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A MILP model for energy system infrastructure planning under scenario-based uncertainty

(DTU Technical Report[*][†])

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April 21, 2023

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Abstract

This document describes a mathematical model for energy system infrastructure planning problems. Its aim is to identify the most socioeconomically desirable energy system configuration for a given problem described within a MILP optimisation framework. The best configuration is identified in a way consistent with net present value calculations for a set of investments and operational needs. The energy system is complemented or created from scratch by the investments, enabling or providing alternative ways for the operational needs to be met. These are expressed as flow requirements in a set of networks that partly describe the energy system. Meeting them may require flows in and out of the system, potentially resulting in expenditures and revenue, in accordance with the respective tariffs. Internal flow distribution proceeds along pre-existing paths or those created through new investments, and is otherwise free, though not necessarily lossless. Losses are path segment-specific, depend on the solution deployed to allow flow along it, and can have static and flow-proportional components. Flows within the system have to be in equilibrium with another and compensate for losses and operational needs. The latter can also be dynamic if specified through modular sets of difference equations and constraints that can also be used to model interactions between networks. These structures are here termed flow converters and can also be introduced through investments. The formulation proposed also includes novelties, namely the structures necessary to use special ordered sets for selecting investments in paths, an alternative way to model static losses without intermediate nodes, and the possibility to define investments in groups of arcs rather than only on individual ones. Other key assumptions include the inexistence of flow delays and the precedence of investments relative to the planning period. Beyond these assumptions, the model design prioritised versatility and organisation over more streamlined approaches, as the model was primarily developed for scientific applications.

Contents

Chapter 1

Introduction

1.1 Motivation

Infrastructure is a term used to describe collective equipments deployed to enable certain activities. Examples include bridges for crossing rivers, waterworks for supplying water, and ports for transshipment. As a decision-making process, it is imbued with technical and political dimensions, each with its own forms of uncertainty. It also tends to involve more resources, larger investments, longer lifespans, more environmental consequences, more participated processes and more external scrutiny compared to more personal decisions, whereas the benefits may be more distributed. As such, anticipating the outcomes of investments in infrastructure, and planning them, can and perhaps should be multi-disciplinary enterprises requiring greater care than individual decisions.

Software tools can bring about improved infrastructure planning. One way is through transparency, since the results and methods might be more easily reproduced and reviewed. Another way is by automating cumbersome and repetitive tasks, the potential benefits of which include time savings, cost reductions, fewer human errors and higher reliability by using more advanced methods. Among them one finds optimisation, the process through which more suitable alternatives can be methodically identified, which is the focus of this document.

The purpose here is to describe a mathematical model for the optimisation of energy system infrastructure. It is intended to help identify the most promising set of investments out of those under contention for a given energy system. Though the aim was also to be as inclusive as possible, an early design decision was to adopt mixed-integer linear programming (MILP) as the optimisation framework, which entails modelling limitations (e.g., no quadratic terms). Despite this assumption, the model can address certain classes of investment problems on energy system infrastructure – i.e., the equipments enabling the organised provision of energy – as could previous models making the same assumption. In the literature survey that follows, such efforts are reviewed.

1.2 Literature review

Model development benefited from and assimilated previous efforts. Those identified are primarily studies concerning district heating and cooling investment problems expressed within a MILP framework. As a result, the objectives pursued reflect economic priorities consistent with socio-economic optimisation, as profit is not necessarily the goal [5, 7, 8]. Instead, cost minimisation for a given specification is often the aim, e.g., the provision of end-user demand [4, 15].

The studies surveyed targeted investments on energy transport, storage, and conversion infrastructure. By considering the first of these, the problems take on a spatial dimension reminiscent of network optimisation problems. Söderman (2007) exemplified this in a study on the design of district cooling networks with minimal cost in which the equipments and their locations are part of the decision space [1]. In the study, the equipments included plants, storage units and pipelines. Manfren (2012) discussed a more general approach for energy systems, described using graph theory terminology, that also considered transshipment [2]. Subsequently, Dorfner et al. (2014, 2017) proposed MILP formulations for designing district cooling systems using directed (2014) and undirected (2017) graphs, wherein arcs were defined as having static and flow-dependent losses [3, 4]. Röder et al. (2021) followed up on these efforts by modelling distributed storage to study its effect on district heating network layouts [6]. In turn, Bordin et al. (2016) prioritised hydraulic considerations and pre-existing conditions in a MILP model for designing district heating networks. The study also discusses the potential for new features and their trade-offs, namely between versatility, model accuracy and computational performance [5].

Computational performance concerns surrounding long-term energy planning abound in the literature. Among studies using MILP, the issue has been framed around the number of decision variables needed to model such problems, particularly binary ones [1]. Renaldi and Friedrich (2017) suggested adjusting equipment models and temporal discretisations as ways to mitigate the issue, and explored the latter [10]. Gabrielli et al. (2017) proposed using separate optimisations for the design and operational aspects [12]. Kuriyan and Shah (2019) explored this aspect further and considered major and minor temporal scales for investments and operational decisions, respectively [11]. On the data side, model reduction techniques have been explored to facilitate this [12, 13].

Other approaches have also been considered. Manfren (2012) suggested the use of special ordered sets [2]. Dorfner et al. (2017) proposed using relaxed flow equilibrium constraints [4]. Others investigated the use of evolutionary meta-heuristics [13–17]. These were found to outperform MILP solvers in a few studies [15, 17]. The increased modelling freedom was also noted [17].

The potential of flexibility and integration synergies has also received attention. Söderman and Ahtila (2010) proposed a MILP model for exploring synergies between heating and cooling systems, namely via investments in heat exchangers [20]. Hilpert et al. (2018) described a model allowing inter-network synergies to be considered [18]. Dominkovic et al. (2020) proposed incorporating short-term flexibility in system planning through a feedback loop [9].

The subject of protection against uncertainty has also been addressed, though primarily using two-stage stochastic programming. Lambert et al. (2016) considered it for designing a district heating network under energy price and heat demand uncertainty [7]. Mavromatidis et al. (2018) complemented that effort by also considering energy demand and solar radiation uncertainty in designing a distributed energy system [19], whereas Egberts et al. (2020) only considered demand uncertainty for planning district heating networks [16]. In contrast, Akbari et al. (2014, 2016) used robust counterparts to evaluate the impact of conversativeness on designs for distributed energy systems [21, 22].

1.3 Objectives

The literature on energy system planning models suggested an opportunity for integration, simplification and synthesis. These assertions were prompted by observations on the overlap between the various models, the unused opportunities for abstraction (e.g., between various technologies and forms of energy), the lack of a comprehensive approach for handling synergies and flexibility, and the general direction of model development (e.g., decoupling investment and operational decisions). Based on these, a more general model was conceptualised.

The mathematical model proposed has two main objectives. The first is to generalise energy system infrastructure planning problems within a MILP framework. Doing so can be beneficial for the following reasons: easier validation and benchmarking, as results become easier to compare and reproduce; improved task separation, by making it easier for researchers to focus on what they do best; extendability, since novelties become easier to recognise and integrate; greater versatility, as the degrees of freedom are preserved to accommodate all cases; and, greater clarity, since by addressing the general case the problem structure is laid bare, and the development of algorithmic approaches made easier. The second objective concerns the identification of structures that can be taken advantage of for enhanced computational performance. In pursuing this aim, more palatable compromises between accuracy and computational cost may be attained for this class of problems or peculiar instances thereof.

1.4 Approach

The objectives outlined were approached in the following way. First, emphasis was put on providing a generic formulation for the elements common to all problems rather than discriminating the various possibilities (e.g., forms of energy) and their peculiarities, since those can be accommodated through extensions. A focus on functionality and modularity was also observed in deriving the formulation, so as to keep the elements relevant and optional, in addition to a general preoccupation around the separation between model and data. In this regard, design choices were avoided whenever possible in order to keep the model flexible. This means that infeasibility is not precluded and that data

curation (parameters and sets) should not be neglected. Consistency was also prioritised over engineering sleights of hand that can have unintended consequences, particularly when problem size renders verification more difficult. A fourth consideration had to do with the completeness of the description, by explicitly including alternative ways of modelling certain aspects, since that can facilitate understanding their repercussions. Its side-effect is a longer model description, even if previous points can counter that tendency.

1.5 Contributions

The MILP model proposed amounts to a two-stage stochastic program for socioeconomic optimisation. It is suitable for evaluating investments in energy system infrastructure based on the net present value. The energy system is partly defined by networks with normal and transshipment nodes and arcs distinguished by their directionality (directed or undirected), pre-existence (new or pre-existing) and losses (static and flow-proportional). These distinctions can already be found in the literature and are integrated into the model.

A few novelties were also introduced in the model. In addition to the general integration effort, the formulation describes static losses in arcs without relying on intermediate nodes. Another novelty concerns the possibility of selecting investments for a multitude of arcs as if it were just one – a generalisation of investments in individual arcs. A third contribution relates to the identification of structures compatible with special ordered sets of type one (SOS1), which are highlighted in the model. The final contribution has to do with the introduction of structures termed flow converters, which refer to sets of difference equations and associated constraints that can be utilised to model behaviours ranging from interactions between different networks to dynamic loads.

A few aspects were also left out of the model, such as transport delays. Converter controls were also left out though they can be added as an extension. Investment scheduling was also not modelled explicitly, though it can be considered in an ad-hoc manner. Another omission is robust programming, since it requires defining uncertainty sets and can be tackled separately.

1.6 Structure

Considering the extent of this document, some clarification around its structure might be pertinent. Following this introductory chapter, the model is introduced gradually, first by describing its components via the main sets used in it, and later through its objective function, variables and constraints (Chapter 2). Subsequently, a chapter describing a few examples is provided to demonstrate its options and applications (Chapter 3), which is followed by some concluding remarks (Chapter 4). Then, the nomenclature is described in detail (Chapter 5), after which the funding sources and individual contributions are discriminated (Acknowledgements). The last chapter is dedicated to the bibliography.

Chapter 2

Mathematical model

2.1 Overview

The model can be briefly described through the sets it uses. These are used to keep the model organised, modular and scalable. The following sections introduce some of the key parts of the model using the relevant sets and examples.

2.1.1 Nomenclature

The model description follows a set of nomenclature conventions. Sets are identified via alphabetic capital letters, one per member dimension, with all members having the same dimensionality: most sets are written as one capital letter (e.g., Q), for one-dimensional members (e.g., $q \in Q$), but a few sets use more capital letters (e.g., GLL) for multi-dimensional members (e.g., $(g, l, l^*) \in GLL$). Set members are identified via lower case equivalents of the letters identifying the set that they are tentatively members of (e.g., $g \in G$). When more than one member needs to be referenced in the same constraint, markers are used to differentiate them (e.g., $g \in G, g^* \in G$). The same capital letter(s) can also be used for different sets if there are subscripts and/or superscripts: the former are used for indexation (e.g., $l \in L_0$), whereas the latter concern fixed categories, often involving shortened names (e.g., $l \in L_0^{imp}$) for ease of understanding.

The aforementioned logic for subscripts and superscripts also applies to variables and parameters. Variables for quantities with an intended technical meaning within this class of investment problems use acronyms in capital letters: $CAPEX$, $SDNCF$, EFR, IFC, EF, and IF. Variables for general quantities use lower case alphabetic letters after t: u, v, w, x, and y. Variables concerning differences between the previously-cited variables use a Δ prefix (e.g., Δx). Variables for binary decisions also use greek letters but in lower case: δ, ξ, and ζ. In turn, parameters use lower case alphabetic letters with one exception (η). Moreover, the letters used for parameters only overlap with those used for variables when they concern the same quantity (e.g., initial conditions).

2.1.2 Problem

The investment problem at the heart of the model considers a given planning horizon divided into sequential reporting periods. Their number and duration are defined through data: $\forall p \in P = \{1, 2, ..., |P|\}$. Evaluating how the energy systems perform during each is done through operational performance assessments: $\forall q \in Q$. One of these may be used for a specific reporting period or multiple. The reporting periods covered by a given assessment q are given by the set P_q, a subset of P, (2.1). In this way, the assessments can cover the entire planning horizon, possibly more than once, since it is also possible to consider alternative scenarios with a given probability: $c_q^{wgt}, \forall q \in Q$. Among other things, this allows for the evaluation of different discount rates.

$$P_q \subseteq P, \forall q \in Q \tag{2.1}$$

Each assessment q is also associated with an independent time scale composed of sequential intervals: $\forall k \in K_q = \{1, 2, ..., |K_q|\}$. The relative weight of a given interval k within a given reporting period p is predetermined and given by a parameter: $c_{q,p,k}^{time}, \forall q \in Q, \forall k \in K_q = \{1, 2, ..., |K_q|\}, \forall p \in P_q$. By introducing these parameters, it becomes possible to assign a greater importance to certain intervals, or sequences thereof, possibly leading to the exclusion of others to obtain smaller problems that are deemed sufficiently accurate. Some of the possibilities offered by this formulation are presented in Tables 2.1-2.5 for problems with three reporting periods ($P = \{1, 2, 3\}$).

The examples illustrate different ways to use the model for a yet unknown energy system. Note also that the time scales and respective discretisations are defined by data in the form of sets and coefficients to keep the model as versatile as possible. Accordingly, it is possible to adjust the planning period time scale without changing the assessment time scale and vice-versa. The following section discusses how the underlying energy system is modelled using only the operational performance assessment time scale.

2.1.3 Energy system

Energy systems are modelled as networks that interact with one another through systems henceforth referred to as converters. The result of these interactions is present in the objective function through the $SDNCF$, which translates operational performance considerations. The objective function is also sensitive to the energy system configuration through the $CAPEX$, which take into account which network layouts and converters were selected. In what follows, the networks and converters used to model energy systems are described.

Networks

An energy system must have at least one network, each with at least one node. Networks are identified through membership in the set G: $\forall g \in G$. Nodes on a

Table 2.1: Problem with three reporting periods relying on one assessment with two intervals: all reporting periods rely on the same assessment.

$q \in Q$	$p \in P_q$?			P_q	K_q
	1	2	3		
1	✓	✓	✓	$\{1,2,3\}$	$\{1,2\}$

Table 2.2: Problem with three reporting periods relying on two assessments with two intervals: one assessment for the first period and another for the rest.

$q \in Q$	$p \in P_q$?			P_q	K_q
	1	2	3		
1	✓	×	×	$\{1\}$	$\{1,2\}$
2	×	✓	✓	$\{2,3\}$	$\{1,2\}$

Table 2.3: Problem with three reporting periods relying on two assessments with two and three intervals: one assessment for the first two periods with two intervals; and another for last period with three intervals.

$q \in Q$	$p \in P_q$?			P_q	K_q
	1	2	3		
1	✓	✓	×	$\{1,2\}$	$\{1,2\}$
2	×	×	✓	$\{3\}$	$\{1,2,3\}$

Table 2.4: Problem with three reporting periods relying on two assessments with two intervals: each assessment is used for all three reporting periods.

$q \in Q$	$p \in P_q$?			P_q	K_q
	1	2	3		
1	✓	✓	✓	$\{1,2,3\}$	$\{1,2\}$
2	✓	✓	✓	$\{1,2,3\}$	$\{1,2\}$

Table 2.5: Problem with three reporting periods relying on six assessments with two intervals: each reporting period is covered by two assessments.

$q \in Q$	$p \in P_q$?			P_q	K_q
	1	2	3		
1	✓	×	×	$\{1\}$	$\{1,2\}$
2	✓	×	×	$\{1\}$	$\{1,2\}$
3	×	✓	×	$\{2\}$	$\{1,2\}$
4	×	✓	×	$\{2\}$	$\{1,2\}$
5	×	×	✓	$\{3\}$	$\{1,2\}$
6	×	×	✓	$\{3\}$	$\{1,2\}$

network g cannot be in other networks and are identified through membership in the set L_g: $\forall l \in L_g$. Two special types of nodes are worth mentioning: import and export nodes. These are identified through membership in the sets L_g^{imp} and L_g^{exp}, respectively, both of which are subsets of L_g, (2.2)-(2.3). At the same time, a network cannot consist of only import nodes or only export nodes, but one that has both is valid (i.e., to evaluate import-export routes), as are all combinations involving other types of nodes. These other types of nodes cannot be import or export nodes and are identified solely through data: waypoint nodes require no incoming or outgoing flows; source nodes require outgoing flows or local sinks; sink nodes require incoming flows or local sources. One other point about nodes is that they cannot be optional, unlike arcs, though they may have no impact on the solution (e.g., waypoint nodes).

$$L_g^{imp} \subset L_g, \forall g \in G \tag{2.2}$$

$$L_g^{exp} \subset L_g, \forall g \in G \tag{2.3}$$

Arcs connect two distinct nodes to allow flows between them. If the flows are one-directional, the arc should be directed. The other option is to use undirected arcs to consider reversible flows – one flow sense per interval. An arc cannot be both directed and undirected but these can exist in parallel, (2.4). Directed arcs allowing flow from node l to node l^* on network g are identified via the set J_{g,l,l^*}^{dir}. In turn, undirected arcs between nodes l and l^* on network g are identified through membership in the sets J_{g,l,l^*}^{und} or $J_{g,l^*,l}^{und}$, but not both simultaneously. If j is a member of J_{g,l,l^*}^{und}, then the nominal direction for that undirected arc is from l to l^*, which is the one that must be used in related sets.

$$
\begin{aligned}
J_{g,l,l^*}^{dir} \cap \left(J_{g,l,l^*}^{und} \cup J_{g,l^*,l}^{und} \right) = \emptyset, & \forall g \in G, \\
& \forall l \in L_g \setminus L_g^{exp}, \\
& \forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right)
\end{aligned}
\tag{2.4}
$$

Arcs cannot exist between any and all pairs of nodes. Directed arcs can link any pair of distinct nodes as long as the start node is not an export node, and the end node is not an import node, (2.5)-(2.7). This is so because self-loops provide no additional functionality and because import and export nodes are meant respectively for incoming and outgoing flows only. Consequently, undirected arcs cannot interact with import and export nodes directly, be it as start or as end nodes, (2.8)-(2.9). The alternative is to introduce intermediate nodes and connect these to the import or export nodes via directed arcs.

$$
\begin{aligned}
J_{g,l,l}^{dir} \cup \left(J_{g,l,l}^{und} \cup J_{g,l,l}^{und} \right) = \emptyset, & \forall g \in G, \\
& \forall l \in L_g
\end{aligned}
\tag{2.5}
$$

$$J_{g,l,l^*}^{dir} = \emptyset, \forall g \in G,$$
$$\forall l \in L_g^{exp}, \tag{2.6}$$
$$\forall l^* \in L_g$$

$$J_{g,l,l^*}^{dir} = \emptyset, \forall g \in G,$$
$$\forall l \in L_g, \tag{2.7}$$
$$\forall l^* \in L_g^{imp}$$

$$J_{g,l,l^*}^{und} \cup J_{g,l^*,l}^{und} = \emptyset, \forall g \in G,$$
$$\forall l \in L_g^{imp}, \tag{2.8}$$
$$\forall l^* \in L_g$$

$$J_{g,l,l^*}^{und} \cup J_{g,l^*,l}^{und} = \emptyset, \forall g \in G,$$
$$\forall l \in L_g, \tag{2.9}$$
$$\forall l^* \in L_g^{exp}$$

Arcs can also be pre-existing or new, (2.10). Pre-existing arcs have been dimensioned and must be part of any and all feasible configurations, though if and to what extent they are to be utilised is unknown. They are identified through membership in the set J_{g,l,l^*}^{pre}, whereas new arcs are identified via J_{g,l,l^*}^{new}.

$$J_{g,l,l^*}^{new} \cap J_{g,l,l^*}^{pre} = \emptyset, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp}, \tag{2.10}$$
$$\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right)$$

Since pre-existing arcs have already been dimensioned, their flow limitations must be known. They can have finite flow limits, if they are directed or undirected, though the former can also be specified as having no flow limit. Directed arcs without flow limits are identified through the set J_{g,l,l^*}^{inf}, (2.11).

$$J_{g,l,l^*}^{inf} \subseteq J_{g,l,l^*}^{dir} \cap J_{g,l,l^*}^{pre}, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp}, \tag{2.11}$$
$$\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right)$$

New arcs have not been dimensioned in advance and can induce capital costs if they are, but not otherwise. Dimensioning a new arc implies selecting the nominal flow amplitude and one arc option out of possibly several, each with

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independent flow limits (more below). New arcs can be dimensioned individually or in groups. In the former case, the act of selecting options is independent for each arc. In the latter case, the same option is selected for all arcs in the group, though their operational performance remains independent. A new arc j between nodes l and l^* on network g is selected as part of a group if it is a member of J_{g,l,l^*}^{col}. Alternatively, it must be a member of J_{g,l,l^*}^{sgl}, (2.12)-(2.14).

$$
\begin{aligned}
J_{g,l,l^*}^{col} \subseteq J_{g,l,l^*}^{new}, &\forall g \in G, \\
&\forall l \in L_g \setminus L_g^{exp}, \\
&\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right)
\end{aligned} \tag{2.12}
$$

$$
\begin{aligned}
J_{g,l,l^*}^{sgl} \subseteq J_{g,l,l^*}^{new}, &\forall g \in G, \\
&\forall l \in L_g \setminus L_g^{exp}, \\
&\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right)
\end{aligned} \tag{2.13}
$$

$$
\begin{aligned}
J_{g,l,l^*}^{sgl} \cap J_{g,l,l^*}^{col} = \emptyset, &\forall g \in G, \\
&\forall l \in L_g \setminus L_g^{exp}, \\
&\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right)
\end{aligned} \tag{2.14}
$$

A group of arcs t can refer to any number of arcs on any network but one arc cannot appear in more than one group. The set for all groups of arcs is T. Members of a group $t \in T$ are identified via the set $GLLJ_t^{col}$, all of which must correspond to new arcs selected as a group, (2.15)-(2.16). For undirected arcs, only the nominal direction can be used for inclusion in a group.

$$
\begin{aligned}
J_{g,l,l^*}^{col} = \left\{ j \in J_{g,l,l^*}^{new} : \exists t \in T, (g,l,l^*,j) \in GLLJ_t^{col} \right\}, \\
\forall g \in G, \\
\forall l \in L_g \setminus L_g^{exp}, \\
\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right)
\end{aligned} \tag{2.15}
$$

$$
GLLJ_t^{col} \cap GLLJ_{t^*}^{col} = \emptyset, \forall t \in T, \forall t^* \in T \setminus \{t\} \tag{2.16}
$$

Different sets are used depending on whether arcs are dimensioned individually or in groups. In the former case, the options for arc $j \in J_{g,l,l^*}^{sgl}$ are the members of $H_{g,l,l^*,j}$. If j is also undirected, it is identified via the nominal direction, as described above, which applies to J_{g,l,l^*}^{new}, J_{g,l,l^*}^{sgl} and $H_{g,l,l^*,j}$. For a group of arcs t, the options common to all arcs in that group are given by H_t.

