



## Estimating Waves Through Measured Ship Responses

Nielsen, U. D.; Brodtkorb, A.H. ; Iseki, T.; Jensen, J. J.; Mittendorf, M.; Mounet, R.E.G. ; Sørensen, A. J. ; Takami, T.

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# Estimating Waves Through Measured Ship Responses

**Ulrik Dam Nielsen**

Section for Fluid Mechanics, Coastal and Maritime Engineering,  
DTU Construct (Dept. of Civil and Mechanical Engineering)  
**Technical University of Denmark**

# Many thanks to all my co-authors

**A.H. Brodtkorb<sup>2</sup>, T. Iseki<sup>3</sup>, J.J. Jensen<sup>1</sup>, M. Mittendorf<sup>1</sup>,  
R.E.G. Mounet<sup>1,2</sup>, A.J. Sørensen<sup>2</sup>, T. Takami<sup>4</sup>**

<sup>1</sup>DTU Construct, Technical University of Denmark, Kgs. Lyngby, Denmark

<sup>2</sup>NTNU AMOS, Norwegian University of Science and Technology, Trondheim, Norway

<sup>3</sup>Tokyo University of Marine Science and Technology, Tokyo, Japan

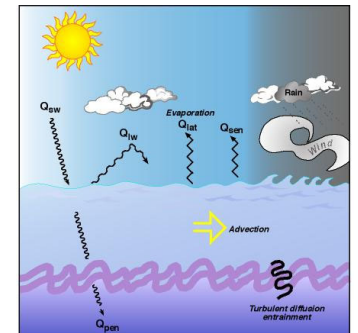
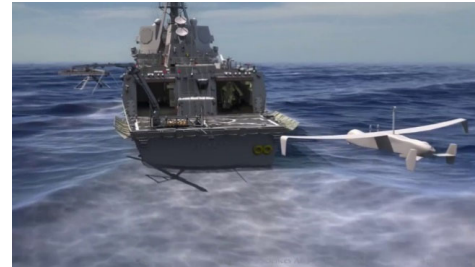
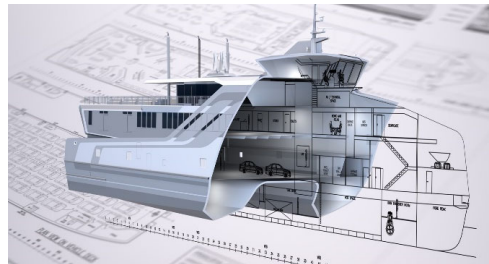
<sup>4</sup>National Maritime Research Institute, Tokyo, Japan

## This talk presents...

- A brief outline of **past an ongoing activities** related to the use of ships as sailing wave buoys
  - Emphasis on a **hybrid approach** (“ML-informed physics-based approach”)
- Just a few glimpses of the associated mathematical theory
- Many slides... so I will try to be swift; but just **reach out** and we can continue the discussions later

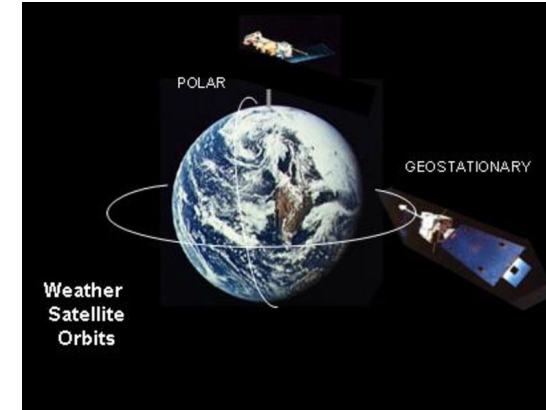
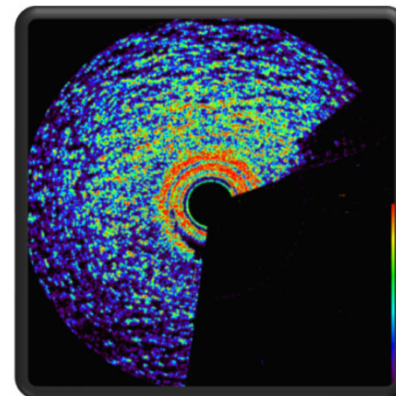
# Introduction: Why do we want to *measure* waves?

- **Because waves are the fundamental driver of most of the *processes*\* we are concerned about** (as maritime engineers, naval architects, or ocean scientists)
  - \* Analysing wave-structure interactions before/during/after operations
  - \* Collecting historical data for design and rule specifications
  - \* Understanding mechanisms of surface-water mixing and air-sea fluxes



# Introduction: How can we *estimate* waves?

- Well-known means for sea state **estimation**
  - In-situ buoys
  - Remote sensing (aircraft or satellite)
  - Wave radar (X-band marine radar)
  - ...
- All with their own pros and cons
- The specific use-case (previous slide) may set different requirements to availability and updating-frequency



# Analogy to a wave buoy



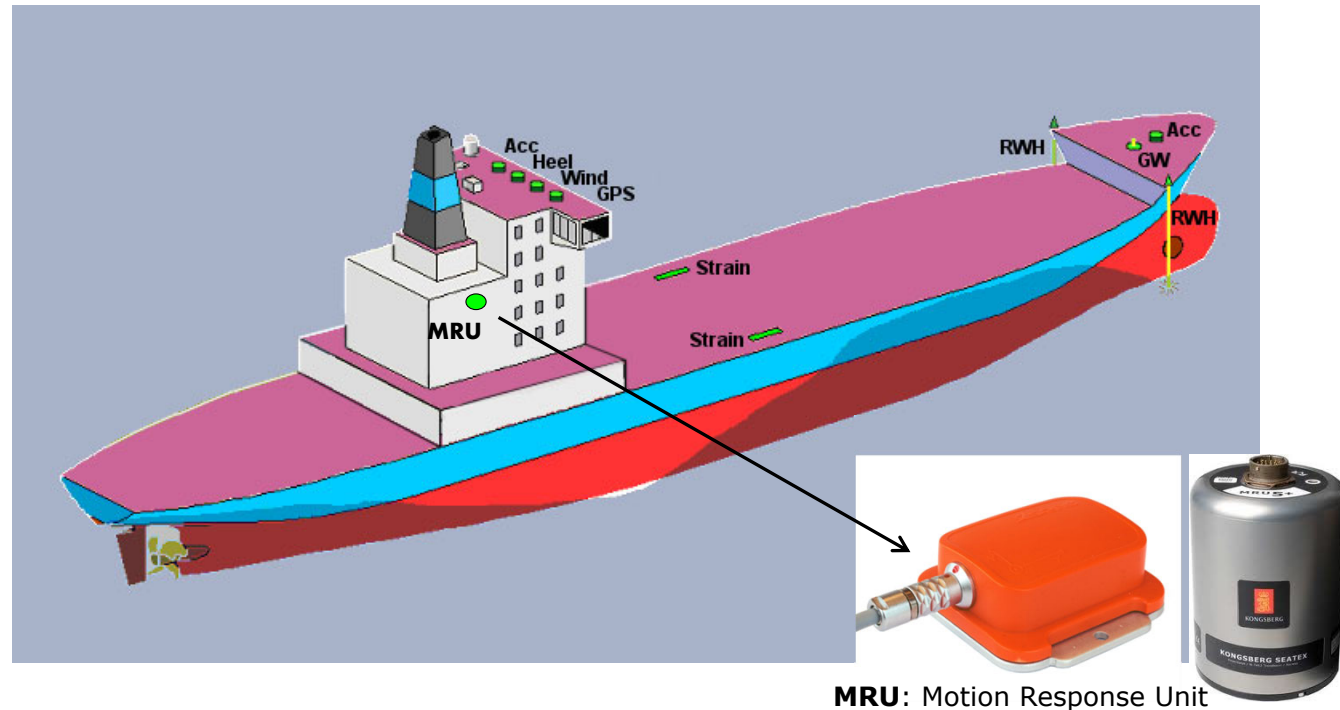
- Measurements of the buoy's motions are processed to give the wave spectrum in real-time and at the buoy's exact position.
- Can we do the same with a ship...?? Yes!

