mathcal{S}_x,\boldsymbol{\Omega}\in\mathcal{S}_\Omega,\boldsymbol{\theta}\in\mathcal{S}_\theta,V_r\in[V_1,V_m]}{\text{maximize}} AEP(\mathbf{x},\boldsymbol{\Omega},\boldsymbol{\theta}) \tag{4a}$$

$$\text{subject to}\qquad P(\Omega_i,\theta_i,V_i,\mathbf{x}) \le P_r,\ i=1,\ldots,m \tag{4b}$$

$$\theta_i \le \theta_{i+1}, \qquad i=1,\ldots,m-1 \tag{4c}$$

$$\Omega_i = \Omega_m, \qquad \forall i,\ V_i > V_r \tag{4d}$$

$$P(\Omega_m,\theta_1,V_r,\mathbf{x}) = P_r \tag{4e}$$

$$\Omega_1 V_r = \Omega_m V_1 \tag{4f}$$

Different constraints are set in order to have equivalence between problems (3) and (4). The equivalence comes from the fact that the objective function is based on power. If the optimization seeks to maximize the power, the regulation strategy in the one-level formulation will necessarily follow the maximum power coefficient when possible, that is, when the wind speed is below rated. The regulation strategy follows $(\theta^{II},\lambda^{II})$, just as in the nested formulation. Above rated wind speed, the rated power is accessible and therefore the power is maintained at rated. Equations (4d) and (4e) define the value of the rated wind speed and the rotational speed in region III. Equation (4f) ensures that the tip speed ratios used for the power output at the wind speeds V_1 and V_r are the same. Since the same pitch angle is used below V_r, the two equations enforce that the power coefficient is the same for the two wind speeds. It implicitly defines V_r as the wind speed where rated power is reached while following the maximum C_P. In addition, Ω_m is defined to correspond to the optimal tip speed ratio. Finally, above V_r, the equality constraints (4b) and (4c) enforce that the pitch increases and the rotational speed is kept constant. More details about the equivalence between the two formulations can be found in Appendix B.

In the set of constraints on the rotational speed, represented by Equation (4d), the number of constraints depends on the rated wind-speed V_r. This results in a non-standard nonlinear optimization problem with a varying set of constraints, which is particularly difficult to solve. In order to avoid this issue, an estimate of the rated wind speed V_r^{est} is used, and Equation (5) is implemented instead.

$$\Omega_i = \Omega_m, \ \forall i \ V_i > V_r^{est} \tag{5}$$

As long as V_r and V_r^{est} belong to the same wind speed interval $[V_q, V_{q+1}]$, it is equivalent to use Equation (4d) or Equation (5) for the constraint. In practice, a value for the rated wind speed estimate can be obtained by first using the rated wind speed of the initial design, and then run the optimization with a coarse wind speed discretization.

2.3.3 | AEP maximization with thrust penalty

In the second set of optimization problems, a penalty term on the thrust and constraints on the tip speed are added. The penalty term represents a load alleviation objective, since high thrust is associated with high ultimate loads and leads to higher fatigue damage. The one-level and nested problems are no longer equivalent and can give different optimal designs. The objective function now contains two competing objectives: maximizing AEP and minimizing the thrust distribution.

$$f(\boldsymbol{x},\boldsymbol{\Omega},\boldsymbol{\theta}) = AEP(\boldsymbol{x},\boldsymbol{\Omega},\boldsymbol{\theta}) - \frac{\eta}{m}\sum_{i=1}^{m} T(\Omega_i,\theta_i,V_i,\boldsymbol{x}) \tag{6}$$

The penalty parameter $\eta > 0$ is a tuning parameter and does not have a specific meaning on the physical system. It can be interpreted as a measure of how much the optimization will value a reduction of average thrust compared with an increase in AEP. The nested problem resembling Problem (3) with a modified objective function and the additional tip speed constraints isbn

$$\underset{\boldsymbol{x} \in \mathcal{S}_x}{\text{maximize}} \ f(\boldsymbol{x},\boldsymbol{\Omega}(\boldsymbol{x}),\boldsymbol{\theta}(\boldsymbol{x})) \tag{7a}$$

$$\text{subject to } \boldsymbol{\Omega}(\boldsymbol{x}) \in \mathcal{S}_\Omega, \boldsymbol{\theta}(\boldsymbol{x}) \in \mathcal{S}_\theta, V_r(\boldsymbol{x}) \in [V_1, V_m] \tag{7b}$$

$$\Omega_i(\boldsymbol{x}) R \leq v_{tip}^{max}, \quad i = 1,\ldots,m \tag{7c}$$

$$\text{with} \qquad \boldsymbol{\Omega}(\boldsymbol{x}), \boldsymbol{\theta}(\boldsymbol{x}), V_r(\boldsymbol{x}) \text{ calculated using Equations (3c) to (3f)} \tag{7d}$$

In Problem (7), the regulation strategy is calculated in the same way as in Problem (3). The one-level AEP maximization problem with thrust penalty is reported below.

$$\underset{\boldsymbol{x} \in \mathcal{S}_x, \boldsymbol{\Omega} \in \mathcal{S}_\Omega, \boldsymbol{\theta} \in \mathcal{S}_\theta}{\text{maximize}} \quad f(\boldsymbol{x},\boldsymbol{\Omega},\boldsymbol{\theta}) \tag{8a}$$

$$\text{subject to} \quad \Omega_i R \leq v_{tip}^{max}, \qquad i = 1,\ldots,m \tag{8b}$$

$$P(\Omega_i,\theta_i,V_i,\boldsymbol{x}) \leq P_r, \ i = 1,\ldots,m \tag{8c}$$

Contrary to Problem (4), the regulation strategy is not constrained to follow a constant rotational speed or a pitch-to-feather strategy in the rated region. As discussed earlier, there are infinite solutions along the isoline for C_P that are valid solutions for region III. In Problem (4) where the Const-Ω regulation strategy is used, the solution is made unique by fixing the rotational speed above rated wind speed. Relaxing this constraint makes the problem ill-posed; that is, the optimum is possibly not unique. For this reason, the one-level formulation needs to be constructed carefully to guarantee that the optimum is unique. In Problem (8), introducing the thrust penalty as a secondary objective leads to a unique solution again, as explained in Section 2.2. It is possible to make the problem well-posed without a penalty term but by adjusting the problem formulation in a different way. This is an open question, and the solution should be adapted to the wind turbine design requirements.

3 | NUMERICAL METHODS AND IMPLEMENTATION

Different numerical methods are used in this work, represented in Figure 5 and explained below. The upper level optimization of Problems (3) and (7) and the one-level optimization problems (4) and (8) are solved using the interior point line search filter method implemented in IPOPT[26] version 3.12. This framework is chosen because it is widely used in numerous applications and is updated on a regular basis. Further details on the numerical methods implemented can be found in Nocedal and Wright.[27]

3.1 | Solving the BEM equations

The BEM equations are particularly difficult to solve due to their non-linear nature. The fixed-point algorithm is commonly used in the field of wind energy, but can have convergence problems.[28] Brent's algorithm has been used to solve the BEM equation with very good performance compared with other algorithms.[29] However, it is difficult to extend this algorithm to multi-disciplinary problems. Instead, Newton's method is chosen in this work because of its advantageous convergence properties.

More precisely, the algorithm used to solve the BEM equation is a Newton-Raphson method with backtracking line search combined with a trust region. It is necessary to implement a trust region to ensure that the algorithm stays in the feasible domain of the BEM equations. In addition, the Newton-Raphson algorithm can fall into an unstable region. This type of exception is handled by using non-linear successive over-relaxation[30] once a certain number of iterations is reached.

