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Deconvolution of heat sources for application in thermoelectric micro four-point probe measurements

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ABSTRACT

Joule heating is a primary phenomenon responsible for increasing temperature in electronic devices, and consequently, decreasing the lifetime and performance of electronic devices. However, this unwanted Joule heating can itself be used as a local source of heat to map the temperature-dependent material properties. Recently, micro four-point probe (M4PP) showed a promising potential for characterizing the temperature coefficient of resistance (TCR) and the Seebeck coefficient using Joule heating resulting from the measurement current. Here, we use M4PP to estimate the micrometer scale, relative temperature profile resulting from a single heat source. We introduce a triplet of four-point voltages measured at the second harmonic frequency, to deconvolute the thermoelectric voltage from the three individual heat sources involved. This paper tests and documents the validity of the proposed scheme in the 1–40 μ m range on highly doped single crystal Si at 300 K, supporting predominantly Fourier heat transport at these scales. The method of deconvolution of heat sources reduces the complexity in evaluation of length- and time-dependent measurements, specifically used in the characterization of thermoelectric properties. The proposed method may also facilitate a more profound understanding of heat transport on the mesoscopic scale.

1. Introduction

Increasing temperature due to Joule heating in electronic devices is a persistent problem for thermal management, consequently reducing the reliability of these devices [1]. For instance, Joule heating in low dimensional materials in combination with their limited thermal conductance often leads to overheating and thermal failure [2]. However, Joule heating has also been used to characterize thermal properties of materials via various techniques like the 3ω method [3,4], Raman spectroscopy [5,6], reflectance thermometry with a focused laser [7] and scanning thermal microscopy (SThM) [8].

Here, we introduce the micro four-point probe (M4PP) [9] as a tool to estimate the relative temperature profile resulting from a single heat source. An M4PP is fabricated in a similar fashion as an atomic force microscope probe. It consists of four or more micro-scale metal-coated Si cantilevers extending from silicon die. The electrode pitch of an M4PP ranges from sub-micrometer [10] to millimeters [11]. During a measurement, each electrode exerts a contact force on the order of 10^{-5} N which is five orders of magnitude lower than that of conventional four-point probe [12]. M4PPs can be used to perform a

wide range of physical measurements such as sheet resistance [13,14], resistivity [15], tunneling magneto-resistance [16], carrier mobility and carrier density [17]. Measurements are performed using a lockin amplifier, and recent studies have shown a great potential using higher harmonics for evaluating the temperature-related properties such as the temperature coefficient of resistance (TCR) [18], the ratio of Seebeck coefficient α_s to the thermal conductivity κ [19], and most recently the thermal diffusivity [20]. However, the temperature model used in these studies relies on Fourier's law, which has not yet been validated spatially for M4PP measurements. Most importantly, thermal analysis in these measurements is challenging due to the involvement of two point-like heat sources, i.e. Joule heating at the two current contacts.

In this article, we develop an experimental method based on M4PP for investigating the relative temperature profile from a single heat source using the phenomena of Joule heating. The method is developed for deconvolving two heat sources, by linear combination of three separate four-point measurements. The concept is verified on B-doped single crystal Si at 300 K.

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2. Theory

In a four-point probe measurement, where a current I is forced through the sample via two electrodes and the voltage difference V between two other electrodes is measured, significant advantages are gained by use of an alternating current (AC) i.e. $\operatorname{Re}(I_0e^{i\omega t}) = I_0 \cos(\omega t)$, where I_0 is amplitude, ω the angular frequency of the current and t is time. The excitation and voltage detection become less prone to interference from noise, and more information can be gained by analyzing higher harmonics. Due to inherent non-ohmic effects in the sample, the measured voltage may consist of various harmonics:

$$V(t) = \operatorname{Re} \sum_{n=0}^{\infty} R_{n\omega} I_0 e^{\mathrm{i}\phi_n} e^{\mathrm{i}n\omega t} = \sum_{n=0}^{\infty} V_{n\omega}.$$
 (1)

Here, the transfer resistance $R_{n\omega}$ is the magnitude and ϕ_n is the phase angle of the transfer "impedance" [15]. The primary signal in a fourpoint measurement is the ohmic signal at 1 ω . The rise in the temperature of the sample due to Joule heating will be proportional to the current squared ($\propto I^2$), and as a consequence of the Seebeck effect, a voltage harmonics at 2ω arises and can be measured [19].