## Inherent complexities:

- Geometry
- Size (low-pass filter)
- Forward speed

# The wave buoy analogy (= *WBA*)

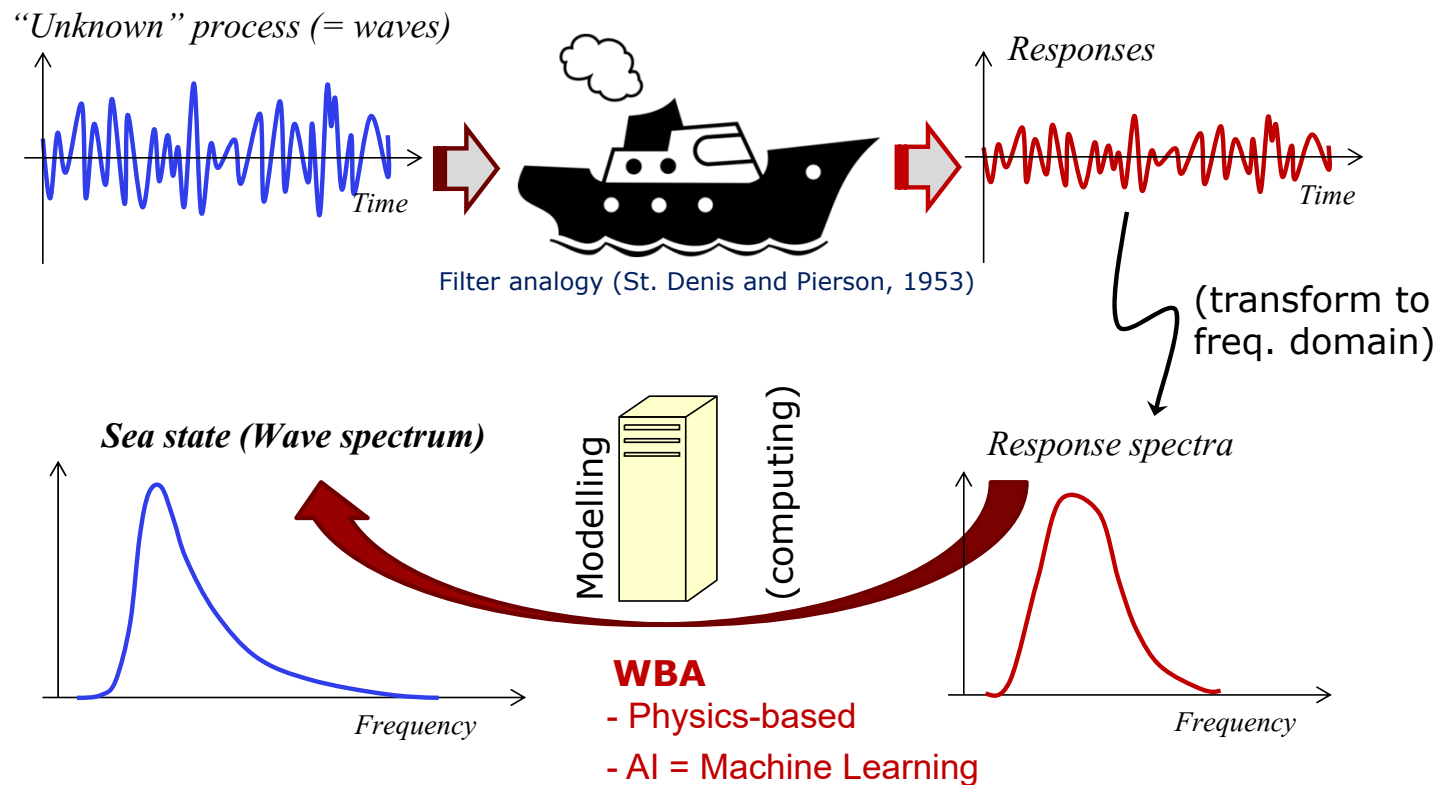
- A ship is like a wave buoy; it responds to the waves... (and has plenty of sensors installed)



**MRU:** Motion Response Unit



# Estimating waves through measured ship responses in a nutshell



## Physics-based framework

The governing equation system is based on a comparison of response spectra...

**Measure-  
ments:**



**Response  
spectra**

- Motions  
(e.g. heave, roll, pitch)
- Accelerations
- VBM

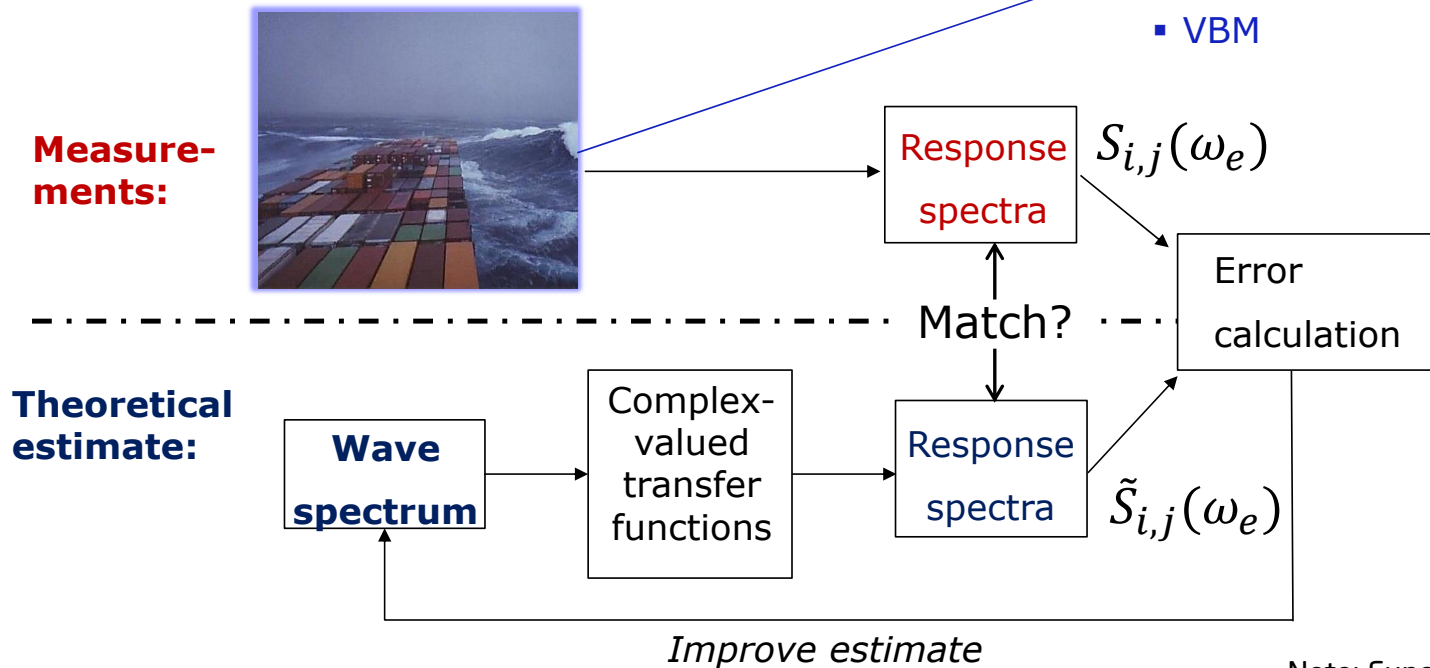
$$i, j = \{z, \phi, \theta\}$$

$$S_{i,j}(\omega_e)$$

# Physics-based framework

The governing equation system is based on a comparison of response spectra...

