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A generalized objective function based on weight coefficient for topology-finding of tensegrity structures

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\begin{abstract}
This paper proposes a generalized objective function for the topology-finding of tensegrity structures to be able to assign selection priorities to different members and efficiently find multiple tensegrity structures through a single ground structure. The generalized objective function is constructed by the sum of the product of member internal forces and weight coefficients. The member weight coefficients can be defined and adjusted freely to change the selection priorities of different members in the topology-finding process. By adjusting the weight coefficients, different tensegrity structures can be generated. The weight coefficients can be determined by the designer according to the practical design requirements and preferences, e.g., member length limitations, member position requirements. In addition, a circular computing strategy is proposed for the weight coefficient adjustment to efficiently obtain a large number of tensegrity structures through a single ground structure. The topology design of typical regular tensegrity structures, as well as irregular ellipsoid tensegrity structures, are carried out to demonstrate the effectiveness of the proposed method. Furthermore, by using the proposed method, multiple novel tensegrity structures based on common Archimedes polyhedrons have been found; detailed information (e.g., member connectives, self-stress) are given as an open database for future investigation and applications.
\end{abstract}

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1. Introduction

Tensegrity structures are self-stressed structural systems composed of continuous tensile members (e.g., cables) and discontinuous compressive members (e.g., struts) [1]. Their integrity is maintained by self-balanced prestress in the tensile members (e.g., cables) and compressive members (e.g., struts) [1]. Since their invention, tensegrity structures have been widely concerned in many engineering and scientific fields, such as civil engineering [2,3], robotics [4], biomechanics [5,6], art [7], and material [8].

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Tensegrity structures are a special type of prestressed systems; one of the basic properties of tensegrity structures is the self-equilibrium, which means that the structure should be in equilibrium state under the member forces even if there is no external load is applied. This equilibrium is called the “initial state” of a tensegrity structure. Another basic property is the number limitation of struts connected to a single node, which means that a class-k tensegrity requires that at most k struts should be connected to a single node. This is called “class-k condition” of a tensegrity structure. In addition, cables in tension and struts in compression must be satisfied in the initial state, which is called unilateral member force condition.

The first and foremost step for tensegrity applications is the form design, i.e., to obtain a tensegrity system satisfying necessary requirements mentioned above. Tensegrity form design can be divided into the following three categories: form-finding [9–11], force-finding [12,13], and topology-finding [14,15]. Specifically, the topology-finding (also referred to as topology optimization [16,17] or topology design [18,19]) of tensegrity structures is an emerging scheme proposed in recent years that is a powerful tool to design tensegrity structures with specified shapes. Similar to the topology optimization of conventional structures, member connectivities are treated as the main variables in the topology-finding of tensegrity structures.

Most existing tensegrity topology-finding methods transform tensegrity topology design into optimization problems. Paul et al. [20] and Gan et al. [21] both treated tensegrity topology design as optimization problems and use a genetic algorithm to explore new patterns of member topology. Lee and Lee [22] proposed a topology-finding method in which the strut layout is firstly manually selected in a fully connected network based on given nodes, and then a force-finding procedure is conducted through an optimization model. In the optimization model, the Euclidean norm of the equilibrium equation is treated as the objective function while unilateral member force condition is formulated as constraints; then, a stochastic heuristic algorithm is employed to solve the optimization problem.

Apart from the methods mentioned above, another category of tensegrity topology-finding method usually formulates tensile structure topology design as mathematical programming, especially mixed-integer programming (MIP), models. Ehara and Kanno [18] firstly proposed a two-step tensegrity topology design method based on MIP, in which 0-1 integer variables were used to describe the member connectivities. In the two-step method, the connectivities of struts are obtained in the first step, and then the connectivities of cables are determined in the second step through the minimization of the number of cables. Kanno [23] transformed the two-step topology design scheme into a one-step MIP-based optimization model which is able to find the global optimal solution of the problem and use the total length of cables as the objective function. Both methods mentioned above only consider the tensegrity structures with only one strut connecting to a node. Xu et al. [15] extended the one-step method in Kanno [23] to be able to deal with tensegrity structures with more than one strut connecting to a node and multi-objective optimization problems [24]. Liu and Paulino [25] improved the one-step method in [15,23] by removing the 0-1 integer variables corresponding to the cables and adopting a new objective function to maximize the sum of member internal forces. Compared with previous methods, this model has higher computational efficiency and is conducive to obtaining symmetric structures. Wang, et al. [19] explore the topology-finding of general tensegrity structures with rigid bodies by using the objective function proposed by Liu and Paulino [25] and a new objective to get more even member force distribution.

As mentioned above, in the existing tensile structure topology-finding models, different objective functions have been proposed to find tensegrity structures satisfying different design targets. In the topology-finding process, each candidate member in the ground structure has the same priority to be selected. However, in practical application, different members usually have different priorities according to the design requirements. For example, the members in specified positions or with a specific length range may be preferred in the design process, which means that some specified members should have higher priorities than others. This design requirement cannot be realized in the existing optimization models.

In addition, though topology design strategy offers a new way to design novel tensegrity structures, the existing topology design methods are all based on a single-processing strategy, i.e., only one solution can be obtained in each calculation. Also, in the existing topology-finding models, different objective functions have been proposed to find tensegrity structures satisfying different design targets, none of which provides a systematic way to find various structure forms from a single ground structure. Considering that not all the design requirements such as aesthetics property, manufacturing complexity, and other specific needs can be expressed mathematically and explicitly in the optimization model, a simple but effective solution to address this issue is to obtain various structure forms that satisfy the basic design requirements from a given ground structure, which can offer the designer more freedom to post-evaluate and select the optimal one from the candidates. In the existing methods, only one tensegrity structure can be found through a single computation and thus it is not efficient to obtain multiple tensegrity structures from the same ground structure. Therefore, a systematic approach to constructing a large number of tensegrity forms is necessary but has not been studied yet.

This study aims to propose an effective approach to allow the designer to be able to obtain multiple tensegrity forms from a single given ground structure. The approach is still based on a topology-finding scheme and is formulated into a MIP-based optimization model, but a generalized objective function for the optimization is proposed to allow the designer to assign selection priorities to different members and efficiently find multiple tensegrity structures through a single ground structure. Different weight coefficients can be assigned to different members in the topology-finding process according to the design preferences. By adjusting the weight coefficients, different tensegrity structures can be generated. In addition, a circular computing strategy is proposed for the weight coefficient adjustment to efficiently obtain a large number of tensegrity structures through the same ground structure.
Compared to previous studies, the main contributions of this work are summarized as follows:

- A generalized objective function based on member weight coefficient is proposed to be able to assign selection priorities to different members in the topology-finding process.
- A circular computing strategy for the member weight coefficient adjustment is proposed to efficiently obtain a large number of tensegrity structures through the same ground structure.
- Ball-like tensegrity structures are widely applied in various fields. The discovery of novel forms of ball-like tensegrity is of great significance. By using the proposed method, multiple novel ball-like tensegrity structures based on common Archimedes polyhedrons are found; and their detailed information (e.g., member connectives, internal forces) are given as an open database for tensegrity community for future investigation and applications.

