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Deterministic Quantum Phase Estimation beyond N00N States

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The modern scientific method is critically dependent on precision measurements of physical parameters. A classic example is the measurement of the optical phase enabled by optical interferometry, where the error on the measured phase is conventionally bounded by the so-called Heisenberg limit. To achieve phase estimation at the Heisenberg limit, it has been common to consider protocols based on highly complex N00N states of light. However, despite decades of research and several experimental explorations, there has been no demonstration of deterministic phase estimation with N00N states reaching the Heisenberg limit or even surpassing the shot noise limit. Here we use a deterministic phase estimation scheme based on a source of Gaussian squeezed vacuum states and high-efficiency homodyne detection to obtain phase estimates with an extreme sensitivity that significantly surpasses the shot noise limit and even beats the conventional Heisenberg limit as well as the performance of a pure N00N state protocol. Using a high-efficiency setup with a total loss of about 11%, we achieve a Fisher information of 15.8(6) rad^{-2} per photon—a significant increase in performance compared to state of the art and beyond an ideal six photon N00N state scheme. This work represents an important achievement in quantum metrology, and it opens the door to future quantum sensing technologies for the interrogation of light-sensitive biological systems.

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It is of fundamental interest and practical relevance to investigate the ultimate bounds on the precision in estimating a phase [1,2]. According to classical (that is, approximate) theories of light, phase estimation can, in principle, be carried out with an arbitrary precision, but due to the inherent corpuscular quantum nature of light phase measurements will in reality be limited in precision—a precision that depends on the probing quantum state of light. If nonentangled states are used, the ultimate precision limit is the shot-noise limit (SNL), where the sensitivity σ is $1/2\sqrt{\langle\hat{\mathbf{n}}\rangle}$, with $\langle\hat{\mathbf{n}}\rangle$ being the average number of photons traversing the sample. By exploiting entangled states, it is possible to reach the Heisenberg limit (sometimes referred to as the weak Heisenberg limit [3–6]), $1/2\langle\hat{\mathbf{n}}\rangle$, exhibiting superior scaling.

One of the most celebrated quantum states for reaching the ultimate Heisenberg limit—often referred to as the optimal state for loss-free sensing—is the so-called N00N state [7]: $|\Psi_{\mathrm{N00N}}\rangle = 1/\sqrt{2}(|N\rangle|0\rangle + |0\rangle|N\rangle)$ which represents an optical state that is a superposition of N photons across two optical modes. In this case, the Heisenberg limit becomes $1/N$, which is an absolute limit for states with a definite photon number such as the N00N state. Figure 1(a) illustrates different phase sensing schemes and their sensitivity scaling.

Although a large number of experimental realizations on phase estimation with N00N states have been reported [8–15], as of today, only two experiments have been able to obtain a sensitivity that violates the SNL [12,15], and even in those realizations, the SNL was only beaten by using a probabilistic, but heralded, source of two-photon N00N states. Because of the high complexity in generating the N00N state and their extreme fragility to loss, it is unlikely that N00N states will in practice be able to reach the Heisenberg limit, or even beat the SNL, for high photon numbers.

It has been known for decades that the SNL can be more easily surpassed using squeezed states of light [16–19], which by now has also been realized in several phase estimation experiments [20–25]. However, in most of those experiments, squeezed light is combined with a bright coherent state in an interferometric measurement by which the sensitivity is often limited to $\sqrt{V_-}/2\sqrt{\langle\hat{\mathbf{n}}\rangle}$ (where V_- is the variance of the squeezed state quadrature normalized to the variance of the vacuum state) [26]. Although this gives a constant factor improvement over the shot noise limit, it it still far from attaining the Heisenberg limit. However, it is known that, by using states without a definite number of photons such as the squeezed state, it is in fact possible to not only reach the Heisenberg limit but also surpass it by a constant factor. Indeed, a number of schemes that violate the Heisenberg limit based on squeezed vacuum and different detection strategies have been suggested but not yet demonstrated [17,28–30].