- Motions  
(e.g. heave, roll, pitch)
- Accelerations
- VBM



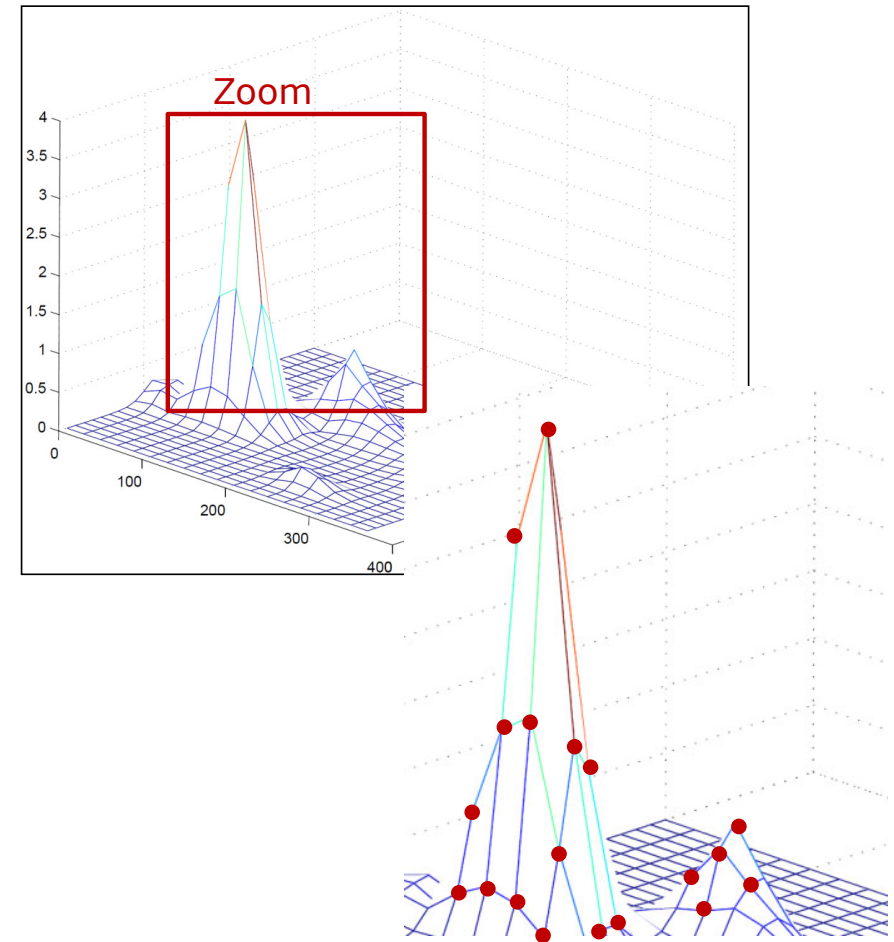
$$\tilde{S}_{i,j}(\omega_{e,l}) = \int_0^{2\pi} \left\langle \Phi_i(\omega, \beta) \overline{\Phi_j(\omega, \beta)} E(\omega, \mu) \frac{d\omega}{d\omega_e} \right\rangle_{\omega_{e,l}} d\mu$$

Note: Fundamentally, expressions originate from the governing physics (Newton's 2<sup>nd</sup> law) → **physics-based** approach.

# Solving the problem

$$\min \sum_{i,j} \sum_{l=1}^L \left| S_{i,j}(\omega_{e,l}) - \int_0^{2\pi} \left\langle \Phi_i(\omega, \beta) \overline{\Phi_j(\omega, \beta)} E(\omega, \mu) \frac{d\omega}{d\omega_e} \right\rangle_{\omega_{e,l}} d\mu \right|^2$$

- ❑ **Underdetermined equation system solved by a Bayesian (non-parametric) method**  
solving directly for all of the unknown (discrete) spectral ordinates of the spectrum  $E(\omega, \mu)$ , requiring regularization.
- ❑ **Regularization, i.e. smoothing** (minimization of the second order difference) of the spectrum in the two dimensions
- ❑ Solution obtained as the **compromise between “goodness of fit” and smoothness**
- ❑ A glimpse of the theory...



# Solving the problem

$$\min \sum_{i,j} \sum_{l=1}^L \left| S_{i,j}(\omega_{e,l}) - \int_0^{2\pi} \left\langle \Phi_i(\omega, \beta) \overline{\Phi_j(\omega, \beta)} E(\omega, \mu) \frac{d\omega}{d\omega_e} \right\rangle_{\omega_{e,l}} d\mu \right|^2$$

- ❑ **Bayesian (non-parametric) method** solving directly for all  $K \cdot M$  unknowns, requiring regularization.
- ❑ **Parametric method** based on optimisation of *wave parameters* ( $H_s, T_z, \beta, \dots$ ) in a summation of parameterised wave spectrum formulations (e.g. JONSWAP) and a directional spreading function.
- ❑ **Spectral-residual calculation** solving directly for  $M$  unknowns while 'optimising' for the mean heading  $\beta$  used as input for a directional spreading function. (essentially a combination of the other two)

**Note: In the general case, forward speed is different from zero →  
Consideration of the Doppler shift!**

# Bayesian method (1)

- Governing equation (matrix notation) established by minimization

$$\chi^2(\mathbf{x}) \equiv \|\mathbf{A}\mathbf{f}(\mathbf{x}) - \mathbf{b}\|^2$$

introducing a non-negativity constraint  $\mathbf{f}(\mathbf{x}) = \exp(\mathbf{x})$ .

- Regularization (smoothing) based on minimization of the second order derivative (difference)

$$\varepsilon_{1mn}^2 = \sum_{n=1}^N \sum_{m=1}^M (x_{m-1,n} - 2x_{m,n} + x_{m+1,n})^2; \quad (x_{0,n} = x_{M,n}, \quad x_{M+1,n} = x_{1,n})$$

$$\varepsilon_{2mn}^2 = \sum_{m=1}^M \sum_{n=2}^{N-1} (x_{m,n-1} - 2x_{m,n} + x_{m,n+1})^2$$

$$\varepsilon_{1mn}^2 = \mathbf{x}^T \mathbf{H}_1 \mathbf{x}$$

$$\varepsilon_{2mn}^2 = \mathbf{x}^T \mathbf{H}_2 \mathbf{x}$$

## Bayesian method (2)

- Principally, the solution is obtained from the minimization of

$$S(\mathbf{x}) = \|\mathbf{A}\mathbf{f}(\mathbf{x}) - \mathbf{b}\|^2 + \mathbf{x}^T(u^2\mathbf{H}_1 + v^2\mathbf{H}_2)\mathbf{x}$$

which is equivalent to maximizing the (posterior):

$$p(\mathbf{x}|u, v, \sigma^2) = c \left( \frac{1}{2\pi\sigma^2} \right)^{\frac{P+KM}{2}} |\det(u^2\mathbf{H}_1 + v^2\mathbf{H}_2)|^{1/2} \exp \left( -\frac{1}{2\sigma^2} S(\mathbf{x}) \right)$$

- The solution is controlled by the hyperparameters  $u$  and  $v$ ; and the *optimum* solution is obtained for minimum of 'A Bayesian Information Criterion':

$$\text{ABIC} = -2 \ln \int p(\mathbf{x}|u, v, \sigma^2) d\mathbf{x}$$

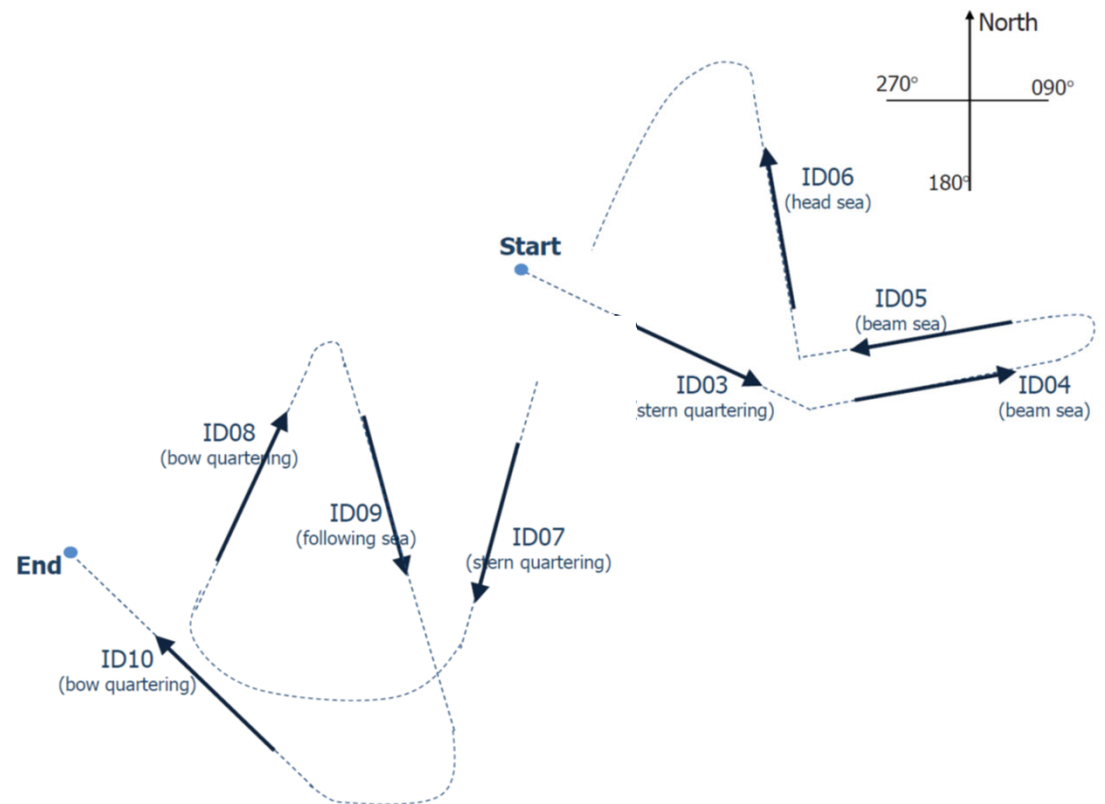
# Results (... by the “original”, *un*conditioned method)