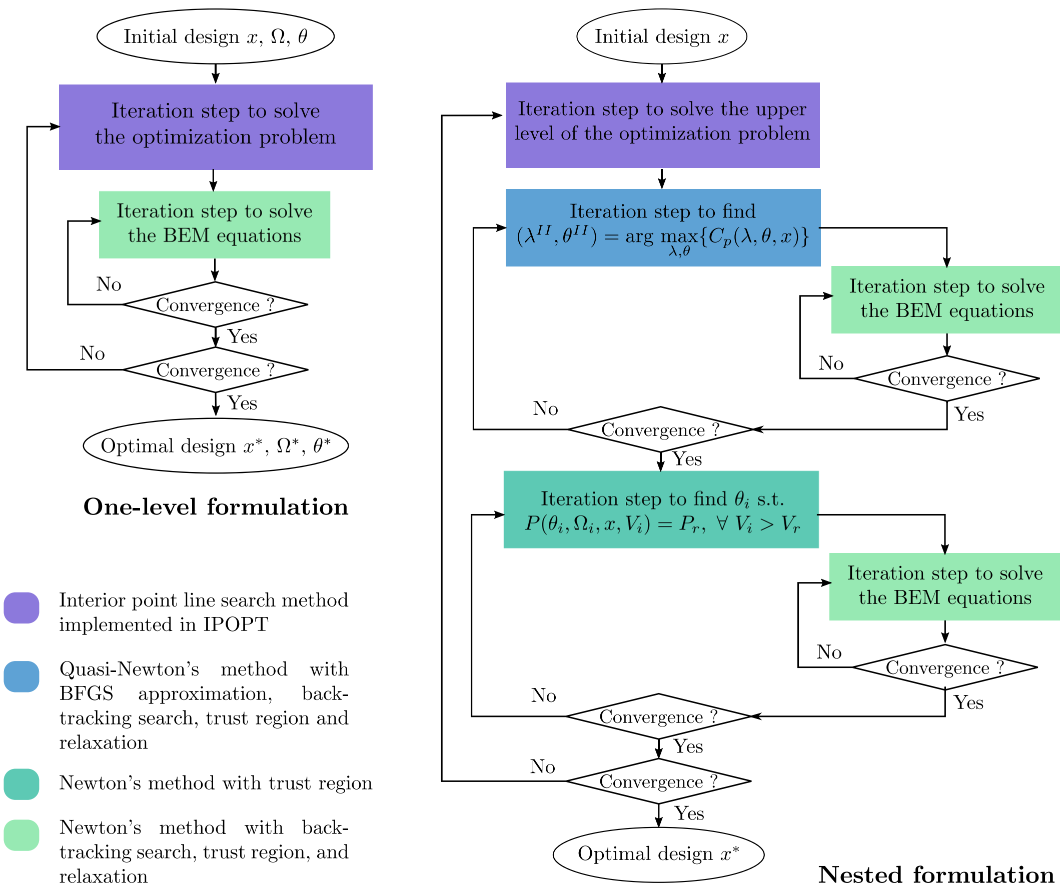

FIGURE 5 Numerical methods used in the one-level and nested optimization problems

The Newton-Raphson method uses the Jacobian of the residual equation, which is computed exactly. The Jacobian can become poorly conditioned, for example, when the angle of attack has high values. In that case, we use an algorithm based on the Levenberg-Marquardt algorithm[31] to recover the step direction.

3.2 | C_P maximization problem

The problem of finding the maximum power coefficient (Equation (3d)) is solved with a quasi-Newton's method.[27] In this problem, the algorithm is applied to find the root of the norm of the gradient of C_P. Evaluating exactly the Hessian of C_P using finite-difference is computationally expensive, Instead, we use the BFGS approximation where estimation of the Hessian is improved at each iteration using the first-order derivatives of C_P. Here as well, we use a trust region, backtracking line search and relaxation to improve the practical convergence properties of the method.

3.3 | Inverse problem to find the pitch angle

Equation (1) is solved using a Newton-Raphson method combined with a trust region.

3.4 | Design sensitivity

The aerodynamic model described in Section 2.1 is implemented so that analytic gradients can be computed. In the one-level problems, the gradient of the objective function and of the constraints are computed using analytic gradients by default.

In the nested formulation problems, the C_P maximization problem and the inverse problem on the pitch angle are solved using analytic gradients. However, in the upper level of the optimization problem, the gradient of the objective function is computed using central finite-difference (FD). Implementing analytic gradient for the upper level requires the second-order sensitivity on the aerodynamic model. Calculating second-order gradient is difficult both computationally and implementation-wise. Even if a significant speed-up is expected when using analytic gradient in the upper optimization, it does not balance with the significant implementation effort required.

4 | RESULTS

This section contains the results of three numerical experiments based on the optimization problems described in the Section 2. First, the nested and one-level AEP maximization problems (3) and (4) are compared for computational efforts and optimization performances. Next, the AEP maximization problems with thrust penalty ((7) and (8)) are compared in order to underline the difference in optimal designs for the two formulations in the presence of a secondary objective. Finally, a third experiment is conducted to show the flexibility of the one-level formulation, by adding constraints to the AEP maximization problem with thrust penalty. Table 1 reports the parameters common to the three numerical experiments. The wind speeds chosen are uniformly discretized between V_{in} and V_{out}. The airfoil data is based on the FFA-W3-XXX series.[32] The blades have the relative thickness distribution of the IEA 10 MW reference wind turbine.[33] The initial design of the blade is based on the aerodynamic design of the IEA 10 MW reference wind turbine. In addition, the chord and twist distribution are constrained so that the chord at the blade root has the same value as the initial design, and the twist at the blade tip is set to zero. There is no pre-bend, pre-cone or shaft tilt. The optimizations are run on a high-performance computer on one node with 32 cores used, with a AMD EPYC 7351 processor running at 2.9 GHz using 8GB of RAM.

4.1 | Computational effort

The two optimization problems compared are described by Equations (3) and (4). Table 2 reports the performances of the optimization algorithm when applied to the nested and one-level formulations, respectively, for a number of wind speeds $m = 22$. The number of constraints listed includes bounds on the design variables, linear and non-linear constraints. The optimization with the one-level formulation is run with analytic gradients and with gradients approximated with central finite-differences in order to have a fair comparison with the nested formulation that runs only with finite-difference approximations. The number of design variables and constraints is higher in the one-level formulation because (i) the rotational speed and pitch angle are added as design variables and (ii) constraints are added corresponding to Equations (4b) to (4f). The five constraints in the nested formulation correspond to bounds on the rotational speed and pitch angle vectors and on the rated wind speed

TABLE 1 Parameters of the optimization problems

Description	Notation	Value
Rated power	P_r	10.0 MW
Rotor radius	R	99 m
Hub radius	R_h	2.3 m
Cut-in wind speed	V_{in}	4 m/s
Cut-out wind speed	V_{out}	25 m/s
Number of blade elements	n	20
Air density	ρ	1.225 kg/m^3

TABLE 2 Characteristics of the optimization process for the nested formulation and one-level formulation with analytic gradient and finite-difference gradient for the AEP maximization problems (3) and (4) with $m = 22$

	Nested	One-level	One-level (FD)
No. of variables	11	56	56
No. of constraints	5	66	66
No. of function calls	56	289	186
No. of gradient calls	40	76	73
No. of interior-point iterations	39	75	72
Total time (s)	23,595.9	910.5	40,519.1