In this Letter, we show that by employing squeezed vacuum as a probe and a simple quadrature detector as the measurement device, phase estimates beyond the

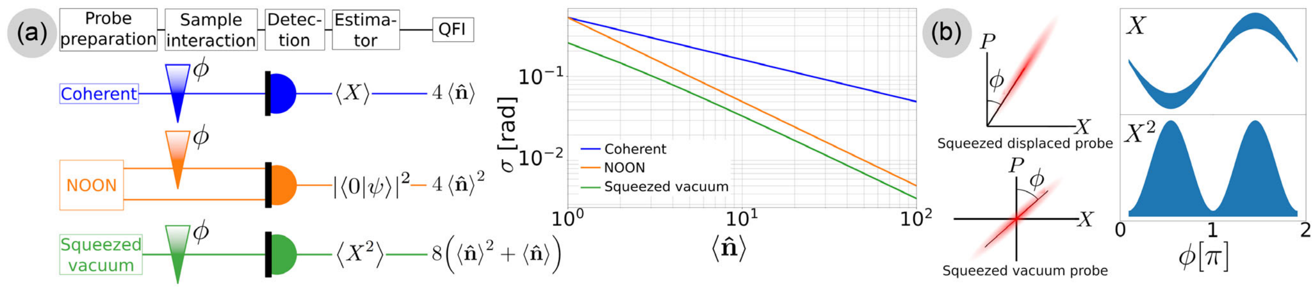

FIG. 1. Principles and limits of quantum phase estimation. (a) Left: Schematics of three different phase estimation schemes. A quantum state of light undergoes a phase shift that is measured with either a homodyne detector (HD) or a N00N state detector (involving photon counters) from which estimators are used to estimate the phase shift. Note that the N00N state scheme is based on a two-beam interferometer in which only half of the photons traverse the sample. We therefore use the conservative sensitivity bound of $1/2\langle\hat{\mathbf{n}}\rangle$ (where $N=2\langle\hat{\mathbf{n}}\rangle$) for the comparison to our approach. Right: The optimal sensitivities σ for the three schemes. (b) Phase space pictures of a displaced squeezed state and a vacuum squeezed state, and the resulting quadrature measurements as a function of the phase. The phase is estimated using the estimators $\langle X\rangle$ or $\langle X^2\rangle$ for the displaced squeezed state and vacuum squeezed state, respectively.

Heisenberg limit as well as the ideal N00N state protocol can be attained. In our demonstration, we outperform the SNL for all photon numbers up to an average of 40 photons, while beating the Heisenberg limit for up to around three photons. This corresponds to surpassing the performance of an ideal six-photon N00N state. We also note that, in contrast to previous N00N state realizations, our scheme is not based on probabilistic sources of light or any post-selection of the measurement outcomes.

The conventional approach to squeezing-enhanced phase estimation is based on displaced squeezed states undergoing phase shifts that are estimated using a phase-referenced homodyne detector measuring the quadrature $\hat{\mathbf{X}}$. The estimator of the phase $\hat{\phi}$ is then constructed from $\langle\hat{\mathbf{X}}\rangle$ with a quadrature uncertainty that depends on the actual phase as illustrated in Fig. 1(b): The best phase estimate is achieved when the response (derivative of $\langle\hat{\mathbf{X}}\rangle$) is maximized and the noise is minimized which, in this case, occurs midfringe (at the phases $\phi=n\pi$, where $n=0,1,2,\ldots$). Using instead squeezed vacuum as the probe, the measurement of $\langle\hat{\mathbf{X}}\rangle$ does not yield information about the phase, since in this case $\langle\hat{\mathbf{X}}\rangle=0$, but if we use $\langle\hat{\mathbf{X}}^2\rangle$ to form an estimator of ϕ, the information is revealed.

