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*Published in:*

Extended Abstract Book of the 9th JSTP International Seminar on Precision Forging

*Publication date:*

2024

*Document Version*

Publisher's PDF, also known as Version of record

[Link back to DTU Orbit](#)

*Citation (APA):*

Zwicker, M., Bay, N., & Nielsen, C. V. (2024). An Analytical Solution to the Strain Space Limitation in Asperity Flattening Experiments Combining Normal Loading with Biaxial In-Plane Strains. In *Extended Abstract Book of the 9th JSTP International Seminar on Precision Forging* (pp. 55-56). The Japan Society for Technology of Plasticity .

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# An Analytical Solution to the Strain Space Limitation in Asperity Flattening Experiments Combining Normal Loading with Biaxial In-Plane Strains

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## Abstract

Friction in metal forming is largely determined by the evolution of the workpiece's surface topography, i.e. by flattening of surface asperities due to normal loading combined with, among others, subsurface strains, sliding and friction. Zwicker et al. [1,2] conducted experiments with model asperities under combined normal loading and a variation of bi-axial in-plane strain conditions on a newly developed test set-up. Both studies could achieve asperity flattening under balanced biaxial stretching and plane strain, but not uni-axial tension. Up to this point the explanation for the limitation of the strain space was merely qualitative, namely the competition of narrowing due to elongation, widening due to normal loading, and a resistance to width change caused by friction between asperities and tool. In this work, an analytical solution based on basic plasticity is presented to further explain the strain space limitation.

**Keywords** : Sheet forming, asperity flattening, tribology, friction

## 1. Introduction

Friction is present in all industrial forming applications, and its understanding is crucial to process stability and optimization. In sheet metal forming, friction is largely determined by asperity contact and subsequent asperity flattening upon contact between sheet and tool. Surface asperities flatten due a combination of normal loading with for example sliding or subsurface strains. The understanding of the evolution of the local contact regions between sheet and die are crucial to estimating friction. Asperity flattening can be investigated on real surfaces or on emulated surfaces, which are exposed to normal loading with or without additional influencing factors like sliding or subsurface strains.

To increase the understanding of asperity flattening, experiments with bi-axial strain conditions are desirable, as those are present in almost all sheet forming operations. Zwicker et. al [1] presented a new test principle, inspired by the Marciniak-Kuczynski test principle, which allows the combination of normal loading and bi-axial strain conditions. Experimental results by Zwicker et. al [1,2] indicate that it is difficult to achieve bi-axial strain states, where a lateral contraction of the work piece, such as the strain state in uni-axial tension, can be achieved, as presented in Fig. 1(a), where major as function of minor in-plane strains are presented. This effect is rooted in the competition between three effects, namely narrowing due to elongation, widening due to normal loading, and a resistance to width change caused by friction between asperities and tool. Zwicker et. al [2] varied the specimen geometry to enable lateral contraction, as presented in Fig. 1(b), which did not result in a strain condition beyond plane strain. To further validate this phenomenon, the authors present an analytical solution based on basic plasticity, which derives the principal in-plane strain ratio for in-plane, uni-axial tension with normal loading and allows for comparing it to a pure uni-axial case.

## 2. Analytical solution

In order to understand whether the absence of the uni-axial case in the presented experiments is induced by factors like stiffness of the test geometry or the fact that model asperities were tested, it was decided to derive an analytical solution for the tested case. In a forming-limit diagram, where the discussed cases can be considered, uni-axial tension is characterized by the principal in-plane strain ratio  $s$  of  $-1/2$  under the assumption of isotropy. Here, the principal strain ratio is based on Levy-Mises' flow rule as shown in Eq. (1), where  $d\varepsilon_y$  and  $d\varepsilon_x$  are the in-plane strain increments and

$\sigma'_x$  and  $\sigma'_y$  are the in-plane deviatoric stresses. To understand how a normal pressure influences the principal in-plane strain ratio during uni-axial tension combined with normal loading, the in-plane deviatoric stresses need to be constructed for the specific case, as shown in Fig. 1(c).

$$s = \frac{d\varepsilon_y}{d\varepsilon_x} = \frac{\sigma'_y}{\sigma'_x} \quad (1)$$

