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Loss models for long Josephson junctions

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A general model for loss mechanisms in long Josephson junctions is presented. An expression for the zero-field step is found for a junction of overlap type by means of a perturbation method. Comparison between analytic solution and perturbation result shows good agreement.

Fluxon dynamics in long Josephson junctions have mainly been investigated theoretically by means of analytical methods,[1,2] perturbation analysis,[3–5] and numerical simulations;[6–8] only recently experiments[9,10] have been performed which, however, agree almost perfectly with the theory. In Ref. 10 results from the perturbation theory for fluxon motion in the modified sine-Gordon equation are compared with experiments on long Josephson junctions of small damping constants. This comparison leads to a determination of the damping constant at various temperatures. Especially for low temperatures the experiments show that the model possibly should include the effect of surface impedance. The authors, however, note that other corrections could replace or supplement this term such as nonlinearities in the shunt conductance or inclusion of a $\cos\phi$ damping. The purpose of the present Brief Report is to present a model which takes into account nonlinearities in the shunt conductance, the $\cos\phi$ conduction, and the surface impedance. For this model we perform a perturbation analysis to obtain an expression for the stationary velocity of the fluxon and compare this result with an exact solution which can be obtained in a special case.

This equation describing fluxon (2π-kink) in long Josephson junctions is assumed to be the perturbed sine-Gordon equation

$$\phi_{xx}-\phi_{tt}=\sin\phi+\eta+\sum_i \alpha_i\phi_t|\phi_t|^{i-1}(1+\epsilon\cos\phi)-\beta\phi_{xxt} \tag{1}$$

with appropriate boundary conditions. We refer to Ref. 7 for a detailed description of the model. Here we only discuss the damping mechanisms. The third term on the right side represents the current through the shunt conductance (due to quasiparticle tunneling current) being a nonlinear function of voltage as proposed in Ref. 12 for the small junction. For $i=1$, $\alpha_1=(1/\beta_c)^{1/2}$, where β_c is the McCumber parameter. The parameter represents the conductance due to quasiparticle interference current—physical reality requires $|\epsilon|\leq 1$. The last term on the right side represents surface impedance damping while η represents the uniformly distributed bias current. We note that this model represents the pure overlap junction where the boundary conditions are $\phi_x(0,t)=\phi_x(l,t)=0$ because no self-fields are present at the junction ends. For the in-line junction the antisymmetrical bias current distribution results in $\eta=0$ and self-fields give rise to boundary conditions $\phi_x(0,t)=-\phi_x(l,t)=H$ which provide energy input.

In the following we use the definition of the momentum to derive the approximate stationary velocity of the fluxon. This velocity corresponds to the voltage of the first zero-field step. The normalized momentum is defined by

$$P=-\tfrac{1}{8}\int\phi_x\phi_t\,dx \ . \tag{2}$$

Differentiation of P with respect to time and use of (1) yields

$$\frac{dP}{dt}=-\tfrac{1}{8}\left(\tfrac{1}{2}\phi_t^2+\tfrac{1}{2}\phi_x^2+\cos\phi-\eta\phi\right)_{x=-\infty}^{x=\infty}-\tfrac{1}{8}\int\phi_x\left[\sum_i\alpha_i\phi_t|\phi_t|^{i-1}(1+\epsilon\cos\phi)-\beta\phi_{xxt}\right]dx \ . \tag{3}$$

Now, assuming ϕ is given by[11] $\phi=-\sin^{-1}\eta+\phi^k(x,t)$, where ϕ^k is the 2π-kink solution to the pure sine-Gordon equation, we get

$$P=P_k=u_\infty\gamma(u_\infty)$$

and

$$\phi_t=\phi_t^k=2P\sin(\tfrac{1}{2}\phi^k) \ , \tag{4}$$

where $\gamma(u_\infty)$ is the Lorentz factor and u_∞ is the stationary

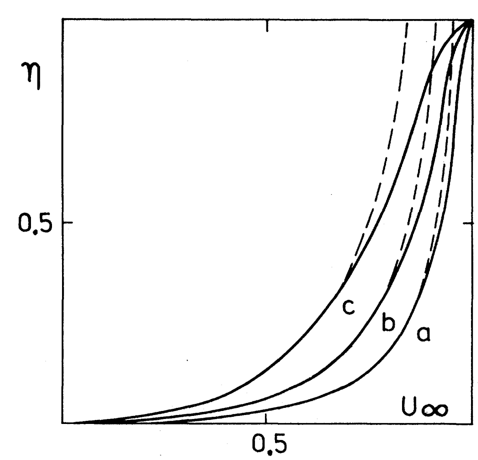

FIG. 1. Comparison between expression obtained by perturbation method and exact solution to Eq. (1). Parameter values $i=2$, $\alpha_1=0$, $\epsilon=0$, and $\beta=0$, and $\alpha_2=0.05$ (curve a), 0.1 (curve b), and 0.2 (curve c). Full curves represent the exact solution, dashed curves represent the result from the perturbation method. Good agreement is observed for $\eta\lesssim 0.4$.

velocity. Insertion of (4) into (3) gives

$$\frac{dP_k}{dt}=\frac{\pi}{4}\eta-P_k\left[\alpha_1\left(1-\frac{\epsilon}{3}(1-\eta^2)^{1/2}\right)-\frac{\beta}{3}\right]-P_k^2\alpha_2\frac{\pi}{2}\left(1-\frac{\epsilon}{2}(1-\eta^2)^{1/2}\right)$$
$$-P_k^3\left[\alpha_3\frac{8}{3}\left(1-\frac{3}{5}\epsilon(1-\eta^2)^{1/2}\right)-\frac{\beta}{3}\right]-\frac{1}{2}\sum\alpha_i(2P_k)^i\left[C_i\left(1+\epsilon(1-\eta^2)^{1/2}\right)-2\epsilon(1-\eta^2)^{1/2}C_{i+2}\right]\ , \tag{5a}$$

where

$$C_i=\begin{cases}\dfrac{\pi}{2}\dfrac{i!}{2^i[(\frac{1}{2}i)!]^2}, & i \text{ even}\ ,\\[2ex] \dfrac{2^{i-1}\{[\frac{1}{2}(i-1)]!\}^2}{i!}, & i \text{ odd}\ .\end{cases} \tag{5b}$$

For the stationary fluxon we have $dP_k/dt=0$. Thus Eq. (5) is an expression for the IV characteristics of the first zero-field step. From (5) it is seen that a term $\alpha_3\phi_t^3$ influences the zero-field step in the same way as a $-\beta\phi_{xxt}$ term. Further, we remark that the $\cos\phi$ term introduces the term $(1-\eta^2)^{1/2}$ in (5).

In order to check the perturbation method we compare the result with an exact solution. For $i=2$, $\alpha_1=0$, $\epsilon=0$, and $\beta=0$ the 2π-kink solution to Eq. (1) is given in Ref. 2. With use of this solution the IV characteristic can be written

$$\eta=\frac{2\alpha_2P_k^2}{(1+4\alpha_2^2P_k^4)^{1/2}}\ , \tag{6}$$

while Eq. (5) gives

$$\eta=2\alpha_2P_k^2\ . \tag{7}$$

In Fig. 1 we compare the two expressions. Good agreement is observed for $\eta\lesssim 0.4$.

Finally, we remark that the influence of plasma waves on the 2π-kink motion is beyond the scope of this Brief Report but has been examined to some extent for $i=1$ in Ref. 11.

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