In this case, however, the phase shift for which the response is the largest is not coinciding with the phases with minimum noise (at $\phi=n\pi$) and thus a trade-off needs to be found for which the sensitivity is optimized. The trade-off is optimized for the phases $\phi_{\mathrm{est,opt}}=\arccos(\tanh 2r)/2+n\pi$ at which the Fisher information is maximized; $F=2\sinh^2(2r)$ where r is the squeezing parameter. The outlined phase estimation strategy is rather noisy (see Supplemental Material [31]), so in the actual implementation as discussed below we use Bayesian estimation of the posterior distribution $p(\phi|\{x\})$.

From the Fisher information, we find the sensitivity $\sigma_{\mathrm{sqz}}=1/\sqrt{2\sinh^2(2r)}$, which can be expressed in terms of the average photon number,

$$\sigma_{\mathrm{sqz}}=\frac{1}{\sqrt{8(\langle\hat{\mathbf{n}}\rangle^2+\langle\hat{\mathbf{n}}\rangle)}}. \tag{1}$$

Here we assume a pure squeezed state; for impure squeezed states, see the Supplemental Material [31]. This expression surpasses the Heisenberg limit, and moreover, it saturates the quantum Cramér-Rao bound, which means that the scheme with homodyne detection of squeezed vacuum is optimal among all possible measurement strategies. In addition to being optimal among all Gaussian states, it is also clear that it beats the complex estimation strategy of using non-Gaussian N00N states as $\sigma_{\mathrm{sqz}}<1/2\langle\hat{\mathbf{n}}\rangle$ for all $\langle\hat{\mathbf{n}}\rangle$.

A simplified schematic of the experimental setup is shown in Fig. 2(a) (see Supplemental Material [31] for details). We employ type 0 parametric down-conversion in a high-quality optical cavity to produce squeezed vacuum in a single spatial mode at the wavelength of 1550 nm. The squeezed vacuum state then experiences a phase shift of ϕ (relative to a reference) before its X quadrature is measured by a homodyne detector. At this detector, the squeezed mode interferes with a phase-referenced local oscillator mode at a balanced beam splitter, the two outputs are detected with high-efficiency photodiodes, and the resulting currents are subtracted, amplified, and fed to a computer for phase estimation and analysis.

By paying careful attention to the design and implementation of the source and the detectors, the total absorption and scattering loss was kept below 11% including the loss associated with the source, the propagation, and the detector. As a result, we produce squeezed states with a maximum of 9.0 dB of squeezing at a sideband frequency of 5 MHz with a bandwidth of 1 MHz [see Fig. 2(b)]. In future experiments, the bandwidth can be easily increased to allow for faster measurements.

Because of the absorption and scattering losses, the produced squeezed vacuum state is not pure, which

