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MINIMISING THE UNCERTAINITIES IN THE CALCULATION OF STROBOSCOPIC EFFECT VISIBILITY MEASURE

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Abstract

The applied numerical calculations to determine the stroboscopic effect visibility measure (SVM) of a specific temporal light modulation waveform will influence the uncertainty of the determined value. An investigation of the achievable uncertainty under application of zero-padding and quadratic interpolation for peak finding is performed. This is done for sinusoidal and square verification waveforms outlined in IEC TR 63158 and demonstrates that using the referenced Stroboscopic effect visibility measure toolbox gives deviations from the reference values of up to 2.2%. Applying a combined optimised method of zero-padding and quadratic interpolation deviations less than 0.05% are achieved and yielding consistent results over different sampling frequencies. The verification waveforms are generated on a programmable light modulator and measurement results are shown.

Keywords: Temporal light modulation, Stroboscopic effect visibility measure, discrete Fourier transform, peak estimation, interpolation, zero-padding

1 Introduction

LED technology and Solid State Lighting has renewed the interest in temporal light modulation (TLM) and related temporal light artefacts. The stroboscopic effect is such an artefact and Perz et al. conducted perception experiments to develop a measure for predicting the visibility of the stroboscopic effect occurring in temporally modulated light systems (Perz et al., 2015). This is called the stroboscopic effect visibility measure, $M_{VS}$, and is defined by CIE in 2016 (CIE, 2016) and a test method for undertaking measurements for determination of SVM is described in the technical report, IEC TR 63158 (IEC, 2018). These two reports make reference to a recommended Stroboscopic effect visibility measure toolbox (Banerjee, 2022) available at Matlab Central for the calculation of SVM. In the technical report “Visual Aspects of Time-Modulated Lighting Systems” (CIE, 2022) the definition of the temporal contrast threshold function is slightly different, but it still refers to the same toolbox as the other documents.

Verification waveforms are given in Annex A.5 of (IEC, 2018) for the verification of the implementation of the calculation of SVM. It includes five square and four sinusoidal waveforms defined with modulation frequency, modulation depth and the analytical exact reference values of SVM. Using the Matlab toolbox (Banerjee, 2022) for calculation of the SVM for the verification waveforms yields large uncertainties, i.e. relative deviations from the reference SVM values, in the order of 0.2% - 2.2%. The deviation varies with sampling frequencies varying from 20 kHz to 50 kHz. So, this implies that large uncertainties are associated with the numerical calculation of SVM and it is conceivable that the application of this calculation method to actual (complex) temporal light waveforms could give rise to even larger uncertainties. The problem arises from the method of peak finding used by the Matlab toolbox. It employs zero-padding to the first nearest power of 2 for the input to the Discrete Fourier Transform (DFT) and followed by a simple, maximum-value search function “findpeak”, that results in significant errors in its identification of peak amplitudes and peak frequencies. These inaccurate frequency and magnitude pairs then pass to the calculation of SVM resulting in large uncertainties.

The objective of this study is to reduce the error in calculated results for SVM of these verification waveforms, while also improving the reproducibility across different sampling frequencies. The uncertainty on SVM calculations by this method can be minimised by
improving the accuracy of the estimation of peak frequency and amplitude in the DFT, specifically through:

- using extended zero-padding yielding narrower frequency bins in the DFT calculation and using a nearest bin peak estimation method
- introducing minimum thresholds for peak identification, to discard peaks arising from signal noise
- using a quadratic interpolation method, using the three DFT frequency bins around each peak in the DFT for better peak estimation.

It is important to reduce the uncertainty associated with the numerical calculations of SVM as this is only one of many uncertainty components in the measurement of SVM. Also important is that recommended methods for the calculation of SVM offer minimal uncertainties that remain consistent across a range of temporal light waveform measurement parameters (e.g., sampling frequency and duration). Without such consistency, it is challenging to produce reliable results at a high level of proficiency across laboratories that employ different commercial and in-house temporal light modulation measurement systems.