## Physics-based framework (Part 1)

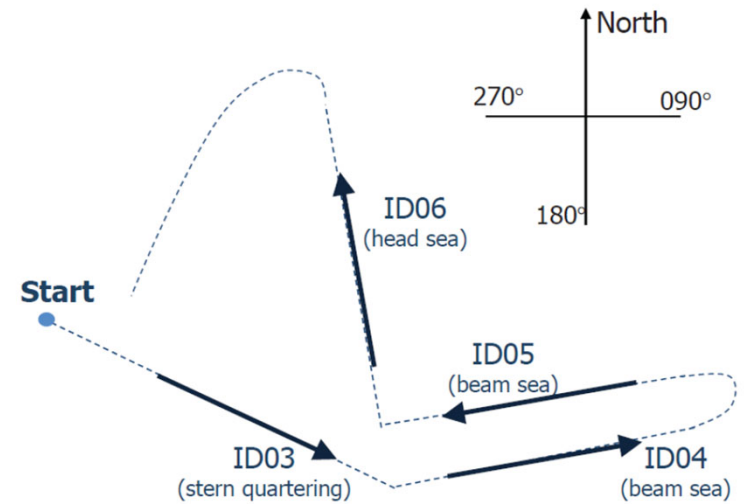
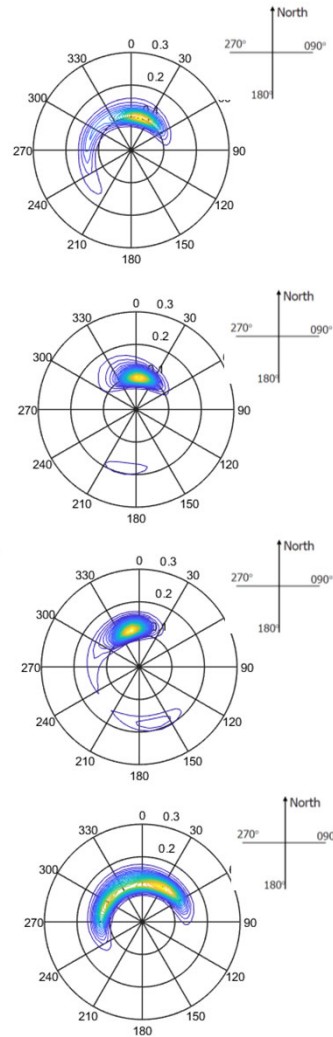
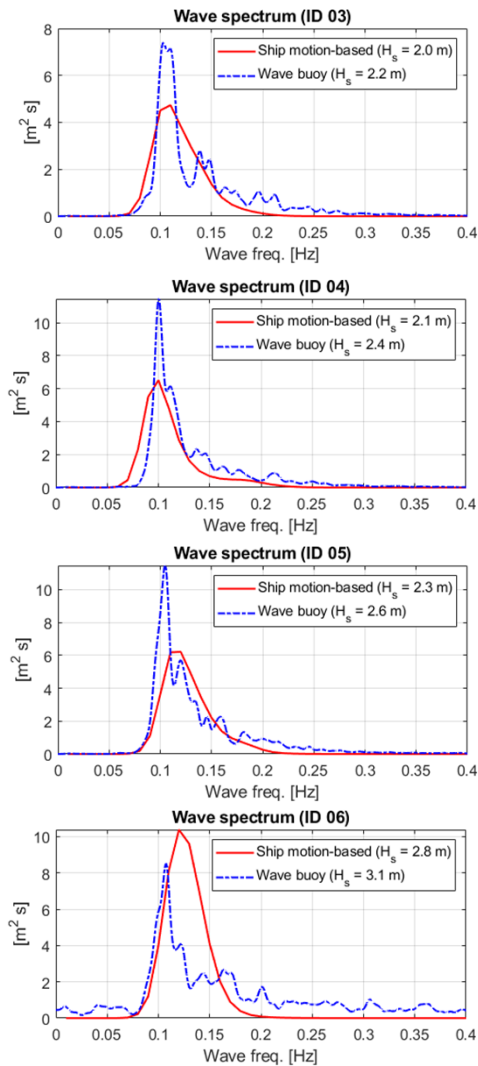
- **Combination of measured data *and* transfer functions**
- Comparison between estimates by a wave buoy and corresponding results using ship motions



Nielsen et al. (2018)







- Similar agreements for the other four cases (ID07 – ID10)... and for the parametric approach even better agreements are obtained (cf. appendix)
- **Main take-away:** Good estimates can be expected under “controlled” conditions where uncertainties in operational parameters are small → RAOs are reliable(!)

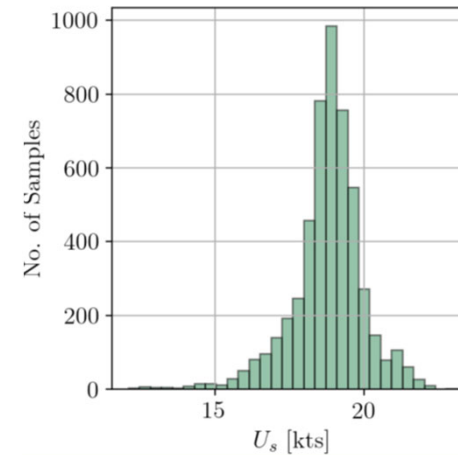
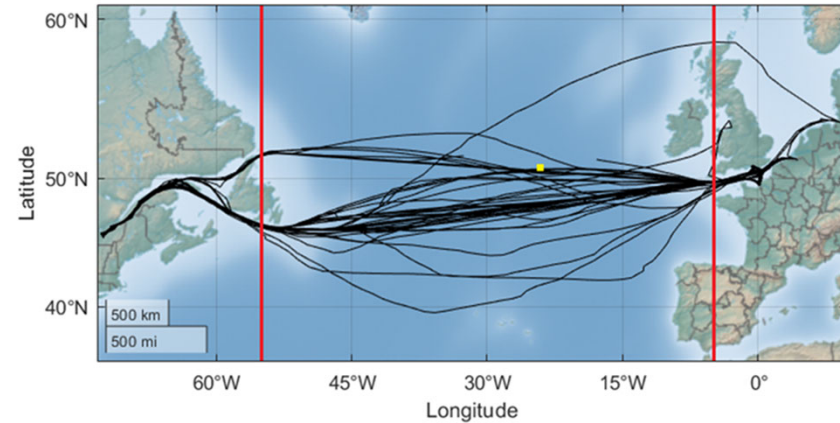
# Results (... by the "original", *un*conditioned method)

## Physics-based framework (Part 2)

- **Combination of measured data *and* transfer functions**
- Comparison with wave radar (Wavex)
- Nearly two years of operational data
- No info about loading condition ( $T_{\text{transit}} = 9.5 \text{ m}$ )

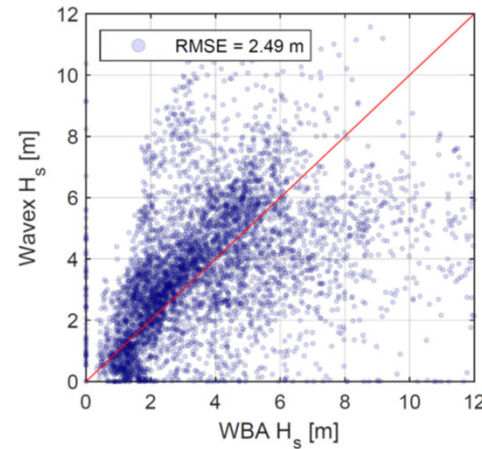


Nielsen et al. (2023)

