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Internal motion within pulsating pure-quartic soliton molecules in a fiber laser

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ABSTRACT
Various striking nonlinear evolution dynamics have been observed in mode-locked fiber lasers owing to the high peak power of the solitons. Herein, we numerically demonstrate the dynamical generation of dispersion-dependent pure-quartic solitons (PQS) molecules in a fiber laser. We discover the evolution from a single-pulse initial condition to different types of pulsating PQS molecules with enhanced net fourth-order dispersion, indicating that the internal motion and energy exchange can be both quasi-periodical and periodical. Furthermore, we reveal that the generation of these two types of pulsating PQS molecules is associated with a gain competition effect between the two sub pulses during mode-locking. Finally, we propose a new method that can achieve the transition between a loose PQS molecule and a tight PQS molecule. Enlightened by the numerical results, we speculate that more internal motion within the PQS molecule will be discovered, which will promote a deep insight into the physical mechanism behind.

1. Introduction

Optical solitons, based on balancing the nonlinear effect and dispersion in the propagation media such as the optical fiber, have been researched and applied widely in many fields including optical communications, optical imaging, nonlinear optics, and so on [1–4]. One significant research topic related to optical solitons is the emission from a mode-locked fiber laser, which results from the interplay between the quadratic (second order) dispersion and the self-phase modulation (SPM) [5–6]. Owing to the soliton area theorem \(E \propto |\beta_4|/(\gamma \tau_0)\), where \(E\), \(\beta_4\), \(n_2\), \(\gamma\), and \(\tau_0\) respectively represent the pulse energy, the quadratic dispersion coefficient, the nonlinear coefficient, and the pulse width, the energy of this kind of soliton in a fiber laser is limited to \(\sim 0.1\) nJ. Thus, various methods have been adopted to improve the emission pulse energy inside fiber lasers, such as the dispersion-management engineering and the Mamyshev oscillator concept [7–10].

In recent years, studies showed that pure-quartic solitons (PQSs) exhibit a striking advantage over quadratic solitons in terms of the energy-width scaling, which can be expressed as \(E \propto |\beta_4|/(\gamma \tau^2)\), where \(\beta_4\) is the fourth-order dispersion (FOD) [11]. Depending on the balance between the negative FOD and the positive SPM effect, the PQS opens a whole new world of research for generating high-power soliton pulses from a fiber laser. A remarkable characteristic of the PQS, which is different from the quadratic soliton, is that the interaction between FOD and SPM can induce oscillating tails in the time domain and a flattened shape in the frequency domain [12]. Another advantage is that PQSs have an approximately Gaussian temporal profile, which is much more beneficial for the amplification with higher peak power than the traditional soliton pulse [13,14]. Nevertheless, the challenge in the PQS generation is to develop suitable platforms with significant FOD over a large bandwidth.

To date, the physical mechanism behind PQSs is mainly studied by

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numerical simulation. Karlsson et al. presented an exact localized analytical solution for the pulse propagation in optical fibers including both the anomalous second order dispersion and FOD [15]. Tam et al. numerically studied the stationary and dynamical properties of the PQS by solving a generalized nonlinear Schrödinger eq. [11]. Qian et al. investigated the dissipative PQS generation in a positive FOD cavity [16]. The first experimental demonstration of the PQS fiber laser was in 2020 by Runge et al., who adopted a pulse shaper to produce an arbitrary phase mask, which was then used to adjust the net cavity dispersion [12].

