An Extension to a Filter Implementation of a Local Quadratic Surface for Image Noise Estimation

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Abstract

Based on regression analysis this paper gives a description for simple image filter design. Specifically 3×3 filter implementations of a quadratic surface, residuals from this surface, gradients and the Laplacian are given. For the residual a 5×5 filter is given also. It is shown that the 3×3 filter for the residual gives low values for horizontal and vertical lines and edges as opposed to diagonal ones. Therefore an extension including a rotated version of the filter for the residual to ensure low values for lines and edges in all directions is suggested. It is also shown that the 5×5 filter for the residual does not give low values for lines and edges in any direction. The performance of six noise models including the ones mentioned above are compared. Based on visual inspection of results from an example using a generated image (with all directions and many spatial frequencies represented) it is concluded that if striping is to be considered as a part of the noise, the residual from a 3×3 median filter seems best. If we are interested in a salt-and-pepper noise estimator the proposed extension to the 3×3 filter for the residual from a quadratic surface seems best. Simple statistics and autocorrelations in the estimated noise images support these findings.

1. Introduction

An important task in image analysis is the estimation of noise content. This is often done by means of methods that estimate the noise directly from the data such as residuals from mean or median filters. In this paper filter implementations of residuals from a quadratic surface in small (3×3 and 5×5) windows are given. Also 3×3 filters for gradients and the Laplacian are given. A quadratic surface is chosen for its expected ability to adapt to both dark and bright lines and edges in all directions. A potential drawback of this otherwise desired characteristic is 1) the adaption to the horizontal (across-track) striping often present in data from whisk-broom scanners such as the (spaceborne) Landsat Thematic Mapper (TM) and the Airborne Visible and Infra-Red Imaging Spectrometer (AVIRIS), and the Digital Airborne Imaging Spectrometer (DAIS), and 2) the adaption to vertical (along-track) striping often present in data from push-broom scanners such as the (spaceborne) SPOT High Resolution Visible (HRV) and the Compact Airborne Spectrographic Imager (*casi*), and the (airborne) Reflective Optics System Imaging Spectrometer (ROSIS). For a description and comparison of six noise estimators including the mean and median filters see [9]. Space limitations allow neither examples with real world data nor the application of the noise models to e.g. comparisons between the MAF/MNF transformation, [11, 4, 1, 3, 8, 6, 10, 7], and the principal components (PC) transformation.

2. Quadratic Surface in a 3×3 window

In this section filters based on a quadratic surface in a 3×3 window are deduced. This is done by means of regression analysis.

2.1. Residuals

Consider a 3×3 univariate image $\boldsymbol{Z}$ with the following numbering of pixels

Z_1	Z_2	Z_3
Z_4	Z_5	Z_6
Z_7	Z_8	Z_9

where Z_i denotes the pixel value at location $[X_i, Y_i]^T$. We wish to predict the pixel values Z_i by a quadratic surface which is chosen for its expected ability to adapt to both dark and bright lines and edges in all directions present in the 3×3 image

$$\begin{aligned} Z_i &= \theta_0 + \theta_1 X_i + \theta_2 Y_i + \theta_3 X_i^2 + \theta_4 Y_i^2 + \theta_5 X_i Y_i + \varepsilon_i, \\ i &= 1, \ldots, 9 \end{aligned}$$