In 2019 the European Commission set requirements for the maximum value of SVM for the intrinsic performance of light emission light from LED and OLED light sources. This has been revised in 2021 (EC, 2021) and the requirement is that SVM ≤ 0.9 at full-load from September 2021, and from 1 September 2024 it is stricter: SVM ≤ 0.4 at full-load. Therefore, there is a growing demand for test laboratories to be able to make consistent and reproducible SVM measurements with low uncertainties, to determine compliance with the legislation.

With the increasing demands for testing laboratory accreditation for TLM quantities, the International Energy Agency (IEA) 4E Solid State Lighting Annex is planning on conducting an Interlaboratory Comparison (IC 2022) for the measurement of TLM of solid-state lighting (SSL) products. Designed to meet proficiency testing requirements, IC 2022 is organised to compare measured TLM waveform data and calculated SVM as well as short term flicker index (PstLM) of comparison artefacts. It is therefore critical to the endeavours of IC 2022, to resolve the identified issues in the referenced method for calculation of SVM values and achieve accurate SVM results with minimal uncertainties prior to commencement of IC 2022.

The SVM calculation and peak estimation methods in the DFT are explained in section 2, together with a description of the verification waveforms for SVM calculation. A Matlab® function combining zero-padding with both nearest-to-peak bin method and the quadratic interpolation (QI) method for peak estimation is implemented. Results on the simulated verification waveforms are shown using this new function in section 3. Results are also given for measured waveforms on a programmable light generator.

2 Method

In temporal light analysis the measured or simulated temporal waveform \( E(t) \) is normalised to the mean value. The numerical calculation of the SVM is performed in the frequency domain. The waveform is normally multiplied by a window function in order to reduce spectral leakage. A Hanning window is used here. The magnitude spectrum is calculated using the Discrete Fourier Transform (DFT) of the windowed waveform.

In Figure 1 a sketch of the situation of peak estimation in the magnitude spectrum is illustrated. The continuous dashed blue line is the Discrete-Time Fourier Transform (DTFT) of a Hanning-windowed sinusoid. The maximum of the DTFT is associated with the frequency and amplitude of the sinusoidal component in the waveform \( E(t) \). It is marked by a black dot and given by the frequency \( f \), and amplitude \( y \).

The stroboscopic visibility measure is defined as (IEC, 2018)

\[
M_{VS} = \sqrt[3.7]{\sum_{m=1}^{N_z} \left( \frac{c_m}{t_m} \right)^{3.7}}
\]

where
\( C_m \) is the relative amplitude of the \( m \)-th Fourier component of the normalised waveform \( E(t) \);
\( T_m \) is the visibility threshold for the stroboscopic effect for a sine wave at the frequency, \( f_m \) of the \( m \)-th Fourier component.

The visibility threshold is given as

\[
T(f) = \frac{1}{1 + e^{-a(f-b)}} + 20e^{-f/10}
\]

where

- \( f \) is the frequency in Hz;
- \( a = 0.00518 \);
- \( b = 306.6 \).

The numerical calculation of SVM therefore involves both the frequency and amplitude of the Fourier components with highest sensitivity being to the accuracy of fundamental frequency and amplitude due to its significant influence by way of the function’s high Minkowski norm value of 3.7. The uncertainty of the SVM therefore directly depends on the uncertainty of the determination of each of the peak frequencies and amplitudes in the DFT of the waveform. Figure 1 illustrates the process for determination of a peak by sampling of the DTFT at the frequencies indexed by \( k \), illustrated as red circles with indexes from \( k_p-2 \) to \( k_p+2 \).

![Figure 1 Sketch of the situation of peak estimation in the magnitude spectrum around a peak (black dot) in the DTFT (blue dashed line), showing frequency bins (k) and amplitudes (y) of the DFT (red dots and circles).](image)

A simple way of estimating sinusoid parameters is to find the peak in the DFT magnitude spectrum and use its frequency and amplitude as estimates, this is called nearest bin method. According to Figure 1 this would be using \((k_p, y_p)\), where \( y_p \) is the amplitude at the frequency bin, as an estimate for \((f, y_f)\). The estimate derived from the nearest bin method can have up to \(\pm 0.5 \) bins of error.

This simple peak estimation can be refined by increasing zero padding before evaluation of the DFT and/or by single function evaluation on the amplitude at the \( k_p \) bin and at the neighbour bins \( k_{p-1} \) and \( k_{p+1} \). The two methods are explained in the following sections.