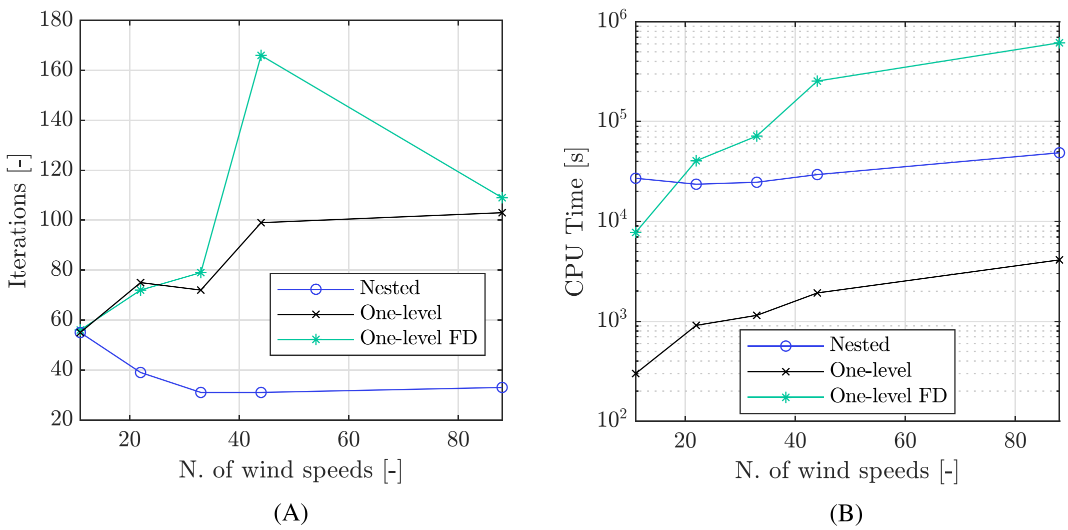

FIGURE 6 Characteristics of the optimization process for the nested and one-level formulations of the AEP maximization problem for different number of wind speeds. (A) Number of interior point iterations and (B) CPU time

(Equation (3b)). The rotational speed and pitch angle are constructed to be monotonically increasing following Equation (1); therefore, it is sufficient to apply the boundary condition only on the first and last element of both quantities. This allows to reduce the computational effort of the nested formulation.

Figure 6A,B reports the number of iterations and CPU time for the two formulations and for different values of m. When using the one-level formulation, the optimization algorithm converges with more iterations with and without analytic gradient compared with the nested formulation. This is because in the nested formulation, at each iteration, the lower optimization level finds the maximum C_P and the corresponding regulation strategy. In other words, part of the computational effort is delegated towards the lower level of the optimization. Instead, in the one-level formulation, the optimization algorithm needs to find simultaneously the planform design and the regulation strategy, which results in more complexity and thus more iterations. In addition, the increase in iterations when the gradient is calculated using finite-difference is likely due to inaccuracies introduced in the derivative approximation.

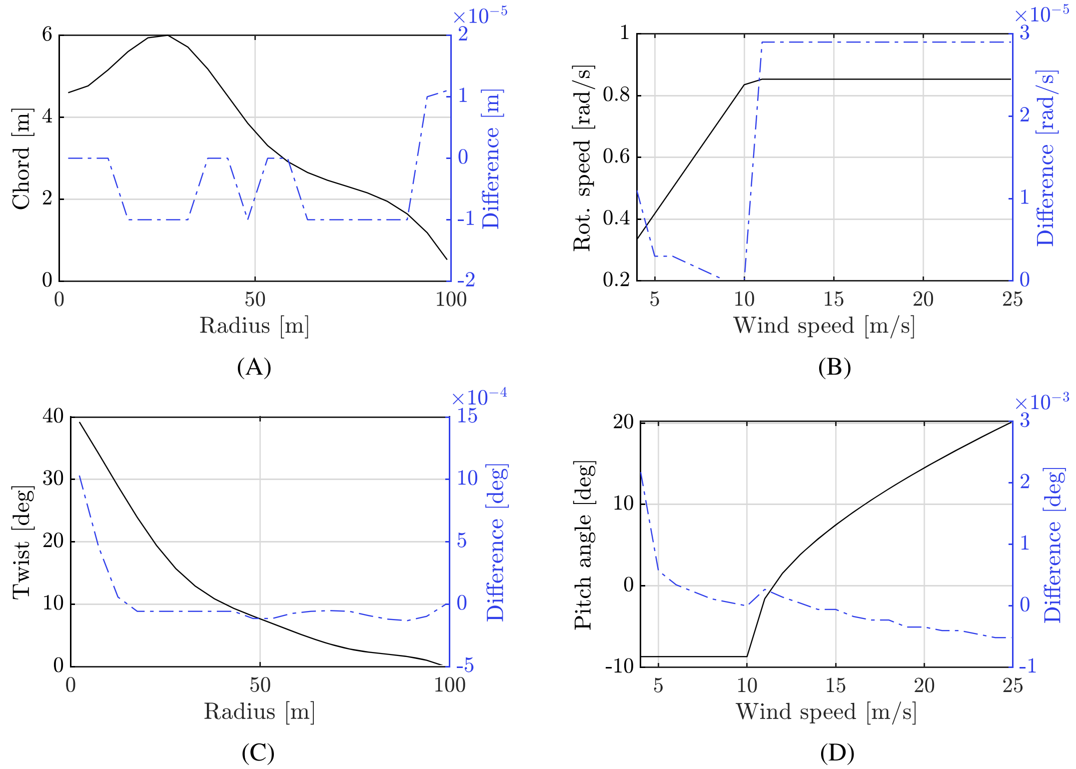

FIGURE 7 Optimal designs for the nested and one-level formulations of the AEP maximization problems (7) and (8). The difference between the two optimal design is shown as a dot-dotted line. (A) Chord distribution, (B) rotational speed strategy, (C) twist angle distribution, and (D) pitch angle strategy

In terms of computational time, the optimization with the one-level formulation converges in 15 min compared with 6 h 33 min with the nested formulation and 11 h 15 min for the one-level formulation with finite-difference gradient, counted in CPU time and for $m = 22$. The computational time is significantly increased by using finite-difference gradient approximation. In the case of the nested formulation, the computational time is still 25 times higher than the one-level formulation, even though the number of constraints and the number of design variables is significantly lower. The difference in CPU time decreases when the number of wind speeds m increases, but stays large. This is because the parameter m directly impacts the number of design variables in the one-level formulation, whereas it only impacts the lower level of the optimization in the nested formulation.

As mentioned in Section 3, implementing analytic gradients in the nested formulation is difficult, while it is straightforward for the one-level formulation. The one-level formulation should therefore be preferred over the nested formulation considering the large speed-up brought by the use of analytic gradients.

Figure 7 reports the optimal design found for the nested and one-level formulations described by Equations (3) and (4). Since the two designs are visually identical, only one curve is shown along with the difference between the two designs. This is an expected result since the one-level optimization problem is constructed to be equivalent to the nested problem. The two designs are not exactly identical because the feasibility and optimality tolerances are small but larger than zero. The rotational speed and pitch angle evolution follow the Const-Ω strategy. The optimal AEP is 34.176 GWh. These results show that the one-level approach can still give a Const-Ω regulation strategy for a properly posed problem.

4.2 | Comparison of the nested and one-level formulations in the presence of a thrust penalty

The second numerical experiment compares the nested and one-level formulations in Equations (7) and (8) for an AEP maximization problem with thrust penalty. The upper bound on the rotational speed is set so that the maximum tip speed is $v_{tip}^{max} = 90$ m/s. The scaling parameter is set to $\eta = 0.355$ GWh/N. The number of wind speeds is set to $m = 22$.