where θ_i are parameters to be estimated. In matrix notation we get

$$\boldsymbol{Z} = \begin{bmatrix} 1 & X_1 & Y_1 & X_1^2 & Y_1^2 & X_1Y_1 \\ 1 & X_2 & Y_2 & X_2^2 & Y_2^2 & X_2Y_2 \\ 1 & X_3 & Y_3 & X_3^2 & Y_3^2 & X_3Y_3 \\ 1 & X_4 & Y_4 & X_4^2 & Y_4^2 & X_4Y_4 \\ 1 & X_5 & Y_5 & X_5^2 & Y_5^2 & X_5Y_5 \\ 1 & X_6 & Y_6 & X_6^2 & Y_6^2 & X_6Y_6 \\ 1 & X_7 & Y_7 & X_7^2 & Y_7^2 & X_7Y_7 \\ 1 & X_8 & Y_8 & X_8^2 & Y_8^2 & X_8Y_8 \\ 1 & X_9 & Y_9 & X_9^2 & Y_9^2 & X_9Y_9 \end{bmatrix} \boldsymbol{\theta} + \boldsymbol{\varepsilon} = \boldsymbol{X\theta} + \boldsymbol{\varepsilon}$$

where $\boldsymbol{Z} = [Z_1, \ldots, Z_9]^T$, $\boldsymbol{\theta} = [\theta_0, \ldots, \theta_5]^T$ and $\boldsymbol{\varepsilon} = [\varepsilon_1, \ldots, \varepsilon_9]^T$. For the norm of the residuals $\boldsymbol{\varepsilon}$ which have dispersion $\boldsymbol{\Sigma}$ we get

$$||\boldsymbol{\varepsilon}||^2 = \boldsymbol{\varepsilon}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\varepsilon} = (\boldsymbol{Z} - \boldsymbol{X\theta})^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{Z} - \boldsymbol{X\theta})$$

which is a quadratic function in $\boldsymbol{\theta}$. To minimise we differentiate and set the derivative to zero

$$\frac{\partial}{\partial \boldsymbol{\theta}} ||\boldsymbol{\varepsilon}||^2 = 2\boldsymbol{X}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{X\theta} - 2\boldsymbol{X}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{Z} = \boldsymbol{0}$$

$$\boldsymbol{\theta} = (\boldsymbol{X}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{X})^{-1} \boldsymbol{X}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{Z}.$$

If we think of the 3×3 image as a window moving over a larger image to make local estimates of a quadratic surface and consider the center pixel Z_5 as the origin of a coordinate system that moves with the window, we get

$$\begin{aligned} Z_5 &= \begin{bmatrix} 1 & X_5 & Y_5 & X_5^2 & Y_5^2 & X_5Y_5 \end{bmatrix} \boldsymbol{\theta} + \varepsilon_5 \\ &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \boldsymbol{\theta} + \varepsilon_5 \\ &= \theta_0 + \varepsilon_5. \end{aligned}$$

We see that with fixed $\boldsymbol{\Sigma}$ we can implement the quadratic surface estimator for the center pixel as a filter: multiply the first row of $(\boldsymbol{X}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{X})^{-1} \boldsymbol{X}^T \boldsymbol{\Sigma}^{-1}$ with $\boldsymbol{Z}$ to obtain θ_0. With $\boldsymbol{\Sigma} = \sigma^2 \boldsymbol{I}$ where σ^2 is the variance of ε_i and $\boldsymbol{I}$ is the identity matrix we get (Gaussian or other weights can be applied through another choice of $\boldsymbol{\Sigma}$)

$$\boldsymbol{X} = \begin{bmatrix} 1 & -1 & 1 & 1 & 1 & -1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & -1 & -1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$$

$$\boldsymbol{X}^T \boldsymbol{X} = \begin{bmatrix} 9 & 0 & 0 & 6 & 6 & 0 \\ 0 & 6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 & 0 \\ 6 & 0 & 0 & 6 & 4 & 0 \\ 6 & 0 & 0 & 4 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

$$(\boldsymbol{X}^T \boldsymbol{X})^{-1} = \begin{bmatrix} \frac{5}{9} & 0 & 0 & -\frac{1}{3} & -\frac{1}{3} & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\ -\frac{1}{3} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ -\frac{1}{3} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \end{bmatrix}.$$