New arcs are further defined as being either mandatory or optional if one of the options available has to be selected or not, respectively. A new arc j

between nodes l and l^* on network g is mandatory if it is a member of J_{g,l,l^*}^{mdt}, (2.17). The alternative is to be regarded as optional through membership in J_{g,l,l^*}^{opt}, (2.18)-(2.19). This distinction also applies to groups of arcs: a group of arcs t is mandatory if it is a member of T^{mdt}, and optional if it is a member of T^{opt}, (2.20)-(2.23). Note that the groups have to be defined with this in mind since it affects all arcs in the group. That is, no group can contain both optional and mandatory arcs, only optional or mandatory arcs can be in a given group.

$$J_{g,l,l^*}^{mdt} \subseteq J_{g,l,l^*}^{new}, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp},$$
$$\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\}\right) \tag{2.17}$$

$$J_{g,l,l^*}^{opt} \subseteq J_{g,l,l^*}^{new}, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp},$$
$$\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\}\right) \tag{2.18}$$

$$J_{g,l,l^*}^{mdt} \cap J_{g,l,l^*}^{opt} = \emptyset, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp},$$
$$\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\}\right) \tag{2.19}$$

$$GLLJ_t^{col} \subseteq \left\{ g \in G, l \in L_g \setminus L_g^{exp}, l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\}\right), \\ j \in J_{g,l,l^*}^{new} : j \in J_{g,l,l^*}^{opt} \right\}, \forall t \in T^{opt} \tag{2.20}$$

$$GLLJ_t^{col} \subseteq \left\{ g \in G, l \in L_g \setminus L_g^{exp}, l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\}\right), \\ j \in J_{g,l,l^*}^{new} : j \in J_{g,l,l^*}^{mdt} \right\}, \forall t \in T^{mdt} \tag{2.21}$$

$$T = T^{opt} \cup T^{mdt} \tag{2.22}$$

$$T^{mdt} \cap T^{opt} = \emptyset \tag{2.23}$$

Arcs can also have static losses in addition to the standard flow-proportional ones. An arc j between nodes l and l^* on network g with static losses is identified

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through the set J_{g,l,l^*}^{stt}. If the arc is also undirected, it can only be found in the set for the nominal direction. Static losses are flow-independent and defined per arc. As a result, they do not vary as a function of the flow sense.

$$
\begin{aligned}
J_{g,l,l^*}^{stt} \subseteq J_{g,l,l^*}^{dir} \cup J_{g,l,l^*}^{und}, &\forall g \in G, \\
&\forall l \in L_g \setminus L_g^{exp}, \\
&\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right)
\end{aligned}
\tag{2.24}
$$

Converters

Converters ($\forall i \in I$) consist of independent sets of difference equations that can interact with any number of nodes on any number of networks. Their equations are consistent with discretised solutions to linear ordinary differential equations described in state space. Accordingly, they are described using variables corresponding to the signals conventionally used to describe dynamic systems, namely inputs ($\forall m \in M_i$), states ($\forall n \in N_i$) and outputs ($\forall r \in R_i$). Among these, only inputs and outputs impact networks directly, though all can induce capital costs, should their respective nominal amplitudes be dimensionable.

Dimensioning nominal amplitudes is possible with new ($i \in I^{new}$) as opposed to pre-existing ($i \in I^{pre}$) converters, (2.26)-(2.25), though the possibilities differ depending on the signal. States and outputs variables are generally unconstrained and for this reason their limits can be defined using positive ($n \in N_i^{dim,pos}, r \in R_i^{dim,pos}$) and negative ($n \in N_i^{dim,neg}, r \in R_i^{dim,neg}$) nominal amplitudes, independent of one another or made to match ($n \in N_i^{dim,eq}, r \in R_i^{dim,eq}$), (2.27)-(2.30). In turn, input amplitude limits are defined using one nominal amplitude, seeing as the inputs are non-negative, (2.31)-(2.32).

$$
I^{pre} \subseteq I \tag{2.25}
$$

$$
I^{new} \subseteq I \tag{2.26}
$$

$$
N_i^{dim,pos} \cup N_i^{dim,neg} \subseteq N_i, \forall i \in I \tag{2.27}
$$

$$
R_i^{dim,pos} \cup R_i^{dim,neg} \subseteq R_i, \forall i \in I \tag{2.28}
$$

$$
N_i^{dim,eq} \subseteq N_i^{dim,pos} \cap N_i^{dim,neg}, \forall i \in I \tag{2.29}
$$

$$R_i^{dim,eq} \subseteq R_i^{dim,pos} \cap R_i^{dim,neg}, \forall i \in I \tag{2.30}$$

$$M_i^{nnr} \subseteq M_i, \forall i \in I \tag{2.31}$$

$$M_i^{dim} \subseteq M_i^{nnr}, \forall i \in I \tag{2.32}$$

$$M_i^{dim} = \emptyset, \forall i \in I^{pre} \tag{2.33}$$

$$N_i^{dim,pos} \cup N_i^{dim,neg} = \emptyset, \forall i \in I^{pre} \tag{2.34}$$

$$R_i^{dim,pos} \cup R_i^{dim,neg} = \emptyset, \forall i \in I^{pre} \tag{2.35}$$

Alternatively, non-dimensionable signals can have predetermined fixed bounds ($m \in M_i^{fix}, n \in N_i^{fix}, r \in R_i^{fix}$) or they can be binary, though only in the case of inputs ($m \in M_i^{bin}$), (2.36)-(2.42). Note that these signals exist in all converters but dimensionable signals can only exist in new converters.

$$M_i^{fix} \subseteq M_i, \forall i \in I \tag{2.36}$$

$$N_i^{fix} \subseteq N_i, \forall i \in I \tag{2.37}$$

$$R_i^{fix} \subseteq R_i, \forall i \in I \tag{2.38}$$

$$M_i^{bin} \subseteq M_i, \forall i \in I \tag{2.39}$$

$$N_i^{fix} \cap \left(N_i^{dim,pos} \cup N_i^{dim,neg} \right) = \emptyset, \forall i \in I \tag{2.40}$$

$$R_i^{fix} \cap \left(R_i^{dim,pos} \cup R_i^{dim,neg} \right) = \emptyset, \forall i \in I \tag{2.41}$$

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$$M_i^{fix} \cap M_i^{dim} \cap M_i^{bin} = \emptyset, \forall i \in I \tag{2.42}$$

For pre-existing converters, no capital costs can be induced (sunk costs), yet all converters can impact the operating results. This is possible via the inputs and outputs of each converter for any time interval, period and assessment. Separately, states can also impact the operational performance in two ways. One is through violations of high or low state references, indicated via the sets $N_i^{ref,hgh}$ and $N_i^{ref,low}$, respectively, (2.43)-(2.44). The other is through positive or negative state variations during a given reporting period via the sets $N_i^{pos,var}$ and $N_i^{neg,var}$, also respectively, (2.45)-(2.46). The first kind indicates failure to meet standards and the second represents burden shifting onto subsequent reporting periods. Both can be used to represent negative externalities, whereas the aforementioned converter outputs may be used to account for positive or negative externalities (e.g., CO_2 emissions). In turn, the impact of converter inputs on operating results is mostly meant to account for operating costs.

$$N_i^{ref,hgh} \subseteq N_i, \forall i \in I \tag{2.43}$$

$$N_i^{ref,low} \subseteq N_i, \forall i \in I \tag{2.44}$$

$$N_i^{pos,var} \subseteq M_i, \forall i \in I \tag{2.45}$$

$$N_i^{neg,var} \subseteq N_i, \forall i \in I \tag{2.46}$$

2.1.4   Implementation options

The formulation proposed includes redundant ways to achieve a given outcome. This is meant to enable and facilitate comparisons. For the moment, this only concerns different approaches for modelling static losses in arcs and for multiple-choice decisions using special ordered sets of type 1 (SOS1).

Static losses

Modelling static losses is possible in several mutually-exclusive ways depending on where those losses appear and on the flow sense*. For directed arcs, static losses can appear in the start ($j \in J_{g,l,l^*}^{stt,dep}$) or in the end node ($j \in J_{g,l,l^*}^{stt,arr}$), (2.47)-(2.49). For undirected arcs, the same is true but the static losses can appear in different nodes depending on the flow sense, prompting four alternative

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*A different approach is to consider intermediate nodes [6].

implementations: the losses appear either in the start ($j \in J_{g,l,l^*}^{stt,dep}$) or in the end ($j \in J_{g,l,l^*}^{stt,arr}$) node, defined in relation to the nominal flow sense, and thus irrespective of the actual flow sense; or, the losses appear upstream ($j \in J_{g,l,l^*}^{stt,us}$) or downstream ($j \in J_{g,l,l^*}^{stt,ds}$) in relation to the flow sense, (2.50)-(2.52). In this last case, and with losses placed upstream, they appear in A when flow is from A to B and in B when flow is from B to A. If downstream, they appear in B when flow is from A to B and in A when flow is from B to A.

Some of aforementioned options cannot be used with every arc. Directed arcs can have static losses except those between import and export nodes, (2.53) [†]. Beyond that limitation, directed arcs cannot have static losses placed upstream, if the source node is an import node, nor downstream if the end node is an export node, (2.54)-(2.55). In turn, all options are available for undirected arcs since these cannot involve neither import nor export nodes.

$$J_{g,l,l^*}^{stt,dep} \subseteq J_{g,l,l^*}^{stt}, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp},$$
$$\forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right) \tag{2.47}$$

$$J_{g,l,l^*}^{stt,arr} \subseteq J_{g,l,l^*}^{stt}, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp},$$
$$\forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right) \tag{2.48}$$

$$J_{g,l,l^*}^{stt,dep} \cap J_{g,l,l^*}^{stt,arr} = \emptyset, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp},$$
$$\forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right) \tag{2.49}$$

$$J_{g,l,l^*}^{stt,us} \subseteq J_{g,l,l^*}^{stt} \cap J_{g,l,l^*}^{und}, \forall g \in G,$$
$$\forall l \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp}\right),$$
$$\forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp} \cup L_g^{exp}\right) \tag{2.50}$$

$$J_{g,l,l^*}^{stt,ds} \subseteq J_{g,l,l^*}^{stt} \cap J_{g,l,l^*}^{und}, \forall g \in G,$$
$$\forall l \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp}\right),$$
$$\forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp} \cup L_g^{exp}\right) \tag{2.51}$$

[†]A workaround is to use a pre-existing lossless arc between either end and an intermediate node, where static losses can then be placed to achieve the same result.

$$J_{g,l,l^*}^{stt,ds} \cap J_{g,l,l^*}^{stt,us} \cap J_{g,l,l^*}^{stt,dep} = \emptyset, \forall g \in G,$$
$$\forall l \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp} \right), \tag{2.52}$$
$$\forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp} \cup L_g^{exp} \right)$$

$$J_{g,l,l^*}^{stt} = \emptyset, \forall g \in G,$$
$$\forall l \in L_g^{imp}, \tag{2.53}$$
$$\forall l^* \in L_g^{exp}$$

$$J_{g,l,l^*}^{stt,dep} = \emptyset, \forall g \in G,$$
$$\forall l \in L_g^{imp}, \tag{2.54}$$
$$\forall l^* \in L_g \setminus L_g^{imp}$$

$$J_{g,l,l^*}^{stt,arr} = \emptyset, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp}, \tag{2.55}$$
$$\forall l^* \in L_g^{exp}$$

Special ordered sets of type 1

Investments in arcs require binary decisions. A natural way to model this is to declare binary decision variables for each option. The variables for the options available within an arc or group of arcs can also be declared members of a SOS1, since at most one is non-zero, (2.56, 2.60). If the arcs under consideration are also mandatory, then exactly one of variables has to be non-zero. In that case, the variables do not need to be binary but can instead simply be non-negative real, (2.57, 2.61). Therefore, there are three options to model investment decisions: using binary variables, with or without being declared members of a SOS1; using non-negative real variables, if they concern mandatory arcs and are declared members of a SOS1. The same logic applies to both individual arcs, (2.56)-(2.59), and groups of arcs, (2.60)-(2.63). Another aspect concerns the weights with which to declare the SOS1s, for which there are multiple options. Since it has more to do with the solver, the subject is not addressed here.

$$J_{g,l,l^*}^{arc,sos} \subseteq J_{g,l,l^*}^{sgl}, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp}, \tag{2.56}$$
$$\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right)$$

$$J_{g,l,l^*}^{arc,nnr} \subseteq J_{g,l,l^*}^{arc,sos} \cap J_{g,l,l^*}^{mdt}, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp}, \tag{2.57}$$
$$\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right)$$

$$J_{g,l,l^*}^{arc,bin} \cup J_{g,l,l^*}^{arc,nnr} = J_{g,l,l^*}^{sgl}, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp}, \tag{2.58}$$
$$\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right)$$

$$J_{g,l,l^*}^{arc,bin} \cap J_{g,l,l^*}^{arc,nnr} = \emptyset, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp}, \tag{2.59}$$
$$\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right)$$

$$T^{arc,sos} \subseteq T \tag{2.60}$$

$$T^{arc,nnr} \subseteq T^{arc,sos} \cap T^{mdt} \tag{2.61}$$

$$T^{arc,bin} \cup T^{arc,nnr} = T \tag{2.62}$$

$$T^{arc,bin} \cap T^{arc,nnr} = \emptyset \tag{2.63}$$

Deciding which way the flow is going on undirected arcs also involves mutually-exclusive binary decisions. The flow sense can only be from A to B or from B to A at any given time, which means the decisions can be modelled using two binary variables per undirected arc and time interval. These variables can also be declared a member of a SOS1, since at most one can be non-zero, (2.64). Other than a potential impact on solver performance, there is no difference here between requiring one or at most one variable to be non-zero. Therefore, if we assume the former and declare the variables for both senses of a given arc to be members of a SOS1, the variables can simply be declared to be non-negative real, (2.65). As such, there are three ways to model which way the flow is going in undirected arcs, (2.66)-(2.67): using binary variables, with or without SOS1; or using non-negative real variables, as members of a SOS1. About the weights, there are only two effective options since there are only two senses per arc: either favoring the nominal direction or the reverse one. Since this mostly concerns the solver, the subject is not addressed here in greater detail.

$$J_{g,l,l^*}^{sns,sos} \subseteq J_{g,l,l^*}^{und}, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp}, \tag{2.64}$$
$$\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right)$$

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$$J_{g,l,l^*}^{sns,nnr} \subseteq J_{g,l,l^*}^{sns,sos}, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp}, \tag{2.65}$$
$$\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right)$$

$$J_{g,l,l^*}^{sns,bin} \cup J_{g,l,l^*}^{sns,nnr} = J_{g,l,l^*}^{und}, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp}, \tag{2.66}$$
$$\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right)$$

$$J_{g,l,l^*}^{sns,bin} \cap J_{g,l,l^*}^{sns,nnr} = \emptyset, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp}, \tag{2.67}$$
$$\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right)$$

Special ordered sets may only prove advantageous if its members are not subject to other discrete conditions. In order to evaluate this possibility, it is necessary to prevent SOS1 members from being used directly in the same constraints as members of other SOS1 or other binary variables. Examples include the constraints requiring that one flow sense be picked if an undirected arc has been selected (i.e., arc selection and flow sense indicator variables in the same constraint) or those limiting the number of arcs between two nodes (i.e., arc selection variables for different arcs in the same constraint). One approach to deal with this is to create an interface between SOS1 and these constraints by introducing one variable per selection process. This only needs to involve new and optional arcs, or groups of arcs, since the investment decisions for new and mandatory arcs or groups thereof are predictable, as is the effect of pre-existing arcs, (2.68). As for the type of variables, they are assumed to be non-negative real to remain consistent with the rationale for their introduction. If arc j between nodes l and l^* is a member of J_{g,l,l^*}^{int}, then an interface variable will be used to hold information about whether or not the arc was selected. This is valid for both directed and undirected arcs, the latter using the nominal sense only. If an interface is also required for selecting the group of arcs t, then t is a member of T^{int}, which is a subset of T^{opt} (2.69).

$$J_{g,l,l^*}^{int} \subseteq J_{g,l,l^*}^{opt}, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp}, \tag{2.68}$$
$$\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right)$$

$$T^{int} = \left\{ t \in T^{opt} : \forall (g, l, l^*, j) \in GLLJ_t^{col}, j \in J_{g,l,l^*}^{int} \right\} \tag{2.69}$$

2.1.5 Key assumptions

The model can be characterised as follows:

- Two-stage socioeconomic optimisation within MILP framework
- Investments are decided through a modified net present value
- Capital costs, operational results and externalities are considered
- Capital costs arise due to investments on infrastructure
- Operational costs and revenue are possible through imports and exports
- Piece-wise linear functions for import and export prices
- The planning horizon is divided into several reporting periods
- At least one operational performance assessment is needed per period
- Operational performance assessments are used for technical evaluations
- No investment scheduling (all investments precede the planning period)
- Infrastructure is available throughout the planning horizon
- No flow delays (flows reach their destination within a time interval)
- System boundaries are defined by import and export nodes
- Flow equilibria at each internal node and time interval
- Arcs allow for flows between nodes in the same network
- Nodes can be sinks, sources or waypoints during any time interval
- Arcs can be pre-existing or new, and then optional or mandatory
- Investments in arcs can be decided individually or in groups
- Arcs can be directed (unidirectional) or undirected (bidirectional)
- Arcs can have flow-proportional and flow-independent losses
- Each arc can only have one nominal capacity (same for both flow senses)
- Static losses are isotropic (the same for both flow senses)
- New arcs can be dimensioned individually or as a group
- Flow-proportional losses can differ by flow sense (undirected arcs only)
- Flow converters can impact any internal node on any network
- Flow converters can be dynamic flow sinks, sources or both (e.g., storage)

2.2 Objective function

The objective function is defined to be consistent with net present value calculations. This is defined as the sum of the discounted net cash flows ($SDNCF_q$) for all the assessments ($\forall q \in Q$) minus the initial investments (CAPEX).

$$\max \left(\sum_{q \in Q} c_q^{wgt} SDNCF_q - CAPEX \right) \tag{2.70}$$

$$CAPEX \geq 0 \tag{2.71}$$

Note: all variables are free (real) unless explicitly stated otherwise.

2.2.1 Capital costs

Capital expenditures arise if optional converters and (new) arcs are to be deployed. Note that costs associated with shared import or export infrastructure are not explicitly modelled, though they can be considered through an additional (waypoint) node and an arc between it and import or export nodes.

$$CAPEX \geq \sum_{i \in I^{new}} CAPEX_i^{cvt} + \sum_{t \in T} CAPEX_t^{arc,col} +$$
$$\sum_{g \in G} \sum_{l \in L_g \setminus L_g^{exp}} \sum_{l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right)} \sum_{j \in J_{g,l,l^*}^{sgl}} CAPEX_{g,l,l^*,j}^{arc,sgl} \tag{2.72}$$

$$CAPEX_i^{cvt} \geq 0, \forall i \in I^{new} \tag{2.73}$$

$$\begin{aligned} CAPEX_{g,l,l^*,j}^{arc,sgl} \geq 0, &\forall g \in G, \\ &\forall l \in L_g \setminus L_g^{exp}, \\ &\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right), \\ &\forall j \in J_{g,l,l^*}^{sgl} \end{aligned} \tag{2.74}$$

$$CAPEX_t^{arc,col} \geq 0, \forall t \in T \tag{2.75}$$

Converters

Optional converters induce capital expenditures if they are deployed. Each converter has a minimum cost and extra costs dependent on the amplitudes of inputs, outputs and states, but only if these signals are dimensionable.

$$
\begin{aligned}
CAPEX_i^{cvt} \geq {}& c_i^{cvt,min}\delta_i^{cvt,inv} + \sum_{m \in M_i^{dim}} c_{i,m}^{cvt,u} u_{i,m}^{amp} + \\
& \sum_{r \in R_i^{dim,pos}} c_{i,r}^{cvt,y,pos} y_{i,r}^{amp,pos} + \\
& \sum_{r \in R_i^{dim,neg}\backslash R_i^{dim,eq}} c_{i,r}^{cvt,y,neg} y_{i,r}^{amp,neg} + \\
& \sum_{n \in N_i^{dim,pos}} c_{i,n}^{cvt,x,pos} x_{i,n}^{amp,pos} + \\
& \sum_{n \in N_i^{dim,neg}\backslash N_i^{dim,eq}} c_{i,n}^{cvt,x,neg} x_{i,n}^{amp,neg}, \forall i \in I^{new}
\end{aligned}
\tag{2.76}
$$

Arcs

New arcs impose capital costs if they are selected. Each has a minimum cost plus extra costs dependent on the nominal flow amplitude.

$$
\begin{aligned}
& CAPEX_{g,l,l^*,j}^{arc,sgl} \geq c_{g,l,l^*,j}^{arc,amp} v_{g,l,l^*,j}^{amp} + \sum_{h \in H_{g,l,l^*,j}} c_{g,l,l^*,j,h}^{arc,min} \delta_{g,l,l^*,j,h}, \\
& \forall g \in G, \\
& \forall l \in L_g \backslash L_g^{exp}, \\
& \forall l^* \in L_g \backslash \left(L_g^{imp} \cup \{l\} \right), \\
& \forall j \in J_{g,l,l^*}^{sgl}
\end{aligned}
\tag{2.77}
$$

$$
CAPEX_t^{arc,col} \geq c_t^{arc,amp} v_t^{amp} + \sum_{h \in H_t} c_{t,h}^{arc,min} \delta_{t,h}^{arc,inv}, \forall t \in T
\tag{2.78}
$$

2.2.2   Operational performance

The operational performance of each energy system configuration is determined through the sum of discounted net cash flows for each assessment. These reflect the operating result for each of the reporting periods and the respective discount factors. The operational result itself depends on transshipment flows and

converter-related costs meant to encompass operating costs and externalities.