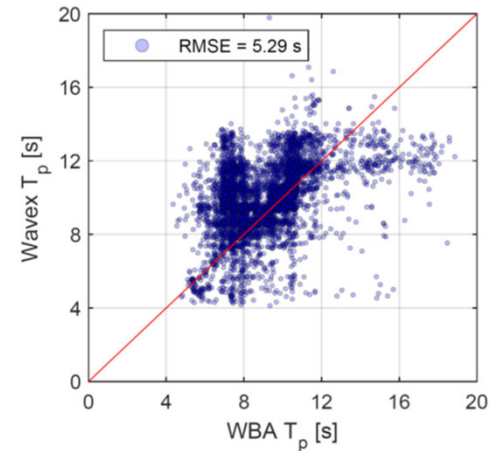


## Results (Part 2)

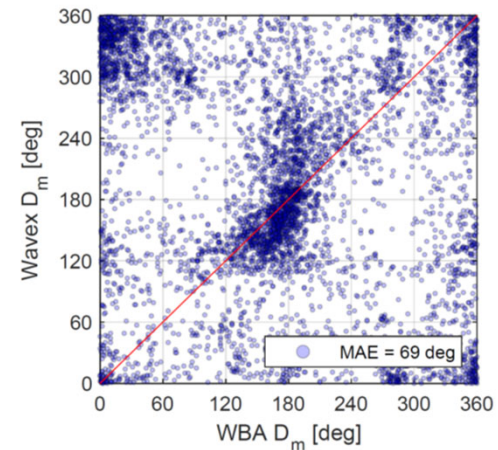
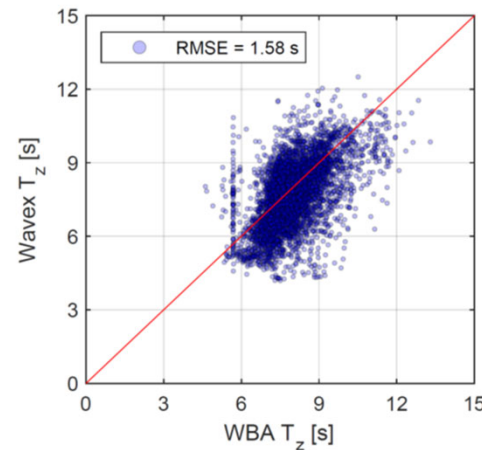
- Physics-based (= 'WBA') vs. Wavex results...
- NB. *All* data is considered.
- Can we improve the estimates by the WBA by combining results from ML and physics-based?
- Using a *hybrid framework*...
- First, however, what is the motivation for such an approach?



(a) Significant wave height.



(b) Peak period.

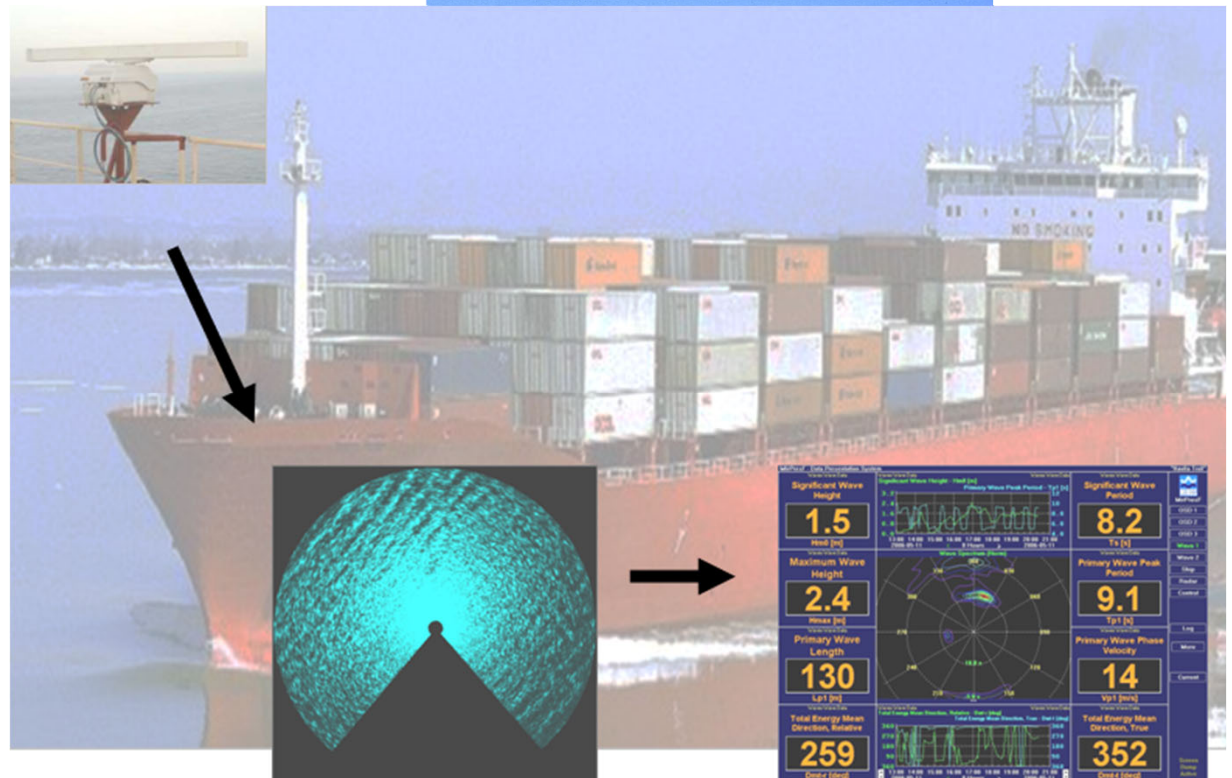


# Machine learning-based framework

## Machine Learning (ML)

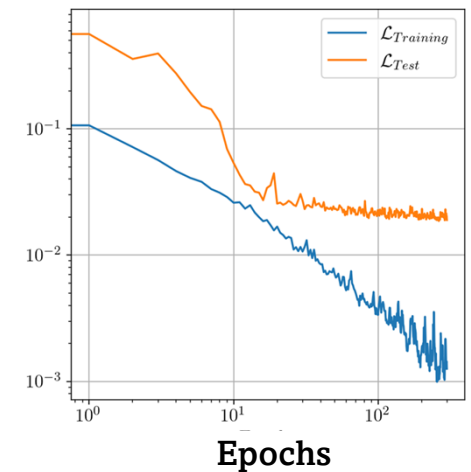
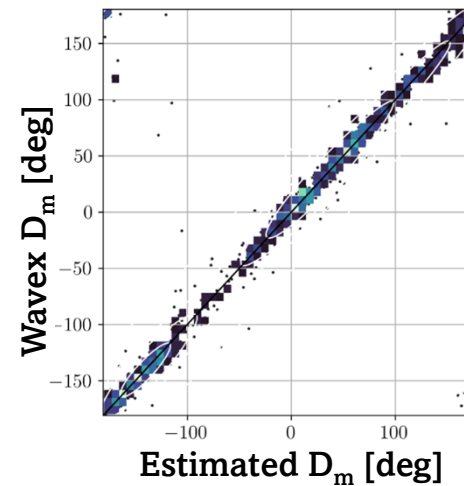
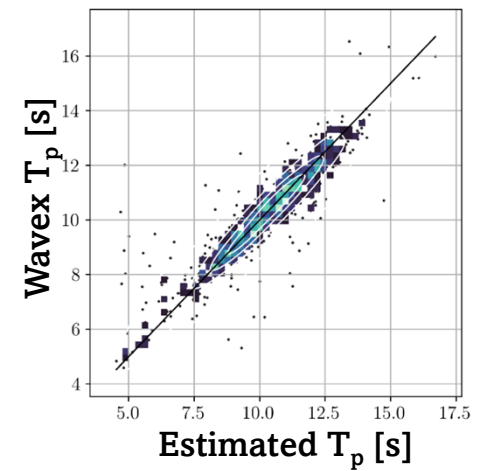
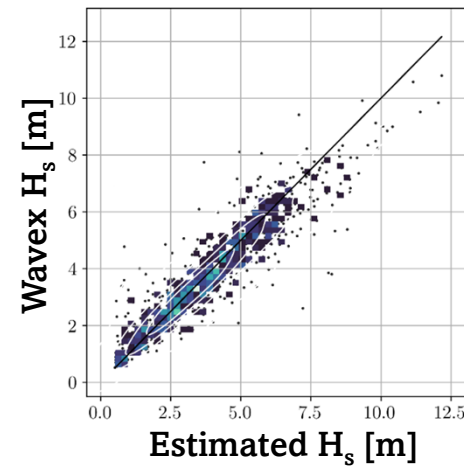
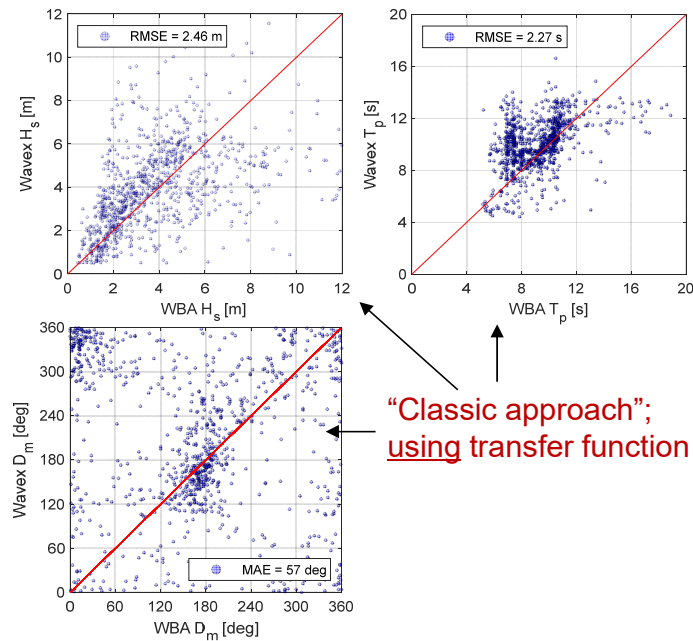
- Training of **convolutional neural networks**; based on the following
  - Two years of ship telemetry data:
    - **MRU** located close to COG
    - Accelerations
    - Strains
    - Speed
  - **Wave data obtained by wave radar** (WaveX from Miros)
  - Targets:  $H_s$ ,  $T_p$ ,  $D_m$  (or  $\beta$ )