In this case we are interested in θ_0 so we need the first row of $(\boldsymbol{X}^T \boldsymbol{X})^{-1} \boldsymbol{X}^T$ only (below we shall need more rows)

$$(\boldsymbol{X}^T \boldsymbol{X})^{-1} \boldsymbol{X}^T = \qquad (1)$$

$$\begin{bmatrix} -\frac{1}{9} & \frac{2}{9} & -\frac{1}{9} & \frac{2}{9} & \frac{5}{9} & \frac{2}{9} & -\frac{1}{9} & \frac{2}{9} & -\frac{1}{9} \\ -\frac{1}{6} & 0 & \frac{1}{6} & -\frac{1}{6} & 0 & \frac{1}{6} & -\frac{1}{6} & 0 & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & -\frac{1}{6} & -\frac{1}{6} & -\frac{1}{6} \\ \frac{1}{6} & -\frac{1}{3} & \frac{1}{6} & \frac{1}{6} & -\frac{1}{3} & \frac{1}{6} & \frac{1}{6} & -\frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ -\frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 & -\frac{1}{4} \end{bmatrix}.$$

Because of the special structure of $(\boldsymbol{X}^T \boldsymbol{X})^{-1}$ (the three zeros in columns two, three and six of the first row) and $\boldsymbol{X}^T$ (rows two, three and six corresponding to the X-, Y- and XY-terms in the estimator) θ_0 corresponding to the first row of $(\boldsymbol{X}^T \boldsymbol{X})^{-1} \boldsymbol{X}^T$ is independent of the X-, Y- and XY-terms.

The estimator for Z_5 which we call $\hat{Z}_5$ is

$$\hat{Z}_5 = \begin{bmatrix} -\frac{1}{9} & \frac{2}{9} & -\frac{1}{9} & \frac{2}{9} & \frac{5}{9} & \frac{2}{9} & -\frac{1}{9} & \frac{2}{9} & -\frac{1}{9} \end{bmatrix} \boldsymbol{Z}$$

or written as a filter for $\boldsymbol{Z}$

$$\begin{array}{|c|c|c|} \hline -\frac{1}{9} & \frac{2}{9} & -\frac{1}{9} \\ \hline \frac{2}{9} & \frac{5}{9} & \frac{2}{9} \\ \hline -\frac{1}{9} & \frac{2}{9} & -\frac{1}{9} \\ \hline \end{array}.$$

For the residual $\varepsilon_5 = Z_5 - \hat{Z}_5$ we get

$$\begin{array}{|c|c|c|} \hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} - \frac{1}{9} \begin{array}{|c|c|c|} \hline -1 & 2 & -1 \\ \hline 2 & 5 & 2 \\ \hline -1 & 2 & -1 \\ \hline \end{array} = \frac{1}{9} \begin{array}{|c|c|c|} \hline 1 & -2 & 1 \\ \hline -2 & 4 & -2 \\ \hline 1 & -2 & 1 \\ \hline \end{array}$$

which is separable. We note that the weights add to zero and that all rows and columns have weights that add to zero. Hence horizontal and vertical ideal one-pixel wide lines and ideal edges have residuals equal to zero. Diagonal ones do not. The filter for the residual is proportional to the filter postulated in [5]. The filter weights are independent of the chosen coordinate system.

2.2. Gradients

To obtain gradients in the column and row directions we find

$$\frac{\partial Z_5}{\partial X_5} = \theta_1 + 2\theta_3 X_5 + \theta_5 Y_5 = \theta_1$$

$$\frac{\partial Z_5}{\partial Y_5} = \theta_2 + 2\theta_4 Y_5 + \theta_5 X_5 = \theta_2.$$

From equation 1 we get the filters

$$\frac{1}{6}\begin{array}{|r|r|r|}\hline -1 & 0 & 1 \\ \hline -1 & 0 & 1 \\ \hline -1 & 0 & 1 \\ \hline \end{array} \quad \text{and} \quad \frac{1}{6}\begin{array}{|r|r|r|}\hline 1 & 1 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -1 & -1 \\ \hline \end{array}.$$