2.1. Ohmic signal

When a current I(t) is forced through a point into a homogeneous half-space, the electric potential ϕ at a location **r** due to the point current source at **r**₀ is [21]

$$\phi(\mathbf{r},\mathbf{r}_0,t) = \frac{\rho I(t)}{2\pi |\mathbf{r} - \mathbf{r}_0|},\tag{2}$$

where ρ is the resistivity of the sample. In a four-point measurement, the potential difference $\binom{uv}{pq}V$ across two voltage pins located at \mathbf{r}_{p} and \mathbf{r}_{q} due to the current flow between another two electrodes located at \mathbf{r}_{u} and \mathbf{r}_{v} is given by [21]

$${}^{uv}_{pq}V(t) = \frac{\rho I(t)}{2\pi} \left(\frac{1}{|\mathbf{r}_p - \mathbf{r}_u|} - \frac{1}{|\mathbf{r}_q - \mathbf{r}_u|} - \frac{1}{|\mathbf{r}_p - \mathbf{r}_v|} + \frac{1}{|\mathbf{r}_q - \mathbf{r}_v|} \right).$$
(3)

For each unique triplet combination of the current pins across three pairs of electrodes at \mathbf{r}_u , \mathbf{r}_v and \mathbf{r}_w , while recording the voltage difference across electrodes at fixed \mathbf{r}_p and \mathbf{r}_q ; one can obtain the first harmonic voltage identity

$${}^{uv}_{pa}V_{1\omega}(t) + {}^{vw}_{pa}V_{1\omega}(t) = {}^{uw}_{pa}V_{1\omega}(t),$$
(4)

as illustrated in Fig. 1a and verified with Eq. (3).

The first harmonic resistance can be used to correct for in-line geometrical errors. As electrodes get in contact with the probed surface, their actual positions might differ from the nominal values, introducing errors in the estimated properties. A way to reduce these errors substantially is to perform exhaustive sub-sampling of four-point configurations, and minimization of observed vs. model resistances [16]. Hence, the distances used in the rest of the article are electrically regressed values instead of nominal values of the probe design.

2.2. Thermoelectric signal

Eq. (4) describes the first harmonic voltage combination of three configurations in relation to each other. Similarly, the second harmonic of the same three configurations can be used to estimate the relative temperature profile due to a single heater.

The injection of current at a point, leading to the electrical potential of Eq. (3), implies a purely radial current density $(|\mathbf{J}(\mathbf{r},\mathbf{r}_0,t)| = I(t)/(2\pi|\mathbf{r}-\mathbf{r}_0|^2))$, which causes the Joule heating $\rho|\mathbf{J}(\mathbf{r},\mathbf{r}_0,t)|^2 \propto |\mathbf{r}-\mathbf{r}_0|^{-4}$. The resulting Joule heating decays rapidly with distance from the current injection point. Thus, at a distance $|\mathbf{r}-\mathbf{r}_0| \gg$ contact radius, Joule heating will appear to originate from a point-like heat source.



Fig. 1. (a) Finite element method (FEM) simulation of M4PP measurement on bulk Si (top panel), shows the high current density in the vicinity of the contacts, and the resulting electric potential (color bar). The schematic below illustrates Eq. (4) (top to bottom), and shows the first harmonic voltage $(V_{1\omega})$, corresponding to a triplet of four-point measurement, in which the voltage electrode (p, q) is fixed, while the current pins are interchanged between u, v and w. (b) FEM simulation of M4PP measurement on bulk Si (top panel), shows the heat dissipation around the current contacts and its conduction into the substrate (color bar). The schematic below illustrates Eq. (9) and shows the measurements whereas the voltage electrodes (p, q) are fixed while the heat pins are interchanged between u, v and w. The different Q is the power dissipated in each contact due to Joule heating, which depends on contact resistance and current via $R_v I^2$.