Mittendorf et al. (2022)



# Results using ML

- Assessment based on *test* data (20 %); from Mittendorf et al. (2022)
- No use of transfer functions





# Wave spectrum estimation **conditioned** on output from machine learning

- **Motivation:**

- 1) Physics-based gives the detailed 2D wave spectrum (ML does *not*)
- 2) ML provides reliable estimates of wave *parameters* (Physics-based does not *always*; wave direction appears to be the most difficult)

- **Methodology:**

- **Constrain the wave spectrum estimate**
- Formulation of additional equations based on output from the Machine Learning model (Mittendorf et al., 2022); concatenated into the governing equation system:

$$\chi^2(\mathbf{x}) \equiv \|\mathbf{A}\mathbf{f}(\mathbf{x}) - \mathbf{b}\|^2$$

$$\begin{aligned} \int \int \tilde{E}(\omega, \mu) d\omega d\mu &= \frac{1}{16} \hat{H}_s^2 \\ \int \int \tilde{E}(\omega, \mu) \sin \mu d\omega d\mu &= \hat{d} \\ \int \int \tilde{E}(\omega, \mu) \cos \mu d\omega d\mu &= \hat{c} \end{aligned}$$

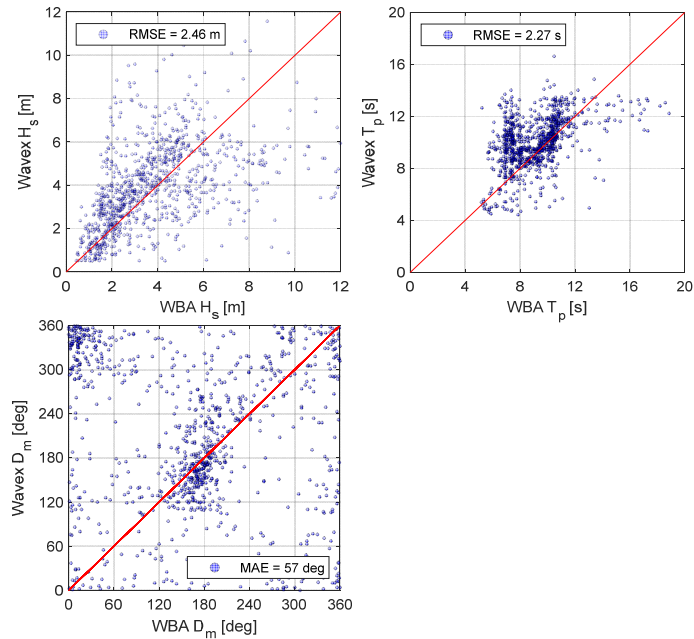
$$D_m = \arctan(d/c)$$

Nielsen et al. (2023)

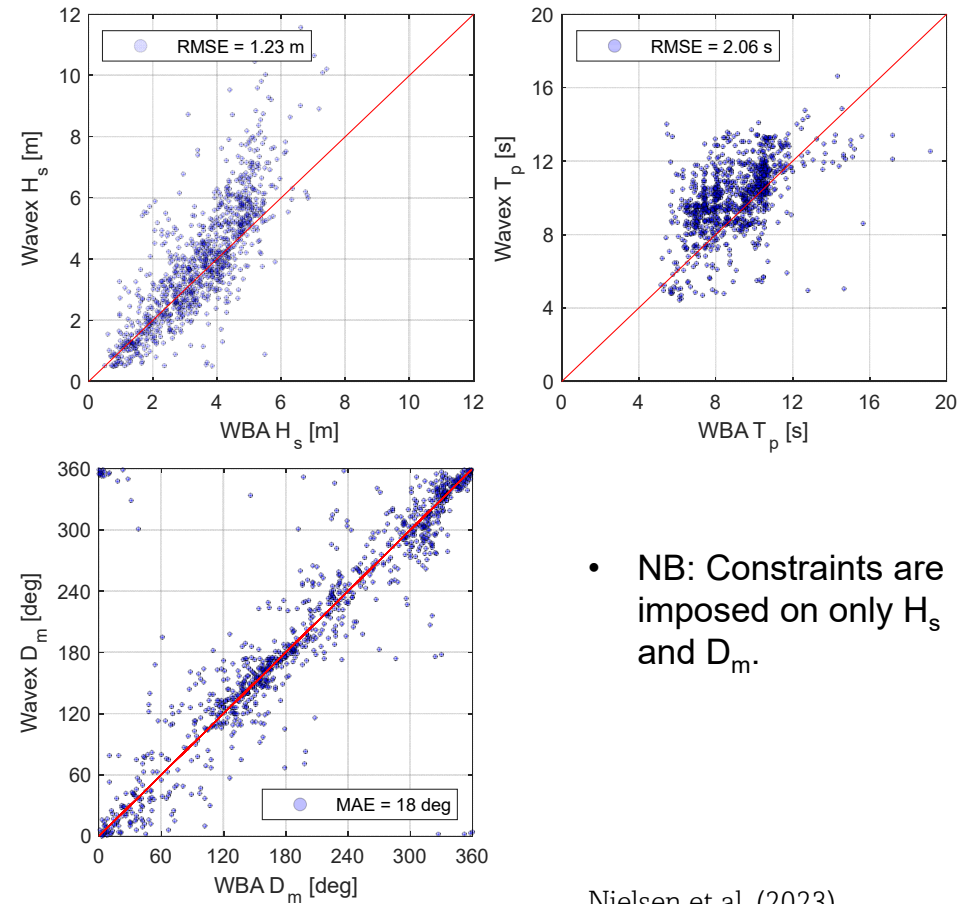
# Wave spectrum estim. conditioned on ML

## • Results

### Unconditioned estimates



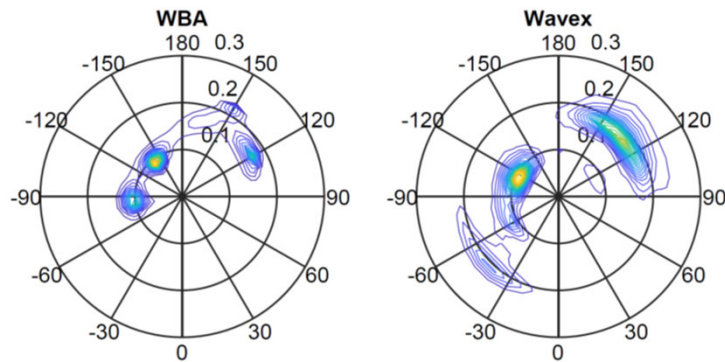
### Conditioned estimates



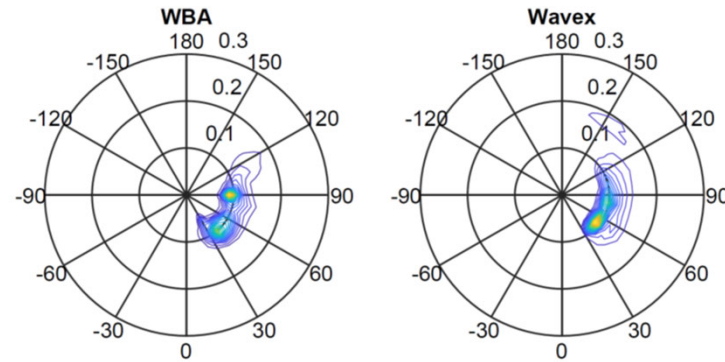
- NB: Constraints are imposed on only  $H_s$  and  $D_m$ .