The optimal designs for the two problems are reported in Figure 8 and in Table 3. The two problems are not equivalent and therefore do not have the same optimum. The one-level problem reaches an objective function value higher than the nested problem, by lowering the average thrust. The regulation strategy is constrained to follow the Const-Ω strategy in the nested problem while it is not constrained in the one-level

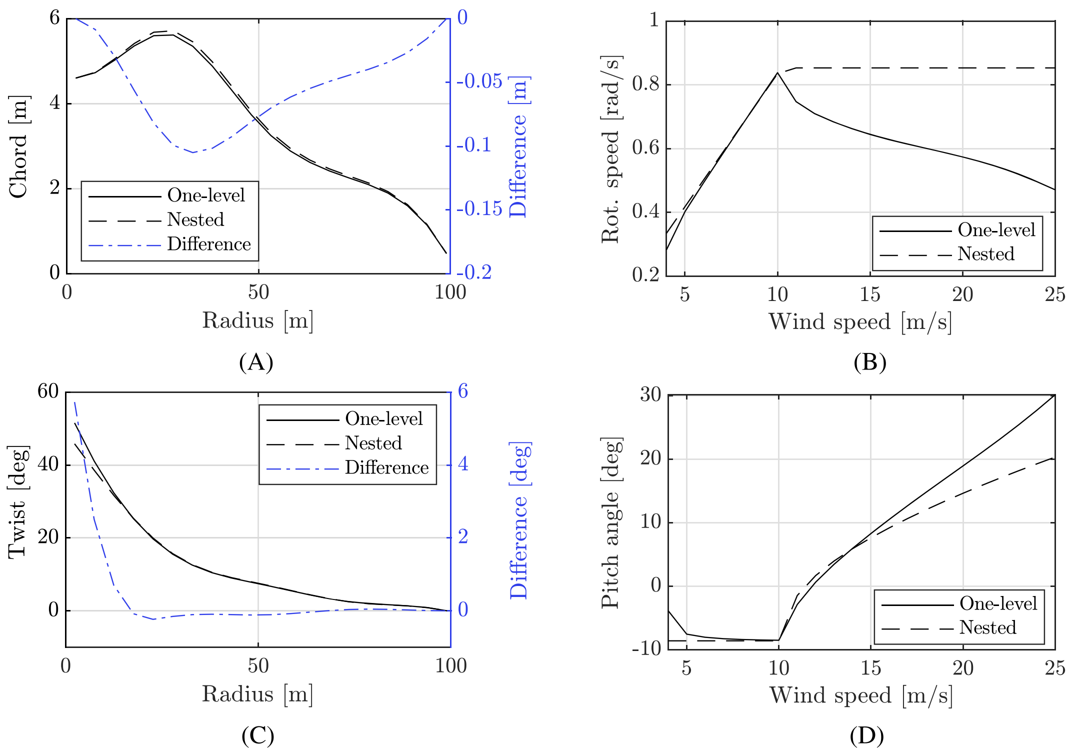

FIGURE 8 Optimal design for the nested and one-level formulations of the AEP maximization problem with thrust penalty (Problems (7) and (8)). (A) Chord distribution, (B) rotational speed strategy, (C) twist angle distribution, and (D) pitch angle strategy

TABLE 3 Characteristics of the optimal designs for the nested and one-level AEP maximization problems with thrust penalty (Problems (7) and (8))

	One-level	Nested
Objective function value	9.534	9.532
AEP (GWh)	34.173	34.175
Average T (kN)	747.078	772.511
C_P^{II} (-)	0.49707	0.49717
λ^{II} (-)	8.295	8.264
θ^{II} (deg)	−8.521	−8.610

strategy, giving more design freedom. Figure 9 shows the two regulation strategies as functions of the tip-speed ratio and the pitch angle, along with the contour plots of the power coefficient and the thrust coefficient. The regulation strategy starts at the maximal power coefficient in the case of the nested formulation. In the one-level formulation, the regulation strategy does not reach the maximum power coefficient: this illustrates the fact that this design reaches a balance between maximizing power and reducing thrust. In addition, the figure shows how the two strategies follow a Const-Ω and Min-C_T strategies respectively above rated (see Figure 3).

The reduction in thrust in the one-level formulation comes from different features of the optimal regulation strategy

- The blade is pitched at low wind speeds
- The rotational speed decreases in the rated power region in order to track the minimum thrust coefficient (as reported on Figure 9)
- The planform is designed with a co-design objective. A change in the planform design changes the C_P and C_T surfaces, while a change in the power regulation strategy changes the trajectory followed on these surfaces. The co-design approach allows to change both in order to find the best ratio of power over thrust in the entire operational range.

Despite not being constrained, the optimal regulation strategy of the one-level problem looks realistic: it can be fitted with a continuous function and has few changes of monotonicity.

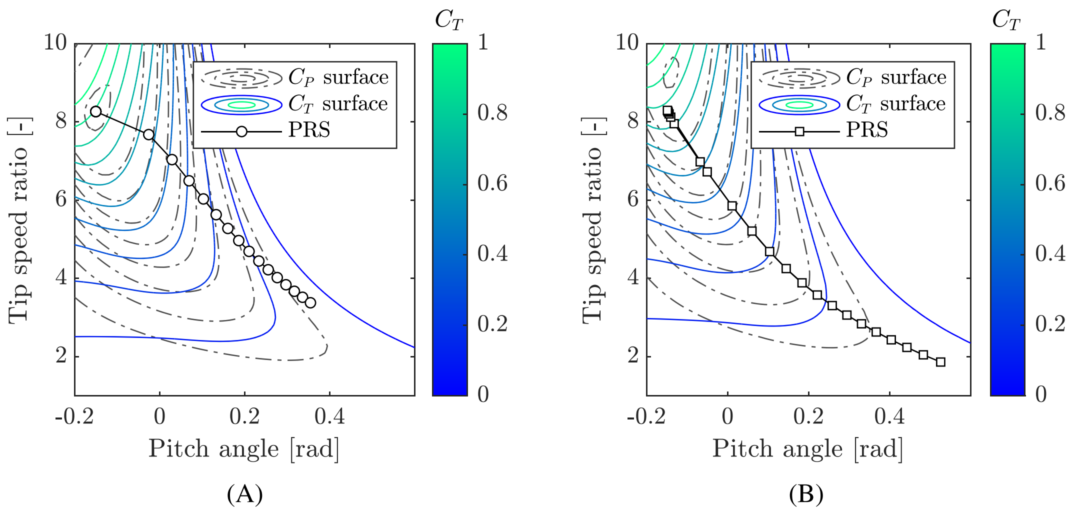

FIGURE 9 Trajectory of the power regulation strategy (PRS) of the optimal design represented on the C_P contour lines (dotted line) and the C_T contour lines (full line) for the nested and the one-level formulations of the AEP maximization problem (Problems (7) and (8)). (A) Nested formulation and (B) one-level formulation

While the power distribution is almost identical between the two cases, the thrust distribution is on average 3% lower in the one-level case. The difference in thrust is small but not negligible. This is due to the fact that the thrust and power are tightly coupled.

The optimum design for the one-level formulation has the same thrust peak as in the nested approach, even though the peak could be eliminated by a peak-shaving strategy. It means that the thrust reduction possible by a peak-shaving strategy is not worth the power loss. A peak-shaving strategy could appear if the parameter η is increased, or if the thrust around the rated wind speed has more weight in the penalty term.

In addition, the reduction of rotational speed presented leads to a significant increase in the rotor torque. From an integrated design perspective, this thrust reduction may not be justified considering the increase in the loads on the gearbox. However, these results demonstrate that the one-level approach can successfully incorporate a secondary objective in the design process. A more realistic problem formulation could include the ultimate loads or fatigue damage instead of thrust, as this is more relevant to rotor design.