2.3. The Laplacian

To obtain the Laplacian we find

$$\frac{\partial^2 Z_5}{\partial X_5^2} = 2\theta_3 \quad , \quad \frac{\partial^2 Z_5}{\partial Y_5^2} = 2\theta_4 \quad \text{and} \quad \nabla^2 = 2(\theta_3 + \theta_4).$$

From equation 1 we get the filter

$$\frac{1}{3}\begin{array}{|r|r|r|}\hline 2 & -1 & 2 \\ \hline -1 & -4 & -1 \\ \hline 2 & -1 & 2 \\ \hline \end{array}.$$

3. Quadratic Surface in a 5×5 window

As in the 3×3 case for a 5×5 window with this pixel numbering

Z_1	Z_2	Z_3	Z_4	Z_5
Z_6	Z_7	Z_8	Z_9	Z_{10}
Z_{11}	Z_{12}	Z_{13}	Z_{14}	Z_{15}
Z_{16}	Z_{17}	Z_{18}	Z_{19}	Z_{20}
Z_{21}	Z_{22}	Z_{23}	Z_{24}	Z_{25}

we get for the center pixel $Z_{13} = \theta_0 + \varepsilon_{13}$ with $\boldsymbol{Z} = [Z_1, \ldots, Z_{25}]^T$, $\boldsymbol{\theta} = [\theta_0, \ldots, \theta_5]^T = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{Z}$ ($\boldsymbol{X}$ is now 25 rows by 6 columns), $\boldsymbol{\varepsilon} = [\varepsilon_1, \ldots, \varepsilon_{25}]^T$.

For the residual $\varepsilon_{13} = Z_{13} - \hat{Z}_{13}$ we get

$$\frac{1}{175}\begin{array}{|r|r|r|r|r|}\hline 13 & -2 & -7 & -2 & 13 \\ \hline -2 & -17 & -22 & -17 & -2 \\ \hline -7 & -22 & 148 & -22 & -7 \\ \hline -2 & -17 & -22 & -17 & -2 \\ \hline 13 & -2 & -7 & -2 & 13 \\ \hline \end{array}.$$

We note that the weights add to zero and that no rows or columns have weights that add to zero. We see that we do not obtain the same desired filter characteristics as with the 3×3 filter.

In a similar fashion we could obtain filters for gradients etc. with other filter sizes.

4. An Extension to the 3×3 filter

In the 3×3 case we saw that horizontal and vertical lines and edges have low residuals as opposed to diagonal ones. Therefore the following extension to the simple 3×3 filter is suggested: first apply the simple filter derived above, then apply the same filter rotated 45° and use the residual closer to zero. This ensures low residuals for both horizontal, vertical and diagonal lines and edges.

5. Example

An example with a generated image is shown. The generated image features all directions and many spatial frequencies. Results from six noise estimators are shown, namely

1. simple differencing with the immediate north and east neighbours corresponding to this 2×2 filter (with the center pixel placed in the lower left corner)

   $$\frac{1}{2}\begin{array}{|r|r|}\hline -1 & 0 \\ \hline 2 & -1 \\ \hline \end{array};$$

   this is used in [2] which is a commercial software package that offers an MNF transformation;

2. residual from a 3×3 mean filter

   $$\frac{1}{9}\begin{array}{|r|r|r|}\hline -1 & -1 & -1 \\ \hline -1 & 8 & -1 \\ \hline -1 & -1 & -1 \\ \hline \end{array};$$

   it is seen that this filter results in high residuals for lines and edges in all directions;

3. residual from a 3×3 median filter;

4. residual from the 3×3 filter for a quadratic surface derived above

   $$\frac{1}{9}\begin{array}{|r|r|r|}\hline 1 & -2 & 1 \\ \hline -2 & 4 & -2 \\ \hline 1 & -2 & 1 \\ \hline \end{array};$$

5. residual from the 3×3 filter for a quadratic surface derived above rotated 45°

   $$\frac{1}{9}\begin{array}{|r|r|r|}\hline -2 & 1 & -2 \\ \hline 1 & 4 & 1 \\ \hline -2 & 1 & -2 \\ \hline \end{array};$$

   it is seen that this filter results in low residuals for diagonal lines and edges;

6. residual from the extended 3×3 filter for a quadratic surface proposed in this paper (i.e., choose the residual closer to 0 from the two filters mentioned immediately above).