Nielsen et al. (2023)

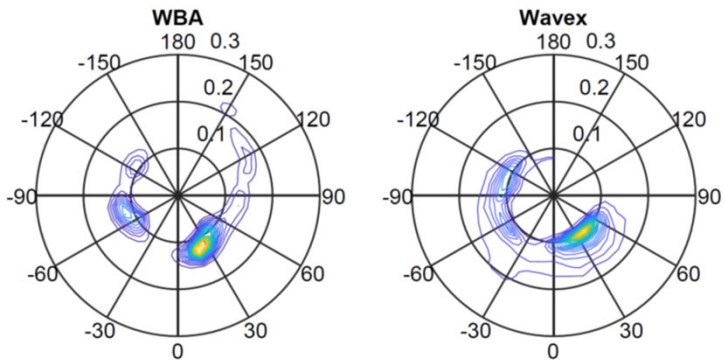
# Directional wave spectra (arbitrary outcomes)



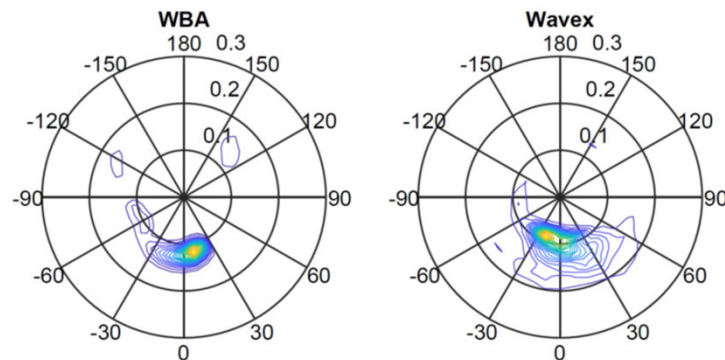
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(b) UTC: 15-09-2008 02:30.



(c) UTC: 30-12-2008 04:30.

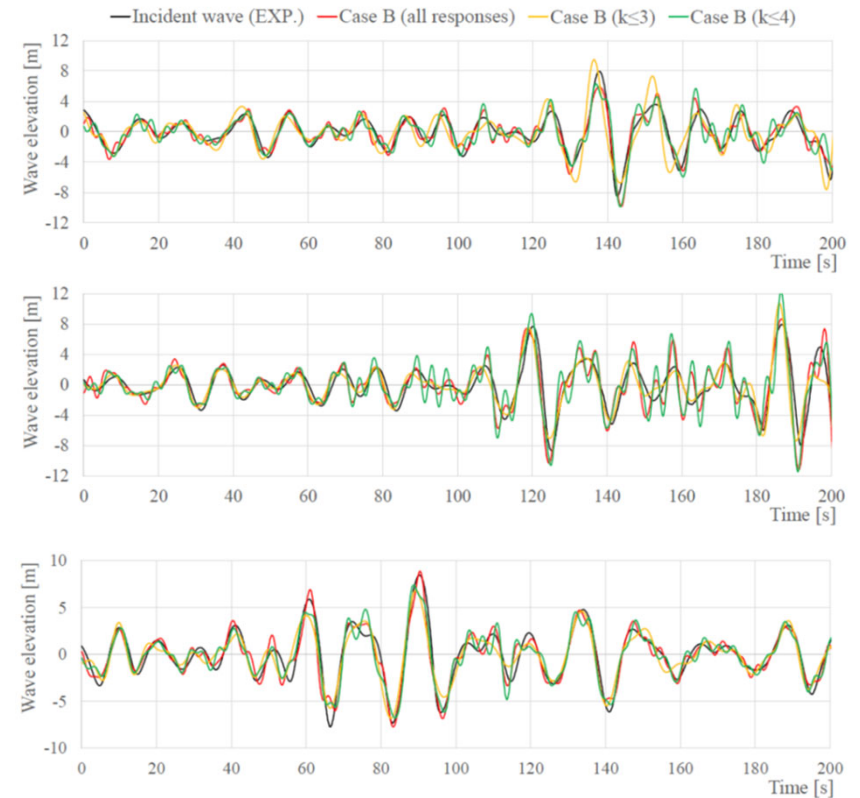
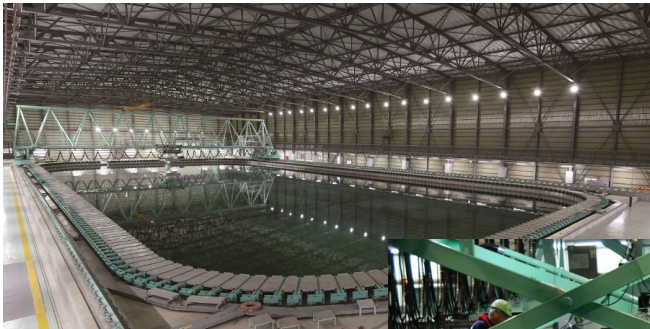


(d) UTC: 27-02-2009 13:30.



# Future applications (1): Reconstructing the encountered surface elevation

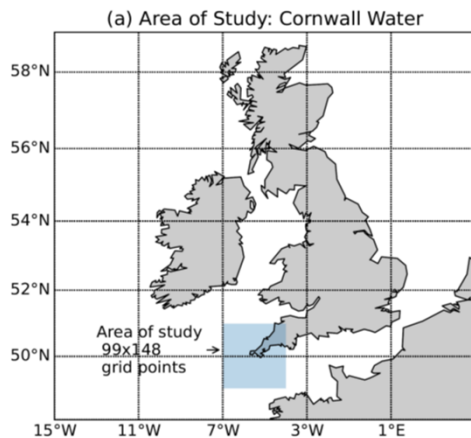
- Investigations made with experimental data (seakeeping model tests)



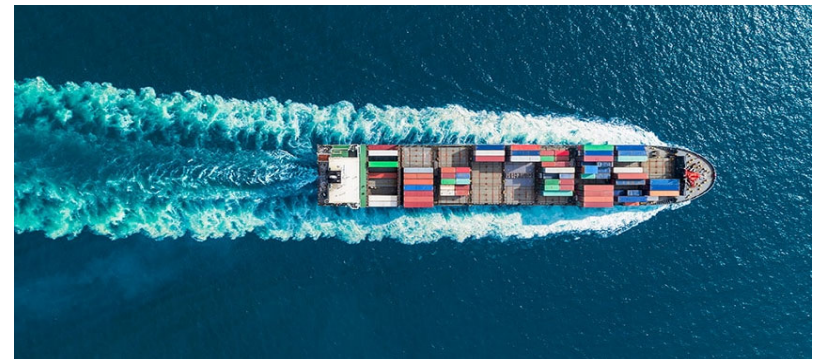
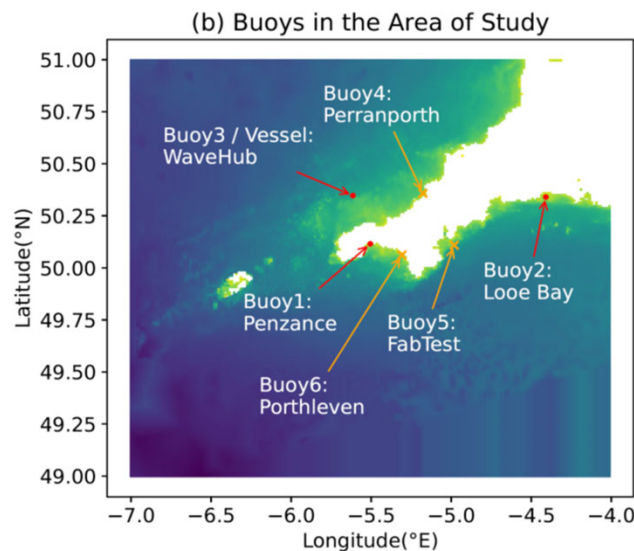
Takami et al. (2022,2023)

# Future applications (2a): Spatial Wave Data from a Network of Buoys and Ships

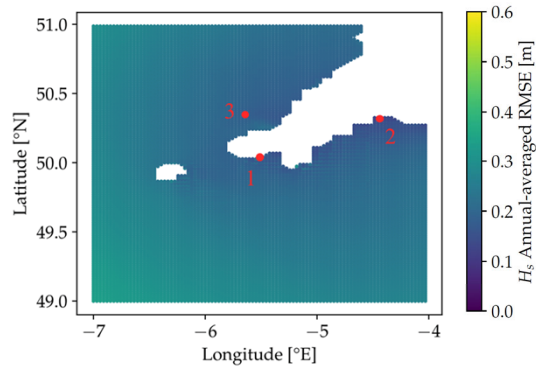
- Nowcasting as well as **forecasting of waves on a large-scale geographical domains using multiple observation platforms**, including ships
- Assessment of wave energy resources, operational windows, ship routing, assimilation (weather + waves),  
...