4.3 | Power regulation strategy defined by the optimization's constraints and objective

The third numerical experiments objective is to show how the regulation strategy design changes with the addition of constraints with the one-level formulation. Constraints are added on the AEP maximization problem with thrust penalty and with the one-level formulation (Equation (8)). The rotational speed is bounded between 6.0 RPM and $v_{\text{tip}}^{\max}/R$. The thrust is constrained to be below 1.2 kN in order to reproduce a peak-shaving regulation strategy. The number of wind speeds is set to $m = 88$ to increase the precision of the optimal regulation strategy.

Figure 10 reports the optimal regulation strategy and the corresponding thrust and power curves for this problem. Different regions of operations can be identified depending on which constraints are active. In the first region $V \in [4, 6.8]$ m/s, the rotational speed lower bound constraint is active. The blade is pitched in this region in order to respect the rotational speed constraint and reduce the thrust. In the second region $V \in [6.8, 8.6]$ m/s, none of the nonlinear constraints are active, and the regulation strategy follows a constant pitch and a rotational speed increasing linearly with the wind speed. In the third region $V \in [8.6, 11]$ m/s, the thrust constraint is active, which changes the evolution of both the pitch angle and rotational speed. In the fourth region $V \in [11, 22.3]$ m/s, the power constraint is active, and the power regulation strategy minimizes the thrust. Finally, a fifth region $V \in [22.3, 25]$ m/s can be identified where the lower bound on the rotational speed is active.

The third region is a region of peak-shaving. It is designed automatically during the optimization process, from the maximum value of the thrust.

If additional constraints are added, additional regions of operation can appear. For example, a constraint on the maximum torque would limit the rotational speed at high wind speeds, and therefore change the evolution of the pitch angle.

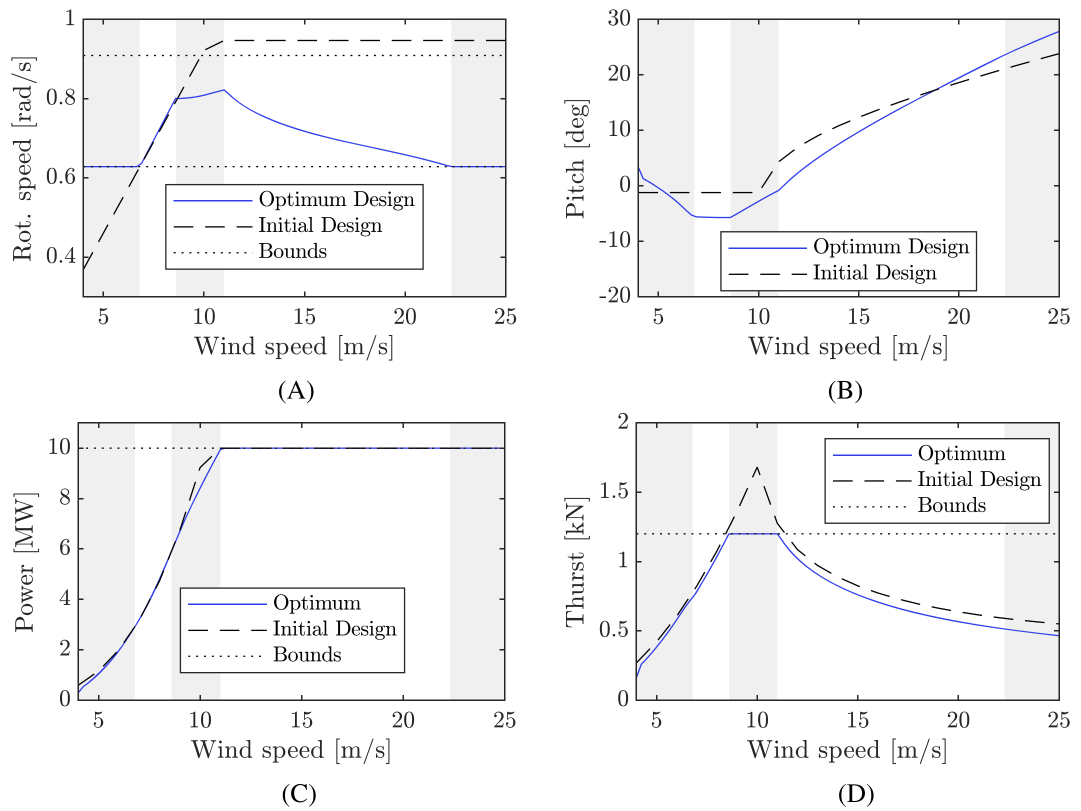

FIGURE 10 Optimal designs obtained with the one-level AEP maximization problem with thrust penalty and constraints on the rotational speed and maximum thrust. Identified regions of operation are represented by shaded areas. (A) Rotational speed strategy, (B) pitch angle strategy, (C) power curve, and (D) thrust curve

5 | DISCUSSION AND POTENTIAL EXTENSIONS

The results presented in this work suggest that using the one-level approach can reduce significantly the computational time of the optimization process. The high computational time required by wind turbine optimization limits the number of design variables and therefore the precision of the final design. It can also be an obstacle for tackling highly coupled problem such as those that arise from integrated design of wind turbines. Reducing the computational time in the aerodynamic design would help overcome these problems.

This method has potential to be beneficial for aero-structural optimization of wind turbine when structural constraints such as stress, strain or fatigue are present. Modern wind turbine designs are now additionally driven by fatigue damage and ultimate load requirements. Further work should focus on how the change of regulation strategy can impact the fatigue damage at high wind speeds. In addition, the fact that the one-level formulation requires less computational effort than the nested formulation could be used to reduce the computational time for existing and future multi-disciplinary frameworks.

The results of this work can also be used in a systems engineering perspective. Large rotors are not based on AEP maximization but on the levelized cost of energy (LCOE), which in-turn balances the power output with load alleviation. This work presents an additional tool for load alleviation above rated in the context of multi-disciplinary design optimization of wind turbines. When there is more design freedom, it is possible to find new and innovative methods for load alleviation that take advantage of how the different load alleviation technologies work together.

The application of the one-level formulation is studied with a simplified problem in this work. First, the relative thickness is not included as a design variable, even though it is common practice in wind turbine blade design optimization. In addition, the regulation strategy is only studied from the perspective of aerodynamic performances. In reality, the regulation strategy needs to be designed with considerations on the generator constraints and on the requirements for power production and grid stability. Further studies on the design of the regulation strategy should focus on its application to industrially relevant problems. This is important in order to understand how this co-design approach can be efficiently used to improve the global design of the wind turbine and not only its aerodynamic aspect. Considering that the one-level approach takes advantage of the couplings between control and the other discipline, there is a potential to improve both the aerodynamics and the structural properties if it is applied to a more complex problem.

The regulation strategy interacts with the control laws. Therefore, constraints on the control laws can be translated into requirements on the regulation strategy. In this work, such requirements are not taken in account. In the Const-Ω strategy, the regulation strategy is often associated with two decoupled controllers: a torque/power controller and a pitch controller.[34] The two controllers can be decoupled because there is no wind speed window where both the pitch angle and the rotational speed need to be adjusted. When the regulation strategy follows a different strategy, the pitch angle and the rotational speed need to be adjusted at the same time and therefore it is not possible to use two decoupled controllers. Instead, a more advanced control law needs to be used, for example, the power-tracking curve controller described by Bottasso et al.[10]

The change of formulation from nested to one-level brings additional challenges. The implementation of the one-level formulation requires that the constraints and the design variables can be easily modified in the wind turbine optimization framework. This level of flexibility is not guaranteed, considering that many state-of-the-art frameworks in the field use black-box analysis code made for numerical simulations. If the design of the regulation strategy is already integrated in the analysis code, it can be very difficult to remove this feature while keeping the rest of the code robust.