The remainder of the paper is organized as follows. Section 2 defines the problem to be studied. Section 3 establishes the topology-finding optimization model. Section 4 proposes the circular computing strategy for the member weight coefficient adjustment. Section 5 carries out typical numerical examples to verify the proposed approach. Finally, Section 6 discusses and concludes the study. In addition, the novel tensegrity structures found through common Archimedes polyhedrons and their detailed information are given in Appendix 2.

2. Problem definition

Fig. 1 illustrates the tensegrity topology-finding procedure adopted in this study. Firstly, some isolated nodes are given in space (Fig. 1a), and the topology-finding problem is to obtain a large number of symmetrical tensegrities generated through the given nodes. The topology-finding is based on the ground structure method [26]. The ground structure is generated by connecting each pair of the given nodes (Fig. 1b), then based on a topology-finding algorithm, necessary members are selected to serve as cables or struts and the other members are deleted, finally desired tensegrity structure can be obtained (Fig. 1c).

To obtain various different tensegrity from the same ground structure, different weights will be assigned to different members in order to adjust the selection priorities of different members in the topology-finding process. The values of the weight coefficients can be determined according to practical design preferences and tensegrity structures with different topologies can be obtained by adjusting the weight coefficients. Fig. 1 conceptually shows that the topology of the cables and struts can be changed by adjusting the weight coefficients and different tensegrity structures can be found from the same ground structure.

In the following sections, topology-finding models based on an optimization scheme are firstly developed and then a circular computing strategy for the weight coefficient adjustment is proposed to efficiently obtain a large number of tensegrity structures through the same ground structure.

![Diagram](image)

**Fig. 1.** Topology-finding diagram (a) the given nodes; (b) the ground structure; (c) the result of adjusting $\omega_1$ and $\omega_3$; (d) the result of adjusting $\omega_1$ and $\omega_2$; (e) the result of adjusting $\omega_2$ and $\omega_3$ (a weight coefficient refers to a group of members rather than a single member).
3. Optimization model

3.1. Variable definition and member classification

Use \( n \) and \( m \) to denote the number of nodes and members in the ground structure, respectively; use \( V = \{1, ..., n\} \) and \( E = \{1, ..., m\} \) to denote the set of node indexes and member indexes, respectively; use \( E_S, E_C \), and \( E_N \) to denote the set of struts, cables, and removed members, respectively. Then the relationship of \( E, E_S, E_C \), and \( E_N \) can be expressed as
\[
E_S \cup E_C \cup E_N = E
\]
(1)
Use binary variable \( x_i \) to describe the state of member \( i \) as [18]
\[
\begin{align*}
    x_i = 1 & \iff \text{member } i \text{ is strut} \\
    x_i = 0 & \iff \text{member } i \text{ cable or removal}
\end{align*}
\]
(2)

3.2. Constraints

3.2.1. Equilibrium condition

Use \( t = (t_1, t_2, ..., t_m) \) to denote the vector of member internal force, then it should satisfy the equilibrium condition, i.e.,
\[
At = 0
\]
(3)
where \( A \) is the equilibrium matrix of the system, more details about the concept of \( A \) can be referred in [27].

3.2.2. Member internal force expression

The member internal force vector \( t \) is replaced by the difference of two non-negative vector \( t^c \) and \( t^s \), i.e., \( t = t^c - t^s \). This definition decouples the internal force as a combination of tension part and compression part. By using this definition, another two constraints should be introduced [25], i.e.,
\[
\begin{align*}
    t &= t^c - t^s \\
    0 &\leq t^c \leq x \leq 1
\end{align*}
\]
where vector \( t^c \) corresponds to the tension forces and \( t^s \) corresponds to the compression forces.

3.2.3. Class-k condition

According to the general definition of tensegrity structures [28], the number of struts connected to each node should not exceed \( k \), i.e.,
\[
\sum_{i \in E_j} x_i \leq k, \forall j \in V
\]
(5)
where \( E_j \) denotes the set of members connected to node \( j \) in the ground structure.

3.2.4. Member intersection constraint

The intersection of members, especially struts, will bring inconvenience to structure fabrication. According to the variable defined in Section 3.1, the intersection between struts can be avoided through the following constraints
\[
x_i + x_{i'} \leq 1, \forall (i, i') \in E_{\text{cross}}
\]
(6)
where \( E_{\text{cross}} \) is the set of intersecting member groups in the ground structure.

Since the existence of cables is not expressed by binary variables, Eq. (6) cannot be applied to control the intersection between cables and struts. In this paper, a new constraint is proposed to address the intersection problem of cables and struts.
\[
\begin{align*}
    x_i + t_{i'} &\leq 1 \\
    x_{i'} + t_i &\leq 1
\end{align*}
\]
(7)
Constraint Eq. (6) implies that binary variables \( x_i \) and \( x_{i'} \) cannot equal to 1 simultaneously, which means that intersecting member \( i \) and \( i' \) cannot serve as struts simultaneously. The effectiveness of Eq. (7) to avoid the intersection between struts and cables can be proved as follows.

If \( x_i = 1 \) (or \( x_{i'} = 1 \)), from Eq. (7) we have \( t_{i'} \leq 0 \) (or \( t_i \leq 0 \)); since member \( i' \) (or member \( i \)) cannot be a strut (Eq. (6)), i.e., \( t_{i'} \geq 0 \) (or \( t_i \geq 0 \)); therefore, member \( i' \) (or member \( i \)) cannot serve as a cable and \( t_{i'} = 0 \) (or \( t_i = 0 \)) must be satisfied. If \( x_i = 0 \) (or \( x_{i'} = 0 \)), from Eq. (7) we have \( t_{i'} \leq 1 \) (or \( t_i \leq 1 \)); in this case, member \( i' \) (or member \( i \)) can be either a cable or be removed and thus member intersection will not happen.

Note that the intersection of cables (tensile members) is not considered here because in practical tensegrity structures the tensile members are much thinner than struts and typically flexible enough to handle intersections (collisions).
3.2.5. Constraints for strut length type

Generally, regular structures applied in practice tend to have fewer member length types. Take the strut length type as an example, suppose that the number of member length types in the ground structure is \( p \), and use binary variable \( v_i (r = 1, \ldots, p) \) to denote that whether the members with the \( r \)th length type is used as struts in the final obtained tensegrity structure, i.e.,

\[
\begin{align*}
    v_r = 1, & \text{ at least one member with } r \text{th length type is used as strut} \\
    v_r = 0, & \text{ no member with } r \text{th length type is used as strut}
\end{align*}
\]  

(8)

To realize Eq. (8), the following constraints are introduced

\[
\sum_{i=1}^{q_r} x_i \leq v_r \cdot q_r, \forall i \in E_r
\]

(9)

where \( E_r \) is the set of members with the \( r \)th length type and \( q_r \) is the number of elements in \( E_r \).