$$
\begin{aligned}
SDNCF_q = \sum_{p \in P_q} c_{q,p}^{df} \Bigg[\sum_{k \in K_q} c_{q,p,k}^{time} \Bigg(\\
\sum_{g \in G} \Bigg(\sum_{l \in L_g^{exp}} EFR_{g,l,q,p,k} - \sum_{l^* \in L_g^{imp}} IFC_{g,l^*,q,p,k} \Bigg) + \\
\sum_{i \in I} \Bigg(\sum_{m \in M_i} c_{i,m,q,p,k}^{opex,u} u_{i,m,q,k} + \sum_{r \in R_i} c_{i,r,q,p,k}^{opex,y} y_{i,r,q,k} + \\
\sum_{n \in N_i^{ref,hgh}} c_{i,n,q,p,k}^{ref,hgh} \Delta x_{i,n,q,k}^{ref,hgh} + \sum_{n \in N_i^{ref,low}} c_{i,n,q,p,k}^{ref,low} \Delta x_{i,n,q,k}^{ref,low} \Bigg) \Bigg) + \\
\sum_{i \in I} \Bigg(\sum_{n \in N_i^{pos,var}} c_{i,n,q,p}^{pos,var} \Delta x_{i,n,q}^{pos,var} + \\
\sum_{n \in N_i^{neg,var}} c_{i,n,q,p}^{neg,var} \Delta x_{i,n,q}^{neg,var} \Bigg) \Bigg], \forall q \in Q
\end{aligned}
$$

(2.79)

Export revenue and import costs

The export revenue and import costs are determined for export and import nodes, respectively, for each performance assessment and time interval. They are given as the product of prices and flow volumes, be it for imports or exports. Price functions are defined per volume segment in a piece-wise linear manner. All such segments must entail finite flow volumes except those which allow for the highest flow volumes (i.e., the last segment). In those cases, flow volume limits are optional as long as the functions for imports and exports are monotonically-increasing (convex: lower prices for low flow volumes) and monotonically-decreasing (concave: higher prices for low flow volumes), respectively, with regard to volume. This is to ensure the correct usage of the price segments, which is only possible without finite bounds in those cases.

$$
EFR_{g,l,q,p,k} \geq 0, \forall g \in G, \forall l \in L_g^{exp}, \forall q \in Q, \forall p \in P_q, \forall k \in K_q
$$

(2.80)

$$
IFC_{g,l,q,p,k} \geq 0, \forall g \in G, \forall l \in L_g^{imp}, \forall q \in Q, \forall p \in P_q, \forall k \in K_q
$$

(2.81)

$$EFR_{g,l,q,p,k} = \sum_{s \in S_{g,l,q,p,k}} p_{g,l,q,p,k,s} EF_{g,l,q,p,k,s}, \forall g \in G,$$

$$\forall l \in L_g^{exp},$$
$$\forall q \in Q,$$
$$\forall p \in P_q, \qquad (2.82)$$
$$\forall k \in K_q$$

$$IFC_{g,l,q,p,k} = \sum_{s \in S_{g,l,q,p,k}} p_{g,l,q,p,k,s} IF_{g,l,q,p,k,s}, \forall g \in G,$$

$$\forall l \in L_g^{imp},$$
$$\forall q \in Q,$$
$$\forall p \in P_q, \qquad (2.83)$$
$$\forall k \in K_q$$

Imports and exports

Imports and exports correspond to the sum of the flows going in and out of the system. These are given for each import and export node, respectively, by performance assessment, period, time interval and price segment.

$$\sum_{s \in S_{g,l,q,p,k}} EF_{g,l,q,p,k,s} = \sum_{l^* \in L_g \setminus L_g^{exp}} \sum_{j^* \in J_{g,l^*,l}^{dir}} \eta_{g,l^*,l,j^*,q,k} v_{g,l^*,l,j^*,q,k},$$
$$\forall g \in G, \forall l \in L_g^{exp}, \forall q \in Q, \forall p \in P_q, \forall k \in K_q \quad (2.84)$$

$$\sum_{s \in S_{g,l,q,p,k}} IF_{g,l,q,p,k,s} = \sum_{l^* \in L_g \setminus L_g^{imp}} \sum_{j \in J_{g,l,l^*}^{dir}} v_{g,l,l^*,j,q,k},$$
$$\forall g \in G, \forall l \in L_g^{imp}, \forall q \in Q, \forall p \in P_q, \forall k \in K_q \qquad (2.85)$$

$$EF_{g,l,q,p,k,s} \geq 0, \forall g \in G,$$
$$\forall l \in L_g^{exp},$$
$$\forall q \in Q,$$
$$\forall p \in P_q, \qquad (2.86)$$
$$\forall k \in K_q,$$
$$\forall s \in S_{g,l,q,p,k}$$

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$$IF_{g,l,q,p,k,s} \geq 0, \forall g \in G,$$
$$\forall l \in L_g^{imp},$$
$$\forall q \in Q,$$
$$\forall p \in P_q, \tag{2.87}$$
$$\forall k \in K_q,$$
$$\forall s \in S_{g,l,q,p,k}$$

$$EF_{g,l,q,p,k,s} \leq v_{g,l,q,p,k,s}^{max}, \forall g \in G,$$
$$\forall l \in L_g^{exp},$$
$$\forall q \in Q,$$
$$\forall p \in P_q, \tag{2.88}$$
$$\forall k \in K_q,$$
$$\forall s \in S_{g,l,q,p,k}^{fin}$$

$$IF_{g,l,q,p,k,s} \leq v_{g,l,q,p,k,s}^{max}, \forall g \in G,$$
$$\forall l \in L_g^{imp},$$
$$\forall q \in Q,$$
$$\forall p \in P_q, \tag{2.89}$$
$$\forall k \in K_q,$$
$$\forall s \in S_{g,l,q,p,k}^{fin}$$

$$S_{g,l,q,p,k}^{fin} \subseteq S_{g,l,q,p,k}, \forall g \in G,$$
$$\forall l \in L_g^{imp} \cup L_g^{exp},$$
$$\forall q \in Q, \tag{2.90}$$
$$\forall p \in P_q,$$
$$\forall k \in K_q$$

Note: only directed arcs can reach import and export nodes.

Piece-wise linear price functions

The previous formulation is enough when the piece-wise linear price functions for imports and exports are monotonically-increasing (convex) and monotonically-decreasing (concave), respectively, with respect to volume. When these conditions are not met, additional constraints and variables are needed to ensure the segments are used right: the preceding segments must be used completely first.

2.3 Directed and undirected flows

2.3.1 Arc investment variables

The decision to invest in a new arc requires one variable per arc and arc option, if the arc is to be selected individually. If not, one variable per arc option and group are necessary. These variables can be binary or, if they are associated with mandatory arcs and members of a special ordered sets of type one, non-negative real. This applies to both directed and undirected arcs.

$$
\begin{aligned}
\delta^{arc,inv}_{g,l,l^*,j,h} \in \{0,1\}, &\forall g \in G, \\
&\forall l \in L_g \setminus L^{exp}_g, \\
&\forall l^* \in L_g \setminus \left(L^{imp}_g \cup \{l\} \right), \\
&\forall j \in J^{arc,bin}_{g,l,l^*}, \\
&\forall h \in H_{g,l,l^*,j}
\end{aligned}
\tag{2.91}
$$

$$
\begin{aligned}
\delta^{arc,inv}_{g,l,l^*,j,h} \geq 0, &\forall g \in G, \\
&\forall l \in L_g \setminus L^{exp}_g, \\
&\forall l^* \in L_g \setminus \left(L^{imp}_g \cup \{l\} \right), \\
&\forall j \in J^{arc,nnr}_{g,l,l^*}, \\
&\forall h \in H_{g,l,l^*,j}
\end{aligned}
\tag{2.92}
$$

$$
\delta^{arc,inv}_{t,h} \in \{0,1\}, \forall t \in T^{arc,bin}, \forall h \in H_t
\tag{2.93}
$$

$$
\delta^{arc,inv}_{t,h} \geq 0, \forall t \in T^{arc,nnr}, \forall h \in H_t
\tag{2.94}
$$

2.3.2 Arc flow variables

Each arc requires one flow variable per arc direction (1 for directed, 2 for undirected arcs), assessment and time interval. This applies to all arcs, including pre-existing. Undirected arcs have two flow variables, one for each direction. Note that if j represents an undirected arc between nodes l and l^* on network g, it is a member of J^{und}_{g,l,l^*} or $J^{und}_{g,l^*,l}$ but not both simultaneously.

$$
\begin{aligned}
v_{g,l,l^*,j,q,k} \geq 0, &\forall g \in G, \\
&\forall l \in L_g \setminus L^{exp}_g, \\
&\forall l^* \in L_g \setminus \left(L^{imp}_g \cup \{l\} \right), \\
&\forall j \in J^{dir}_{g,l,l^*} \cup \left(J^{und}_{g,l,l^*} \cup J^{und}_{g,l^*,l} \right), \\
&\forall q \in Q, \\
&\forall k \in K_q
\end{aligned}
\tag{2.95}
$$

2.3.3 Arc flow amplitude variables

New arcs selected individually require one non-negative real flow amplitude variable. In turn, arcs selected in groups require one variable per group.

$$
\begin{aligned}
v_{g,l,l^*,j}^{amp} \geq 0, &\forall g \in G, \\
&\forall l \in L_g \setminus L_g^{exp}, \\
&\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right), \\
&\forall j \in J_{g,l,l^*}^{sgl}
\end{aligned}
\tag{2.96}
$$

$$
v_t^{amp} \geq 0, \forall t \in T
\tag{2.97}
$$

2.3.4 Flow sense variables

Two flow sense variables are needed per undirected arc, interval and assessment. These can be binary or non-negative real if identified as members of an SOS1.

$$
\begin{aligned}
\zeta_{g,l,l^*,j,q,k}^{sns} \in \{0,1\}, &\forall g \in G, \\
&\forall l \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp} \right), \\
&\forall l^* \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp} \cup \{l\} \right), \\
&\forall j \in J_{g,l,l^*}^{sns,bin} \cup J_{g,l^*,l}^{sns,bin}, \\
&\forall q \in Q, \\
&\forall k \in K_q
\end{aligned}
\tag{2.98}
$$

$$
\begin{aligned}
\zeta_{g,l,l^*,j,q,k}^{sns} \geq 0, &\forall g \in G, \\
&\forall l \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp} \right), \\
&\forall l^* \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp} \cup \{l\} \right), \\
&\forall j \in J_{g,l,l^*}^{sns,nnr} \cup J_{g,l^*,l}^{sns,nnr}, \\
&\forall q \in Q, \\
&\forall k \in K_q
\end{aligned}
\tag{2.99}
$$

2.3.5 Static loss variables for new arcs

Modelling static losses requires one non-negative real variable per new arc, assessment and time interval. For pre-existing arcs, a parameter is used instead.

$$w_{g,l,l^*,j,q,k} \geq 0, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp},$$
$$\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\}\right),$$
$$\forall j \in J_{g,l,l^*}^{stt} \cap J_{g,l,l^*}^{new}, \tag{2.100}$$
$$\forall q \in Q,$$
$$\forall k \in K_q$$

2.3.6 Static loss variables for each flow sense

Modelling static losses also requires one non-negative real variable per undirected arc, flow sense, assessment and time interval.

$$w_{g,l,l^*,j,q,k}^{sns} \geq 0, \forall g \in G,$$
$$\forall l \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp}\right),$$
$$\forall l^* \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp} \cup \{l\}\right),$$
$$\forall j \in \left(J_{g,l,l^*}^{stt} \cap J_{g,l,l^*}^{und}\right) \cup \left(J_{g,l^*,l}^{stt} \cap J_{g,l^*,l}^{und}\right), \tag{2.101}$$
$$\forall q \in Q,$$
$$\forall k \in K_q$$

2.3.7 Interface variables

One non-negative real variable is used to interface each investment decision, be it for individual arcs or groups, but the arcs can only be new and optional.

$$\xi_{g,l,l^*,j}^{arc,inv} \geq 0, \forall g \in G,$$
$$\forall l \in L_g \setminus \left(L_g^{exp}\right),$$
$$\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\}\right), \tag{2.102}$$
$$\forall j \in J_{g,l,l^*}^{int} \cap J_{g,l,l^*}^{sgl}$$

$$\xi_t^{arc,inv} \geq 0, \forall t \in T^{int} \tag{2.103}$$

2.3.8 Interface equations

$$\xi_{g,l,l^*,j}^{arc,inv} = \sum_{h \in H_{g,l,l^*,j}} \delta_{g,l,l^*,j,h}^{arc,inv}, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp},$$
$$\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\}\right), \tag{2.104}$$
$$\forall j \in J_{g,l,l^*}^{int} \cap J_{g,l,l^*}^{sgl}$$

$$\xi_t^{arc,inv} = \sum_{h \in H_t} \delta_{t,h}^{arc,inv}, \forall t \in T^{int} \tag{2.105}$$

2.3.9 New yet optional arcs

New yet optional arcs and groups of arcs may be part of a feasible solution.

$$\sum_{h \in H_{g,l,l^*,j}} \delta_{g,l,l^*,j,h}^{arc,inv} \leq 1, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp},$$
$$\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right), \tag{2.106}$$
$$\forall j \in J_{g,l,l^*}^{opt} \cap J_{g,l,l^*}^{sgl} \setminus J_{g,l,l^*}^{int}$$

$$\xi_{g,l,l^*,j}^{arc,inv} \leq 1, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp},$$
$$\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right), \tag{2.107}$$
$$\forall j \in J_{g,l,l^*}^{int} \cap J_{g,l,l^*}^{sgl}$$

$$\sum_{h \in H_t} \delta_{t,h}^{arc,inv} \leq 1, \forall t \in T^{opt} \setminus T^{int} \tag{2.108}$$

$$\xi_t^{arc,inv} \leq 1, \forall t \in T^{int} \tag{2.109}$$

2.3.10 New yet mandatory arcs

New yet mandatory arcs of groups of arcs have to be part of any feasible solution.

$$\sum_{h \in H_{g,l,l^*,j}} \delta_{g,l,l^*,j,h}^{arc,inv} = 1, \forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp},$$
$$\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right), \tag{2.110}$$
$$\forall j \in J_{g,l,l^*}^{mdt} \cap J_{g,l,l^*}^{sgl}$$

$$\sum_{h \in H_t} \delta_{t,h}^{arc,inv} = 1, \forall t \in T^{mdt} \tag{2.111}$$

2.3.11 Nominal flow amplitudes depend on arc options

Nominal flow amplitudes for new arcs are determined by arc technologies.

$$v_{g,l,l^*,j}^{amp} \leq \sum_{h \in H_{g,l,l^*,j}} v_{g,l,l^*,j,h}^{amp,max} \delta_{g,l,l^*,j,h}^{arc,inv}, \forall g \in G,$$

$$\forall l \in L_g \setminus L_g^{exp},$$
$$\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right),$$
$$\forall j \in J_{g,l,l^*}^{sgl}$$

$$\tag{2.112}$$

$$v_t^{amp} \leq \sum_{h \in H_t} v_{t,h}^{amp,max} \delta_{t,h}^{arc,inv}, \forall t \in T \tag{2.113}$$

2.3.12 Flow senses are mutually-exclusive

Undirected arcs cannot have flows in both directions simultaneously.

$$\zeta_{g,l,l^*,j,q,k}^{sns} + \zeta_{g,l^*,l,j,q,k}^{sns} = \xi_{g,l,l^*,j}^{arc,inv}, \forall g \in G,$$

$$\forall l \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp} \right),$$
$$\forall l^* \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp} \cup \{l\} \right),$$
$$\forall j \in J_{g,l,l^*}^{und} \cap J_{g,l,l^*}^{int} \cap J_{g,l,l^*}^{sgl},$$
$$\forall q \in Q,$$
$$\forall k \in K_q$$

$$\tag{2.114}$$

$$\zeta_{g,l,l^*,j^*,q,k}^{sns} + \zeta_{g,l^*,l,j^*,q,k}^{sns} = \xi_t^{arc,inv}, \forall t \in T^{int},$$

$$\forall (g,l,l^*,j) \in GLLJ_t^{col},$$
$$\forall j^* \in \{j\} \cap J_{g,l,l^*}^{und},$$
$$\forall q \in Q,$$
$$\forall k \in K_q$$

$$\tag{2.115}$$

$$\zeta_{g,l,l^*,j,q,k}^{sns} + \zeta_{g,l^*,l,j,q,k}^{sns} = \sum_{h \in H_{g,l,l^*,j}} \delta_{g,l,l^*,j,h}^{arc,inv},$$

$$\forall g \in G,$$
$$\forall l \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp} \right),$$
$$\forall l^* \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp} \cup \{l\} \right),$$
$$\forall j \in J_{g,l,l^*}^{und} \cap J_{g,l,l^*}^{opt} \cap J_{g,l,l^*}^{sgl} \setminus J_{g,l,l^*}^{int},$$
$$\forall q \in Q,$$
$$\forall k \in K_q$$

$$\tag{2.116}$$

$$\zeta_{g,l,l^*,j^*,q,k}^{sns} + \zeta_{g,l,l^*,l,j^*,q,k}^{sns} = \sum_{h \in H_t} \delta_{t,h}^{arc,inv}, \forall t \in T^{opt} \setminus T^{int},$$

$$\begin{aligned} &\forall (g,l,l^*,j) \in GLLJ_t^{col}, \\ &\forall j^* \in \{j\} \cap J_{g,l,l^*}^{und}, \\ &\forall q \in Q, \\ &\forall k \in K_q \end{aligned} \quad (2.117)$$

$$\zeta_{g,l,l^*,j,q,k}^{sns} + \zeta_{g,l,l^*,l,j,q,k}^{sns} = 1, \forall g \in G,$$

$$\begin{aligned} &\forall l \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp} \right), \\ &\forall l^* \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp} \cup \{l\} \right), \\ &\forall j \in J_{g,l,l^*}^{und} \cap \left(J_{g,l,l^*}^{pre} \cup J_{g,l,l^*}^{mdt} \right), \\ &\forall q \in Q, \\ &\forall k \in K_q \end{aligned} \quad (2.118)$$

2.3.13 Flows through new undirected arcs have to be consistent with the respective flow sense

Flows through new undirected arcs in a given direction have a positive upper limit if the flow is along that direction, or are zero if the flow is in the opposite sense. The positive upper limit should be consistent with the nominal flow amplitudes possible among the options available. This logic applies if those arcs are selected individually, (2.119)-(2.120), or as a group, (2.121)-(2.122).

$$v_{g,l,l^*,j,q,k} \leq \left(\max_{h \in H_{g,l,l^*,j}} v_{g,l,l^*,j,h}^{amp,max} f_{g,l,l^*,j,q,k}^{amp,v} \right.$$

$$\left. - \sum_{j^* \in \{j\} \cap \left(J_{g,l,l^*}^{stt,dep} \cup J_{g,l,l^*}^{stt,us} \right)} w_{g,l,l^*,j^*,h,q,k}^{new} \right) \zeta_{g,l,l^*,j,q,k}^{sns},$$

$$\begin{aligned} &\forall g \in G, \\ &\forall l \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp} \right), \\ &\forall l^* \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp} \cup \{l\} \right), \\ &\forall j \in J_{g,l,l^*}^{und} \cap J_{g,l,l^*}^{sgl}, \\ &\forall q \in Q, \\ &\forall k \in K_q \end{aligned} \quad (2.119)$$

$$
\begin{aligned}
v_{g,l,l^*,j,q,k} \leq \Bigg(&\max_{h \in H_{g,l^*,l,j}} v^{amp,max}_{g,l^*,l,j,h} f^{amp,v}_{g,l^*,l,j,q,k} \\
&- \sum_{j^* \in \{j\} \cap \left(J^{stt,arr}_{g,l^*,l} \cup J^{stt,us}_{g,l^*,l} \right)} w^{new}_{g,l^*,l,j^*,h,q,k} \Bigg) \zeta^{sns}_{g,l,l^*,j,q,k}, \\
&\forall g \in G, \\
&\forall l \in L_g \setminus \left(L^{exp}_g \cup L^{imp}_g \right), \\
&\forall l^* \in L_g \setminus \left(L^{exp}_g \cup L^{imp}_g \cup \{l\} \right), \\
&\forall j \in J^{und}_{g,l^*,l} \cap J^{sgl}_{g,l^*,l}, \\
&\forall q \in Q, \\
&\forall k \in K_q
\end{aligned}
\tag{2.120}
$$

$$
\begin{aligned}
v_{g,l,l^*,j^*,q,k} \leq \Bigg(&\max_{h \in H_t} v^{amp,max}_{t,h} f^{amp,v}_{g,l,l^*,j^*,q,k} \\
&- \sum_{j^\square \in \{j^*\} \cap \left(J^{stt,dep}_{g,l,l^*} \cup J^{stt,us}_{g,l,l^*} \right)} w^{new}_{g,l,l^*,j^\square,h,q,k} \Bigg) \zeta^{sns}_{g,l,l^*,j^*,q,k}, \\
&\forall t \in T, \\
&\forall (g,l,l^*,j) \in GLLJ^{col}_t, \\
&\forall j^* \in \{j\} \cap J^{und}_{g,l,l^*}, \\
&\forall q \in Q, \\
&\forall k \in K_q
\end{aligned}
\tag{2.121}
$$

$$
\begin{aligned}
v_{g,l,l^*,j^*,q,k} \leq \Bigg(&\max_{h \in H_t} v^{amp,max}_{t,h} f^{amp,v}_{g,l^*,l,j^*,q,k} \\
&- \sum_{j^\square \in \{j^*\} \cap \left(J^{stt,arr}_{g,l^*,l} \cup J^{stt,us}_{g,l^*,l} \right)} w^{new}_{g,l^*,l,j^\square,h,q,k} \Bigg) \zeta^{sns}_{g,l,l^*,j^*,q,k}, \\
&\forall t \in T, \\
&\forall (g,l^*,l,j) \in GLLJ^{col}_t, \\
&\forall j^* \in \{j\} \cap J^{und}_{g,l^*,l}, \\
&\forall q \in Q, \\
&\forall k \in K_q
\end{aligned}
\tag{2.122}
$$

2.3.14   Fixed flow limits for directed arcs

Fixed upper bounds are only needed for directed arcs and if they are finite-capacity and pre-existing. For new arcs, the same result can be achieved by using a single option and adjusting the bound through the f parameter.

$$
\begin{aligned}
v_{g,l,l^*,j,q,k} \leq \overline{v_{g,l,l^*,j,q,k}}, &\forall g \in G, \\
&\forall l \in L_g \setminus L_g^{exp}, \\
&\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right), \\
&\forall j \in J_{g,l,l^*}^{dir} \cap J_{g,l,l^*}^{pre} \setminus J_{g,l,l^*}^{inf}, \\
&\forall q \in Q, \\
&\forall k \in K_q
\end{aligned}
\tag{2.123}
$$