Under appreciate conditions, two or more pulses simultaneously propagating in a fiber laser cavity can interact and bind together, resulting in the so-called optical soliton molecule [17,18]. In a soliton molecule, the most relevant degrees of freedom for evolution dynamics are the pulse separation and phase evolution among the internal pulses. By constructing the laser cavity with different interplay among nonlinearity, dispersion, gain and loss, various ultrafast evolutionary processes have been demonstrated numerically and observed experimentally using the time-stretch dispersive Fourier-transform (DFT) technique. Peng et al. demonstrated the periodical evolution of the soliton molecule in a normal-dispersion fiber laser by observing the exchange energies during the propagation [19]. Xia et al. reported a so-called shaking molecule can be achieved by adjusting the net FOD. These findings suggest that the PQS molecules can exist and what properties they would have. The special temporal profile of PQSs with the oscillating tails could lead to novel internal dynamics in terms of relative motion and energy exchange. However, few studies have focused on the dynamics of a PQS molecule during the formation and propagation in fiber laser systems so far. Ying et al. demonstrated the generation of a gain-dependent PQS molecule, but the influence of the net FOD on the PQS molecule generation and its evolution dynamics were not mentioned [22]. Therefore, it is desirable to discover more intriguing and intricate nonlinear evolutionary behaviors of a PQS molecule and gain a deeper understanding of PQSs in a fiber laser.

In this work, we numerically investigate the internal motion of a FOD-dependent pulsating PQS molecule in a mode-locked fiber laser. Both the quasi-periodical and periodical pulsating evolutionary dynamics of the PQS molecule output from the mode-locked fiber laser can be observed, which result from the intrinsic temporal oscillating tail feature of the PQS. We demonstrate that the generation of these two pulsating behaviors is related to the gain competition between the sub pulses. Furthermore, the transition between loose and tight PQS molecule can be achieved by adjusting the net FOD. These findings suggest that the PQS fiber laser could also be a promising platform for manipulating the internal motions within a PQS molecule.

2. Simulation model

The PQS mode-locked fiber laser in the simulation system is schematically shown in Fig. 1, which mainly consists of four parts: the Er-doped fiber (EDF), the saturable absorber (SA), the output coupler, and the single-mode fiber (SMF). 10 % of the oscillating laser is output from the cavity. Pulse propagation within the cavity is described by the generalized nonlinear Schrödinger eq. [16]:

$$\frac{\partial A}{\partial z} = - \frac{i \beta_2}{2} \frac{\partial^2 A}{\partial t^2} + \frac{i \beta_3}{6} \frac{\partial^3 A}{\partial t^3} + i \beta_4 \frac{\partial^4 A}{\partial t^4} + \frac{g}{2} A + \frac{\alpha}{2} A + \frac{g}{2} \frac{\partial^2 A}{\partial t^2} + i \gamma |A|^2 A$$

where $A$ is the slowly varying amplitude of the pulse envelope; $\beta_2$ is the second-order dispersion; $\beta_3$ is the third-order dispersion; $\beta_4$ is the FOD; $t$ is the time in a retarded frame of reference moving at the group velocity of the laser pulses; $\gamma$ is the nonlinear coefficient; $\alpha$ is the cavity loss; $\Omega$ is the 3-dB bandwidth of a gain medium; $g$ is the saturable gain of the fiber, $W$ is the 3-dB bandwidth of a gain medium; $g_0$ is the small signal gain and $E_{sat}$ is the saturation energy of the EDF. The equation can also be applied when pulses enter the passive fiber where $g_0$ is set to be 0. The Raman effect is neglected because we operate at low peak powers everywhere in the cavity and self-steepening is neglected because we operate with pulses longer than 1 ps.

Here, the mode-locking operation in the cavity is achieved by employing the SA with the following power-dependent transfer function [23]:

$$T = 1 - \frac{\alpha_0}{1 + \frac{E_{sat}}{E_{sat}}}$$

where $T$ is the transmittance of the SA; $\alpha_0$ is the modulation depth of the SA; $E_{sat}$ is the saturation power of the SA.