A parameter \( B_r \) is used to describe the number of strut length types in the final structure [23], then the relationship between \( v_r \) and \( B_r \) can be expressed as

\[
\sum_{r=1}^{p} v_r = \overline{B}
\]

(10)

To improve the customizable design capabilities, members with specific length types in the ground structure can be designated as struts. For example, if only the members with the \( p \)th length type are required to be served as struts, then \( v_r=1(r=p) \) and \( v_r=0(r \neq p) \) can be introduced. Note that the cable length types can also be controlled in a similar way if needed.

3.3. Objective function

Basic requirements to constitute a tensegrity structure have been formulated into constraints in the above sections. Recalling that the aim of the proposed topology-finding model is to find as many tensegrity structure forms as possible from a given ground structure, hence the objective function should have the ability to be tuned easily to result in various tensegrity structures with different topologies. Following the objective proposed in [25], the sum of the member internal forces is treated as the objective function. However, unlike the objective function in [25] that directly sum all the internal forces, which means that all the members have the same priority during the selection process, different weights are assigned to the members by multiplying a weight coefficient to each member force. In this way, the weight coefficients can be easily tuned to change the selection priorities of the members, which gives the objective function the ability to lead to various tensegrity topologies. Then, accomplished by a customized and strategic circular computing algorithm proposed in Section 4, multiple tensegrity structures can be effectively found from a given ground structure.

Without loss of generality, assume that the \( i \)th member in the ground structure has a weight coefficient \( \omega_i \) corresponding to its internal forces \( t_i \) in the topology-finding process, then the generalized objective function is defined as

\[
f = \sum_{i=1}^{m} \omega_i \times t_i
\]

(11)

Specifically, if all the members have the same weight coefficient, then the objective function can be reduced to

\[
f = \sum_{i=1}^{m} t_i
\]

(12)

Note that the internal force values (i.e., \( t_i \)) in the objective function are all scalers but not vectors, hence the equilibrium state of the found structure does not mean that the sum of the member forces equal to zero. The equilibrium state of the structure is described by the equilibrium equation which has been formulated into a constraint (i.e., Eq. (3)) by using the equilibrium matrix \( A \) that contains the geometry information (e.g., lengths and directions) of the members [27].

The sum of the non-weighted member forces has been proved to be an efficient objective function for tensegrity topology-finding problem to generate symmetrical and stable structure forms [25], and the objective function proposed in this study is an improvement of the method to assign different weights to different members to allow the adjustment of member selection priorities and provides the ability to find various different tensegrity topologies; but it should be noted that the summation of the (weighted) member forces is just adopted as a mathematical and effective strategy for the tensegrity topology-finding problem and there is no exact physical or engineering meaning for the objective function value.

In the topology-finding process, the objective function can either be maximized or minimized, according to design requirements. Take the hexadecagon tensegrity with \( \omega_i = 1, \forall i \in E \) in Fig. 2 as an example, according to the force balance relation at the node, the cables having larger angles with the strut should have larger internal tension forces to balance the compression force. So external cables (Fig. 2a) having larger angles with the strut are preferred when the sum of forces is
maximized and the internal cables (Fig. 2d) having smaller angles with the strut are preferred when the sum of forces is minimized.

If the objective function Eq. (12) with identical member weight coefficients is adopted, only one extreme structure (either structure having the largest or smallest average angle between cables and struts) can be found in the topology-finding, and other cases lying between the two extreme cases will be neglected in the topology-finding process. By assigning different weight coefficients to the members, specified members can have higher selection priorities in the topology-finding process, which cannot consider the two extreme cases but also other intermediate cases.

### 3.4. Weight coefficient

This section will qualitatively study the influence of weight coefficients on the optimization results, and more detailed analysis can be referred to in Appendix 1. The weight coefficients of different members can be determined according to the design requirements. For example, members with different lengths can have different weight coefficients, members at different positions can have different weight coefficients. In this study, the member length is adopted as a determining factor to assign weight coefficients. Assume that the members in the ground structure with the same length have the same weight coefficient, then the objective function can be written as

\[
    f = \sum_{r=1}^{p} \sum_{i=1}^{q_r} \omega_r \times t_i, i \in E_r
\]  

where \( p \) denotes the number of member length type in ground structure; \( q_r \) denotes the number of members in \( r^{th} \) length type in ground structure; \( E_r \) denotes the set of \( r^{th} \) length type in ground structure; \( \omega_r \) is used to denote the weight coefficient of \( r^{th} \) length type.
Without loss of generality, the objective function Eq. (13) is to be minimized in the topology-finding process, and assume that the weight coefficients satisfy the relationship

\[ 0 < \omega_1 < \omega_2 < \cdots < \omega_p \]  

(14)

As shown in Fig. 3, to minimize the objective function, the tension forces should choose a smaller weight coefficient, while the compression forces should choose a larger weight coefficient. Using this relationship, the designer can assign a higher priority to the desired member to be served as a cable by multiplying its force by a smaller weight coefficient, while assign a higher priority to the desired member to be served as a strut by multiplying its force by a larger weight coefficient.

The purpose of weight coefficients is to change the selection priority of members. This means that the absolute size of the weight coefficients is not important, and the real importance is the relative size between the weight coefficients. In order to separate the weight coefficients of two adjacent priorities as far as possible, it means that

\[ 0 << \omega_1 << \omega_2 << \cdots << \omega_p \]  

(15)

a heuristic weight coefficient assignment strategy is proposed as

\[ \omega_r = \alpha^{b_r} \]  

(16)

where \( \alpha \) is a positive number that greater than 1, such as 3, and \( p_r \) denotes the order of the \( r \)th length type. On the selection of \( \alpha \), trial computations indicate that a too small \( \alpha \) will lead to a small gap between the two adjacent weight coefficients, and it is difficult to adjust the weight coefficients to assign higher priority to the expected members; on the contrary, a too large \( \alpha \) will generate excessive weight coefficients that will cause numerical errors in the computation. Therefore, \( \alpha = 3 \) is finally adopted, which has been proved to be a robust value for the computations.

For ground structures with irregular node coordinates, the number of member length types is often large, especially for the ground structures with a large number of nodes. To reduce the computational burden of the weight coefficient calculation, we can replace the fixed-length with interval length; then the members in each length interval could be assigned the same weight coefficient.

In structural fabrication, designers usually prefer the member lengths to lie in a specified range due to restrictions on fabrication and equipment. To realize this purpose, the weight coefficients can be assigned as

\[ \omega_r = \alpha^{|p_{\text{core}} - p_r|} \]  

(17)

where \( p_{\text{core}} \) denote the order of center length interval that the designer preferred. From Eq. (17), if the length of a member is far from the center length interval, it will have a larger weight coefficient and thus have a small priority to be selected in the force-finding process.