For consistency, the following should be observed:

$$
\overline{v_{g,l,l^*,j,q,k}} = f_{g,l,l^*,j,q,k}^{amp,v} v_{g,l,l^*,j}^{amp} - \sum_{j^* \in \{j\} \cap J_{g,l,l^*}^{stt,dep}} w_{g,l,l^*,j^*,q,k},
$$
$$
\begin{aligned}
&\forall g \in G, \\
&\forall l \in L_g \setminus L_g^{exp}, \\
&\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right), \\
&\forall j \in J_{g,l,l^*}^{dir} \cap J_{g,l,l^*}^{pre} \setminus J_{g,l,l^*}^{inf}, \\
&\forall q \in Q, \\
&\forall k \in K_q
\end{aligned}
\tag{2.124}
$$

2.3.15   Flows through pre-existing undirected arcs have to be consistent with the respective flow sense

Flows through pre-existing undirected arcs in a given direction are subject to a positive upper limit if the flow is in that direction, or zero if it is in the opposite.

$$
\begin{aligned}
v_{g,l,l^*,j,q,k} \leq \overline{v_{g,l,l^*,j,q,k}} \zeta_{g,l,l^*,j,q,k}^{sns}, &\forall g \in G, \\
&\forall l \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp} \right), \\
&\forall l^* \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp} \cup \{l\} \right), \\
&\forall j \in \left(J_{g,l,l^*}^{und} \cap J_{g,l,l^*}^{pre} \right) \cup \left(J_{g,l^*,l}^{und} \cap J_{g,l^*,l}^{pre} \right), \\
&\forall q \in Q, \\
&\forall k \in K_q
\end{aligned}
$$

$$
\tag{2.125}
$$

For consistency, the following should be observed:

$$\overline{v_{g,l,l^*,j,q,k}} = f_{g,l,l^*,j,q,k}^{amp,v} v_{g,l,l^*,j}^{amp} - \sum_{j^* \in \{j\} \cap \left(J_{g,l,l^*}^{stt,dep} \cup J_{g,l,l^*}^{stt,us} \right)} w_{g,l,l^*,j^*,q,k},$$

$$\forall g \in G,$$
$$\forall l \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp} \right),$$
$$\forall l^* \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp} \cup \{l\} \right),$$
$$\forall j \in J_{g,l,l^*}^{und} \cap J_{g,l,l^*}^{pre},$$
$$\forall q \in Q,$$
$$\forall k \in K_q$$

(2.126)

$$\overline{v_{g,l,l^*,j,q,k}} = f_{g,l^*,l,j,q,k}^{amp,v} v_{g,l^*,l,j}^{amp} - \sum_{j^* \in \{j\} \cap \left(J_{g,l^*,l}^{stt,arr} \cup J_{g,l^*,l}^{stt,us} \right)} w_{g,l^*,l,j^*,q,k},$$

$$\forall g \in G,$$
$$\forall l \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp} \right),$$
$$\forall l^* \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp} \cup \{l\} \right),$$
$$\forall j \in J_{g,l^*,l}^{und} \cap J_{g,l^*,l}^{pre},$$
$$\forall q \in Q,$$
$$\forall k \in K_q$$

(2.127)

2.3.16 Flow limits for new arcs

Flows through new arcs have to observe flow limits modulated by the nominal flow amplitude and adjusted by static losses, if any and depending on the flow sense, (2.128)-(2.129). For arcs selected as a group, the limits are not group-specific, though the nominal flow amplitude is, (2.130)-(2.131).

$$v_{g,l,l^*,j,q,k} \leq f_{g,l,l^*,j,q,k}^{amp,v} v_{g,l,l^*,j}^{amp} - \sum_{j^* \in \{j\} \cap \left(J_{g,l,l^*}^{stt,dep} \cup J_{g,l,l^*}^{stt,us} \right)} w_{g,l,l^*,j^*,q,k},$$

$$\forall g \in G,$$
$$\forall l \in L_g \setminus L_g^{exp},$$
$$\forall l^* \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right),$$
$$\forall j \in J_{g,l,l^*}^{sgl},$$
$$\forall q \in Q,$$
$$\forall k \in K_q$$

(2.128)

$$v_{g,l,l^*,j,q,k} \leq f^{amp,v}_{g,l^*,l,j,q,k} v^{amp}_{g,l^*,l,j} - \sum_{j^* \in \{j\} \cap \left(J^{stt,arr}_{g,l^*,l} \cup J^{stt,us}_{g,l^*,l}\right)} w_{g,l^*,l,j^*,q,k},$$

$$\forall g \in G,$$
$$\forall l \in L_g \setminus \left(L^{exp}_g \cup L^{imp}_g\right),$$
$$\forall l^* \in L_g \setminus \left(L^{exp}_g \cup L^{imp}_g \cup \{l\}\right),$$
$$\forall j \in J^{sgl}_{g,l^*,l} \cap J^{und}_{g,l^*,l},$$
$$\forall q \in Q,$$
$$\forall k \in K_q$$

$$(2.129)$$

$$v_{g,l,l^*,j,q,k} \leq f^{amp,v}_{g,l,l^*,j,q,k} v^{amp}_t - \sum_{j^* \in \{j\} \cap \left(J^{stt,dep}_{g,l,l^*} \cup J^{stt,us}_{g,l,l^*}\right)} w_{g,l,l^*,j^*,q,k},$$

$$\forall t \in T,$$
$$\forall (g, l, l^*, j) \in GLLJ^{col}_t,$$
$$\forall q \in Q,$$
$$\forall k \in K_q$$

$$(2.130)$$

$$v_{g,l,l^*,j^*,q,k} \leq f^{amp,v}_{g,l^*,l,j^*,q,k} v^{amp}_t - \sum_{j^\square \in \{j^*\} \cap \left(J^{stt,arr}_{g,l^*,l} \cup J^{stt,us}_{g,l^*,l}\right)} w_{g,l^*,l,j^\square,q,k},$$

$$\forall t \in T,$$
$$\forall (g, l^*, l, j) \in GLLJ^{col}_t,$$
$$\forall j^* \in \{j\} \cap J^{und}_{g,l^*,l},$$
$$\forall q \in Q,$$
$$\forall k \in K_q$$

$$(2.131)$$

2.3.17 Static losses only exist if the arc is selected

Static losses for each assessment and time interval are determined by the arc and option selected. For pre-existing arcs, the static losses are parameters.

$$
w_{g,l,l^*,j,q,k} = \sum_{h \in H_{g,l,l^*,j}} w^{new}_{g,l,l^*,j,h,q,k} \delta^{arc,inv}_{g,l,l^*,j,h},
$$

$$
\forall g \in G,
$$
$$
\forall l \in L_g \setminus L^{exp}_g,
$$
$$
\forall l^* \in L_g \setminus \left(L^{imp}_g \cup \{l\} \right),
$$
$$
\forall j \in J^{stt}_{g,l,l^*} \cap J^{sgl}_{g,l,l^*},
$$
$$
\forall q \in Q,
$$
$$
\forall k \in K_q
$$

(2.132)

$$
w_{g,l,l^*,j^*,q,k} = \sum_{h \in H_t} w^{new}_{g,l,l^*,j^*,h,q,k} \delta^{arc,inv}_{t,h},
$$

$$
\forall t \in T,
$$
$$
\forall (g,l,l^*,j) \in GLLJ^{col}_t,
$$
$$
\forall j^* \in \{j\} \cap J^{stt}_{g,l,l^*},
$$
$$
\forall q \in Q,
$$
$$
\forall k \in K_q
$$

(2.133)

2.3.18 Downstream losses must be compensated by arc

When static losses are placed downstream in the relation to the flow, the flows through the arc must be enough to compensate for the losses. For directed arcs, this amounts to a minimum flow requirement equivalent to the losses, (2.134). For undirected arcs, the minimum flow limit exists only when the flow sense makes the losses appear downstream, (2.135)-(2.136). Without enforcing this, the losses could be compensated by flows from other arcs ending in the same node and not by flows through the original arc. However, if there are no such arcs, these constraints become redundant and can be left out. The condition to make the constraints redundant for an arc j between l and l^* is:

$$\bigcup_{l^\circ \in L_g} \left(J^{dir}_{g,l^\circ,l^*} \cup J^{und}_{g,l^\circ,l^*} \cup J^{und}_{g,l^*,l^\circ} \right) \setminus \{j\} = \emptyset$$

$$
\begin{aligned}
v_{g,l,l^*,j,q,k} \geq w_{g,l,l^*,j,q,k}, & \forall g \in G, \\
& \forall l \in L_g \setminus L_g^{exp}, \\
& \forall l^* \in L_g \setminus \left(L_g^{imp} \cup L_g^{exp} \cup \{l\} \right), \\
& \forall j \in J^{dir}_{g,l,l^*} \cap J^{stt,arr}_{g,l,l^*}, \\
& \forall q \in Q, \\
& \forall k \in K_q
\end{aligned}
\tag{2.134}
$$

$$
\begin{aligned}
& v_{g,l,l^*,j,q,k} \geq w^{sns}_{g,l,l^*,j,q,k}, \\
& \forall g \in G, \\
& \forall l \in L_g \setminus \left(L_g^{imp} \cup L_g^{exp} \right), \\
& \forall l^* \in L_g \setminus \left(L_g^{imp} \cup L_g^{exp} \cup \{l\} \right), \\
& \forall j \in \left(J^{und}_{g,l,l^*} \cap J^{stt,ds}_{g,l,l^*} \right) \cup \left(J^{und}_{g,l^*,l} \cap J^{stt,ds}_{g,l^*,l} \right), \\
& \forall q \in Q, \\
& \forall k \in K_q
\end{aligned}
\tag{2.135}
$$

$$
\begin{aligned}
& v_{g,l,l^*,j,q,k} \geq w^{sns}_{g,l,l^*,j,q,k}, \\
& \forall g \in G, \\
& \forall l \in L_g \setminus \left(L_g^{imp} \cup L_g^{exp} \right), \\
& \forall l^* \in L_g \setminus \left(L_g^{imp} \cup L_g^{exp} \cup \{l\} \right), \\
& \forall j \in \left(J^{und}_{g,l,l^*} \cap J^{stt,arr}_{g,l,l^*} \right) \cup \left(J^{und}_{g,l^*,l} \cap J^{stt,dep}_{g,l^*,l} \right), \\
& \forall q \in Q, \\
& \forall k \in K_q
\end{aligned}
\tag{2.136}
$$

Note: export nodes are automatically ruled out as end nodes since those arcs cannot have losses placed downstream; import and export nodes are also ruled out when considering undirected arcs because these are incompatible.

2.3.19 Static losses for pre-existing undirected arcs

$$
\begin{aligned}
w^{sns}_{g,l,l^*,j,q,k} = w_{g,l,l^*,j,q,k} \zeta^{sns}_{g,l,l^*,j,q,k}, & \forall g \in G, \\
& \forall l \in L_g \setminus \left(L_g^{imp} \cup L_g^{exp} \right), \\
& \forall l^* \in L_g \setminus \left(L_g^{imp} \cup L_g^{exp} \cup \{l\} \right), \\
& \forall j \in J^{und}_{g,l,l^*} \cap J^{stt}_{g,l,l^*} \cap J^{pre}_{g,l,l^*}, \\
& \forall q \in Q, \\
& \forall k \in K_q
\end{aligned}
\tag{2.137}
$$

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$$w^{sns}_{g,l,l^*,j,q,k} = w_{g,l^*,l,j,q,k}\zeta^{sns}_{g,l,l^*,j,q,k}, \forall g \in G,$$
$$\forall l \in L_g \setminus \left(L^{imp}_g \cup L^{exp}_g\right),$$
$$\forall l^* \in L_g \setminus \left(L^{imp}_g \cup L^{exp}_g \cup \{l\}\right),$$
$$\forall j \in J^{und}_{g,l^*,l} \cap J^{stt}_{g,l^*,l} \cap J^{pre}_{g,l^*,l},$$
$$\forall q \in Q,$$
$$\forall k \in K_q \tag{2.138}$$

2.3.20 Static losses for new undirected arcs

$$w^{sns}_{g,l,l^*,j,q,k} \leq \left(\max_{h \in H_{g,l,l^*,j}} w^{new}_{g,l,l^*,j,h,q,k}\right)\zeta^{sns}_{g,l,l^*,j,q,k},$$
$$\forall g \in G,$$
$$\forall l \in L_g \setminus \left(L^{imp}_g \cup L^{exp}_g\right),$$
$$\forall l^* \in L_g \setminus \left(L^{imp}_g \cup L^{exp}_g \cup \{l\}\right),$$
$$\forall j \in J^{und}_{g,l,l^*} \cap J^{stt}_{g,l,l^*} \cap J^{sgl}_{g,l,l^*},$$
$$\forall q \in Q,$$
$$\forall k \in K_q \tag{2.139}$$

$$w^{sns}_{g,l,l^*,j,q,k} \leq \left(\max_{h \in H_{g,l^*,l,j}} w^{new}_{g,l^*,l,j,h,q,k}\right)\zeta^{sns}_{g,l,l^*,j,q,k},$$
$$\forall g \in G,$$
$$\forall l \in L_g \setminus \left(L^{imp}_g \cup L^{exp}_g\right),$$
$$\forall l^* \in L_g \setminus \left(L^{imp}_g \cup L^{exp}_g \cup \{l\}\right),$$
$$\forall j \in J^{und}_{g,l^*,l} \cap J^{stt}_{g,l^*,l} \cap J^{sgl}_{g,l^*,l},$$
$$\forall q \in Q,$$
$$\forall k \in K_q \tag{2.140}$$

$$w^{sns}_{g,l,l^*,j,q,k} \leq w_{g,l,l^*,j,q,k}, \forall g \in G,$$
$$\forall l \in L_g \setminus \left(L^{imp}_g \cup L^{exp}_g\right),$$
$$\forall l^* \in L_g \setminus \left(L^{imp}_g \cup L^{exp}_g \cup \{l\}\right),$$
$$\forall j \in J^{und}_{g,l,l^*} \cap J^{stt}_{g,l,l^*} \cap J^{new}_{g,l,l^*},$$
$$\forall q \in Q,$$
$$\forall k \in K_q \tag{2.141}$$

$$w^{sns}_{g,l,l^*,j,q,k} \leq w_{g,l^*,l,j,q,k}, \forall g \in G,$$
$$\forall l \in L_g \setminus \left(L^{imp}_g \cup L^{exp}_g \right),$$
$$\forall l^* \in L_g \setminus \left(L^{imp}_g \cup L^{exp}_g \cup \{l\} \right),$$
$$\forall j \in J^{und}_{g,l^*,l} \cap J^{stt}_{g,l^*,l} \cap J^{new}_{g,l^*,l}, \quad (2.142)$$
$$\forall q \in Q,$$
$$\forall k \in K_q$$

$$w^{sns}_{g,l,l^*,j,q,k} \geq w_{g,l,l^*,j,q,k} - \left(\max_{h \in H_{g,l,l^*,j}} w^{new}_{g,l,l^*,j,h,q,k} \right) \zeta^{sns}_{g,l^*,l,j,q,k},$$
$$\forall g \in G,$$
$$\forall l \in L_g \setminus \left(L^{imp}_g \cup L^{exp}_g \right),$$
$$\forall l^* \in L_g \setminus \left(L^{imp}_g \cup L^{exp}_g \cup \{l\} \right), \quad (2.143)$$
$$\forall j \in J^{und}_{g,l,l^*} \cap J^{stt}_{g,l,l^*} \cap J^{sgl}_{g,l,l^*},$$
$$\forall q \in Q,$$
$$\forall k \in K_q$$

$$w^{sns}_{g,l,l^*,j,q,k} \geq w_{g,l^*,l,j,q,k} - \left(\max_{h \in H_{g,l^*,l,j}} w^{new}_{g,l^*,l,j,h,q,k} \right) \zeta^{sns}_{g,l^*,l,j,q,k},$$
$$\forall g \in G,$$
$$\forall l \in L_g \setminus \left(L^{imp}_g \cup L^{exp}_g \right),$$
$$\forall l^* \in L_g \setminus \left(L^{imp}_g \cup L^{exp}_g \cup \{l\} \right), \quad (2.144)$$
$$\forall j \in J^{und}_{g,l^*,l} \cap J^{stt}_{g,l^*,l} \cap J^{sgl}_{g,l^*,l},$$
$$\forall q \in Q,$$
$$\forall k \in K_q$$

$$w^{sns}_{g,l,l^*,j^*,q,k} \leq \left(\max_{h \in H_t} w^{new}_{g,l,l^*,j^*,h,q,k} \right) \zeta^{sns}_{g,l,l^*,j^*,q,k},$$
$$\forall t \in T,$$
$$\forall (g,l,l^*,j) \in GLLJ^{col}_t,$$
$$\forall j^* \in \{j\} \cap J^{und}_{g,l,l^*} \cap J^{stt}_{g,l,l^*}, \quad (2.145)$$
$$\forall q \in Q,$$
$$\forall k \in K_q$$

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$$w_{g,l,l^*,j^*,q,k}^{sns} \leq \left(\max_{h \in H_t} w_{g,l^*,l,j^*,h,q,k}^{new} \right) \zeta_{g,l,l^*,j^*,q,k}^{sns},$$

$$\forall t \in T,$$

$$\forall (g,l,l^*,j) \in GLLJ_t^{col}, \tag{2.146}$$

$$\forall j^* \in \{j\} \cap J_{g,l^*,l}^{und} \cap J_{g,l^*,l}^{stt},$$

$$\forall q \in Q,$$

$$\forall k \in K_q$$

$$w_{g,l,l^*,j^*,q,k}^{sns} \geq w_{g,l,l^*,j^*,q,k} - \left(\max_{h \in H_t} w_{g,l,l^*,j^*,h,q,k}^{new} \right) \zeta_{g,l^*,l,j^*,q,k}^{sns},$$

$$\forall t \in T,$$

$$\forall (g,l,l^*,j) \in GLLJ_t^{col}, \tag{2.147}$$

$$\forall j^* \in \{j\} \cap J_{g,l,l^*}^{und} \cap J_{g,l,l^*}^{stt},$$

$$\forall q \in Q,$$

$$\forall k \in K_q$$

$$w_{g,l,l^*,j^*,q,k}^{sns} \geq w_{g,l^*,l,j^*,q,k} - \left(\max_{h \in H_t} w_{g,l^*,l,j^*,h,q,k}^{new} \right) \zeta_{g,l^*,l,j^*,q,k}^{sns},$$

$$\forall t \in T,$$

$$\forall (g,l,l^*,j) \in GLLJ_t^{col}, \tag{2.148}$$

$$\forall j^* \in \{j\} \cap J_{g,l^*,l}^{und} \cap J_{g,l^*,l}^{stt},$$

$$\forall q \in Q,$$

$$\forall k \in K_q$$

2.4 Converters

2.4.1 Inputs

Input variables

A given input m can be modelled as a non-negative real ($m \in M_i^{nnr}, \forall i \in I$) or as a binary variable ($m \in M_i^{bin}, \forall i \in I$). Binary inputs cannot be dimensioned and therefore do not induce capital costs, unlike non-negative real inputs.

$$
\begin{aligned}
u_{i,m,q,k} \geq 0, &\forall i \in I, \\
&\forall m \in M_i^{nnr}, \\
&\forall q \in Q, \\
&\forall k \in K_q
\end{aligned}
\tag{2.149}
$$

$$
\begin{aligned}
u_{i,m,q,k} \in \{0,1\}, &\forall i \in I, \\
&\forall m \in M_i^{bin}, \\
&\forall q \in Q, \\
&\forall k \in K_q
\end{aligned}
\tag{2.150}
$$

Dimensionable nominal input amplitude

Dimensionable inputs ($m \in M_i^{dim}$) require a nominal amplitude variable. This variable is non-negative real and can only be associated with new converters.

$$
\begin{aligned}
u_{i,m}^{amp} \geq 0, &\forall i \in I^{new}, \\
&\forall m \in M_i^{dim}
\end{aligned}
\tag{2.151}
$$

Maximum input modulated by the nominal amplitude

For new converters, dimensionable inputs cannot exceed the value stipulated by a linear function of the nominal amplitude.

$$
\begin{aligned}
u_{i,m,q,k} \leq f_{i,m,q,k}^{amp,u} u_{i,m}^{amp}, &\forall i \in I^{new}, \\
&\forall m \in M_i^{dim}, \\
&\forall q \in Q, \\
&\forall k \in K_q
\end{aligned}
\tag{2.152}
$$

Maximum nominal input amplitude

The maximum nominal input amplitude is equal to a given parameter if the converter is selected, and zero otherwise.

$$u_{i,m}^{amp} \leq u_{i,m}^{amp,max} \delta_i^{cvt,inv}, \forall i \in I^{new},$$
$$\forall m \in M_i^{dim} \tag{2.153}$$

Fixed upper bounds for non-binary non-dimensionable inputs

Inputs in pre-existing converters can have fixed upper bounds, unless they are dimensionable (redundant) or binary (the bounds are predefined).

$$u_{i,m,q,k} \leq \overline{u_{i,m,q,k}}, \forall i \in I^{pre},$$
$$\forall m \in M_i^{fix},$$
$$\forall q \in Q, \tag{2.154}$$
$$\forall k \in K_q$$

Maximum input for non-dimensionable inputs in new converters

Non-dimensionable inputs for new converters should be zero if that converter has not been selected, and otherwise limited to some predefined value.

$$u_{i,m,q,k} \leq \overline{u_{i,m,q,k}} \delta_i^{cvt,inv}, \forall i \in I^{new},$$
$$\forall m \in M_i^{fix},$$
$$\forall q \in Q, \tag{2.155}$$
$$\forall k \in K_q$$

$$u_{i,m,q,k} \leq \delta_i^{cvt,inv}, \forall i \in I^{new},$$
$$\forall m \in M_i^{bin},$$
$$\forall q \in Q, \tag{2.156}$$
$$\forall k \in K_q$$

Note: binary variables on pre-existing converters require no upper bounds.