In the modeling we use the following parameters for our simulations:

- $\beta_2 = 0 \text{ ps}^2/\text{km}$; $\beta_3 = 0 \text{ ps}^3/\text{km}$; $g_0 = 900/\text{km}$; $\alpha_0 = 53 \%$; $E_{sat} = 120 \text{ W}$; $\Omega = 49 \text{ mm}$; $\alpha = 0.2 \text{ dB/km}$, $\gamma_{EDF} = 15/(\text{W km})$. These values are reasonable for a real gain fiber and SA [24,25]. The combination of $g_0$ and $E_{sat}$ can provide enough gain for the laser amplification. The specific parameter of $E_{sat}$ will be presented in the next part. High modulation depth of the SA can promote the bound-state multi-soliton complexes generation [26]. The pulse propagation in the cavity is solved with the 2nd order split-step Fourier method. The time window size is 64 ps which consists of 4096 points. The pulses will sequentially pass through EDF, SA, OC, and SMF in our simulation. The length of the EDF and SMF are 2 m and 5 m, respectively, whose step length is $1 \times 10^{-3} \text{ m}$. 

3. Results and discussions

3.1. Single PQS

Stable single PQSs, whose pulse shape and spectrum remain the same after 150 roundtrips, can be obtained easily as shown in Fig. 2 by adjusting the simulation parameters, in which $\beta_{2,\text{EDF}}, \beta_{4,\text{SMF}}$, and $E_{sat}$ are set as 10 $\text{ps}^2/\text{km}$, $-15 \text{ ps}^3/\text{km}$, and 0.01 nJ, respectively. Fig. 2(a) and (b) illustrate the emission pulse shape and the optical spectrum evolution within 2000 roundtrips on a logarithmic scale, with the initial condition of a weak pulse: $A = \text{sqrt}(P_0) \times \text{sech}(t/\tau_0)$. Where $P_0$ is 0.0001 W and $\tau_0$ is 2 ps. The stable single PQS can be obtained after the 150th roundtrip. Fig. 2(c) illustrates the emission pulse shape at the 2000th
roundtrip. Obvious oscillating tails can be found at the two sides of the temporal profile on logarithmic scale, indicating the unique characteristic of the PQS [27]. Fig. 2(d) shows the corresponding emission optical spectrum at the 2000th roundtrip which has a 9.2-nm 3-dB bandwidth.

3.2. Quasi-periodical pulsating PQS molecule

For a real fiber laser, the increase in pump power can promote the pulse split and form the soliton molecule. To generate the PQS molecule, the saturation energy of the EDF corresponding to the pump power is then increased to 0.05 nJ to promote the soliton splitting. And several kinds of PQS molecule evolutionary dynamics can be observed by optimizing the FOD of the SMF, which mainly include the quasi-periodical pulsating PQS and the periodical pulsating PQS. Firstly, the quasi-periodical pulsating PQS molecule will be introduced in this part. The quasi-periodical pulsating PQS molecule obtained with $E_{sat} = 0.05$ nJ is shown in Fig. 3. Fig. 3(a) shows the pulse optical spectrum evolution within 4000 roundtrips. It is obvious that the emission optical spectrum has interference fringes, which verifies the PQS molecule generation [28]. What’s more, the emission optical spectrum shows instability along with the evolution in the cavity since the spectrum changes with the increase of the roundtrip times. In order to have a deep insight into the evolution dynamics of the PQS molecule, the corresponding temporal evolution is plotted in Fig. 3(b). The single pulse splits into two pulses rapidly after a few roundtrips. In addition, the two pulses are bound to each other with a quasi-periodical pulsating state in the subsequent spread. The temporal state also confirms the spectral interferogram. The two pulses nearly have the same intensity, which is a consequence of the gain competition between the two solitons [29–30]. However, the two sub pulses are not stable but have a consecutive evolution in terms of their central positions in the time domain. Furthermore, the moving distance of the central positions is not a constant. Another interesting phenomenon is that the bound two pulses in a PQS molecule also have a certain degree of independence for their own evolution when sharing the result of the gain competition. For example, the left pulse first appears a large moving distance and starts a long period since the 1520th roundtrip, and then changes to periodic small-large value. On the contrary, the right pulse first appears a small moving distance and a shorter period since the 1520th roundtrip, and then changes to periodic large-small value.