3.5. Topology-finding model

Based on the constraints and the objective function introduced above, the topology-finding model can be expressed as follows

\[
\begin{align*}
\min f &= \sum_{r=1}^{p} \sum_{i=1}^{q_r} \omega_r \times t_i, \quad i \in E_r \\
\text{s.t.} \quad &A \mathbf{t} = \mathbf{0} \\
&\sum_{i \in \mathcal{E}_i} x_i \leq k, \quad \forall j \in \mathcal{V} \\
&x_i + x_{i'} \leq 1, \quad \forall (i, i') \in \mathcal{E}_{\text{cross}} \\
&x_i + t_i \leq 1, \quad \forall (i, i') \in \mathcal{E}_{\text{cross}} \\
&m = t^c - t^c \\
&0 \leq t^c \leq \mathbf{x} \leq 1 \\
&\sum_{r=1}^{p} v_r = \bar{B} \\
&v_r \leq \sum_{i=1}^{q_r} x_i \leq v_r \cdot q_r, \quad \forall i \in E_r
\end{align*}
\]

(18)

In some cases, the strut layout might be specified by the designer in advance and thus should never change during the design. For the topology-finding with given strut connectivities, the model can be simplified. Because the strut connectivities have been specified, the member internal force constraints can be simplified as

\[
\begin{align*}
t_i &\geq 0, \quad \forall i \in \mathcal{E}_C \cup \mathcal{E}_N \\
t_i &\leq -1, \quad \forall i \in \mathcal{E}_S
\end{align*}
\]

(19)
where \( t_i \leq -1 (\forall i \in E_S) \) is used to ensure that the feasible solution space is bounded [25].

For the member intersection constraints, on the one hand, the crossing between struts will not exist; on the other hand, the crossing between struts and cables can be avoided by ensuring that the intersecting members are removed. Therefore, Eqs. (6) and (7) can be further simplified to

\[
t_i = 0, \ \forall (i, j) \in E_{cross} \& j \in E_S
\] (20)

For the objective function, the part corresponding to the struts will be a constant and thus can be eliminated in the calculation. Therefore, the objective function can be further simplified as

\[
f = \sum_{r=1}^{p} \sum_{i=1}^{q_r} \omega_r \times t_i, \ \ i \in E_I \cap (E_C \cup E_N)
\] (21)

Based on the constraints and the objective function introduced above, the topology-finding model with given strut connectivities can be expressed as follows

\[
\begin{align*}
\min f &= \sum_{r=1}^{p} \sum_{i=1}^{q_r} \omega_r \times t_i, \ \ i \in E_I \cap (E_C \cup E_N) \\
\text{s.t.} \quad &At = 0 \\
& t_i \geq 0, \ \forall i \in E_C \cup E_N \\
& t_i \leq -1, \ \forall i \in E_S \\
& t_i = 0, \ \forall (i, j) \in E_{cross} \& j \in E_S
\end{align*}
\] (22)

Eqs. (18) and (22) show the topology-finding models with unspecified and specified struts, respectively. In both the two models, the objective function and all the constraints are linear with respect to the optimization variables, hence Eq. (18) is a mixed integer linear programming (MILP) model that can be solved efficiently to globally optimum through a branch and bound algorithm together with a simplex algorithm [29] and Eq. (22) is a linear programming (LP) model that can be solved efficiently to globally optimum through a simplex algorithm [29].

4. Circular computing strategy

For regular tensegrity structures, the number of cable length types used is far less than the number of member types in the ground structure, i.e.,

\[
p' \ll p
\] (23)

where \( p' \) denotes the number of cable length types used for a tensegrity structure. Taking the hexadecagon tensegrity in Fig. 4 as an example, when the second type of members are desired to serve as cables, as long as \( \omega_2 \) is the smallest, i.e., the second type of members have the highest priority to be selected, all the six weight coefficient assignment cases will lead to the same result. Therefore, to obtain as more structures as possible from the same ground structure, it is not necessary to consider all the cases of the permutations of the weight coefficients.

Based on this analysis, we define a variable \( FW \) to denote the number of weight coefficients for cycle operation in the computation and use \( R(FW=i) \) to denote the number of structures obtained in the \( i^{th} \) cycle. Then, the computation will stop after all the cycles are finished or the following condition is satisfied

\[
R(FW) = R(FW - 1)
\] (24)

When Eq. (24) is satisfied in the cycle operation, the result of adjusting the \( FW^{th} \) weight coefficient is identical to that of \( (FW-1)^{th} \). According to Eq. (15) the cost of increasing the one-level weight coefficient is huge, that is, each introduction of a new type of member will greatly increase the value of objective function, and the objective function has the characteristics of using as fewer types of members as possible. Since the former circle is the subloop of the latter circle, it means that the introduction of the \( FW^{th} \) member is not enough to offset the cost caused by the increase of weight, and up to \( (FW-1)^{th} \) members used in structure. Since the introduction of \( FW^{th} \) member has become a huge cost, it is impossible that the introduction of \( (FW+1)^{th} \) to \( p^{th} \) will result in the optimal structure.
Different from the full enumeration method, the circular strategy can significantly reduce the number of computational cycles. We only need to change the first few weight coefficients in the cycle, and the remaining are still assigned through the ascending length rule. The total number of computations (denote as CYC) that will be carried out for each FW is

$$\text{CYC} = C_1^p \times C_{p-1}^1 \times \cdots \times C_{p+1-FW}^1$$

(25)

The pseudo-code of the topology-finding using the circular computing strategy algorithm is shown in Algorithm 1.

5. Numerical examples

In the following numerical examples, YALMIP toolbox [30] and Matlab Ver. R2019a [31] are used to construct the optimization models, and Gurobi Optimizer Ver. 9.11 [32] is used to solve the models. All the parameters are set as default values in the computations. As analyzed in Section 3.5, the optimization model is either MILP or LP and the global optimum can be theoretically obtained. But considering that numerical computation is adopted in Gurobi, if the optimality gap of a problem is smaller than the default convergence threshold, the solution is deemed as a global optimum solution under the stated numerical accuracy. All the computations are carried out on a PC with a 2.40GHz Inter(R) Core(TM) i5-9300H CPU and 16.0 GB RAM. In addition, since the tensegrity stability condition [33,34] is not considered in the topology-finding process, a stability post-check will be carried out for each obtained system.

5.1. Prismatic tensegrity

In this section, prismatic tensegrity is adopted as an example to illustrate the topology-finding process. The nodes of prismatic tensegrity located at the vertices of a twisted prism with each base face being a regular $N$-gon. As shown in
**Fig. 5.** The ground structure of octagonal prism that can be divided into 8 types by length.