The generated 256×256 image which features all directions and many spatial frequencies is shown in Figure 1. The left column shows the generated image itself (top) and the generated image with pseudo-random, independent, zero-mean, Gaussian noise with a signal-to-noise ratio equal to one added (bottom). The right column shows the corresponding noise images as estimated by means of the residual from a quadratic surface in a 5×5 window. Table 1 shows simple statistics and autocorrelations between E-W, N-S, SW-NE, SE-NW neighbours and their mean value in these images.

Figures 2 and 3 show noise as estimated from the six 3×3 filters given above stretched linearly between minimum and maximum, Figure 2 on the image in Figure 1 top-left, and Figure 3 on the image in Figure 1 bottom-left. Tables 2 and 3 show simple statistics and autocorrelations between E-W, N-S, SW-NE, SE-NW neighbours and their mean value in the images in Figures 2 and 3, respectively.

6. Discussion and Conclusions

Figure 1 shows that the 5×5 quadratic surface residual filter does not give low values in any direction for intermediate and high spatial frequencies. Figures 2 and 3 show that 1) noise model 1 leaves more visual structure than the other noise models, noise estimated from models 1, 2, 3 and 5 contain horizontal and vertical striping (which for model 5 complies with the filter weights), noise estimated from model 4 contains diagonal structure for intermediate and high spatial frequencies, noise estimated from model 5 contains low and intermediate spatial frequency diagonal structure, and noise estimated from model 6 contains intermediate spatial frequency diagonal structure; 2) for noise model 4 we get low estimates in the E-W and N-S directions for all spatial frequencies also with severe noise present (the signal-to-noise ratio is one); 3) for noise model 5 we get low estimates in the SW-NE and SE-NW directions for high spatial frequencies; and 4) for noise model 6 we get a combination of the characteristics of models 4 and 5.

Remarks based on inspection are supported by the simple statistics and autocorrelations calculated. If striping is to be considered as a part of the noise, model 3 seems to be the best noise detector since it picks up both salt-and-pepper noise and striping in all directions at many spatial frequencies. The suggested noise model 6 seems to be the best salt-and-pepper noise detector.

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Table 1. Generated image, top: simple statistics, bottom: autocorrelations between E-W, N-S, SW-NE, SE-NW neighbours and their mean value.

Image	Mean	Stddev	Min	Max	
top-left	–0.00	0.71	–1.00	1.00	
top-right	–0.00	0.56	–1.32	1.31	
bot-left	–0.00	1.00	–3.69	3.65	
bot-right	–0.00	0.85	–3.58	3.70	
Image	E-W	N-S	SW-NE	SE-NW	Mean
top-left	0.37	0.37	0.14	0.14	0.25
top-right	0.07	0.07	–0.06	–0.06	0.01
bot-left	0.18	0.19	0.07	0.07	0.13
bot-right	–0.08	–0.07	–0.11	–0.11	–0.09

Table 2. Generated image noise estimates, top: simple statistics, bottom: autocorrelations between E-W, N-S, SW-NE, SE-NW neighbours and their mean value.