### 2.1 Zero-padding/Nulling

Adding zeros to the signal and performing a larger DFT creates a greater number of, and narrower frequency bins and hence lower errors in the peak estimation using nearest bin method. This is called nulling or zero-padding. Applying zero-padding to achieve the nearest
power of 2 samples above the recorded sample size, i.e. the length of the discrete waveform $E(t)$, is used to enhance frequency bin resolution of the Fast Fourier Transform (FFT). But this results in peak frequencies away from integer numbers. This is done in the SVM toolbox and using the nearest bin method for peak estimation results in large errors for the verification waveforms given in (IEC, 2018) which will be shown in the Results section.

Zero-padding corresponds to sampling of the DTFT, the dashed blue line in Figure 1, with finer frequency bins. Increasing the number of nulling, NFFT+, (i.e. an increasing number, NFFT+, of powers of two), will reduce the error in peak estimation when using the nearest bin method. For example, a 1 s duration signal at 50 kHz the frequency bin width is equal to 0.38Hz/NFFT+, and the error in frequency estimation will be $\pm (0.5 \times 0.38) \text{ Hz}/NFFT+$. For NFFT+ = 7 the error in frequency estimation becomes $\pm 0.055$ Hz.

Zero-padding a signal does not reveal more information about the spectrum, but it only interpolates between the frequency bins that would occur when no zero-padding is applied. To increase the spectral resolution, longer durations of real measurements are necessary. A longer measurement obtains more information from the measured signal, and a narrower frequency bin.

Zero-padding increases the calculation load drastically and may limit real time evaluation functionality of SVM measurements in an instrument.

### 2.2 Quadratic interpolation

The peak finding may be refined without a considerable calculation load using interpolation methods around the maximum points in the magnitude spectrum. Quadratic-Interpolation FFT (QIFFT or QI) over the three DFT bins surrounding magnitude peaks improves the estimation of the peak frequency and amplitude, and it is applicable to any window type (Werner and Germain, 2016).

This has been implemented in the SVM calculation algorithm used here. It uses the peak found by nearest bin method $(k_p, y_p)$, and its lower and upper neighbour bins $(k_{p-1}, y_{p-1})$ and $(k_{p+1}, y_{p+1})$, as illustrated by the full red circle and its unfilled red circle neighbours in Figure 1.

The method assumes that the underlying DTFT of a window transform is reasonably smooth, and fits a parabola to the magnitude spectrum and determines the vertex of this parabola as a refined estimate of the true peak. This is shown in Figure 1 by the red solid line and the peak estimate marked by the full green circle $(f_{QI}, y_{QI})$ is given by (Werner and Germain, 2016):

$$f_{QI} = k_p + \frac{1}{2} \frac{y_{p-1} - y_{p+1}}{y_{p-1} - 2y_p + y_{p+1}}$$

$$y_{QI} = y_p - \frac{1}{8} \frac{(y_{p-1} - y_{p+1})^2}{y_{p-1} - 2y_p + y_{p+1}}$$

The QI method is inexpensive with regards to calculation load/time and it can be combined seamlessly with zero padding to further increase their accuracy. These combined methods have been applied in this work on the specific verification waveforms.

A Matlab® function for the calculation of the SVM has been written, that allows for setting the zero-padding number, NFFT+, the threshold for peak finding and implementation of the quadratic interpolation. The function gives the calculated value of SVM using NFFT+ with both the nearest bin method and the quadratic interpolation (QI) method. These will be compared in section 3 Results on both simulated and measured waveforms.