**Fig. 6.** Prismatic tensegrity structures obtained using weight coefficients corresponding to different sequences.

Fig. 5, d and h are the diameter of the circumcircle and height of two parallel planes of the prism, respectively. Without loss of generality, let h/d = 1. In this example, the convergence criterion Eq. (24) is adopted. There are 8 kinds of member lengths in the ground structure, each of which corresponds to a specific position form. In ascending order of member length, members with length types Nos.1–8 are shown in Fig. 5.

To control the length of struts, let $p_8=8$, and only 1–7 kinds of length members are circularly operated. Fig. 6 shows the results obtained by using different weight coefficients. Table 1 shows the summary information of the obtained structures.
In Table 1, $m_t$ and $m_c$ refer to the number of struts and cables in the obtained structures, respectively; $sm$ and $mm$ denote the number of self-stress modes and infinitesimal mechanism modes, respectively. Take Fig. 6a as an example, the orders and weight coefficients corresponding to members with different length types are appointed as follows:

$$p_1 = 1, p_2 = 3, p_3 = 4, p_4 = 2, p_5 = 5, p_6 = 6, p_7 = 7, p_8 = 8$$

$$\omega_1 = 3^1, \omega_2 = 3^3, \omega_3 = 3^4, \omega_4 = 3^2, \omega_5 = 3^5, \omega_6 = 3^6, \omega_7 = 3^7, \omega_8 = 3^8$$

According to Fig. 6 and Table 1, each structure is only composed of two types of members. When we put these two types of members in the first and second place in the sequence (i.e., the two types of members have the first two smallest weight coefficients), the computations will converge to the structure composed of the corresponding types of members. This proves the effectiveness of the objective function on the control of priority of specified members.

Note that the result obtained when the weight coefficients are assigned according to the ascending order of member length (i.e., sequence 12,345,687) is the same as the sequence 14,235,678 (Fig. 6a). This indicates that using different orders for the members could lead to the same structure. Duplicate structures can be deleted from the results in post-processing.

The weight coefficients can also be used to control the member selection for struts. In previous examples, we always assign the largest weight coefficient to the members with the longest length. Next we assign the largest weight coefficient to members with the penultimate longest length, i.e., using sequence 12,345,687 to carry out the topology-finding, then the corresponding members (i.e., members of length type No. 7) are finally selected as struts (Fig. 7). Therefore, the selection priority of compression members and tension members can both be controlled through fine-tuning the weight coefficients.

It can be observed from the results that the members with lower weight coefficients are preferred for cables while those with larger weight coefficients are preferred for struts in the topology-finding process, which is consistent with the analysis in Section 3.4. Compared with using total force maximization or total force minimization as the objective function, multiple tensegrity structures can be obtained through the same ground structure based on the generalized objective function.

Table 2 shows the summary information of the computation results for $FW = 3–6$ in the circular computing. According to the convergence criterion, 252 cycles are performed and the computation time is 459 s. Even though the CYCs for $FW = 4–6$ are much more than the CYCs of $FW = 2$ and 3, the numbers of structure types finally obtained are the same. This proves that the strategy proposed in Section 4 is effective, i.e., we just need to adjust the first several weight coefficients to avoid the permutations of all the weight coefficients. This also indicates that convergence criterion Eq. (24) can effectively reduce the computational cycles and thus improve the efficiency to find multiple structures through the same ground structure.
Table 2
Summary for circular computing of $FW = 2–6$.

<table>
<thead>
<tr>
<th>FW</th>
<th>CYC</th>
<th>Time (s)</th>
<th>Number of structures obtained</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>42</td>
<td>79</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>210</td>
<td>380</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>840</td>
<td>1610</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>2520</td>
<td>5836</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>5040</td>
<td>11,086</td>
<td>12</td>
</tr>
</tbody>
</table>

![Diagram](image)

**Fig. 8.** Geometry and graphic result of tower tensegrity structure obtained through weight coefficients in ascending order of length (a) geometry; (b) graphic result.

Table 3
Summary information of tower tensegrity structures.

<table>
<thead>
<tr>
<th>Figure</th>
<th>$n$</th>
<th>$m_r$</th>
<th>$m_s$</th>
<th>$sm$</th>
<th>$mm$</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 8b</td>
<td>24</td>
<td>12</td>
<td>60</td>
<td>6</td>
<td>0</td>
<td>Super-stable</td>
</tr>
<tr>
<td>Fig. 9a</td>
<td>54</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>Prestress-stable</td>
</tr>
<tr>
<td>Fig. 9b</td>
<td>48</td>
<td>1</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>Prestress-stable</td>
</tr>
<tr>
<td>Fig. 9c</td>
<td>48</td>
<td>2</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>Prestress-stable</td>
</tr>
<tr>
<td>Fig. 9d</td>
<td>54</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>Prestress-stable</td>
</tr>
</tbody>
</table>

5.2. Tower tensegrity

**Fig. 8a** shows a two-layer tower with an overlapping height $h_d$. $\alpha_1$ and $\alpha_2$ denote the twisting angles of the waisted prisms, and $\beta$ is the relative rotation between the two prisms. $r$ and $h$ are the circumradius and the height of two parallel planes of the prism, respectively. To better compare the results, we adopted the same strut connectivities as [25] and used the simplified model Eq. (22).

The ground structure contains 25 kinds of length members corresponding to different positions. As shown in **Fig. 8b**, when the ascending order of member length is adopted to assign the weight coefficients, the obtained result is consistent with that obtained through total force maximization [25]. By using the circular computing strategy, the calculation went through 13,800 cycles in 9407 s and a total of 287 symmetrical and stable tensegrity structures (including 86 super-stable tensegrity structures) were obtained. **Fig. 9** shows some of the results and **Table 3** summarized the information of the solutions.

5.3. Spherical tensegrity

In this section, a small rhombicosidodecahedron (**Fig. 10**) is used for topology-finding to design spherical tensegrity structures. Assume that the small rhombicosidodecahedron has a side length of $L$. Existing methods in [18,15,25] are discussed and compared with the proposed method.

Note that the existing methods adopted different objective functions. The design objectives in [18,15] are to minimize the total number of cables and minimize the total length of cables, respectively, and the design objective in [25] is to maximize the total member forces. Both the topology-finding modes in [18,15] introduce binary variables to describe the cable topology, which causes the methods not to be applicable to the topology design of large-scale tensegrity structures. For example, for the case considered in this example, the ground structure contains 60 nodes and 1770 candidate members, which
Fig. 9. Different tower tensegrity structures.

Fig. 10. Geometry of small rhombicosidodecahedron.
Fig. 11. Graphic results of small rhombicosidodecahedron tensegrity.
Table 4
Summary information of small rhombicosidodecahedron tensegrity.