2.4.2 Outputs

Output variables

The outputs for a given converter i a given time interval k and assessment q are defined using unconstrained (or free) real variables.

$$y_{i,r,q,k} \in \mathbb{R}, \forall i \in I, \forall r \in R_i, \forall q \in Q, \forall k \in K_q \tag{2.157}$$

Output equations

The outputs are defined as linear functions of states and inputs.

$$y_{i,r,q,k} = \sum_{n \in N_i} c^{eq,y}_{i,r,n,q,k} x_{i,n,q,k} + \sum_{m \in M_i} d^{eq,y}_{i,r,m,q,k} u_{i,m,q,k} + e^{eq,y}_{i,r,q,k},$$
$$\forall i \in I, \forall r \in R_i, \forall q \in Q, \forall k \in K_q \tag{2.158}$$

Positive output amplitude variables

New converters can have outputs whose positive amplitudes are dimensionable. A non-negative real variable is needed for that per dimensionable output.

$$y^{amp,pos}_{i,r} \geq 0, \forall i \in I^{new},$$
$$\forall r \in R^{dim,pos}_i \tag{2.159}$$

Negative output amplitude variables

New converters can have outputs whose negative amplitudes are dimensionable. A non-negative real variable per output is needed. If a positive amplitude is also dimensionable and equal to the negative amplitude, then no variable is needed.

$$y^{amp,neg}_{i,r} \geq 0, \forall i \in I^{new},$$
$$\forall r \in R^{dim,neg}_i \setminus R^{dim,eq}_i \tag{2.160}$$

Positive output limits if dimensionable

Dimensionable outputs are constrained using the nominal positive amplitude.

$$y_{i,r,q,k} \leq f^{amp,y}_{i,r,q,k} y^{amp,pos}_{i,r}, \forall i \in I^{new},$$
$$\forall r \in R^{dim,pos}_i,$$
$$\forall q \in Q,$$
$$\forall k \in K_q \tag{2.161}$$

Negative output limits if dimensionable

Dimensionable outputs are constrained using the nominal negative amplitude, (2.162). If the positive and negative nominal amplitudes are dimensionable and made to match, the common nominal amplitude is used instead, (2.163).

$$
\begin{aligned}
-y_{i,r}^{amp,neg} f_{i,r,q,k}^{amp,y} \leq y_{i,r,q,k}, &\forall i \in I^{new}, \\
&\forall r \in R_i^{dim,neg} \setminus R_i^{dim,eq}, \\
&\forall q \in Q, \\
&\forall k \in K_q
\end{aligned}
\tag{2.162}
$$

$$
\begin{aligned}
-y_{i,r}^{amp,pos} f_{i,r,q,k}^{amp,y} \leq y_{i,r,q,k}, &\forall i \in I^{new}, \\
&\forall r \in R_i^{dim,eq}, \\
&\forall q \in Q, \\
&\forall k \in K_q
\end{aligned}
\tag{2.163}
$$

Positive output amplitude limits

Dimensionable positive amplitudes must be zero if the converter is not selected.

$$
\begin{aligned}
y_{i,r}^{amp,pos} \leq \overline{y_{i,r}^{amp,pos}} \delta_i^{cvt,inv}, &\forall i \in I^{new}, \\
&\forall r \in R_i^{dim,pos}
\end{aligned}
\tag{2.164}
$$

Negative output amplitude limits

Dimensionable negative amplitudes must be zero if the converter is not selected. If the positive and negative amplitudes are dimensionable and made to match, the constraint is not needed, since only the common amplitude is used.

$$
\begin{aligned}
y_{i,r}^{amp,neg} \leq \overline{y_{i,r}^{amp,neg}} \delta_i^{cvt,inv}, &\forall i \in I^{new}, \\
&\forall r \in R_i^{dim,neg} \setminus R_i^{dim,eq}
\end{aligned}
\tag{2.165}
$$

Fixed bounds for output variables

Outputs can be constrained using fixed upper and lower bounds.

$$
\begin{aligned}
\underline{y_{i,r,q,k}} \leq y_{i,r,q,k} \leq \overline{y_{i,r,q,k}}, &\forall i \in I^{pre}, \\
&\forall r \in R_i^{fix}, \\
&\forall q \in Q, \\
&\forall k \in K_q
\end{aligned}
\tag{2.166}
$$

$$\underline{y_{i,r,q,k}}\delta_i^{cvt,inv} \leq y_{i,r,q,k} \leq \overline{y_{i,r,q,k}}\delta_i^{cvt,inv}, \forall i \in I^{new},$$
$$\forall r \in R_i^{fix},$$
$$\forall q \in Q, \tag{2.167}$$
$$\forall k \in K_q$$

2.4.3 States

State variables

States are modelled as unconstrained (or free) real variables.

$$x_{i,n,q,k} \in \mathbb{R}, \forall i \in I, \forall n \in N_i, \forall q \in Q, \forall k \in K_q \tag{2.168}$$

State equations

States are defined as linear functions of states and inputs.

$$x_{i,n,q,k} = \sum_{n^* \in N_i} a^{eq,x}_{i,n^*,n,q,k} x_{i,n^*,q,k-1} + \sum_{m \in M_i} b^{eq,x}_{i,n,m,q,k} u_{i,m,q,k} + e^{eq,x}_{i,n,q,k},$$
$$\forall i \in I, \forall n \in N_i, \forall q \in Q, \forall k \in K_q \tag{2.169}$$

Fixed bounds for state variables

States can be constrained using fixed upper and lower bounds.

$$\underline{x_{i,n,q,k}} \leq x_{i,n,q,k} \leq \overline{x_{i,n,q,k}}, \forall i \in I^{pre},$$
$$\forall n \in N_i^{fix},$$
$$\forall q \in Q, \tag{2.170}$$
$$\forall k \in K_q$$

$$\underline{x_{i,n,q,k}} \delta_i^{cvt,inv} \leq x_{i,n,q,k} \leq \overline{x_{i,n,q,k}} \delta_i^{cvt,inv}, \forall i \in I^{new},$$
$$\forall n \in N_i^{fix},$$
$$\forall q \in Q, \tag{2.171}$$
$$\forall k \in K_q$$

Nominal positive state amplitude variables

$$x_{i,n}^{amp,pos} \geq 0, \forall i \in I^{new},$$
$$\forall n \in N_i^{dim,pos} \tag{2.172}$$

Nominal negative state amplitude variables

$$x_{i,n}^{amp,neg} \geq 0, \forall i \in I^{new},$$
$$\forall n \in N_i^{dim,neg} \setminus N_i^{dim,eq} \tag{2.173}$$

Positive state limit if dimensionable

$$x_{i,n,q,k} \leq f_{i,n,q,k}^{amp,x} x_{i,n}^{amp,pos}, \forall i \in I^{new},$$
$$\forall n \in N_i^{dim,pos},$$
$$\forall q \in Q,$$
$$\forall k \in K_q \tag{2.174}$$

Negative state limit if dimensionable

$$-x_{i,n}^{amp,neg} f_{i,n,q,k}^{amp,x} \leq x_{i,n,q,k}, \forall i \in I^{new},$$
$$\forall n \in N_i^{dim,neg} \setminus N_i^{dim,eq},$$
$$\forall q \in Q,$$
$$\forall k \in K_q \tag{2.175}$$

$$-x_{i,n}^{amp,pos} f_{i,n,q,k}^{amp,x} \leq x_{i,n,q,k}, \forall i \in I^{new},$$
$$\forall n \in N_i^{dim,eq},$$
$$\forall q \in Q,$$
$$\forall k \in K_q \tag{2.176}$$

Nominal positive state amplitude limits

$$x_{i,n}^{amp,pos} \leq \overline{x_{i,n}^{amp,pos}} \delta_i^{cvt,inv}, \forall i \in I^{new},$$
$$\forall n \in N_i^{dim,pos} \tag{2.177}$$

Nominal negative state amplitude limits

$$x_{i,n}^{amp,neg} \leq \overline{x_{i,n}^{amp,neg}} \delta_i^{cvt,inv}, \forall i \in I^{new},$$
$$\forall n \in N_i^{dim,neg} \setminus N_i^{dim,eq} \tag{2.178}$$

Positive state variation variables

$$\Delta x_{i,n,q}^{pos} \geq 0, \forall i \in I,$$
$$\forall n \in N_i^{pos,var},$$
$$\forall q \in Q \tag{2.179}$$

Negative state variation variables

$$\Delta x_{i,n,q}^{neg} \geq 0, \forall i \in I,$$
$$\forall n \in N_i^{neg,var},$$
$$\forall q \in Q \tag{2.180}$$

Positive state variation determination

$$x_{i,n,q,|K|} - x_{i,n,q,0} \le \Delta x_{i,n,q}^{pos}, \forall i \in I,$$
$$\forall n \in N_i^{pos,var}, \tag{2.181}$$
$$\forall q \in Q$$

Note: $x_{i,n,0}$ is the initial condition.

Negative state variation determination

$$x_{i,n,q,|K|} - x_{i,n,q,0} \ge -\Delta x_{i,n,q}^{neg}, \forall i \in I,$$
$$\forall n \in N_i^{neg,var}, \tag{2.182}$$
$$\forall q \in Q$$

Note: $x_{i,n,0}$ is the initial condition.

Violation of upper references

$$x_{i,n,q,k} - x_{i,n,q,k}^{ref,hgh} \le \Delta x_{i,n,q,k}^{ref,hgh}, \forall i \in I,$$
$$\forall n \in N_i^{ref,hgh},$$
$$\forall q \in Q, \tag{2.183}$$
$$\forall k \in K_q$$

Violation of lower references

$$x_{i,n,q,k}^{ref,low} - x_{i,n,q,k} \le \Delta x_{i,n,q,k}^{ref,low}, \forall i \in I,$$
$$\forall n \in N_i^{ref,low},$$
$$\forall q \in Q, \tag{2.184}$$
$$\forall k \in K_q$$

Upper reference violation variables

$$\Delta x_{i,n,q,k}^{ref,hgh} \ge 0, \forall i \in I,$$
$$\forall n \in N_i^{ref,hgh},$$
$$\forall q \in Q, \tag{2.185}$$
$$\forall k \in K_q$$

Lower reference violation variables

$$\Delta x_{i,n,q,k}^{ref,low} \ge 0, \forall i \in I,$$
$$\forall n \in N_i^{ref,low},$$
$$\forall q \in Q, \tag{2.186}$$
$$\forall k \in K_q$$

2.5 Network constraints

2.5.1 Flow equilibrium at each internal node

Flow equilibria is required at all times in internal nodes. In practice, this means that for each node excluding import and export nodes, and for each time interval, the balance between effective incoming and outgoing flows plus converter effects must match the base flow. The term effective used in the previous sentence is meant to account for the effects of static and flow-proportional losses.

$$
\begin{aligned}
v_{g,l,q,k}^{base} =& \sum_{l^\circ \in L_g \backslash (L_g^{exp} \cup \{l\})} \sum_{j^\circ \in J_{g,l^\circ,l}^{dir} \cup J_{g,l,l^\circ}^{und}} \eta_{g,l^\circ,l,j^\circ,q,k} v_{g,l^\circ,l,j^\circ,q,k} \\
&- \sum_{l^\bullet \in L_g \backslash (L_g^{imp} \cup \{l\})} \sum_{j^\bullet \in J_{g,l,l^\bullet}^{dir} \cup J_{g,l,l^\bullet}^{und}} v_{g,l,l^\bullet,j^\bullet,q,k} \\
&- \sum_{l^\star \in L_g \backslash (L_g^{exp} \cup \{l\})} \sum_{j^\star \in J_{g,l^\star,l}^{stt,arr} \cap J_{g,l^\star,l}^{dir}} \eta_{g,l^\star,l,j^\star,q,k} w_{g,l^\star,l,j^\star,q,k} \\
&- \sum_{l^\diamond \in L_g \backslash (L_g^{imp} \cup \{l\})} \sum_{j^\diamond \in J_{g,l,l^\diamond}^{stt,dep} \cap J_{g,l,l^\diamond}^{dir}} w_{g,l,l^\diamond,j^\diamond,q,k} \\
&- \sum_{l^\star \in L_g \backslash (L_g^{exp} \cup L_g^{imp} \cup \{l\})} \sum_{j^\star \in J_{g,l,l^\star}^{und} \cap J_{g,l,l^\star}^{stt,dep}} \\
&\qquad\qquad \left(\eta_{g,l^\star,l,j^\star,q,k} w_{g,l^\star,l,j^\star,q,k}^{sns} + w_{g,l,l^\star,j^\star,q,k}^{sns} \right) \\
&- \sum_{l^* \in L_g \backslash (L_g^{exp} \cup L_g^{imp} \cup \{l\})} \sum_{j^* \in J_{g,l^*,l}^{und} \cap J_{g,l^*,l}^{stt,arr}} \\
&\qquad\qquad \left(\eta_{g,l^*,l,j^*,q,k} w_{g,l^*,l,j^*,q,k}^{sns} + w_{g,l,l^*,j^*,q,k}^{sns} \right) \\
&- \sum_{l^\circ \in L_g \backslash (L_g^{exp} \cup L_g^{imp} \cup \{l\})} \sum_{j^\circ \in J_{g,l,l^\circ}^{und} \cap J_{g,l,l^\circ}^{stt,ds}} \eta_{g,l^\circ,l,j^\circ,q,k} w_{g,l^\circ,l,j^\circ,q,k}^{sns} \\
&- \sum_{l^\circ \in L_g \backslash (L_g^{exp} \cup L_g^{imp} \cup \{l\})} \sum_{j^\circ \in J_{g,l^\circ,l}^{und} \cap J_{g,l^\circ,l}^{stt,ds}} \eta_{g,l^\circ,l,j^\circ,q,k} w_{g,l^\circ,l,j^\circ,q,k}^{sns} \\
&- \sum_{l^\square \in L_g \backslash (L_g^{exp} \cup L_g^{imp} \cup \{l\})} \sum_{j^\square \in J_{g,l,l^\square}^{und} \cap J_{g,l,l^\square}^{stt,us}} w_{g,l,l^\square,j^\square,q,k}^{sns} \\
&- \sum_{l^\square \in L_g \backslash (L_g^{exp} \cup L_g^{imp} \cup \{l\})} \sum_{j^\square \in J_{g,l^\square,l}^{und} \cup J_{g,l^\square,l}^{stt,us}} w_{g,l,l^\square,j^\square,q,k}^{sns} \\
&+ \sum_{i \in I} \left(\sum_{r \in R_i} a_{g,l,i,r,q,k}^{node,y} y_{i,r,q,k} + \sum_{m \in M_i} a_{g,l,i,m,q,k}^{node,u} u_{i,m,q,k} \right) \\
& \forall g \in G, \forall l \in L_g \backslash \left(L_g^{imp} \cup L_g^{exp} \right), \forall q \in Q, \forall k \in K_q
\end{aligned} \tag{2.187}
$$

2.5.2 Only one incoming directed arc without outgoing arcs

Internal nodes can be set up to be incompatible with having multiple directed incoming arcs unless there are outgoing arcs too. This prevents one node from being an end node for multiple branches. It does not apply in the presence of undirected arcs. For each node, the constraint can be summarised as follows: the number of incoming arcs must be lower than or equal to one, if there are no outgoing arcs; if there are outgoing arcs, the constraint is ignored.

$$NIDA_{g,l} \leq 1 + a_{g,l}^{max,in} NOA_{g,l}, \forall g \in G, \forall l \in L_g^{max,in} \tag{2.188}$$

$$
\begin{aligned}
NOA_{g,l} = \sum_{l^\diamond \in L_g \setminus \left(L_g^{imp} \cup \{l\}\right)} \Bigg(
&\sum_{t \in T^{int}} \Bigg(\sum_{j \in J_{g,l,l^\diamond}^{col}} |GLLJ_t^{col} \cap \{(g,l,l^\diamond,j)\}| + \\
&\sum_{j \in J_{g,l^\diamond,l}^{col} \cap J_{g,l^\diamond,l}^{und}} |GLLJ_t^{col} \cap \{(g,l^\diamond,l,j)\}| \Bigg) \xi_t^{arc,inv} + \\
&\sum_{t \in T^{opt} \setminus T^{int}} \Bigg(\sum_{j \in J_{g,l,l^\diamond}^{col}} |GLLJ_t^{col} \cap \{(g,l,l^\diamond,j)\}| + \\
&\qquad \sum_{j \in J_{g,l^\diamond,l}^{col} \cap J_{g,l^\diamond,l}^{und}} |GLLJ_t^{col} \cap \{(g,l^\diamond,l,j)\}| \Bigg) \sum_{h \in H_t} \delta_{t,h}^{arc,inv} + \\
&\sum_{j^\diamond \in J_{g,l,l^\diamond}^{int} \cap J_{g,l,l^\diamond}^{sgl}} \xi_{g,l,l^\diamond,j^\diamond}^{arc,inv} + \\
&\sum_{j^\diamond \in J_{g,l^\diamond,l}^{und} \cap J_{g,l^\diamond,l}^{int} \cap J_{g,l^\diamond,l}^{sgl}} \xi_{g,l^\diamond,l,j^\diamond}^{arc,inv} + \\
&\sum_{j^\star \in J_{g,l,l^\diamond}^{opt} \cap J_{g,l,l^\diamond}^{sgl} \setminus J_{g,l,l^\diamond}^{int}} \sum_{h^\star \in H_{g,l,l^\diamond,j^\star}} \delta_{g,l,l^\diamond,j^\star,h^\star}^{arc,inv} + \\
&\sum_{j^* \in J_{g,l^\diamond,l}^{und} \cap J_{g,l^\diamond,l}^{opt} \cap J_{g,l^\diamond,l}^{sgl} \setminus J_{g,l^\diamond,l}^{int}} \sum_{h^* \in H_{g,l^\diamond,l,j^*}} \delta_{g,l^\diamond,l,j^*,h^*}^{arc,inv} + \\
&|J_{g,l,l^\diamond}^{pre}| + |J_{g,l,l^\diamond}^{mdt}| + |J_{g,l^\diamond,l}^{pre} \cap J_{g,l^\diamond,l}^{und}| + |J_{g,l^\diamond,l}^{mdt} \cap J_{g,l^\diamond,l}^{und}| \Bigg), \\
&\forall g \in G, \forall l \in L_g^{max,in}
\end{aligned}
\tag{2.189}
$$

$$NIDA_{g,l} = \sum_{l^\circ \in L_g \setminus (L_g^{exp} \cup \{l\})} \Bigg($$

$$\sum_{t \in T^{int}} \left(\sum_{j \in J_{g,l^\circ,l}^{dir} \cap J_{g,l^\circ,l}^{col}} |GLLJ_t^{col} \cap \{(g,l^\circ,l,j)\}| \right) \xi_t^{arc,inv} +$$

$$\sum_{t \in T^{opt} \setminus T^{int}} \left(\sum_{j \in J_{g,l^\circ,l}^{dir} \cap J_{g,l^\circ,l}^{col}} |GLLJ_t^{col} \cap \{(g,l^\circ,l,j)\}| \right) \sum_{h \in H_t} \delta_{t,h}^{arc,inv} +$$

$$|J_{g,l^\circ,l}^{dir} \cap J_{g,l^\circ,l}^{pre}| + |J_{g,l^\circ,l}^{dir} \cap J_{g,l^\circ,l}^{mdt}| +$$

$$\sum_{j^\circ \in J_{g,l^\circ,l}^{dir} \cap J_{g,l^\circ,l}^{int} \cap J_{g,l^\circ,l}^{sgl}} \xi_{g,l^\circ,l,j^\circ}^{arc,inv} +$$

$$\sum_{j^\bullet \in J_{g,l^\circ,l}^{dir} \cap J_{g,l^\circ,l}^{opt} \cap J_{g,l^\circ,l}^{sgl} \setminus J_{g,l^\circ,l}^{int}} \sum_{h^\circ \in H_{g,l^\circ,l,j^\bullet}} \delta_{g,l^\circ,l,j^\bullet,h^\circ}^{arc,inv} \Bigg), \forall g \in G, \forall l \in L_g^{max,in}$$

$$(2.190)$$

Note: the big-M values need to larger than or equal to the number of incoming directed arcs minus one.

$$a_{g,l}^{max,in} + 1 \geq \sum_{l^\circ \in L_g \setminus (L_g^{exp} \cup \{l\})} |J_{g,l^\circ,l}^{dir}|, \forall g \in G, \forall l \in L_g^{max,in} \qquad (2.191)$$

Note:

$$L_g^{max,in} \subseteq L_g \setminus L_g^{imp}, \forall g \in G \qquad (2.192)$$

2.5.3 Only one outgoing directed arc without incoming arcs

Internal nodes can be set up to be incompatible with having multiple directed outgoing arcs unless there are incoming arcs too. This prevents one node from being a start node for multiple branches. It does not apply in the presence of undirected arcs. For each node, the constraint can be summarised as follows: the sum of outgoing directed arcs must be lower than or equal to one, if there are no incoming arcs; if there are incoming arcs, the constraint is ignored.