The total power P(t) dissipated due to Joule heating is well described by

$$P(t) = R_c (I_0 \cos(\omega t))^2,$$
(5)

where R_c is a contact resistance, which depends on the actual contact radius. Eq. (5) can be expressed as the real part of the complex power $\tilde{P}_{n\omega}(t) = P_{0,n\omega}e^{inot}$, where the amplitude is $P_{0,n\omega} = \frac{1}{2}R_c I_0^2$ for $n \in \{0,2\}$ and otherwise $P_{0,n\omega} = 0$. The temperature increase resulting from Joule heating can now be calculated by solving the heat equation for heat diffusion from a point power source into bulk half-space [22,23],

$$\tilde{T}_{n\omega}(\mathbf{r},\mathbf{r}_{0},t) = \frac{\tilde{P_{n\omega}}(t)}{2\pi\kappa|\mathbf{r}-\mathbf{r}_{0}|}e^{-(1+i)\sqrt{\frac{n\omega}{2D}}|\mathbf{r}-\mathbf{r}_{0}|}.$$
(6)

Here, *D* is the thermal diffusivity and κ is the thermal conductivity of the sample. In M4PP measurement, the second harmonic temperature

difference between two points \mathbf{r}_p and \mathbf{r}_q due to a heater at \mathbf{r}_u can be calculated using two instances of Eq. (6):

$${}^{u}_{pq}\Delta\tilde{T}_{2\omega}(t) = \tilde{T}_{2\omega}(\mathbf{r}_{p}, \mathbf{r}_{u}, t) - \tilde{T}_{2\omega}(\mathbf{r}_{q}, \mathbf{r}_{u}, t).$$
(7)

The principle of superposition is valid for an approximated point heat source, and consequently the four-point temperature difference ${}_{pq}^{uv}\Delta \tilde{T}_{2\omega}(t)$ between points \mathbf{r}_p and \mathbf{r}_q with heat injection at points \mathbf{r}_u and \mathbf{r}_v is:

$${}^{uv}_{pq}\Delta\tilde{T}_{2\omega}(t) = {}^{u}_{pq}\Delta\tilde{T}_{2\omega}(t) + {}^{v}_{pq}\Delta\tilde{T}_{2\omega}(t).$$
(8)

Analogously to the derivation of Eq. (4) from a linear combination of three instances of Eq. (3), a linear combination of three instances of Eq. (8) leads

$${}^{uv}_{pq}\Delta\tilde{T}_{2\omega}(t) + {}^{vw}_{pq}\Delta\tilde{T}_{2\omega}(t) - {}^{uv}_{pq}\Delta\tilde{T}_{2\omega}(t) = 2 {}^{v}_{pq}\Delta\tilde{T}_{2\omega}(t),$$
(9)

i.e. the temperature difference from a deconvolved heat source as illustrated in Fig. 1b. The second harmonic temperature difference across the voltage electrodes can be measured via the Seebeck effect [19],

$${}^{uv}_{pq}V_{2\omega}(t) = -(\alpha_s - \alpha_e) \operatorname{Re}^{uv}_{pq} \Delta \tilde{T}_{2\omega}(t)$$
(10)

where α_s and α_e are the Seebeck coefficients of the sample and the electrodes, respectively. To extract the experimental thermoelectric response due to a single heater located at \mathbf{r}_v ,