To further explore the pulsating PQS molecule, the evolution dynamics of the two pulses between the 1520th and 2490th roundtrip in Fig. 3(b) are zoomed in to present corresponding details. It is obvious that the left pulse, firstly, have 5 times periodic pulsating with large moving distance and period, while the right one has a smaller moving distance and period. And then, the moving distance of the left pulse turns into small, while the right one turns into a periodic-like pulsating with large moving distance. In addition, the pulsating process of the two sub pulses seems complementary. Based on the above behavior of the two sub pulses, the PQS molecule formed in this case is called the quasi-periodical pulsating one.

Furthermore, the temporal separation distance between the centers
of these two pulses is retrieved from pulse shape evolution trace and shown in Fig. 4(a). According to the calculated results, the temporal evolution is in a state of dynamic equilibrium and can be regarded as stable since the distance between two pulses is characterized by a small fluctuation ($\pm 0.4$ ps) around the fitting line (12.5 ps). The energy distribution of the two sub pulses within the soliton molecule inside the laser cavity shares the same rule and is illustrated in Fig. 4(b). Although the energy of the two sub pulses is in a dynamic evolution, they fluctuate

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Fig. 3. Mode-locked quasi-periodical pulsating PQS molecule. (a) Optical spectrum and (b) pulse shape evolution within 4000 roundtrips. $\beta_4^{\text{EDF}}, \beta_4^{\text{SMF}}$, and $E_{\text{sat}}$ are 10 ps$^2$/km, $-15$ ps$^2$/km, and 0.05 nJ.

Fig. 4. (a) The distance between the two pulses within the quasi-periodical pulsating PQS molecule. (b) The separate and total energy evolution of the two pulses within the quasi-periodical pulsating PQS molecule.
slightly around a fixed value. The inset presents a clear evolution process of the energy distribution among the PQS molecule between the 2500th to 3000th roundtrip and demonstrates that the energy evolution of the two sub pulses are approximately complementary, i.e., the two sub pulses evolves in a contrary tend. To note that, according to the energy evolution, the two pulses with the nearly equal energy intermittently gain advantage in the gain competition. However, for the quasi-periodical pulsating PQS molecule, the total energy evolves in a quasi-periodical fluctuation as shown in the green line and the fluctuation seems smaller than the separated two sub pulses. Fig. 5 shows the evolution of the phase difference between pulses in the soliton molecule. The phase difference is oscillating with the flipping nature from one value to another abruptly, which had been reported in Ref. 31 and 32. Furthermore, it is obvious that the flipping range changes with the evolution of the roundtrips, which causes the quasi-pulsating behavior of the PQS molecule.

3.3. Periodical pulsating PQS molecule

With the enhancement of the $\beta_{4\text{-SMF}}$, the periodical pulsating PQS molecule can be obtained. Herein, the periodical evolution of the PQS molecule pulse is obtained at $\beta_{4\text{-SMF}} = -24 \text{ ps}^4/\text{km}$ while the other parameters are the same as part 3.2, and the spectrum is shown in Fig. 6(a). The equally spaced interference fringe can be observed easily from it, which is similar to the quasi-periodical pulsating PQS molecule. The pulse shape evolution of the periodical pulsating PQS molecule is shown in Fig. 6(b). It can be concluded that the PQS molecule has a smaller pulsation period. The evolution process of the two sub pulses between the 2500th to 2600th roundtrip is also zoomed in. The two sub pulses have an equal moving distance and period, which demonstrates the periodical pulsating PQS molecule generation.