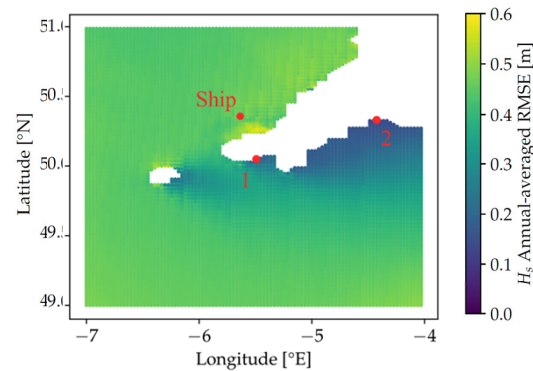
Mounet et al. (2023)



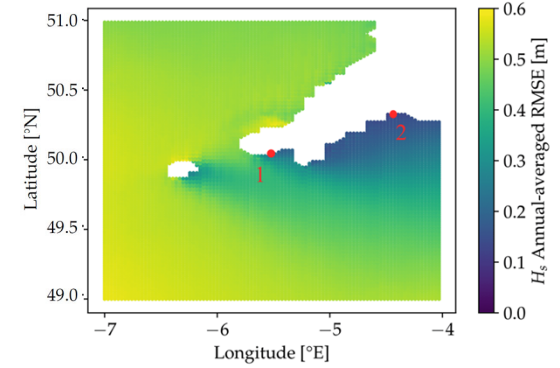
# Future applications (2b): Spatial Wave Data from a Network of Buoys and Ships



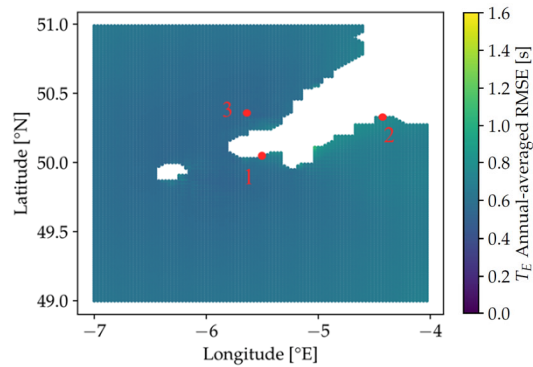
(a) Model 1,  $H_s$ -RMSE.



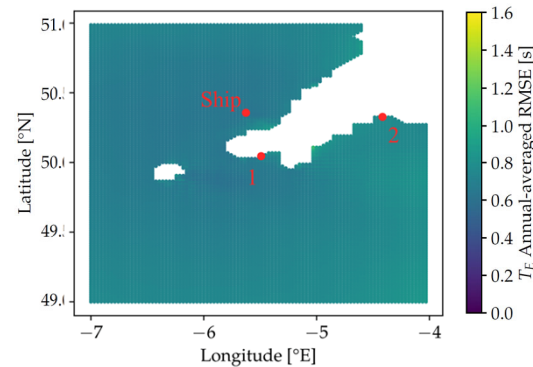
(c) Model 2,  $H_s$ -RMSE.



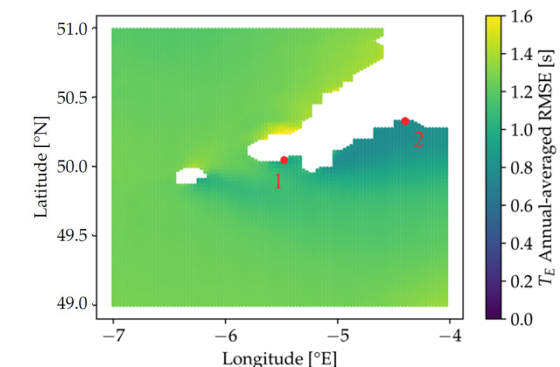
(e) Model 3,  $H_s$ -RMSE.



(b) Model 1,  $T_E$ -RMSE.



(d) Model 2,  $T_E$ -RMSE.



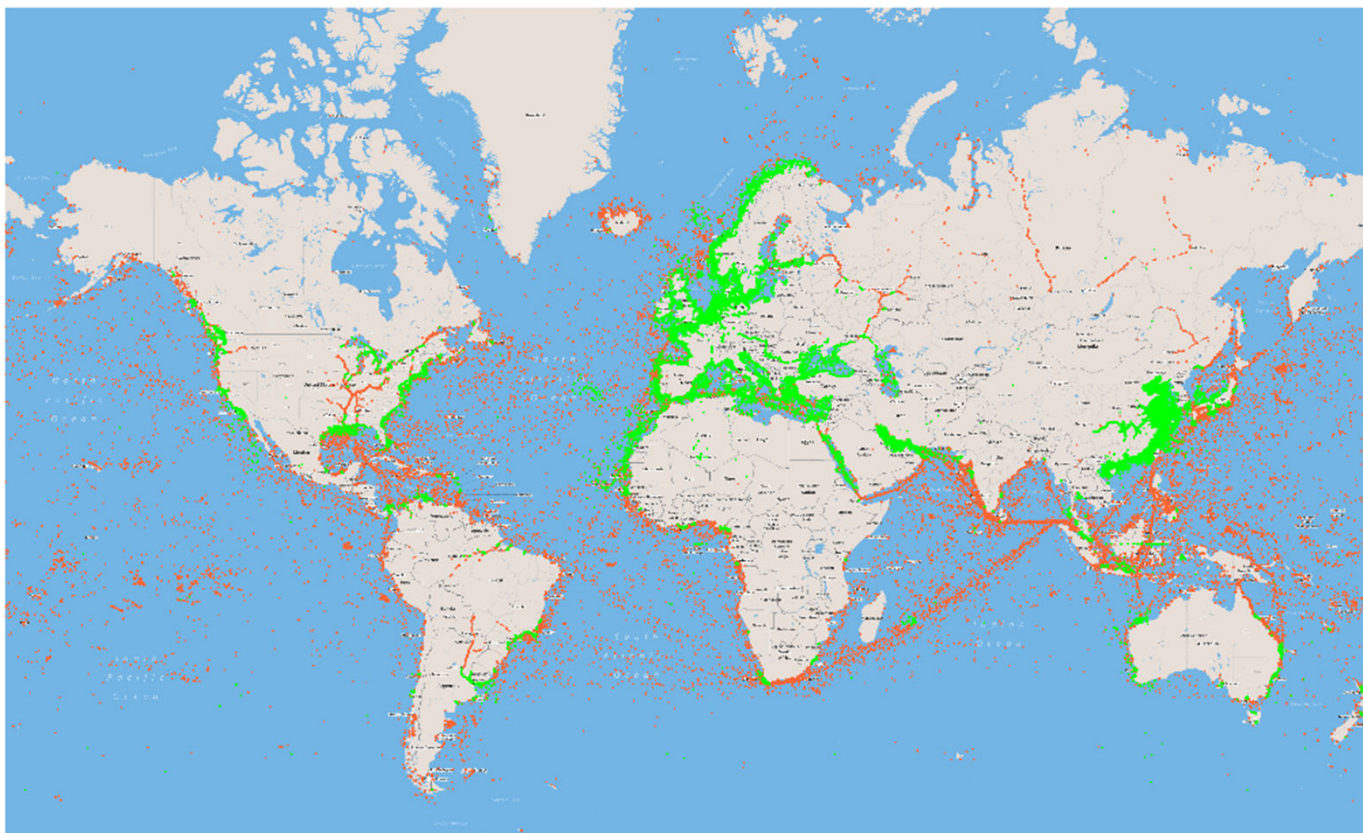
(f) Model 3,  $T_E$ -RMSE.

# Closing remarks

- DTU has conducted work on the use of ships as sailing wave buoys for the past two decades
- Generally, *the wave buoy analogy provides reasonable results, with fair agreement* compared to other means for wave estimation (remote sensing, wave radar, buoy measurements, spectral wave models); despite “inherent” complexities (hull geometry, relative size, forward speed)
- The use of *transfer function requires detailed and exact knowledge* about the operational condition (notably loading condition and speed); if not available *significant uncertainty* can exist
- *Machine learning methods* - with no need for transfer functions - have shown to *yield good results for wave parameters*; but the (directional) wave spectrum is not available
- A *hybrid approach* (“machine learning-informed physics-based”) is appealing since scarcity of data and other inherent problems related to data can be partly mitigated via use of transfer functions
- The wave buoy analogy appears attractive, considering the *large number of ships* operating around; future application using *multiple ships in a network*



## The potential...



A **snapshot of vessel positions around the world's ocean** based on data from AIS (green: terrestrial, red: satellites)

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A/S D/S Orient's Fond



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Questions?



Contact  
Ulrik Dam Nielsen  
[udni@dtu.dk](mailto:udni@dtu.dk)