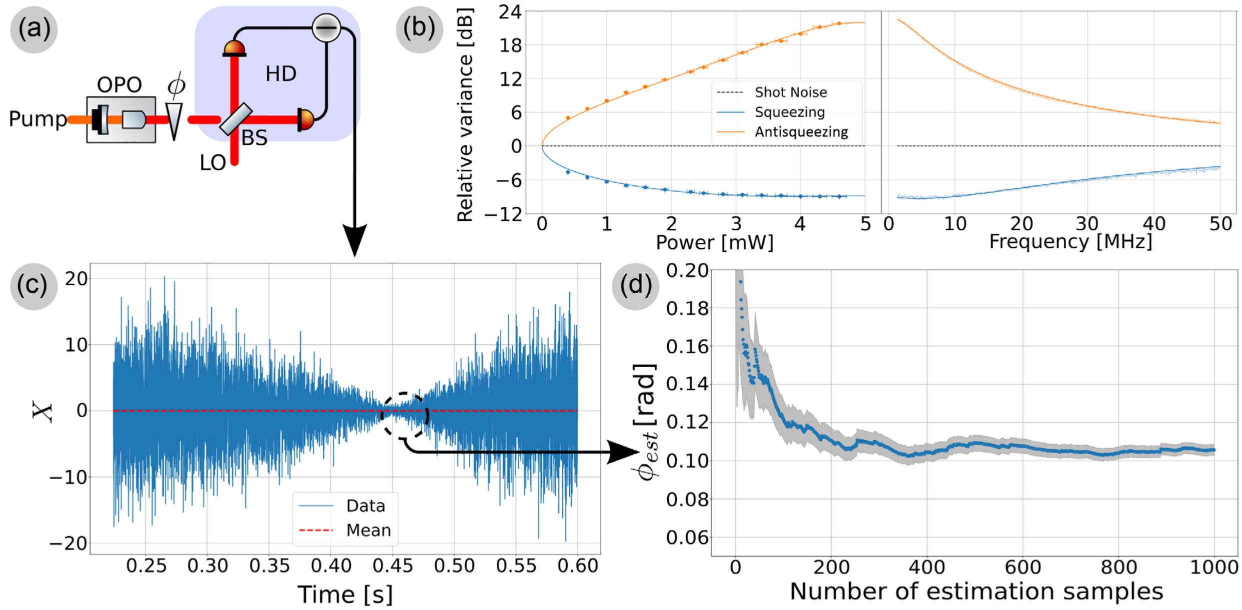

FIG. 2. Experimental scheme and measurement method. (a) Schematic of the experimental setup comprising an optical parametric oscillator (OPO) for squeezed light generation and a high-efficiency homodyne detector with a controllable local oscillator (LO). As the estimated phase shift is relative between the squeezed vacuum and the local oscillator, in the experimental realization, we imposed the phase shift onto the local oscillator. (b) Squeezed and antisqueezed variances relative to the shot noise variance as a function of pump power at a sideband frequency of 5 MHz (left) and frequency at a pump power of 3.5 mW (right). (c) Quadrature measurement outcomes. The data are acquired while slowly varying the phase of the local oscillator and down-mixed to a 5 MHz sideband frequency with a bandwidth of 1 MHz. (d) An example of how the estimated phase converges as a function of homodyne samples used in the Bayesian estimation process. The gray area marks the standard deviation.

eventually leads to a deviation from Heisenberg scaling of the sensitivity. To estimate the phase ϕ and the associated uncertainty, we conduct $M = 1000$ quadrature measurements for each phase setting, thereby producing a collection of 1000 data points $\{x\}_M$. An example of the measured quadrature X for different phases are presented in Fig. 2(c). From these measurements, we find the likelihood of acquiring the set $\{x\}_M$ conditioned on the phase ϕ: $P(\{x\}_M|\phi) = \Pi_{i=1}^M P(x_i|\phi)$. The individual measurements are sampled from a Gaussian distribution, $P(x|\phi) = \exp[-x^2/2V(\phi)]/\sqrt{2\pi V(\phi)}$, with variance $V(\phi) = V_- \cos^2(\phi) + V_+ \sin^2(\phi)$, where V_+ and V_- are the antisqueezed and squeezed variances, respectively. Using the Bayes theorem, we find the probability distribution for the phase conditioned on the measurement outcomes (the *a posteriori* probability distribution): $P(\phi|\{x\}_M) = P(\{x\}_M|\phi)P(\phi)/P(\{x\}_M)$, where $P(\{x\}_M)$ is a normalization factor and $P(\phi) = 2/\pi$ is the *a priori* probability distribution of the phase (assumed to be flat in the range $[0, \pi/2]$). In Fig. 2(d), we plot the *a posteriori* distribution for different values of M, illustrating the gradual Bayesian updating of the phase estimate. We then determine the estimated phase as the argument of the maximum value of $P(\phi|\{x\}_M)$ [see inset in Fig. 2(d)] and the associated phase uncertainty by the width of the distribution. These results are summarized in Fig. 3(a) for $\langle\hat{\mathbf{n}}\rangle = 1.8$ and in a polar plot representation in Fig. 3(b) for different average photon numbers. It is clear that the phase uncertainty decreases with increasing photon number (which we realize by varying the squeezing degree) and that it is optimized at specific phases [see Fig. 3(c)]. The best operating principle of the system is thus to measure small phase shifts relative to the measurement angle for which the phase variance is smallest. In Fig. 3(d), we plot the sensitivity optimized over the phase for different photon numbers, and we clearly observe performance beyond the Heisenberg limit and the ideal N00N state for photons up to around 3 (corresponding to a six-photon N00N state), as well as beyond the SNL and the loss-adapted N00N state limit for photons up to around 40.