$$NODA_{g,l} \leq 1 + a_{g,l}^{max,out} NIA_{g,l}, \forall g \in G, \forall l \in L_g^{max,out} \tag{2.193}$$

$$
NIA_{g,l} = \sum_{l^\diamond \in L_g \setminus (L_g^{exp} \cup \{l\})} \Bigg(
$$
$$
\sum_{t \in T^{int}} \Bigg(\sum_{j \in J_{g,l^\diamond,l}^{col}} |GLLJ_t^{col} \cap \{(g,l^\diamond,l,j)\}| +
$$
$$
\sum_{j \in J_{g,l,l^\diamond}^{col} \cap J_{g,l,l^\diamond}^{und}} |GLLJ_t^{col} \cap \{(g,l,l^\diamond,j)\}| \Bigg) \xi_t^{arc,inv} +
$$
$$
\sum_{t \in T^{opt} \setminus T^{int}} \Bigg(\sum_{j \in J_{g,l^\diamond,l}^{col}} |GLLJ_t^{col} \cap \{(g,l^\diamond,l,j)\}| +
$$
$$
\sum_{j \in J_{g,l,l^\diamond}^{col} \cap J_{g,l,l^\diamond}^{und}} |GLLJ_t^{col} \cap \{(g,l,l^\diamond,j)\}| \Bigg) \sum_{h \in H_t} \delta_{t,h}^{arc,inv} + \tag{2.194}
$$
$$
\sum_{j^\diamond \in J_{g,l^\diamond,l}^{int} \cap J_{g,l^\diamond,l}^{sgl}} \xi_{g,l^\diamond,l,j^\diamond}^{arc,inv} +
$$
$$
\sum_{j^\diamond \in J_{g,l,l^\diamond}^{und} \cap J_{g,l,l^\diamond}^{int} \cap J_{g,l,l^\diamond}^{sgl}} \xi_{g,l,l^\diamond,j^\diamond}^{arc,inv} +
$$
$$
\sum_{j^\star \in J_{g,l^\diamond,l}^{opt} \cap J_{g,l^\diamond,l}^{sgl} \setminus J_{g,l^\diamond,l}^{int}} \sum_{h^\star \in H_{g,l^\diamond,l,j^\star}} \delta_{g,l^\diamond,l,j^\star,h^\star}^{arc,inv} +
$$
$$
\sum_{j^* \in J_{g,l,l^\diamond}^{und} \cap J_{g,l,l^\diamond}^{opt} \cap J_{g,l,l^\diamond}^{sgl} \setminus J_{g,l,l^\diamond}^{int}} \sum_{h^* \in H_{g,l,l^\diamond,j^*}} \delta_{g,l,l^\diamond,j^*,h^*}^{arc,inv} +
$$
$$
|J_{g,l^\diamond,l}^{pre}| + |J_{g,l^\diamond,l}^{mdt}| + |J_{g,l,l^\diamond}^{pre} \cap J_{g,l,l^\diamond}^{und}| + |J_{g,l,l^\diamond}^{mdt} \cap J_{g,l,l^\diamond}^{und}| \Bigg),
$$
$$
\forall g \in G, \forall l \in L_g^{max,out}
$$

$$NODA_{g,l} = \sum_{l^\circ \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right)} \Bigg($$

$$\sum_{t \in T^{int}} \left(\sum_{j \in J_{g,l,l^\circ}^{dir} \cap J_{g,l,l^\circ}^{col}} |GLLJ_t^{col} \cap \{(g,l,l^\circ,j)\}| \right) \xi_t^{arc,inv} +$$

$$\sum_{t \in T^{opt} \setminus T^{int}} \left(\sum_{j \in J_{g,l,l^\circ}^{dir} \cap J_{g,l,l^\circ}^{col}} |GLLJ_t^{col} \cap \{(g,l,l^\circ,j)\}| \right) \sum_{h \in H_t} \delta_{t,h}^{arc,inv} +$$

$$|J_{g,l,l^\circ}^{dir} \cap J_{g,l,l^\circ}^{pre}| + |J_{g,l,l^\circ}^{dir} \cap J_{g,l,l^\circ}^{mdt}| +$$

$$\sum_{j^\circ \in J_{g,l,l^\circ}^{dir} \cap J_{g,l,l^\circ}^{int} \cap J_{g,l,l^\circ}^{sgl}} \xi_{g,l,l^\circ,j^\circ}^{arc,inv} +$$

$$\sum_{j^\bullet \in J_{g,l,l^\circ}^{dir} \cap J_{g,l,l^\circ}^{opt} \cap J_{g,l,l^\circ}^{sgl} \setminus J_{g,l,l^\circ}^{int}} \sum_{h^\circ \in H_{g,l,l^\circ,j^\bullet}} \delta_{g,l,l^\circ,j^\bullet,h^\circ}^{arc,inv} \Bigg), \forall g \in G, \forall l \in L_g^{max,out}$$

$$(2.195)$$

Note: the big-M values need to larger than or equal to the number of outgoing directed arcs minus one.

$$a_{g,l}^{max,out} + 1 \geq \sum_{l^\circ \in L_g \setminus \left(L_g^{imp} \cup \{l\} \right)} |J_{g,l,l^\circ}^{dir}|, \forall g \in G, \forall l \in L_g^{max,out} \qquad (2.196)$$

Note:

$$L_g^{max,out} \subseteq L_g \setminus L_g^{exp}, \forall g \in G \qquad (2.197)$$

2.5.4 Limited number of parallel arcs per direction

The number of arcs between two nodes in a given direction and in a given network can be limited. The limit also applies to undirected arcs, which will be counted once per direction with limits: if there are limits between nodes A and B and vice-versa, and an undirected arc exists between them, then that arc influences the limits from A to B and from B to A. The constraint is defined so as to take advantage of model structures, (2.198): pre-existing and new mandatory arcs are accounted for through data, since they have to exist; new optional arcs rely on the decision variables for their options unless they are interfaced, in which case interface variables are used; the same is true for groups of arcs.

$$
\sum_{t \in T^{int}} \left(\sum_{j \in J^{col}_{g,l,l^*}} |GLLJ^{col}_t \cap \{(g,l,l^*,j)\}| + \right.
$$

$$
\left. \sum_{j \in J^{col}_{g,l^*,l} \cap J^{und}_{g,l^*,l}} |GLLJ^{col}_t \cap \{(g,l^*,l,j)\}| \right) \xi^{arc,inv}_t +
$$

$$
\sum_{t \in T^{opt} \setminus T^{int}} \left(\sum_{j \in J^{col}_{g,l,l^*}} |GLLJ^{col}_t \cap \{(g,l,l^*,j)\}| + \right.
$$

$$
\left. \sum_{j \in J^{col}_{g,l^*,l} \cap J^{und}_{g,l^*,l}} |GLLJ^{col}_t \cap \{(g,l^*,l,j)\}| \right) \sum_{h \in H_t} \delta^{arc,inv}_{t,h} +
$$

$$
\sum_{j \in J^{int}_{g,l,l^*} \cap J^{sgl}_{g,l,l^*}} \xi^{arc,inv}_{g,l,l^*,j} + \tag{2.198}
$$

$$
\sum_{j \in J^{opt}_{g,l,l^*} \cap J^{sgl}_{g,l,l^*} \setminus J^{int}_{g,l,l^*}} \sum_{h \in H_{g,l,l^*,j}} \delta^{arc,inv}_{g,l,l^*,j,h} +
$$

$$
\sum_{j \in J^{und}_{g,l^*,l} \cap J^{int}_{g,l^*,l} \cap J^{sgl}_{g,l^*,l}} \xi^{arc,inv}_{g,l^*,l,j} +
$$

$$
\sum_{j \in J^{und}_{g,l^*,l} \cap J^{opt}_{g,l^*,l} \cap J^{sgl}_{g,l^*,l} \setminus J^{int}_{g,l^*,l}} \sum_{h \in H_{g,l^*,l,j}} \delta^{arc,inv}_{g,l^*,l,j,h} +
$$

$$
|J^{pre}_{g,l,l^*}| + |J^{und}_{g,l^*,l} \cap J^{pre}_{g,l^*,l}| + |J^{mdt}_{g,l,l^*}| + |J^{und}_{g,l^*,l} \cap J^{mdt}_{g,l^*,l}| \leq a^{arc,max}_{g,l,l^*},
$$

$$
\forall (g,l,l^*) \in GLL^{arc,max},
$$

Note:

$$
GLL^{arc,max} \subseteq GLL \tag{2.199}
$$

$$
GLL = \left\{ (g,l,l^*) : g \in G, l \in L_g \setminus L^{exp}_g, l^* \in L_g \setminus \left(L^{imp}_g \cup \{l\} \right) \right) \right\} \tag{2.200}
$$

Chapter 3

Examples

This chapter is dedicated to explaining the model through examples.

3.1 Static losses

3.1.1 Directed arcs

Consider a problem as in Figure 3.1 involving a network g with three nodes: one import node, IMP $\in L_g^{imp}$; and two regular nodes, A $\in L_g$ and B $\in L_g$. Node A is a waypoint node ($v_{g,A,q,k}^{base} = 0$) during a given interval k and assessment q, whereas node B is a sink node requiring an incoming flow of 0.2 ($v_{g,B,q,k}^{base} = 0.2$). There are directed arcs from IMP to A ($IA \in J_{g,IMP,A}^{dir}$) and from A to B ($AB \in J_{g,A,B}^{dir}$). The former has infinite capacity and induces no losses, static ($J_{g,IMP,A}^{stt} = \emptyset$) or otherwise ($\eta_{g,IMP,A,IA,q,k} = 1$). The latter has an efficiency of 80% ($\eta_{g,A,B,AB,q,k} = 0.8$), unit capacity ($v_{g,A,B,AB}^{amp} = 1$, $f_{g,A,B,AB,q,k} = 1$) and static losses ($AB \in J_{g,A,B}^{stt}$) of 0.1 during time interval k and assessment q ($w_{g,A,B,AB,q,k} = 0.1$). Assume for now that those static losses are placed upstream in A ($AB \in J_{g,A,B}^{stt,dep}$). Let us observe what flow equilibrium (2.187) entails for nodes A and B during time interval k and assessment q:

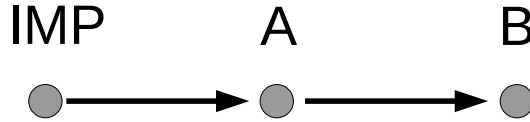Node A: $0 = v_{g,IMP,A,IA,q,k} - v_{g,A,B,AB,q,k} - 0.1$

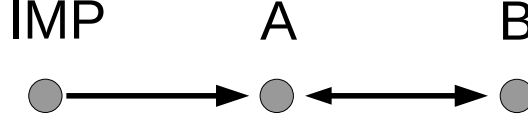Figure 3.1: Network with 3 nodes (IMP, A and B) and 2 directed arcs (IA and AB)

Figure 3.2: Network with 3 nodes (IMP, A and B), 1 directed arc (IA) and 1 undirected arc (AB)

Node B: $0.2 = 0.8 \times v_{g,A,B,AB,q,k}$

From the above, it is clear that $v_{g,A,B,AB,q,k}$ must be equal to 0.25 and that $v_{g,IMP,A,IA,q,k}$ must be 0.35 to account for the static losses in A. The flow between A and B is also below the maximum allowed, which is lowered relative to the specified value due to the static losses being modelled upstream, (2.128): $v_{g,A,B,AB,q,k} \leq 1 - 0.1 = 0.9$. This means that maximum useful flow that can reach B is 0.72 ($0 \leq v_{g,B,q,k}^{base} \leq 0.72$), in which case the flow leaving A would be 0.9 and the flow entering A would be exactly 1. Now consider that static losses are instead placed downstream in B ($AB \notin J_{g,A,B}^{stt,dep}, AB \in J_{g,A,B}^{stt,arr}$):

Node A: $0 = v_{g,IMP,A,IA,q,k} - v_{g,A,B,AB,q,k}$

Node B: $0.2 = 0.8 \times v_{g,A,B,AB,q,k} - 0.8 \times 0.1$

In this case, the flow through IA and AB must both be 0.35, none of which violate flow limits. The limits themselves do not need to be adjusted since the static losses are placed downstream in B, making the maximum flow entering and leaving A exactly 1, and 0.72 the maximum useful flow that could reach B. These are the same results for useful flow reaching B (via AB) and for flow reaching A (via IA) as if the static losses (for AB) were placed upstream in A, even though the actual flow through AB differs. The conclusion is that placing static losses upstream or downstream has the same outcome, from no flow up to the maximum permitted. Note also that since there are no alternative ways to supply flow to B, the constraint (2.134) is redundant, as it would otherwise impose a minimum flow of 0.1 through the arc to compensate for its static losses.

3.1.2 Undirected arcs

Losses are placed upstream

Consider now that the arc between A and B is instead undirected ($AB \notin J_{g,A,B}^{dir}, AB \in J_{g,A,B}^{und}, AB \notin J_{g,B,A}^{und}$), as shown in Figure 3.2, is pre-existing ($AB \in J_{g,A,B}^{pre}$), has the same efficiency in both directions, and its static losses are placed upstream in relation to the flow sense ($AB \in J_{g,A,B}^{stt,us}$). If all else is the same as in the examples given in Section 3.1.1, the results are the same since the flow can only be from A to B ($\zeta_{g,A,B,AB,q,k}^{sns} = 1, \zeta_{g,B,A,AB,q,k}^{sns} = 0$):

Node A:

$$v_{g,A,q,k}^{base} = v_{g,IMP,A,IA,q,k} + \eta_{g,B,A,AB,q,k} v_{g,B,A,AB,q,k} - v_{g,A,B,AB,q,k}$$
$$- w_{g,A,B,AB,q,k}^{sns}$$

Node B:

$$v_{g,B,q,k}^{base} = \eta_{g,A,B,AB,q,k} v_{g,A,B,AB,q,k} - v_{g,B,A,AB,q,k} - w_{g,B,A,AB,q,k}^{sns}$$

with:

$$w_{g,A,B,AB,q,k}^{sns} = w_{g,A,B,AB,q,k} \zeta_{g,A,B,AB,q,k}^{sns}$$
$$w_{g,B,A,AB,q,k}^{sns} = w_{g,A,B,AB,q,k} \zeta_{g,B,A,AB,q,k}^{sns}$$

The solution requires the flow through AB to be 0.25 and the flow through IA to be 0.35, the latter to compensate for the static losses in A. The amplitude allowed is also lower than the nominal one (0.9 instead of 1.0) due to (2.125), since losses appear before the arc, exactly as in the example for directed arcs.

Consider then another interval, $k+1$. During this interval, node B acts as a source ($v_{g,B,q,k+1}^{base} = -0.6$), node A as a sink ($v_{g,A,q,k+1}^{base} = 0.4$), the arc efficiency from B to A is 50% ($\eta_{g,B,A,AB,q,k} = 0.5$), static losses are placed upstream ($AB \in J_{g,A,B}^{stt,us}$), and all other parameters remain the same. The simplified balances for A and B during interval $k+1$ and assessment q are:

Node A:

$$0.4 = v_{g,IMP,A,IA,q,k+1} + 0.5 \times v_{g,B,A,AB,q,k+1} - v_{g,A,B,AB,q,k+1}$$
$$- 0.1 \times \zeta_{g,A,B,AB,q,k+1}^{sns}$$

Node B:

$$-0.6 = 0.8 \times v_{g,A,B,AB,q,k+1} - v_{g,B,A,AB,q,k+1} - 0.1 \times \zeta_{g,B,A,AB,q,k+1}^{sns}$$

In this case, the flow has to be from B to A ($\zeta_{g,B,A,AB,q,k}^{sns} = 1$), since there is no other way to deal with a source in B. Consequently, the static losses appear in B, as B is upstream relative to the flow sense. Solving the equation shows that the flow through AB must be equal to 0.5 but only 0.25 makes it to A, due to the 50 % efficiency from B to A, and due to this 0.15 must be imported from node IMP. Since the losses are upstream, the flow amplitude is also lower than the nominal one through (2.125), though of no consequence in this example. Next, the focus is on demonstrating how the three other approaches for modelling static losses in undirected arcs produce exactly the same results.

Losses are placed downstream

Consider now that the static losses are instead placed downstream relative to the flow sense ($AB \in J_{g,A,B}^{stt,ds}$). The balances for nodes A and B become:

Node A:

$$v_{g,A,q,k}^{base} = v_{g,IMP,A,IA,q,k} + \eta_{g,B,A,AB,q,k} v_{g,B,A,AB,q,k} - v_{g,A,B,AB,q,k}$$
$$- \eta_{g,B,A,AB,q,k} w_{g,B,A,AB,q,k}^{sns}$$

Node B:

$$v_{g,B,q,k}^{base} = \eta_{g,A,B,AB,q,k} v_{g,A,B,AB,q,k} - v_{g,B,A,AB,q,k}$$
$$- \eta_{g,A,B,AB,q,k} w_{g,A,B,AB,q,k}^{sns}$$

In the situation above, the flow has to be from A to B ($\zeta_{g,A,B,AB,q,k}^{sns} = 1$), yet the static losses are instead in B. As such, the flow through AB and IA is 0.35, exactly as in the second example with directed arcs in Section 3.1.1. The constraint (2.135) requiring a minimum flow through the arc is also observed ($v_{g,A,B,AB,q,k} \geq 0.1$), though redundant due to a lack of alternative flow paths leading to B. Consider now the same case during interval $k+1$ and assessment q using the data defined in the previous example except the modelling approach:

Node A:

$$0.4 = v_{g,IMP,A,IA,q,k+1} + 0.5 \times v_{g,B,A,AB,q,k+1} - v_{g,A,B,AB,q,k+1}$$
$$- 0.5 \times 0.1 \times \zeta_{g,B,A,AB,q,k+1}^{sns}$$

Node B:

$$-0.6 = 0.8 \times v_{g,A,B,AB,q,k+1} - v_{g,B,A,AB,q,k+1} - 0.8 \times 0.1 \times \zeta_{g,A,B,AB,q,k+1}^{sns}$$

The solution requires the flow to be from B to A, since there is no other way to deal with a source of flow in B. Consequently, the losses appear in A (downstream relative to the flow sense). Note that the loss term amounts to 0.05 due to the arc efficiency for flows from B to A. This means that the flow through AB is 0.6, of which 0.3 reach A, forcing 0.15 to be imported through IMP, which is the same results obtained with losses placed upstream relative to the flow sense. It is also the case that (2.135) is observed ($v_{g,B,A,AB,q,k} \geq 0.1$), despite being redundant, as the alternative ways to compensate for the static losses (namely via IA) would lead to infeasibility (due to the flows from B).

Losses in the nominal source node

Another alternative is to place static losses in a single node, irrespective of the flow sense. Consider then the example of Figure 3.2 but assuming that static losses are placed in the source node for the nominal flow sense ($j \in J_{g,l,l^*}^{stt,dep}$), that is, in node A. For interval k and assessment q, that means:

Node A:

$$
\begin{aligned}
v_{g,A,q,k}^{base} =& v_{g,IMP,A,IA,q,k} + \eta_{g,B,A,AB,q,k} v_{g,B,A,AB,q,k} - v_{g,A,B,AB,q,k} \\
& - w_{g,A,B,AB,q,k}^{sns} - \eta_{g,B,A,AB,q,k} w_{g,B,A,AB,q,k}^{sns}
\end{aligned}
$$

Node B:

$$
v_{g,B,q,k}^{base} = \eta_{g,A,B,AB,q,k} v_{g,A,B,AB,q,k} - v_{g,B,A,AB,q,k}
$$

The solution requires that the flow along AB be from A to B and equal to 0.25, which forces means the losses appear in A without any adjustment. In turn, this forces 0.35 to be imported through IA, which is the same as in previous examples. The amplitude limit reduction due to the losses being upstream (2.126) is also the same as in previous examples ($v_{g,A,B,AB,q,k} \leq 1 - 0.1$) and equally of no consequence. Now let us consider interval $k+1$ for assessment q:

Node A:

$$
\begin{aligned}
0.4 =& v_{g,IMP,A,IA,q,k+1} + 0.5 \times v_{g,B,A,AB,q,k+1} - v_{g,A,B,AB,q,k+1} \\
& - 0.1 \times \zeta_{g,A,B,AB,q,k+1}^{sns} - 0.5 \times 0.1 \times \zeta_{g,B,A,AB,q,k+1}^{sns}
\end{aligned}
$$

Node B:

$$
-0.6 = 0.8 \times v_{g,A,B,AB,q,k+1} - v_{g,B,A,AB,q,k+1}
$$

In this case, the flow is from B to A, as there is no other viable alternative, and equal to 0.6, of which 0.3 reach node A. Since the flow is from B to A, and the losses are in A, the loss term is equal to 0.05 due to the arc efficiency in that direction, which forces 0.15 to be imported through IA. The minimum flow requirement imposed via (2.136) is also observed ($v_{g,A,B,AB,q,k} \geq 0.1$). In conclusion, these results match those obtained with other modelling approaches.

Losses in the nominal end node

The modelling approach that is left considering relies on placing losses in the nominal end node, irrespective of the flow sense ($j \in J_{g,l,l^*}^{stt,arr}$), that is, in node B. For interval k and assessment q, that translates into:

Node A:

$$v_{g,A,q,k}^{base} = v_{g,IMP,A,IA,q,k} + \eta_{g,B,A,AB,q,k} v_{g,B,A,AB,q,k} - v_{g,A,B,AB,q,k}$$

Node B:

$$v_{g,B,q,k}^{base} = \eta_{g,A,B,AB,q,k} v_{g,A,B,AB,q,k} - v_{g,B,A,AB,q,k}$$
$$- \eta_{g,A,B,AB,q,k} w_{g,A,B,AB,q,k}^{sns} - w_{g,B,A,AB,q,k}^{sns}$$

The conditions make clear that the flow must be from A to B. In turn, this means the losses appear in B adjusted by the respective arc efficiency from A to B. The result is that losses amount to 0.08 in B, which means the flow through AB must be 0.35 to provide 0.28 in B. Consequently, the flow through IA must also be 0.35 (lossless arc). Since the losses are downstream, a minimum flow requirement in (2.136) is imposed and met ($v_{g,A,B,AB,q,k} \geq 0.1$). These are effectively the same results as obtained using other approaches.

Consider then the following interval $(k+1)$ for the same assessment (q):

Node A:

$$0.4 = v_{g,IMP,A,IA,q,k} + 0.5 \times v_{g,B,A,AB,q,k} - v_{g,A,B,AB,q,k}$$

Node B:

$$-0.6 = 0.8 \times v_{g,A,B,AB,q,k+1} - v_{g,B,A,AB,q,k+1}$$
$$- 0.8 \times 0.1 \times \zeta_{g,A,B,AB,q,k+1}^{sns} - 0.1 \times \zeta_{g,B,A,AB,q,k+1}^{sns}$$

In this case, the flow must be from B to A, for lack of viable alternatives. The consequence is that the losses appear in B without being affected by the arc efficiency. The loss term adds up to 0.1, which means the flow through AB must be 0.5, 50% of which arrive in A, leaving a gap of 0.15 to be provided through IA. Since the losses appear upstream, the amplitude limit must be reduced via (2.127), though it produces does not affect the outcome: $v_{g,B,A,AB,q,k} \leq 0.9$. As with previous examples, the results are the same as if using other approaches.

Summary

The previous sections illustrate the use of different ways to model static losses in undirected arcs. For convenience, Table 3.1 summarises their implications.

3.2 Converters

This section provides examples of problems involving converters.

Table 3.1: Approaches available to model static losses in an undirected arc j between nodes A and B on network g : 1) in the nominal source node, A ($j \in J_{g,A,B}^{stt,dep}$); 2) in the nominal end node, B ($j \in J_{g,A,B}^{stt,arr}$); 3) downstream relative to the flow sense ($j \in J_{g,A,B}^{stt,ds}$); or, 4) upstream ($j \in J_{g,A,B}^{stt,us}$).