$$\begin{aligned} {}^{v}_{p}V_{2\omega}(t) &= {}^{v}_{pq}\Delta V_{2\omega}(t) + {}^{v}_{q}V_{2\omega}^{+}(t) \\ &= {}^{uv}_{pq}V_{2\omega}(t) + {}^{vw}_{pq}V_{2\omega}(\omega,t) - {}^{uw}_{pq}V_{2\omega}(t) \\ &= {}^{uv}_{2\omega}V_{2\omega}(t) + {}^{vw}_{pq}V_{2\omega}(\omega,t) - {}^{uw}_{pq}V_{2\omega}(t) \\ &= {}^{v}_{2\omega}V_{2\omega}(t) + {}^{v}_{pq}V_{2\omega}(\omega,t) - {}^{uw}_{pq}V_{2\omega}(t) \\ &= {}^{uv}_{2\omega}V_{2\omega}(t) + {}^{v}_{pq}V_{2\omega}(\omega,t) - {}^{uw}_{pq}V_{2\omega}(t) \\ &= {}^{v}_{2\omega}V_{2\omega}(t) + {}^{v}_{pq}V_{2\omega}(t) + {}^{v}_{pq}V_{2\omega}(\omega,t) - {}^{uw}_{pq}V_{2\omega}(t) \\ &= {}^{v}_{2\omega}V_{2\omega}(t) + {}^{v}_{pq}V_{2\omega}(\omega,t) - {}^{uw}_{pq}V_{2\omega}(t) \\ &= {}^{v}_{2\omega}V_{2\omega}(t) + {}^{v}_{pq}V_{2\omega}(\omega,t) - {}^{v}_{pq}V_{2\omega}(t) \\ &= {}^{v}_{2\omega}V_{2\omega}(t) + {}^{v}_{pq}V_{2\omega}(\omega,t) - {}^{v}_{pq}V_{2\omega}(t) \\ &= {}^{v}_{2\omega}V_{2\omega}(t) + {}^{v}_{pq}V_{2\omega}(t) + {}^{v}_{pq}V_{2\omega}(\omega,t) - {}^{v}_{pq}V_{2\omega}(t) \\ &= {}^{v}_{2\omega}V_{2\omega}(t) + {}^{v}_{2\omega}V_{2\omega}(t) + {}^{v}_{2\omega}V_{2\omega}(t) \\ &= {}^{v}_{2\omega}V_{2\omega}(t) + {}^{v}_{2\omega}V_{2\omega}(t) + {}^{v}_{2\omega}V_{2\omega}(t) + {}^{v}_{2\omega}V_{2\omega}(t) \\ &= {}^{v}_{2\omega}V_{2\omega}(t) + {}^{v}_{2\omega}V_{2\omega}(t) + {}^{v}_{2\omega}V_{2\omega}(t) + {}^{v}_{2\omega}V_{2\omega}(t) \\ &= {}^{v}_{2\omega}V_{2\omega}(t) + {}^{v}_{2\omega}V_{2\omega}(t) + {}^{v}_{2\omega}V_{2\omega}(t) + {}^{v}_{2\omega}V_{2\omega}(t) \\ &= {}^{v}_{2\omega}V_{2\omega}(t) + {}^{v}_{2\omega$$

Here, ${}^v_p V_{2\omega}(t)$ is the thermoelectric potential increase due to a single heater and ${}^v_q V^{\dagger}_{2\omega}(t) = -(\alpha_s - \alpha_e) \operatorname{Re} \tilde{T}_{2\omega}(\mathbf{r}_q, \mathbf{r}_v, t)$ is an estimated correction potential assuming Fourier's law and macroscopic material properties.

In our studied material, the Seebeck coefficient has only a weak temperature dependence [24], while our experiments are performed at room temperature with a small gradient of temperature in a localized area, leading us to assume a constant Seebeck coefficient. This is further supported by numerical simulations in section 4.1 of Ref. [19]. Hence, the Thomson effect is minor and can be neglected. TCR and Peltier's effects mainly spread into 1st and 3rd harmonic of the measured signal, leaving the 2nd harmonic signal unaffected [25]. Therefore, in our analysis, we have neglected the consequence of these effects.

3. Methods and materials

3.1. Measurements

Measurements were performed on a CAPRES A300 microRSP metrology tool using an internal lock-in amplifier recording higher harmonic signals [15,19]. The first harmonic is dominated by the ohmic signal (cf. Eq. (3)), which corresponds to the voltage output of Eq. (3) and is used to determine the material's resistivity [25]. The second harmonic is dominated by the thermoelectric voltage arising from Joule heating induced by the measurement current [19] cf. Eq. (10).

Eq. (11) can also be expressed in terms of the contact resistance and material properties,

$${}_{p}^{\nu}V_{2\omega,rms} = -(\alpha_{s} - \alpha_{e})\frac{I_{rms}^{2}R_{c}}{2\pi\kappa d}.$$
(12)

Here, I_{rms} is the experimental root mean square current, R_c is the contact resistance of the single heater calculated using two-point resistance [18], and $d = |\mathbf{r}_p - \mathbf{r}_v|$. Eq. (12) will hold true for the low-frequency limit [15] (for our experiment, $D = 6.8 \times 10^{-5} \text{ m}^2 \text{s}^{-1}$, f = 3 Hz, max $(|\mathbf{r} - \mathbf{r}_0|) = 37 \mu \text{m}$, which results in a value of $\sqrt{\frac{n\omega}{2D}} |\mathbf{r} - \mathbf{r}_0| \approx 2.0 \times 10^{-4} \ll 1$).