Since our states are being produced and measured deterministically, we are in a position to perform real-time measurements of a dynamically varying phase with near-ultimate precision. To do this, we probe an induced 3 kHz phase modulation as well as other low-frequency phase noise components with our sensitive probe, which in these measurements contain six photons and preset (and locked with a bandwidth of less than approximately 1 kHz) to the optimal phase. The frequency spectrum of the measured phase signal and noise is shown in Fig. 4(a) and the real-time estimate of the dynamically varying phase is shown in Fig. 4(b) for $M = 100$. By enlarging a certain time interval, the 3 kHz signal becomes visible [Fig. 4(b) enlarged area].

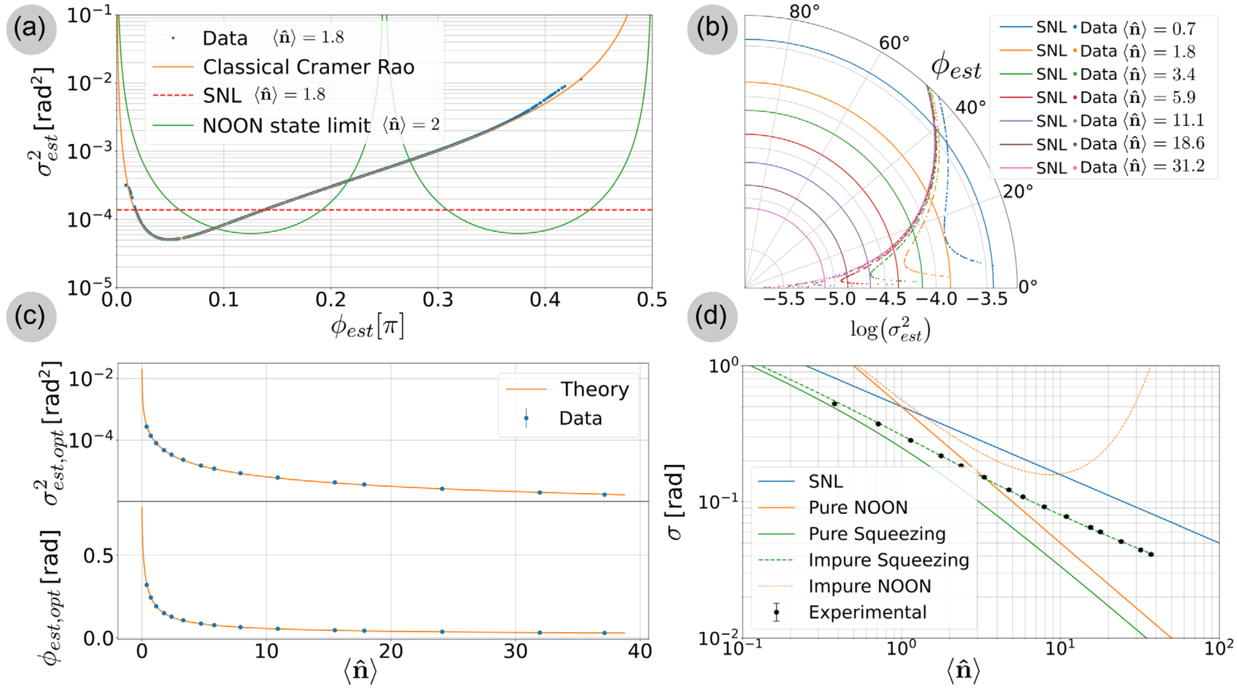

FIG. 3. Quantum phase estimation results. (a) The variance of the phase estimate σ^2_{est} based on 1000 quadrature samples of a squeezed vacuum state with 1.8 photons on average. This is compared to the SNL and the limit for a lossless N00N state with $2\langle\hat{\mathbf{n}}\rangle = N = 4$. (b) The variance of the phase estimated for different average photon numbers represented in a polar diagram and compared to the SNLs of the respective realizations (the curves are color coded). It is clear from these plots that the variance is minimized for certain phases. (c) The optimal estimated phases $\phi_{\mathrm{est,opt}}$ with corresponding estimation variance $\sigma^2_{\mathrm{est,opt}}$ are presented for different photon numbers and compared with theory. (d) The experimental sensitivities σ versus photon numbers are presented and related to the theoretical predictions for the SNL, squeezed vacuum limit, and the N00N state limit. We include theoretical predictions for the ideal limits and the practical limits with 11% loss as measured in our system.