Approach	Flow sense	Lossy node	Minimum arc flow?	Reduced amplitude?	Restrictions
$j \in J_{g,A,B}^{stt,dep}$	A → B	A	×	✓	(2.126), (2.128)
	B → A	A	✓	×	(2.136)
$j \in J_{g,A,B}^{stt,arr}$	A → B	B	✓	×	(2.136)
	B → A	B	×	✓	(2.127),(2.129)
$j \in J_{g,A,B}^{stt,ds}$	A → B	B	✓	×	(2.135)
	B → A	A	✓	×	(2.135)
$j \in J_{g,A,B}^{stt,us}$	A → B	A	×	✓	(2.126), (2.128)
	B → A	B	×	✓	(2.127), (2.129)

3.2.1 Converters as dynamic sinks

Consider a network $G1$ with one import node IMP, one regular node A and one pre-existing (directed) lossless infinite capacity arc connecting them ($IA \in J_{G1,IMP,A}^{inf}$), as illustrated in Figure 3.3. Consider also a pre-existing converter C, with one binary input ($M1 \in M_C^{bin}$) and one state ($N1 \in N_C^{fix}$) bounded between 18 and 22, and whose initial condition is 18 ($x_{C,N1,q,0} = 18$). The converter acts as a dynamic sink on node A: $a_{G1,A,M1,C,q,k}^{node,u} = -1, \forall q \in Q, \forall k \in K_q$. The following constraints define this part of the problem:

- State equations

$$x_{C,N1,q,k} = 0.95 \times x_{C,N1,q,k-1} + 3 \times u_{C,M1,q,k},$$
$$\forall q \in Q, \forall k \in K_q$$

- State bounds

$$18 \leq x_{C,N1,q,k} \leq 22, \forall q \in Q, \forall k \in K_q$$

- Node balances:

$$0 = v_{G1,IMP,A,IA,q,k} - u_{C,M1,q,k}, \forall q \in Q, \forall k \in K_q$$

Considering the initial condition, it is clear that the input needs to be active during the first time interval if the state is to remain between 18 and 22: if $u_{C,M1,q,0} = 0$, then $x_{C,N1,q,1} = 17.1$; if $u_{C,M1,q,0} = 1$, then $x_{C,N1,q,1} = 20.1$. This also means that a flow of 1 will be imported through IMP. For the subsequent interval, the input has to be remain inactive: if $u_{C,M1,q,1} = 0$, then $x_{C,N1,q,2} = 19.095$; if $u_{C,M1,q,1} = 1$, then $x_{C,N1,q,2} = 22.095$, which is above
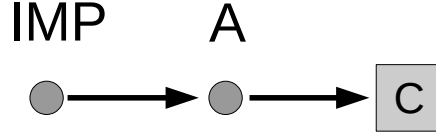
Figure 3.3: Network G1 with 2 nodes (IMP and A), 1 directed arc (IA) and 1 converter (C)

the upper bound. In contrast, the next interval presents two viable paths: if $u_{C,M1,q,2} = 0$, then $x_{C,N1,q,3} = 18.14025$; if $u_{C,M1,q,2} = 1$, then $x_{C,N1,q,3} = 21.14025$. The best decision is dependent on import prices and on whether or not the state can be kept within its bounds in subsequent intervals. This logic repeats itself in subsequent intervals, which may reveal several viable state trajectories. The point here is to demonstrate that converters can be defined to act as dynamic sinks whose states need to be kept under control.

3.2.2 Converters as dynamic sources

Converters can also be used as dynamic sources. Consider another network $G2$ with one regular node B, one export node EXP, and one pre-existing (directed) lossless infinite capacity arc connecting them ($BE \in J^{inf}_{G2,B,EXP}$). A converter C with one bounded state ($N1 \in N^{fix}_C$) and one binary input ($M1 \in M^{bin}_C$) interacts with node B on network $G2$: $a^{node,u}_{G2,B,M1,C,q,k} = 1, \forall q \in Q, \forall k \in K_q$. The following constraints materialise this part of the problem:

- State equations

$$x_{C,N1,q,k} = 0.98 \times x_{C,N1,q,k-1} - 5 \times u_{C,M1,q,k} + e^{eq,x}_{C,N1,q,k},$$
$$\forall q \in Q, \forall k \in K_q$$

- State bounds

$$70 \leq x_{C,N1,q,k} \leq 95, \forall q \in Q, \forall k \in K_q$$

- Node balances:

$$0 = -v_{G2,B,EXP,BE,q,k} + u_{C,M1,q,k}, \forall q \in Q, \forall k \in K_q$$

The constraints above describe a problem similar to the one in Section 3.2.1, though in this case the converter acts as a source in node B. The flow created must then be exported through the arc BE and node EXP, possibly generating revenue. Since state $N1$ is bounded, the flow is limited. This demonstrates the use of converters as sources whose performance depends on their internal states.
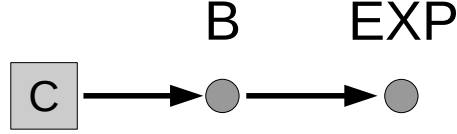
63

Figure 3.4: Network G2 with 2 nodes (EXP and B), 1 directed arc (BE) and 1 converter (C)

3.2.3 Sink-to-source converter

Consider now networks $G1$ and $G2$ from Sections 3.2.1 and 3.2.2 together with a newly-defined converter C, as shown in Figure 3.5. The converter C is defined as having one bounded state ($N1 \in N_C^{fix}$) and two binary inputs ($M1, M2 \in M_C^{bin}$). One of the inputs ($M1$) interacts with node A and the other ($M2$) interacts with node B. The coefficients are such that using the former requires imports from node IMP ($a_{G1,A,M1,C,q,k}^{node,u} = -1, \forall q \in Q, \forall k \in K_q$), whereas using the latter leads to exports through node EXP ($a_{G2,B,M2,C,q,k}^{node,u} = 1, \forall q \in Q, \forall k \in K_q$). Note that since the inputs are binary, there is no risk of getting an unbounded solution should the export prices exceed the import ones. The following constraints describe this behaviour:

- State equations

$$x_{C,N1,q,k} = 0.99 \times x_{C,N1,q,k-1} + 0.05 \times u_{C,M1,q,k} - 0.05 \times u_{C,M2,q,k},$$
$$\forall q \in Q, \forall k \in K_q$$

- State bounds

$$0.2 \leq x_{C,N1,q,k} \leq 1.0, \forall q \in Q, \forall k \in K_q$$

- Node A:

$$0 = v_{G1,IMP,A,IA,q,k} - u_{C,M1,q,k}, \forall q \in Q, \forall k \in K_q$$

- Node B:

$$0 = -v_{G2,B,EXP,BE,q,k} + u_{C,M2,q,k}, \forall q \in Q, \forall k \in K_q$$

In this example, two otherwise independent networks interact with one another through a converter. It draws flow from one network ($G1$) to create it in the other ($G2$), potentially taking advantage of price differences to improve the operational result. On a different note, the example has a low dimensionality but there are no limits to the number of states, inputs and outputs, nor to the number of nodes with which the converter interacts on any given network.
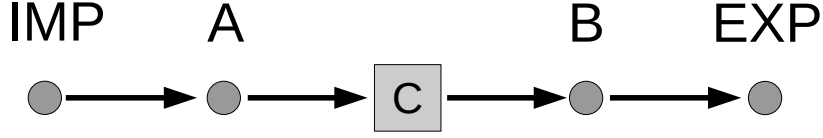
Figure 3.5: Two networks (G1 and G2) connected through a converter (C)

Figure 3.6: A network ($G1$) with one import node (IMP), one regular node (A) and one directed arc between them (IA)

3.3 Problems

3.3.1 Single network, single arc problem

Consider a problem relying on one assessment ($Q = \{0\}$) for two reporting periods ($P = P_0 = \{1, 2\}$). Since there is only one assessment, it has probability 1 ($c_0^{wgt} = 1$) and all periods rely on the same number of intervals ($K_0 = \{1, 2, 3\}$). The problem itself concerns a network $G1$ with one import node IMP, one regular node A and one optional directed arc between them ($IA \in J_{G1,IMP,A}^{dir} \cap J_{G1,IMP,A}^{opt}$), as shown in Figure 3.6. The arc has one option ($H_{G1,IMP,A,IA} = \{4\}$) with an amplitude-dependent cost ($c_{G1,IMP,A,IA}^{arc,amp} = 1.0$, $c_{G1,IMP,A,IA,4}^{arc,min} = 2.0$) and specifications ($v_{G1,IMP,A,IA,4}^{amp,max} = 3.0$; $\forall k \in K_0$: $f_{G1,IMP,A,IA,0,k}^{amp,v} = 1.0$, $\eta_{G1,IMP,A,IA,0,k} = 0.5$) that allow the flows needs for node A to be met ($v_{G1,A,0,1}^{base} = 0.5$, $v_{G1,A,0,2}^{base} = 0.0$, $v_{G1,A,0,3}^{base} = 1.0$). Meeting them requires imports and the prices for node IMP follow a single-segment no-volume-limit tariff ($\forall q \in Q, \forall p \in P_q, \forall k \in K_q$: $S_{G1,IMP,q,p,k} = \{5\}$, $S_{G1,IMP,q,p,k}^{fin} = \emptyset$), in which prices are positive and constant ($p_{G1,IMP,q,p,k,s} = 1.0, \forall q \in Q, \forall p \in P_q, \forall k \in K_q, \forall s \in S_{G1,IMP,q,p,k}$). Similarly, the weights for each time interval and period are invariant ($\forall q \in Q, \forall p \in P_q, \forall k \in K_q$: $c_{q,p,k}^{time} = 1$). In turn, the discount factors are based on a discount rate of 3.5%

$(\forall q \in Q: c_{q,1}^{df} = 0.966, c_{q,2}^{df} = 0.934)$. This problem can be formulated as follows:

$$
\begin{aligned}
\max \quad & (SDNCF_0 - CAPEX) \\
\text{s.t.} \quad & CAPEX \geq CAPEX_{G1,IMP,A,IA}^{arc} \\
& CAPEX_{G1,IMP,A,IA}^{arc} \geq v_{G1,IMP,A,IA}^{amp} + 2.0 \times \delta_{G1,IMP,A,IA,4}^{arc,inv} \\
& SDNCF_0 = -0.966 \left(IFC_{G1,IMP,0,1,1} + IFC_{G1,IMP,0,1,2} + \right. \\
& \qquad\qquad\qquad\qquad \left. IFC_{G1,IMP,0,1,3} \right) \\
& \qquad\quad - 0.934 \left(IFC_{G1,IMP,0,2,1} + IFC_{G1,IMP,0,2,2} + \right. \\
& \qquad\qquad\qquad\qquad \left. IFC_{G1,IMP,0,2,3} \right) \\
& 0.5 = 0.5 \times v_{G1,IMP,A,IA,0,1} \\
& 0.0 = 0.5 \times v_{G1,IMP,A,IA,0,2} \\
& 1.0 = 0.5 \times v_{G1,IMP,A,IA,0,3} \\
& IFC_{G1,IMP,0,p,k} = IF_{G1,IMP,0,p,k,5}, \forall p \in P_0, \forall k \in K_0 \\
& IF_{G1,IMP,0,p,k,5} = v_{G1,IMP,A,IA,0,k}, \forall p \in P_0, \forall k \in K_0 \\
& v_{G1,IMP,A,IA}^{amp} \leq 3.0 \times \delta_{G1,IMP,A,IA,4}^{arc,inv} \\
& v_{G1,IMP,A,IA,0,k} \leq v_{G1,IMP,A,IA}^{amp}, \forall k \in K_0 \\
& v_{G1,IMP,A,IA}^{amp} \geq 0 \\
& CAPEX \geq 0 \\
& CAPEX_{G1,IMP,A,IA}^{arc} \geq 0 \\
& IFC_{G1,IMP,0,p,k} \geq 0, \forall p \in P_0, \forall k \in K_0 \\
& IF_{G1,IMP,0,p,k,5} \geq 0, \forall p \in P_0, \forall k \in K_0 \\
& v_{G1,IMP,A,IA,0,k} \geq 0, \forall k \in K_0 \\
& \delta_{G1,IMP,A,IA,4}^{arc,inv} \in \{0,1\}
\end{aligned}
$$

The solution requires that the arc IA be installed and that its amplitude be at least two $(v_{G1,IMP,A,IA}^{amp} \geq 2.0)$. Since there is no benefit of having an amplitude higher than two, capital expenditures equal four $(CAPEX = 4)$ and the flows through IA have to be 1.0, 0.0 and 2.0 during the intervals 1, 2 and 3, respectively. The objective function value is then -9.7 for the optimal solution.

3.3.2 Two-scenario problem

Consider another problem in which the details of the previous one are used for the first out of two assessments $(Q = \{0,6\})$ associated with different probabilities $(c_0^{wgt} = 0.7, c_6^{wgt} = 0.3)$. The second one covers three periods $(P_6 = \{1,2,3\})$ and considers only two time intervals $(K_6 = \{1,2\})$. Despite this, the arc cost function and performance are the same $(\forall q \in Q, \forall k \in K_q: f_{G1,IMP,A,IA,q,k}^{amp,v} = 1.0, \eta_{G1,IMP,A,IA,q,k} = 0.5)$, as is the tariff for node IMP $(\forall q \in Q, \forall k \in K_q: S_{G1,IMP,q,p,k} = \{5\}, S_{G1,IMP,q,p,k}^{fin} = \emptyset, p_{G1,IMP,q,p,k,5} =$

1.0). Conversely, the flow needs in node A differ in the second assessment $(v^{base}_{G1,A,6,1} = 1.25, v^{base}_{G1,A,6,2} = 0.3)$ though the time weights are also invariant $(\forall k \in K_6, \forall p \in P_6: c^{time}_{6,p,k} = 1.0)$. The problem can be represented as:

$$\max \left(0.7 \times SDNCF_0 + 0.3 \times SDNCF_6 - CAPEX\right)$$

$$s.t. \ CAPEX \geq CAPEX^{arc}_{G1,IMP,A,IA}$$

$$CAPEX^{arc}_{G1,IMP,A,IA} \geq v^{amp}_{G1,IMP,A,IA} + 2.0 \times \delta^{arc,inv}_{G1,IMP,A,IA,4}$$

$$SDNCF_0 = -0.966 \left(IFC_{G1,IMP,0,1,1} + IFC_{G1,IMP,0,1,2} + \right.$$
$$\left. IFC_{G1,IMP,0,1,3}\right)$$
$$- 0.934 \left(IFC_{G1,IMP,0,2,1} + IFC_{G1,IMP,0,2,2} + \right.$$
$$\left. IFC_{G1,IMP,0,2,3}\right)$$

$$SDNCF_6 = -0.966 \left(IFC_{G1,IMP,6,1,1} + IFC_{G1,IMP,6,1,2}\right)$$
$$- 0.934 \left(IFC_{G1,IMP,6,2,1} + IFC_{G1,IMP,6,2,2}\right)$$
$$- 0.902 \left(IFC_{G1,IMP,6,3,1} + IFC_{G1,IMP,6,3,2}\right)$$

$$0.5 = 0.5 \times v_{G1,IMP,A,IA,0,1}$$
$$0.0 = 0.5 \times v_{G1,IMP,A,IA,0,2}$$
$$1.0 = 0.5 \times v_{G1,IMP,A,IA,0,3}$$
$$1.25 = 0.5 \times v_{G1,IMP,A,IA,6,1}$$
$$0.30 = 0.5 \times v_{G1,IMP,A,IA,6,2}$$
$$IFC_{G1,IMP,q,p,k} = IF_{G1,IMP,q,p,k,5}, \forall q \in Q, \forall p \in P_q, \forall k \in K_q$$
$$IF_{G1,IMP,q,p,k,5} = v_{G1,IMP,A,IA,q,p,k}, \forall q \in Q, \forall p \in P_q, \forall k \in K_q$$
$$v^{amp}_{G1,IMP,A,IA} \leq 3.0 \times \delta^{arc,inv}_{G1,IMP,A,IA,4}$$
$$v_{G1,IMP,A,IA,q,k} \leq v^{amp}_{G1,IMP,A,IA}, \forall q \in Q, \forall k \in K_q$$
$$v^{amp}_{G1,IMP,A,IA} \geq 0$$
$$CAPEX \geq 0$$
$$CAPEX^{arc}_{G1,IMP,A,IA} \geq 0$$
$$IFC_{G1,IMP,q,p,k} \geq 0, \forall q \in Q, \forall p \in P_q, \forall k \in K_q$$
$$IF_{G1,IMP,q,p,k,5} \geq 0, \forall q \in Q, \forall p \in P_q, \forall k \in K_q$$
$$v_{g,IMP,A,IA,q,k} \geq 0, \forall q \in Q, \forall k \in K_q$$
$$\delta^{arc,inv}_{G1,IMP,A,IA,4} \in \{0,1\}$$

The solution to the aforementioned problem also requires the arc IA, though in this case with an amplitude of at least 2.5 to accommodate the second scenario $(v^{amp}_{G1,IMP,A,IA} \geq 2.5)$. Since amplitudes are penalised, and no advantage can be obtained from selecting a higher amplitude, 2.5 is the optimal amplitude $(CAPEX = 4.5)$. The flows through arc IA are the same for assessment 0 and the following for assessment 6: 2.5 and 0.6 for intervals 1 and 2, respectively. Consequently, the objective function value is -11.096 for the optimal solution.

Chapter 4

Concluding remarks

The present document began to be written as a reference for a model that soon became too long and fastidious for integral publication under the standard scientific article format. It has since gained more descriptive passages, examples and an introduction, in a bid to make it easier to understand.

The mathematical model described in these pages has been designed to address socio-economic investment problems concerning energy system infrastructure. Its aim is to determine the most advantageous set of investments out of those under consideration to create or complement an energy system. The investments allow for new operational realities within the energy system and their combined effect determines the socio-economic performance. The energy system itself is defined using temporal and spatial dimensions and formulated by drawing on graph theory to represent network flows and generic difference equations for other processes. The general idea was to arrive at a scalable, versatile and technologically-agnostic model suitable for a wide range of problems.

The model has been validated through a software implementation. The validation has included minimalist and realistic problems. Some of the former have been described in Section 3 – the entire set was deemed inadequate for inclusion here. As for the latter, these will be discussed in a separate publication focusing on validation and computational performance.

Computational performance can easily become an issue with this model, if it is applied without a clear understanding of its limitations. The curse of dimensionality and combinatorial explosion are not new phenomena in optimisation studies and there is no reason not to expect them to affect this model. The implication is that one should not expect it to perform well with numerous scenarios, large networks, described with a high temporal resolution and considering an exhaustive number of technological alternatives everywhere. In practice, compromises will likely have to be made and it is to the users to determine where to make them for each problem – a corollary of the separation between model and data. The model is being made available despite this possibility, in the hope that others may find it useful or discover ways to complement or correct it.

Chapter 5

Nomenclature

5.1 Sets

5.1.1 General

Q set of operational performance assessments relied upon

P set of reporting periods within the planning horizon

P_q set of reporting periods relying on assessment q, $\forall q \in Q$

K_q set of time intervals used in the assessment q, $\forall q \in Q$

5.1.2 Networks

G set of networks under consideration

L_g set of node locations for network g, $\forall g \in G$

L_g^{imp} set of import nodes for network g, $\forall g \in G$

L_g^{exp} set of export nodes for network g, $\forall g \in G$

$L_g^{max,in}$ set of nodes on network g incompatible with having more than one incoming arc unless there are outgoing arcs too, $\forall g \in G$

$L_g^{max,out}$ set of nodes on network g incompatible with having more than one outgoing arc unless there are incoming arcs too, $\forall g \in G$

$GLL^{arc,max}$ set of network and node pair tuples whose members have to observe a maximum number of arcs in the specified direction

GLL — set of network and node pair tuples

$S_{g,l,q,p,k}$ — set of segments for node l on network g during period p and interval k of assessment q, $\forall g \in G, \forall l \in L_g^{imp} \cup L_g^{exp}, \forall q \in Q, \forall p \in P_q, \forall k \in K_q$

$S_{g,l,q,p,k}^{fin}$ — subset of $S_{g,l,q,p,k}$ for segments having finite maximum volumes, $\forall g \in G, \forall l \in L_g^{imp} \cup L_g^{exp}, \forall q \in Q, \forall p \in P_q, \forall k \in K_q$

J_{g,l,l^*}^{dir} — set of directed arcs for flows from node l to node l^* on network g, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$

J_{g,l,l^*}^{und} — set of undirected arcs between nodes l and l^* on network g which are identified via the flow sense defined from node l to node l^*, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$

J_{g,l,l^*}^{pre} — set of pre-existing arcs allowing flow between nodes l and l^* on network g, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$

J_{g,l,l^*}^{inf} — set of pre-existing directed arcs allowing unrestrained flows from node l to node l^* on network g, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$

J_{g,l,l^*}^{new} — set of new arcs between nodes l and l^* on network g, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$

J_{g,l,l^*}^{sgl} — set of individually-selected arcs between nodes l and l^* on network g, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$

J_{g,l,l^*}^{col} — set of group-selected arcs between nodes l and l^* on network g, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$

J_{g,l,l^*}^{stt} — set of arcs between nodes l and l^* on network g modelled as having static losses, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$

$J_{g,l,l^*}^{stt,dep}$ — set of arcs between nodes l and l^* on network g whose static losses appear in l, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$

$J_{g,l,l^*}^{stt,arr}$ — set of arcs between nodes l and l^* on network g whose static losses appear in l^*, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$

$J_{g,l,l^*}^{stt,us}$ — set of arcs between nodes l and l^* on network g whose static losses appear upstream, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$

$J_{g,l,l^*}^{stt,ds}$ — set of arcs between nodes l and l^* on network g whose static losses appear downstream, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$

$J_{g,l,l^*}^{sns,sos}$ set of undirected arcs between nodes l and l^* on network g relying on special ordered sets of type one for flow sense determination, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$

$J_{g,l,l^*}^{sns,bin}$ set of undirected arcs between nodes l and l^* on network g using binary variables to decide the flow senses during each interval, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$

$J_{g,l,l^*}^{sns,nnr}$ set of undirected arcs between nodes l and l^* on network g using non-negative real variables to select the respective flow senses, $\forall g \in G, \forall l \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp}\right), \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp} \cup L_g^{exp}\right)$

J_{g,l,l^*}^{mdt} set of new mandatory arcs between nodes l and l^* on network g, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$

J_{g,l,l^*}^{opt} set of new optional arcs between nodes l and l^* on network g, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$

J_{g,l,l^*}^{int} set of arcs between nodes l and l^* on network g whose selection is to be interfaced, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$

$J_{g,l,l^*}^{arc,bin}$ set of new arcs between nodes l and l^* on network g whose options are to be selected using binary variables, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$

$J_{g,l,l^*}^{arc,nnr}$ set of new arcs between nodes l and l^* on network g whose options are to be selected using non-negative real variables, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$

$J_{g,l,l^*}^{arc,sos}$ set of new arcs between nodes l and l^* on network g whose options are to be selected using special ordered sets of type one (SOS1), $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$

$H_{g,l,l^*,j}$ set of options for new arc j between nodes l and l^* on network g, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right), \forall j \in J_{g,l,l^*}^{sgl}$

T set of arc groups that are to be selected together

T^{mdt} set of arc groups for mandatory arcs

T^{opt} set of arc groups for optional arcs

T^{int} set of arc groups whose selection is to be interfaced

$T^{arc,sos}$ set of arc groups whose selection relies on SOS1

$T^{arc,bin}$ set of arc groups whose selection uses binary variables

$T^{arc,nnr}$ set of arc groups whose selection uses non-negative real variables

$GLLJ_t^{col}$ set of network, node pair and arc tuples in group t, $\forall t \in T$