For the measurements, we used an L8PP variable pitch probe, which has eight electrodes (cf. Fig. 2a). The electrodes of L8PP are made of poly-silicon coated with a 100 nm Au layer serving as the current



Fig. 2. (a) Scanning electron microscope image of the L8PP probe, which comprises a silicon body with L-shaped cantilever electrodes. It has eight electrodes, out of which any subset of four can be used for measurement (Courtesy: Lior Shiv). (b) Each triplet configuration corresponds to a set of three measurements with fixed voltage pin positions (p, q), and three unique pairs of current pins u, v and w (cf. Fig. 1b). (c) The output of each triplet in panel (b) corresponds to one measurement point (blue circle) cf. Eq. (12), and the extracted temperature from the measured thermoelectric voltage on a secondary (right) *Y*-axis. The solid-black line corresponds to the expected model temperature using the reference α_s , κ . The dashed line (red) illustrates the location of a single heater i.e. v is fixed to one electrode.

carrier. Measurements were performed at room temperature with a frequency of 3 Hz and I_{rms} of 1.8 mA. For the calibration of lead resistance, reference measurements were performed on a thick Ni slab both before, and after the measurement with the same measurement parameters as those on Si. Monitored lead resistance and two-point resistance were later used to calculate individual contact resistance (R_c), as detailed in Ref. [18]. A selection of triplet configurations (defining voltage and current pins) is demonstrated in Fig. 2b, where each horizontal line corresponds to a set of three measurements with fixed voltage pins (open-circles) and three unique pairs of current pins (triangles).

Table 1

List of doping concentration and thermoelectric properties [19] (at 300 K) of B-doped single crystal Si. The uncertainty on the accuracy of measured reference values of α and κ is estimated to be 5% [28,29].

| $N ({\rm cm}^{-3})$ | α_s ($\mu V K^{-1}$) | κ (W m^{-1} K^{-1}) | TCR (% K^{-1}) |
|----------------------|-------------------------------|----------------------------|-------------------|
| 5.54×10^{18} | 632 | 128.3 | 0.18 |



Fig. 3. Distance dependence (*d*) of the normalized thermoelectric voltage (circles, left-hand side of Eq. (12) normalized with power), measured at I_{rms} of 1.8 mA. The solid-black line corresponds to the model value calculated from Fourier's law using reference parameters. The relative temperature profile includes information from all the possible triplets (i.e. $v \in 1, 2, ..., 8$). Inset shows the spatial distribution of the extracted $(\alpha_s - \alpha_c)/\kappa$ each with its standard deviation; the horizontal line with shaded area marks to the reference $(\alpha_s - \alpha_c)/\kappa$ with ±5% uncertainty.

3.2. Sample

For demonstration of the proposed method in this article, we have chosen a well-characterized B-doped single crystal Si sample (lab code 4790 in Ref. [19]). Its macroscale properties (Table 1) include α_s as characterized using the slope method [26] and κ was characterized using the transient planar source (TPS) [27]. The value of the Seebeck coefficient ($\alpha_e = 2.5 \mu$ V K⁻¹) of electrodes was adopted from Refs. [30, 31].

4. Results and discussion

All results presented here were obtained from a single engage, in which 234 four-point probe configurations were measured and the results are representative of repeated measurements. Using the first harmonic identity (Eq. (4)), we evaluate the ratio of the LHS to the RHS of Eq. (4) for all triplet configurations, and as expected the mean ratio is unity with a standard deviation of < 0.1% and a maximum deviation of < 0.5%. This result is in line with previously reported M4PP repeatability and reproducibility with a relative standard deviation of < 0.1% for sheet resistance measurements on Si [13]. Thus, even with a large measurement current and deliberate contact heating, the measurement precision of the first harmonic appears unaffected. In order to demonstrate the proposed single heater scheme (cf. Fig. 1b), we evaluate the subset of triplet configurations for a fixed heater position as illustrated in Fig. 2c. As described in the theory, for each triplet configuration, a thermoelectric potential is calculated via Eq. (11). The extracted thermoelectric potentials are normalized with the power dissipated in the contact $(I_{rms}^2 R_c)$, and reported in Fig. 2c (left axis), along with temperature extracted from the measured thermoelectric voltage assuming a mean contact resistance and using macroscopic parameters (right axis). The solid-black line corresponds to the predicted

normalized potential by Fourier's law using parameters from Table 1. The contact resistance R_c is calculated individually for each point contact and ranges from 500–1100 Ω . As expected from diffusive heat transport, the measured values are higher close to the heater position (red line) and fall inversely with distance. Qualitatively, these results agree with the expected model for a single-point heater very accurately, with no systematic offset. The relative difference between experimental potentials and the model is larger at longer distances from the heater, which is due to the signal amplitude approaching zero.