To implement the one-level formulation to an existing problem with a nested formulation, the first task is to expose the regulation strategy design variables to the top-level optimization. Second, the inner optimization loop must also be removed. It is advised to test this modified formulation on Problem (4) to verify that the changes still give the expected solution with a Const-Ω regulation strategy. After which, one is free to add additional operational constraints.

6 | CONCLUSIONS

This study presents models to introduce increased design freedom in setting the regulation strategy in the aerodynamic design optimization of variable speed, variable pitch wind turbines using a one-level formulation. Different optimization problems in both the one-level and the nested formulations are studied. The design variables are the twist and chord distribution, as well as the pitch angle and rotational speed evolution for the one-level formulation. Constraints are enforced on the integrated aerodynamic rotor forces. These are calculated using the blade element method. Several conclusions can be drawn from this study:

- A conventional regulation strategy can still emerge in a properly posed problem using the one-level formulation
- The one-level formulation converges in less time for the studied AEP maximization problem instances compared with the nested formulation. This is due to the use of analytic gradients that is facilitated in the one-level formulation.
- Including the regulation strategy in the rated power production region as a design variable can make the optimization problem ill-posed. One solution is to add a secondary objective, for example, penalty on the thrust, to the objective function so that the choice of regulation strategy is driven to a unique point in the design space.
- The one-level formulation allows greater freedom in the regulation strategy compared with the nested formulation. Using this formulation on an AEP maximization problem with thrust penalty and maximum thrust constraint, the optimal regulation strategy shows a region of peak-shaving and a region of thrust minimization at high wind speeds.
- The one-level formulation can automatically identify new regions in optimal regulation strategy. The new regions of operations are defined explicitly by the constraints of the optimization problem.

REPLICATION OF RESULTS

The required blade data is from the IEA 10MW reference wind turbine and is publicly available.[33] The analysis equations are described in this manuscript and in the cited literature. The problem formulations, the problem data, parameters and tolerances are sufficiently well described in the manuscript to allow replication of the numerical results.

The characteristics of the optimization used in this paper are publicly available.[35] The data can be used to validate the results presented. This validation should be feasible with any analysis framework implementing the blade element method as described in Section 2.1 and Appendix A.

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CONFLICT OF INTEREST

The authors state that there are no conflicts of interest.

PEER REVIEW

The peer review history for this article is available at https://publons.com/publon/10.1002/we.2769.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are openly available in figshare at https://doi.org/10.11583/DTU.17033267.

NOMENCLATURE

$\boldsymbol{x}$	vector of the spline control points describing the chord and twist distribution (-)
$\boldsymbol{\theta}$	vector of pitch angles (rad)
$\boldsymbol{\Omega}$	vector of rotational speeds (rad/s)
V_r, V_i	rated wind speed and wind speed at *i*th position in the operational range (m/s)
λ	tip speed ratio (-)
P, P_r	aerodynamic power and rated power (W)
T	rotor thrust (N)
C_P, C_P^{II}	power coefficient and maximum power coefficient (-)
θ^{II}	pitch angle corresponding to the maximum power coefficient (rad)
λ^{II}	tip speed ratio corresponding the the maximum power coefficient (-)
C_T	thrust coefficient (-)
n	number of blade elements
ρ	air density (kg/m^3)
m	number of wind speeds considered in the operational range
R	rotor radius (m)
A	rotor area (m^2)
AEP	annual energy production (Wh)
v_{tip}^{max}	maximum allowed tip speed (m/s)

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A: EQUATIONS USED FOR THE AERODYNAMIC MODEL

This appendix reports in details the aerodynamic model implemented for this work. The equations are based on a reduced form of the BEM theory introduced by Madsen et al.[21]

Each blade is discretized into $n-1$ elements along its span, from hub to rotor radius. The BEM equations for a given element j are described below, in order to highlight the dependency on the axial and tangential induction factors (a_j, a_j'), the pitch angle θ and the rotational speed Ω. The aerodynamic forces $f_{x,j}$ and $f_{y,j}$ on the blade elements are calculated from data describing the lift and drag aerodynamic coefficients c_l and c_d, the norm of the relative wind speed V_{rel} and the chord c_j.

$$f_{x,j} = \frac{1}{2}\rho c_j V_{\text{rel}}^2 (c_d(\alpha_i)\cos(\phi_j) - c_l(\alpha_j)\sin(\phi_j)) \tag{C1a}$$

$$f_{y,j} = \frac{1}{2}\rho c_j V_{\text{rel}}^2 (c_d(\alpha_j)\sin(\phi_j) + c_l(\alpha_j)\cos(\phi_j)) \tag{C1b}$$

where ρ is the air density. The relative wind speed, the local flow angle ϕ_j and the angle of attack α_j depend on the local induction factors (a_j, a_j') and on the rotational speed Ω. In addition, the angle of attack depends on the pitch angle θ and the local structural twist angle β_j as:

$$\alpha_j = \phi_j - \theta - \beta_j \tag{C2}$$

The local torque coefficient $C_{Q,j}$, the local thrust coefficient $C_{T,j}$, the Prandtl tip correction F_j and the corrected local thrust coefficient $C_{TF,j}$ are calculated as

$$C_{Q,j} = \frac{n_B f_{x,j}}{\rho V^2 \pi r_j}\; C_{T,j} = \frac{n_B f_{y,j}}{\rho V^2 \pi r_j} \tag{C3}$$

$$F_j = \frac{2}{\pi}\arccos\left(\exp\left(-\frac{n_B(R-r_j)}{2r_j\sin\phi_j}\right)\right) C_{TF,j} = \frac{C_{T,j}}{F_j} \tag{C4}$$

where $n_B = 3$ is the number of blades. Then, the local induction factors are computed as

$$a_j = \begin{cases} k_3 C_{TF,j}^3 + k_2 C_{TF,j}^2 + k_1 C_{TF,j} + k_0 & \text{if } C_{TF,j} < 2.5 \\ k_5 C_{TF,j} + k_4 & \text{otherwise} \end{cases} \tag{C5}$$

$$a'_j = \frac{C_{Q,j} V}{4(1-a_j) r_j \Omega} \tag{C6}$$

The coefficients k_i, for $i = 0,..,5$ are obtained with a polynomial fit of experimental data and their values can be found in the work by Madsen et al.[21]

Solving the BEM equations means finding the value of the induction factors a_j and a'_j satisfying Equations (C1) to (C6). Following Equations (C5) and (C6), the thrust and torque coefficients $C_{TF,j}$ and $C_{Q,j}$ depend non-linearly on a_j and a'_j. Therefore, the problem needs to be solved using an iterative numerical method. This is described in Section 3.

Once the BEM equations are solved, the power P and the thrust T are calculated with the equations below. They can also be obtained from the power and thrust coefficients C_P and C_T, where R is the rotor radius and $\lambda = \frac{\Omega R}{V}$ is the tip speed ratio.

$$P(\Omega,\theta,V) = \Omega n_B \sum_{j=1}^{n-1} f_{x,j} \frac{(r_j + r_{j+1})}{2}(r_{j+1} - r_j) = \frac{1}{2}\rho\pi R^2 V^3 C_P(\lambda,\theta) \tag{C7}$$

$$T(\Omega,\theta,V) = n_B \sum_{j=1}^{n-1} f_{y,j} \frac{(r_j + r_{j+1})}{2}(r_{j+1} - r_j) = \frac{1}{2}\rho\pi R^2 V^2 C_T(\lambda,\theta) \tag{C8}$$

B: ON THE EQUIVALENCE BETWEEN THE NESTED AND ONE-LEVEL FORMULATIONS FOR THE AEP MAXIMIZATION PROBLEM

In this appendix, we investigate under what conditions and to what extent the two presented AEP maximization problems (3) and (4) are related. For readability, we re-state the two problems and the most relevant sub-problems below as Problem (C9) and Problem (C11) respectively. More precisely, we show that under suitable assumptions, if $\boldsymbol{x}$ is feasible for the nested problem (C9), then $(\boldsymbol{x},\boldsymbol{\Omega}(\boldsymbol{x}),\boldsymbol{\theta}(\boldsymbol{x}),V_r(\boldsymbol{x}))$ is feasible for the one-level problem (C11). We also show that if $(\boldsymbol{x}^*,\boldsymbol{\Omega}^*,\boldsymbol{\theta}^*,V_r^*)$ is an optimal solution of the one-level problem (C11), then $\boldsymbol{x}^*$ is feasible for the nested problem (C9). Here, $\Omega(\boldsymbol{x}),\theta(\boldsymbol{x})$ and $V_r(\boldsymbol{x})$ refer to the power regulation strategy and rated wind speed calculated for the Const-Ω strategy, that is, following Equations (C9e) and (C10).