H_t set of arc options for group t, $\forall t \in T$

5.1.3 Converters

I	set of all flow converters
I^{pre}	set of pre-existing flow converters
I^{new}	set of new flow converters
M_i	set of inputs for converter i, $\forall i \in I$
M_i^{nnr}	set of non-negative real inputs for converter i, $\forall i \in I$
M_i^{bin}	set of binary inputs for converter i, $\forall i \in I$
M_i^{dim}	set of dimensionable inputs for converter i, $\forall i \in I$
M_i^{fix}	set of bounded inputs for converter i, $\forall i \in I$
R_i	set of outputs for converter i, $\forall i \in I$
$R_i^{dim,pos}$	set of outputs for converter i whose nominal positive amplitudes are dimensionable, $\forall i \in I$
$R_i^{dim,neg}$	set of outputs for converter i whose nominal negative amplitudes are dimensionable, $\forall i \in I$
$R_i^{dim,eq}$	set of outputs for converter i whose dimensionable nominal positive and negative amplitudes have to match, $\forall i \in I$
R_i^{fix}	set of bounded outputs for converter i, $\forall i \in I$
N_i	set of states for converter i, $\forall i \in I$
N_i^{fix}	set of bounded states for converter i, $\forall i \in I$
$N_i^{dim,pos}$	set of states for converter i whose nominal positive amplitudes are dimensionable, $\forall i \in I$
$N_i^{dim,neg}$	set of states for converter i whose nominal negative amplitudes are dimensionable, $\forall i \in I$
$N_i^{dim,eq}$	set of states for converter i whose dimensionable nominal positive and negative amplitudes have to match, $\forall i \in I$

$N_i^{pos,var}$ set of states for converter i capable of inducing penalties for state increases over the span of a reporting period, $\forall i \in I$

$N_i^{neg,var}$ set of states for converter i capable of inducing penalties for state decreases over the span of a reporting period, $\forall i \in I$

$N_i^{ref,hgh}$ set of states for converter i capable of inducing penalties due to violations of upper state references, $\forall i \in I$

$N_i^{ref,low}$ set of states for converter i capable of inducing penalties due to violations of lower state references, $\forall i \in I$

5.2 Variables

5.2.1 Objective Function

$CAPEX$ Capital expenditures prior to the planning horizon begins

$SDNCF_q$ Sum of the discounted net cash flows for the reporting periods covered by assessment q, $\forall q \in Q$

$EFR_{g,l,q,p,k}$ Revenue for flow exported through node l on network g during interval k within assessment q and period p, $\forall g \in G, \forall l \in L_g^{exp}$, $\forall q \in Q$, $\forall p \in P_q$, $\forall k \in K_q$

$IFC_{g,l,q,p,k}$ Cost for flows imported through node l on network g during interval k within assessment q and period p, $\forall g \in G, \forall l \in L_g^{imp}$, $\forall q \in Q$, $\forall p \in P_q$, $\forall k \in K_q$

$EF_{g,l,q,p,k,s}$ Flow exported through node l on network g during interval k of assessment q and period p and according to segment s, $\forall g \in G, \forall l \in L_g^{exp}$, $\forall q \in Q, \forall p \in P_q, \forall k \in K_q, \forall s \in S_{g,l,q,p,k}$

$IF_{g,l,q,p,k,s}$ Flow imported through node l on network g during interval k of assessment q and period p and according to segment s, $\forall g \in G, \forall l \in L_g^{imp}$, $\forall q \in Q, \forall p \in P_q, \forall k \in K_q, \forall s \in S_{g,l,q,p,k}$

$CAPEX_i^{cvt}$ Capital expenditures due to converter i, $\forall i \in I^{new}$

$CAPEX_{g,l,l^*,j}^{arc,sgl}$ Capital costs due to arc j between nodes l and l^* on network g, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$, $\forall j \in J_{g,l,l^*}^{sgl}$

$CAPEX_t^{arc,col}$ Capital costs due to arc group t, $\forall t \in T$

5.2.2 Networks

$\delta_{g,l,l^*,j,h}^{arc,inv}$ investment decision for arc j with option h between nodes l and l^* on network g, $\forall g \in G$, $\forall l \in L_g \setminus L_g^{exp}$, $\forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$, $\forall j \in J_{g,l,l^*}^{sgl}, \forall h \in H_{g,l,l^*,j}$

$v_{g,l,l^*,j,q,k}$ flow from node l to l^* through arc j on network g during interval k of assessment $q, \forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus (\{l\} \cup L_g^{imp})$, $\forall j \in J_{g,l,l^*}^{dir} \cup J_{g,l,l^*}^{und} \cup J_{g,l^*,l}^{und}, \forall q \in Q, \forall k \in K_q$

$v_{g,l,l^*,j}^{amp}$ nominal flow amplitude for arc j between nodes l and l^* on network g, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right), \forall j \in J_{g,l,l^*}^{sgl}$

$w_{g,l,l^*,j,q,k}$ static losses for new arc j between nodes l and l^* on network g during interval k of assessment q, $\forall q \in Q, \forall k \in K_q, \forall g \in G$, $\forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right), \forall j \in J_{g,l,l^*}^{stt} \cap J_{g,l,l^*}^{new}$

$w_{g,l,l^*,j,q,k}^{sns}$ static losses for undirected arc j when the flow is from node l to l^* on network g during interval k of assessment q, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}$, $\forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right), \forall j \in \left(J_{g,l,l^*}^{stt} \cap J_{g,l,l^*}^{und}\right) \cup \left(J_{g,l^*,l}^{stt} \cap J_{g,l^*,l}^{und}\right)$, $\forall q \in Q, \forall k \in K_q$

$\xi_{g,l,l^*,j}^{arc,inv}$ interface variable for new arc j between l and l^* on network g, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right), \forall j \in J_{g,l,l^*}^{int}$

$\zeta_{g,l,l^*,j,q,k}^{sns}$ flow sense indicator for flows from node l to node l^* on network g via arc j during interval k of assessment q, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}$, $\forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right), \forall j \in J_{g,l,l^*}^{und} \cup J_{g,l^*,l}^{und}, \forall q \in Q, \forall k \in K_q$

$\delta_{t,h}^{arc,inv}$ investment decision for arc group t with option h, $\forall t \in T, \forall h \in H_t$

$\xi_t^{arc,inv}$ interface variable for arc group t, $\forall t \in T^{int}$

v_t^{amp} nominal flow amplitude for arc group t, $\forall t \in T$

5.2.3 Converters

$\delta_i^{cvt,inv}$ investment decision for converter i, $\forall i \in I^{new}$

$u_{i,m,q,k}$ input m of converter i during time interval k of assessment q, $\forall i \in I, \forall m \in M_i, \forall q \in Q, \forall k \in K_q$

$u_{i,m}^{amp}$ nominal amplitude for input m of converter i, $\forall i \in I^{new}, \forall m \in M_i^{dim}$

$y_{i,r,q,k}$ output r of converter i during interval k of assessment q, $\forall i \in I$, $\forall r \in R_i, \forall q \in Q, \forall k \in K_q$

$y_{i,r}^{amp,pos}$ nominal (positive) amplitude for output r of converter i, $\forall i \in I^{new}$, $\forall r \in R_i^{dim,pos}$

$y_{i,r}^{amp,neg}$ nominal negative amplitude for output r of converter i, if not made to match the positive one, $\forall i \in I^{new}$, $\forall r \in R_i^{dim,neg} \setminus R_i^{dim,eq}$

$x_{i,n,q,k}$ state n of converter i during interval k of assessment q, $\forall i \in I$, $\forall n \in N_i, \forall q \in Q, \forall k \in K_q$

$x_{i,n}^{amp,pos}$ nominal (positive) amplitude for state n of converter i, $\forall i \in I$, $\forall n \in N_i^{dim,pos}$

$x_{i,n}^{amp,neg}$ nominal negative amplitude for state n of converter i, if not made to match the positive one, $\forall i \in I$, $\forall n \in N_i^{dim,neg} \setminus N_i^{dim,eq}$

$\Delta x_{i,n,q,k}^{ref,hgh}$ upper reference violation for state n of converter i during interval k of assessment q, $\forall i \in I$, $\forall n \in N_i^{ref,hgh}, \forall q \in Q, \forall k \in K_q$

$\Delta x_{i,n,q,k}^{ref,low}$ lower reference violation for state n of converter i during interval k of assessment q, $\forall i \in I$, $\forall n \in N_i^{ref,low}, \forall q \in Q, \forall k \in K_q$

$\Delta x_{i,n,q}^{pos,var}$ positive variation of state n on converter i within assessment q, $\forall i \in I$, $\forall n \in N_i^{pos,var}, \forall q \in Q$

$\Delta x_{i,n,q}^{neg,var}$ negative variation of state n on converter i within assessment q, $\forall i \in I$, $\forall n \in N_i^{neg,var}, \forall q \in Q$

5.3 Parameters

5.3.1 Objective function

c_q^{wgt} — weight of assessment q, $\forall q \in Q$

$c_{q,p}^{df}$ — discount factor for period p and assessment q, $\forall q \in Q, \forall p \in P_q$

$c_{q,p,k}^{time}$ — relative weight of interval k relative to period p within assessment q, $\forall q \in Q, \forall p \in P_q, \forall k \in K_q$

$c_{g,l,l^*,j,h}^{arc,min}$ — minimum cost of new arc j with option h for flows between nodes l and l^* on network $g, \forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp} \right)$, $\forall j \in J_{g,l,l^*}^{new}, \forall h \in H_{g,l,l^*,j}$

$c_{g,l,l^*,j}^{arc,var}$ — unit flow amplitude cost for arc j between nodes l and l^* on network $g, \forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp} \right), \forall j \in J_{g,l,l^*}^{new}$

$c_{i,m,q,p,k}^{opex,u}$ — unit price of using input m on converter i during interval k and period p of assessment q, $\forall i \in I, \forall m \in M_i, \forall q \in Q, \forall k \in K_q, \forall p \in P_q$

$c_{i,r,q,p,k}^{opex,y}$ — unit price associated with output r on converter i during interval k and period p of assessment q, $\forall i \in I, \forall r \in R_i, \forall q \in Q, \forall k \in K_q, \forall p \in P_q$

$c_{i,n,q,p}^{pos,var}$ — unit cost of the positive variation of state n on converter i during period p of assessment q, $\forall i \in I, \forall n \in N_i^{pos,var}, \forall q \in Q, \forall p \in P_q$

$c_{i,n,q,p}^{neg,var}$ — unit cost of the negative variation of state n on converter i during period p of assessment q, $\forall i \in I, \forall n \in N_i^{neg,var}, \forall q \in Q, \forall p \in P_q$

$c_{i,n,q,p,k}^{ref,hgh}$ — unit cost of upper state reference violations of state n on converter i during interval k and period p of assessment q, $\forall i \in I, \forall n \in N_i^{ref,hgh}$, $\forall q \in Q, \forall k \in K_q, \forall p \in P_q$

$c_{i,n,q,p,k}^{ref,low}$ — unit cost of lower state reference violations of state n on converter i during interval k and period p of assessment q, $\forall i \in I, \forall n \in N_i^{ref,low}$, $\forall q \in Q, \forall k \in K_q, \forall p \in P_q$

$c_i^{cvt,min}$ — minimum cost of installing flow converter i, $\forall i \in I^{new}$

$p_{g,l,q,p,k,s}$ price of resource through node l on network g during interval k within assessment q and period p, and according to segment s, $\forall g \in G, \forall l \in L_g^{imp} \cup L_g^{exp}, \forall q \in Q, \forall p \in P_q, \forall k \in K_q, \forall s \in S_{g,l,q,p,k}$

$v_{g,l,q,p,k,s}^{max}$ maximum volume permitted through node l on network g during interval k of assessment q and period p according to segment s, $\forall g \in G, \forall l \in L_g^{imp} \cup L_g^{exp}, \forall q \in Q, \forall p \in P_q, \forall k \in K_q, \forall s \in S_{g,l,q,p,k}^{fin}$

$c_{i,m}^{cvt,u}$ unit cost of the nominal amplitude for input m on flow converter i, $\forall i \in I^{new}, \forall m \in M_i^{dim}$

$c_{i,n}^{cvt,x,pos}$ unit cost of the nominal (positive) amplitude for state n on flow converter i, $\forall i \in I^{new}, \forall n \in N_i^{dim,pos}$

$c_{i,n}^{cvt,x,neg}$ unit cost of the nominal negative amplitude for state n on flow converter i, $\forall i \in I^{new}, \forall n \in N_i^{dim,neg} \setminus N_i^{dim,eq}$

$c_{i,r}^{cvt,y,pos}$ unit cost of the nominal (positive) amplitude for output r on flow converter i, $\forall i \in I^{new}, \forall r \in R_i^{dim,pos}$

$c_{i,r}^{cvt,y,neg}$ unit cost of the nominal negative amplitude for output r on flow converter i, $\forall i \in I^{new}, \forall r \in R_i^{dim,neg} \setminus R_i^{dim,eq}$

$c_{t,h}^{arc,min}$ minimum cost of arc group t with option h, $\forall t \in T, \forall h \in H_t$

$c_t^{arc,var}$ unit flow amplitude cost for arc group t, $\forall t \in T$

5.3.2 Networks

$v_{g,l,q,k}^{base}$ base flow component in node l on network g during interval k and assessment q, $\forall g \in G, \forall l \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp}\right), \forall q \in Q, \forall k \in K_q$

$a_{g,l,l^*}^{max,arc}$ maximum permitted number of arcs allowing flow from node l to node l^* on network g, $\forall(g,l,l^*) \in GLL^{arc,max}$

$a_{g,l,i,m,q,k}^{node,u}$ gain for input m within converter i on node l in network g during interval k and assessment q, $\forall g \in G, \forall l \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp}\right)$, $\forall i \in I, \forall m \in M_i, \forall q \in Q, \forall k \in K_q$

$a_{g,l,i,r,q,k}^{node,y}$ gain for output r within converter i on node l in network g during interval k and assessment q, $\forall g \in G, \forall l \in L_g \setminus \left(L_g^{exp} \cup L_g^{imp}\right)$, $\forall i \in I, \forall m \in M_i, \forall q \in Q, \forall k \in K_q$

$a_{g,l}^{max,in}$ big M value for the disjunctive constraint limiting the number of incoming arcs for node l on network g, $\forall g \in G, \forall l \in L_g^{max,in}$

$a_{g,l}^{max,out}$ big M value for the disjunctive constraint limiting the number of outgoing arcs for node l on network g, $\forall g \in G, \forall l \in L_g^{max,out}$

5.3.3 Arcs

$\eta_{g,l,l^*,j,q,k}$ — efficiency of arc j from node l to node l^* on network g during interval k within assessment q, $\forall q \in Q, \forall g \in G, \forall l \in L_g \setminus L_g^{exp}$, $\forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right), \forall j \in J_{g,l,l^*}^{dir} \cup J_{g,l,l^*}^{und} \cup J_{g,l^*,l}^{und}, \forall k \in K_q$

$v_{g,l,l^*,j}^{amp}$ — nominal amplitude for pre-existing arc j between nodes l and l^* on network g, $\forall g \in G, \ \forall l \in L_g \setminus L_g^{exp}, \ \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$, $\forall j \in J_{g,l,l^*}^{pre} \setminus J_{g,l,l^*}^{inf}$

$v_{g,l,l^*,j,h}^{amp,max}$ — maximum nominal amplitude for new arc j with option h that is between nodes l and l^* on network g, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}$, $\forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right), \forall j \in J_{g,l,l^*}^{sgl}, \forall h \in H_{g,l,l^*,j}$

$\overline{v_{g,l,l^*,j,q,k}}$ — maximum amplitude for flows through pre-existing arc j between nodes l and l^* on network g during interval k and assessment q, $\forall q \in Q, \forall k \in K_q, \forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$, $\forall j \in \left(J_{g,l,l^*}^{und} \cup J_{g,l^*,l}^{und}\right) \cap \left(J_{g,l,l^*}^{pre} \cup J_{g,l^*,l}^{pre}\right) \cup \left(J_{g,l,l^*}^{dir} \cap J_{g,l,l^*}^{pre} \setminus J_{g,l,l^*}^{inf}\right)$

$f_{g,l,l^*,j,q,k}^{amp,v}$ — amplitude adjustment coefficient for arc j between nodes l and l^* on network g during interval k of assessment q, $\forall q \in Q, \forall k \in K_q$, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right), \forall j \in J_{g,l,l^*}^{dir} \cup J_{g,l,l^*}^{und}$

$w_{g,l,l^*,j,q,k}$ — Static losses for pre-existing arc j between nodes l and l^* on network g during interval k of assessment q, $\forall q \in Q, \forall k \in K_q$, $\forall g \in G, \forall l \in L_g \setminus L_g^{exp}, \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right), \forall j \in J_{g,l,l^*}^{stt} \cap J_{g,l,l^*}^{pre}$

$w_{g,l,l^*,j,h,q,k}^{new}$ — Static losses for new arc j with option h between nodes l and l^* on network g during interval k within assessment q, $\forall q \in Q$, $\forall k \in K_q, \ \forall g \in G, \ \forall l \in L_g \setminus L_g^{exp}, \ \forall l^* \in L_g \setminus \left(\{l\} \cup L_g^{imp}\right)$, $\forall j \in J_{g,l,l^*}^{stt} \cap J_{g,l,l^*}^{new}, \forall h \in H_{g,l,l^*,j}$

$v_{t,h}^{amp,max}$ — maximum nominal amplitude of arc group t with option h, $\forall t \in T, h \in H_t$

5.3.4 Converters

$\overline{u_{i,m,q,k}}$ — upper bound for input m linked to converter i for interval k and assessment q, $\forall i \in I, \forall m \in M_i^{fix}, \forall q \in Q, \forall k \in K_q$

$u_{i,m}^{amp,max}$ — maximum nominal amplitude for input m on converter i, $\forall i \in I^{new}$, $\forall m \in M_i^{dim}$

$f_{i,m,q,k}^{amp,u}$ — amplitude correction coefficient of input m and converter i during interval k and assessment q, $\forall i \in I^{new}, \forall m \in M_i^{dim}, \forall q \in Q, \forall k \in K_q$

$a_{i,n,n^*,q,k}^{eq,x}$ — coefficient for the effect of state n^* on state n of converter i during interval k of assessment q, $\forall i \in I, \forall n \in N_i, \forall n^* \in N_i, \forall q \in Q, \forall k \in K_q$

$b_{i,n,m,q,k}^{eq,x}$ — coefficient for the effect of input m on state n of converter i during interval k of assessment q, $\forall i \in I, \forall n \in N_i, \forall m \in M_i, \forall q \in Q, \forall k \in K_q$

$e_{i,n,q,k}^{eq,x}$ — constant in the equation for state n of converter i during interval k of assessment q, $\forall i \in I, \forall n \in N_i, \forall q \in Q, \forall k \in K_q$

$\overline{x_{i,n,q,k}}$ — upper bound for state n on converter i during time interval k within assessment q, $\forall i \in I, \forall n \in N_i, \forall q \in Q, \forall k \in K_q$

$\underline{x_{i,n,q,k}}$ — lower bound for state n on converter i during time interval k within assessment q, $\forall i \in I, \forall n \in N_i, \forall q \in Q, \forall k \in K_q$

$\overline{x_{i,n}^{amp,pos}}$ — maximum nominal (positive) amplitude for state n on converter i, $\forall i \in I, \forall n \in N_i^{dim,pos}, \forall q \in Q, \forall k \in K_q$

$\overline{x_{i,n}^{amp,neg}}$ — maximum nominal negative amplitude for state n on converter i, $\forall i \in I, \forall n \in N_i^{dim,neg} \setminus N_i^{dim,eq}, \forall q \in Q, \forall k \in K_q$

$f_{i,n,q,k}^{amp,x}$ — amplitude correction coefficient for state n on converter i during interval k and assessment q, $\forall i \in I, \forall n \in N_i, \forall q \in Q, \forall k \in K_q$

$x_{i,n,q,0}$ — initial condition for state n on converter i for assessment q, $\forall i \in I$, $\forall n \in N_i, \forall q \in Q$

$x_{i,n,q,k}^{ref,hgh}$ — upper reference for state n on converter i during interval k within assessment q, $\forall i \in I, \forall n \in N_i^{ref,hgh}, \forall q \in Q, \forall k \in K_q$

$x_{i,n,q,k}^{ref,low}$ — lower reference for state n on converter i during interval k within assessment q, $\forall i \in I, \forall n \in N_i^{ref,low}, \forall q \in Q, \forall k \in K_q$

$c^{eq,y}_{i,r,n,q,k}$ coefficient for the effect of state n on output r of converter i during interval k of assessment q, $\forall i \in I$, $\forall r \in R_i, \forall n \in N_i, \forall q \in Q, \forall k \in K_q$

$d^{eq,y}_{i,r,m,q,k}$ coefficient for the effect of input m on output r of converter i during interval k of assessment q, $\forall i \in I$, $\forall r \in R_i, \forall m \in M_i, \forall q \in Q, \forall k \in K_q$

$e^{eq,y}_{i,r,q,k}$ constant in the equation for output n of converter i during interval k of assessment q, $\forall i \in I$, $\forall r \in R_i, \forall q \in Q, \forall k \in K_q$

$\overline{y_{i,r,q,k}}$ upper bound for output r on converter i during time interval k of assessment q, $\forall i \in I$, $\forall r \in R_i, \forall q \in Q, \forall k \in K_q$

$\underline{y_{i,r,q,k}}$ lower bound for output r on converter i during time interval k of assessment q, $\forall i \in I$, $\forall r \in R_i, \forall q \in Q, \forall k \in K_q$

$\overline{y^{amp,pos}_{i,r}}$ maximum nominal (positive) amplitude for output r on converter i, $\forall i \in I$, $\forall r \in R^{dim,pos}_i, \forall q \in Q, \forall k \in K_q$

$\overline{y^{amp,neg}_{i,r}}$ maximum nominal negative amplitude for output r on converter i, $\forall i \in I$, $\forall r \in R^{dim,neg}_i \setminus R^{dim,eq}_i, \forall q \in Q, \forall k \in K_q$

$f^{amp,y}_{i,r,q,k}$ amplitude correction coefficient for output r on converter i during interval k and assessment q, $\forall i \in I$, $\forall r \in R_i, \forall q \in Q, \forall k \in K_q$

Acknowledgements

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