### 2.3 Verification waveforms

In Annex A.5 in (IEC, 2018) five square and four sinusoidal waveforms are defined for the verification of the implementation of the calculation of SVM, The waveforms are given by

$$E_{sq}(t) = 1 + m_{ver} \text{ sign}(\sin(2\pi f_{vert}t)) \quad \text{and} \quad E_{sin}(t) = 1 + m_{ver} \sin(2\pi f_{vert}t)$$

(5)
for the square and sinusoidal waveform, respectively. Here $m_{ver}$ is the verification modulation depth and $f_{ver}$ is the verification frequency. In Table A.1 in (IEC, 2018) the parameters for the verification waveforms are given. Modulation depths $m_{ver}$ are calculated for the verification waveforms to give analytical exact reference values $M_{VS}^E$ of SVM using

$$m_{ver} = M_{VS}^E T(f_{ver}),$$

and

$$m_{ver} = M_{VS}^E \sqrt{\frac{3.7 \sum_{n=1}^{N_{f_{ver}}} \pi^2 (f_{ver} \pm n f_{ver})^{3.7}}{N_{f_{ver}}}}$$

for the square and sinusoidal waveforms, respectively. In Table 1 the parameters of the nine verification waveforms are listed with higher number of digits than in Table A.1 in (IEC, 2018). This is necessary when investigating small deviations.

### Table 1 – Specification of waveform type, modulation frequency, modulation depth and reference SVM value for the verification waveforms

<table>
<thead>
<tr>
<th>Verification waveform</th>
<th>Type of modulation</th>
<th>Modulation frequency $f_{ver}$ [Hz]</th>
<th>Modulation Depth $m_{ver}$</th>
<th>Reference SVM $M_{VS}^E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>VW-sq1</td>
<td>Square</td>
<td>99</td>
<td>0.200 492 00</td>
<td>1,000 000</td>
</tr>
<tr>
<td>VW-sq2</td>
<td>Square</td>
<td>100</td>
<td>0.201 190 76</td>
<td>1,000 000</td>
</tr>
<tr>
<td>VW-sq3</td>
<td>Square</td>
<td>100</td>
<td>0.020 119 076</td>
<td>0,100 000</td>
</tr>
<tr>
<td>VW-sq4</td>
<td>Square</td>
<td>100</td>
<td>0.804 763 04</td>
<td>4,000 000</td>
</tr>
<tr>
<td>VW-sq5</td>
<td>Square</td>
<td>101</td>
<td>0.201 898 57</td>
<td>1,000 000</td>
</tr>
<tr>
<td>VW-sn1</td>
<td>Sinusoidal</td>
<td>32</td>
<td>1</td>
<td>0,990 565</td>
</tr>
<tr>
<td>VW-sn2</td>
<td>Sinusoidal</td>
<td>100</td>
<td>0.256 275 33</td>
<td>1,000 000</td>
</tr>
<tr>
<td>VW-sn3</td>
<td>Sinusoidal</td>
<td>500</td>
<td>0.731 414 69</td>
<td>1,000 000</td>
</tr>
<tr>
<td>VW-sn4</td>
<td>Sinusoidal</td>
<td>1900</td>
<td>0.999 739 80</td>
<td>1,000 000</td>
</tr>
</tbody>
</table>

### 3 Results

The verification waveforms given above have been used for calculations of simulated waveforms and they have been implemented on a programmable light generator for subsequent measurements. Both simulated and measured waveforms are used to investigate the uncertainty of the SVM calculation using the developed Matlab® function.

#### 3.1 Simulated waveforms

Calculations of SVM and the deviations from the reference values are performed for each of the verification waveforms, specified in Eq. (5), with increasing zero padding, NFFT+. Calculations are done using simple peak finding with the nearest bin method and using quadratic interpolation, QI. This is done for sampling frequencies of 20 kHz and 50 kHz. The results are shown in Figure 2 for the five square verification waveforms and in Figure 3 for the four sinusoidal verification waveforms. Here the blue markers are for nearest bin method and the red markers and for quadratic interpolation (QI) in peak estimation.

Using the toolbox (Banerjee, 2022) for calculation of the SVM corresponds to the blue markers at NFFT+ equal to 1, and it can be observed that these values have the largest deviations ranging from 0.2% to 2.2%. It is also seen that the deviation varies considerably with the sampling frequency applied in the simulation.

Increasing number of nulling, shown up to NFFT+ = 7, reduces the deviation and the values converge to zero. It is found that deviations of less than 0.05% are achieved for NFFT+ ≥ 4 or
5 dependent on the waveform. Applying quadratic interpolation is observed to markedly reduce the deviation for all values of NFFT+. It can be seen by looking at the red markers and lines that deviations of less than 0.05% can be achieved with NFFT+ equal to 2 or 3 dependent on the waveform.