We assume that for every feasible design $\boldsymbol{x} \in \mathcal{S}_x$, $\boldsymbol{\Omega}(\boldsymbol{x})$ and $\boldsymbol{\theta}(\boldsymbol{x})$ are uniquely defined and that $\boldsymbol{\Omega}(\boldsymbol{x}) \in \mathcal{S}_\Omega$, and $\boldsymbol{\theta}(\boldsymbol{x}) \in \mathcal{S}_\theta$. The bounds on the rotational speed and the pitch angle are assumed to be chosen large enough as not to influence the optimization. In addition, the power coefficient $C_P(\lambda,\theta)$ is assumed to be a continuously differentiable concave function of the pitch angle and the tip speed ratio such that the C_P maximization problem has a unique solution satisfying standard first-order optimality conditions. Empirical evidence suggests that this is an acceptable assumption for many problems from wind energy applications.

The nested AEP maximization problem is

$$\underset{\boldsymbol{x} \in \mathcal{S}_x}{\text{maximize}}\, AEP(\boldsymbol{x},\boldsymbol{\Omega}(\boldsymbol{x}),\boldsymbol{\theta}(\boldsymbol{x})) \tag{C9a}$$

$$\text{subject to } \boldsymbol{\Omega}(\boldsymbol{x}) \in \mathcal{S}_\Omega, \boldsymbol{\theta}(\boldsymbol{x}) \in \mathcal{S}_\theta, V_r(\boldsymbol{x}) \in [V_1, V_m] \tag{C9b}$$

$$\text{with } (\lambda^{II}(\boldsymbol{x}),\theta^{II}(\boldsymbol{x})) = \arg\max_{\lambda,\theta}\{C_p(\lambda,\theta,\boldsymbol{x})\} \tag{C9c}$$

$$C_p^{II}(\boldsymbol{x}) = \max_{\lambda,\theta}\{C_p(\lambda,\theta,\boldsymbol{x})\} \tag{C9d}$$

$$\text{and } V_r(\boldsymbol{x}) = \sqrt[3]{\frac{2P_r}{\rho A C_p^{II}(\boldsymbol{x})}} \tag{C9e}$$

$$\text{and } (\boldsymbol{\Omega}(\boldsymbol{x}),\boldsymbol{\theta}(\boldsymbol{x})) = \mathcal{F}(\lambda^{II}(\boldsymbol{x}),\theta^{II}(\boldsymbol{x}),V_r(\boldsymbol{x}),\boldsymbol{x}) \tag{C9f}$$

The pitch angle and rotational speeds come from

$$\mathcal{F} : (\theta^{II},\lambda^{II},V_r,\boldsymbol{x}) \mapsto \begin{cases} \theta_i = \theta^{II} & \text{and} \quad \Omega_i = \lambda^{II}\dfrac{V_i}{R} \quad \text{if} \quad V_i \le V_r \\ P(\Omega_i,\theta_i,V_i,\boldsymbol{x}) = P_r, \theta_i > \theta^{II} & \text{and} \quad \Omega_i = \lambda^{II}\dfrac{V_r}{R} \quad \text{if} \quad V_i > V_r \end{cases} \tag{C10}$$

The corresponding one-level AEP maximization problem is

$$\underset{\boldsymbol{x} \in \mathcal{S}_x, \boldsymbol{\Omega} \in \mathcal{S}_\Omega, \boldsymbol{\theta} \in \mathcal{S}_\theta, V_r \in [V_1,V_m]}{\text{maximize}} AEP(\boldsymbol{x},\boldsymbol{\Omega},\boldsymbol{\theta}) \tag{C11a}$$

$$\text{subject to} \quad P(\Omega_i,\theta_i,V_i,\boldsymbol{x}) \le P_r, i = 1,\ldots,m \tag{C11b}$$

$$\theta_i \le \theta_{i+1}, i = 1,\ldots,m-1 \tag{C11c}$$

$$\Omega_i = \Omega_m, \forall i, V_i > V_r \tag{C11d}$$

$$P(\Omega_m,\theta_1,V_r,\boldsymbol{x}) = P_r \tag{C11e}$$

$$V_r\Omega_1 = V_1\Omega_m \tag{C11f}$$

The feasible set of the nested problem is in the feasible for the one-level problem

Let $\boldsymbol{x}$ be feasible for the nested problem (C9) and $\boldsymbol{\Omega}(\boldsymbol{x}),\boldsymbol{\theta}(\boldsymbol{x}),V_r(\boldsymbol{x})$ the corresponding rotational speed, pitch angle control set points and rated wind speed. We show that $(\boldsymbol{x},\boldsymbol{\Omega}(\boldsymbol{x}),\boldsymbol{\theta}(\boldsymbol{x}),V_r(\boldsymbol{x}))$ satisfies all constraints of the one-level problem (C11).

In Problems (C9) and (C11), the bounds on the design variable $\boldsymbol{x}$ are the same. In addition, Problem (C9) constrains the rotational speed, pitch angle, and rotational speed to be within the bounds of Problem (C11). Therefore $\Omega(\boldsymbol{x}),\boldsymbol{\theta}(\boldsymbol{x}),V_r(\boldsymbol{x})$ are within the bounds of Problem (C11).

The rotational speed and pitch angle control set points are calculated from Equation (C10). By definition of $\mathcal{F}$, the following conditions are satisfied:

$$\Omega_i(\boldsymbol{x}) = \Omega_m(\boldsymbol{x}),\ \forall i, V_i > V_r$$
$$V_r(\boldsymbol{x})\Omega_1(\boldsymbol{x}) = V_1(\boldsymbol{x})\Omega_m(\boldsymbol{x})$$

The constraint (C11c) in the one-level problem states that the pitch angle should be increasing with the wind speed. The pitch angle evolution satisfies $\theta_i(\boldsymbol{x}) = \theta^{II}(\boldsymbol{x}) = \theta_1(\boldsymbol{x})$ for $V_i \le V_r(\boldsymbol{x})$. Below rated wind speed, the pitch angle is constant and therefore the constraint is satisfied. Above rated wind speed, the pitch angle satisfies $\theta_i(\boldsymbol{x}) \ge \theta^{II}(\boldsymbol{x})$ and is calculated as the solution of an inverse problem on the power. Considering the structure of the power coefficient surface, there are always only two possible choice of pitch angle to reach a certain power, and therefore a certain power coefficient at a given wind speed and at fixed rotational speed. There is one solution above θ^{II} and the other one is below. When the wind speed increases, and when the target power coefficient decreases, the two solutions get further away from θ^{II}. In other words, the pitch angle is monotonically increasing for the pitch-to-feather strategy and monotonically decreasing for the pitch-to-stall strategy. Therefore, since $\boldsymbol{\theta}(\boldsymbol{x})$ is calculated with the pitch-to-feather strategy, it is increasing above rated. Therefore, the pitch angle satisfies the constraint (C11c) on the entire operational range:

$$\theta_i(\boldsymbol{x}) \le \theta_{i+1}(\boldsymbol{x}), i = 1,\ldots,m-1$$

The power production below rated wind speed is

$$\begin{aligned} P(\Omega_i(\mathbf{x}),\theta_i(\mathbf{x}),V_i,\mathbf{x}) = &\ P(\lambda^{II}(\mathbf{x})\frac{V_i}{R},\theta^{II}(\mathbf{x}),V_i,\mathbf{x}) \\ = &\ \frac{1}{2}\rho A V_i^3 C_P^{II}(\mathbf{x}) \\ &\le \frac{1}{2}\rho A V_r(\mathbf{x})^3 C_P^{II}(\mathbf{x}) \\ &\le P_r \end{aligned}$$

The power production above rated is equal to P_r, by construction of $\boldsymbol{\Omega}(\mathbf{x})$ and $\boldsymbol{\theta}(\mathbf{x})$. Therefore, the constraints on the power (C11b) is satisfied.