Model	Mean	Stddev	Min	Max	
1	0.00	0.64	–1.76	1.76	
2	–0.00	0.51	–1.05	1.05	
3	–0.00	0.47	–1.51	1.40	
4	–0.00	0.20	–0.97	1.05	
5	0.00	0.38	–1.03	1.03	
6	0.00	0.14	–0.50	0.49	
Model	E-W	N-S	SW-NE	SE-NW	Mean
1	0.12	0.12	0.21	–0.23	0.06
2	0.15	0.15	–0.08	–0.08	0.04
3	0.12	0.12	–0.10	–0.10	0.01
4	–0.19	–0.19	0.04	0.04	–0.08
5	0.29	0.29	–0.12	–0.12	0.09
6	0.00	0.00	–0.06	–0.06	–0.03

Table 3. Generated image (with noise) noise estimates, top: simple statistics, bottom: autocorrelations between E-W, N-S, SW-NE, SE-NW neighbours and their mean value.

Model	Mean	Stddev	Min	Max	
1	0.00	1.08	–4.59	4.46	
2	–0.00	0.84	–3.47	3.70	
3	0.00	0.85	–3.83	4.11	
4	0.00	0.51	–2.28	2.17	
5	0.00	0.60	–2.53	2.22	
6	0.00	0.33	–1.65	1.80	
Model	E-W	N-S	SW-NE	SE-NW	Mean
1	–0.17	–0.17	0.19	–0.08	–0.06
2	–0.05	–0.04	–0.15	–0.15	–0.10
3	–0.03	–0.03	–0.12	–0.12	–0.07
4	–0.59	–0.59	0.38	0.38	–0.11
5	0.11	0.12	–0.27	–0.28	–0.08
6	–0.11	–0.10	0.02	0.02	–0.04

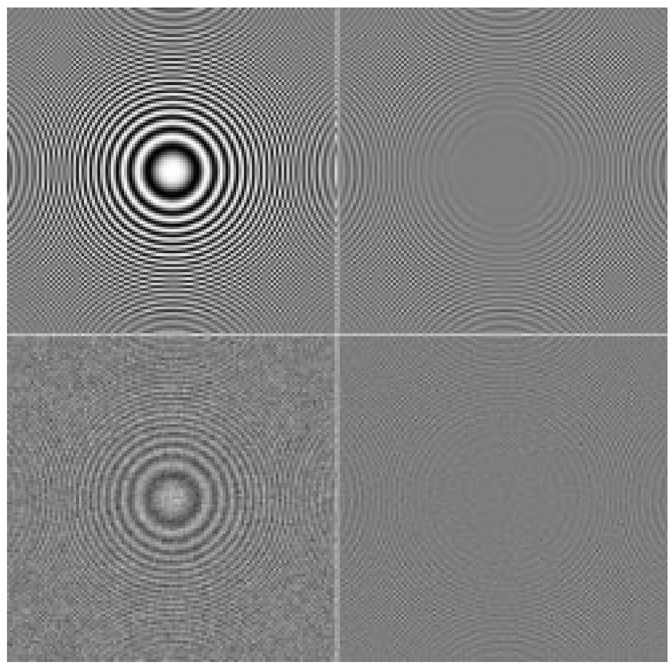

Figure 1. Generated image (top-left), corresponding 5×5 noise image (top-right), generated image with noise added (bottom-left), corresponding 5×5 noise image (bottom-right), all with linear stretching between minimum and maximum.