From selective subsampling of triplet configurations, it is possible to assign the heater spot to any of the electrode positions. Analogous to Fig. 2c, the resulting relative potential due to the combination of all possible heater positions, is plotted against the distance *d* in Fig. 3. The result is equivalent to a detailed relative temperature profile due to a single heater, assuming a constant relative Seebeck coefficient in the contacts. Similar to Fig. 2c, in Fig. 3, the solid black line corresponds to Fourier's law using macroscopic reference parameters (cf. Table 1). The model line passes through the data and shows excellent agreement ($r^2 = 0.98$) between the analytical and the experimental result obtained with M4PP.

From Eq. (12), we can estimate the ratio $(\alpha_s - \alpha_e)/\kappa$, which is calculated using same data as in the main Fig. 3, and plotted as inset in Fig. 3. The horizontal line and shaded area correspond to the reference value of $(\alpha_s - \alpha_e)/\kappa$ with an error of ±5%. Using an electrode pitch down to 1 µm, enables us to estimate the ratio $(\alpha_s - \alpha_e)/\kappa$ on the same material as in our previous study [19] but with an order of magnitude smaller spatial scale.

In the present study the electrode material is Au and thus the probe $\alpha_e = 2.5 \ \mu V \ K^{-1}$ is an order of magnitude smaller than that of Ni electrodes used in our previous study [19]. The error bar for the estimates of $(\alpha_s - \alpha_e)/\kappa$ corresponds to the standard deviation from multiple points of identical distance. To first order, these estimates, made at a single current of 1.8 mA agree reasonably well with its independent macro scale reference value. However, measurements at a single high current can show offsets of up to ~ 10%, due to a combination of cold finger and TCR effects [18,19]. Secondary effects, such as a progressive deviation from Fourier's law may be expected when the characteristic length scales are comparable to the phonon mean free path [32–34]. However, a rigorous evaluation of these effects goes beyond the scope of this study and will be reported elsewhere.

The 1 ω triplet identity (cf. Eq. (4)) and 2 ω single heater reduction scheme are independent of probe geometry. However, the determination of the relative temperature profile requires accurate knowledge of the actual electrode positions. This is especially relevant for small electrode distances e.g. $\leq 5 \,\mu$ m. Geometrical corrections are insignificant for large pitch (e.g. $\geq 10 \,\mu$ m) measurements [19].

The significance of the model-based correction (term ${}^{v}_{q}V^{\dagger}_{2\omega}(t)$ of Eq. (11)) is relaxed when the distance to the far voltage pin increases. Designing probes with larger pitch variations will allow for measurements without prior knowledge of the sample's thermal properties. Although, on our Si sample we find the temperature profile to fit the expected result from a diffusive heat transport. Our method may open up the possibility of studying non-diffusive heat transport. This could be possible by reducing the electrode pitch or by performing measurements at a cryogenic temperature where non-diffusive transport is maybe more pronounced.

5. Conclusion

In a recent study, we have shown the potential of a micro fourpoint probe (M4PP) to measure thermoelectrical material properties. The oscillating measurement current is used to generate a temperature gradient in the proximity of the current electrodes, and the thermal response is probed via the Seebeck effect (at the second harmonic).

With five electrodes in contact with a sample, and a triplet of four-point configurations, we define an identity for both the first and second harmonic voltages. The 1 ω identity is verified to be reproducible within a relative standard deviation of < 0.1%, whereas the 2 ω identity enables to effectively isolate the thermoelectric signal arising from a single heat source. Furthermore, our single heater reduction scheme is used to evaluate the relative temperature profile due to a heater on a bulk single crystal of a highly B-doped Si sample. Within the uncertainty of the measurements, it is not possible to verify the extent of potential deviations from Fourier's law. Our study solidifies the accuracy of the temperature model used in M4PP and demonstrates the α_s/κ ratio extraction for electrode separation down to 1 µm, an order of magnitude smaller than previously reported.

The concept to isolate a single heater is general and is independent of geometry. It may apply to any type of heat source as long as the temperature rise can be described by a superposition of heat sources. Most importantly, the method may greatly reduce the complexity of evaluating thermoelectric properties e.g. thermal diffusivity and the ratio of Seebeck coefficient to thermal conductivity.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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