The principle in-plane stresses,  $\sigma_x$ ,  $\sigma_y$  and  $\sigma_z$ , corresponding to the case in Fig. 1(c) are

$$\sigma_x; \sigma_y = 0; \sigma_z = -q, \quad (2)$$

and let the normal pressure by a fraction,  $\eta$ , of the flow stress  $\sigma_f$

$$q = \eta\sigma_f. \quad (3)$$

By assuming the material to be isotropic, the principal in-plane stress  $\sigma_x$  can be derived from the von Mises yield criterion,

$$\sigma_f = \sqrt{\frac{1}{2}[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2]} \quad (4)$$

resulting in

$$\sigma_x = \frac{1}{2}(-\eta + \sqrt{4 - 3\eta^2})\sigma_f. \quad (5)$$

To construct the in-plane deviatoric stresses, the hydrostatic stress,

$$\sigma_m = \frac{\sigma_x + \sigma_y + \sigma_z}{3} = \left(-\frac{\eta}{2} + \frac{1}{6}\sqrt{4 - 3\eta^2}\right)\sigma_f \quad (6)$$

needs to be subtracted from the total in-plane stresses,

$$\sigma'_x = \sigma_x - \sigma_m = \frac{1}{3}\sqrt{4 - 3\eta^2}\sigma_f \quad (7)$$

$$\sigma'_y = \sigma_y - \sigma_m = -\sigma_m \quad (8)$$

Based on the in-plane deviatoric stresses, the principal in-plane strain ratios for pure uni-axial tension can be compared to in-plane uni-axial tension combined with normal loading. Zwicker et al. [1,2] conducted their experiments with pure aluminum AA1050-H14 with a flow of stress of approximately 100 MPa in the relevant strain interval and with a normal pressure of 37 MPa in all experiments, resulting in  $\eta = 0.37$  and in a principal in-plane strain ratio  $s = -0.2$ . As can be seen in Fig. 1(a), the principal in-plane strain ratio is increased as pressure is added, indicated by the arrow labelled with 1, and it can be concluded that the analytical solution confirms the observed trend. The fact that the solutions does not result in plane strain is due to neglecting friction and stiffness of the specimen design, which can be assumed to move the strain path to plane strain, indicated by the arrow labelled with 2.

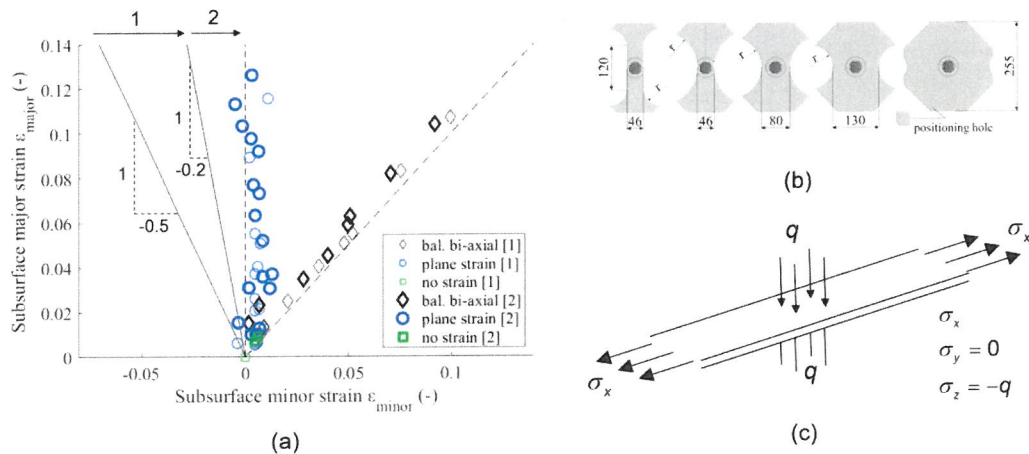


Figure 1: Presentation of experimental results by Zwicker et al. [1,2], the test specimens used by Zwicker et al. [2], and a sketch of the stresses and pressures in the in-plane uni-axial tension case combined with normal loading.

## References

- [1] Zwicker, MFR., Bay, N., Nielsen, CV., 2023, A new test for sheet metal asperity flattening under varying subsurface strain conditions. Tribology International. 180, 108249.
- [2] Zwicker, MFR., Arinbjarnar, U. Knoll, M., Bay, N., Nielsen, CV., 2023, Check for updates Flattening of Pyramidal Asperities Under Combined Normal Loading and In-Plane Biaxial Straining. Current Trends in the Technology of Plasticity (Proceedings of the 14th International Conference on the Technology of Plasticity (ICTP 2023)). 12-19.