<table>
<thead>
<tr>
<th>Figure</th>
<th>$m_t$</th>
<th>$m_c$</th>
<th>$sm$</th>
<th>$mm$</th>
<th>Stability</th>
<th>Sum of cable length ($\times L$)</th>
<th>Evenness of internal force of cables ($\times 10^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 11a</td>
<td>30</td>
<td>120</td>
<td>1</td>
<td>25</td>
<td>Prestress stability</td>
<td>210.98</td>
<td>14.16</td>
</tr>
<tr>
<td>Fig. 11b</td>
<td>120</td>
<td>1</td>
<td>25</td>
<td></td>
<td>Prestress stability</td>
<td>233.01</td>
<td>21.29</td>
</tr>
<tr>
<td>Fig. 11c</td>
<td>120</td>
<td>1</td>
<td>25</td>
<td></td>
<td>Prestress stability</td>
<td>227.29</td>
<td>28.85</td>
</tr>
<tr>
<td>Fig. 11d</td>
<td>120</td>
<td>1</td>
<td>25</td>
<td></td>
<td>Prestress stability</td>
<td>236.40</td>
<td>22.27</td>
</tr>
<tr>
<td>Fig. 11e</td>
<td>120</td>
<td>1</td>
<td>25</td>
<td></td>
<td>Super stability</td>
<td>144.87</td>
<td>47.07</td>
</tr>
<tr>
<td>Fig. 11f</td>
<td>120</td>
<td>1</td>
<td>25</td>
<td></td>
<td>Prestress stability</td>
<td>181.92</td>
<td>46.90</td>
</tr>
<tr>
<td>Fig. 11g</td>
<td>120</td>
<td>1</td>
<td>25</td>
<td></td>
<td>Prestress stability</td>
<td>210.98</td>
<td>13.71</td>
</tr>
<tr>
<td>Fig. 11h</td>
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<td>1</td>
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<td>Prestress stability</td>
<td>233.00</td>
<td>13.67</td>
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<tr>
<td>Fig. 11i</td>
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<td>1</td>
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<td></td>
<td>Prestress stability</td>
<td>248.06</td>
<td>24.37</td>
</tr>
<tr>
<td>Fig. 11j</td>
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<td>1</td>
<td>25</td>
<td></td>
<td>Prestress stability</td>
<td>264.37</td>
<td>13.11</td>
</tr>
<tr>
<td>Fig. 11k</td>
<td>120</td>
<td>1</td>
<td>25</td>
<td></td>
<td>Prestress stability</td>
<td>270.08</td>
<td>11.61</td>
</tr>
<tr>
<td>Fig. 11l</td>
<td>120</td>
<td>1</td>
<td>25</td>
<td></td>
<td>Prestress stability</td>
<td>273.48</td>
<td>15.17</td>
</tr>
<tr>
<td>Fig. 11m</td>
<td>120</td>
<td>1</td>
<td>25</td>
<td></td>
<td>Prestress stability</td>
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<td>6.89</td>
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<tr>
<td>Fig. 11n</td>
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<td>1</td>
<td>25</td>
<td></td>
<td>Prestress stability</td>
<td>270.10</td>
<td>10.66</td>
</tr>
<tr>
<td>Fig. 11o</td>
<td>120</td>
<td>1</td>
<td>25</td>
<td></td>
<td>Prestress stability</td>
<td>218.05</td>
<td>15.55</td>
</tr>
<tr>
<td>Fig. 11p</td>
<td>150</td>
<td>9</td>
<td>3</td>
<td></td>
<td>Prestress stability</td>
<td>342.92</td>
<td>2.15</td>
</tr>
</tbody>
</table>

![Fig. 12. The convergence history of topology-finding of a small rhombicosidodecahedron tensegrity structure.](image)

is difficult to be handled by the two methods. Trial computations show that the models generated through \[15,18\] cannot be solved in a reasonable time; this indicates that the two design objectives (i.e., cable number and length minimization) cannot be realized through the methods for the case considered in this example.

The computational efficiency of the method in \[25\] is better because the binary variables for the cable topology are discarded, which significantly reduces the complexity of the topology-finding model. For the case considered in this example, trial computation shows that a feasible tensegrity can be found in a reasonable time. However, as analyzed in Section 3.3, force maximization can only lead to one solution; if other design objectives need to be considered, the method might become not applicable. For example, cable number or length maximization cannot be realized by the method in \[25\] because the number and total length of cables cannot be accurately expressed without using binary variables.

The method proposed in this study offers a solution to address the problem mentioned above. On the one hand, cable topology is not described by binary variables and thus the computational efficiency is significantly better than the methods in \[15,18\]. On the other hand, the generalized objective function together with the circular computing strategy allow the topology-finding to find multiple feasible structures through the ground structure; the obtained structures can be used as a database for the designer to select the best ones that satisfy different design objectives.

For the case considered in this example, a total of 321 symmetrical and stable tensegrity structures have been found by using the proposed method. Fifteen different systems have been shown in Fig. 11. The summary information is given in Table 4. Note that the cable number, total length, force evenness, etc. are obtained through a post-analysis after the structures are obtained. Best designs corresponding to different objective functions can be selected from the structures. For example, if the tensegrity structures with minimum number of cables are preferred, the structure in Fig. 11a–o can be selected; if the tensegrity structures with shorter cable total length are preferred, the structure in Fig. 11e can be selected; if the structure with more even cable forces are preferred, the structure in Fig. 11p can be selected (Eq.(28) is used as the
Fig. 13. Geometrical dimensions and random nodes distributed on the surface of an ellipsoid (a) front view; (b) top view.

Fig. 14. Graphic results of irregular ellipsoid tensegrity in different conditions (a) assigning weight coefficient in ascending order of length; (b) and (c) assigning weight coefficient according to Eq. (17).

Fig. 15. The range of cable length of irregular ellipsoid tensegrity.
evaluation index of cable force uniformity).

\[
g = \frac{\sum_{i \in E_c} (t_i - \bar{t})^2}{m_c}
\]

where \( \bar{t} = \sum_{i \in E_c} t_i / m_c \) and \( \bar{t} \) denote the average value of cable internal forces.

Considering that spherical tensegrity is an important category of tensegrity structures, common Archimedean polyhedrons are adopted for the topology-finding and multiple novel spherical tensegrity structures have been found. The detailed information (e.g., member connectives, self-stress) of the structures are given as an open database for future investigation and applications.

Take the topology-finding of a small rhombicosidodecahedron tensegrity structure (Fig. 26(b)) as example, the strut lengths are specified, which results in a MILP problem. The convergence history is shown in Fig. 12. Once the upper bound (the current best feasible solution that has been obtained) equal to the lower bound (the current best solution to the relaxation problem) within the default numerical accuracy, the solution is obtained. The result shows that the final gap between the upper bound and lower bound is 0.00% and thus the obtained solution can be treated as the global optimal solution within the default numerical accuracy.