Finally, the constraint on the power production at $V = V_r(\mathbf{x})$ is satisfied because

$$\begin{aligned} P(\Omega_m(\mathbf{x}),\theta_1(\mathbf{x}),V_r(\mathbf{x}),\mathbf{x}) &= P(\lambda^{II}(\mathbf{x})\frac{V_r(\mathbf{x})}{R},\theta^{II}(\mathbf{x}),V_r(\mathbf{x}),\mathbf{x}) \\ &= \frac{1}{2}\rho A V_r(\mathbf{x})^3 C_P^{II}(\mathbf{x}) \\ &= P_r \end{aligned}.$$

Therefore, if $\mathbf{x}$ is feasible for the nested problem (C9), then $(\mathbf{x},\boldsymbol{\Omega}(\mathbf{x}),\boldsymbol{\theta}(\mathbf{x}),V_r(\mathbf{x}))$ is feasible for the one-level problem (C11).

A solution of the one-level problem is feasible for the nested problem

Let $(\mathbf{x},\boldsymbol{\Omega},\boldsymbol{\theta},V_r)$ be an optimal solution for the one-level problem (C11). We want to show that $\mathbf{x}$ is feasible for the nested problem (C9) and that the rotational speed, pitch angle, and rated wind speed correspond to the one calculated following a Const-Ω regulation strategy, that is,

$$(\boldsymbol{\Omega},\boldsymbol{\theta}) = \mathcal{F}(\theta^{II}(\mathbf{x}),\lambda^{II}(\mathbf{x}),V_r(\mathbf{x}))$$

and

$$V_r = V_r(\mathbf{x}) = \sqrt[3]{\frac{2P_r}{\rho A C_p^{II}(\mathbf{x})}}$$

First, the bounds on the design variables in Problem (C11) correspond to the bounds in Problem (C9), as explained in Section (B.1).

If $(\mathbf{x},\boldsymbol{\Omega},\boldsymbol{\theta},V_r)$ is an optimal solution for Problem (C11), then there is wind speed above which the power constraint is active. This observation comes from the fact that if the rated power is reachable at $V_r \in [V_1, V_m]$, then for all wind speeds above V_r, the rated power is also reachable.

In other words, for all $V_k \ge V_r$, there is θ_k,Ω_k such that $P(\Omega_k,\theta_k,V_k,\mathbf{x}) = P_r$. If there is a wind speed above V_r such that the power of the optimal design is strictly below P_r, then it is a contradiction with the fact that the design is optimal because a higher AEP is reachable.

Therefore, the operational range can be divided into two regions, below rated and above rated. The wind speed at which the divide occurs is noted V_r^*. Note that $V_r^* \le V_r$.

For the region below rated, it is important to note that for a given wind speed and the given design $\mathbf{x}$, the power output has a maximum bound:

$$P(\Omega_i,\theta_i,V_i,\mathbf{x}) \le \frac{1}{2}\rho A V_i^3 C_P^{II}(\mathbf{x}) \tag{C12}$$

In addition, by concavity of C_P, the maximum power coefficient is reached by a unique pair $(\lambda^{II}(\mathbf{x}),\theta^{II}(\mathbf{x}))$ and reachable following the assumption on $\mathcal{S}_x,\mathcal{S}_\Omega,\mathcal{S}_\theta$. Since C_P^{II} is reachable and by assumption that $(\mathbf{x},\boldsymbol{\Omega},\boldsymbol{\theta},V_r)$ is optimal, then it is reached by the optimal solution. One has:

$$\theta_i = \theta^{II} \text{ and } \Omega_i = \lambda^{II}\frac{V_i}{R} \quad \text{for } V_i \le V_r^* \tag{C13}$$

Therefore, the regulation strategy below rated follow the maximum power coefficient, identical to a Const-Ω strategy.

By definition, V_r^* is the first wind speed at which the rated power is reached while following the maximum C_P. Therefore, one has:

$$P\left(\lambda^{II}\frac{V_r^*}{R},\theta_1,V_r^*,\mathbf{x}\right)=P_r=\frac{1}{2}\rho A(V_r^*)^3C_P^{II} \tag{C14}$$

For wind speeds above V_r, the rotational speed is constant following Equation (C11d). Equation (C11f) gives the value of the rotational speed:

$$\Omega_m=V_r\frac{\Omega_1}{V_1}=V_r\frac{\lambda^{II}}{R} \tag{C15}$$

The power output at the wind speed V_r is also equal to rated power because of the constraint (C11e). The expression can be developed to highlight the dependency on the power coefficient, using the value of the rotational speed Ω_m from Equation (C15). By construction of Ω_m, the power coefficient is equal to the maximum power coefficient.

$$P\left(\lambda^{II}\frac{V_r}{R},\theta_1,V_r,\mathbf{x}\right)=P_r=\frac{1}{2}\rho AV_r^3C_P(\lambda^{II},\theta^{II},\mathbf{x})=\frac{1}{2}\rho AV_r^3C_P^{II} \tag{C16}$$

Using Equations (C14) and (C16), one has $V_r=V_r^*=\sqrt[3]{\frac{2P_r}{\rho AC_p^{II}(\mathbf{x})}}=V_r(x)$.

In addition, these results show that the rotational speed $\boldsymbol{\Omega}$ follows a Const-Ω strategy for the entire operational range: below rated it follows the tip-speed ratio giving the maximum power coefficient and is constant above rated.

Finally, consider the pitch angle evolution above V_r. The constraint (C11c) on the pitch angle ensures that the choice of pitch angle in the rated region follows the pitch-to-feather strategy. In addition, the power above rated is equal to P_r. As explained in the previous section, it is equivalent to have the constraint $\theta_i\le\theta_{i+1}, i=1,..,m-1$ or the constraint $\theta^{II}\le\theta_i, i=1,..,m$. Therefore, the pitch angle evolution above rated is equal to the one calculated with the Const-Ω strategy.

Conclusion

In the two previous section, we have shown:

- If $\boldsymbol{x}$ is feasible for the nested problem (C9), then $(\mathbf{x},\boldsymbol{\Omega}(\mathbf{x}),\boldsymbol{\theta}(\mathbf{x}),V_r(\mathbf{x}))$ is feasible for the one-level problem (C11).
- If $(\boldsymbol{x}^*,\boldsymbol{\Omega}^*,\boldsymbol{\theta}^*,V_r^*)$ is an optimal solution of the one-level problem (C11), then $\boldsymbol{x}^*$ is feasible for the nested problem (C9).

Following the first point, a solution optimal for Problem (C9) is feasible for Problem (C11).

The one-level problem and the nested problem have the same objective function. Therefore, the optimal solution for the one-level problem is also optimal for the nested problem.

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