The only exception is for the square verification waveforms VW-sq3 and VW-sq4, for which the deviation converges to a value different from zero and slightly higher than 0.05%. The reason for this has not been identified.

![Graphs showing deviation from reference value of SVM as a function of NFFT+ for the five square verification waveforms. Calculations are done at sampling frequency of 20 kHz and 50 kHz, and with no interpolation (-) and with quadratic interpolation (QI).]
Figure 3 Calculated deviation from reference value of SVM as a function of the zero-padding number (NFFT+) for the four sinusoidal verification waveforms. Calculations are done at sampling frequency of 20 kHz and 50 kHz, and with no interpolation (■) and with quadratic interpolation (QI).

3.2 Measured waveforms

The verification waveforms used in the methods section have been implemented on a programmable light generator. With a 500 lm average luminous flux, the nine verification waveforms specified in Eq. (5) have been generated. They have been applied and measured using a custom built temporal light measurement system based on a ø10mm Si-detector with photometric filter and diffuser, a variable gain transimpedance amplifier and an analog to digital converter with a maximum sampling frequency of 100 kHz.

The waveforms are measured at various sample frequencies of 20 kHz and 50 kHz over 1s duration. The SVM values are calculated with the developed Matlab® function. In Figure 4 the measurement results are shown as the deviation from the reference SVM value of the verification waveforms. The solid colour bars are for 50 kHz and and the striped are for 20 kHz. The blue bars are calculated according to the toolbox (Banerjee, 2022) and the orange bars are for the optimized method described here using NFFT+ of 3 and using quadratic interpolation in peak estimation.
Figure 4 Measured and calculated deviation from the reference SVM value of the verification waveforms implemented on a programmable light generator.

The measurements and calculations of SVM shows that using the toolbox there are large variations for the two sampling frequencies applied. This is not the case for the optimised method, where the values are which is consistent with the observations on the simulated verification waveforms.

The measured values using the optimised method shows a deviation from the reference value of SVM that varies from +0.4% to -1.5% This uncertainty that may come from the programming and generation of the temporal light modulated signal and the measurement of it.

4 Conclusion

The calculation of the stroboscopic effect visibility measure (SVM) and related peak estimation has been described. It is shown how the uncertainty in peak estimation, frequency and amplitude, and hence SVM calculation can be minimised. This is done using increased zero-padding in the DFT calculation and/or using quadratic interpolation on the nearest bin and its two neighbour frequency bins in the DFT.

Simulation of the square and sinusoidal verification waveforms have shown that deviations from the reference value of SVM, of less than 0.05% can be achieved using zero-padding with NFFT+ ≥ 4 or 5 dependent on the waveform. Applying quadratic interpolation in the peak estimation is shown to markedly reduce the deviation for all values of NFFT+, and that deviations of less than 0.05% can be achieved with NFFT+ = to 2 or 3 dependent on the waveform.

Since increasing zero-padding increases the numerical calculation load markedly, and the quadratic interpolation is inexpensive in only requiring a few multiplications, an optimal SVM calculation would be: a combined approach with limited zero padding of NFFT+ equal to 2 or 3 and using quadratic interpolation. This would be a recommended calculation method for SVM offering minimal uncertainties that remain consistent across different temporal light waveform measurement parameters such as the sampling frequency.

The method has been shown to reduce the uncertainty in the SVM numerical calculation as seen using the referenced toolbox (Banerjee, 2022) and it also removes the large variations in SVM calculation as a function of sampling frequency.
Measurements on verification waveforms implemented on a programmable light generator have been carried out and calculation using the toolbox and optimised method described here shows a much higher consistency over the applied sampling frequencies using the optimised method. The measurements reveal an uncertainty of up to 1.5% that may be associated with the programmed generation and the measurement of the temporal modulated light.

Using the optimised method described, it is possible to produce reliable results that see agreement across laboratories that employ different commercial and in-house temporal light measurement systems. This is important in the preparation for the International Energy Agency (IEA) 4E Solid State Lighting Annex IC 2022 on TLM measurement.

Acknowledgement

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