Fig. 7(a) shows the temporal separation distance of the two pulses, which also has the fluctuation around a fixed value according to the fitting line. Compared with the quasi-periodical pulsating PQS molecule, the periodical pulsating PQS molecule has a persistent and smaller pulsating period and moving distance in time domain. The energy distributions of the two sub pulses and the soliton molecule inside the laser cavity are shown in Fig. 7(b). Different from the energy evolution of the quasi-periodical pulsating PQS molecule, the pulse energy of the periodical one fluctuates periodical around a fixed value but with periodical smaller fluctuations. In terms of the separated pulse energy, the pulsating of two pulses shows a more obvious periodicity and the two pulses are still mutually compensated along with the pulse evolution. In particular, the left pulse always maintains a competitive advantage according to the energy, although the gain competition occurs along the propagation and eventually causes the periodic evolution. It should also be noted that the mismatch of the group velocity and phase velocity within the PQS molecule occurs here and results in the deflection of pulses in time domain, in which case the corresponding spectrum still maintains a consistent periodicity. Therefore, we can regard this pulsed state as a balanced mode-locking one in which the periodical PQS molecule is obtained. One can observe from Fig. 8 that the phase difference of pulses is flipping from $\sim -2$ to $26 \text{ rad}$ and vice versa nearly all the time, which results in the generation of the periodical pulsating PQS.

3.4. Discussions

According to our results, different cavity parameters can converge to different attractors for various emission states of the PQS and the PQS molecule. The FOD, like the second-order dispersion for the quadratic soliton, also plays an important role in the pulse evolution and is especially highlighted in the research of PQSs. Compared with the quadratic soliton, the PQS presents the similar but unique evolutionary properties when considering the linear and nonlinear interplay in a fiber laser system. The PQS will split once the FOD is failed to balance the nonlinear effect. Once the pulse splits, the sub pulses will suffer from the interaction force between them, and the new evolution will occur until the new balance is built. Thus, the soliton molecule will be observed consecutive or vanished depending on certain cavity parameters. In our simulation process, $E_{sat}$ and the nonlinear coefficient also have been increased with the purpose to generate three or more pulses bound together but failed, which may be due to that the broader temporal trail of the pulses can result in a more complex energy exchange process. Thus, only the two PQS molecules are discussed in this paper, which implies that the generation of a PQS molecule has the very different ultrafast dynamics from the quadratic soliton molecule.

In general, the PQS has oscillating tails in time domain, contributing to more possibility of energy exchange within the soliton molecule than that in a quadratic soliton molecule. In our simulation process, the quasi-periodical and periodical pulsating PQS molecule can be observed with the increased $E_{sat}$ and the enhanced $\beta_{4\text{-SMF}}$. On the contrary, no stable PQS molecule can be obtained, which also proves the strong and consistent internal energy exchange. The strictly stable PQS molecule may be obtained in the future work with the other appropriate parameters to reduce the interaction force and offset the energy exchange. As for the quasi-periodical pulsating PQS molecule, the energy evolution of the two pulses have an opposite trend along with round-trip times but have the same averaged value, which illustrates that the soliton energy quantum effect also acts equally to the quasi-periodical pulsating PQS molecule case.

![Fig. 5. Phase difference between pulses as a function of roundtrips in the quasi-periodical pulsating PQS.](image-url)
The quasi-periodical pulsating behavior of PQS molecule, such as the temporal separation between two sub pulses and the fluctuation of the pulse energy, is the consequence of a quasi-balance among the FOD, SPM, gain and loss. In the quasi-periodical PQS molecule, the gain competition cannot bind the sub pulses together by a certain periodical interaction force. Furthermore, the enhanced interaction caused by more FOD in the same cavity can promote the quasi-periodical pulsating PQS molecule into a periodical one. This indicates that the periodic

Fig. 6. Mode-locked periodical pulsating PQS molecule. (a) Optical spectrum and (b) pulse shape evolution within 2000 roundtrips. $\beta_{4,\text{EDF}}, \beta_{4,\text{SMF}}$, and $E_{\text{sat}}$ are $10$ ps$^4$/km, $-24$ ps$^4$/km, and 0.05 nJ.