### 5.4. Irregular ellipsoid tensegrity

Irregular nodes can also be adopted to construct tensegrity structures. In this section, irregular nodes distributed on the surface of an ellipsoid are used to generate a ground structure for the topology-finding. The ground structure contains 80 nodes and 3160 members. The number of member length types is 2098. The members in the ground structure are divided into 40 interval groups according to their lengths, and the strut lengths are limited to be smaller than 2/3D where D is the length of the major axis of the ellipsoid.

When Eq. (16) is used to calculate the weight coefficients, the solution is shown in Fig. 14a; when Eq. (17) is adopted and \( p_{core} = 7 \) and \( p_{core} = 13 \) are used to calculate the weight coefficients, the solution is shown in Fig. 14b and c, respectively. Table 5 summaries the information of the three structures. When the weight coefficients are assigned according to the ascending order of member length, the obtained cable lengths are distributed between 0.11D and 0.33D; when the weight coefficients are assigned according to Eq. (17) and \( p_{core} = 7 \) and \( p_{core} = 13 \) are used, the obtained cable lengths are distributed in the rage of 0.20D-0.41D and 0.35D-0.53D, respectively. Fig. 15 illustrates the number of cables of different lengths and relative length ratio after normal distribution fitting in the tensegrity structures.

As shown in Fig. 14, the position distributions of the cables in the three structures are very different, which also results in different cable length distributions (Fig. 15). This result verifies that the proposed method can customize the length distribution of the cable by adjusting the weight coefficients, which is of positive significance for the need to control the member length distribution in a tensegrity structure [35,36].

### 6. Conclusion

A generalized objective function based on weight coefficients is proposed in this study. Compared with other objective functions employed in existing methods, the generalized objective function is a more effective tool to customize member selection priority in the topology-finding process through the proper assignment of member weight coefficients. Different tensegrity structures can be obtained from the same ground structure by adjusting the weight coefficients. Based on the rule of ascending order of member length, the weight coefficients can be appropriately determined to control the member length distribution of the final tensegrity structure. A circular computing strategy is proposed to quickly and efficiently find as many feasible tensegrity structures as possible, and thus a large number of tensegrity structures can be obtained from a given ground structure.

Compared with existing tensegrity topology-finding methods, the proposed method not only has good computational efficiency for the topology design of large-scale tensegrity structures but also has the ability to obtain as many feasible tensegrity structures as possible from a given ground structure. These features offer the designer more flexibility and freedom to customize the design according to various design requirements. By using the proposed method, multiple novel tensegrity structures have been found through common Archimedes polyhedrons; the detailed information is given as an open database for future investigation and applications, which is another meaningful contribution of this work.
Future work could investigate other rules to determine the weight coefficients to explore more novel tensegrity structures satisfying different design requirements. Besides, this study only carries out numerical investigations on tensegrity topology design, future work will focus on the mechanical and experimental study as well as practical applications of the novel tensegrity structures obtained through the proposed method.

**Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

**Data availability**

Data will be made available on request.

**Acknowledgments**

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**Supplementary materials**

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.apm.2022.10.038.

**Appendix 1. Influence of adjusting weight coefficients**

In this section, the relation between the weight coefficients and the optimization results is discussed. The analysis focuses on the relation between the cables and the corresponding weight coefficients under the same strut layout, which is the main problem solved by the topology-finding model. Since the objective function tends to assign larger compression forces to the struts and considering that the compression forces of struts are constrained in Eq. (4) (i.e., the absolute value of strut compression forces cannot be larger than 1.0), the struts will have as larger compression values as possible. In the following proof, the compression force of the strut is assumed to be a fixed value and for brevity, only two groups of members that can maintain an equilibrium state together with the strut are considered. When there are more than two groups of members, a similar way can be used for pairwise comparison and analysis.

**Fig. 16** gives a general diagram of the member forces based on a right-handed Cartesian coordinate system with the origin located at node O. The X-axis coincides with the direction of strut. It is assumed that for a given compression force \( t_s \), there are two groups of cables \( C_{i1}, C_{i2},...,C_{i1n_i}, C_1,...,C_G \) and \( C_{i1}, C_{i2},...,C_{i2n_i}, C_1,...,C_G \) that can satisfy the equilibrium condition together with the strut. Common members \( C_1,...,C_G \) are the cables shared by the two groups, and there are \( n_1 \) and \( n_2 \) cables except the common members in the first and second group, respectively. The internal forces of the cables in each

![Diagram 1](image1.png)

**Fig. 16.** Force diagram (\( t_s \) denotes the strut compression force; \( t_l \) denotes the tension force of the \( j \)th component in group \( i \); \( \alpha_i \) denotes the angle between the X-axis and the direction of cable \( C_i \); \( \alpha_i \) denotes the angle between the X-axis and strut \( S \), that is \( \alpha_i = 0 \); rotating clockwise from the direction of member to the X-axis is defined as positive direction of angle; from the hinged node \( O \) to the outside is the positive direction of internal forces; cable \( C_i \) is a common member of groups 1 and 2).
group are \( t_{11}, t_{12}, \ldots, t_{1n_1}, t_{1(n_1+1)}, \ldots, t_{1(n_1+g)}, t_{21}, t_{22}, \ldots, t_{2n_2}, t_{2(n_2+1)}, \ldots, t_{2(n_2+g)} \), respectively. Further, it is assumed that neither group contains a subset of cables that can satisfy the equilibrium condition together with the strut. Then from the equilibrium of the forces at node 0, we have

\[
\begin{align*}
\sum_{j=1}^{n_1+g} t_{ij} \cos \alpha_{ij} &= t^i, \; i \in \{1, 2\} \\
\sum_{j=1}^{n_1+g} t_{ij} \sin \alpha_{ij} &= 0, \; i \in \{1, 2\}
\end{align*}
\]

(29)

(30)

where \( t^i \) is the union force of the \( i \)th group member in the X-axis direction. According to the superposition principle, the internal forces of common members are given by

\[
\overline{t}_j = t_{1(n_1+j)} + t_{2(n_2+j)}, \; j \in \{1, 2, \ldots, g\}
\]

(31)

where \( \overline{t}_j \) denotes the internal force of cable \( C_j \). At the same time, the angle between the two groups of common members is the same, that is

\[
\cos \alpha_j = \cos \alpha_{1(n_1+j)} = \cos \alpha_{2(n_1+j)}, \; j \in \{1, 2, \ldots, g\}
\]

(32)

where \( \alpha_j \) denotes the angle between the X-axis and cable \( C_j \). Similarly, the weight coefficients of the common members satisfy that

\[
\omega_j = \omega_{1(n_1+j)} = \omega_{2(n_2+j)}, \; j \in \{1, 2, \ldots, g\}
\]

(33)

The equilibrium along X-axis direction leads to

\[
t^1 + t^2 + t_s = 0
\]