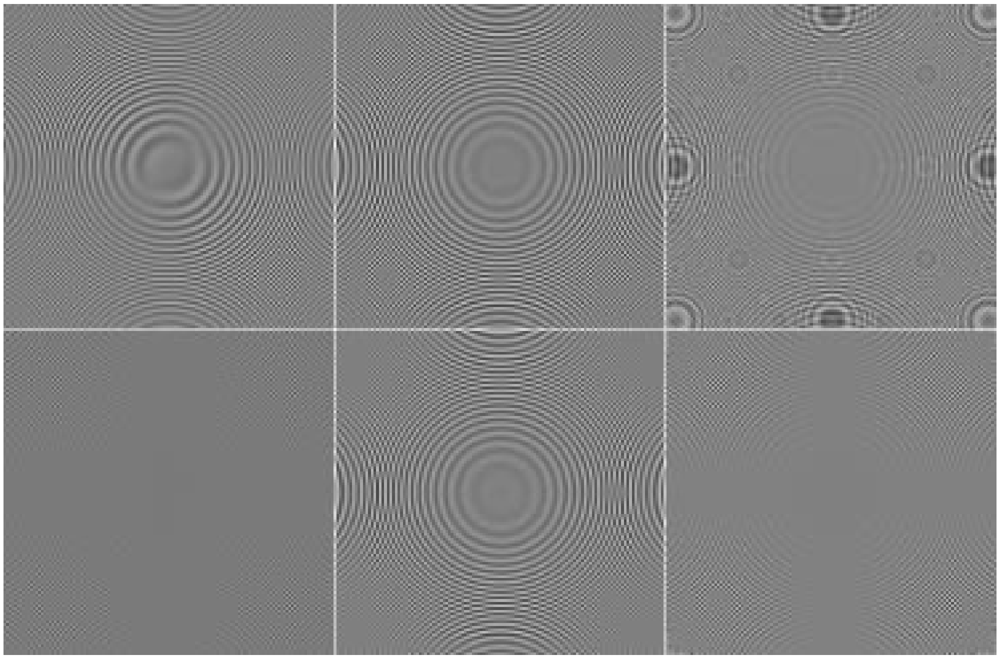

Figure 2. Generated - row-wise, noise as estimated from the six models mentioned above, linear stretching between minimum and maximum.

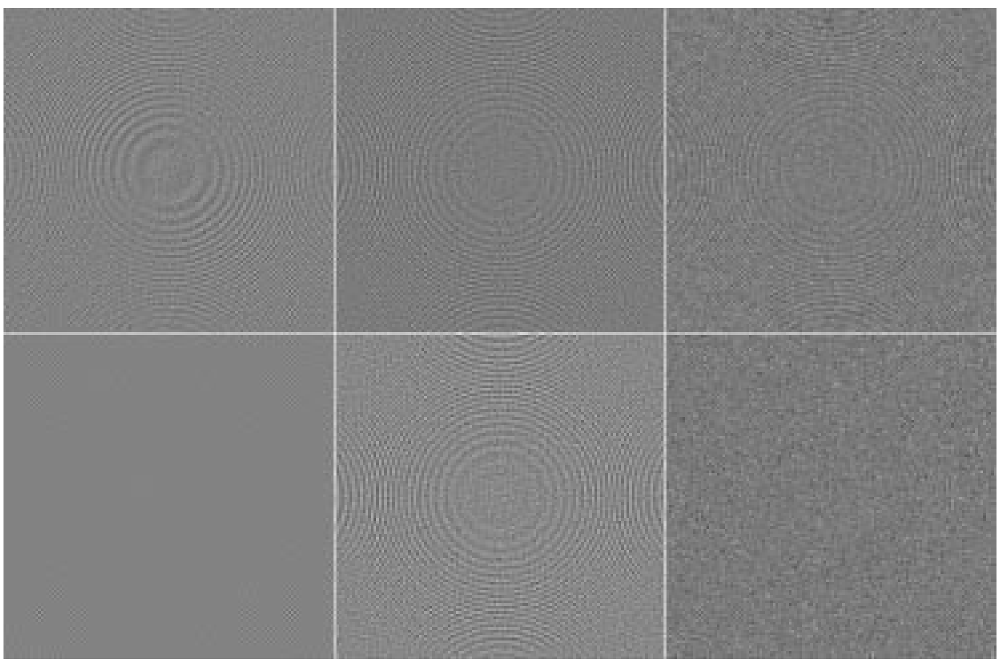

Figure 3. Generated with noise - row-wise, noise as estimated from the six models mentioned above, linear stretching between minimum and maximum.