Fig. 7. (a) The distance between the two pulses within the periodical pulsating PQS molecule. (b) The separate and total energy evolution of the two pulses within the periodical pulsating PQS molecule.
energy exchange between the two sub pulses can be achieved through overlapping the temporal oscillating tails. But the rebalanced state is still dynamic according to the different and periodic energy of the two sub pulses, which means the generation of pulsating PQS molecule is associate with the gain competition between two solitons. Based on the deduction, the FOD of the SMF is enhanced to be $-27 \text{ ps}^4/\text{km}$ and the generated periodical pulsating PQS molecule is shown in Fig. 9(a). It has a smaller pulsating period and the more compact temporal structure as shown in the inset compared with Fig. 5. Fig. 9(b) shows the pulse shape on a relative strength at the 1000th roundtrip, in which the intensity difference between the two pulse turns into larger. Based on the comparison among Figs. 3, 6, and 9, we may conclude that the gain competition between the sub pulses within a PQS molecule directly affects the stability of the PQS molecule in terms of the PQS evolution. Further investigation shows that the phase difference between the two solitons also flips with the roundtrip, as shown in Fig. 10, which features a complete equal oscillating amplitude and period.

Obviously, it can be concluded from our results that the bound state, in terms of the temporal distance and phase difference between two solitons, is quite different when only changing the FOD. These calculations reveal that the FOD directly contributes to the bound force between sub pulses when balancing nonlinearity for building the soliton molecule. Herein, the FOD of the SMF with values of $-15$, $-24$, $-27 \text{ ps}^4/\text{km}$ (corresponding to net FOD values of $-0.055 \text{ ps}^4$, $-0.1 \text{ ps}^4$, and $-0.115 \text{ ps}^4$, respectively) causes the pulse distances of 12.8 ps, 6.2 ps and 1.1 ps respectively, and the attraction between sub solitons seems to be enhanced during this process. To further investigate the impact of FOD, the distances between the two solitons under different FOD values are calculated and plotted in Fig. 11. It shows a maximum value of 13.7 ps under $-17 \text{ ps}^4/\text{km}$ FOD and a minimum value of 1.1 ps under $-27 \text{ ps}^4/\text{km}$ FOD. The temporal distance characteristics are shown in two main distribution regions, with some mutations resulting from several values of FOD. The simulation results show that the transition between a loose PQS molecule and a tight PQS molecule can be achieved by adjusting the FOD in the laser cavity.

4. Conclusion

In conclusion, we have numerically studied the internal motion within the FOD-dependent pulsating PQS molecule in a mode-locked fiber laser. Optimizing the balance relationship among FOD, SPM, gain, and loss, significant quasi-periodical and periodical pulsating...
dynamics of the PQS molecule can be observed, which result from the energy exchange and redistribution between the sub pulses. Our study has revealed that the internal motion within a pulsating PQS molecule is tightly related to the gain competition effect. The quasi-periodical PQS dynamic is characterized by two pulses with the same intensity, while the periodical one has an intensity difference between the two sub pulses. The gain competition between the sub pulses, determined by the linear and nonlinear interplay in the fiber laser system, directly affects the stability of the PQS molecule. In addition, the loose and tight PQS molecules with different internal phase distribution can be obtained separately by optimizing the net FOD. All these findings provide new perspectives into dynamic behavior of PQS molecule and highlight the potential applications.

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CRediT authorship contribution statement

Song Yang: Conceptualization, Writing – original draft, Writing – review & editing, Visualization. Zhiwei Zhu: Methodology, Software, Writing – original draft, Visualization. Yaoyao Qi: Validation, Investigation. Lei Jin: Validation, Investigation. Li Li: Validation, Investigation. Xuechun Lin: Conceptualization, Writing – review & editing, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data underlyng the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

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