In summary, we have demonstrated phase sensing close to the ultimate limit, beating the ideal N00N state phase sensing scheme—often viewed as the optimal phase sensing strategy—with up to about three photons using solely squeezed vacuum and homodyne detection. To the best of our knowledge this is the best single shot sensitivity per photon achieved in any optical phase sensing experiment: The directly measured Fisher information per photon in our scheme is 15.8(6) rad^{-2}, which should be contrasted to the Fisher information of the best N00N state experiment of $\sim$6.1 rad^{-2} [12].

Because of the intrinsic nature of squeezed light, we have demonstrated violations of the Heisenberg limit for only a small range of phases, making this protocol best suited for measuring small phase shifts. It can, however, easily be extended to phases covering the entire range of $[0, \pi/2]$ by

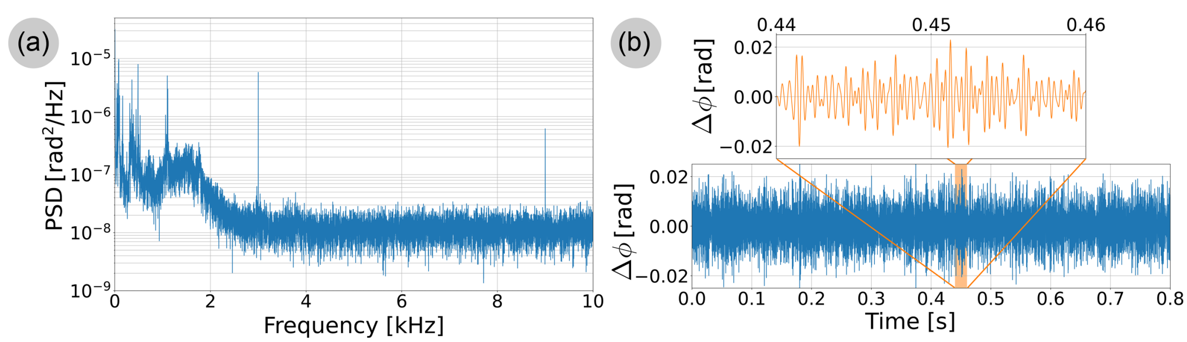

FIG. 4. Quantum-enhanced tracking of a phase signal. (a) Power spectral density (PSD) of a measurement of an estimated dynamically varying phase signal using squeezed vacuum (with six photons) and Bayesian inference. A 3 kHz induced signal as well as some low-frequency noise is apparent. (b) Time trace of the same signal, but bandpass filtered at 3 kHz with a 2 kHz bandwidth. The enlargement of the time trace as well as the frequency spectrum clearly shows the 3 kHz modulation. The y axis $\Delta\phi$ is the relative phase shifts compared to the preset measurement phase.

making use of an adaptive feedback approach [40]. We also note that, by combining our strategy with a multipass metrology protocol [41,42], the sensitivity can be improved even further, as in this case Heisenberg scaling will also apply to the number of measured samples [43]. The development and realization of a practical and loss-tolerant phase sensing scheme that beats the performance of any other current phase sensing strategy is not only of fundamental interest, but is also of practical relevance in phase sensing scenarios, where a low photon flux is needed to avoid the change of dynamics of the interrogated, potentially light-sensitive sample, such as atomic [44], molecular [45], and biological [46–49] systems.

The data that support the plots within this paper and other findings of this study are available from the corresponding authors upon reasonable request.

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