(34)

i.e.,

\[
\sum_{i=1}^{2} \sum_{j=1}^{n_1+g} t_{ij} \cos \alpha_{ij} + t_s \cos \alpha_s = 0
\]

(35)

According to the definition of angle \( \alpha_s \), we have

\[
\cos \alpha_s = 1
\]

(36)

The objective function is given by

\[
f = \sum_{i=1}^{2} \sum_{j=1}^{n_1+g} t_{ij} \omega_{ij} + t_s \omega_s
\]

(37)

Then considering Eqs. (35) and (36), the objective function can be written as

\[
f = \sum_{i=1}^{2} \sum_{j=1}^{n_1+g} t_{ij} \cos \alpha_{ij} + \sum_{i=1}^{2} \sum_{j=1}^{n_1+g} t_{ij} (\omega_{ij} - \cos \alpha_{ij}) + t_s \cos \alpha_s + t_s (\omega_s - \cos \alpha_s)
\]

\[
= \sum_{i=1}^{2} \sum_{j=1}^{n_1+g} t_{ij} (\omega_{ij} - \cos \alpha_{ij}) + t_s (\omega_s - 1)
\]

(38)

Based on the above fundamental formulas, next we can analyze the influences of adjusting the weight coefficients on the optimization results.

(a) Setting a large weight coefficient to a non-common member
Without loss of generality, assume the first member in the first group is assigned a sufficiently large weight coefficient, i.e.,

\[
\omega_{11} \gg \omega_{ij}, \; \forall (i, j) \neq (1, 1)
\]

(39)

Because \( \cos \alpha \in [-1, 1] \), we have

\[
(\omega_{11} - \cos \alpha_{11}) \gg (\omega_{ij} - \cos \alpha_{ij}), \; \forall (i, j) \neq (1, 1)
\]

(40)

Because the maximum absolute value of the strut force is bounded to be smaller than 1.0, the corresponding cable forces \( t_{ij} (> 0) \) in the equilibrium state will not be too large, therefore, the influence of member internal forces on the objective function value is much smaller than the weight coefficients. Based on this, it can be assumed that the objective function value is mainly governed by term \( t_{11} \omega_{11} \); if \( t_{11} \neq 0 \), the objective function will have a sufficiently large value, while if \( t_{11} = 0 \), the objective function could have a smaller value. Therefore, the objective function can achieve the minimum value
of $\sum_{j=1}^{n_1+g} t_{ij}(\omega_{2j} - \cos \alpha_{2j}) + t_{ij}(\omega_j - 1)$ when $t_{ij} = 0$ ($j = 1, \ldots, n_1$) is satisfied, i.e., the first group of members are discarded while the second group of members are selected.

Above analysis indicates that the selection priority of the members in a group can be reduced by assigning a relatively large weight coefficient to one of the non-common members in the group.

(b) Setting a large weight coefficient to a common member

Without loss of generality, assume that the first common member is assigned a sufficiently large weight coefficient, i.e.,

$$\omega_1 = \omega_{1(n_1+1)} = \omega_{2(n_2+1)} >> \omega_{ij} > 1, \quad \forall (i, j) \neq (1, n_1+1) \& (i, j) \neq (2, n_2+1)$$

Then following a similar analysis as above, if $t_{1(n_1+1)} > t_{2(n_2+1)}$, the objective function will achieve the minimum value of $\sum_{j=1}^{n_1+g} t_{ij}(\omega_{2j} - \cos \alpha_{2j}) + t_{ij}(\omega_j - 1)$ when $t_{ij} = 0$ ($j = 1, \ldots, n_1$), i.e., the first group of members are discarded while the second group of members are selected; if $t_{1(n_1+1)} < t_{2(n_2+1)}$, the objective function will achieve the minimum value of $\sum_{j=1}^{n_1+g} t_{ij}(\omega_{1j} - \cos \alpha_{1j}) + t_{ij}(\omega_j - 1)$ when $t_{ij} = 0$ ($j = 1, \ldots, n_2$), i.e., the second group of members are discarded while the first group of members are selected.

Above analysis shows that increasing the weight coefficient of a common member will reduce the selection priority of the group with relatively larger common member internal forces.

Based on the above analysis, the member weight coefficients can be systematically adjusted to change the selection priorities of the members in different groups, which allows us to find multiple tensegrity structures from the same ground structure by tuning the member weight coefficients. More details on the strategy of adjusting the weight coefficients can be referred to in Section 4.

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Table 6
Numerical results of Figs. 17–27.

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Appendix 2. Regular spherical tensegrities based on Archimedes polyhedrons

Regular spherical tensegrities based on different Archimedes polyhedrons have been found using the proposed method. Considering the practical application potential of symmetrical and regular tensegrity structures, only parameter $B = 1$ is adopted for the topology-finding, which means that each obtained structure has only one strut length type. For simplification, the weight coefficients are arranged in ascending order of length. If necessary, the readers can use other parameter and weight coefficient settings to investigate different novel tensegrity structures by using the proposed method.

Figs. 17–27 gives graphic illustrations of the tensegrity structures. Table 6 gives the summary information (e.g., number of members, self-stress modes, infinitesimal modes, stability) of the tensegrity structures. Considering the paper length limitation, more detailed information regarding the nodal coordinates, member connectivities, and internal forces of the tensegrity structures are given in Supplementary Information.

![Fig. 17. Graphic result of truncated tetrahedron assigned to weight coefficients in ascending order of length (a) geometry; (b) tensegrity.](image)

![Fig. 18. Graphic result of cuboctahedron assigned to weight coefficients in ascending order of length (a) geometry; (b) tensegrity.](image)
Fig. 19. Graphic results of truncated cube assigned to weight coefficients in ascending order of length (a) geometry; (b)–(d) tensegrity.

Fig. 20. Graphic results of truncated octahedron assigned to weight coefficients in ascending order of length (a) geometry; (b)–(d) tensegrity.
Fig. 21. Graphic result of small rhombicuboctahedron based on member length ascending order (a) geometry; (b) tensegrity.

Fig. 22. Graphic results of great rhombicuboctahedron assigned to weight coefficients in ascending order of length (a) geometry; (b)–(f) tensegrity.
Fig. 23. Graphic results of snub cube assigned to weight coefficients in ascending order of length (a) geometry; (b)–(e) tensegrity.

Fig. 24. Graphic results of truncated dodecahedron assigned to weight coefficients in ascending order of length (a) geometry; (b)–(c) tensegrity.
Fig. 25. Graphic results of truncated icosahedron assigned to weight coefficients in ascending order of length (a) geometry; (b)–(e) tensegrity.

Fig. 26. Graphic results of small rhombicosidodecahedral assigned to weight coefficients in ascending order of length (a) geometry; (b)–(d) tensegrity.
Fig. 27. Graphic results of snub dodecahedral assigned to weight coefficients in ascending order of length (a) geometry; (b)–(i) tensegrity.
References


